Testing Allocative Efficiency of Zero-Intelligence Traders under Extreme Demand and Supply Conditions

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Francesca De Maggis

1 Summary

The purpose of this research paper is to investigate the study conducted by Gode and Sunder (1993) and deeply criticized by Cliff et al. (1997) on the dynamics governing a double-auction market. If a simple agent-based model is presented by the former, a revisited version is implemented by the latter. Cliff et al., through their agent-based model, display the successful achievement of the theoretical equilibrium price also in extreme market cases (with flat demand and/or flat supply curves). With our analysis we implement a dynamic agent-based model showing that the theoretical market equilibrium can not be achieved when both supply and demand curves are flat.

2 Introduction

Inspired by the work of Gode and Sunder's (1993) and Cliff (1997), we developed our own analysis of the dynamics characterising a double-auction market, where bidders' and buyers' behaviours are modeled with an agent-based approach.
Moreover, we studied how quickly the market reaches its equilibrium (when this happens) if either the sole demand or the sole supply or both of them lose their flexibility, thus becoming flat curves.

We build our simulated double-auction market by populating it with agents acting as the zero-intelligence-constrained (ZI-C) traders described in the study by Gode and Sunder's (1993), and with a proper algorithm we establish when demand and supply meet. This simulates the order book dynamics and is a reinterpretation of the algorithm provided by David Ritzwoller, Blake LeBaron (2016).

The goal of this paper will be therefore to explore the undoubtfully valid criticisms of Cliff et al. to Gode and Sunder by studying price convergences in a ZIT model faithful to the original one.

3 Background

The convergence to the equilibrium price in markets populated by human traders has been studied and proved by Smith (1962), whose research represented the starting point of all the subsequent relevant economic literature.

In particular, Gode and Sunder (1993) build on this by showing how agents trading with the sole concern of not entering deals resulting in a final loss (called *zero-intelligence constrained agents, ZI-C*) effectively imitate human behavior. This leads to the conclusion that market equilibrium is reached whenever, and solely if, there is an efficient resource allocation imputable to the institutional design of the markets, thus highlighting the uselessness of considering human-specific features, such as past experience, personal knowledge, smartness or determination, when modeling the dynamics of agentsâĂŹ trading choices.

Four years after Gode and Sunder's findings, Cliff et al. illustrate a series of logical fallacies built in their 1993 work, showing that in most of the cases the expected value of prices at which transactions are executed by ZI-C agents are different from the true equilibrium price. After building the probability distribution function of transaction prices, they highlight the built-in convergence to equilibrium imputable to the peculiar form of the price CDF itself, whose peak occurs exactly at the equilibrium price of maximum allocative efficiency. Figure 1 shows the qualitative process used by Cliff et al. to determine the shape of the transaction price CDF, obtained by the intersection of the PDFs of bid and ask prices of demand and offer respectively. The conclusion is that Gode and Sunder's model leads to highly efficient allocation not of the institutional design of the market, but because the CDF of transaction prices happens to be centered on the allocative equilibrium price itself.

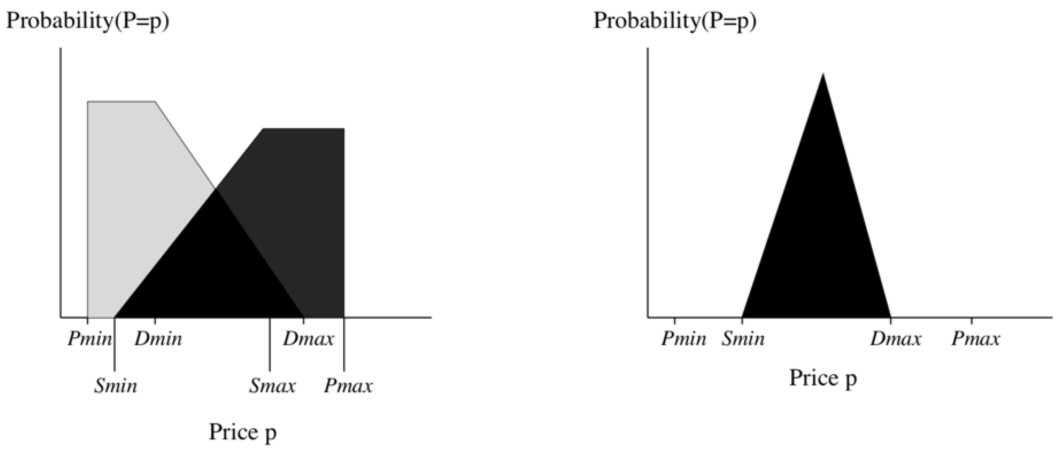


Figure 1: *On the left, the PDFs of the bid prices (Grey) and the offer prices (Black). On the right, the PDF of transaction prices obtained from the intersection of the two PDFs.*

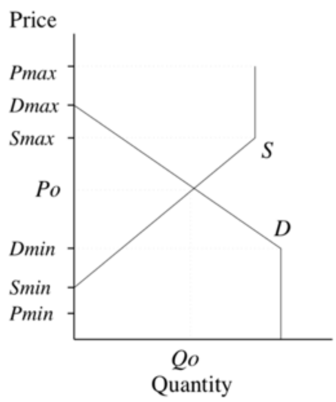


Figure 2: *Supply and Demand curves model for the ZIT model. Credit: Cliff et al.*

Cliff et al. support their thesis by analytically deriving a price to which the ZIT model converges in situations of flat supply and demand curves, through integration of the PDF of transaction prices. These prices are shown in Figure 2, with *Dmax* being the maximum value a buyer is willing to pay for the asset among all buyers, *P0* the optimal price, and *Smin* the lowest offer a seller is willing to accept among all sellers.

$$\textit{Standard case}: E(P) = P_0$$

$$\textit{Flat supply case}: E(P) = P_0 + \tfrac{1}{3} \times (D_{max} - P_0)$$

$$\textit{Flat supply and demand case, with excess supply}: E(P) = \tfrac{1}{2} \times (P_0 - D_{max})$$

$$\textit{Flat supply and demand case, with excess supply}: E(P) = \tfrac{1}{2} \times (P_0 + S_{min})$$

The authors then validate using a modified version of the model from Gode and Sunder: in this setting, Cliff et al. structure the simulation in trading days, in which each day sees 11 transactions happening. The results obtained this way are coherent with the price convergences computed analytically.

Nonetheless, despite the brilliant analysis of Cliff et al. both in qualitative and quantitative terms, we have a series of criticisms to raise. First, as mentioned by the authors themselves, the CDFs used to compute the convergence values change right after the first trade, which means that their analytical price limit is derived solely on the supply and demand curves of t = 0. Second, we believe that the model used diverges excessively from the concepts illustrated in Gode and Sunder and, therefore, can not be considered as a valid instruments to make generalizations.

4 Model

To study market allocation efficiency, we developed and coded our own agent-based model based on the logic used by Gode and Sunder in their 1993 paper and on the interpretation that Ritzwoller and LeBaron (2016) gave to it.

Our modeled market is populated by two types of agents, and the number of agents labeled under each class (*buyers* and *sellers*) is an input of our resulting model.
Buyers are interested in buying a given asset, while Sellers are interested in selling it. For simplicity, we assume that all agents buy/sell only one unit of the asset, and then exit the market once they have taken part in a transaction.

Sellers are characterized by two values: a unique ID number to identify them and a *Cost* value, that determines the minimum price they are willing to sell the asset at. The *Cost* value is determined individually for each seller by sampling from a uniform distribution U(1, *maxCost*), where *maxCost* is an exogenous parameter defined by the user. Buyers, just like sellers, have two values: a unique ID number to identify them and a *Price* value, that determines the maximum price they are willing to buy the asset at. The *Price* value is determined individually for each buyer by sampling from a uniform U(1, *maxPrice*), where *maxPrice* is an exogenous parameter defined by the user.

The model works through what Gode and Sunder named 'Double Auction'. At t = 0, an order book is initialized in the form of a dictionary. This order book stores two values: *LowestAsk* and *HighestOffer*.

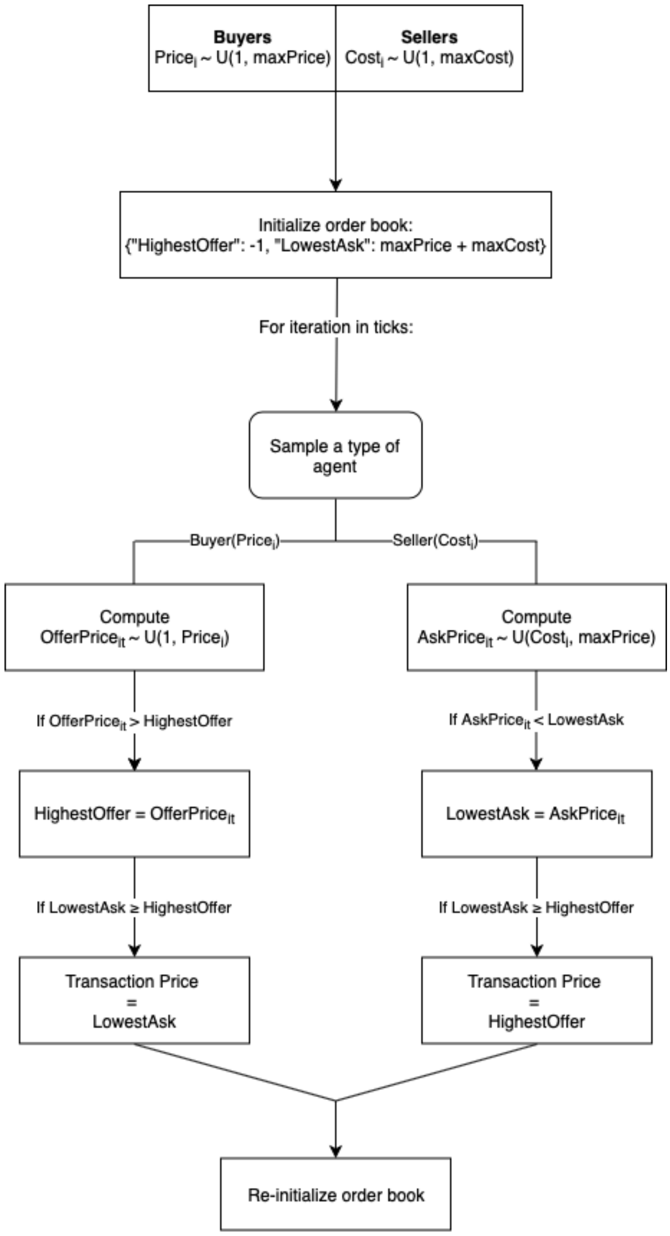


Figure 3: *Flowchart of the model proposed by Gode and Sunder*

At each time step t, the model randomly picks a class label (*buyer* or *seller*) and, accordingly to the class chosen, it randomly picks one agent belonging to the selected class.

- If the selected agent is a buyer: the selected agent i bids for the asset in the open auction, with an $OfferPrice_{i,t}$ that is defined by sampling on a uniform $U(1,Price_i)$. If the bid is higher than the order book's *HighestOffer*, the order book's *HighestOffer* becomes equal to $OfferPrice_{i,t}$.

- If the selected agent is a seller: the selected agent i tries to sell the asset in the open auction, with an $AskPrice_{i,t}$ that is defined on a uniform $U(Cost_i, maxPrice)$. If the requested price is lower than the order book's *LowestAsk*, the order book's *LowestAsk* becomes equal to $AskPrice_{i,t}$.

The process iterates until *LowestAsk* $\geq$ *HighestOffer*. At that point a transaction occurs: if the last agent to update the order book was a buyer, the transaction price recorded is *LowestAsk*. If the last agent to update the order book was a seller, the transaction price recorded is *HighestOffer*. The idea is that if a buyer arrives on the market and sees the asset on sale at a price that he deems convenient, he will buy it at the on-sale price. Else, if a seller arrives on the market and sees a bid for the asset that is greater than the price he is willing to put it on sale at, he will accept the bid and sell it for that amount.
After each transaction the order book is reset, and the transacting agents are removed from the model. Then the cycle continues for an arbitrary number of iterations.

5 Results

5.1 Base case

In order to represent the demand and the supply curves with respect to the base case (fully flexible demand and supply), we set *maxPrice*, *maxCost* and number of iterations equal to \$50, \$25 and 10'000, respectively.

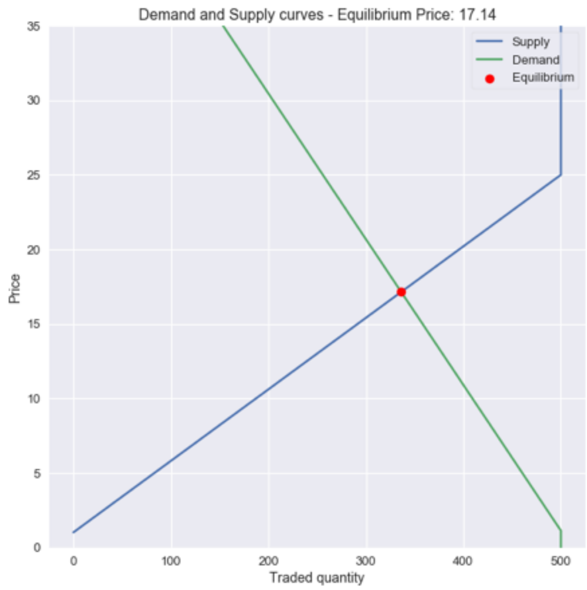


Figure 4: *Demand and Supply curves with maxPrice = 50, maxCost = 25, time = 10'000*

As it emerges from Figure 4, the demand and the supply curves intersect each other at the theoretical equilibrium price of $17.14. Moreover, it is worth highlighting that in correspondence of the 500th traded unit both curves become perfectly rigid, simply because buyers/sellers cannot buy/sell more units of the asset that the number of buyer/sellers. Below we can see the results of the simulation ran for 10'000 iterations.

From Figure 5 and Figure 6 we can see that prices converge to the theoretical equilibrium price, but still slightly deviate from it, in line with what illustrated by Gode and Sunder. We can double check this by computing the average transaction prices per day over an arbitrary number of iterations. As expected, we see high levels of allocative efficiency but not perfect ones, for all the *maxPrice* < *maxCost* and the *maxCost* > *maxPrice* scenarios.

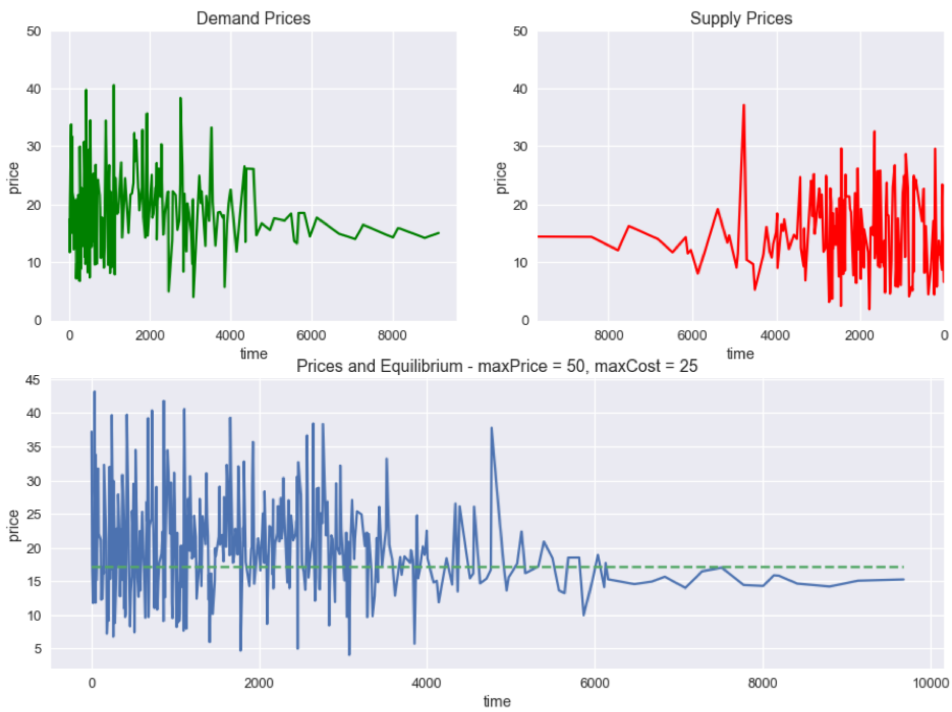


Figure 5: *Base case - Bid (green line) and Offer (red line) prices of executed transactions over time. Transactions prices aggregated over time (blue line).*

Figure 6: *Average transactions prices over time for different parameter combinations.*

5.2 Flat Supply case

Here we start tackling the scenarios proposed by Cliff et al. The first scenario that they study to criticise Gode and Sunder is one in which the supply curve is flat, with S_{min} = S_{max} = P_0.

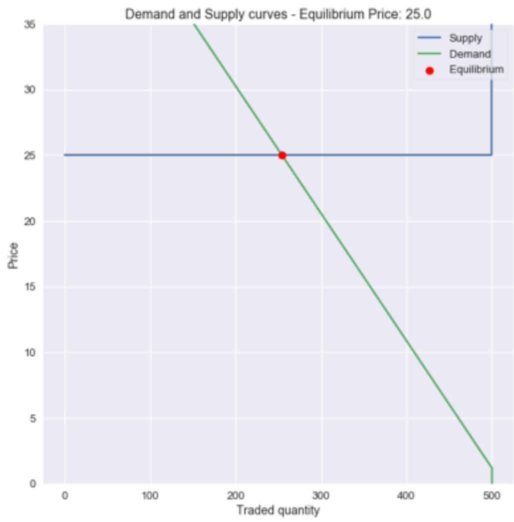
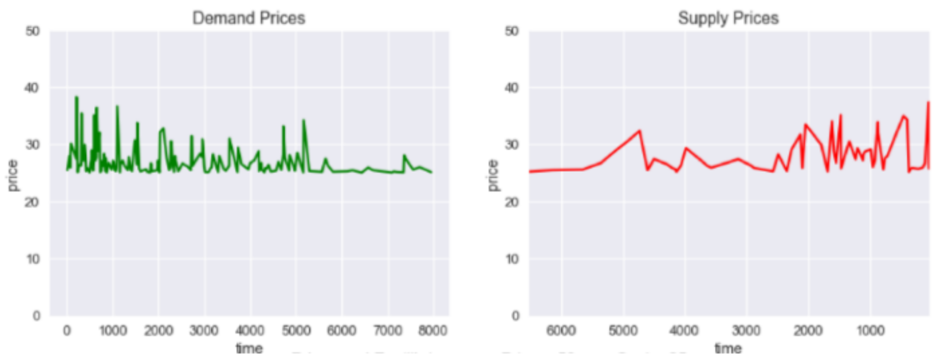


Figure 7: *Demand and Supply curves with maxPrice = 50, maxCost = 25, time = 8'000*



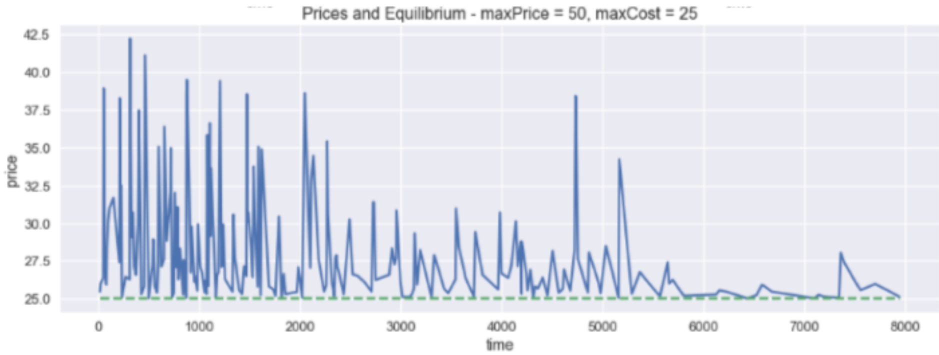


Figure 8: *Flat Supply case - Bid (green line) and Offer (red line) prices of executed transactions over time. Transactions prices aggregated over time (blue line).*

Eyeballing the results of the single simulation, one thing we can note is the reduced density of transactions. Of course, given the now rigid preferences of Sellers, buyers will have a harder time in coming up with offers that can satisfy them. Apparently, it seems the price converges even more to the equilibrium price, whereas Cliff et al. forecasted a convergence to: $E(P) = P_0 + \frac{1}{3} \times (D_{max} - P_0)$.

The aggregate results generalize our findings in the single-run model as shown in Figure 9.

Figure 9: *Average transactions prices over time - Flat Supply Case*

Here we can see that transaction prices converge to the theoretical equilibrium price and not to the convergence forecasted by Cliff et al. The only time transaction prices come close to the price value proposed by Cliff et al.'s equilibrium is during the very first transactions.

The implications of these findings cast strong doubts on the analytical computation of the convergence price: what found by Cliff et al. is a time-static price equilibrium that does not consider the subsequent transformations in the supply and demand curves that occur as agents enter trades and exit the model. Moreover, the validation process based on their model seems to be built ad-hoc to validate their findings, artificially limiting the number of transactions to 11 per day in order to keep the timeframe restricted.

5.3 Flat Demand and Supply case

The same findings can be generalized for the flat demand and supply case with excess supply, setting *Cost* = 30 and *Price* = P_0 = 25, with n_{buyers} = 300 and $n_{sellers}$ = 500.

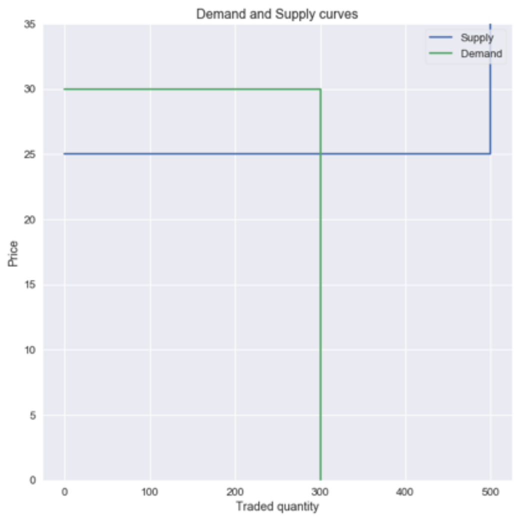


Figure 10: *Demand and Supply curves with Price = 25, Cost = 25, time = 8'000*

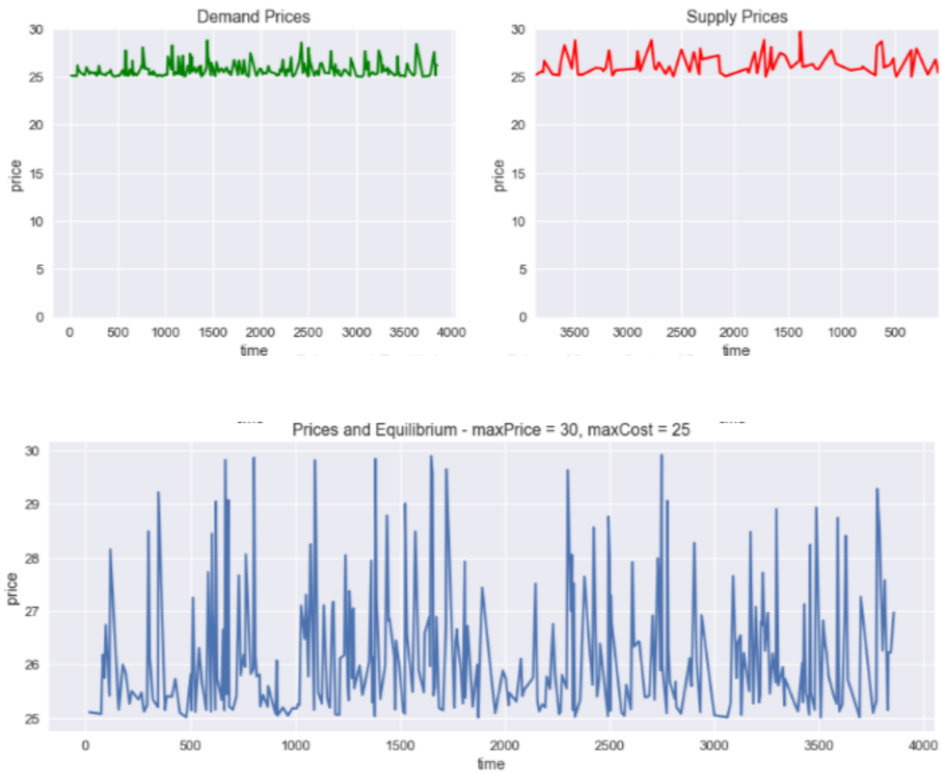


Figure 11: *Flat Demand and Supply case - Bid (green line) and Offer (red line) prices of executed transactions over time. Transactions prices aggregated over time (blue line).*

Here, we see transaction prices never converging to the theoretical equilibrium, but rather randomly fluctuating in the interval [25, 30], since agents randomly sample bids and ask prices on the market, occasionally managing to engage in transactions. The same findings apply to the aggregated case, where the average price stabilizes between the two limits theorized by both studies. In this case, the conclusions of both Gode and Sunder and Cliff et al. show no application in the standard setting.

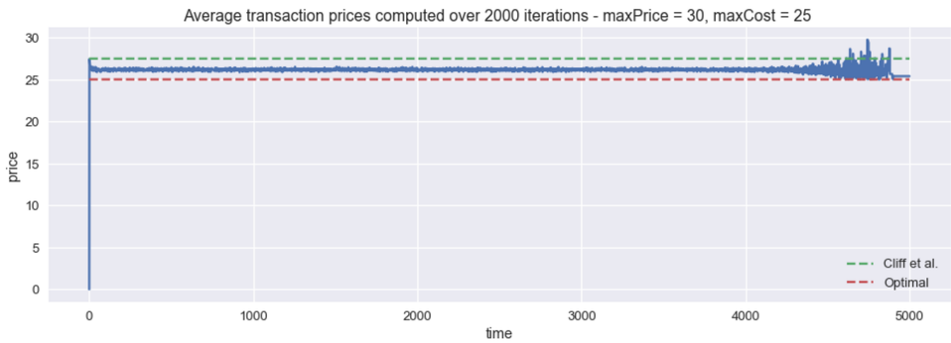


Figure 12: *Average transactions prices over time - Flat Demand and Supply Case*

One thing worth noting is that the number of transactions in Figure 12 gets really rare around the 4000 tick, because all sellers manage to sell their assets by the 4000th tick.

6 Conclusions

By deeply analyzing the criticism moved against Gode and Sunder by Cliff et al., we realized that the convergence to the theoretical market price can not be achieved by bidders and buyers operating in a double-auction market modeled with an agent-based approach when extreme market cases show up (case where both demand curve and supply curve are flat).

We also identified the reason of Cliff et al. questionable conclusions in their time-static mathematical analysis validated on a time-constrained model which allows only eleven transactions per day. In fact, we found the model proposed by Gode and Sunder to be a good and well-interpretable representation of double-auction markets, thus insisting on its implementation to both counter-argue Cliff et al. conclusions and show that the ever-renowned capability of zero-intelligence traders in reaching market equilibrium thanks to their allocative efficiency does not apply in extreme market cases.

7 References

- Gode, Sunder (1993), *Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality*
- David Ritzwoller, Blake Lebaron (2016), *Trading with Zero Intelligence Agents Model without Marshallian Path*
- Vernon L. Smith (1962), *An Experimental Study of Competitive Market Behavior*