

Assignment #7

Physics 2 Spring 2020
Instructor: Prof. Dirk Bouwmeester
Due: 05/17 5pm PST

Each problem is worth three points. Award yourself three points if you get the formula and number correct, two points if you made a numerical error, or one point if you got the wrong final equation but got the basic idea correct.

1 Force of Gas on Box

Four moles of an ideal gas are in a rigid cubical box with sides of length 0.25m. What is the force that the gas exerts on each of the six sides when the gas temperature is 25 degrees Celsius?

$$PV = nRT, P = \frac{F}{A}, F = PA$$
$$F = \frac{nRT \times A}{V}$$
$$= \frac{4 \times 8.31 \times (273+25)}{0.25} = 39622.08 \text{ N}$$

2 How Close Together are Gas Molecules?

Consider an ideal gas at 30 degrees Celsius and 1.5 atm pressure. To get some idea how closed these molecules are to each other, imagine them to be uniformly spaced with each molecule at the center of a small cube. What is the length of an edge of each cube if adjacent cubes touch but don't overlap? For scale note that a typical spacing of atoms in solids is around 0.3nm.

$$PV = nRT$$
$$V = \frac{nRT}{P}$$
$$V = \frac{1 \times 8.31 \times (273+30)}{1.5 \times 10^5}$$
$$V = 0.0167862 \text{ m}^3$$
$$\sqrt[3]{\frac{0.0167862}{6.02 \times 10^{23}}} \Rightarrow 3.03 \times 10^{-9} \text{ m}$$

1.5 × 10⁵ Pa

3.03 nm 10 times of in solid

3 Martian Climate

The atmosphere of Mars is mostly Carbon Dioxide (which has molar mass 45g/mol) under a pressure of 600 Pa, which we will take to be constant. In many places the temperature varies from 0 degrees Celsius to -100 degrees Celsius in winter. Over the course of a Martian year, what are the ranges of the (a) Rms speeds of the CO₂ molecules, and (b) the density (in mol/m³) in the atmosphere?

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$
$$\sqrt{\frac{3 \times 8.31 \times 273}{45 \times 10^{-3}}} = 389.9 \text{ m/s}$$
$$\sqrt{\frac{3 \times 8.31 \times 173}{45 \times 10^{-3}}} = 309.6 \text{ m/s}$$
$$PV = nRT$$
$$\frac{n}{V} = \frac{P}{RT}$$
$$= \frac{600}{8.31 \times 273} = 0.2645 \text{ mol/m}^3$$
$$= \frac{600}{8.31 \times 173} = 0.4174 \text{ mol/m}^3$$
$$0.4174 - 0.2645 = 0.15$$

4 Heat Capacity

Perfectly rigid containers each hold n moles of ideal gas, one being Helium and the other being Carbon Dioxide. Heat is applied to the Helium system to increase the temperature of the Helium by 3 degrees Celsius. How many degrees will the same heat raise the temperature of the Carbon Dioxide? Note that the heat capacities (at constant volume) are given in the textbook.

$$Q = nC_v \Delta T \quad Q \text{ is the same for both gases}$$

$$\text{He: } Q = n \times \frac{3}{2} R \Delta T$$

$$\text{CO}_2: \quad Q = n \times 3.93 \times R \Delta T_1$$

$$\frac{3}{2} \Delta T = 3.93 \Delta T_1$$
$$\frac{4.5}{3.93} = \Delta T_1$$
$$\Delta T_1 = 1.246$$

5 Cumulus Clouds

Puffy cumulus clouds (made of water) occur at lower altitudes in the atmosphere. Wispy cirrus clouds (made of ice) only occur at higher altitudes. Find the altitude above sea level for which only cirrus clouds can occur. Use the expression $T=20^\circ C - 0.005^\circ C / 1000 m^y$, where y is the altitude above sea level.

$$\text{Ice at } T=0, \frac{20}{0.005} = 4000 \text{ km}$$

6 Automobile Tire

An automobile tire has a volume of $0.02m^3$ on a cold day when the temperature of the air in the tire is $4^\circ C$ and atmospheric pressure is 1.01 atm. Under these conditions the gauge pressure is measured to be 1.8 atm. After the car is driven on the highway for 30min the temperature of the air has risen to $50^\circ C$ and the volume has risen to $0.025m^3$. What then is the gauge pressure?

$$\frac{PV}{T} = \frac{P_1 V_1}{T_1}$$

$$\frac{1.8 \times 0.02 \times (50 + 273)}{(4 + 273) \times 0.025} = P_1$$

$$P_1 = 1.619 \text{ Pa}$$

$$PV = nRT$$

$$PV = PV$$

7 Tank of Water

A large tank of water has a hose connected to it. The tank is sealed at the top and has compressed air between the water surface and the top. When the water height h has the value 4.0m, the absolute pressure p of the compressed air is $4 \times 10^5 \text{ Pa}$. Assume that the air above the water expands at constant temperature, and take the atmospheric pressure to be $1 \times 10^5 \text{ Pa}$. What is the speed with which water flows out of the hose when $h=3\text{m}$?

$$P_1 + \frac{1}{2} \rho V^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho V^2 + \rho gh_2$$

$$\frac{(h-4)}{(h-3)} 4 \times 10^5 + 1 \times 10^5 \times 9.81 \times 3 = 1 \times 10^5 + 500V^2 + 9.81 \times 10^3$$

$$\sqrt{\frac{(h-4)4 \times 10^5 + 1 \times 10^5 \times 9.81 \times 10^3}{500}} = V$$

$$V = 29.3 \text{ m/s}$$

$$P_1(h-4)A = P_2(h-3)A$$

$$\frac{P_1(h-4)}{(h-3)} = P_2$$

$$4 \times 10^5 \times \frac{(h-4)}{(h-3)} = P_2$$

8 Insect Collisions

A cubical cage 1.5m on each side contains 3000 content bees, each flying randomly at 1.5 m/s. Treating the bees as spheres of diameter 2cm, (a) How far does the typical bee travel between collisions? (b) What is the average time between collisions? (c) How many collisions per second does a bee make?

$$\lambda = \sqrt{t_N} = \frac{1.5^3}{4\pi \sqrt{2}(0.01)^2 \times 3000} \approx 0.6330 \text{ m}$$

$$\frac{0.6330}{1.5} = 0.4220 \text{ s}$$

$$n = \frac{1}{t_p} \frac{1}{0.4220} = 2.370 \text{ collisions/sec}$$