# Assignment #5

Physics 2 Spring 2020
Instructor: Prof. Dirk Bouwmeester
Due: 05/03/20 5pm PST

Comments: Each problem is worth three points. If the problem has multiple parts the points breakdown is delineated in the problem.

## 1 Simple Harmonic Motion

A machine part is undergoing SHM with a frequency of 3.30 Hz and amplitude 1.80 cm. How long does it take the part to go from x = 0 to x = -1.80 cm ?

$\omega = 2\pi f \quad 6.6\pi t,\ 1.8\sin(6.6\pi t)$

$\frac{\sin^{-1}(0)}{6.6\pi} - \frac{\sin^{-1}\left(-\frac{1.8}{1.8}\right)}{6.6\pi} = \frac{5}{66}\,s$

## 2 Air-track Glider

In a physics lab, you attach a 0.300 kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 3.60 s. Find the spring's force constant.

$F = kx \quad ma = kx - 0.3g$

$-a = \frac{k}{m}x - g$

$x'' + \frac{k}{m}x = g$

$n^2 + \frac{k}{m} = 0$

$n = \sqrt{\frac{k}{m}}\,i$

$\Rightarrow \mathrm{cis}\left(\sqrt{\frac{k}{m}}\,t\right)$

$\omega = \sqrt{\frac{k}{m}}$

half period = 3.6 s
Full period = 7.2 s

$\frac{2\pi}{7.2} = \sqrt{\frac{k}{0.3}}$

$k = 0.2285$

## 3 Oscillating Mass

A mass is oscillating with amplitude A at the end of a spring. How far (in terms of A) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

$K_e = K_E$

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

Say: $x = A\sin(\omega t)$ and $\therefore v = A\omega\cos(\omega t)$, $A\omega = \max v$

$kA^2\sin^2(\omega t) = mA^2\omega^2\cos^2(\omega t)$

$\tan^2(\omega t) = 1$

$\omega t = \pm\frac{\pi}{4}$, $x = A\sin\frac{\pi}{4}$

$x = \frac{\sqrt{2}}{2}A$

## 4 Torsion

You want to find the moment of inertia of a complicated machine part about an axis through its center of mass. You suspend it from a wire along this axis. The wire has a torsion constant of $0.390 N\ m/rad$. You twist the part a small amount about this axis and let it go, timing 108 oscillations in 330 s. What is its moment of inertia?

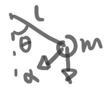
$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$

$k/(2\pi f)^2 = I$

$0.390/\left(4\pi^2\left(\frac{108}{330}\right)^2\right) = I = 0.092\ kgm^2$

## 5 Space Explorer

After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 45.0 cm. She finds that the pendulum makes 75 complete swings in 105 s. What is the value of g on this planet?



$ml^2\alpha = mg\sin\theta\, l$ $\quad \sin\theta \approx \theta$

$\alpha l = g\theta$ $\quad \omega = \sqrt{\frac{g}{l}}$

$\alpha = \frac{g}{l}\theta$

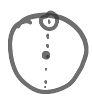
$2\pi f = \omega$

$l \times 4\pi^2 \times (\frac{75}{105})^2 = g$

$g \approx 9.064$

## 6 Physical Pendulum

We want to hang a thin hoop on a horizontal nail and have the hoop make one complete small-angle oscillation each 3.0 s. What must the hoop's radius be?



$T = 2\pi\sqrt{\frac{I}{mgd}}$ $\quad \omega = \sqrt{\frac{g}{l}}$ $\quad l = 2r$

$2\pi f = \sqrt{\frac{g}{2r}}$

$2\pi\sqrt{\frac{r}{g}} = T = 3$

$r \approx 2.236m$ $-1$

$\frac{2.236}{2} \approx 1.12$

## 7 Three Stars Problem

An object is undergoing SHM with period 0.600 s and amplitude 9.00 cm. At t = 0 the object is instantaneously at rest at x = 9.00 cm. Calculate the time it takes the object to go from x = 9.00 cm to x = -4.50 cm.

$T = 0.6 = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{.6} = \frac{\pi}{0.3}$

$9\cos(\frac{\pi}{3}t) = x$

$\frac{0.3}{\pi}\left(\cos^{-1}(\frac{-4.5}{9}) - \cos^{-1}(1)\right) = 0.2s$

## 8 The Silently Ringing Bell

A large, 36.0 kg bell is hung from a wooden beam so it can swing back and forth with negligible friction. The bell's center of mass is 0.55 m below the pivot. The bell's moment of inertia about an axis at the pivot is 36.0 kg $\cdot m^2$. The clapper is a small, 2.8 kg mass attached to one end of a slender rod of length L and negligible mass. The other end of the rod is attached to the inside of the bell; the rod can swing freely about the same axis as the bell. What should be the length L of the clapper rod for the bell to ring silently — that is, for the period of oscillation for the bell to equal that of the clapper?

$I_T = 2.8(L+0.55)^2 + 36$

$I\alpha = 2.8g(L+0.55)\sin\theta + 36g\times 0.55\sin\theta$ $\quad \sin\theta \approx \theta$

$\alpha = \frac{(2.8(L+0.55)+36\times 0.55)g}{2.8(L+0.55)^2}\theta$

$\alpha = \frac{21.34+2.8L}{2.8(L^2+0.3025+1.1L)}\theta$

$T = 2\pi\sqrt{\frac{36}{36g\times L}}$

$T = 2\pi\sqrt{\frac{1}{9.81\times 0.55}}$

$\omega = \sqrt{0.55g}$

$\omega = \sqrt{\frac{g}{L+0.55}}$ $\Rightarrow \frac{20}{11} = L+0.55$ $\quad$ 1.82

$L = 1.268m$