University of California Santa Barbara

Homework #4

Introduction to EE

ECE 3

Instructor: Christos Thrampoulidis

Worked Solutions

First & Last Name: ______________________________

Perm Number: ____________________

For instructor use:

Question:	1	2	3	Total
Points:	15	14	6	35
Score:				

Instructions:

- Write your name in the space provided above.
- Please consult the Syllabus for HW grading and late-submission policies.
- Please first solve the problem in draft paper and present your **complete** final answer in *clean* form on the space provided. **Answer all of the questions in the spaces provided. Answers are expected to be succinct but complete.** Please only use extra space (attach to the end of your submission) if absolutely necessary.
- The HW is **due Tuesday November 26 10:00am sharp**.
- The HW set includes **Programming Assignments** marked as **(P.A.)**. Create a new Jupyter notebook and write code for each one of them. Execute the code to obtain the desired results (e.g., plot of the signals). Upload the notebook **including the code and its output** on GauchoSpace. **Attach a printed copy of your code.**
- **Return a paper copy of your HW to the homework box in HF.**
- **The returned copy should include a printout of your code for the Programming assignments. Also upload your P.A.s to GauchoSpace.**

1. **Problem 1 [Shhhh]. (P.A.)** The goal of this exercise is to demonstrate the use of the running-average filter to **denoise** a signal. Specifically, consider the following *uncorrupted* finite-length discrete signal $s[n]$:

$$s[n] = \begin{cases} 2ne^{-0.1n} & , 0 \leq n \leq 40, \\ 0 & , \text{otherwise.} \end{cases}$$

The signal $\{s[n]\}_n$ is corrupted by *additive random noise* resulting in a new *noisy* signal $x[n]$ as follows. For every $0 \leq n \leq 40$, $x[n] = s[n] + z[n]$ where $z[n]$ is a random number (formally: a random variable). In particular, $z[n]$ can take the values $+1/2$ or $-1/2$ with equal probability. You can think of it this way. At each time index n, nature tosses a (fair) coin. If the outcome is head then $x[n] = s[n] + \frac{1}{2}$. Otherwise, $x[n] = s[n] - \frac{1}{2}$.

This could model the noise of a measurement system. Imagine a scenario where the original signal $\{s[n]\}_n$ is unknown to us. Instead, we only have access to the noisy measurements $\{x[n]\}_n$.

We will use a running-average system to reconstruct the original signal.

Write Python code as requested in each one of the following parts. Return a notebook with your code. Your submission should include code that is bug-free and should show the desired outputs (such as requested plots). Use headlines to distinguish between different parts.

(a) (2 points) Use Python to make a stem plot of the signal $s[n]$.

(b) (2 points) Use the function "numpy.random.rand" to generate a random signal $\{z[n]\}_{n=0}^{40}$ as shown below. [1]

Make a stem plot of the vector z (eqv. of the noise signal $z[n]$) to make sure that it only takes values $+1/2$ and $-1/2$ as desired. The output of "numpy.random.rand" is a (pseudo)-random number. Thus, every time you run your code, you get a different vector z and a different stem plot.

```python
import numpy as np

rand_uniform = np.random.rand(40,1)
z = 1*(rand_uniform>0.5) - 0.5
```

(c) (2 points) Use Python to compute and plot of the signal $x[n] = s[n] + z[n],\ n \in \mathbb{Z}$.

(d) (1 point) Make a second plot that shows both $\{s[n]\}_n$ and $\{x[n]\}_n$.

(e) (5 points) Let $\{y[n]\}_n$ be the output of a 5-point causal running average filter with input $\{x[n]\}_n$. Plot the signals $\{y[n]\}_n$ and $\{x[n]\}_n$ together to test whether the output of the filter is reasonably close to the original uncorrupted signal.

(f) (3 points) What is the effect of the 5-point running average filter on the noise signal $\{z[n]\}$? Plot the output of the filter with $\{z[n]\}$ as its input. Below, give a short explanation of what you observe and why this is the case.

[1] See here for details: (https://docs.scipy.org/doc/numpy-1.14.1/reference/generated/numpy.random.rand.html. Do not worry about the details of this implementation for now. If you are curious come to my office hours and ask.

2. **Problem 2 [LTI].** For each of the following systems determine whether or not the system is (1) linear, (2) time-invariant, and (3) causal. *Explain your answers!*

(a) (2 points) $y[n] = x[n] - x[n-1]$

Linear: Let any $x_1[n], x_2[n], \alpha, \beta \in \mathbb{R}$

Then; $T\{\alpha x_1[n] + \beta x_2[n]\} = \alpha x_1[n] + \beta x_2[n] + \alpha x_1[n-1] + \beta x_2[n-1]$

$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = \alpha x_1[n] + \alpha x_1[n-1] + \beta x_2[n] + \beta x_2[n-1]$

$\Rightarrow T\{\alpha x_1 + \beta x_2\} = \alpha T\{x_1\} + \beta T\{x_2\}$

TI: Let any $x[n]$ and $n_0 \in \mathbb{Z}$. Call $y[n] = T\{x[n]\} = x[n] - x[n-1]$

$T\{x[n-x_0]\} = x[n-n_0] + x[n-n_0-1]$

$y[n-n_0] = x[n-n_0] + x[n-1-n_0]$

$\Rightarrow T\{x[n-n_0]\} = y[n-n_0]$

Causal: since output $y[n]$ at time n only depends on present & past inputs

(b) (2 points) $y[n] = |x[n]|$

Not linear: Let $x_1[n] = \delta[n], x_2[n] = 0, \alpha = -1, \beta = 4$

Then, $T\{\alpha x_1[n] + \beta x_2[n]\} = T\{-\delta[n]\} = |-\delta[n]| = \delta[n]$

$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = (-1) T\{\delta[n]\} + 4T\{0\} = -\delta[n]$

$\Rightarrow \neq$

TI: Let any $x[n]$ and $n_0 \in \mathbb{Z}$. Call $y[n] = T\{x[n]\} = |x[n]|$

$T\{x[n-n_0]\} = |x[n-n_0]| = y[n-n_0]$

Causal: Clear, same reason as (a).

(c) (2 points) $y[n] = x[n]\cos(0.2\pi n)$

Linear: $T\{\alpha x_1[n] + \beta x_2[n]\} = (\alpha x_1[n] + \beta x_2[n]) \cos(0.2\pi n)$

$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = \alpha x_1[n] \cos(0.2\pi n) + \beta x_2[n] \cos(0.2\pi n)$

$\Rightarrow =$

Not TI: Let $x[n] = \delta[n]$ and $y[n] = \delta[n] \cdot \cos(0.2\pi n)$

Then $T\{x[n-1]\} = T\{\delta[n-1]\} = \delta[n-1] \cdot \cos(0.2\pi n) \overset{n=1}{\Longrightarrow} T\{x[n-1]\}\big|_{n=1} = \cos(0.2\pi)$

$y[n-1] = \delta[n-1] \cdot \cos(0.2\pi n - 0.2\pi) \overset{n=1}{\Longrightarrow} y[n-1]\big|_{n=1} = \cos(0) = 1$

$\Rightarrow T\{x[n-1]\} \neq y[n-1]$

Causal: Yes. Same as ~~(a)~~ (b)

(d) (2 points) $y[n] = Ax[n] + B$, where A and B are *nonzero* constants.

Not linear: $x_1[n] = 0, x_2[n] = 0, \alpha = \beta = 2$

$T\{\alpha x_1[n] + \beta x_2[n]\} = T\{0\} = B$

$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = 2B + 2B = 4B$

$\Rightarrow \neq$

TI: $T\{x[n-n_0]\} = Ax[n-n_0] + B$

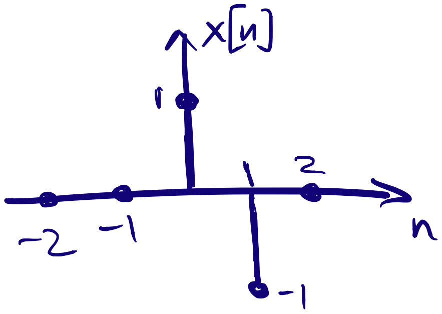
$y[n-n_0] = Ax[n-n_0] + B$

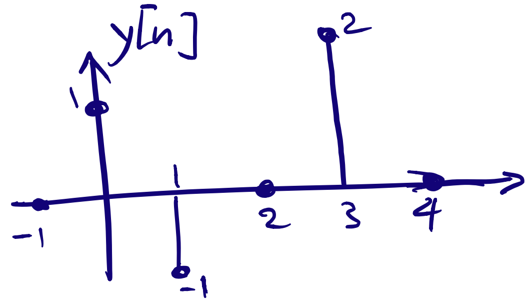
$\Rightarrow =$

Causal: Yes; output depends only on present values.

For the next two parts, suppose that $\mathcal{T}$ is a linear and time-invariant system whose exact input-output relation is *unknown*. However, the system is tested by running some inputs into the system and then observing the output signals. Specifically, when $x[n] = \delta[n] - \delta[n-1]$ is applied as the input, then the observed output is $y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$. Here, $\{\delta[n]\}_n$ denotes the unit impulse sequence.

(e) (2 points) Make plots (on paper) of the input and output signals $x[n]$ and $y[n]$ defined above.





(f) (4 points) Use linearity and time-invariance to compute and plot the output of the system when the input is

$$\widetilde{x}[n] = 7\delta[n] - 7\delta[n-2].$$

Note that $\widetilde{x}[n] = 7(\delta[n] - \delta[n-1]) + 7(\delta[n-1] - \delta[n-2])$

$= 7x[n] + 7x[n-1]$

By Linearity:

$\mathcal{T}\{\widetilde{x}[n]\} = 7\mathcal{T}\{x[n]\} + 7\mathcal{T}\{x[n-1]\}$

$= 7y[n] + 7\mathcal{T}\{x[n-1]\}$

By Time Invariance:

$\mathcal{T}\{x[n-1]\} = y[n-1]$

Combined,

$\mathcal{T}\{\widetilde{x}[n]\} = 7y[n] + 7y[n-1] =$

$= 7\delta[n] + 14\delta[n-3] - 7\delta[n-2] + 14\delta[n-4].$

——— *End of HW #1* ———