University of California Santa Barbara

Homework #1

Introduction to EE

ECE 3

Instructor: Christos Thrampoulidis

First & Last Name: ______________________________

Perm Number: ____________________

For instructor use:

Question:	1	2	3	4	Total
Points:	9	8	18	15	50
Score:					

Instructions:

- Write your name in the space provided above.
- Please consult the Syllabus for HW grading and late-submission policies.
- Please first solve the problem in draft paper and present your **complete** final answer in *clean* form on the space provided. **Answer all of the questions in the spaces provided. Answers are expected to be succinct but complete.** Please only use extra space (attach to the end of your submission) if absolutely necessary.
- The HW is **due Friday October 11 9:00am sharp**.
- The problem statements might be long. This does not seem that the problems are hard to solve! The questions are designed to guide you through the problem step by step. Give them a shot. Read the lecture notes. Talk to your classmates. Ask for help in the lab and in the office hours if you need!
- The HW set includes **Programming Assignments** marked as **(P.A.)**. Create a new Jupyter notebook and write code for each one of them. Execute the code to obtain the desired results (e.g., plot of the signals). Upload the notebook **including the code and its output** on GauchoSpace.
- For this HW, you are expected to upload **two** notebooks. The first one contains your answers to problems 3b, 3d, 3e. The second one contains your work in Problem 4.
- **Return a paper copy of your HW to the homework box in HF. Upload your P.A.s to Gauchospace.**

1. **Problem 1 [Jargon].**

   (a) (3 points) Give a definition for the term "*signal*" as used in Signal Processing (SP). Give two examples of signals. Are they continuous or discrete? Also, briefly describe why you might want to process these signals.

   (b) (2 points) What is "*sampling*" and why is it important in SP?

   (c) (1 point) Explain in your own words what is the "*spectrum*" of a signal.

   (d) (3 points) Give two examples where it is beneficial to think of the signal in the "*frequency domain*".

2. **Problem 2 [Building blocks].**

(a) (3 points) The phase of a sinusoid can be related to *time shift* as follows:

$$s(t) = A\cos(2\pi f_0 t + \phi) = A\cos(2\pi f_0(t - t_1)).$$

Assume the period of the signals is $T_0 = 8$s. *Explain* whether the following are True or False.

1. "When $t_1 = -2$s, the value of the phase is $\phi = 3\pi/4$."

Call $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} = \frac{\pi}{4}\,\frac{\text{rad}}{\text{s}}$. Then, $\omega_0 t_1 = \frac{\pi}{4}(-2) = -\frac{\pi}{2}$

Clearly $A\cos(\omega_0 t + \frac{\pi}{2}) \neq A\cos(\omega_0 t + \frac{3\pi}{4}) \Rightarrow$ $\underline{\text{FALSE}}$

2. "When $t_1 = 3$s, the value of the phase is $\phi = 3\pi/4$."

Here, $\omega_0 t_1 = \frac{\pi}{4}\cdot 3 = \frac{3\pi}{4}$

Clearly, $A\cos(\omega_0 t - \frac{3\pi}{4}) \neq A\cos(\omega_0 t + \frac{3\pi}{4}) \Rightarrow$ $\underline{\text{FALSE}}$

3. "When $t_1 = 7$s, the value of the phase is $\phi = \pi/4$."

Here, $\omega_0 t_1 = \frac{\pi}{4}\cdot 7 = \frac{7\pi}{4} = 2\pi - \frac{\pi}{4}$

By periodicity of cosine: $A\cos(\omega_0 t + \frac{7\pi}{4}) = A\cos(\omega_0 t - \frac{\pi}{4} + 2\pi)$
$= A\cos(\omega_0 t - \frac{\pi}{4}) \Rightarrow$ $\underline{\text{TRUE}}$

(b) (2 points) Let $A, B, f_0 \in \mathbb{R}$ be given. Find C and ϕ (in terms of the given variables) such that

$$A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t) = C\cos(2\pi f_0 t + \phi), \qquad \text{for all} \quad t \in \mathbb{R}.$$

Recall the tricogonometric identity: $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

Specifically, $C\cos(\omega_0 t + \phi) = C\cos(\phi)\cdot\cos(\omega_0 t) - C\sin(\phi)\cdot\sin(\omega_0 t)$

Hence, we just need to choose C and ϕ s.t.

$$\left\{\begin{array}{l} C\cos\phi = A \\ C\sin\phi = B \end{array}\right\} \iff \left\{\begin{array}{l} C = \sqrt{A^2+B^2} \\ \phi = \arctan(-\frac{B}{A}) \end{array}\right.$$

(c) (3 points) Let $f_0, \{A_k\}_{k=1}^N, \{\phi_k\}_{k=1}^N \in \mathbb{R}$ and $N \in \mathbb{N}$ be given. Consider the signal

$$s(t) = \sum_{k=1}^{N} A_k\cos(2\pi f_0 t + \phi_k)$$

that is a linear additive combination of sinusoidal signals of the same frequency, but different amplitudes and phases. Show that there exist amplitude A and phase ϕ such that $s(t)$ can be equivalently expressed as: $s(t) = A\cos(2\pi f_0 t + \phi)$.

(Hint: You don't need to explicitly compute A and ϕ. Just find a way to argue that such values exist. Part (b) of this problem together with its reverse statement that we saw in class might be useful.)

By the trigonometric identity in part (b):

$$s(t)=\sum_{k=1}^{N}A_k\cos(\omega_0 t+\phi_k)=\sum_{k=1}^{N}\left(A_k\cos\phi_k\cdot\cos(\omega_0 t)-A_k\sin\phi_k\cdot\sin(\omega_0 t)\right)$$

$$=\underbrace{\left(\sum_{k=1}^{N}A_k\cos\phi_k\right)}_{\triangleq C_1}\cdot\cos(\omega_0 t)-\underbrace{\left(\sum_{k=1}^{M}A_k\sin\phi_k\right)}_{\triangleq C_2}\cdot\sin(\omega_0 t)$$

$$=C_1\cdot\cos(\omega_0 t)+C_2\cdot\sin(\omega_0 t)$$

But, by Part (b) $\exists\, A,\phi$ s.t. $C_1\cos(\omega_0 t)+C_2\sin(\omega_0 t)=$

$$=A\cos(\omega_0 t+\phi)$$

Combining the above completes the proof.

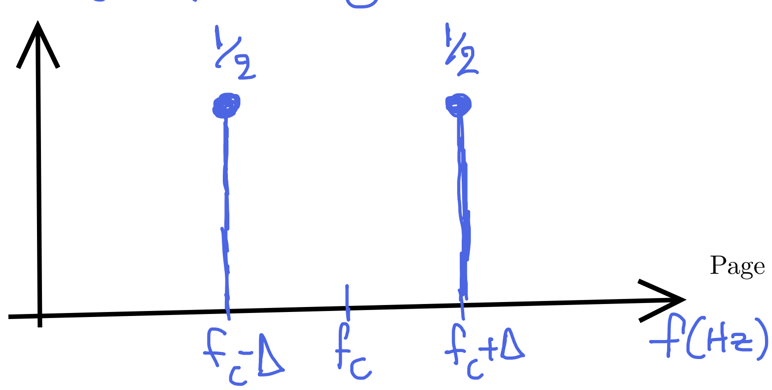
3. **Problem 3 [Modulation].** Let f_c, f_Δ two frequencies such that $f_c \gg f_\Delta$. Consider the signal resulting by multiplying two pure cos-sinusoidal signals with these frequencies.

$$s(t)=\cos(2\pi f_\Delta t)\cos(2\pi f_c t).$$

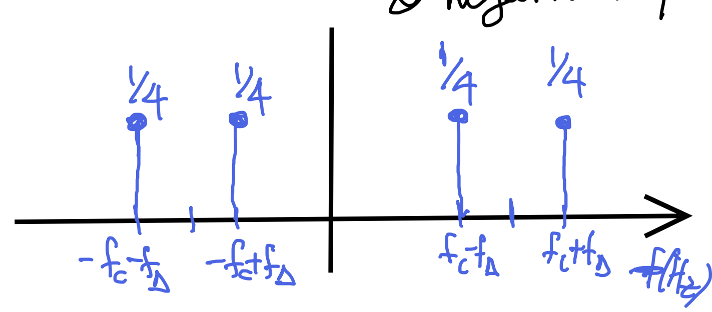
(a) (3 points) Compute the frequency spectrum of the signal $s(t)$ and make a sketch of it.
(Hint: Recalling the trigonometric identities from high-school might be helpful!)

In order to compute & plot the spectrum, we need to express $s(t)$ as an additive linear combination of real sinusoids.
To do this, we use the identity: $2\cos(\alpha)\cos(\beta)=\cos(\alpha-\beta)+\cos(\alpha+\beta)$
We then find that $s(t)=\frac{1}{2}\cos(2\pi(f_c+f_\Delta)t)+\frac{1}{2}\cos(2\pi(f_c-f_\Delta)t)$

We immediately recognize the two frequencies f_c-f_Δ and f_c+f_Δ present in the signal



This is the two-sided spectrum including pos & negative freqs



(b) (3 points) **(P.A.)** Take $f_c = 200$Hz and $f_\Delta = 20$Hz. Use Python to plot the following signals on the same plot: $s(t)$, $\cos(40\pi t)$ and $-\cos(40\pi t)$ for $0 \leq t \leq 100$.

(c) (1 point) The plots of $\cos(40\pi t)$ and $-\cos(40\pi t)$ are called the *envelope* of the signal $s(t)$. Give an explanation for this term.

The two plots define the boundaries inside of which ~~the~~ the higher frequency signal is drawn. These boundaries are called the signal's envelope.

(d) (1 point) **(P.A.)** Can you play the signals $s(t)$ and $\cos(2\pi 200t)$ on your computer? What do you observe?

The signal $s(t)$ appears to fade in and out because the signal envelope is rising & falling periodically. This results in the audible phenomenon called "beating" of tones in music and is used to bring two instruments "in tune".

For the remaining part of the exercise, consider the signal $s(t) = (5 + 4\cos(40\pi t))cos(400\pi t)$.

(e) (1 point) **(P.A.)** Use Python to plot $s(t)$.

(f) (1 point) In reference to the plot you generated above, *explain* if the following statement is True or False: "The effect of multiplying the higher-frequency sinusoid (called the carrier) by the lower-frequency sinusoid is to change the amplitude of the waveform of the former."

True. The lower freq. sinusoid determines the envelope of the signal changing the amplitude.

(g) (2 points) Make a sketch of the frequency spectrum of $s(t)$. How is it different from your answer in part (a)?

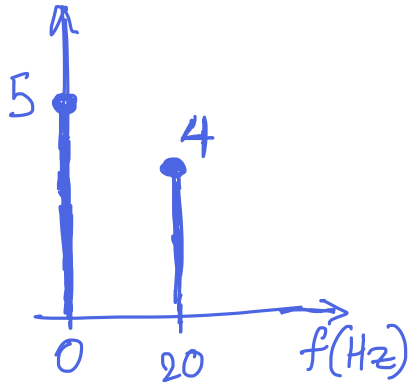
Using same argument as in Part (a), we write

$$s(t) = 5\cos(2\pi t \cdot 200) + 2\cos(2\pi(180)t) + 2\cos(2\pi(220)t)$$

resulting in the (one-sided) line spectrum below:

In contrast to Part (a) the carrier f_c is also present in the signal.

(h) (2 points) Make a sketch of the frequency spectrum of the low-frequency signal $5 + 4cos(40\pi t)$. Compare the result with the spectral plot in the previous part (g).



Comparing to Part (g):
multiplying by the carrier in time domain
<u>shifts</u> the DC-term to the carrier in
the frequency domain.

Multiplying sinusoids is commonplace in communication systems. In particular, the process of multiplying a high-frequency sinusoidal signal by a low-frequency signal (such as voice or music signal) is known as *Amplitute Modulation* (AM). It is exactly the technique used in broadcast AM radio! The AM signal is a product of the form

$$s(t) = v(t)\cos(2\pi f_c t),$$

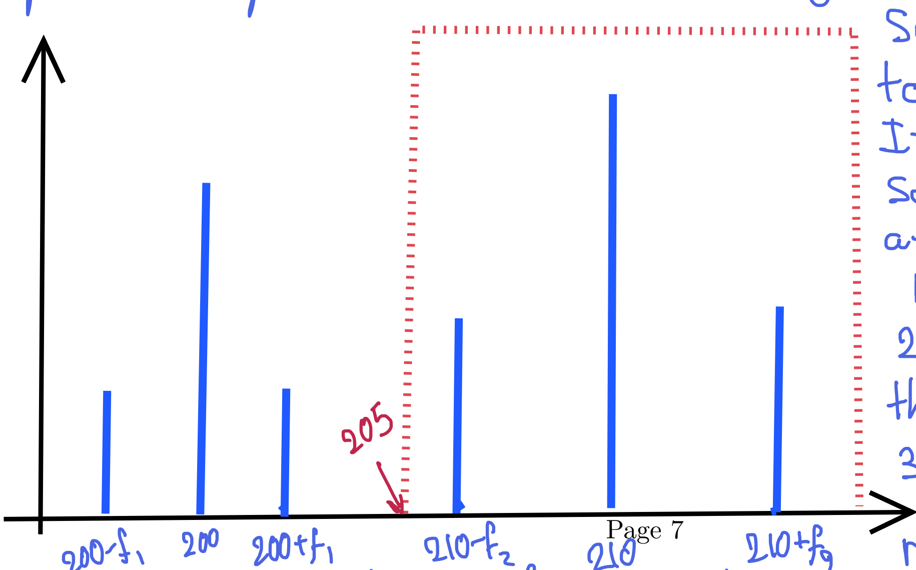
where the frequency f_c (known as the *carrier* frequency) is assumed much higher than the frequencies in $v(t)$ (voice or music signal).

(i) (1 point) The "Santa Barbara News-Press Radio" is on station 1290 kHz. Write an expression for the signals broadcasted by this station and explain the terms involved.

$$s(t) = \underbrace{v(t)}_{\text{voice signal}} \cos(2\pi \underbrace{1290}_{\text{carrier}} t)$$

(j) (3 points) Let $s_1(t) = (A_1 + B_1\cos(2\pi f_1 t))\cos(400\pi t)$ and $s_2(t) = (A_2 + B_2\cos(2\pi f_2 t))\cos(420\pi t)$ be the transmitted signals from two AM radio stations. This (very) simple model allows for the two stations to transmit different information by modifying the corresponding frequencies f_1, f_2 and the amplitudes A_1, A_2, B_1, B_2. Suppose we receive a signal $r(t) = s_1(t) + s_2(t)$. We know the carrier frequencies 200Hz and 210Hz of the two stations, but, at any given instance, we do *not* know the frequencies f_1 and f_2 that the two stations choose to transmit. Argue that it is possible to design a system that is able to reproduce the true content of each station's signal as long as $f_1 + f_2 < 10$Hz.

We plot the spectrum of the recieved signal $r(t)$:



Suppose we want to listen to the station at 210Hz
If $f_1+f_2<10$Hz, then we can do so with the following algorithm:

1. Plot spectrum of $r(t)$
2. Zero-out all freqs but those in the interval $[205, 215]$Hz
3. Find the time-domain representation of the resulting spectrum (only what's in red in the figure).
4. Play that.

Note: the algorithm need not know f_1 or f_2 and is guaranteed to work provided $f_1+f_2<10$Hz!

4. (15 points) **Problem 4 [Find the note]. (P.A.)** Download the files "HW_1_find_the_note.ipynb" and "piano.txt" from GauchoSpace. Open the notebook "HW_1_find_the_note.ipynb" and go through the material. Fill in all the blanks in the code cells (indicated by three dots ...) and write your own lines of code wherever required. When appropriate, answer the questions posed (e.g., "which note was this?"). **Answer everything on the notebook and upload your work on GauchoSpace.**

——— *End of HW #1* ———