Vector Calculus Formulae

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August 2020

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1 Formulae

1.1 Vector Differentiation

1.1.1 For a parameterized curve r(t)

$$\vec{r}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \end{bmatrix} \qquad \frac{d\vec{r}}{dt} = \begin{bmatrix} df_1/dt \\ df_2/dt \\ \vdots \end{bmatrix}$$

1.1.2 Unit Tangent, Normal and Binomial Vector of r

$$\vec{T} = \frac{\vec{r'}(t)}{\left|\vec{r'}(t)\right|} \qquad \vec{N} = \frac{\vec{T'}(t)}{\left|\vec{T'}(t)\right|} \qquad \vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

1.1.3 Gradient, Divergence and Curl

$$\nabla = \begin{bmatrix} d/dx_1 \\ d/dx_2 \\ \vdots \end{bmatrix} \qquad \nabla F = \begin{bmatrix} d(F_1)/dx_1 \\ d(F_2)/dx_2 \\ \vdots \end{bmatrix} \qquad \nabla \cdot F = \frac{d(F_1)}{dx_1} + \frac{d(F_2)}{dx_2} \dots$$

$$\nabla \times f = det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} & \\ d/dx_1 & d/dx_2 & d/dx_3 & \dots \\ F_1 & F_2 & F_3 & \\ & \vdots & & \end{bmatrix}$$

1.2 Vector Integral

1.2.1 Substitution to polar, cylindrical and spherical coordinate system

$$\begin{bmatrix} x & = & r\cos(\theta) \\ y & = & r\sin(\theta) \end{bmatrix} \qquad \begin{bmatrix} x & = & r\cos(\theta) \\ y & = & r\sin(\theta) \\ z & = & z \end{bmatrix} \qquad \begin{bmatrix} x & = & \rho\sin(\phi)\cos(\theta) \\ y & = & \rho\sin(\phi)\sin(\theta) \\ x & = & \rho\cos(\phi) \end{bmatrix}$$

$$dA = rdr\ d\theta \qquad dV = r\ dz\ dr\ d\theta \qquad dV = \rho^2\sin(\phi)\ d\rho\ d\theta\ d\phi$$

1.2.2 Line Integral

$$\int_C f(x,y)ds = \int_a^b f(h(t), g(t)) \left|\vec{r'}(t)\right| dt$$

1.2.3 Line Integral over Vector Field

$$\int\limits_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r'}(t) dt$$

Green's Theorem

$$\oint\limits_C P dx + Q dy = \iint_D \left(\frac{\sigma Q}{\sigma x} - \frac{\sigma P}{\sigma y} \right) dA$$

1.2.4 Surface Integral

$$\vec{r}(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j} + x(u,v)\hat{k}$$

Given $\vec{r}(u,v)$

$$\int\limits_S f(x,y,z) dS = \iint_D f(\vec{r}(u,v) \left| \vec{r_v} \times \vec{r_u} \right| dA$$

If $\vec{r}(x,y) = x\hat{i} + y\hat{j} + g(x,y)\hat{k}$

$$\left| \vec{r_v} \times \vec{r_u} \right| = \sqrt{\left(\frac{\sigma g}{\sigma x} \right)^2 + \left(\frac{\sigma g}{\sigma y} \right)^2 + 1}$$

1.2.5 Surface Integral over Vector Field

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} \; dS = \iint_S \vec{F} \cdot (\vec{r_v} \times \vec{r_u}) \; dA$$

1.2.6 Stoke's theorem

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

1.2.7 Divergence's theorem

$$\int_S \vec{F} \cdot d\vec{S} = \iiint_E (\nabla \cdot \vec{F}) dV$$

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References

[1] Paul's calculus notes, 2003-2020.