University of California Santa Barbara

Midterm

Introduction to EE

ECE 3

Instructor: Christos Thrampoulidis

First & Last Name: ______________________________

Perm Number: ____________________

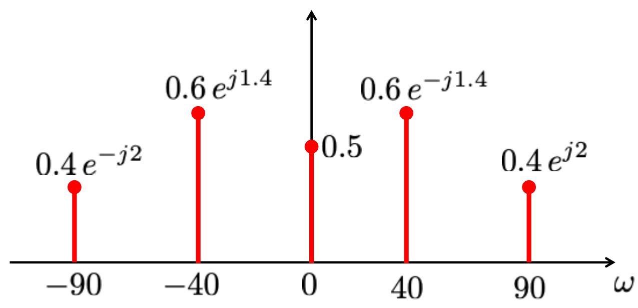
For instructor use:

Question:	1	2	3	Total
Points:	13	9	13	35
Score:				

Instructions:

- Write your name in the space provided above.
- Please first solve the problem in draft paper and present your **complete** final answer in *clean* form on the space provided. **Answer all of the questions in the spaces provided. Answers are expected to be succinct but complete.** Please only use extra space (attach to the end of your submission) if absolutely necessary.
- The duration of the exam is **60 minutes** sharp.
- Exam is closed-book, closed-book and closed-notes (any type of notes!)
- NO use of phone, laptop, calculator or any other electronic device.
- NO collaboration allowed.
- There are 3 Problems in this exam and a total of 7 pages + 3 pages of scratch paper.

1. **Problem 1 [Synthesize it!].** We are given the following frequency representation of a signal $s(t)$. Note that the frequency information on the horizontal axis of the two-sided spectrum is given in terms of the **radial frequency** which has units of **rad/s**. Recall that the radial frequency ω(rad/s) relates to the frequency f(Hz) via the formula $\omega = 2\pi f$.



(a) (3 points) Write a mathematical formula for the signal $s(t)$ in the time domain.

By inspection,

$$s(t) = 0.4e^{-j2}e^{-j90t} + 0.6e^{j1.4}e^{-j40t} + 0.5 + 0.6e^{-j1.4}e^{j40t} + 0.4e^{j2}e^{j90t}$$

(b) (1 point) Is the signal $s(t)$ a *real* signal? Why? Or, why not?

The signal is real because the Fourier coefficient are conjugate symmetric, i.e. magnitudes are symmetric and phases are anti-symmetric.

(See also Part (d))

(c) (1 point) Determine the *DC value* of this signal.

The DC value of the signal is 0.5

i.e., the Fourier coefficient corresponding to the 0 frequency in the spectrum.

(d) (2 points) Can you express the signal $s(t)$ as an additive linear combination of ***real*** *sinusoids*? If yes, do so.

We apply Euler formula in Part (a)

Note that $0.6e^{j1.4t}e^{-j40t} + 0.6e^{-j1.4}e^{j40t} = \frac{1.2}{2}\left(e^{j(40t-1.4)} + e^{-j(40t-1.4)}\right)$

$= 1.2 \cdot \cos(40t - 1.4)$

$0.4e^{-j2}e^{-j90t} + 0.4e^{j2}e^{j90t} = \frac{0.8}{2}\left(e^{j(90t+2)} + e^{-j(90t+2)}\right)$

$= 0.8 \cdot \cos(90t + 2)$

Thus, combining these with Part (a):

$s(t) = 0.5 + 1.2 \cdot \cos(40t - 1.4) + 0.8\cos(90t + 2).$

(e) (2 points) What is the fundamental frequency f_0 **in Hertz** of the signal?

The frequencies present in the signal (other than the DC) are $\pm 40, \pm 90 \frac{rad}{s}$ or $\pm \frac{40}{2\pi}, \pm \frac{90}{2\pi}$ Hz

The fundamental frequency is $f_0 = \gcd\left(\frac{40}{2\pi}, \frac{90}{2\pi}\right) = \frac{10}{2\pi} = \frac{5}{\pi}$ Hz

(f) (2 points) Determine which harmonics (positive and negative) are present.

Note that $\frac{40}{2\pi} = 4 \cdot \frac{5}{\pi} = 4f_0$

and $\frac{90}{2\pi} = 9 \cdot \frac{5}{\pi} = 9f_0$

Hence, the 4th and 9th harmonics are present in the signal. (and their negatives)

(g) (2 points) Is the signal periodic? What is the fundamental period T_0(s) of the signal? That is, which is the shortest possible period?

The signal is periodic with period $T_0 = \frac{1}{f_0} = \frac{\pi}{5}$ sec because from Part (a), we are able to write $s(t)$ as a linear combination of complex exponentials corresponding to harmonics of the fundamatal frequency $f_0 = \frac{5}{\pi}$ Hz.

2. **Problem 2 [Sample it!].** You are given the following real continuous-time periodic signal

$$s(t) = 5 + 12\cos(40t) + 8\sin(90t), \quad \forall t.$$

(a) (2 points) Prove that the signal is periodic with period $T_0 = \pi/5$ sec.

The signal is an aditive linear combination of a DC-term and two sinusoidal signals of frequencies 40 and 90 $\frac{rad}{sec}$ or $\frac{40}{2\pi}$ and $\frac{90}{2\pi}$ Hz. Note that for $f_0 = \frac{5}{\pi}$ Hz: $40 = 4 f_0$, $90 = 9 f_0$

Hence, f_0 is the fundamental freq and $s(t)$ is periodic with period $\frac{\pi}{5}$ sec.

(b) (2 points) Let $s[n]$ be a discrete signal obtained by sampling $s(t)$ with a sampling rate $f_s = \frac{180}{\pi}$ Hz. Write a mathematical expression for the sampled signal $s[n]$.

$$s[n] = s(n \cdot T_s) = s\left(n \cdot \frac{1}{f_s}\right) = s\left(\frac{n\pi}{180}\right) =$$

$$= 5 + 12\cos\left(40 \cdot \frac{\pi}{180} \cdot n\right) + 8 \cdot \sin\left(90 \cdot \frac{\pi}{180} \cdot n\right)$$

$$= 5 + 12 \cdot \cos\left(\frac{2\pi}{9} n\right) + 8 \cdot \sin\left(\frac{\pi}{2} \cdot n\right) \quad \forall n \in \mathbb{Z}$$

(c) (3 points) Is the signal $s[n]$ periodic? In other words, can you find *integer* N such that $s[n] = s[n+N]$?

We need to find integer $N \in \mathbb{Z}$ s.t.

$$s[n+N] = s[n] \Leftrightarrow s\big((n+N)T_s\big) = s(n \cdot T_s) \Leftrightarrow$$

$$\Leftrightarrow s(n \cdot T_s + N \cdot T_s) = s(n \cdot T_s) \quad \forall n \in \mathbb{Z} \quad (*)$$

Since $s(t)$ is periodic with period $T_0 = \frac{\pi}{5}$ sec it holds:

$$s\left(t + \frac{\pi}{5}\right) = s(t) \quad \forall t \in \mathbb{Z}$$

Hence, the desired equation $(*)$ is true provided that

$$N \cdot T_s = \frac{\pi}{5} \Leftrightarrow N = \frac{\pi}{5} \cdot \frac{1}{T_s} = \frac{\pi}{5} \cdot f_s = \frac{\cancel{\pi}}{5} \cdot \frac{180}{\cancel{\pi}} = 36$$

We have shown that the discrete signal is periodic with period $N = 36$.

(d) (2 points) Next, consider a shifted version of the signal $s(t)$ as follows. Let the new signal be defined as follows:

$$x(t) = s\big(t + \pi/5\big), \qquad \forall t.$$

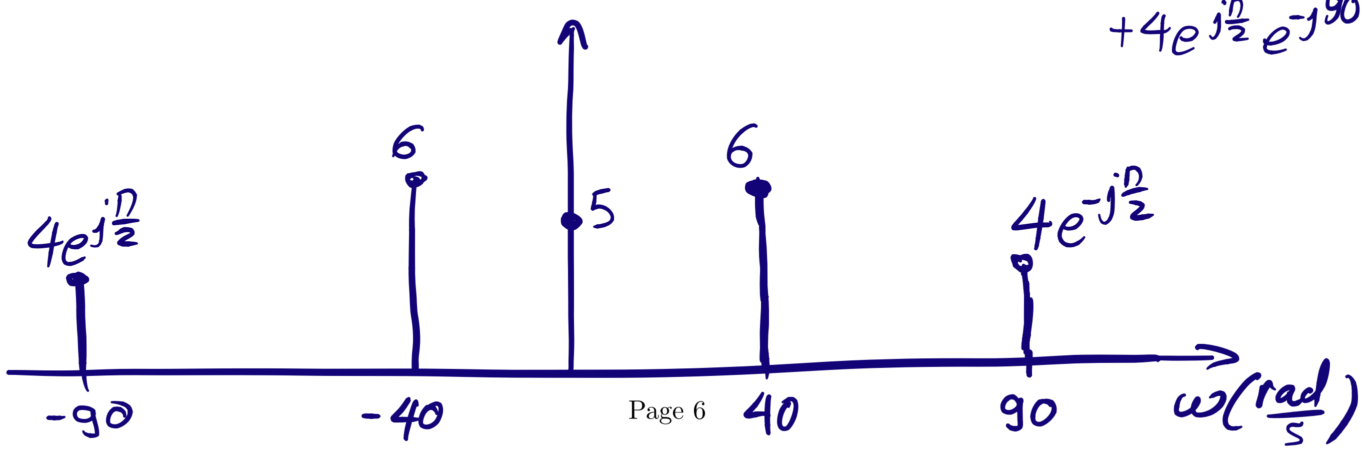
Derive and plot the **two-sided** spectrum of the signal $x(t)$.

Recall that $s(t)$ is periodic with period $T_0 = \frac{\pi}{5}$ sec

Hence $x(t) = s\left(t + \frac{\pi}{5}\right) = s(t) = 5 + 12\cos(40t) + 8\sin(90t) =$

$$= 5 + 12 \cdot \frac{1}{2}\left(e^{j40t} + e^{-j40t}\right) + 8 \cdot \frac{1}{2j}\left(e^{j90t} - e^{-j90t}\right) = 5 + 6e^{j40t} + 6e^{-j40t} - 4je^{j90t} + 4je^{-j90t}$$

$$= 5 + 6e^{j40t} + 6e^{-j40t} + 4e^{-j\frac{\pi}{2}}e^{j90t} + 4e^{j\frac{\pi}{2}}e^{-j90t}$$



3. **Problem 3 [Program it!].** For the following questions let $s(t)$ and $s[n]$ be the signals that you have worked with in **Problem 2**. Fill in the blanks (marked as "...**(BX)**...") in the following piece of code that plots the signal $s(t)$. The sampling frequency here is the same as in Problem 2(a).

   (a) (2 points)
   - ...(B1)... = numpy
   - ...(B2)... = np

   (b) (2 points)
   - ...(B3)... = 270/(2*np.pi)
   - ...(B4)... = t_max * fs

   (c) (4 points)
   - ...(B5)... = np. linspace
   - ...(B6)... = N

   (d) (3 points)
   - ...(B7)... = 5+12*np.cos(40*t)+8*np.sin(90*t)

   (e) (2 points)
   - ...(B8)... = t
   - ...(B9)... = s_t

```
In [ ]: import ...(B1)... as ...(B2)...
        # allows matplotlib charts to be displayed inside the notebook
        %matplotlib inline
        import matplotlib.pyplot as plt


        fs = ...(B3)...                 # sampling frequency (Hz)
        t_max = 0.1                     # signal duration (sec)


        N = int(...(B4)...)                      # number of samples
        t = ...(B5)...(0,t_max,...(B6)...)       # (discretized) time vector
        s_t = ...(B7)...                         # signal


        # Plot signal in time domain
        fig, ax = plt.subplots()
        ax.plot(...(B8)...,...(B9)...)
        ax.set_xlabel("t (sec)")
```

——— *End of Exam – Good luck ☺* ———

Scratch paper

Scratch paper

Scratch paper