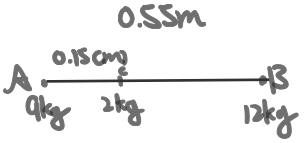
Assignment #4

Physics 2 Spring 2020
Instructor: Prof. Dirk Bouwmeester
Due: 04/26/20 5pm PST

Each problem is worth three points. Award yourself three points if you get the formula and number correct, two points if you made a numerical error, or one point if you got the wrong final equation but got the basic idea correct.

1 Particle Between Two Point Masses

A 9kg point mass and a 12kg point mass are held in place 55cm apart. A particle of mass 2kg is released from a point between the two masses 15cm from the 9kg mass and on the line connecting the two masses. Find the magnitude and direction of the acceleration of the particle.

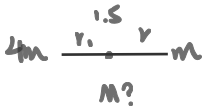


$F_1 = GMm/r^2$ $\quad F_1 + F_2 = F_R = ma$

$F_2 = GMm/r^2$ $\quad \frac{6.67}{10^8} \times 2\left(\frac{9}{0.15^2} - \frac{12}{0.4^2}\right) = a \times 2$, $a \Rightarrow 2.16715 \times 10^{-8}$, left

2 Achieving Gravitational Equilibrium

A particle of mass 4m is located 1.5m from a particle of mass m. Where should you put a third mass M so that the net gravitational force on M due to the two masses is exactly zero?



$\frac{GMm}{r^2} = \frac{GMm}{r_1^2} = 0$ $\quad r + r_1 = 1.5$ $\quad$ hence $r_1 = 1$, $r = 0.5$

$\frac{4m}{r^2} - \frac{m}{r_1^2} = 0$,

$\frac{2}{r} = \frac{1}{r_1}$

$2r_1 = r$ ✓

3 Density of a Moon

One of Saturn's moons, has a radius of 800km and an acceleration due to gravity of 0.3 m/s^2 at it's surface. Calculate (a) its mass and (b) its average density.

$\frac{GM}{r^2} \times 1kg = kg \times a$

$\frac{GM}{r^2} = 0.3$

$M = \frac{0.3 \times (800 \times 1000)^2}{6.67 \times 10^{-11}}$

$M = 2.8786 \times 10^{21}$ kg

Approximate as sphere: $V = \frac{4}{3}\pi r^3$, $\rho = \frac{m}{V}$

$M / \frac{4}{3} \times \pi \times (800 \times 1000)^3$

$= 1542$ kg/m^3 ✓

4 Mass of a Comet

On July 4, 2005, the NASA spacecraft Deep Impact fired a projectile onto the surface of Comet Tempel 1. This comet is about 10km accross. Observations of surface debris released by the impact showed that dust with a speed as low as 1.5m/s was able to escape the comet. (a) Assuming a spherical shape, what is the mass of this comet? (b) How far from the comet's center will this debris be when it has lost 80 percent of it's initial kinetic energy at the surface?

$\frac{1}{2}mv^2 = \frac{GMm}{r}$

$v = \sqrt{\frac{2GM}{R}}$

$\frac{v^2 R}{2G} = M$

$\frac{1.5^2 \times (10000 \times \frac{1}{2})}{2 \times 6.67 \times 10^{-11}} = \frac{1.6867 \times 10^{11}}{2} = 8.433 \times 10^{10}$

$\frac{1}{2}mv^2 \times (1 - 80\%) - \frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{5000}$

r ≈ ~~6250~~ 5003.9 m

typo in calculator

5 Motion of Planets

A star is 5×10^{17}m from the earth and has a mass of 5×10^{29} kg. A planet has been detected in a circular orbit around this star with an orbital radius 6×10^{8}m around the sun. (a) What is the orbital speed of this planet? (b) What is its orbital period?

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \left(\frac{4 \times \pi^2 \times (6\times10^8)^3}{6.67\times10^{-11} \times 5\times10^{29}}\right)^{\frac{1}{2}} = 15990\,s$$

$$T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} = 235766\,m/s$$

(2) $T = 15990\,s$

6 Gravitational System of Rod

A thin uniform rod has length L and mass M. A particle of mass m is placed a distance x from the end of the rod. Calculate the gravitational force of the rod on the particle.

$$\frac{GMm}{r^2}\,\frac{dl}{L} \times M = dm \qquad F = \frac{GmM}{L}\int \frac{dl}{(L+x)^2} \qquad x^{-2}$$

$$dF = \frac{G\,dm\,m}{(L+x)^2} \qquad F = \frac{GmM}{L} \times -(L+x)^{-1}\Big|_0^L$$

$$F = Gm\int \frac{dm}{(L+x)^2} \Rightarrow = \frac{-GmM}{L\times(L+x)}$$

7 Moon Orbiting Planet

Assume that a moon orbits a planet twice as massive as the Earth (5.972×10^{24}kg) in a circular orbit. From the observed orbital period of 25 days, calculate the distance of the moon from the center of the planet. Express the answer in terms of the mass of the earth.

$25 \times 24 \times 3600 = 216000\,s$ $\qquad M_E$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} \qquad \sqrt[3]{\frac{T^2 \times 6.67\times10^{-11} \times M_E}{4\times\pi^2}}$$

$$\sqrt[3]{\frac{77.7988}{\pi^2}}\,M_E$$

8 Percent Difference in Weight

Calculate the percent difference between the weight of a person of mass 70kg measured at sea level versus at a height of 9000m above sea level.

$$F = GMm/r^2$$

$$6.67\times10^{-11} \times 5.972\times10^{24} \times 70 \times \left(\frac{1}{((6371+9)\times10^3)^2} - \frac{1}{(6371\times10^3)^2}\right)$$

$$= -1.937\,N$$

1.937 N lighter $\qquad -1$

Percent

$\frac{1.937}{9.81\times10} \approx 20.2\%$