Assignment #8

Physics 2 Spring 2020
Instructor: Prof. Dirk Bouwmeester
`Due: 05/24/20 5pm PST`

Comments: Each problem is worth three points. If the problem has multiple parts the points breakdown is delineated in the problem.

1 Internal Energy

Five moles of an ideal monatomic gas with an initial temperature of 117 C expand and, in the process, absorb 1800 J of heat and do 2700 J of work. What is the final temperature of the gas?

$$1800-2700 = 1.5\times 5\times 8.31\times (T_1-T_2)$$

$$T_2 = \frac{2\times(1800-2700)}{3\times 5\times 8.314}+117, \quad T_2 = 102.56\,°c$$

2 Adiabatic Expansion

Five moles of monatomic ideal gas have initial pressure 2.70×10^3 Pa and initial volume 2.10 m^3. While undergoing an adiabatic expansion, the gas does 1380 J of work. What is the final pressure of the gas after the expansion?

$$V_2=\left(\frac{P_1V_1^{\gamma}}{P_2}\right)^{\frac{1}{\gamma}} \Rightarrow \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}V_1$$

$$P_2=\left(\frac{w(1-\gamma)+P_1V_1}{(P_1)^{\frac{1}{\gamma}}(V_1)}\right)^{\frac{\gamma}{\gamma-1}}$$

≈ 1.732 kPa

3 Basketball

A player bounces a basketball on the floor, compressing it to 75.0% of its original volume. The air (assume it is essentially N_2 gas) inside the ball is originally at 20.0 C and 2.00 atm. The ball's inside diameter is 20.0 cm. (a) What temperature does the air in the ball reach at its maximum compression? Assume the compression is adiabatic and treat the gas as ideal. (b) By how much does the internal energy of the air change between the ball's original state and its maximum compression?

(a) is 1 point, (b) is 2 points

$$V=\frac{4}{3}\pi r^3 = 4.189\times 10^{-3}$$

$$n=\frac{PV}{RT} = 0.347\text{ mol}$$

$$\Delta U = n\frac{5R}{2}\Delta T$$

$$= 257.699\text{ J}$$

$$293\times\left(\frac{1}{0.75}\right)^{0.4} = 328.73\text{ K}$$

$$\approx 55.73\,°c$$

4 A Large Mass of Air

On a warm summer day, a large mass of air (atmospheric pressure 1.01×10^5 Pa) is heated by the ground to 32.0 C and then begins to rise through the cooler surrounding air. (This can be treated approximately as an adiabatic process; why?) Calculate the temperature of the air mass when it has risen to a level at which atmospheric pressure is only 8.00×10^4 Pa. Assume that air is an ideal gas, with g = 1.40. (This rate of cooling for dry, rising air, corresponding to roughly 1 C per 100 m of altitude, is called the dry adiabatic lapse rate.)

Hot air work against gravity and expands up, so adiabatic process.

For adiabatic process,

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$$

$$\frac{PV}{T}=\frac{PV}{T} \quad (PV=nRT)$$

$$\therefore \frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = (32+273)\left(\frac{8\times10^4}{1.01\times10^5}\right)^{\frac{0.4}{1.4}}$$

$$T_2 = 285.399\text{ K}$$

$$\approx 12.35\,°c$$

5 Nitrogen Gas

Nitrogen gas in an expandable container is cooled from 45.0 C to 12.3 C with the pressure held constant at $2.58 * 10^5$ Pa. The total heat liberated by the gas is $2.58 * 10^4$ J. Assume that the gas may be treated as ideal. Find (a) the number of moles of gas; (b) the change in internal energy of the gas; (c) the work done by the gas. (d) How much heat would be liberated by the gas for the same temperature change if the volume were constant?

(a) is 1/2 point, (b) is 1/2 point, (c) is 1 point, (d) is 1 point

$n = \frac{Q}{C\Delta T}$

$= \frac{+2.58\times10^4}{29.07\times(45-12.3)}$

$= 27.14$ mol

$\Delta U = nC_v\Delta T$

$= 27.14\times20.76\,(12.3-45)$

$= -18.42$ kJ

$W = Q - \Delta U$

$= -2.58\times10^4 - (-18.42)\times10^3$

$= -7.38$ kJ

$Q = 0 + \Delta U$

$= -18.42$ kJ

6 Air Pump

An air pump has a cylinder 0.300 m long with a movable piston. The pump is used to compress air from the atmosphere (at absolute pressure $1.01 * 10^5$ Pa) into a very large tank at $3.00 * 10^5$ Pa gauge pressure. (For air, C_V = 20.8 J/mol *K.) (a) The piston begins the compression stroke at the open end of the cylinder. How far down the length of the cylinder has the piston moved when air first begins to flow from the cylinder into the tank? Assume that the compression is adiabatic. (b) If the air is taken into the pump at 30.0 C, what is the temperature of the compressed air? (c) How much work does the pump do in putting 25.0 mol of air into the tank?

(a) is 1 point, (b) is 1 point, (c) is 1 point

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$P_1(h_1A_1)^{\gamma} = P_2(h_2A_2)^{\gamma}$

$h_2 = h_1\left(\frac{P_1}{P_2}\right)^{1/\gamma}$

$= 0.3\left(\frac{1.01\times10^5}{4.01\times10^5}\right)^{\frac{1}{1.4}}$

$= 0.112$ m

$0.3-0.112 = 0.188$ m (ΔH)

$T_2 = (30+273)\left(\frac{0.3}{0.188}\right)^{0.4}$

$T_2 = 365.28$ K ≈ 92 °C

$W = nC\Delta T$

$= 25\times20.8\times(365.28-303)$

$= 32385.6$ J

7 Thermodynamic Processes

A monatomic ideal gas expands slowly to twice its original volume, doing 300 J of work in the process. Find the heat added to the gas and the change in internal energy of the gas if the process is (a) isothermal; **(b)** adiabatic; (c) isobaric.

(a) is 1 point, (b) is 1 point, (c) is 1 point

$Q = 300$ J

$\Delta U = 0$

$Q = 0$

$\Delta U = -300$ J

$Q = nC_p\Delta T$

$W = nR\Delta T$

$\Delta T = \frac{W}{nR}$

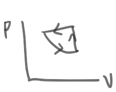
$Q = \frac{5}{2}\times300 = 750$ J

$\Delta U = 750-300 = 450$ J

8 pV Diagrams

A cylinder with a piston contains 0.159 mol of nitrogen at $1.68 * 10^5$ Pa and 320 K. The nitrogen may be treated as an ideal gas. The gas is first compressed isobarically to half its original volume. It then expands adiabatically back to its original volume, and finally it is heated isochorically to its original pressure. (a) Show the series of processes in a pV-diagram. (b) Compute the temperatures at the beginning and end of the adiabatic expansion. (c) Compute the minimum pressure.

(a) is 1 point, (b) is 1 point, (c) is 1 point



$\frac{V}{T} = \frac{nR}{P}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$T_2 = T_1\left(\frac{V_2}{V_1}\right)$

$= 160$ K

$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$

$= 160\times0.5^{0.4}$

$= 121.26$ K

$\frac{121.26}{320}\times1.68\times10^5 = 63660$ pa