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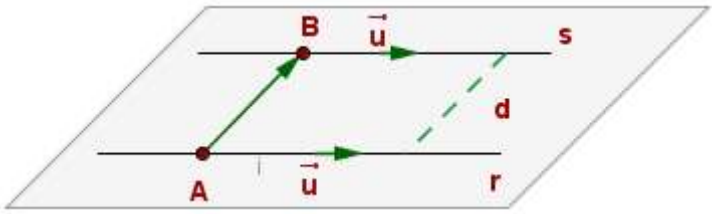
Distance Between Parallel Lines

The distance from a line, r, to another parallel line, s, is the distance from any point from r to s.

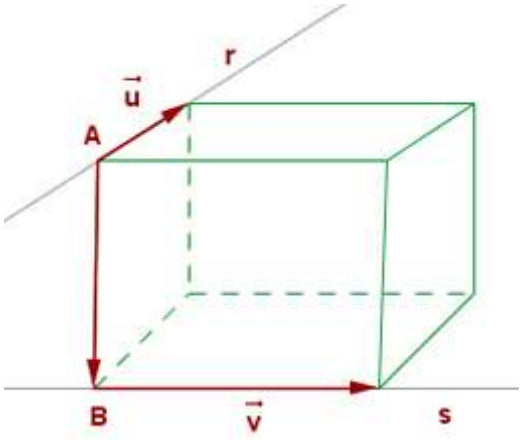


$$d(r,s) = d(A,s) = \frac{|\vec{u} \times \overrightarrow{AB}|}{|\vec{u}|}$$

Distance Between Skew Lines

The distance between skew lines is measured on the common perpendicular.

If $(A,\vec{u})$ and $(B,\vec{v})$ determine the lines r and s.



The vectors $\overrightarrow{AB}, \vec{u}$ and $\vec{v}$ determine the parallelepiped whose height is the distance between the two lines.

The volume of a parallelepiped is $V = A_b \cdot h$.

Given that the volume is the absolute value of the triple product of three vectors and the area of the base is the cross product of the direction vectors of the lines, the height is the distance between two points equal to:

$$d(r,s) = h = \frac{V}{A_b} = \frac{\left[\left[\overrightarrow{AB},\vec{u},\vec{v}\right]\right]}{\left|\vec{u}\times\vec{v}\right|}$$

Example

Find the minimum distance between the following lines:

$$r \equiv \frac{x+8}{2} = \frac{y-10}{3} = \frac{z-6}{1} \qquad s \equiv \frac{x-1}{-1} = \frac{y-1}{2} = \frac{z-1}{4}$$

$$A = (-8,10,6) \quad \vec{u} = (2,3,1)$$

$$\overrightarrow{AB} = (9,-9,-5)$$

$$B = (1,1,1) \qquad \vec{v} = (-1,2,4)$$

$$V = \left|\left[\overrightarrow{AB},\vec{u},\vec{v}\right]\right| = \begin{Vmatrix} 9 & -9 & -5 \\ 2 & 3 & 1 \\ -1 & 2 & 4 \end{Vmatrix} = 136$$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 10\vec{i} - 9\vec{j} + 7\vec{k}$$

$$A_b = \left|\vec{u}\times\vec{v}\right| = \sqrt{10^2 - 9^2 + 7^2} = \sqrt{230}$$

$$h = \frac{136}{\sqrt{230}} = \frac{68\sqrt{230}}{115}$$

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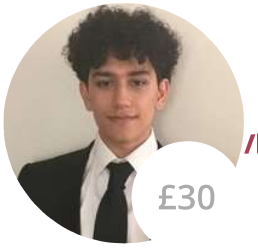

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