Назаров Рустам

Матан ИДЗ-2

М3232 Вар. 13

№1

$$\int_0^{+\infty} \frac{1-a^2x^2}{1+a^2x^2}\cdot\frac{\sin\left(\frac{1}{x}\right)}{x}\,dx =$$

$a>0$

$t=\frac{1}{x}$ $\quad dx=-\frac{dt}{t^2}$

$$= \int_0^{+\infty} \frac{1-\frac{a^2}{t^2}}{1+\frac{a^2}{t^2}}\cdot\frac{\sin(t)}{\frac{1}{t}}\cdot\left(-\frac{dt}{t^2}\right) = -\int_0^{+\infty}\frac{t^2-a^2}{t^2+a^2}\cdot\frac{\sin(t)}{t}\,dt =$$

$$= -\left(\underbrace{\int_0^{+\infty}\frac{t^2}{t^2+a^2}\,\frac{\sin t}{t}\,dt}_{\int_0^{+\infty}\frac{\sin t}{t}dt-\int_0^{+\infty}\frac{a^2}{t^2+a^2}\frac{\sin t}{t}dt} - \int_0^{+\infty}\frac{a^2}{t^2+a^2}\cdot\frac{\sin t}{t}\,dt\right) =$$

$$\int_0^{+\infty}\frac{a^2}{t^2+a^2}\cdot\frac{\sin t}{t}\,dt = \frac{\pi}{2}e^{-a} \;\star$$

Преобразование Лапласа

$$\int_0^{+\infty} f(t)g(t)\,dt = L\{f(t)\}(s)\cdot L\{g(t)\}(s)$$

$f(t)$: $L\left(\frac{\sin t}{t}\right)(s) = \mathrm{arctg}\left(\frac{1}{s}\right)$

$g(t)$: $L\left(\frac{a^2}{t^2+a^2}\right)(s) = \frac{\pi}{2}e^{-as}$

$$\int_0^{+\infty}\frac{a^2}{t^2+a^2}\,\frac{\sin t}{t}\,dt = \mathrm{arctg}\left(\frac{1}{s}\right)\cdot\frac{\pi}{2}\cdot e^{-as} =$$

$s\to 0$ $\quad \mathrm{arctg}\left(\frac{1}{0}\right)=\frac{\pi}{2}$

$$= \boxed{\frac{\pi}{2}\cdot e^{-a}}$$

(см. интеграл Лапласа)

$$= -\left(\underbrace{\int_0^{+\infty}\frac{\sin t}{t}\,dt}_{\boxed{\frac{\pi}{2}}} - 2\underbrace{\int_0^{+\infty}\frac{a^2}{t^2+a^2}\cdot\frac{\sin(t)}{t}}_{\boxed{\frac{\pi}{2}e^{-a}}\,\star}\right) =$$

$$= -\left(\frac{\pi}{2} - 2\cdot\frac{\pi}{2}e^{-a}\right) = \boxed{\pi e^{-a} - \frac{\pi}{a}}$$

№2

$$\int_0^{+\infty}\frac{\sqrt{x}\,dx}{1+x^3} = \left|\begin{matrix} x=t^{\frac{1}{3}} \\ dx=\frac{1}{3}t^{-2/3}dt\end{matrix}\right| = \int_0^{+\infty}\frac{\sqrt[6]{t}\,dt}{3\sqrt[3]{t^2}(1+t)} = \frac{1}{3}\int_0^{+\infty}\frac{t^{-\frac{1}{2}}}{1+t}\,dt = \;\Rightarrow\; x=\frac{1}{2}\;\; y=\frac{1}{2}$$

$$B(x,y) = \int_0^{+\infty}\frac{t^{x-1}\,dt}{(1+t)^{x+y}}$$

$$= \frac{1}{3}B\left(\frac{1}{2};\frac{1}{2}\right) = \frac{1}{3}\,\frac{\overbrace{\Gamma\left(\tfrac{1}{2}\right)\Gamma\left(\tfrac{1}{2}\right)}^{=\sqrt{\pi}}}{\underbrace{\Gamma(1)}_{1}} = \frac{1}{3}\cdot\sqrt{\pi}\cdot\sqrt{\pi} = \boxed{\frac{\pi}{3}}$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

№3

$$f(t) = \begin{cases}\cos(2t)-1 & t \\ 0 & t\notin[-\pi,\pi]\end{cases}$$

$$F[f][t] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{-i\xi t}\,dt = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi}\underbrace{(\cos(2t)-1)}_{-2\sin^2 t}\underbrace{e^{-i\xi t}}_{\cos\xi t-i\sin\xi t}\,dt = -\frac{2}{\sqrt{2\pi}}\int_{-\pi}^{\pi}\sin^2 t\,(\cos\xi t - i\sin\xi t)\,dt =$$

$\int_{-\pi}^{\pi}$ нечетная $=0!$

четная, четная, нечетная

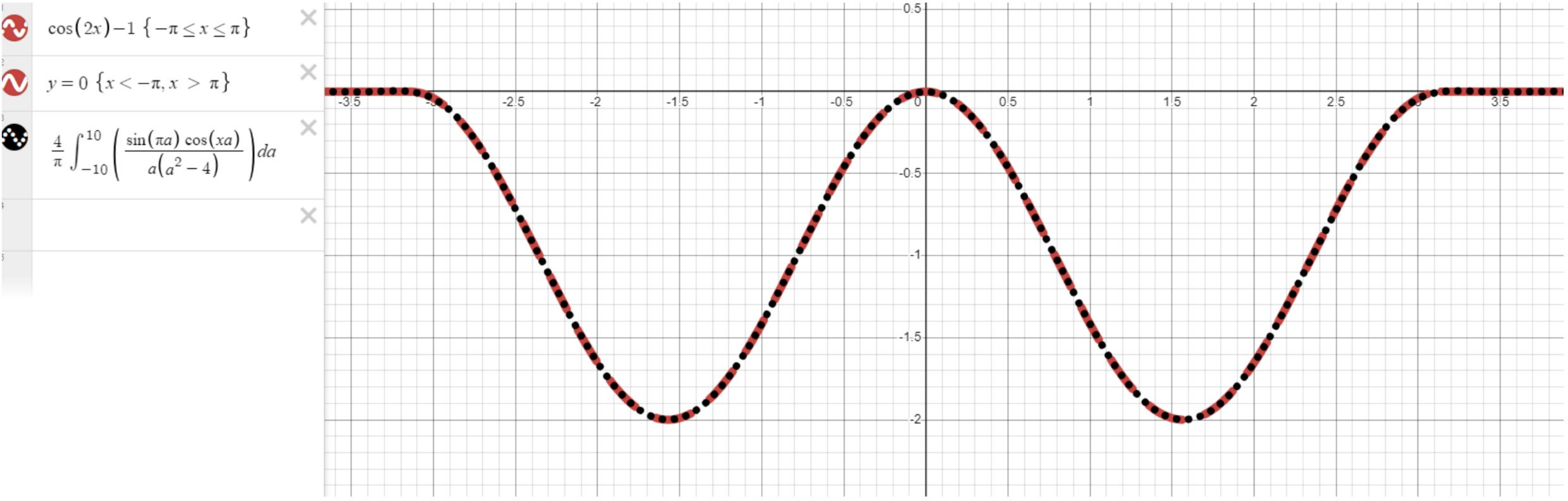
$$= -\frac{2}{\sqrt{2\pi}}\int_{-\pi}^{\pi}\sin^2 t\cdot\cos\xi t\,dt = -\frac{2}{\sqrt{2\pi}}\cdot\left(-\frac{4\sin(\pi\xi)}{\xi^3-4\xi}\right) = \boxed{\frac{4\sqrt{2}\sin(\pi\xi)}{\sqrt{\pi}\cdot\xi\cdot(\xi^2-4)}}$$

Прямое преобразов. Фурье

Обратное преобразование Фурье

$\int_{-\infty}^{\infty}$ нечет $=0$

$$F^{-1}[F[f]][t] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{4\sqrt{2}}{\sqrt{\pi}}\cdot\frac{\sin(\pi\xi)}{\xi(\xi^2-4)}\,\underbrace{e^{i\xi t}}_{\cos\xi t+i\sin\xi t}\,d\xi = \frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{\sin(\pi\xi)}{\xi(\xi^2-4)}\cos\xi t\,d\xi + \frac{4}{\pi}\int_{-\infty}^{\infty}\frac{\sin(\pi\xi)}{\xi(\xi^2-4)}\cdot i\sin\xi t\,d\xi = \boxed{\frac{4}{\pi}\int_{-\infty}^{+\infty}\frac{\sin(\pi\xi)\cdot\cos(\xi t)}{\xi(\xi^2-4)}\,d\xi}$$



N4

$$f(t)=\begin{cases} t & t\in[-a,a] \\ 0 & t\notin[-a,a]\end{cases}$$

$$\int_0^{+\infty}\big(a(\lambda)\cos(\lambda x)+b(\lambda)\sin(\lambda x)\big)d\lambda$$

$$a(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}f(y)\cos(\lambda y)dy$$

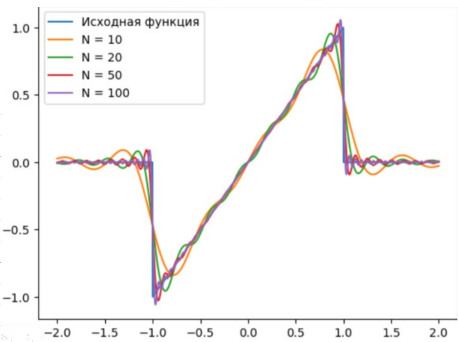
$$b(\lambda)=\frac{1}{\pi}\int_{-\infty}^{+\infty}f(y)\sin(\lambda y)dy$$

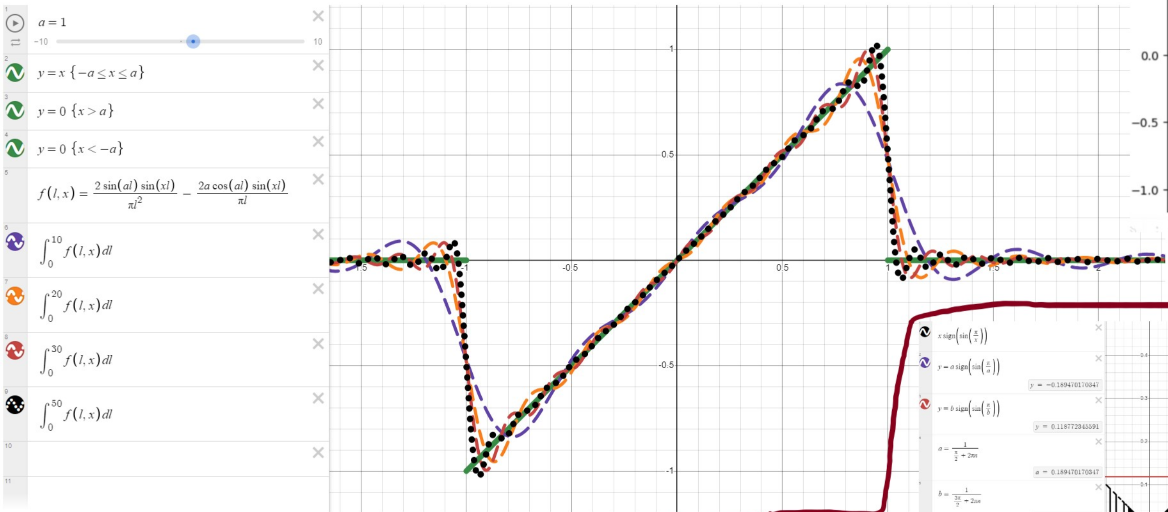
$$a(\lambda)=\frac{1}{\pi}\int_{-a}^{a}y\cos(\lambda y)dy=\frac{1}{\pi}\underbrace{\left(\frac{a\sin\lambda a}{\lambda}+\frac{\cos\lambda a}{\lambda^2}-\frac{-a\sin(-\lambda a)}{\lambda}-\frac{\cos(-\lambda a)}{\lambda^2}\right)}_{0\ (\text{Ноль})}$$

$$b(\lambda)=\frac{1}{\pi}\int_{-a}^{a}y\sin(\lambda y)dy=\frac{1}{\pi}\left(-\frac{a\cos\lambda a}{\lambda}+\frac{\sin\lambda a}{\lambda^2}+\frac{-a\cos(-\lambda a)}{\lambda}-\frac{\sin(-\lambda a)}{\lambda^2}\right)=\frac{2\sin\lambda a}{\pi\lambda^2}-\frac{2a\cos\lambda a}{\pi\lambda}$$

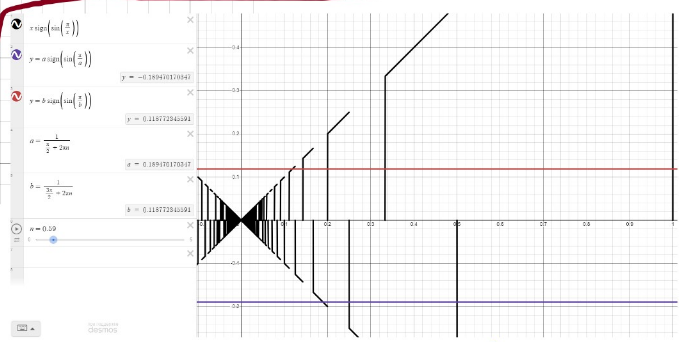
$$\boxed{\int_0^{+\infty}\frac{2\sin(a\lambda)\sin(\lambda x)}{\pi\lambda^2}-\frac{2a\cos(a\lambda)\sin(\lambda x)}{\pi\lambda}d\lambda}$$

Python↓





N5



1)

$$f(x)=\begin{cases}2 \\ x\,\mathrm{sign}\left(\sin\left(\frac{\pi}{x}\right)\right), & 0<x\le 1\end{cases}$$

$$\boxed{\begin{aligned}x_n&=\frac{1}{\frac{\pi}{2}+2\pi n}\\ x_{n+1}&=\frac{1}{3\frac{\pi}{2}+2\pi n}\end{aligned}}$$

$\sin\left(\frac{\pi}{x_n}\right)=1$

$\sin\left(\frac{\pi}{x_{n+1}}\right)=-1$

$$|f(x_n)-f(x_{n+1})|=\left|\frac{1}{\frac{\pi}{2}+2\pi n}+\frac{1}{3\frac{\pi}{2}+2\pi n}\right|\geq\frac{1}{\frac{\pi}{2}+2\pi n}$$

$$V_0^1(f)\geq\sum_{n=1}^{\infty}\frac{1}{\frac{\pi}{2}+2\pi n}\sim\frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}=\infty$$ (гармонич. ряд)

$\measuredangle\{0\}$

$|f(0)-\lim_{x\to 0}f(x)|=|2-0|=2$

$$V_0^1(f)=2+\infty=\infty$$

2) Невозможно, ф-я не монотонна, бесконечное число скачков на интервале [0, 1]. скачки $x\leftrightarrow -x$, беск. осциллирует.