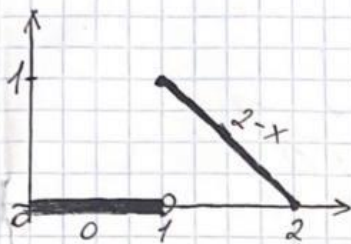


Лабораторная работа №2
Назаров Рустам М3132 Вариант 23

Лабораторная - 2
Вариант 23
Назаров Рустам М3132

$$f(x) = \begin{cases} 0, & x \in [0; 1) \\ 2-x, & x \in [1; 2] \end{cases}$$



$$[0; T] \quad T = 2 \quad l = \frac{T}{2} = 1$$

$\tilde{f}(x)$ - периодическое продолжение
 $f(x)$ на \mathbb{R} с $T = 2$

1) общий тригонометрический ряд

$$\begin{aligned} \tilde{f}(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\tilde{\omega}_n x}{l}\right) + b_n \sin\left(\frac{\tilde{\omega}_n x}{l}\right) = \\ &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\tilde{\omega}_n x) + b_n \sin(\tilde{\omega}_n x) = S(x) \end{aligned}$$

$$a_n = \frac{1}{l} \int_0^T \tilde{f}(x) \cos(\tilde{\omega}_n x) dx$$

$$a_0 = \int_0^2 \tilde{f}(x) \cdot \cos(0) dx = \int_0^2 \tilde{f}(x) dx = \frac{1}{2}$$

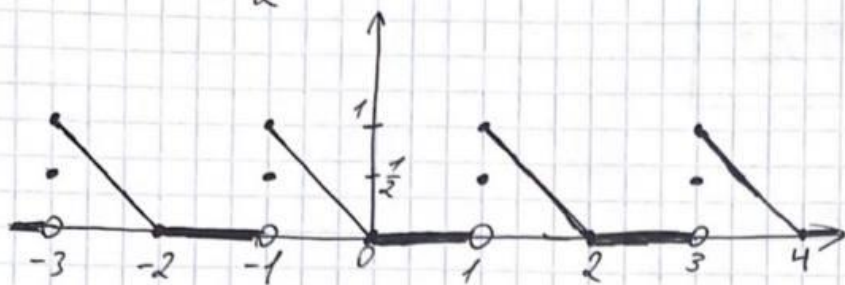
$$\begin{aligned} a_n &= \int_0^2 \tilde{f}(x) \cos(\tilde{\omega}_n x) dx = \int_0^1 0 \cdot \cos(\tilde{\omega}_n x) dx + \int_1^2 (2-x) \cos(\tilde{\omega}_n x) dx = \\ &= \left(\frac{2 \sin(\tilde{\omega}_n x)}{\tilde{\omega}_n} - \frac{x \sin(\tilde{\omega}_n x)}{\tilde{\omega}_n} - \frac{\cos(\tilde{\omega}_n x)}{\tilde{\omega}^2 n^2} \right) \Big|_1^2 = \\ &= \frac{-\cos(2\tilde{\omega}_n) - \tilde{\omega}_n \sin(\tilde{\omega}_n) + \cos(\tilde{\omega}_n)}{\tilde{\omega}^2 n^2} = \boxed{\frac{-1 + (-1)^n}{\tilde{\omega}^2 n^2}} \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{l} \int_0^T \tilde{f}(x) \sin(\tilde{\omega}_n x) dx = \int_1^2 (2-x) \sin(\tilde{\omega}_n x) dx = \\ &= \left(-\frac{2 \cos(\tilde{\omega}_n x)}{\tilde{\omega}_n} + \frac{x \cos(\tilde{\omega}_n x)}{\tilde{\omega}_n} - \frac{\sin(\tilde{\omega}_n x)}{\tilde{\omega}^2 n^2} \right) \Big|_1^2 = \\ &= \frac{-\sin(2\tilde{\omega}_n) + \tilde{\omega}_n \cos(\tilde{\omega}_n) + \sin(\tilde{\omega}_n)}{\tilde{\omega}^2 n^2} = \frac{\tilde{\omega}_n (-1)^n}{\tilde{\omega}^2 n^2} = \boxed{\frac{(-1)^n}{\tilde{\omega}_n}} \end{aligned}$$

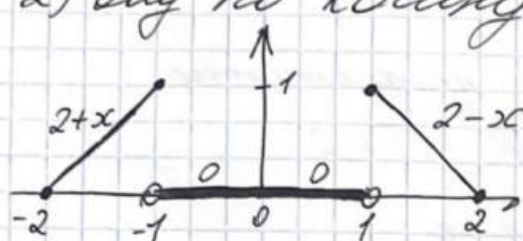
$$S(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\tilde{\omega}^2 n^2} \cos(\tilde{\omega}_n x) + \frac{(-1)^n}{\tilde{\omega}_n} \sin(\tilde{\omega}_n x)$$

$$S(x) = \begin{cases} \tilde{f}(x) & x \neq 2k+1 \\ \frac{1}{2} & x = 2k+1 \end{cases} \quad k \in \mathbb{Z}$$

Продолжаем функцию
четным образом



2) Раз по косинусу



$$f(x) = \begin{cases} 2+x & x \in [-2; -1] \\ 0 & x \in [-1; 1] \\ 2-x & x \in [1; 2] \end{cases}$$

$$T=4 \quad l=2 \quad \left(\frac{T}{2}\right)$$

$$\tilde{f}(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\sqrt{5}}{2} nx\right) = S(x)$$

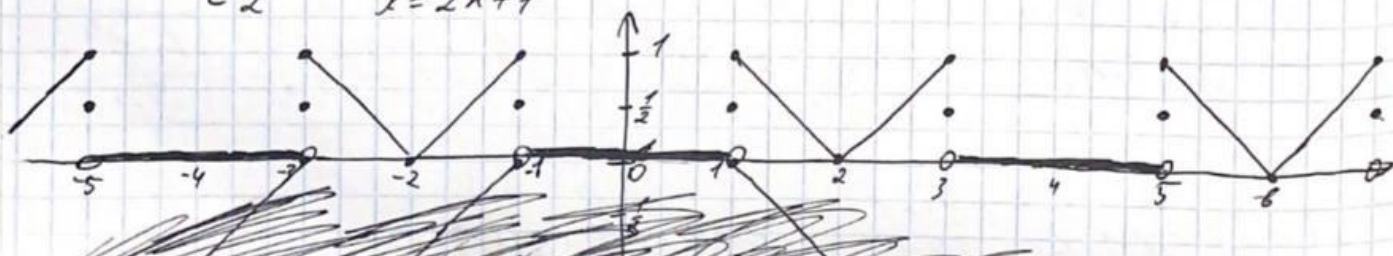
$$a_n = \frac{1}{l} \int_{-l}^l \tilde{f}(x) \cos\left(\frac{\sqrt{5}}{2} nx\right) dx$$

$$a_0 = \frac{1}{2} \int_{-2}^2 \tilde{f}(x) dx = \frac{1}{2} \int_0^2 \tilde{f}(x) dx = \frac{1}{2}$$

$$\begin{aligned} a_n &= \frac{1}{2} \int_{-2}^2 \tilde{f}(x) \cos\left(\frac{\sqrt{5}}{2} nx\right) dx = \frac{1}{2} \int_{-2}^{-1} (2+x) \cos\left(\frac{\sqrt{5}}{2} nx\right) dx + \frac{1}{2} \int_1^2 (2-x) \cos\left(\frac{\sqrt{5}}{2} nx\right) dx \\ &= -\frac{\sin(\frac{\sqrt{5}n}{2})}{\sqrt{5}n} + \frac{2\cos(\frac{\sqrt{5}n}{2})}{\sqrt{5}^2 n^2} - \frac{2\cos(\sqrt{5}n)}{\sqrt{5}^3 n^3} - \frac{\sin(\frac{\sqrt{5}n}{2})}{\sqrt{5}n} + \frac{2\cos(\frac{\sqrt{5}n}{2})}{\sqrt{5}^3 n^3} - \frac{2\cos(\sqrt{5}n)}{\sqrt{5}^2 n^2} \\ &= \frac{-2\sqrt{5}n \sin(\frac{\sqrt{5}n}{2}) + 4\cos(\frac{\sqrt{5}n}{2}) - 4\cos(\sqrt{5}n)}{\sqrt{5}^2 n^2} \\ &= \frac{-2\sqrt{5}n \sin(\frac{\sqrt{5}n}{2}) + 4\cos(\frac{\sqrt{5}n}{2}) - 4 \cdot (-1)^n}{\sqrt{5}^2 n^2} \end{aligned}$$

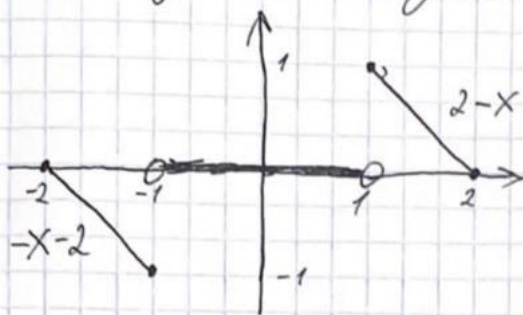
$$S(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{-2\sqrt{5}n \sin(\frac{\sqrt{5}n}{2}) + 4\cos(\frac{\sqrt{5}n}{2}) - 4 \cdot (-1)^n}{\sqrt{5}^2 n^2} \cdot \cos\left(\frac{\sqrt{5}}{2} nx\right)$$

$$S(x) = \begin{cases} \tilde{f}(x) & x \neq 2k+1 \\ \frac{1}{2} & x = 2k+1 \end{cases} \quad k \in \mathbb{Z}$$



3) Заг. на функцията

$$f(x) = \begin{cases} -x-2 & x \in [-2; -1] \\ 0 & x \in (-1; 1) \\ 2-x & x \in [1; 2] \end{cases}$$



$$T=4 \quad l=2$$

$$\tilde{f}(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{\sqrt{\omega}}{2} nx\right) = S(x)$$

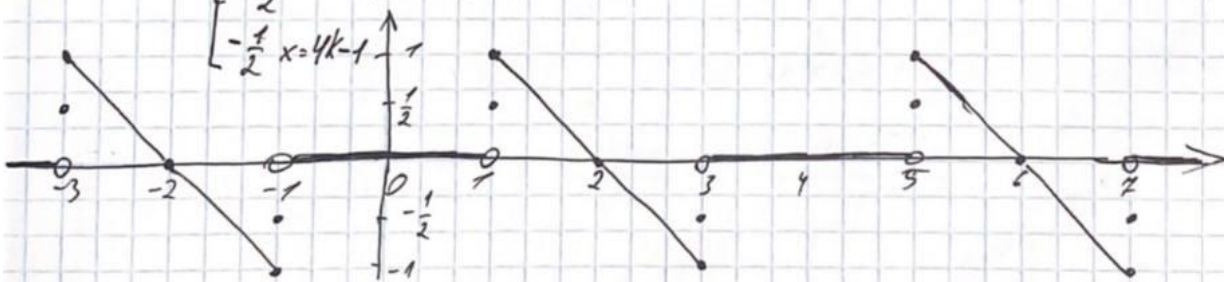
$$b_n = \frac{1}{l} \int_{-l}^l \tilde{f}(x) \sin\left(\frac{\sqrt{\omega}}{2} nx\right) dx = \frac{1}{2} \int_{-2}^{-1} (-x-2) \sin\left(\frac{\sqrt{\omega}}{2} nx\right) dx + \frac{1}{2} \int_1^2 (2-x) \sin\left(\frac{\sqrt{\omega}}{2} nx\right) dx$$

$$= \frac{4 \sin\left(\frac{\sqrt{\omega}}{2} n\right)}{2 \sqrt{\omega}^2 n^2} + \frac{2 \cos\left(\frac{\sqrt{\omega}}{2} n\right)}{2 \sqrt{\omega} n} + \frac{4 \sin\left(\frac{\sqrt{\omega}}{2} n\right)}{2 \sqrt{\omega}^2 n^2} + \frac{2 \cos\left(\frac{\sqrt{\omega}}{2} n\right)}{2 \sqrt{\omega} n} =$$

$$= \frac{4 \sin\left(\frac{\sqrt{\omega}}{2} n\right) + 2 \sqrt{\omega} n \cos\left(\frac{\sqrt{\omega}}{2} n\right)}{\sqrt{\omega}^2 n^2}$$

$$S(x) = \sum_{n=1}^{\infty} \frac{4 \sin\left(\frac{\sqrt{\omega}}{2} n\right) + 2 \sqrt{\omega} n \cos\left(\frac{\sqrt{\omega}}{2} n\right)}{\sqrt{\omega}^2 n^2} \sin\left(\frac{\sqrt{\omega}}{2} nx\right)$$

$$S(x) = \begin{cases} \tilde{f}(x) & x \neq 2k-1 \\ \frac{1}{2} & x = 4k+1 \\ -\frac{1}{2} & x = 4k-1 \end{cases} \quad k \in \mathbb{Z}$$



Я выбрал $1 \leq k \leq 200$ шаг 2

1) Общий тригонометрический ряд

2

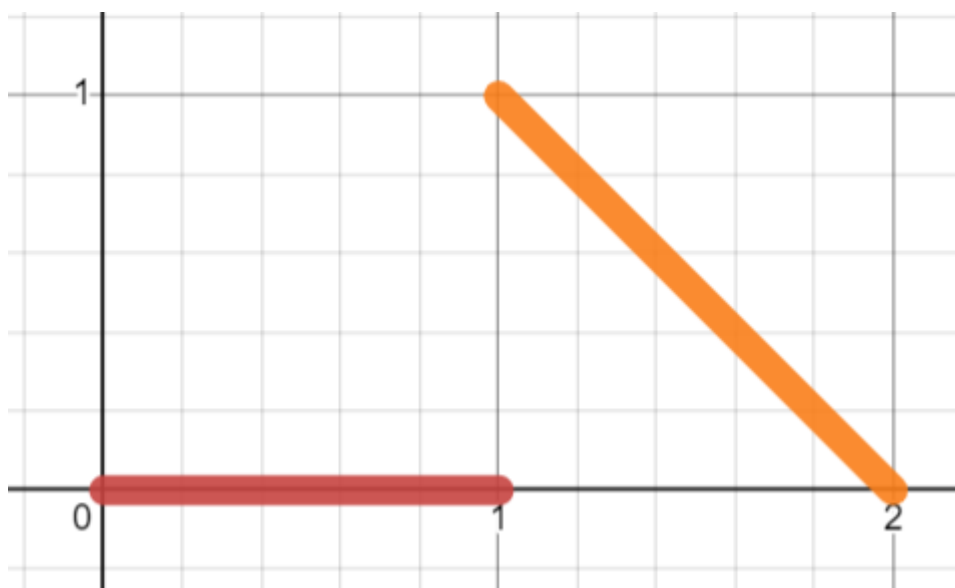
$f(x) = 2 - x \{ 1 \leq x \leq 2 \}$

×

3

$f(x) = 0 \{ 0 \leq x < 1 \}$

×

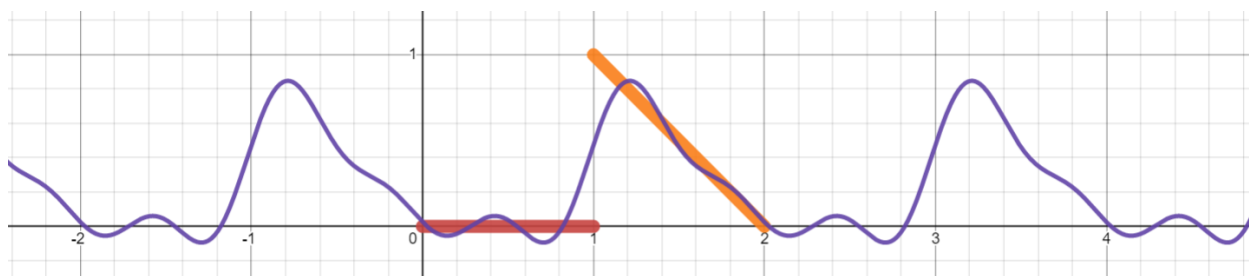


4

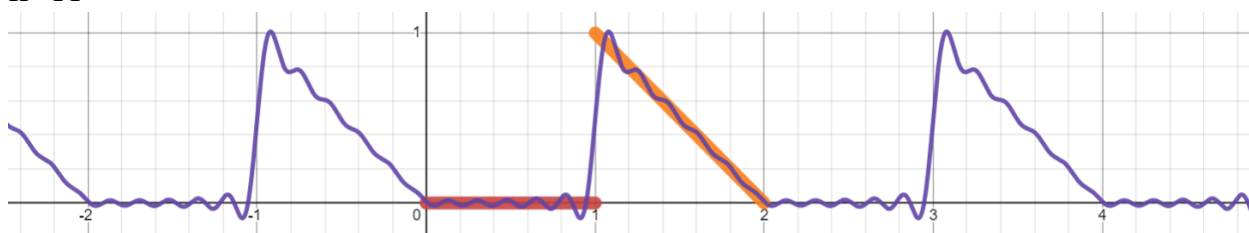
$$S(x) = \frac{1}{4} + \sum_{n=1}^k \left(\frac{((-1)^n - 1)}{\pi^2 n^2} \cos(\pi n x) + \frac{(-1)^n}{\pi n} \sin(\pi n x) \right)$$

×

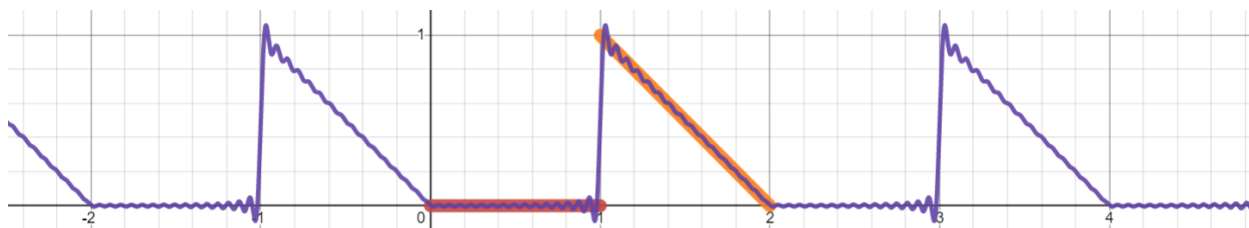
k=3






k=11

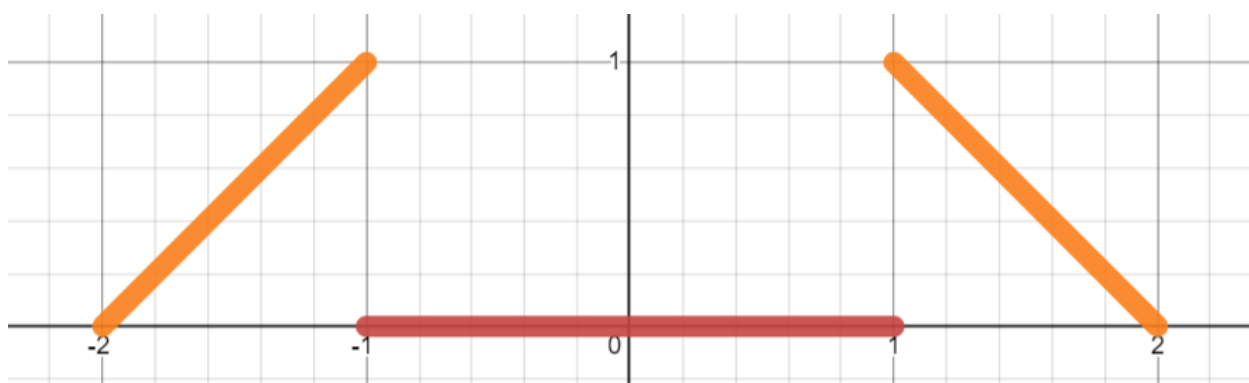


k=31




2) Ряд по косинусам

- | | | |
|---|---|--|
| 5 |  | $f(x) = 2 + x \quad \{-2 \leq x \leq -1\}$ |
| 6 |  | $f(x) = 0 \quad \{-1 < x < 1\}$ |
| 7 |  | $f(x) = 2 - x \quad \{1 \leq x \leq 2\}$ |

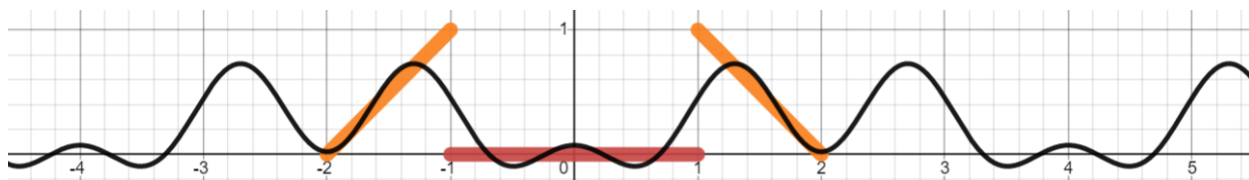


8

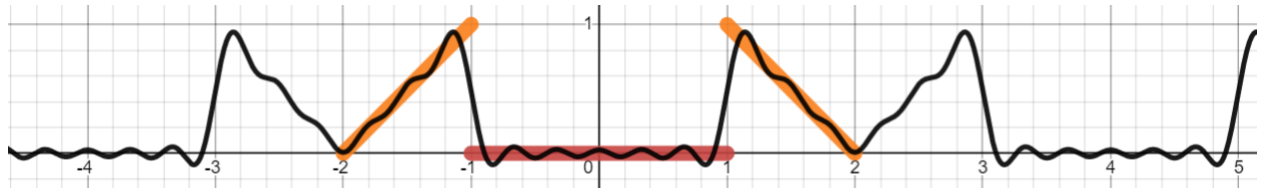


$$S(x) = \frac{1}{4} + \sum_{n=1}^k \left(\frac{-2\pi n \sin\left(\frac{\pi n}{2}\right) - 4(-1)^n + 4\left(\cos\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n^2} \cos\left(\frac{\pi}{2} nx\right) \right)$$

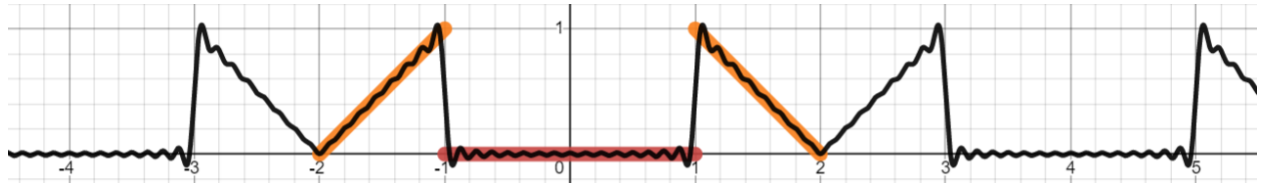
K=3



K=11



K=31



3) Ряд по синусам

9



$$f(x) = -x - 2 \quad \{-2 \leq x \leq -1\}$$

10

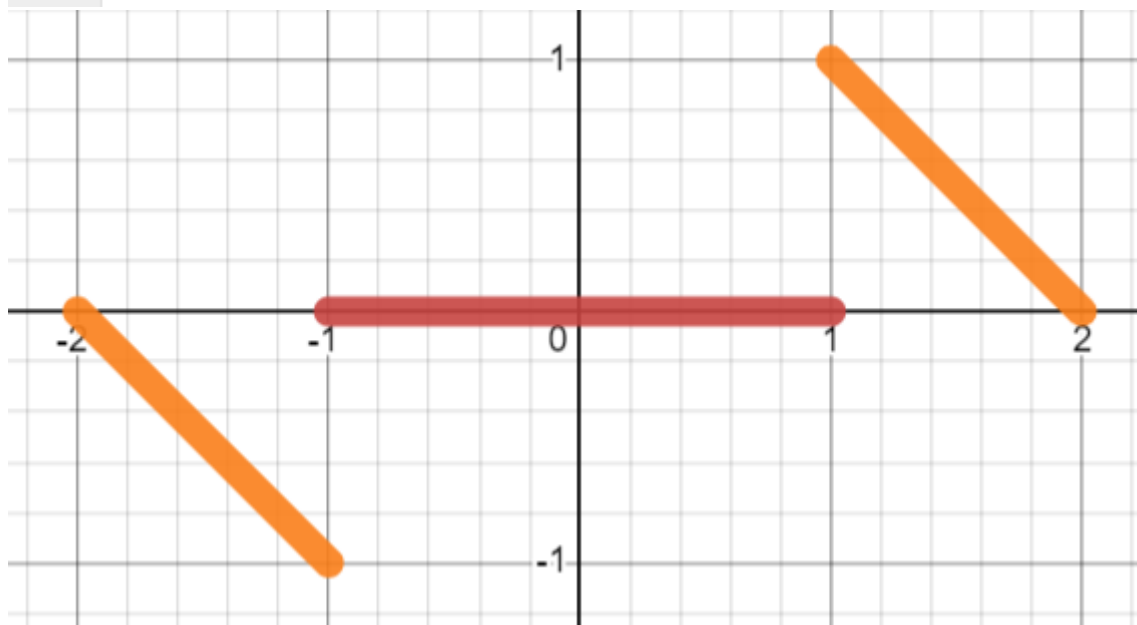


$$f(x) = 0 \quad \{-1 < x < 1\}$$

11



$$f(x) = 2 - x \quad \{1 \leq x \leq 2\}$$

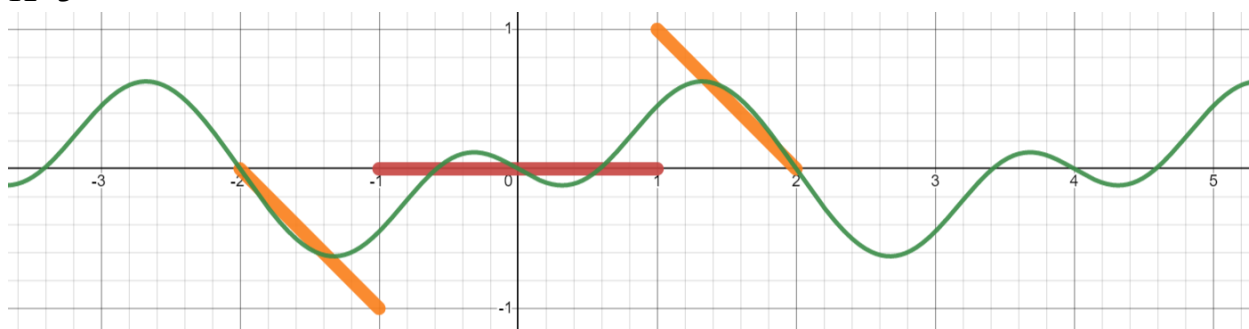


12



$$S(x) = \sum_{n=1}^k \frac{\left(2\pi n \cos\left(\frac{\pi n}{2}\right) + 4 \sin\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n^2} \sin\left(\frac{\pi}{2} nx\right)$$

$K=3$



$K=11$



$K=31$



Ссылка:

<https://www.desmos.com/calculator/dkbd0qjnhv>