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$$\begin{aligned}
 & 1) a \in \mathbb{R}^n, x \in \mathbb{R}^n \\
 & a^T x: \mathbb{R}^n \rightarrow \mathbb{R}^1 \Rightarrow \frac{\partial a^T x}{\partial x} = \nabla_x (a^T x) \\
 & a^T x = (a_1 \ a_2 \ \dots \ a_n) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \sum_{i=1}^n a_i x_i \\
 & \frac{\partial a^T x}{\partial x} = \frac{\partial (\sum_{i=1}^n a_i x_i)}{\partial x} = \begin{pmatrix} \frac{\partial a_1 x_1}{\partial x_1} \\ \frac{\partial a_2 x_2}{\partial x_2} \\ \vdots \\ \frac{\partial a_n x_n}{\partial x_n} \end{pmatrix} = a
 \end{aligned}$$

$$\begin{aligned}
 & 2) A \in \mathbb{R}^{m \times n}, x \in \mathbb{R}^n \\
 & Ax: \mathbb{R}^n \rightarrow \mathbb{R}^m \\
 & \frac{\partial Ax}{\partial x} = \begin{pmatrix} \frac{\partial (\sum_{i=1}^n a_{1i} x_i)}{\partial x_1} & \frac{\partial (\sum_{i=1}^n a_{1i} x_i)}{\partial x_2} & \dots & \frac{\partial (\sum_{i=1}^n a_{1i} x_i)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial (\sum_{i=1}^n a_{mi} x_i)}{\partial x_1} & \frac{\partial (\sum_{i=1}^n a_{mi} x_i)}{\partial x_2} & \dots & \frac{\partial (\sum_{i=1}^n a_{mi} x_i)}{\partial x_n} \end{pmatrix} = \\
 & = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} = A
 \end{aligned}$$

$$3) A \in \mathbb{R}^{n \times n}, x \in \mathbb{R}^n$$

$$x^T A x = \sum_{i=1}^n x_i \sum_{j=1}^n a_{ij} x_j$$

$$\sum_{i=1}^n x_i \sum_{j=1}^n a_{ij} x_j = \frac{\partial (a_{kk} x_k^2 + \sum_{i=1, i \neq k}^n x_i \cdot \sum_{j=1}^n a_{ij} x_j)}{\partial x_k}$$

$$= \frac{\sum_{j=1}^n a_{kj} x_j + \sum_{i=1}^n x_i \sum_{j=1, j \neq k}^n a_{ij} x_j}{\partial x_k} =$$

$$= 2a_{kk} x_k + \sum_{\substack{i=1 \\ i \neq k}}^n a_{ik} x_k + \sum_{\substack{j=1 \\ j \neq k}}^n a_{kj} x_k =$$

$$= \sum_{i=1}^n a_{ik} x_k + \sum_{j=1}^n a_{kj} x_k = \sum_{i=1}^n x_k (a_{ik} + a_{ki})$$

$$\frac{\partial (x^T A x)}{\partial x} = \begin{pmatrix} \sum_{i=1}^n x_i (a_{i1} + a_{1i}) \\ \vdots \\ \sum_{i=1}^n x_i (a_{in} + a_{ni}) \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} + a_{11} & a_{21} + a_{12} & \dots & a_{n1} + a_{1n} \\ \vdots & \ddots & \ddots & \vdots \\ a_{n1} + a_{1n} & \dots & \dots & a_{nn} + a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} =$$

$$= (A + A^T) x$$

$$\text{since } A = A^T \rightarrow \frac{\partial (x^T A x)}{\partial x} = 2Ax$$

$$4) x \in \mathbb{R}^n$$

$$\|x\|^2 = x^T x = \sum_{i=1}^n x_i^2$$

$$\frac{\partial \|x\|^2}{\partial x} = \begin{pmatrix} \frac{\partial \sum_{i=1}^n x_i^2}{\partial x} \\ \vdots \\ \frac{\partial \sum_{i=1}^n x_i^2}{\partial x} \end{pmatrix} = 2x$$

$$5) \frac{\partial g(x)}{\partial x} = \begin{pmatrix} \frac{\partial g(x_1)}{\partial x_1} & \frac{\partial g(x_2)}{\partial x_1} & \dots & \frac{\partial g(x_r)}{\partial x_1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g(x_1)}{\partial x_r} & \dots & \dots & \frac{\partial g(x_r)}{\partial x_r} \end{pmatrix} =$$

$$= \begin{pmatrix} g'(x_1) & 0 & \dots & 0 \\ \vdots & g'(x_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & g'(x_r) \end{pmatrix} = J \circ g(g'(x))$$

$$6) x \in \mathbb{R}^n; h: \mathbb{R}^n \rightarrow \mathbb{R}^m; g: \mathbb{R}^m \rightarrow \mathbb{R}^p$$

$$\frac{\partial g(h(x))}{\partial x} = \begin{pmatrix} \frac{\partial g_1(h(x))}{\partial x_1} & \dots & \frac{\partial g_1(h(x))}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p(h(x))}{\partial x_1} & \dots & \frac{\partial g_p(h(x))}{\partial x_n} \end{pmatrix} \quad (*)$$

$$(*) \frac{\partial g_k(h(x))}{\partial x_i} = \frac{\partial g_k(h(x))}{\partial h_1} \cdot \frac{\partial h_1(x)}{\partial x_i} + \frac{\partial g_k(h(x))}{\partial h_2} \cdot \frac{\partial h_2(x)}{\partial x_i} + \dots + \frac{\partial g_k(h(x))}{\partial h_m} \cdot \frac{\partial h_m(x)}{\partial x_i}$$

$$(*) \quad \frac{\partial g_k(h(x))}{\partial x_i} = \begin{pmatrix} \frac{\partial g_k(h(x))}{\partial h_1} & \frac{\partial g_k(h(x))}{\partial h_2} & \dots & \frac{\partial g_k(h(x))}{\partial h_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_p(h(x))}{\partial h_1} & \frac{\partial g_p(h(x))}{\partial h_2} & \dots & \frac{\partial g_p(h(x))}{\partial h_m} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial h_1(x)}{\partial x_1} & \dots & \frac{\partial h_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m(x)}{\partial x_1} & \dots & \frac{\partial h_m(x)}{\partial x_n} \end{pmatrix} = \frac{\partial g(h(x))}{\partial h} \cdot \frac{\partial h(x)}{\partial x}$$

3.

X	1	1	0	0	-1
y	4	4	0	2	6

$$2) X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & -1 \end{pmatrix} \quad y = \begin{pmatrix} 4 \\ 4 \\ 0 \\ 2 \\ 6 \end{pmatrix}$$

$$X^T X = \begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix}$$

$$X^T y = \begin{pmatrix} 16 \\ 2 \\ 14 \end{pmatrix}$$

$$X^T X \beta = X^T y$$

$$\begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ 14 \end{pmatrix} \Rightarrow \beta = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

$$f(x) = 1 - x + 4x^2$$

3) $\lambda = 1$

$$X^T X + \lambda I = \begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 1 & 3 \\ 1 & 4 & 1 \\ 3 & 1 & 4 \end{pmatrix}$$

$$(X^T X + \lambda I) \beta = X^T y$$

$$\begin{pmatrix} 6 & 1 & 3 \\ 1 & 4 & 1 \\ 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 7.6 \\ 2 \\ 14 \end{pmatrix} \Rightarrow \beta = \begin{pmatrix} 1.5 \\ -0.5 \\ 2.5 \end{pmatrix}$$

$$f(x) = 1.5 - 0.5x + 2.5x^2$$

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X_1	0	1	0	2	2	2	4	3
X_2	-1	0	0	0	1	0	1	2
y	0	0	0	0	0	1	1	1

$$1) \hat{\mu}_0 \{y=0\} = \frac{5}{8} \quad \hat{\mu}_1 \{y=1\} = \frac{3}{8}$$

$$\hat{\mu}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{\mu}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\sum_{i=0}^1 \frac{1}{n_i - 1} \sum (x_i - \hat{\mu}_0) (x_i - \hat{\mu}_0)^T = \frac{1}{4} \left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 0 + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 6 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right) = \frac{1}{4} \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

$$\sum_1 = \frac{1}{2} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$

$$\sum = \frac{1}{6} \begin{pmatrix} 6 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{2}{3} \end{pmatrix}$$

$$\sum_0^{-1} = \begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix}, \quad \sum_1^{-1} = \begin{pmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{pmatrix}$$

$$\sum^{-1} = \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix}$$

$$\begin{aligned} 2) \quad \mathcal{J}_0(x) &= (x_1 \ x_2) \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \\ &= \frac{1}{2} (1 \ 0) \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \ln \frac{5}{8} = \\ &= \left(\frac{8}{5} x_1 - \frac{6}{5} x_2 \right) - \frac{4}{5} + \ln \frac{5}{8} \end{aligned}$$

$$\mathcal{J}_1(x) = \left(\frac{18}{5} x_1 - \frac{6}{5} x_2 \right) - \frac{24}{5} + \ln \frac{3}{8}$$

Разделяющая пов-ть

$$\begin{aligned} &= -\frac{10}{5} x_1 + \frac{20}{5} + \ln 5 - \ln 8 - \ln 3 + \ln 8 = \\ &= -2x_1 + 4 + \ln 5 - \ln 3 = 0 \end{aligned}$$

Квадр. функ. оп-ии

$$\begin{aligned} \mathcal{J}_0(x) &= -\frac{1}{2} \ln\left(\frac{1}{4}\right) - x_1^2 + 2x_1 + 2x_1x_2 - 2x_2^2 - 2x_2 - \\ &= -1 + \ln \frac{5}{8} = -x_1^2 - 2x_2^2 + 2x_1x_2 + 2x_1 - 2x_2 - 1 + \\ &+ \frac{\ln(4)}{2} + \ln \frac{5}{8} \end{aligned}$$

$$\begin{aligned} \mathcal{J}_1(x) &= -\frac{1}{2} \ln \frac{3}{4} - \frac{1}{3} (2x_1^2 + 2x_2^2 - 2x_1x_2 - 10x_1 + \\ &+ 2x_2 + 14) + \ln \frac{3}{8} \end{aligned}$$

Разделяющая пов-ть

$$x_1^2 + x_1(4 - 4x_2) + 4x_2^2 + 4x_2 - 11 + \frac{3}{2} \ln 3 +$$

$$+ 3 \ln 5 = 0$$

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x_1	0	0	1	1	0	0	1	1	1	0
x_2	0	1	0	1	1	1	1	1	1	1
y	0	0	0	0	0	1	1	1	1	1

$$Pr(y=0 | x_1=1, x_2=1) = \frac{Pr(x_1=1 | y=0) \cdot Pr(x_2=1 | y=0) \cdot Pr(y=0)}{Pr(x_1=1, x_2=1)}$$

$$Pr(x_1=1 | y=0) \cdot Pr(x_2=1 | y=0) \cdot Pr(y=0)$$

$$Pr(y=1 | x_1=1, x_2=1) = \frac{Pr(x_1=1 | y=1) \cdot Pr(x_2=1 | y=1) \cdot Pr(y=1)}{Pr(x_1=1, x_2=1)}$$

$$Pr(x_1=1 | y=1) \cdot Pr(x_2=1 | y=1) \cdot Pr(y=1)$$

$$\hat{Pr}(y=0) = \frac{1}{2} ; \hat{Pr}(y=1) = \frac{1}{2} \quad \text{— anp. e ber-mu}$$

$$\hat{Pr}(x_1=0 | y=0) = \frac{3}{5} ; \hat{Pr}(x_1=1 | y=0) = \frac{2}{5} \quad \left. \begin{array}{l} \hat{Pr}(x_1=0 | y=1) = \frac{2}{5} ; \hat{Pr}(x_1=1 | y=1) = \frac{3}{5} \end{array} \right\} \text{yem - e ber-mu}$$

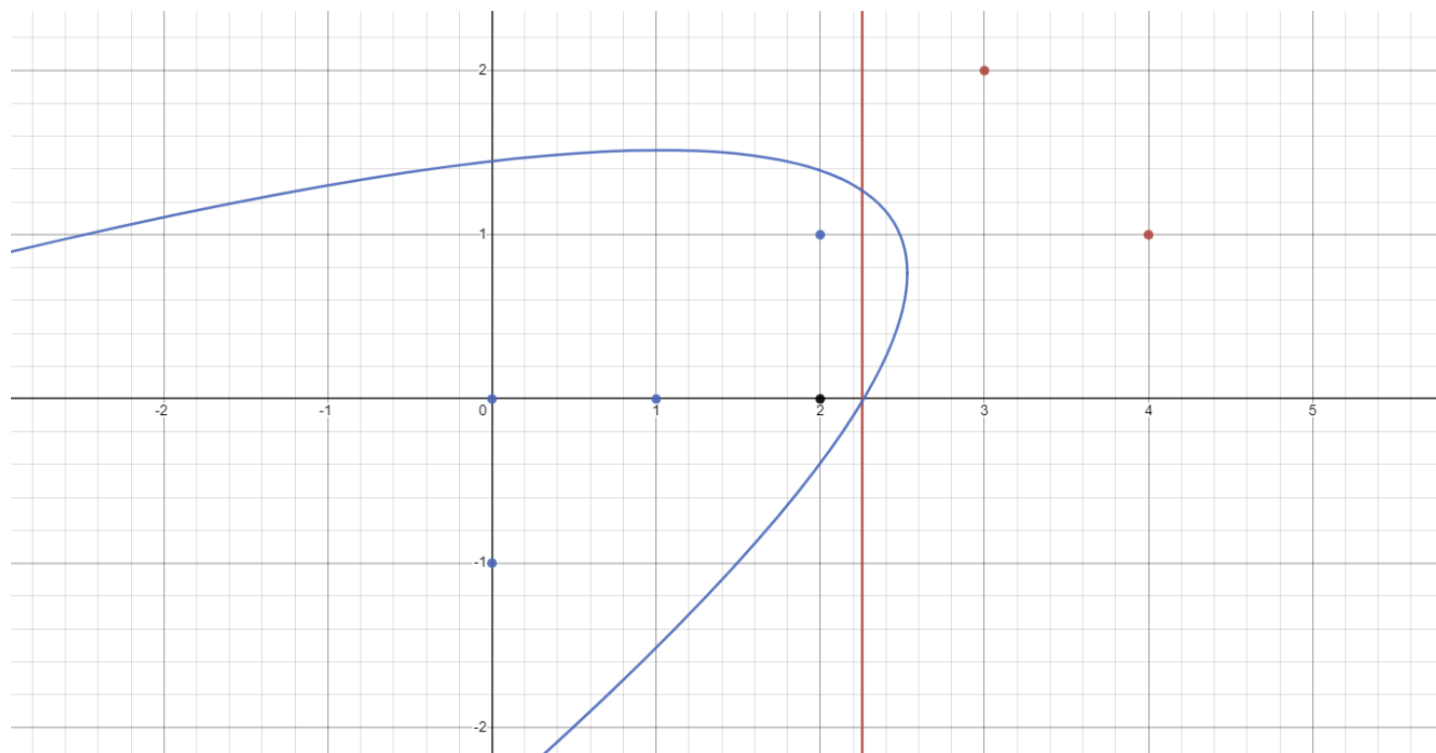
$$\hat{Pr}(x_2=0 | y=0) = \frac{2}{5} ; \hat{Pr}(x_2=1 | y=0) = \frac{3}{5}$$

$$\hat{Pr}(x_2=0 | y=1) = 0 ; \hat{Pr}(x_2=1 | y=1) = 1$$

$$Pr(y=0 | x_1=1, x_2=1) = \frac{4/5 - 3/5 \cdot \frac{1}{2}}{4/50 + 3/10} = \frac{2}{7}$$

$$Pr(y=1 | x_1=1, x_2=1) = \frac{3/5 \cdot 1 - 3/5}{4/50 + 3/10} = \frac{5}{7}$$

Приложение к №9



Приложение к № 15

