Рустамов

$\boxed{1.}$

1) $a \in R^N, x \in R^N$

$a^T x : R^N \to R^1 \Rightarrow \frac{\partial a^T x}{\partial x} = \nabla_x (a^T x)$

$a^T x = (a_1 \ a_2 \ \dots \ a_N) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} = \sum_{i=1}^{N} a_i x_i$

$$\frac{\partial a^T x}{\partial x} = \frac{\partial(\sum_{i=1}^{N} a_i x_i)}{\partial x} = \begin{pmatrix} \frac{\partial a_1 x_1}{\partial x_1} \\ \frac{\partial a_2 x_2}{\partial x_2} \\ \vdots \\ \frac{\partial a_N x_N}{\partial x_N} \end{pmatrix} = a$$

2) $A \in R^{m \times n}, x \in R^n$

$Ax : R^{m \times n} \to R^m$

$$\frac{\partial Ax}{\partial x} = \begin{pmatrix} \frac{\partial(\sum_{i=1}^{N} a_{1i} x_i)}{\partial x_1} & \frac{\partial(\sum_{i=1}^{N} a_{1i} x_i)}{\partial x_2} & \dots & \frac{\partial(\sum_{i=1}^{N} a_{1i} x_i)}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial(\sum_{i=1}^{N} a_{mi} x_i)}{\partial x_1} & \frac{\partial(\sum_{i=1}^{N} a_{mi} x_i)}{\partial x_2} & \dots & \frac{\partial(\sum a_{mi} x_i)}{\partial x_n} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \dots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} = A$$

3) $A \in R^{n\times n}, x \in R^n$

$$x^T A x = \sum_{i=1}^{n} x_i \sum_{j=1}^{n} a_{ij} x_j$$

$$\frac{\sum_{i=1}^{n} x_i \sum_{j=1}^{n} a_{ij} x_j}{\partial x_k} = \frac{\partial\left(a_{kk} x_k^2 + \sum_{i=1, i\neq k}^{n} x_i \cdot \sum_{j=1}^{n} a_{ij} x_j + \sum_{i=1}^{n} x_i \sum_{j=1, j\neq k}^{n} a_{ij} x_j\right)}{\partial x_k} =$$

$$= 2 a_{kk} x_k + \sum_{\substack{i=1,\\ i\neq k}}^{n} a_{ik} x_k + \sum_{\substack{j=1,\\ j\neq k}}^{n} a_{kj} x_k =$$

$$= \sum_{i=1}^{n} a_{ik} x_k + \sum_{j=1}^{n} a_{kj} x_k = \sum_{i=1}^{n} x_k (a_{ik} + a_{ki})$$

$$\frac{\partial (x^T A x)}{\partial x} = \begin{pmatrix} \sum_{i=1}^{n} x_1 (a_{i1} + a_{1i}) \\ \vdots \\ \sum_{i=1}^{n} x_n (a_{in} + a_{ni}) \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11} + a_{11} & a_{21} + a_{12} & \cdots & a_{n1} + a_{1n} \\ \vdots & & \ddots & \\ a_{1n} + a_{n1} & \cdots & & a_{nn} + a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} =$$

$$= (A + A^T)\, x$$

если $A = A^T \rightarrow \frac{\partial (x^T A x)}{\partial x} = 2Ax$

4) $x \in R^n$

$$\|x\|^2 = x^T x = \sum_{i=1}^{n} x_i^2$$

$$\frac{\partial \|x\|^2}{\partial x} = \begin{pmatrix} \frac{\partial \sum_{i=1}^{n} x_i^2}{\partial x} \\ \vdots \\ \frac{\partial \sum_{i=1}^{n} x_i^2}{\partial x} \end{pmatrix} = 2x$$

5) $\frac{\partial g(x)}{\partial x} = \begin{pmatrix} \frac{\partial g(x_1)}{\partial x_1} & \frac{\partial g(x_2)}{\partial x_1} & \dots & \frac{\partial g(x_r)}{\partial x_1} \\ \dots & & & \dots \\ \frac{\partial g(x_1)}{\partial x_r} & \dots & & \frac{\partial g(x_r)}{\partial x_r} \end{pmatrix} =$

$= \begin{pmatrix} g'(x_1) & 0 & \dots & 0 \\ 0 & g'(x_2) & \dots & 0 \\ \dots & & & \\ 0 & & \dots & g'(x_r) \end{pmatrix} = \text{diag}(g'(x))$

6) $x \in R^n$; $h : R^n \to R^m$; $g : R^m \to R^p$

$\frac{\partial g(h(x))}{\partial x} = \begin{pmatrix} \frac{\partial g_1(h(x))}{\partial x_1} & \dots & \frac{\partial g_1(h(x))}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p(h(x))}{\partial x_1} & \dots & \frac{\partial g_p(h(x))}{\partial x_n} \end{pmatrix} =$

$(*)$ $\frac{\partial g_k(h(x))}{\partial x_i} = \frac{\partial g_k(h(x))}{\partial h_1} \cdot \frac{\partial h_1(x)}{\partial x_i} + \frac{\partial g_k(h(x))}{\partial h_2} \cdot \frac{\partial h_2(x)}{\partial x_i} + \dots + \frac{\partial g_k(h(x))}{\partial h_m} \cdot \frac{\partial h_m(x)}{\partial x_i}$

$(*)$
$= \begin{pmatrix} \frac{\partial g_1(h(x))}{\partial h_1} & \frac{\partial g_1(h(x))}{\partial h_2} & \dots & \frac{\partial g_1(h(x))}{\partial h_m} \\ \vdots & & \ddots & \vdots \\ \frac{\partial g_p(h(x))}{\partial h_1} & \frac{\partial g_p(h(x))}{\partial h_2} & \dots & \frac{\partial g_p(h(x))}{\partial h_m} \end{pmatrix} \cdot$

$$\cdot \begin{pmatrix} \frac{\partial h_1(x)}{\partial x_1} & \cdots & \frac{\partial h_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m(x)}{\partial x_1} & \cdots & \frac{\partial h_m(x)}{\partial x_n} \end{pmatrix} = \frac{\partial g(h(x))}{\partial h} \cdot \frac{\partial h(x)}{\partial x}$$

3.

x	1	1	0	0	-1
y	4	4	0	2	6

2) $X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & -1 \end{pmatrix}$ $\quad y = \begin{pmatrix} 4 \\ 4 \\ 0 \\ 2 \\ 6 \end{pmatrix}$

$X^T X = \begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix}$

$X^T y = \begin{pmatrix} 16 \\ 2 \\ 14 \end{pmatrix}$

$X^T X \beta = X^T y$

$\begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ 4 \end{pmatrix} \Rightarrow \beta = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

$f(x) = 1 - x + 4x^2$

3)

$\lambda = 1$

$$X^T X + \lambda I = \begin{pmatrix} 5 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 1 & 3 \\ 1 & 4 & 1 \\ 3 & 1 & 4 \end{pmatrix}$$

$$(X^T X + \lambda I)\beta = X^T y$$

$$\begin{pmatrix} 6 & 1 & 3 \\ 1 & 4 & 1 \\ 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ 14 \end{pmatrix} \Rightarrow \beta = \begin{pmatrix} 1.5 \\ -0.5 \\ 2.5 \end{pmatrix}$$

$$f(x) = 1.5 - 0.5x + 2.5x^2$$

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X_1	0	1	0	2	2	2	4	3
X_2	-1	0	0	0	1	0	1	2
y	0	0	0	0	0	1	1	1

1) $\hat{P}_r\{y=0\} = \frac{5}{8}$ $\hat{P}_r\{y=1\} = \frac{3}{8}$

$\hat{\mu}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mu_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$\hat{\Sigma}_0 \frac{1}{N_0 - 1} \sum (x^i - \hat{\mu}_0)(x^{(i)} - \hat{\mu}_0)^T = \frac{1}{4}\left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 0 + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 6 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0{,}5 \\ 0{,}5 & 0{,}5 \end{pmatrix}$$

$$\hat{\Sigma}_1 = \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0{,}5 \\ 0{,}5 & 1 \end{pmatrix}$$

$$\hat{\Sigma} = \frac{1}{6}\begin{pmatrix} 6 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 2/3 \end{pmatrix}$$

$$\hat{\Sigma}_0^{-1} = \begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix}, \quad \hat{\Sigma}_1^{-1} = \begin{pmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{pmatrix}$$

$$\Sigma^{-1} = \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix}$$

2) $\delta_0(x) = (x_1\ x_2) \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} -$

$- \frac{1}{2} (1\ 0) \begin{pmatrix} 8/5 & -6/5 \\ -6/5 & 12/5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \ln \frac{5}{8} =$

$= \left(\frac{8}{5} x_1 - 6/5\, x_2\right) - 4/5 + \ln \frac{5}{8}$

$\delta_1(x) = \left(\frac{18}{5} x_1 - 6/5\, x_2\right) - \frac{24}{5} + \ln \frac{3}{8}$

Разделяющая пов-ть

$- \frac{10}{5} x_1 + \frac{20}{5} + \ln 5 - \ln 8 - \ln 3 + \ln 8 =$

$= -2x_1 + 4 + \ln 5 - \ln 3 = 0$

Квадр. диск. ф-ии

$\delta_0(x) = -\frac{1}{2} \ln\left(\frac{1}{4}\right) - x_1^2 + 2x_1 + 2x_1x_2 - 2x_2^2 - 2x_2 -$

$- 1 + \ln 5/8 = -x_1^2 - 2x_2^2 + 2x_1x_2 + 2x_1 - 2x_2 - 1 +$

$+ \frac{\ln(4)}{2} + \ln \frac{5}{8}$

$\delta_1(x) = -\frac{1}{2} \ln \frac{3}{4} - \frac{1}{3}\left(2x_1^2 + 2x^2 - 2x_1x_2 - 10x_1 +\right.$

$\left.+ 2x_2 + 14\right) + \ln \frac{3}{8}$

Разделяющая пов-ть

$x_1^2 + x_1(4 - 4x_2) + 4x_2^2 + 4x_2 - 11 + \frac{3}{2} \ln 3 +$

$+3\ln 5=0$

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x_1	0	0	1	1	0	0	1	1	1	0
x_2	0	1	0	1	1	1	1	1	1	1
y	0	0	0	0	0	1	1	1	1	1

$$P_r(y=0 \mid x_1=1, x_2=1) = \frac{P_r(x_1=1 \mid y=0)\cdot P_r(x_2=1 \mid y=0)\cdot P_r(y=0)}{P_r(x_1=1, x_2=1)}$$

$$P_r(y=1 \mid x_1=1, x_2=1) = \frac{P_r(x_1=1 \mid y=1)\cdot P_r(x_2=1 \mid y=1)\cdot P_r(y=1)}{P_r(x_1=1, x_2=1)}$$

$\hat{P}_r(y=0)=\frac{1}{2}$; $\hat{P}_r(y=1)=\frac{1}{2}$ – апр-е вер-ти

$\hat{P}_r(x_1=0 \mid y=0)=\frac{3}{5}$; $\hat{P}_r(x_1=1 \mid y=0)=\frac{2}{5}$

$\hat{P}_r(x_1=0 \mid y=1)=\frac{2}{5}$; $\hat{P}_r(x_1=1 \mid y=1)=\frac{3}{5}$

$\hat{P}_r(x_2=0 \mid y=0)=2/5$; $\hat{P}_r(x_2=1 \mid y=0)=3/5$

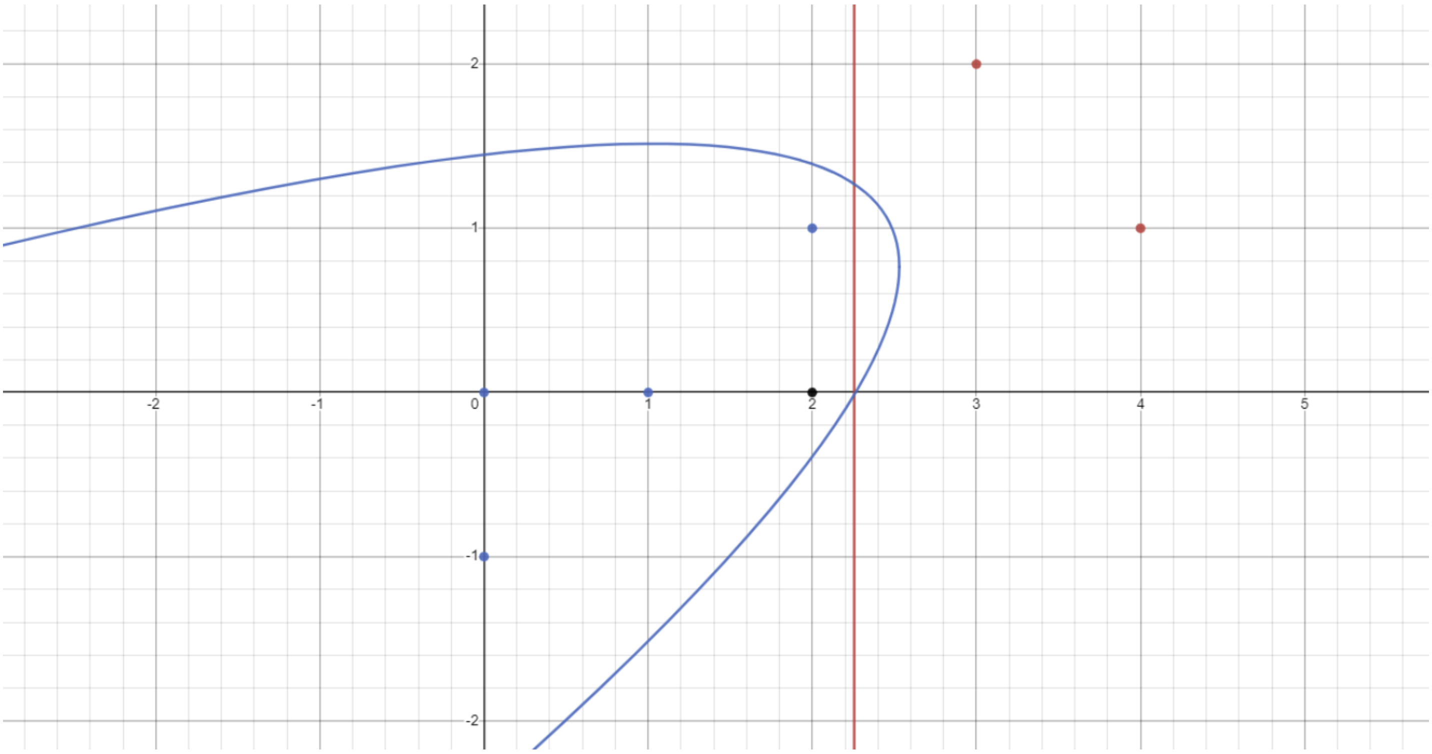
$\hat{P}_r(x_2=0 \mid y=1)=0$; $\hat{P}_r(x_2=1 \mid y=1)=1$

усл-е вер-ти

$$P_r(y=0 \mid x_1=1, x_2=1) = \frac{2/5\cdot 3/5\cdot \frac{1}{2}}{\frac{6}{50}+3/10} = \boxed{\frac{2}{7}}$$

$$P_r(y=1 \mid x_1=1, x_2=1) = \frac{3/5\cdot 1\cdot 1/2}{6/50+3/10} = \boxed{\frac{5}{7}}$$

Приложение к №9



Приложение к № 15