Distributed Newton-Type Methods with Communication Compression and Bernoulli Aggregation.

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Outline

1. The Problem
2. 3PC Compression Mechanism
3. The 3 Special Newton-type Methods
4. Newton-3PC
5. Numerical Experiments

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1. **The Problem**
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The Problem

Strongly convex

clients/devices

2nd order smooth

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} f_i(x) \right\}$$

model parameters

Empirical risk/loss

Local training data

Local loss function $f_i(x) = \mathbb{E}_{\xi \sim \mathcal{D}_i}[f_\xi(x)]$

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3PC Compression Mechanism

$$\mathcal{C}_{\mathbf{H},\mathbf{Y}} : \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{H}\in} \times \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{Y}\in} \times \underbrace{\mathbb{R}^{d\times d}}_{\mathbf{X}\in} \to \mathbb{R}^{d\times d}$$

Parameters

Input

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$$\mathbb{E}\left[\|\mathcal{C}_{\mathbf{H},\mathbf{Y}}(\mathbf{X}) - \mathbf{X}\|_{\mathrm{F}}^2\right] \leq (1-A)\|\mathbf{H}-\mathbf{Y}\|_{\mathrm{F}}^2 + B\|\mathbf{X}-\mathbf{Y}\|_{\mathrm{F}}^2$$

$A \in (0,1]$

$B \geq 0$

3PC Compression Mechanism: Examples

Contractive Compressors:

$$\exists \alpha \in (0,1] : \mathbb{E}\left[\|\mathcal{C}(\mathbf{X}) - \mathbf{X}\|_{\mathrm{F}}^2\right] \leq (1-\alpha)\|\mathbf{X}\|_{\mathrm{F}}^2 \quad \forall \mathbf{X} \in \mathbb{R}^{d\times d}$$

Compressed Lazy Aggregation:

$$\exists \alpha \in (0,1], \zeta \geq 0 : \mathcal{C}_{\mathbf{H},\mathbf{Y}} = \begin{cases} \mathbf{H} + \mathcal{C}(\mathbf{X} - \mathbf{H}) & \text{if } \|\mathbf{H} - \mathbf{X}\|_{\mathrm{F}}^2 > \zeta\|\mathbf{X} - \mathbf{Y}\|_{\mathrm{F}}^2 \\ \mathbf{H} & \text{if otherwise} \end{cases}$$

Contractive compressor with parameter α

Compressed Bernoulli Aggregation:

$$\exists \alpha \in (0,1], p \in (0,1] : \mathcal{C}_{\mathbf{H},\mathbf{Y}} = \begin{cases} \mathbf{H} + \mathcal{C}(\mathbf{X} - \mathbf{H}) & \text{with probability } p \\ \mathbf{H} & \text{with probability } 1-p \end{cases}$$

Outline

Newton Method

$$x^{k+1} = x^k - \Bigg(\frac{1}{n} \sum_{i=1}^{n} \nabla^2 f_i(x^k) \Bigg)^{-1} \Bigg(\frac{1}{n} \sum_{i=1}^{n} \nabla f_i(x^k) \Bigg)$$

Can be computed locally

Expensive to communicate: $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

- $\mathcal{O}(d)$ communication cost per round
- Implementability in practice
- Local quadratic convergence rate independent of the condition number

Hessian Lipschitz constant

$$\|x^{k+1} - x^*\| \le \frac{L}{2\mu} \|x^k - x^*\|^2$$

Strong convexity constant

Local quadratic rate

Newton Star Method

$$x^{k+1} = x^k - \left(\frac{1}{n} \sum_{i=1}^{n} \boxed{\nabla^2 f_i(x^*)} \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^{n} \boxed{\nabla f_i(x^k)} \right)$$

Can NOT be computed locally

Single communication of $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

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Distributed second order methods with fast rates and compressed communication,
ICML 2021.

- ✓ $\mathcal{O}(d)$ communication cost per round
- ✗ Implementability in practice
- ✓ Local quadratic convergence rate independent of the condition number

Hessian Lipschitz constant

$$\|x^{k+1} - x^*\| \leq \frac{L}{2\mu} \|x^k - x^*\|^{2}$$

Strong convexity constant

Local quadratic rate

Newton Zero Method

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^{n} \nabla^2 f_i(x^0)\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} \nabla f_i(x^k)\right)$$

Can be computed locally

Single communication of $\mathcal{O}(d^2)$

Can be computed locally

Easy to communicate: $\mathcal{O}(d)$

- ✓ $\mathcal{O}(d)$ communication cost per round*
- ✓ Implementability in practice
- ✗ Local quadratic convergence rate

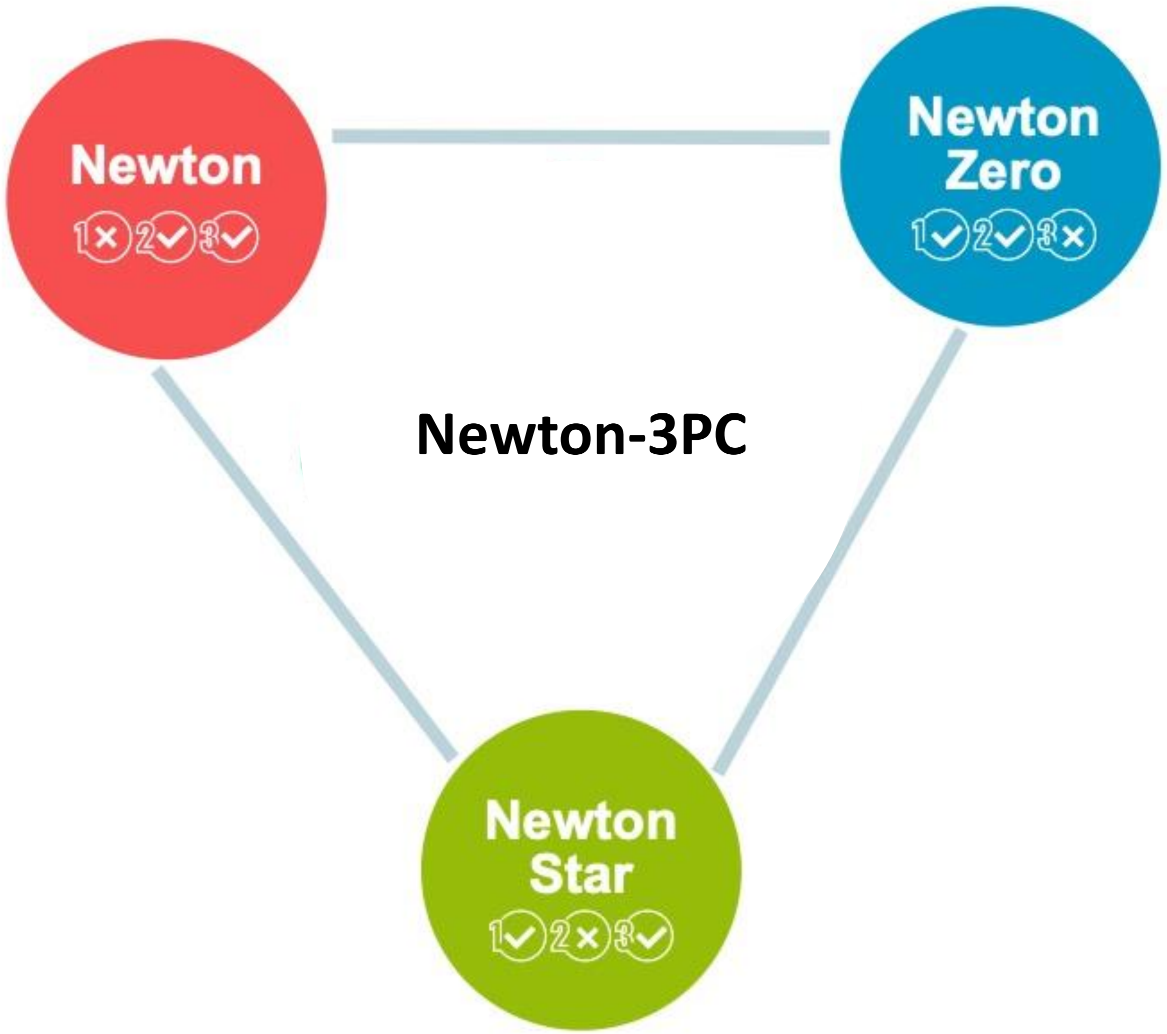
$$\|x^{k+1} - x^*\| \leq \frac{1}{2}\|x^k - x^*\|$$

Local (fixed) linear rate: $\|x^0 - x^*\| \leq \frac{\mu}{2L}$

Strong convexity constant

Hessian Lipschitz constant

“Newton Triangle”



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1. The Problem
2. Brief Comparison with Related Works
3. The 3 Special Newton-type Methods
4. Newton-3PC
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Learning the Optimal Hessian Matrices

Newton Star

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^n \nabla^2 f_i(x^*)\right)^{-1} \nabla f(x^k)$$

Idea! Learn the optimal Hessians $\nabla^2 f_i(x^*)$ in communication efficient manner:

$\boxed{(i)}\ \mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ as $k \to \infty$ $\quad \boxed{(ii)}\ \mathbf{H}_i^{k+1} - \mathbf{H}_i^k$ is compressed

$$x^{k+1} = x^k - \left(\frac{1}{n}\sum_{i=1}^n \mathbf{H}_i^k\right)^{\boxed{-1}} \nabla f(x^k)$$

$$= x^k - \left(\mathbf{H}^k\right)^{\boxed{-1}} \nabla f(x^k)$$

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Distributed second order methods with fast rates and compressed communication,
ICML 2021.

FedNL: Two Options for Updating the Global Model

Option 1

$$x^{k+1} = x^k - \left(\left[\mathbf{H}^k\right]_{\boxed{\mu}}\right)^{-1} \nabla f(x^k)$$

Projection onto the cone of positive definite matrices

Option 2

$$x^{k+1} = x^k - \left(\mathbf{H}^k + \boxed{l^k}\mathbf{I}\right)^{-1} \nabla f(x^k)$$

$$l^k = \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{H}_i^k - \nabla^2 f_i(x^k)\|_{\mathrm{F}}$$

Newton-3PC: New Hessian Learning Technique

$$\mathbf{H}_i^{k+1} = \mathcal{C}_{\mathbf{H}_i^k, \nabla^2 f_i(x^k)}\left(\nabla^2 f_i(x^{k+1})\right)$$

3PC- Compression operator

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FedNL: Hessian Learning Rate Options

Assumption 4.1. *The average loss f is μ-strongly convex, and all local losses $f_i(x)$ have Lipschitz continuous Hessians.*

$$\|\nabla^2 f_i(x) - \nabla^2 f_i(y)\| \quad \leq L_* \|x - y\|,$$

$$\Phi^k := \mathcal{H}^k + 6\left(\frac{1}{A} + 3AB\right) L_{\mathrm{F}}^2 \|x^k - x^*\|^2, \quad \text{where} \quad \mathcal{H}^k := \frac{1}{n}\sum_{i=1}^{n} \|\mathbf{H}_i^k - \nabla^2 f_i(x^*)\|_{\mathrm{F}}^2$$

Can be relaxed just for k=0

Theorem 4.2. *Let Assumption 4.1 hold. Assume $\|x^0 - x^*\| \leq \frac{\mu}{\sqrt{2D}}$ and $\mathcal{H}^k \leq \frac{\mu^2}{4C}$ for all $k \geq 0$. Then,* Newton-3PC *(Algorithm 1) with any 3PC mechanism converges with the following rates:*

$$\|x^k - x^*\|^2 \leq \frac{1}{2^k}\|x^0 - x^*\|^2, \qquad \mathbb{E}\left[\Phi^k\right] \leq \left(1 - \min\left\{\frac{A}{2}, \frac{1}{3}\right\}\right)^k \Phi^0,$$

$$\mathbb{E}\left[\frac{\|x^{k+1} - x^*\|^2}{\|x^k - x^*\|^2}\right] \leq \left(1 - \min\left\{\frac{A}{2}, \frac{1}{3}\right\}\right)^k \left(C + \frac{AD}{12(1+3AB)L_{\mathrm{F}}^2}\right)\frac{\Phi^0}{\mu^2}.$$

Algorithm 1 Newton-3PC (Newton's method with three point compressor)

Input: $x^0 \in \mathbb{R}^d$, $\mathbf{H}_1^0, \ldots, \mathbf{H}_n^0 \in \mathbb{R}^{d \times d}$, $\mathbf{H}^0 := \frac{1}{n}\sum_{i=1}^n \mathbf{H}_i^0$, $l^0 = \frac{1}{n}\sum_{i=1}^n \|\mathbf{H}_i^0 - \nabla^2 f_i(x^0)\|_{\mathrm{F}}$.

on server

  Option 1: $x^{k+1} = x^k - [\mathbf{H}^k]_\mu^{-1} \nabla f(x^k)$

  Option 2: $x^{k+1} = x^k - [\mathbf{H}^k + l^k \mathbf{I}]^{-1} \nabla f(x^k)$

  Broadcast x^{k+1} to all nodes

for each device $i = 1, \ldots, n$ in parallel **do**

  Get x^{k+1} and compute local gradient $\nabla f_i(x^{k+1})$ and local Hessian $\nabla^2 f_i(x^{k+1})$

  Apply 3PC and update local Hessian estimator to $\mathbf{H}_i^{k+1} = \mathcal{C}_{\mathbf{H}_i^k, \nabla^2 f_i(x^k)}\left(\nabla^2 f_i(x^{k+1})\right)$

  Send $\nabla f_i(x^{k+1})$, $\mathbf{H}_i^{k+1}$ and $l_i^{k+1} := \|\mathbf{H}_i^{k+1} - \nabla^2 f_i(x^{k+1})\|_{\mathrm{F}}$ to the server

end for

on server

  Aggregate $\nabla f(x^{k+1}) = \frac{1}{n}\sum_{i=1}^n \nabla f_i(x^{k+1}), \mathbf{H}^{k+1} = \frac{1}{n}\sum_{i=1}^n \mathbf{H}_i^{k+1}, l^{k+1} = \frac{1}{n}\sum_{i=1}^n l_i^{k+1}$

FedNL: Assumptions

FedNL: Local Convergence Theory

$$\|x^k - x^*\|^2 \boxed{\leq} \frac{1}{2^k}\|x^0 - x^*\|^2$$

Local (fixed) linear rate

Lyapunov function

$$\Phi^k = \mathcal{H}^k + 6\left(\frac{1}{\boxed{A}} + 3\boxed{A}\boxed{B}\right) L^2\|x^k - x^*\|^2$$

$$\mathbb{E}[\boxed{\Phi^k}] \boxed{\leq} \left(1 - \min\left\{\frac{\boxed{A}}{2}, \frac{1}{3}\right\}\right)\boxed{\Phi^0}$$

Local linear rates

$x^k \to x^*$

$\mathbf{H}_i^k \to \nabla^2 f_i(x^*)$ for all $i \in [n]$

Constant depending on the choice of the compressor and stepsize

Outline

Experiments: Regularized Logistic Regression

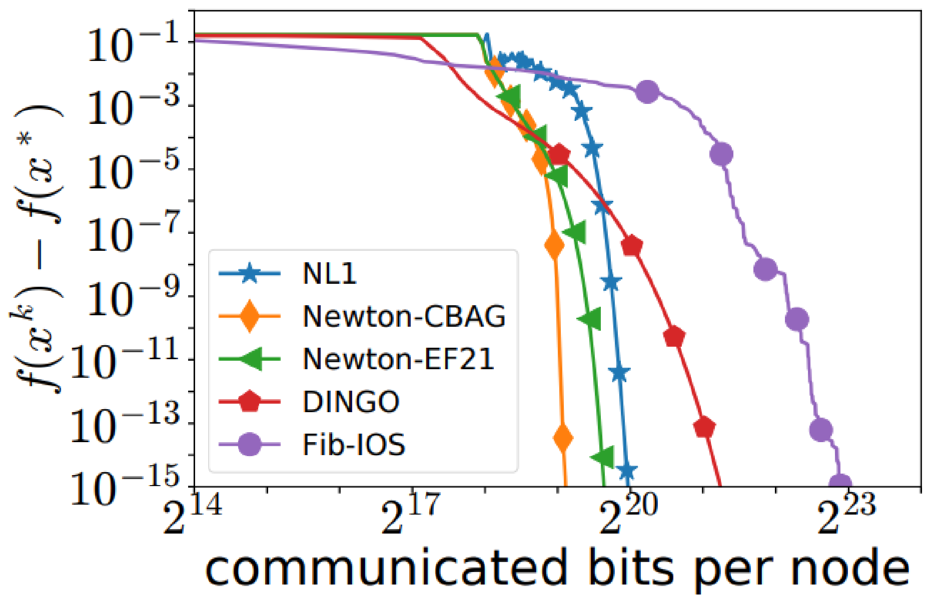
Regularization parameter

Training data points

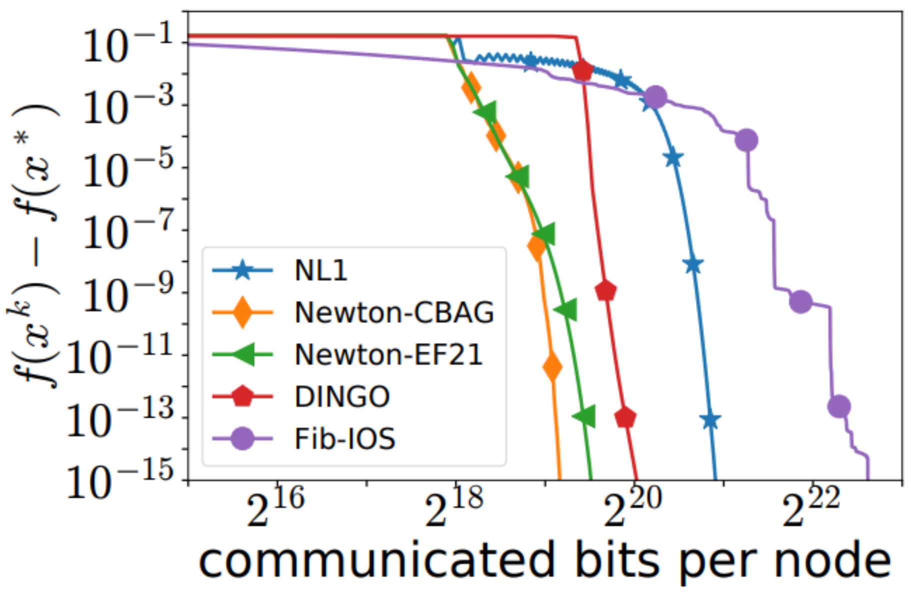
$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{n} \sum_{i=1}^{n} f_i(x) + \frac{\lambda}{2} \|x\|^2 \right\}, \qquad f_i(x) = \frac{1}{m} \sum_{j=1}^{m} \log\left(1 + \exp(-b_{ij} a_{ij}^\top x)\right),$$

where $\{a_{ij}, b_{ij}\}_{j \in [m]}$ are data points at the i-th device. The datasets were taken from LibSVM library [Chang and Lin, 2011]: `a1a`, `a9a`, `w7a`, `w8a`, and `phishing`.

Experiments: Local Comparisong against SOM

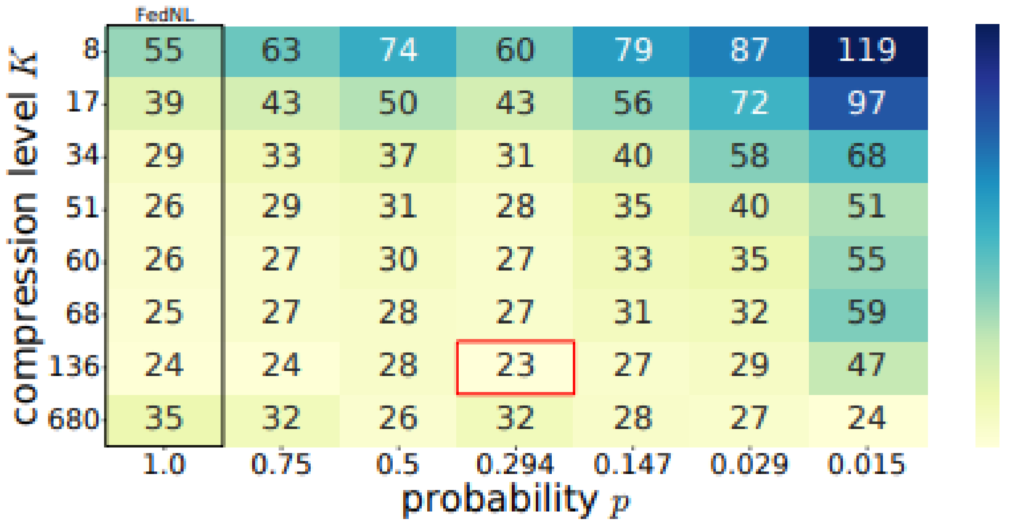


(a) a1a, $\lambda = 10^{-3}$

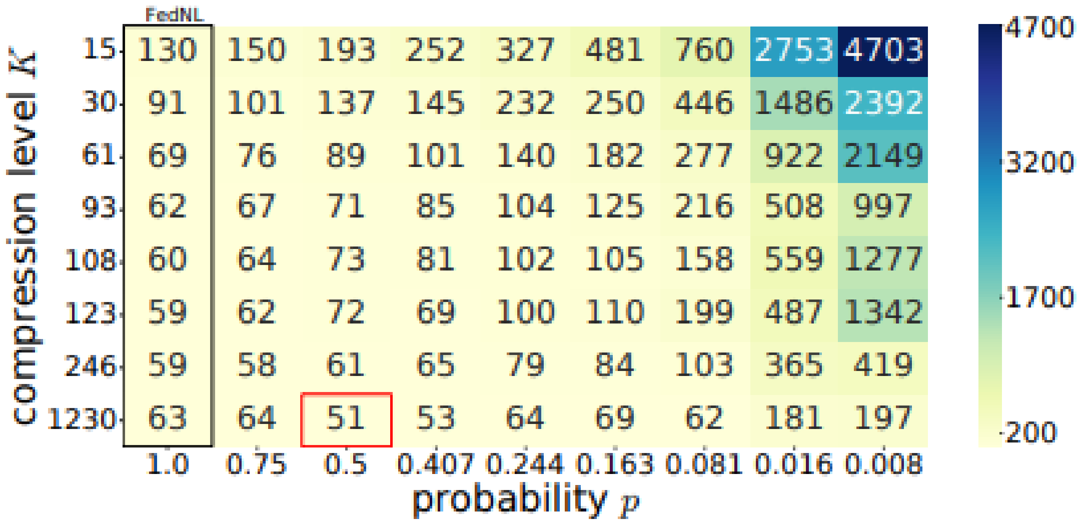


(c) a9a, $\lambda = 10^{-3}$

Experiments: Does Bernoulli Aggregation Bring Benefit?



(e) phishing, $\lambda = 10^{-3}$



(f) a1a, $\lambda = 10^{-4}$

Thank you