Constraint Satisfaction Problem

Representation A

Variables and domains

- Assume $k \geq 0, k < 5$
- the houses can be defined as

$$house(k) = a_k$$

- such that a_k is the kth house from left to right and the houses start at 0
- the color of the kth house can be defined as

$$color(k) \in \{R, G, B, Y, I\}$$

- such that $\{R, G, B, Y, I\}$ are Red, Green, Blue, Yellow, Ivory respectively.
- The nationality of the person in the kth house can be defined as

$$nation(k) \in \{E, S, N, U, J\}$$

- such that $\{E, S, N, U, J\}$ are English, Spanish, Norwegian, Ukranian, and Japanese respectively.
- The candy preference of the person in the kth house can be defined as

$$candy(k) \in \{H, M, K, Sm, Sn\}$$

- such that $\{H, M, K, Sm, Sn\}$ are Hershey's, MilyWay, KitKat, Smileys, and Snickers respectively.
- The drink preference of the person in the kth house can be defined as

$$drink(k) \in \{M, C, T, O, W\}$$

- such that $\{M, C, T, O, W\}$ are Milk, Coffee, Tea, Orange Juice, and Water respectively.
- The pet of the person in the kth house can be defined as

$$pet(k) \in \{D, F, S, H, Z\}$$

- such that $\{D, F, S, H, Z\}$ are Dog, Fox, Snail, Horse, and Zebra respectively.

Constraints

- The Englishman lives in the red house.

$$\forall k \in \mathbb{Z} : (nation(k) = E) \Leftrightarrow (color(k) = R)$$

- The Spaniard owns the dog.

$$\forall k \in \mathbb{Z} : (nation(k) = S) \Leftrightarrow (pet(k) = D)$$

- The Norwegian lives in the first house on the left.

$$nation(0) = N$$

- The green house is immediately to the right of the ivory house.

$$\forall k \in \mathbb{Z} : (color(k) = I) \Rightarrow (color(k+1) = G)$$

- The man who eats Hershey bars lives in the house next to the man with the fox.

$$\forall k \in \mathbb{Z} : (candy(k) = H) \Rightarrow ((pet(k+1) = F) \vee (pet(k-1) = F))$$

- Kit Kats are eaten in the yellow house.

$$\forall k \in \mathbb{Z} : (candy(k) = K) \Leftrightarrow (color(k) = Y)$$

- The Norwegian lives next to the blue house.

$$\forall k \in \mathbb{Z} : (nation(k) = N) \Rightarrow ((color(k+1) = B) \vee (color(k-1) = B))$$

- The Smarties eater owns snails.

$$\forall k \in \mathbb{Z} : (candy(k) = Sm) \Leftrightarrow (pet(k) = S)$$

- The Snickers eater drinks orange juice.

$$\forall k \in \mathbb{Z} : (candy(k) = Sn) \Leftrightarrow (drink(k) = O)$$

- The Ukrainian drinks tea.

$$\forall k \in \mathbb{Z} : (nation(k) = U) \Leftrightarrow (drink(k) = T)$$

- The Japanese eats Milky Ways.

$$\forall k \in \mathbb{Z} : (nation(k) = J) \Leftrightarrow (candy(k) = M)$$

- Kit Kats are eaten in a house next to the house where the horse is kept.

$$\forall k \in \mathbb{Z} : (candy(k) = K) \Rightarrow ((pet(k+1) = H) \vee (pet(k-1) = H))$$

- Coffee is drunk in the green house.

$$\forall k \in \mathbb{Z} : (drink(k) = C) \Leftrightarrow (color(k) = G)$$

- The Milk is drunk in the middle house

$$drink(2) = M$$

Possible Solution using Representation A

$$nation(0) = N; color(0) = Y; pet(0) = F; drink(0) = W; candy(0) = K$$

$$nation(1) = U; color(1) = B; pet(1) = H; drink(1) = T; candy(1) = H$$

$$nation(2) = E; color(2) = R; pet(2) = S; drink(2) = M; candy(2) = Sm$$

$$nation(3) = S; color(3) = I; pet(3) = D; drink(3) = O; candy(3) = Sn$$

$$nation(4) = J; color(0) = G; pet(4) = Z; drink(4) = V; candy(4) = M$$

Representation B

- Given a set of pets we can get any pet as p

$$p \in \{D, F, S, H, Z\}$$

- each p is a tuple describing the pet's owner.

$$(nation, candy, drink, location)$$

$$nation \in \{E, S, N, U, J\}$$

$$candy \in \{H, M, K, Sm, Sn\}$$

$$drink \in \{M, C, T, O, W\}$$

$$positions = \{1, 2, 3, 4, 5\}$$

$$colors = \{red, green, blue, yellow, ivory\}$$

$$location \in colors \times positions$$