CS 475 Logic

1.

I do acknowlege that if we account for all the vocabulary it would increase the number of models in which the result is false and true. But I initially counted those models as redundant as those parts of the vocabulary are not incorporated in the formula.

a.

- There are 3 models
  - If we count the redundant vocabulary there are 6

A	B	C	$((A \wedge B) \vee (B \wedge C))$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	T

b.

- There are 3 models
  - If we count the redundant vocabulary there are 12

A	B	$(A \vee B)$
F	F	F
F	T	T
T	F	T
T	T	T

c.

- There are 4 models
  - If we count the redundant vocabulary there are 8

A	B	C	$(A \Leftrightarrow (B \Leftrightarrow C))$
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	F
T	T	T	T

d.

- There is only 1 model
  - If we count the redundant vocabulary there are 16

D	D
F	F
T	T

2.

a.

- Valid: Yes
  - All models result in True
- Satisfiable: Yes
  - At least one model results in True

smoke	$(smoke \Rightarrow smoke)$
F	T
T	T

b.

- Valid: No
  - Not all models result in True
- Satisfiable: Yes
  - At least one model results in True

fire	smoke	$((smoke \Rightarrow fire) \Rightarrow (\neg smoke \Rightarrow \neg fire))$
F	F	T
F	T	T
T	F	F
T	T	T

c.

- Valid: Yes
  - All models result in True
- Satisfiable: Yes
  - At least one model results in True

fire	heat	smoke	$(((smoke \wedge heat) \Rightarrow fire) \Leftrightarrow ((smoke \Rightarrow fire) \vee (heat \Rightarrow fire)))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

3.

a.

$$((A \wedge B) \vee (B \wedge C)) \Rightarrow (D \wedge E)$$

$$\neg((A \wedge B) \vee (B \wedge C)) \vee (D \wedge E)$$

$$(\neg(A \wedge B) \wedge \neg(B \wedge C)) \vee (D \wedge E)$$

$$((\neg A \vee \neg B) \wedge (\neg B \vee \neg C)) \vee (D \wedge E)$$

$$(D \wedge E) \vee ((\neg A \vee \neg B) \wedge (\neg B \vee \neg C))$$

$$((D \wedge E) \vee (\neg A \vee \neg B)) \wedge ((D \wedge E) \vee (\neg B \vee \neg C))$$

$$((\neg A \vee \neg B) \vee (D \wedge E)) \wedge ((\neg B \vee \neg C) \vee (D \wedge E))$$

$$(\neg A \vee \neg B \vee D) \wedge (\neg A \vee \neg B \vee E) \wedge (\neg B \vee \neg C \vee D) \wedge (\neg B \vee \neg C \vee E)$$

b.

$$((A \vee B) \wedge (B \vee C)) \Rightarrow (D \vee E)$$

$$\neg((A \vee B) \wedge (B \vee C)) \vee (D \vee E)$$

$$(\neg(A \vee B) \vee \neg(B \vee C)) \vee (D \vee E)$$

$$((\neg A \wedge \neg B) \vee (\neg B \wedge \neg C)) \vee (D \vee E)$$

$$((\neg A \wedge \neg B) \vee (\neg B \wedge \neg C)) \vee (D \vee E)$$

$$((\neg B \wedge \neg A) \vee (\neg B \wedge \neg C)) \vee (D \vee E)$$

$$(\neg B \wedge (\neg A \vee \neg C)) \vee (D \vee E)$$

$$(D \vee E) \vee (\neg B \wedge (\neg A \vee \neg C))$$

$$(D \vee E \vee \neg B) \wedge (D \vee E \vee \neg A \vee \neg C)$$

4.

$$((P \Rightarrow Q) \wedge (L \wedge M \Rightarrow P) \wedge (L \wedge B \Rightarrow M) \wedge (A \wedge P \Rightarrow L) \wedge (A \wedge B \Rightarrow L) \wedge A \wedge B) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge (\neg(L \wedge M) \vee P) \wedge (\neg(L \wedge B) \vee M) \wedge (\neg(A \wedge P) \vee L) \wedge (\neg(A \wedge B) \vee L) \wedge A \wedge B) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge (\neg L \vee \neg M \vee P) \wedge (\neg L \vee \neg B \vee M) \wedge (\neg A \vee \neg P) \vee L) \wedge (\neg A \vee \neg B \vee L) \wedge A \wedge B) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge (\neg L \vee \neg M \vee P) \wedge (\neg L \vee \neg B \vee M) \wedge (\neg P) \vee L) \wedge (\neg B \vee L) \wedge B) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge (\neg L \vee \neg M \vee P) \wedge (\neg L \vee M) \wedge (\neg P) \vee L) \wedge L) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge (\neg M \vee P) \wedge M \wedge \neg P) \Rightarrow Q$$

$$((\neg P \vee Q) \wedge P \wedge \neg P) \Rightarrow Q$$

$$Q \Rightarrow Q$$

- $Q \Rightarrow Q$ is valid
- Therefore KB entails Q