

# Fiber-optic sensing of pressure and temperature

G. B. Hocker

The use of a fiber-optic Mach-Zehnder interferometer to measure differences in temperature or pressure between two single-mode fiber arms is described. Temperature or pressure changes are observed as a motion of an optical interference fringe pattern. Values are calculated for the pressure and temperature dependence of the fringe motion. Pressure and temperature measurements are made with the interferometer, and the experimental values for sensitivity are in good agreement with those calculated.

## I. Principles of Operation

A form of Mach-Zehnder interferometer can be made using single-mode optical fibers for the two arms as shown in Fig. 1. If the optical pathlengths of the two arms are nearly equal (to within the coherence length of the source), the light from the two fibers interferes to form a series of bright and dark fringes. A change in the relative phase of the light from one fiber with respect to the other is observed as a displacement of the fringe pattern, a phase change of  $2\pi$  rad causing a displacement of one fringe.

The phase of the light leaving a fiber can be changed by dimensional and/or index of refraction changes in the fiber. Thus, if one fiber is subject to a different strain, pressure, temperature, etc. than the other, this difference appears as a displacement of the fringes and can be measured by this displacement. This is the basic principle of the fiber-optic strain gauge previously reported<sup>1,2</sup> and is similar to a fiber-optic device proposed as an acoustic sensor.<sup>3</sup> In this memo we analyze the sensitivity of this device for pressure and temperature measurement and present experimental results confirming these analyses.

## II. Analysis

The basic quantity to be calculated is the optical phase change per unit fiber length per unit of the physical stimulus considered, that is,  $(\Delta\phi)/(SL)$ , where  $\Delta\phi$  is the phase change in radians,  $L$  is the fiber length, and  $S$  is the stimulus such as pressure, temperature, etc.

This calculation has previously been carried out for the case of longitudinal strain and confirmed by experiment.<sup>1,2</sup>

### A. Pressure Sensitivity

Let  $\epsilon$  = strain,  
 $\sigma$  = stress,  
 $E$  = Young's modulus, and  
 $\mu$  = Poisson's ratio.

Consider a fiber under isotropic stress due to a pressure  $P$ . There are no shear components, and we can write the stress as a 3-component vector:

$$\sigma = \begin{bmatrix} -P \\ -P \\ -P \end{bmatrix}.$$

The strain then can also be written as a vector:

$$\epsilon = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix} = \begin{bmatrix} -P(1-2\mu)/E \\ -P(1-2\mu)/E \\ -P(1-2\mu)/E \end{bmatrix}.$$

Consider a length  $L$  of fiber of core diameter  $D$  and core index  $n$ . Let light of free-space propagation constant  $k_0$  propagate as a single mode of propagation constant  $\beta$ . The phase of the light wave after going through this fiber section is  $\phi = \beta L$ . Straining the fiber causes the phase at the output to shift by

$$\Delta\phi = \beta\Delta L + L\Delta\beta. \quad (1)$$

The first term represents the effect of the physical change of length due to the strain. For the stress considered,

$$\beta\Delta L = \beta\epsilon_z L = -\beta(1-2\mu)L\Delta P/E. \quad (2)$$

The second term, the change in  $\phi$  due to a change in  $\beta$ , can come about from two effects: the strain-optic

The author is with Honeywell Corporate Material Sciences Center, Bloomington, Minnesota 55420.

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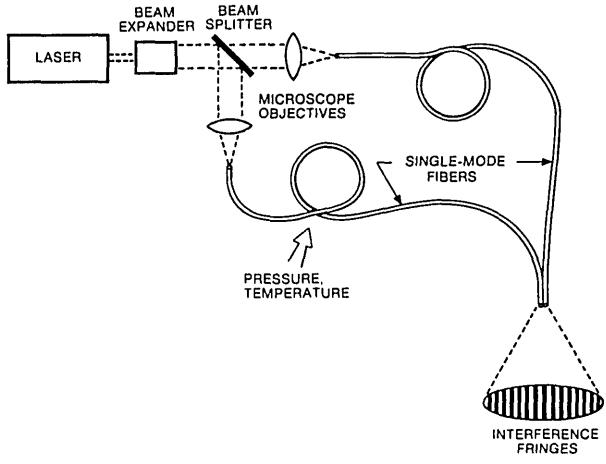


Fig. 1. Single-mode fiber-optic Mach-Zehnder interferometer used for temperature and pressure measurement.

effect whereby the strain changes the refractive index of the fiber, and a waveguide mode dispersion effect due to a change in fiber diameter produced by the strain:

$$L\Delta\beta = L \frac{d\beta}{dn} \Delta n + L \frac{d\beta}{dD} \Delta D. \quad (3)$$

Now  $\beta = n_{\text{eff}}k_0$ , where the effective index  $n_{\text{eff}}$  lies between the core and cladding indices, but since these indices differ by only on the order of 1% we can use  $\beta = nk_0$ . Thus

$$(d\beta)/(dn) = k_0. \quad (4)$$

The strain-optic effect appears as a change in the optical indicatrix

$$\Delta \left( \frac{1}{n^2} \right)_i = \sum_{j=1}^6 p_{ij} \epsilon_j$$

using the standard subscript contraction. With no shear strain  $\epsilon_4 = \epsilon_5 = \epsilon_6 = 0$ . We need only consider the  $i, j = 1, 2, 3$  elements of the strain-optic tensor for a homogeneous isotropic material<sup>4</sup>:

$$P_{ij} = \begin{bmatrix} p_{11} & p_{12} & p_{12} \\ p_{12} & p_{11} & p_{12} \\ p_{12} & p_{12} & p_{11} \end{bmatrix}.$$

Therefore, the change in the optical indicatrix is

$$\Delta \left( \frac{1}{n^2} \right)_{x,y,z} = -p_{11}P(1-2\mu)/E - 2p_{12}P(1-2\mu)/E = -(P/E)(1-2\mu)(p_{11} + 2p_{12}). \quad (5)$$

Light propagating in the  $z$ -direction therefore sees a change in index of

$$\Delta n = -\frac{1}{2}n^3 \Delta \left( \frac{1}{n^2} \right)_{x,y} = \frac{1}{2}n^3(P/E)(1-2\mu)(2p_{12} + p_{11}). \quad (6)$$

The last term in Eq. (3) represents the change in the waveguide mode propagation constant due to a change in fiber diameter. The change in diameter is simply

$$\Delta D = \epsilon_x D = -PD(1-2\mu)/E. \quad (7)$$

The  $d\beta/dD$  term can be evaluated using the normalized parameters describing the waveguide mode<sup>5</sup>:

$$\left. \begin{aligned} b &= \frac{\beta^2/k_0^2 - n_{\text{clad}}^2}{n_{\text{core}}^2 - n_{\text{clad}}^2} \\ V &= k_0 D(n_{\text{core}}^2 - n_{\text{clad}}^2)^{1/2} \end{aligned} \right\}. \quad (8)$$

We can write

$$\frac{d\beta}{dD} = \frac{d\beta}{db} \frac{db}{dV} \frac{dV}{dD}, \quad (9)$$

where

$$\frac{dV}{dD} = k_0(n_{\text{core}}^2 - n_{\text{clad}}^2)^{1/2} = V/D, \quad (10)$$

$$\frac{d\beta}{db} = \frac{(n_{\text{core}}^2 - n_{\text{clad}}^2)k_0^2}{2\beta} = V^2/2\beta D^2, \quad (11)$$

and  $(db)/(dV)$  is the slope of the  $b - V$  dispersion curve at the point which describes the waveguide mode.

Substituting the various expressions above into Eq. (1), we obtain

$$\Delta\phi = -\beta(1-2\mu)LP/E + k_0 n^3 L(P/E)(1-2\mu)(2p_{12} + p_{11})/2 - \frac{LPD(1-2\mu)}{E} (V/D)(V^2/2\beta D^2) \frac{db}{dV}, \quad (12)$$

or, the phase change per unit pressure per unit fiber length is

$$\frac{\Delta\phi}{PL} = -\frac{\beta(1-2\mu)}{E} + \frac{\beta n^2}{2E} (1-2\mu)(2p_{12} + p_{11}) - \frac{V^3(1-2\mu)}{2\beta E D^2} \frac{db}{dV}. \quad (13)$$

It will now be shown that the third term in Eq. (13) is negligible compared with the first two. Parameters representative of a He-Ne laser source and a fused silica fiber are<sup>4,5</sup>

$$\begin{aligned} n &= 1.456, & \lambda &= 0.633 \times 10^{-6} \text{ m}, \\ \beta &= 2\pi n/\lambda = 1.446 \times 10^7 \text{ m}^{-1}, & \mu &= 0.17, \\ E &= 7.0 \times 10^{10} \text{ N/m}^2, & p_{12} &= +0.270, \\ p_{11} &\approx +0.121, \end{aligned}$$

In the single-mode region of the  $b - V$  dispersion curve,  $V \approx 2.4$  and  $db/dV = 0.5$ . (These approximate values are probably correct to  $\sim 20\%$ .) The core diameter might be 4 to 10 free-space wavelengths, so we can choose, say,  $D = 5 \times 10^{-6} \text{ m}$ . Using these values

$$\begin{aligned} \frac{\Delta\phi}{PL} &= -13.63 \times 10^{-5} + 9.55 \times 10^{-5} - 9 \times 10^{-8} \\ &= -4.09 \times 10^{-5} \text{ rad/Pa-m}, \end{aligned} \quad (14)$$

so that a one-fringe displacement requires

$$\Delta PL|_{\text{one fringe}} = 154 \text{ kPa-m} = 22.3 \text{ psi-m}.$$

The third term representing waveguide mode dispersion effects is negligible, so a simplified expression for  $\Delta\phi/PL$  is

$$\frac{\Delta\phi}{PL} = -\frac{\beta(1-2\mu)}{E} + \frac{\beta n^2}{2E} (1-2\mu)(2p_{12} + p_{11}). \quad (15)$$

The value of  $\Delta\phi/PL$  could be quite different for various multicomponent glasses, since Young's modulus

can vary from 5 to  $9 \times 10^{10}$  N/m<sup>2</sup>, Poisson's ratio ranges from 0.17 to 0.32, and the strain-optic coefficients can be in the 0.1–0.3 range.

### B. Temperature Sensitivity

A change in temperature of the fiber  $\Delta T$  changes the optical phase of the light going through it  $\Delta\phi$  due to two effects: the change in fiber length due to thermal expansion or contraction, and the temperature-induced change in the index of refraction. Thus, since  $\phi = nL2\pi/\lambda$ , we can write

$$\frac{\Delta\phi}{\Delta TL} = \frac{2\pi}{\lambda} \left( \frac{n}{L} \frac{dL}{dT} + \frac{dn}{dT} \right), \quad (16)$$

where the effects of fiber diameter changes are neglected as small.

Again taking the case of a He-Ne laser source and fused silica fiber, we use the values<sup>4,6,7</sup>

$$\begin{aligned} \frac{1}{L} \frac{dL}{dT} &= 5 \times 10^{-7}/^\circ\text{C}, & \frac{dn}{dT} &= 10 \times 10^{-6}/^\circ\text{C}, \\ n &= 1.456, & \lambda &= 0.6328 \times 10^{-6}\text{m}, \end{aligned}$$

so that

$$\frac{\Delta\phi}{\Delta TL} = 10^7 \text{ radians}/^\circ\text{C-m}. \quad (17)$$

Stated differently, this is a fringe displacement of 17.0 fringes per  $^\circ\text{C}$  per meter of fiber.

The value for the thermal expansion coefficient and the temperature-dependent refractive index can vary greatly for multicomponent glasses. For example, the thermal expansion coefficient can range from perhaps 4 to  $13 \times 10^{-6}/^\circ\text{C}$ .<sup>8</sup>  $dn/dT$  can actually be positive or negative around room temperature, perhaps -10 to  $+19 \times 10^{-6}/^\circ\text{C}$ .<sup>8</sup> Moreover,  $dn/dT$  is itself a function of temperature and wavelength, and most tabulated values are averaged over a rather large temperature range and given for only a few wavelengths. Therefore, the value of  $\Delta\phi/\Delta TL$  could be quite different from that given in Eq. (17) for other glass compositions.

## III. Experimental Results

The fiber-optic interferometer of Fig. 1 was implemented using a He-Ne laser light source ( $\lambda = 0.633 \mu\text{m}$ ) and two lengths of bare Valtec SM05 step-index optical fiber, single-mode at the laser wavelength. The laser output was expanded, divided by a beam-splitting prism, and the resulting two beams coupled into the two fibers by a pair of  $5\times$  microscope objectives with N.A. = 0.01. Index oil mode strippers were used on both fibers. The output ends of the fibers were placed side by side, so that their axes were parallel and separated by one fiber diameter—approximately 75  $\mu\text{m}$ —and their expanding output beams overlapped. Fiber pairs of 1–2-m length each were used with lengths matched to within 5 cm. The overlapping output beams were observed on a screen, where they formed parallel bright and dark interference fringes.

### A. Temperature Sensitivity

Temperature sensitivity measurements were made with a 1-m length of the fiber in an insulating plastic foam enclosure, which also contained a small resistance heater and thermocouple connected to a temperature controller. Temperature changes  $\Delta T$  of 2–3  $^\circ\text{C}$  were used, and the resulting interference fringe displacements were recorded on video tape for repeated playback and measurement.

The average fringe displacement thus measured was  $13.2 \pm 0.5$  fringes/ $^\circ\text{C-m}$ . The temperature sensitivity calculated in Sec. II.B for pure  $\text{SiO}_2$  was 17.0 fringes/ $^\circ\text{C-m}$ . However, while the numerical values of the parameters used in the calculation were tabulated for pure  $\text{SiO}_2$ , the Valtec fiber consists of a doped  $\text{SiO}_2$  core with  $\text{SiO}_2$  cladding,<sup>9</sup> and the exact parameter values for this material composition are not available. Taking into account this difficulty in comparing calculated and experimental values, we consider the agreement obtained to be good.

### B. Pressure Sensitivity

Pressure sensitivity measurements were made with a 1.5-m length of one fiber in a cylinder, which could be pressurized to 345 kPa (50 psi) with nitrogen. Pressure measurements were made with a gauge reading to 345 Pa (0.05 psi). The fringe displacement was again recorded on video tape for repeated measurement. Pressure changes between 0 and the maximum pressure were made slowly, over a period of several minutes, since fringe displacement caused by the temperature sensitivity of the fiber and temperature changes due to compression or expansion of the gas in the cylinder (about  $1^\circ\text{C}$ ) tended otherwise to overshadow the fringe displacement due to pressure change.

The over-all average of many runs is a pressure sensitivity of one fringe displacement for  $92 \pm 27$  kPa-m ( $13.3 \pm 3.6$  psi-m). The rather large uncertainty is due to the difficulty of measuring the pressure-induced fringe displacement in the presence of larger displacements due to temperature changes. The value predicted in Sec. II.A was 154 kPa-m (22.3 psi-m)/fringe of displacement for pure silica glass. Considering the experimental uncertainty and the possible differences between pure and doped  $\text{SiO}_2$  fibers, this agreement seems reasonable.

## IV. Conclusions

We have shown that a single-mode fiber interferometer can be used to detect and measure temperature and pressure changes in one of the fiber arms. The experimental values of sensitivity are in agreement with those calculated.

The sensitivity of such an interferometer for temperature measurement is high, experimentally 13 fringes/ $^\circ\text{C-m}$ . Disadvantages of this technique include the alignment requirements for the interferometer, the need to observe the motion of optical fringes, and the fact that the measurement is of the change in average temperature of one fiber arm with respect to the other.

The sensitivity of the fiber interferometer to static pressure changes seems low compared with other techniques, and the fringe motion due to pressure changes may often be much less than small motions due to temperature changes. However, for detection of acoustic signals, with a frequency much greater than temperature changes or the thermal response of the fiber, the fiber interferometer scheme may be most useful. Use of a long fiber can provide adequate sensitivity. The fiber can either be coiled in a dimension less than an acoustic wavelength to provide a point sensor, or it may be laid out over many acoustic wavelengths to provide a directional antenna. Detection of the phase-modulated signal (fringe motion) may still be a problem, but possible solutions exist. The device shows promise as a hydrophone,<sup>3</sup> and we have confirmed here the expected numerical value of sensitivity.

## References

1. C. D. Butter and G. B. Hocker, *Appl. Opt.* **17**, 2867 (1978).
2. C. D. Butter and G. B. Hocker, "Fiber Optic Strain Gauge," presented at Electro Optics/Laser 78, 19–21 September 1978, Boston, Mass.
3. J. A. Bucaro *et al.*, *Appl. Opt.* **16**, 1761 (1977).
4. D. A. Pinnow, "Elastooptical Materials," in *Handbook of Lasers*, R. J. Pressley, Ed. (CRC, Cleveland, Ohio, 1971).
5. G. B. Hocker and W. K. Burns, *IEEE J. Quantum Electron.* **QE-11**, 270 (1975); D. B. Keck, in *Fundamentals of Optical Fiber Communications*, M. K. Barnoski, Ed. (Academic, New York, 1976), Chap. 1.
6. "Optical Fused Quartz and Fused Silica," Amersil Inc. publication EM-9227 (1975).
7. D. E. Gray, Ed., *AIP Handbook* (McGraw-Hill, New York, 1972), p. 6–29.
8. G. B. Morey, "Properties of Glass," in *International Critical Tables*, Vol. 2 (McGraw-Hill, New York, 1933).
9. M. Hudson, Valtec Corporation; private communication.

*Meetings Calendar continued from page 1413*

### 1979 October

- 22–26 Nuclear Cross Sections for Technology, Knoxville, Tenn. *W. W. Havens, Jr., 335 E. 45th St., New York, N.Y. 10017*
- 22–26 Advanced Microscopy course, Chicago *N. Daerr, McCrone Res. Inst., 2508 S. Michigan Ave., Chicago, Ill. 60616*
- 29–2 Nov. Applied Polarized Light Microscopy course, Chicago *N. Daerr, McCrone Res. Inst., 2508 S. Michigan Ave., Chicago, Ill. 60616*
- 30–31 Optical Materials for High Power Lasers, 11th Ann. Symp., NBS, Boulder *A. H. Guenther, Air Force Weapons Lab./CA, Kirtland AFB, N.M. 87117*
- 31–2 Nov. Eastern Analytical Symp., Hotel Americana, N.Y.C. *I. L. Simmons, M&T Chemicals, P.O. Box 1104, Rahway, N.J. 07065*

### November

- ? Optical Photonics and Iconics Engineering Mtg., Strasbourg *P. C. Legall, European Photonics Assoc., 3, rue de l'Universite, 67000 Strasbourg, France*

- 12–14 Microscopical Identification of Asbestos course, Chicago *N. Daerr, McCrone Res. Inst., 2508 S. Michigan Ave., Chicago, Ill. 60616*
- 12–16 Scanning Electron Microscopy course, Chicago *N. Daerr, McCrone Res. Inst., 2508 S. Michigan Ave., Chicago, Ill. 60616*
- 12–16 APS Div. of Plasma Physics, Boston *W. W. Havens, Jr., 335 E. 45th St., New York, N.Y. 10017*
- 18–1 Dec. Radiationless Processes, Internat. School of Atomic and Molecular Spectroscopy course, Erice, Italy *B. Di Bartolo, Phys. Dept., Boston Coll., Chestnut, Hill, Mass. 02167*
- 19–21 APS Div. of Fluid Dynamics, Notre Dame, Ind. *W. W. Havens, Jr., 335 E. 45th St., New York, N.Y. 10017*
- 25–30 ASME Winter Mtg., New York *ASME, 345 E. 47th St., New York, N.Y. 10017*

### December

- 10–12 APS Div. of Electron and Atomic Physics, Houston *W. W. Havens, Jr., 335 E. 45th St., New York, N.Y. 10017*
- 10–15 Infrared and Near Millimeter Waves, 4th internat. conf., Americana at Bal Harbour, Miami Beach, Fla. *K. J. Button, MIT, Nat. Magnet Lab., Cambridge, Mass. 02139*
- 17–21 Internat. Conf. of Lasers '79, Orlando, Fla. *V. J. Corcoran, Soc. for Optical and Quantum Electron., P.O. Box 245, McLean, Va. 22101*
- 1980
- ? 6th Internat. Conf. on Vacuum Ultraviolet Radiation Physics, USA (*R. P. Madden, NBS, Washington, D.C. 20234*)

### January

- 6–11 Internat. Winter Conf. 1980 on Developments in Atomic Plasma Spectrochemical Analyses, Inductively Coupled, Microwave, and D.C. Plasma Discharges, San Juan *ICP Information Newsletter, Chemistry—GRC Tower I, U. of Mass., Amherst, Mass. 01003*
- 21–24 APS mtg., Chicago *W. W. Havens, Jr., 335 E. 45th St., New York, N.Y. 10017*
- 28–30 Integrated and Guided Wave Optics, OSA Topical Mtg., Hyatt-Lake Tahoe, Incline Village, Nev. *OSA, 2000 L St. N.W., Suite 620, Washington, D.C. 20036*

### February

- 26–28 Laser and Electro-Optical Systems Conf., Town and Country Hotel, San Diego, Calif. *OSA, 2000 L St. N.W., Suite 620, Washington, D.C. 20036*

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