Simulation Assignment - Spring 2016

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1 Introduction

Utrecht Science park the Uithof is the primary location in Utrecht for research and higher education. A host of institutes, among others Nutricia Research, TNO and the Hubrecht Institute, have major research labs located at the Uithof. In addition this this, the Uithof is host to the Utrecht University campus and the Hogeschool Utrecht University of Applies Sciences, as well as the University Medical Center and the Wilhelmina Children's Hospital. Due to to these institutes being located at the Uithof, it experiences a large in- and efflux of passengers each day. Therefore a well-oiled public transport machine from Utrecht Central Station to the Uithof is of the utmost importance.

In 2016, an estimated 25.000 passengers travel to and from the Uithof from Utrecht Central station (UCS) every day, using the direct bus line which connects the two. However, the number of passengers is expected to grow to 45.000 passengers per day in 2020. Since these expected passenger numbers well exceed the capacity of the bus line, the municipality of Utrecht and the province of Utrecht have decided to renovate the public transport between UCS and the Uithof by replacing the current bus line with a tram line by 2018. Additionally, the tram line should be integrated into the already existing tram lines from UCS to Nieuwegein and Ijsselstein by 2020.

However, due to the expensive nature of this renovation, with a total budget of 440 million euros, the municipality and the province of Utrecht decided that modelling of the tram line was in order to ensure that the tram line will be performing as expected.

In this report, the performance of the Uithoflijn line is analysed, with an emphasis on its potential growth, both in passenger numbers as well as tram frequencies. The posed questions to be answered by the study are:

- How robust is the Uithoflijn to changes in passenger numbers and tram frequencies?
- What are feasible tram frequencies of the Uithoflijn?
- What is the maximum amount of passengers the line can handle?

What measures can be taken to improve the operational performance are also discussed briefly.

2 Problem analysis

2.1 Tram Line Description

In this section, the tram line is explained in more detail. All information in this section was acquired via domain experts J. Jongerius (Province of Utrecht) and M. van Kooten Niekerk (Qbuzz).

2.1.1 Uithoflijn

To increase the capacity of the public transport between Utrecht Central Station and the Uithof, a new tram line will be built.

The new tram line, called the Uithoflijn, will run over a total length of 8 kilometres and will have an expected run time of 17 minutes from UCS to the Uithof. During this run, it will stop at 9 stations. Tables 1 and 2 show data on both distance and driving times to these stations. Additionally, Figure 1 shows a map with the planned tram route.

The operation of the Uithoflijn will be introduced in two phases. In the first phase, directly after finising the tram line in 2018, trams will operate between the Uithof and Utrecht Central Station. In the second phase, which is planned to start in 2020, the Uithofline will be integrated into the already existing tram lines travelling from UCS to Nieuwegein and Ijsselstein. In this phase, trams leave from the Uithof to UCS, travel back to the Uithof and then travel to either Nieuwegein or Ijsselstein via UCS (Figure 2). However, in this report, we will focus on the first phase of the Uithoflijn operation.

Station	Distance (km)	Average driving time (seconds)
Centraal Station	-	-
Vaartsche Rijn	1.4	134
Galgenwaard	3.1	243
Kromme Rijn	0.6	59
Padualaan	0.8	101
Heidelberglaan	0.4	60
UMC	0.4	86
WKZ	0.6	78
P+R De Uithof	0.6	113

Table 1: Information on the route UCS to the Uithof.

Station	Distance (km)	Average driving time (seconds)
P+R De Uithof	-	-
WKZ	0.6	110
UMC	0.6	78
Heidelberglaan	0.4	82
Padualaan	0.4	60
Kromme Rijn	0.8	100
Galgenwaard	0.6	59
Vaartsche Rijn	3.1	243
Centraal Station	1.4	135

Table 2: Information on the route Uithof to UCS.

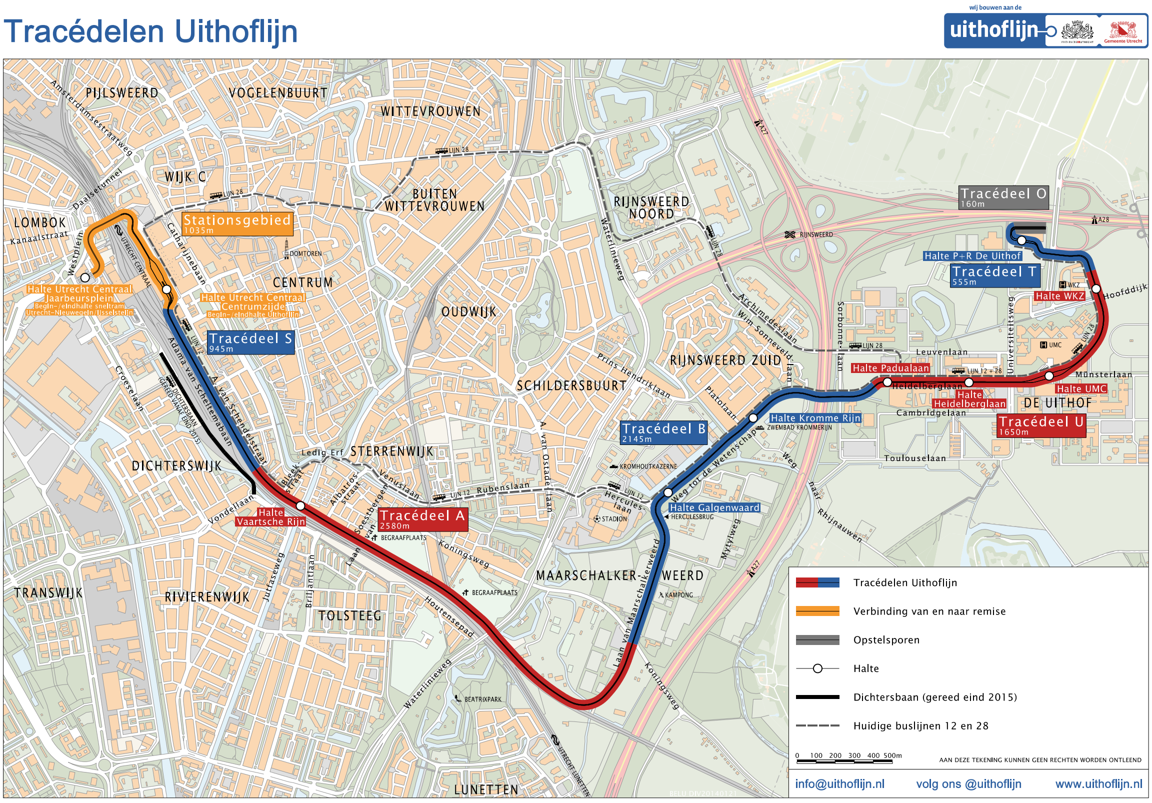

Figure 1: Map of Utrecht showing the planned route of the Uithofline. Projectorganisatie Uithoflijn, 2016

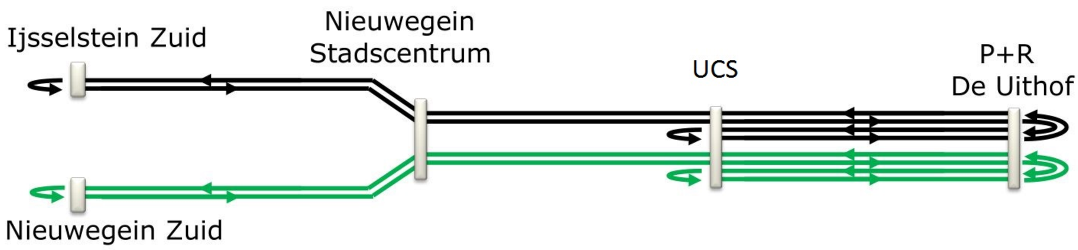

Figure 2: The planned inclusion of the Uithoflijn into the existing tramlines in and around Utrecht in the second phase, planned for 2020. Adapted from J. Jongerius, 2016

The planned schedule for the Uithoflijn can be found in table 3. As shown in this table, this schedule is intended to change over time; the frequency with which trams depart from stations is intended to increase upon integration of the Uithoflijn into the existing tram lines in Utrecht. The trams which will run the Uithoflijn consist of two separate tram units for a total capacity of 420 passengers per combined tram. These schedules will allow for a total passenger capacity of 6720 passengers and 8400 passengers per hour respectively.

Twelve combined trams have been ordered to run the Uithoflijn, with another three separate tram units as reserve.

	low intensity	high intensity
Time	trams/hour	trams/hour
06:00 - 07:00	4	5
07:00 - 19:00	16	20
19:00 - 21:30	4	5

Table 3: Scheduled number of trams per hour in service on the Uithoflijn for both the low and high intensity schedules.

2.1.2 Stations on the Uithoflijn

Two types of stations will be found along the Uithoflijne in phase one. In this section, both will be discussed and their properties explained.

Standard Stations

The first type of station is a regular station. These stations, which will be placed at all stations except for P+R the Uithof and Utrecht Central Station. These stations will have one dedicated platform for either direction; one for trams from the Uithof to UCS and one for trams from UCS to the Uithof. A schematic of such a station is shown in Figure 3. At these stations, the platforms allow for one tram in either direction to embark and disembark passengers; if further trams arrive during this interval, these trams will have to wait for the other tram to leave the station.

Termini

These stations will be found at P+R the Uithof and UCS. These stations have a different layout, which is shown in Figure 4. In this image, three important sub-sections can be discerned. The first of these are the incoming and outgoing track, which, because of the nature of these stations, are on the same side of the station. The second are the two platforms, which are placed side by side or on each side of the track (for simplicity, only the same-side version is shown since there is

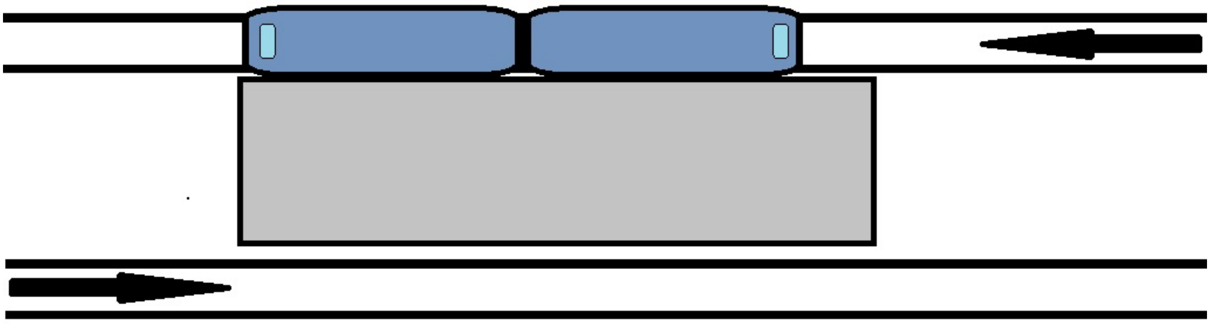

Figure 3: Layout of a standard station along the Uithoflijn route. In grey: the platform on which passengers embark and disembark. The arrows show the direction in which the trams traverse along the tracks.

no practical difference). The third is the diamond crossing, which connects the in- and outgoing rails to the two platforms.

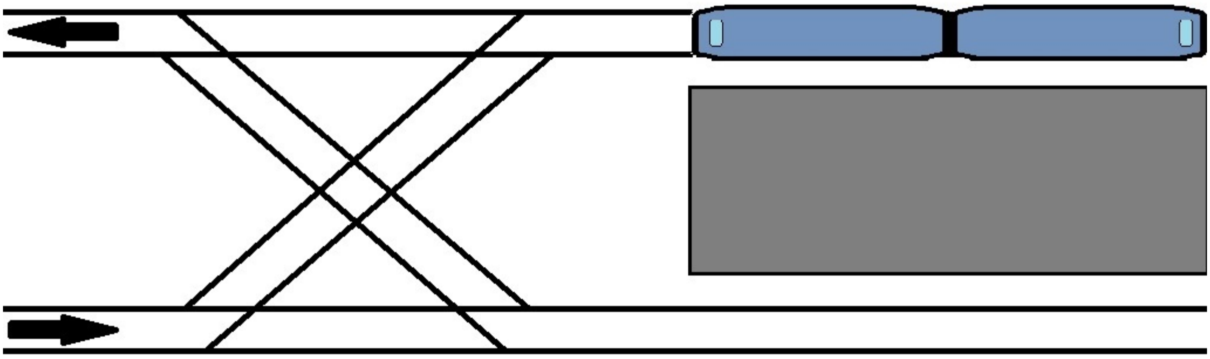

Figure 4: An example of the termini which will be placed at P+R the Uithof and UCS. In grey: the platforms on which passengers embark and disembark. The arrows show the direction in which the trams traverse along the tracks. Notable for the termini is the diamond crossing, which is found between the in- and outgoing rails and the platforms.

Because of this layout, there are some properties to termini which do not exist in the standard stations.

Firstly, since there is a single incoming/outgoing track at each platform, the trams at these platforms will have a turnaround time during which the tram is shut down, the drivers walks to the other end of the tram and starts it up again. This has been estimated to take roughly four minutes. Because of this, there exists a minimal possible dwell time at the termini which is equal to this turnaround time.

Secondly, because of the intersecting tram lines at the diamond crossing, it is necessary for railway sections at this point to be locked for some duration of time. Examples of these situations can be seen in Figure 5.

In the first situation, a tram is attempting to enter the station. If it were to go to platform number 1, it would have to traverse the diamond crossing, in which case both tracks will have to be locked. In contrast: If it were to go to platform 2, only the bottom track would have to be locked.

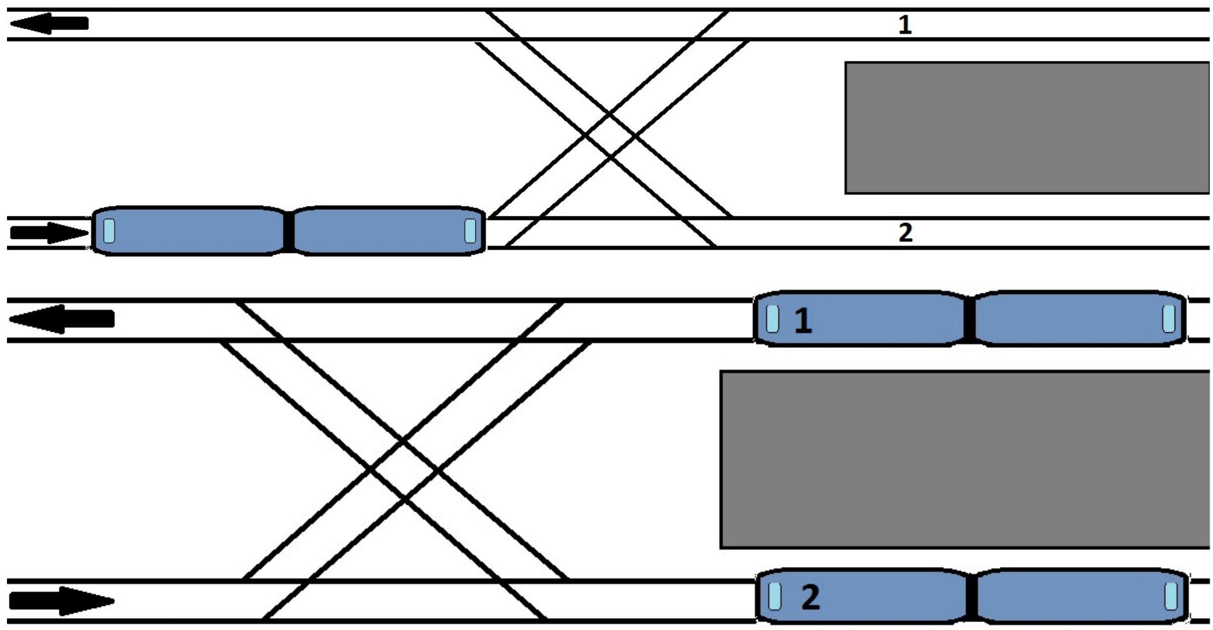

Figure 5: Two possible situations at the termini.

In the second Figure, a tram is trying to leave the station. The reverse of the example above is true here: If the tram at platform one attempts to leave, only the upper track has to be locked, while both tracks would have to be locked in the case of a tram leaving from platform 2.

Because of safety regulations, these locks have a minimum duration of 40 seconds. Other trams which will require access to part of the rail system which is currently locked will have to wait until this lock has been released.

Lastly, there is one difference between the two termini. The terminus P+R the Uithof will be connected to a series of stabling tracks, which will connect to the platforms in the opposite direction of the incoming and outgoing tracks. These stabling tracks will be where trams running on the Uithoflijn will be stationed when not in service.

3 Model Description

The tram line in this report was modelled using a discrete event simulation. This was done due to the similarity to a server-queue problem. In the following section, the model used is described.

3.1 Assumptions

Several assumptions were made to simplify the modelling process. The most important of these are explained below. Assumptions implied by the stochastic models used in the simulation are discussed in the Input Analysis section.

- Only termini have schedules. That is, as soon as a tram has completed embarking and disembarking passengers at a standard station, it is allowed to leave. At a terminus it may have to wait until its scheduled departure time.
- If a tram has to wait for another tram to leave a station before it can enter, it enters the station instantaneously when the blocking tram has left.
- Only one tram is allowed to embark passengers at a station for each direction at any given time
- Trams are only retired to the stabling tracks after the evening rush hours.
- No other causes of delay (e.g: materiel breakdown, extreme weather conditions) than passenger numbers, driving and dwell times are explicitly modelled.
- All passengers still in the tram when it reaches a terminus will leave the tram at that station.
- Passengers will not stop waiting at a station until they enter a tram; there exists no maximum acceptable waiting time.
- Time is modelled on a per second basis; any fractions are rounded to the nearest integer.
- No scheduled departures are dropped because of excessive delays.
- The assumption is made that turnaround time is non-stochastic and will take 4 minutes.

3.2 States

Per station and trams, several variables are kept. These are stated below.

Trams

- Current number of passengers in the tram.
- Arrival time at the station it's currently at.
- Whether or not a tram is currently waiting, either in front of a station or waiting at a terminus to commence embarking passengers or to start transit.
- Where the tram currently is on the track.
- Whether or not the tram has finished embarking and disembarking passengers at the station which it is at.
- Total number of passengers which have embarked and disembarked the tram.

Stations
Stations are modelled as being unidirectional: trams can only exit and enter the station in one single direction. This means that each station is split into two separate station objects: one on the Uithof -> UCS route and one on the UCS -> Uithof route. They keep track of the following variables:

- Whether or not it contains a tram, and if so; which one.
- How many passengers are currently waiting at the station.
- Time of last update of passengers waiting.
- The time of the last tram departure.
- The previous and next station on the tram route.
- Total passengers which have embarked and disembarked at the station.

Termini
In contrast to ordinary stations, termini exist as a single object in the model. This is because there already is only one direction in which trams arrive and depart.

Besides the variables for regular stations, termini also keep track of the following:

- Their stabling tracks (if any) and the trams on them.
- Which rails of the diamond crossing are currently locked
- A list of departures which have been completed.
- For each of the two platforms, whether or not a tram is residing at them, and if there is; which tram it is.
- Whether or not a tram is currently embarking and disembarking passengers at the terminus.

3.3 Event Graphs

In this section, the event graphs and event handlers for standard stations and termini are shown and explained.

3.3.1 Event Graph - Standard Station

Figure 6 shows the event graph of standard stations in the constructed model. It contains four events, which are explained below.

End Transit
When a tram departs from the previous station on the tram line, an End Transit event is created at this station. When called, it will check whether it is the first tram waiting to enter the station, and if it is, whether or not there is already a tram currently embarking or disembarking passengers at the station. If there is, the tram will wait. If there is no other tram currently at the station and no other tram is waiting in front of it, the tram is moved to the station and will immediately start a Start Passenger Transfer event.

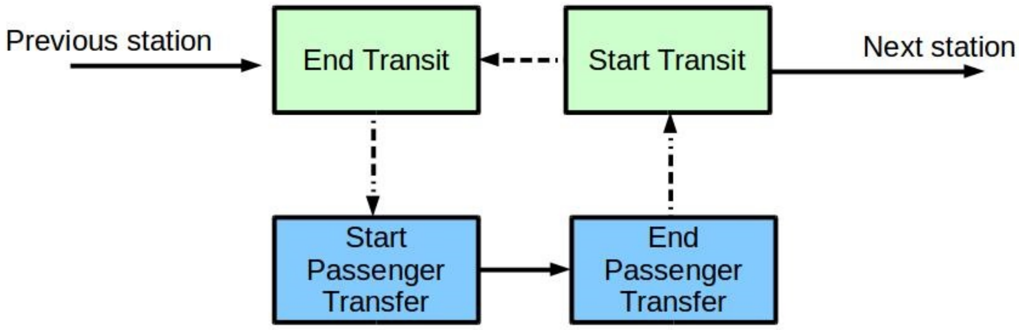

Figure 6: Event graph of standard stations.

Start Passenger Transfer

At the start of passenger transfer, the amount of waiting passengers is updated. The number of passengers leaving the tram is updated, and the number of passengers entering the tram is calculated to be the minimum of the number of passengers waiting at the station and the maximum passenger capacity of the tram (420) minus the number of passengers in the tram after the disembarking passengers have left.

When this is done, an End Passenger Transfer event is scheduled.

End Passenger Transfer

The number of passengers in the tram is updated using the in- and out-going passengers calculated in the Start Passenger Transfer event.

Then, an iterative embarkation (but not disembarkation) is started to take into account passengers arriving in the time that the previous embarkation and disembarkation took. For the runs shown, this iterative process was only done once; tests with multiple iterations showed no differing results.

After this is done, a Start Transit event is started immediately.

Start Transit

The travelling time towards the next station is calculated, and an End Transfer event is scheduled at the next station. The tram is then moved from the station to clear the platform. Then, if there are any trams waiting to enter the station, an End Transit event is scheduled immediately.

3.3.2 Event Graph - Termini

As shown in Figure 7, there are two possible ways to start terminus events. Those, and the other events, are explained below.

Recruit Tram

If the terminus has stabling tracks, 30 seconds before a tram is scheduled to depart from the terminus, it checks whether or not one of the following is true:

- Trams are behind schedule and there is at least one free platform.
- There is one free platform, and the tram on the other platform cannot possibly depart on time due to its turnaround time.

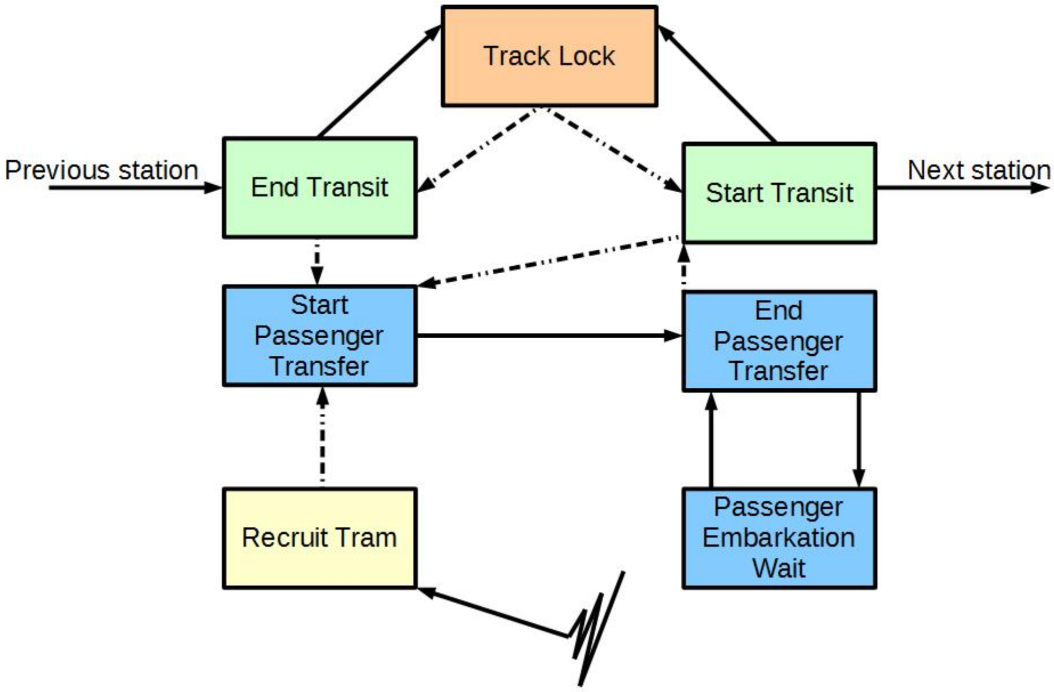

Figure 7: Event graph of termini

- There is currently no tram at the terminus.

If one of these statements is true, a tram is moved from the stabling tracks to the terminus and a Start Passenger Transfer event is scheduled immediately if no other tram is embarking passengers at the terminus.

End Transit
The End Transit events are usually scheduled from the previous station, the same as with standard stations.

During this event, it first checks if it is the first tram in line waiting to enter the station. If it is not, it will wait. If the first platform is free and both rails are unlocked, it will lock both rails and schedule a Rail Lock event for 40 seconds from now and move to the first platform. If this is not the case, it checks the second platform. If it is free and the lower rail is unlocked, it locks the lower rail, schedules a Rail Lock event for 40 seconds from now, and moves to the second platform. Otherwise, the tram will wait.

If the tram has moved to either of the platforms, it will then immediately schedule a Start Passenger Transfer event if no other tram is loading at the terminus.

Start Passenger Transfer
This event works in the exact same way as it does for standard stations.

End Passenger Transfer
As with the End Passenger Transfer event of standard stations, passengers are embarked and disembarked. However, the tram then checks conditions in the following order:

1. If it did not come from a depot, it has to at least wait its turnaround time. If it has not done so yet, it schedules a Passenger Embarkation Wait for the time where it has waited its turnaround time.

2. If the next scheduled departure is more than 5 seconds from now, it schedules a Passenger Embarkation Wait for the time where the next scheduled departure is exactly 5 seconds from now.

3. If neither of the above is the case, it iteratively loads passengers once more and then schedules a Start Departure immediately if the scheduled departure time is already past, or schedules a Start Departure for the scheduled departure time if that is not the case.

Passenger Embarkation Wait
The tram waits until this time point is reached. It then immediately schedules an End Passenger Transfer event to start iteratively embarking passengers.

Start Transit
If the tram is stationed at the first platform, it checks whether or not the upper rail is locked. If it is not, the upper rails is locked and a Track Lock is scheduled for 40 seconds from now. A travelling time is calculated and an End Transit event is scheduled for the next station and the tram is moved from the platform. If the tram is stationed and the second platform, it checks whether or not either rail is locked. If neither of them are, it locks both rails and continues in the manner described above.

It is then checked whether or not there is another tram present at the terminus. If there is, a Start Passenger Transfer event for that tram is scheduled immediately.

Track Lock
When this event occurs, 40 seconds have passed since the track has been locked. It unlocks the track, and then checks whether or not a tram is waiting to depart from the terminus. If there is, it schedules a Start Transit event immediately. If there is a tram waiting to enter the terminus, it schedules an End Transit event immediately.

3.3.3 Simulation initiation and ending

Upon simulation initiation, one Recruit Tram is initiated 30 seconds before each scheduled departure from the terminus at the Uithof.No other events are added to the queue at that time.

Simulations are ended when the event queue has been emptied, which happens after all scheduled departures have been completed and the last tram has returned at the terminus at the Uithof.

3.4 Performance Measures

To answer the questions of the study, stated in the Introduction, we have chosen to measure

1. passenger waiting times,
2. tram delay from schedule, and
3. the proportion of trams that have to wait for another tram to leave a station before it can enter.

The first two are obvious performance measures with the objective at hand. If passengers have to wait long to embark the Uithoflijn, or trams often are delayed compared to their schedule, this is likely to indicate that there are more passengers than the line can handle, there are too few trams available or they depart too infrequent. We have included the last performance measure as well to get insight on how increased tram frequencies might influence the smoothness of the tram ride. If trams run at too high frequencies, they might end up having to wait a lot for the tram in front, diminishing the benefits of high frequencies.

The main measures are averages over the entire span of a day, but for additional insight we have also gathered average waiting times and tram delays for every 30 minute interval between 06:00 and 21:30.

How the waiting times are calculated is described in Appendix - Calculations.

4 Input Analysis

The data available for tuning some of the input parameters of the simulation were:

1. Passenger count data from the bus route 12. This contains data from all bus 12 trips in September 2015 on how many passengers enter and leave at every station. In total there are 3248 trips in direction the Uithof to the Central Station, and 3258 in the other direction.

2. Forecasts of the number of passengers entering and departing a tram at every station in 2020, for time categories full day, morning peak (7:00-9:00) and evening peak (16:00-18:00).

3. Driving times between each stop for the Nieuwegein tram line. For each of the 14 stops, there are 1259 measures of the driving time.

We used this data to determine how to model the following variables:

- Driving time of trams. This is the time a tram uses on travelling from one station to another, without the time it uses while standing still on a stations.

- Passenger arrival rates over the day and at different station.

- Probabilities for a passenger departing at each station for different times of the day.

In addition, a stochastic model for the dwell time of the trams at each station is employed, which depend on the quantities above. The dwell time of a tram is the time from a tram arrives at a station till it leaves. We also used the data to calculate expected dwell time for schedule creation.

In the following sections, we explain our results from this analysis and argue for the stochastic models used in the simulations.

4.1 Driving time of trams

To determine the probability distribution of the driving time for the Uithoflijn, we looked at the driving time data for the Nieuwegein line. These lines have an important difference in that the Uithoflijn will not be blocked by traffic, whereas stretches of the Nieuwegein line can be, hence the latter has more variable driving times. Another difference to note is that the highest average driving time for the Niewegein line is 143 seconds, while it is 243 seconds for the Uithoflijn, which makes for difficulties when trying to generalize the analysis to the Uithoflijn. The last word of warning when it comes to applying these data to the Uithoflijn is that we only have data for 14 distances. For these reasons the analysis of the driving times is based just as much on our judgement of the differences between the lines as formal statistical arguments.

Figure 8 shows density plots of the driving times between each station. Besides a few stations, the driving times look normally distributed. For the reasons in the previous paragraph, we took this as sufficient evidence to assume that the driving times are normally distributed. Figure 17 in the Appendix also show a Q-Q-plot of one of the travel distances.

The variance of the distributions appears to increase as the average driving time increases. This makes sense since a longer distance creates room for more disturbances in the driving time. To find out how the variance is connected to the average driving time, we tried different linear regression models with the standard deviation of the driving time as response and different functions of the average driving time as explanatory variables. Because of the traffic light differences of the lines we removed the five travel distances with highest variance to average ratio, to only use the data that are more comparable to the Uithoflijn. Figure 9 shows the results.

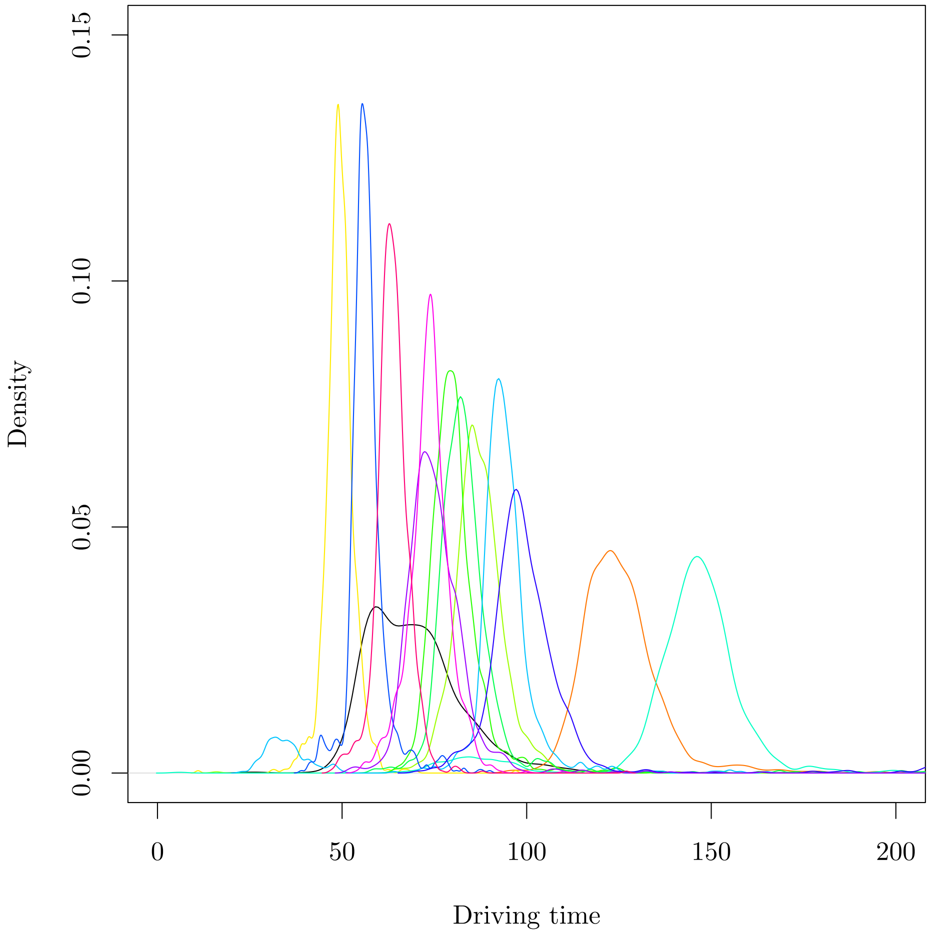

Figure 8: Density plots of the driving times between each station for the Nieuwegein tram line

Because we had so few travel distances to go by, a simple model is usually the safest choice to avoid overfitting. Therefore we decided that the model sd $= \beta_1$avg was the one that would generalize best to the Uithoflijn, with $\beta_1 = 0.108$. Also, since less variance is to be expected at the Uithoflijn and that there are no data on distances greater than 143 seconds, we decided to use the following relation between the average driving time and the standard deviation:

$$\text{sd} = \begin{cases} \beta_1 \text{avg}, & \text{avg} < 150 \\ \beta_1 \cdot 150 + \sqrt{\beta_1(\text{avg} - 150)}, & \text{avg} \geq 150. \end{cases}$$

In words, after average driving time 150 seconds, the increase is dampened by taking the square root of the contribution. The right part of Figure 9 shows the function. This was also done because of pilot runs of the simulation, which showed that by generalizing the linear relationship to the distance of 243 seconds, the travel times would vary too much compared to the rest.

4.2 Dwell time of trams

According to domain expert Marcel van Kooten Niekerk from QBuzz, the appropriate model for dwell time D is a shifted Gamma-distribution parametrized by its shape, mean and minimum value (See Appendix - Calculations for the relationship between this . Specifically,

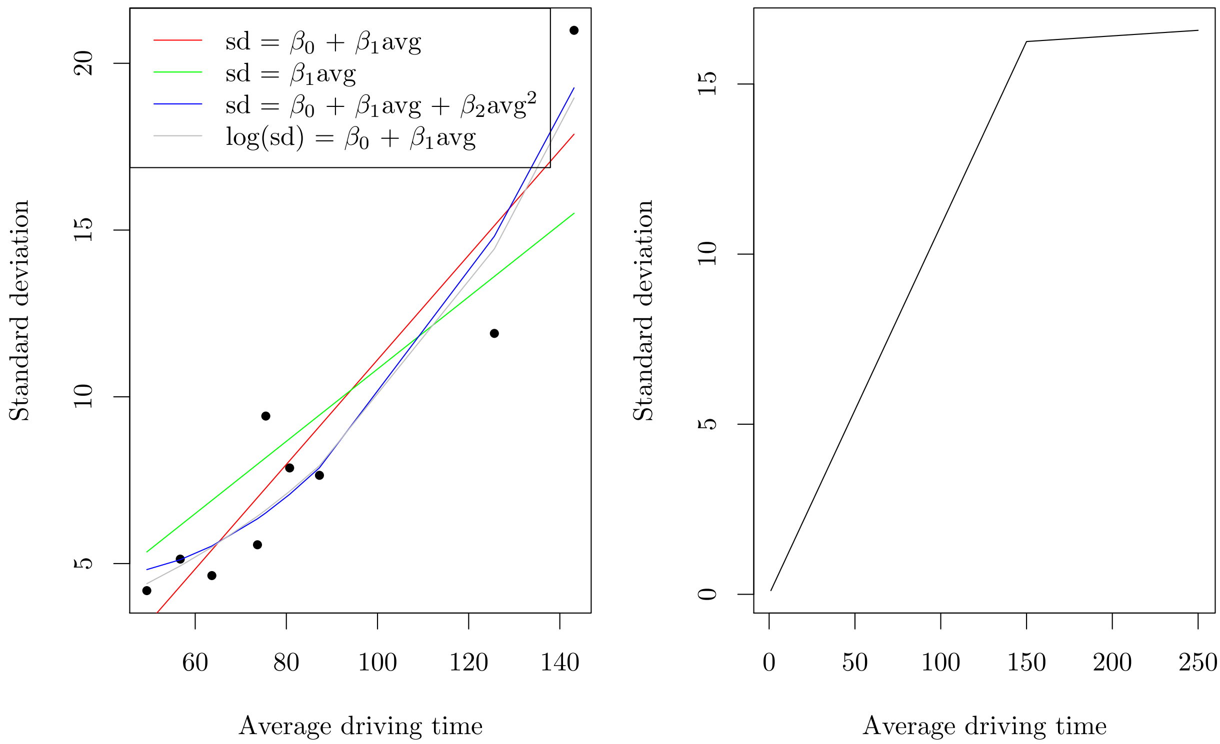

Figure 9: **Left:** The results of regressing the standard deviation of the driving time on different functions of the average with five high variance travel distances removed. **Right:** The model of standard deviation as a function of average driving time used in the simulation.

$D \sim \text{Gamma}(m, \mu, \gamma)$ with

$$\begin{aligned} \mu &:= E[D] = 12.5 + 0.22P_{in} + 0.13P_{out} \\ m &= 2 \\ \gamma &:= \min(D) = 0.8 \cdot \mu, \end{aligned}$$

where P_{in} and P_{out} are the number of passengers entering and leaving the tram respectively.

For the sake of clarity, the relation between the shifted Gamma-distribution and the regular two-parameter Gamma-distribution (as in [1, p. 289]) is that if

$$D \sim \text{Gamma}(m, \mu, \gamma) \text{ and } X \sim \text{Gamma}(m, \mu - \gamma),$$

then $D = \gamma + X$. In this way $E[D] = \gamma + E[X] = \gamma + \mu - \gamma = \mu$ as defined. This is also the relation we have used to generate pseudo-random numbers from the shifted Gamma-distribution.

The expected average dwell time over all stations in direction from De Uithof to the Central Station was calculated from the forecast data to be 17 seconds. The dwell time data was also used in creating timetables for tram departures. To have some leeway in the schedule creation, we decided to multiply this quantity by 1.5.

4.3 Passenger arrivals

To model passenger arrivals we split the task into three parts:

i) Average number of arrivals for the whole tram line per day,

ii) distribution of arrival rate over time and

iii) distribution of arrival rate over stations, depending on time of the day.

The reason for this split is based on the type of data available and an easy way to scale the experiments. For a specific point in time and station, we assume passengers arrive according to a Poisson process. A Poisson process is a standard choice for modelling number of arrivals in a time period. It assumes that passengers arrive independently, one-by-one, with an exponentially distributed time between each arrival. The bus data indicates that the variance of the number of people arriving at each time and station is higher than the mean, which is evidence against a Poisson distribution, but for the sake of not overcomplicating the model, we have still used the Poisson distribution.

However, arguably more important than the manner people arrive, is estimating the rate at which they arrive, because if the mean number of arrivals are correct, it does not really matter if people "actually" arrived in singles or couples in our case.

Since we use different arrival rates over the course of a day, the complete process is a non-homogeneous Poisson process. We chose to let the arrival rates vary over the day because the bus 12 data showed highly varying arrival rates over the course of a day, as Figures 10 and 11 illustrates. Since the bus 12 takes almost the same route as the Uithoflijn will, we found it important to capture the peak periods people want to use the tram line to get realistic estimates of performance. For example, our model take into account that many students at De Uithof have lectures that finish at the exact same time, and therefore arrive at much higher rates than during the lectures.

To estimate the distribution of arrivals over time we first summed the number of arrivals per time over all bus stops. Dividing this by the daily number of arrivals gives a distribution of bus usage per time, which then can be scaled to any daily number of arrivals.

To convert this distribution of bus usage over time into a model for the tram we averaged the average number of arrivals over batches of five time intervals. This stabilizes the average usage some, avoids severe overfitting and ensures that enough data is used in each estimate. When we have data from a month of bus usage, batching the average number of arrivals over five time points gives 150 data points for each estimate of the arrival rate per time period, which should give fairly reliable estimates. Figures 10 and 11 show the resulting batched means used as an estimate of how the arrivals would distribution over the day in the simulation. By knowing how long each of the intervals are, this distribution can be converted into rates per second for instance.

We used the forecast data to calculate how the arrivals distribute over the different stations. Because the forecast contains estimates of how many will arrive at each station over the entire day, during 07:00-09:00 and 16:00-18:00, we calculated how the arrivals would distribute over the stations for the intervals 06:00-07:00, 07:00-09:00, 09:00-16:00, 16:00-18:00 and 18:00-21:30.

From these distributions of arrivals over time and stations, one can generate the number of arrivals for a tram at for example Heidelberglaan in direction of the Central Station at 18:00 in the following way:

1. Say 20000 people use the tram line in a day on average, and let x be time in seconds since the last tram let people enter, r_t the proportion of arrivals that should happen in the current time interval, and r_s the proportion of arrivals that should happen at this station at this time. Also, let l_t be the length in seconds of the interval r_t is the arrival proportion of.

2. Then the expected number of arrivals for this tram, call it $E[N]$, is

$$E[N] = 20000 \cdot \frac{r_t x}{l_t} \cdot r_s$$

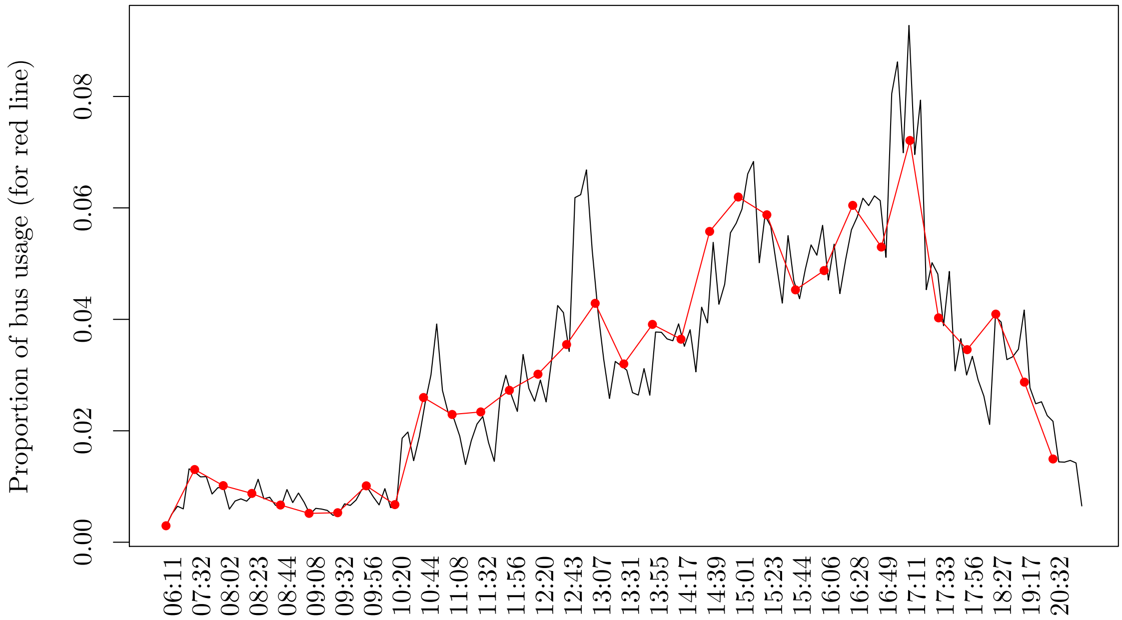

Figure 10: The red line is the distribution of passengers over time used in the simulation in the Uithof -> UCS direction, based on taking batch means of the number of passengers arriving over five by five time points. For comparison, the black line shows an appropriately scaled version of the distribution for all time points.

4.4 Passenger departures

To model how many passengers would depart the tram at every stop, we used a Binomial distribution. What this means is that at every station the tram stops, a trial of leaving or staying is performed for every passenger, and the binomial distribution outputs the number of people leaving. The probabilities of a passenger leaving were calculated from the forecast data for the same intervals as with the arrivals.

The probability of a person leaving the tram at station j is denoted by $P(L_j = 1)$, and not leaving by $P(L_j = 0)$. In this case the probabilities needed as input in the binomial distribution are the conditional probabilities

$$P(L_j = 1 | L_{j-1} = 0, \ldots, L_1 = 0).$$

In other words, the probability of a person leaving at stop j given that the person has not left at an earlier station. See the Appendix - Calculations for how these can be calculated from the given probabilities $P(L_j = 1)$.

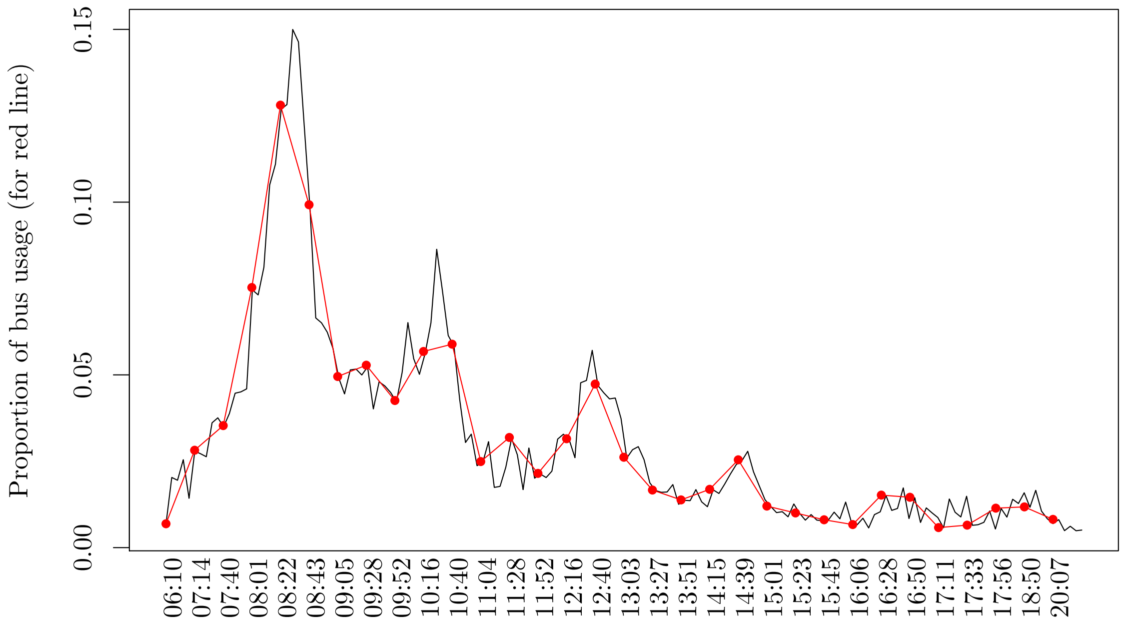

Figure 11: The red line is the distribution of passengers over time used in the simulation in the UCS -> Uithof direction, based on taking batch means of the number of passengers arriving over five by five time points. For comparison, the black line shows an appropriately scaled version of the distribution for all time points.

5 Experiments and results

5.1 Experiments

To study growth scenarios of the Uithoflijn, we conducted experiments for two different tram frequencies and a series of increasing passenger numbers, shown in Table 4. These series were chosen by starting at 20000 passengers in each direction, increasing this number by 2000 for every new run until the average waiting time exceeded 400 seconds. We assumed that an equal amount of people travel in each direction based on the forecast data.

4/16/4, 12 trams	40	44	48	52	56	60	64	68	72	76	80				
5/20/5, 14 trams	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100

Table 4: Experiment setups with total amount of daily passengers in thousands. 4/16/4 means 4 trams/hour between 06:00-07:00, 16 trams/hour between 07:00-19:00 and 4 trams between 19:00-21:00, and similarly for 5/20/5.

In this case, an obvious simulation run length is a day, which makes the simulation a terminating one. To get reliable estimates for every setup, each experiment comprises 365 runs.

Ideally we would have liked to run both experiments with the available 12 trams. The reason for running the 5/20/5 schedule with 14 trams is that 12 trams proved to be too few trams to support this frequency. The average roundabout time, the time between departing from a station and till departing it anew, was calculated to be 45 minutes. When 12 trams leave with 3 minutes in between, all trams are in transit from De Uithof after 36 minutes, implying that there probably are no trams ready to depart again for the next 9 minutes. To further solidify this claim, a full experiment with 12 trams, 40000 passengers and 5/20/5 tram frequency had an average tram delay of 3505 seconds.

45/3 = 15 trams are needed to support the 5/20/5 schedule fully. However, for the sake of keeping the tram number as close to the actual number of available trams we tried to run the experiments with 14 trams, to see how the stochastics influence running slightly too few trams over time. Keep in mind that this is only one more tram carriage than what is available when taking the reserves into account. To illustrate how 15 available trams would compare to 14 for high passenger numbers, we did an experiment with 15 trams, 80000 passengers and 5/20/5 schedule as well.

5.2 Results

The results of our main experiments are presented in Table 5 and Figures 12, 13, 14, 15 and 16.

Empirical confidence intervals are given for all estimates. This means that they are calculated by sorting the 365 estimates from each experiment, then the 2.5 %'th smallest value represent the lower confidence bound and the 97.5%'th smallest value the upper confidence bound. For this amount of runs, these confidence intervals are likely to be a bit wider than confidence intervals based on normal approximations, but since some of the performance measures are highly varying, we found this to be a better representation of the data at hand.

	Waiting time		Schedule deviation		Trams waiting	
Passengers	4/16/4	5/20/5	4/16/4	5/20/5	4/16/4	5/20/5
40000	134 (132, 136)	109 (106, 123)	2 (2, 3)	13 (8, 45)	2.3% (2.1, 2.5)	3.1% (2.4, 6.0)
44000	141 (137, 145)	112 (107, 141)	2 (2, 3)	20 (8, 93)	2.2% (2.0, 2.5)	3.5% (2.4, 10.6)
48000	153 (148, 159)	116 (108, 149)	2 (2, 3)	31 (9, 144)	2.2% (2.0, 2.4)	4.3% (2.4, 11.6)
52000	166 (160, 172)	125 (110, 156)	2 (2, 3)	50 (10, 216)	2.1% (1.9, 2.4)	5.7% (2.4, 12.8)
56000	182 (175, 189)	136 (117, 165)	3 (2, 3)	66 (12, 249)	2.1% (1.9, 2.3)	6.4% (2.4, 12.9)
60000	200 (191, 208)	148 (127, 173)	3 (2, 4)	81 (12, 253)	2.1% (1.8, 2.3)	7.0% (2.4, 12.7)
64000	222 (212, 233)	161 (137, 187)	3 (2, 4)	94 (14, 290)	2.0% (1.8, 2.2)	7.3% (2.4, 12.7)
68000	254 (242, 267)	178 (151, 198)	3 (3, 4)	122 (17, 311)	2.0% (1.7, 2.2)	8.2% (2.6, 13.0)
72000	294 (278, 311)	196 (167, 214)	3 (3, 4)	152 (20, 350)	1.9% (1.7, 2.1)	9.1% (2.8, 12.9)
76000	340 (322, 359)	214 (187, 233)	3 (3, 4)	176 (25, 353)	1.9% (1.7, 2.1)	9.4% (3.4, 12.7)
80000	401 (378, 424)	235 (209, 252)	3 (3, 4)	203 (33, 367)	1.8% (1.6, 2.0)	9.4% (4.0, 12.3)
84000		262 (240, 283)		219 (66, 358)		9.5% (5.6, 12.0)
88000		293 (272, 315)		233 (95, 370)		9.3% (6.0, 11.6)
92000		331 (312, 353)		237 (116, 352)		8.9% (6.6, 11.1)
96000		375 (351, 398)		236 (101, 359)		8.5% (6.2, 10.5)
100000		427 (400, 461)		226 (125, 341)		7.9% (5.7, 9.6)

Table 5: Summary of performance measures. The cells are of the form "estimate (confidence interval)".

5.3 Discussion

The results show that for the planned low intensity schedule (4/16/4 trams per hour), the tram system performs very well for the expected passenger numbers (45.000 passengers per day). Table 5 shows low average deviations from the scheduled departure times, as well as very few trams having to wait to enter the station. Average waiting times are also very close to the expected waiting time of 113 seconds.

However, for higher passenger numbers, passenger waiting times rise sharply, to the point that at 68.000 passengers per day, passengers will on average have to wait for more than the expected inter-arrival time of the trams. Since we do not know what is unacceptable waiting times, we refer the reader to Figure 12 for deciding on when the low frequency schedule is insufficient compared to the high frequency, taking the confidence intervals into account.

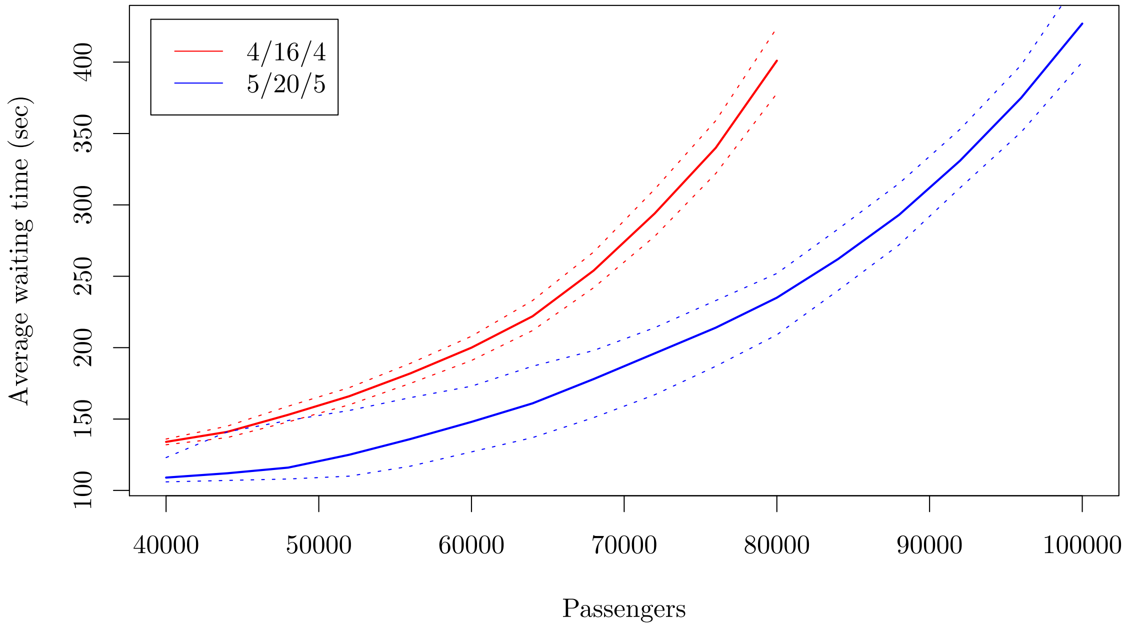

Figure 12: Average waiting times with 95% confidence bounds for the two sets of experiments.

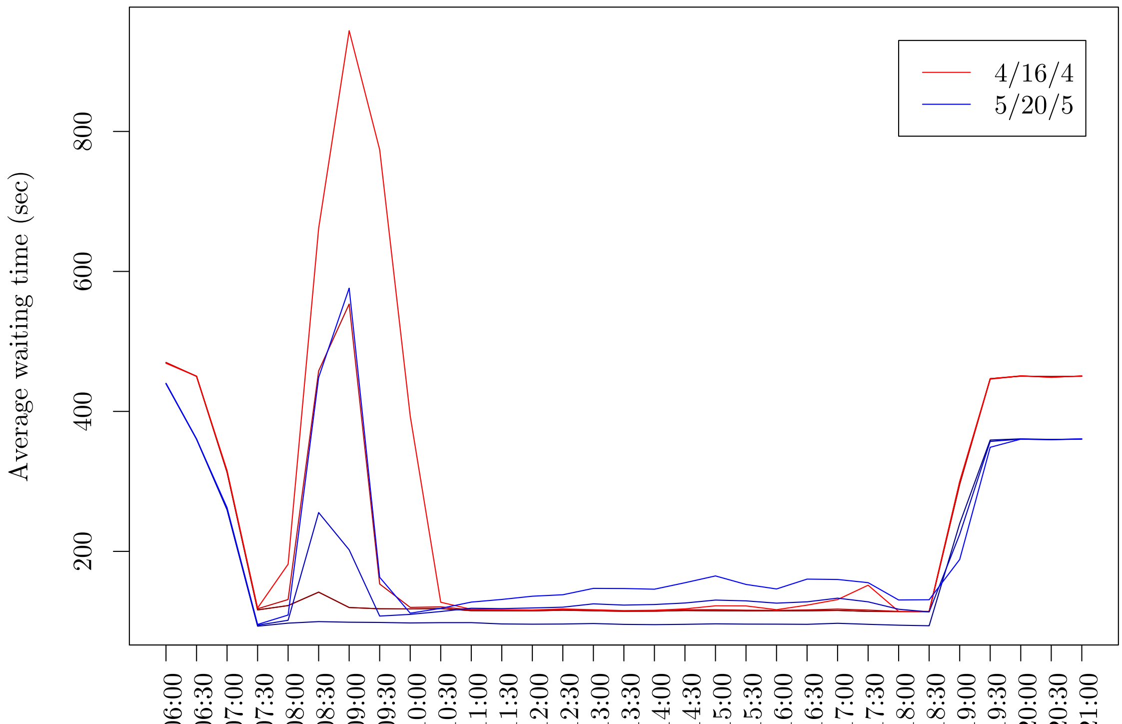

Figure 13: The three red and three blue lines are average waiting times for each 30 minute time interval, for the experiments with 40000, 60000 and 76000 passengers. Naturally, the the higher waiting times represent more passengers. The waiting times with 5/20/5 trams with 76000 passengers are very close to the waiting times for 4/16/4 trams with 60000 passengers.

At this point it is important to note that Figure 13 shows that over the entire day, the waiting times do actually not increase sharply; it's during the morning rush hour where extremely high waiting times are seen which cause the rise of the average waiting time in the entire system.

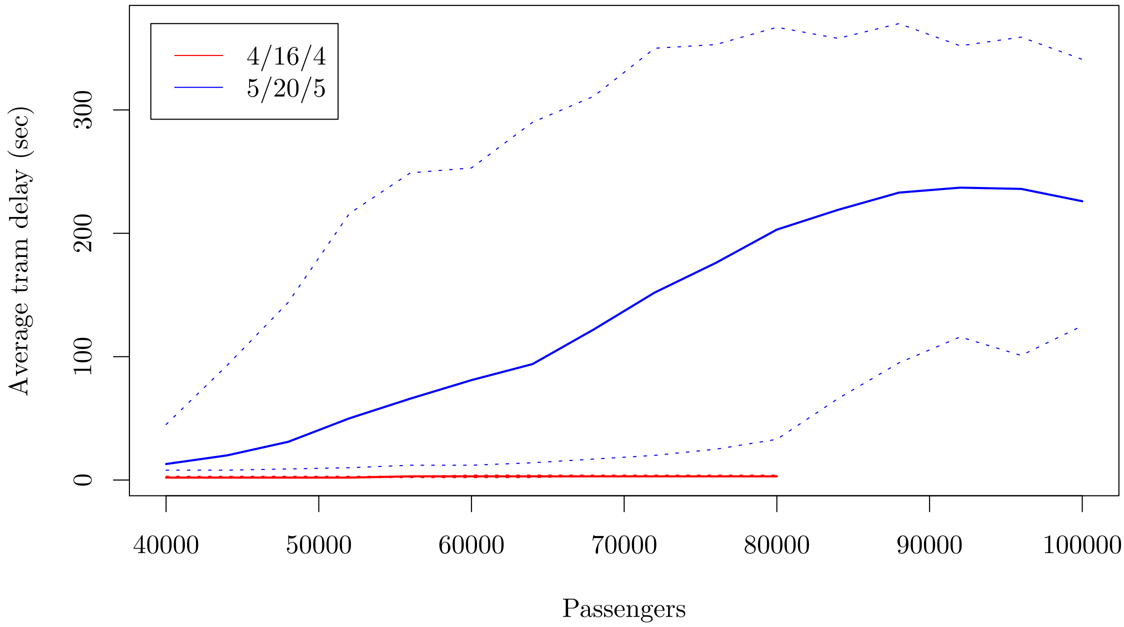

Figure 14: Deviation in seconds from the schedule with 95% confidence bounds as a measure of how punctual the line is for the two sets of experiments.

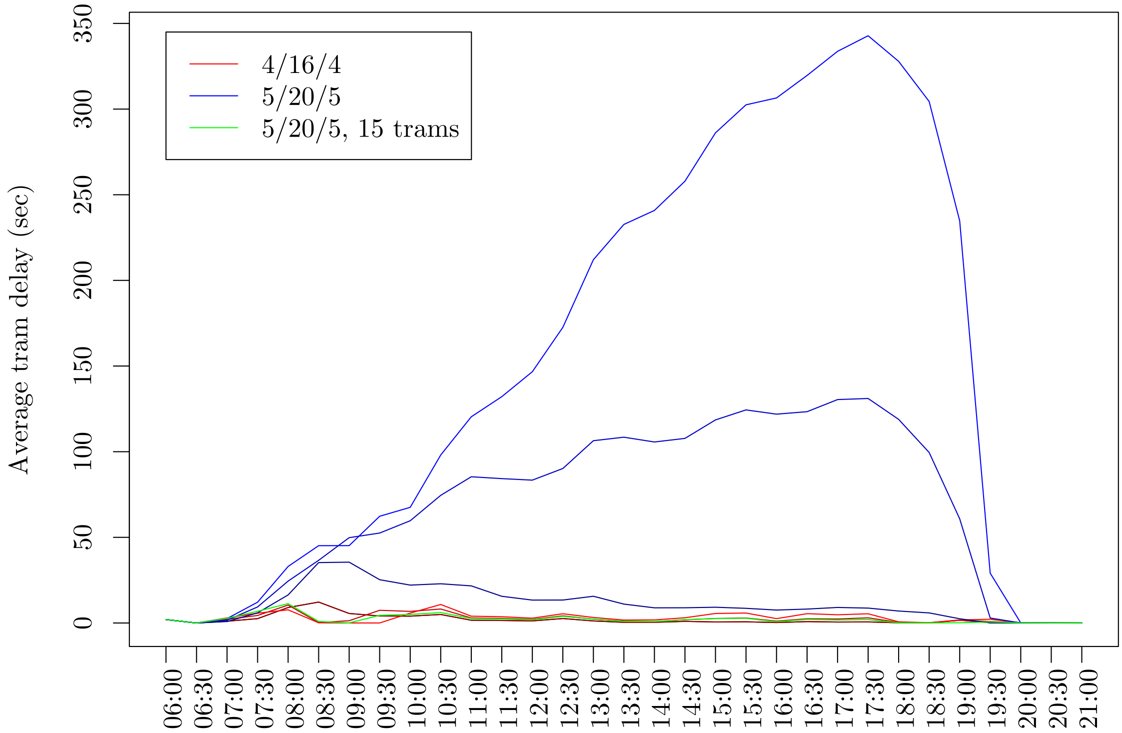

Figure 15: The three red and three blue lines are tram delays for each 30 minute time interval, for the experiments with 40000, 60000 and 76000 passengers. The green line is an experiment with 15 total trams, in stead of 14, with 80000 passengers. With 15 trams there are barely delays.

This is not surprising, seeing how during the morning rush hour nearly 170 passengers arrive per minute, given 68.000 passengers per day. This means that in the inter-departure time given

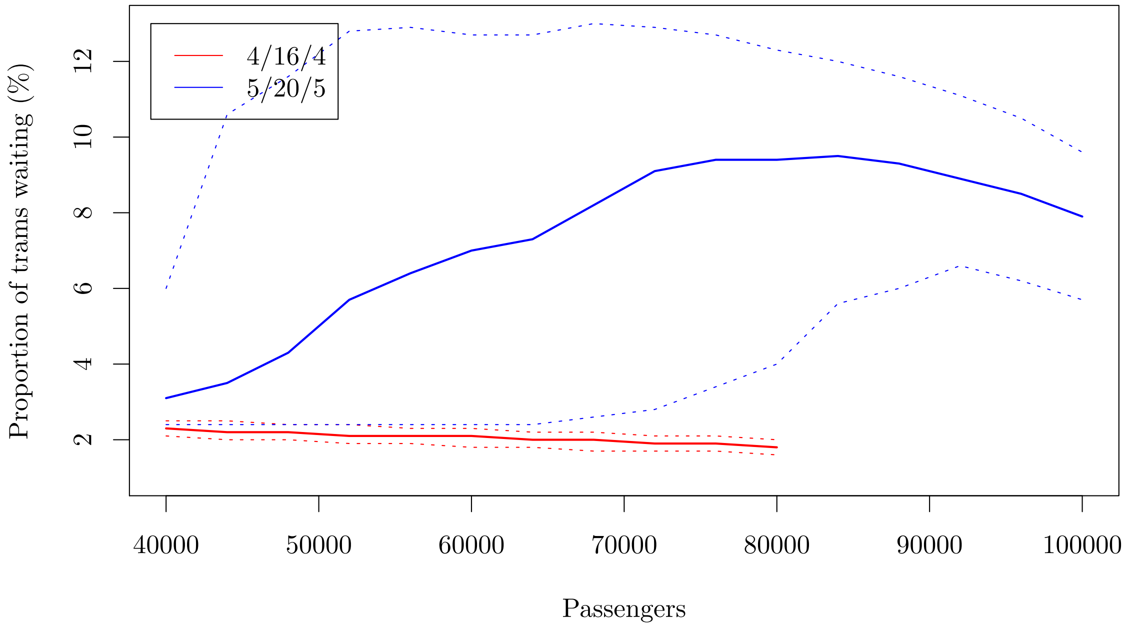

Figure 16: Proportion of tram arrivals at any station that have to wait for another tram to leave before it can go into the station with 95% confidence bounds for the two sets of experiments.

this schedule, more passengers arrive in this interval than fit in the trams.

To be able to cope with these larger amounts of passengers, a high intensity tram schedule is needed, but mostly during the morning rush from 07:30 to 09:30.

Next, the results show that while waiting times stay relatively low with a high frequency tram schedule for even the larger passenger numbers, other performance measures do not. The deviation from the schedule especially rises sharply with increasing passenger numbers (Figure 14). Moreover: Figure 15 shows that this deviation accumulates over the day.

This is the result of there not being enough trams in the system to run this schedule, as discussed in the Experiment section. Because of this disparity between the average inter-departure time a fixed number of trams can support per hour and the inter-departure time given by the schedule, the delay builds up over the day. There appear to be two potential ways to remove this buildup of delays.

The first would be to reduce the roundabout time. Although this could potentially be done by reducing the driving time, this seems infeasible. A better option would be to reduce the dwell time at stations. Although reducing the delay at regular stations is not possible, since trams leave directly after embarking and disembarking passengers, this could be achieved by reducing the turnaround time. However, no simulations have been done to confirm whether or not this solves the issue.

The second option is to increase the number of trams in the system. By running the high intensity schedule with 15 trams instead of 14, the buildup of delays can be prevented (Figure 15).

Nevertheless, another potential issue is seen in with the high intensity schedule: a large percentage of the trams have to wait for another tram to leave the station in most of the runs, even at low passenger counts (table 5, Figure 16). Figure 18 shows that nearly all of these trams have to wait at the termini. The cause of it seems to be a combination of the inter-departure time and the dwell time at the termini, which lead to a very high occupation rate of both of the platforms. This problem may be remedied by either decreasing the turnaround time or increasing the number of platforms at the termini, but this has not been verified.

We also have to comment on the extremely low average tram delay for tram schedules which can be fully supported by the number of available trams. One possible cause is that we leave relatively large amounts of leeway in the scheduling due to taking 150% of the average dwell times at the stations. This means that when a tram arrives at one of the Termini slightly delayed, it will still cause no delay in the departure of the same tram.

Another reason is probably the dynamic recruitment of trams from the stabling tracks. Because of this, trams are exactly recruited to the Uithof Terminus when needed. Earlier simulations using another of the Recruit Tram event showed long delays and waiting times in the 7 to 10 hour interval (Figures 19 and 20 in the Appendix). These were later attributed to trams not being recruited when they were actually required, which resulted in long lasting percolating delays in the system, which were slow to die out. The tram delay estimates will therefore be slightly optimistic, but we found this better than having a defect event which would wrongly influence our results. In addition, this underscores the large delays caused by having too few available trams, and the importance of having a robust schedule for when trams need to be obtained from the stabling track.

6 Conclusions

To assess the robustness of the Uithoflijn to increases in passenger numbers as well as tram frequencies, we modelled the tram line and performed simulations over a set of parameter ranges.

The simulation experiments showed that in terms of both waiting times and punctuality, the schedule with 4/16/4 trams is robust to increasing passenger numbers from the predicted 45000 passengers using the tram in 2020. Furthermore, it appears capable to support much larger passenger numbers. Compared to 2020, only an estimated one minute extra waiting time is expected with 36% more passengers.

As more and more passengers use the tram line, it is during the morning rush (07:30-09:30) from the Central Station that waiting times accumulate. Capacity problems thus seem to be solvable by increasing the capacity only at these times.

Feasible tram frequencies depend on the number of available trams and the average roundabout time of the tram line, which in turn depends on dwell time, driving time and number of passengers. To avoid large deviations from the schedule, we recommend that there are enough available trams to fully support the schedule. Average roundabout time divided by the inter-departure time of a proposed schedule can be taken as a first guideline for how many trams are needed.

A last conclusion that can be drawn from the simulations is the importance of optimally scheduling the recruitment of trams from the stabling tracks.

Future research

Although the results show that the Uithoflijn should be well capable of handling the forecast amount of passengers in 2020, and even well beyond, these simulations have not included the second phase of the Uithoflijn, in which this tram line will be integrated into two existing tram lines in and around Utrecht. To find potential problems and pitfalls of the operation of the integrated tram lines, further modelling and simulations are required.

Furthermore, as discussed in previous sections, our simulations show the importance of a robust schedule for obtaining trams from the stabling tracks. In our study, a dynamic on-demand method was implemented to obtain these trams. However, this is not feasible for the actual tram line. Therefore, further studies to obtain good, robust schedules for this would be advisable.

Lastly, in this study, no attention was given to the robustness of the system to perturbations such as short tram breakdowns. However, Figures 19 and 20 in the Appendix do indicate that delays have a tendency to die out slowly; delayed trams tend to be delayed for the next few rounds they make. Furthermore: it is possible that such delays would percolate and spread throughout the system and cause delays over large periods of time.

To ensure robustness against such perturbations, further modelling should be done to assess the robustness of the Uithoflijn against these perturbations, as well as methods of relieving and preventing them.

7 Appendix - Interview minutes

Capacity of trams: 2x210.

Last/first time of tram: 6:00 - 21:30.

Number of trams:

- 12 trams of 2 carriages
- 3 in reserve

Modelling: Only tram between ucs and AZU.

Rush hour: No real rush hours?

New answer: Trams to operate:

- 6:7 : 4 per hour
- 7:19: 16 per hour / 20 per hour
- 19: 21:30 : 4/ hour

Questions they want answered:

- How far can we increase the frequency of trams?
- Are the tram numbers feasible and the one above.
- Look at growth scenarios (more trams, more people).

Time embarking/disembarking?
Gamma distribution with parameters:
avg := d = 12.5 secs + 0.22 * passengers in + 0.13 * passengers out.
k = 2.
min: 0.8 * d.

Tram data: Driving time between adjacent stops.

Tram failures?

- Trams cannot overtake; IE: traffic stops.
- Get repaired.
- No need to simulate it.

Data on website: Expected passengers per hour of new stops. Use data from bus 12 for passenger distr over day etc.

Minimum distance between trams: 100 metres.

One tram per platform.

Dwelling time at end points?

- 4 minutes.

- In reality: can be done in three minutes.
- Travelling time included.

End point capacity: 2 tracks, one tram per track.

No depot at UCS.

When delayed: 40 seconds driving time to platform?

Trams stop at each stop.

Time table? Can design one yourself. Frequency as input, build time table from that.

Distances between stops in Nieuwegein? On website.

Start from AZU, last tram also from azu and then back.

Trams who can depart from platform before they should according to schedule? They will wait for the schedule. Assume they never leave too early.

All 12 start at AZU.

Time of day: No influence on driving time.

No station capacities for number of passengers.

Passengers do not leave after waiting for a long time.

Switches occupied of a tram is on there. Keep this in mind.

8 Appendix - Figures

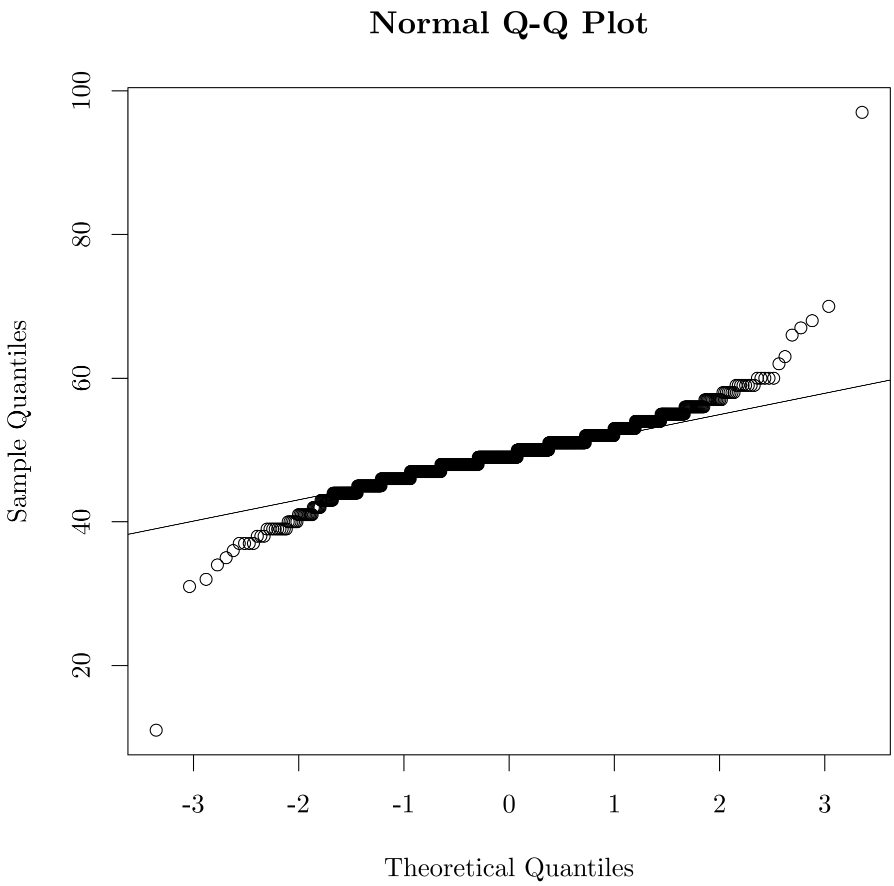

Figure 17: Q-Q-plot of driving time data for Meiplein. Not obviously normal distribution, but there could be explanations for the outliers.

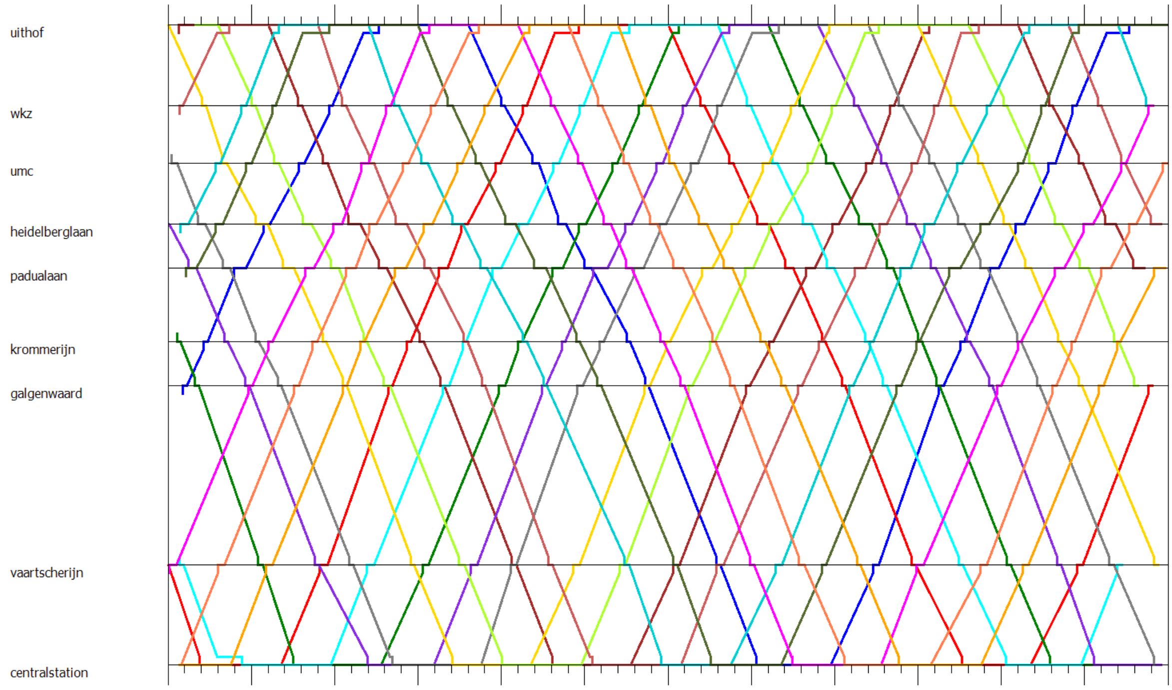

Figure 18: A timeplot of a high intensity schedule simulation for verification purposes. Time shown: 11:00 to 12:00.

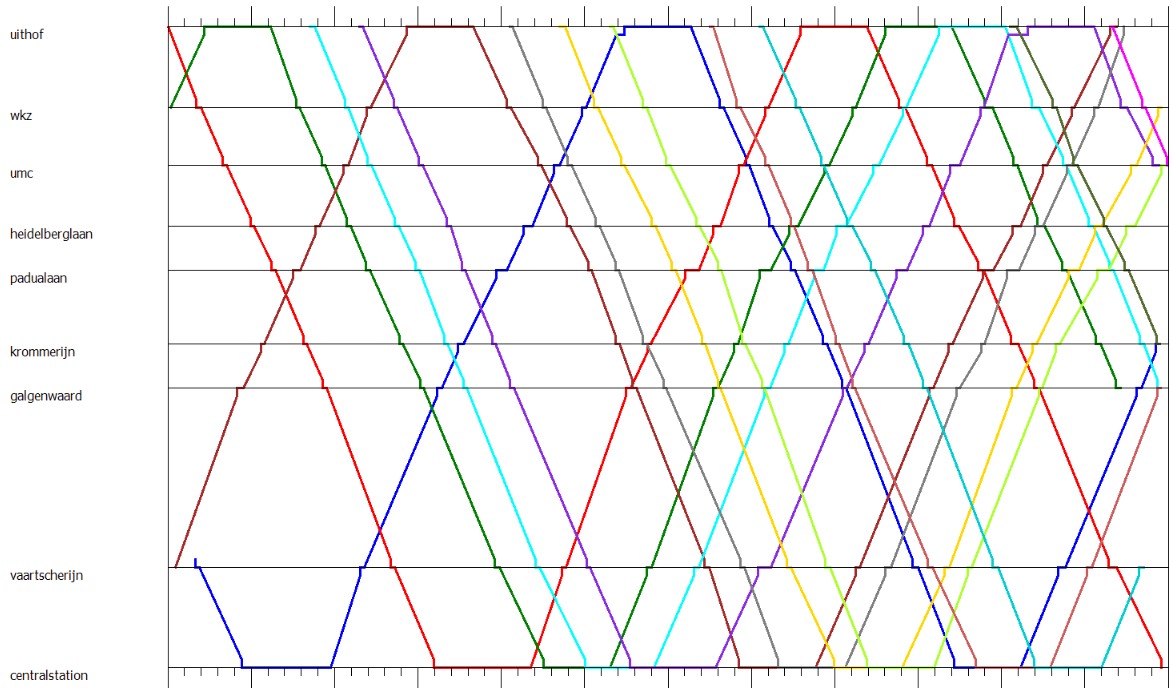

Figure 19: A timeplot of a high intensity schedule simulation with 80000 passenger for an **older event handler of tram recruitment**. Time shown: 07:00-08:00.

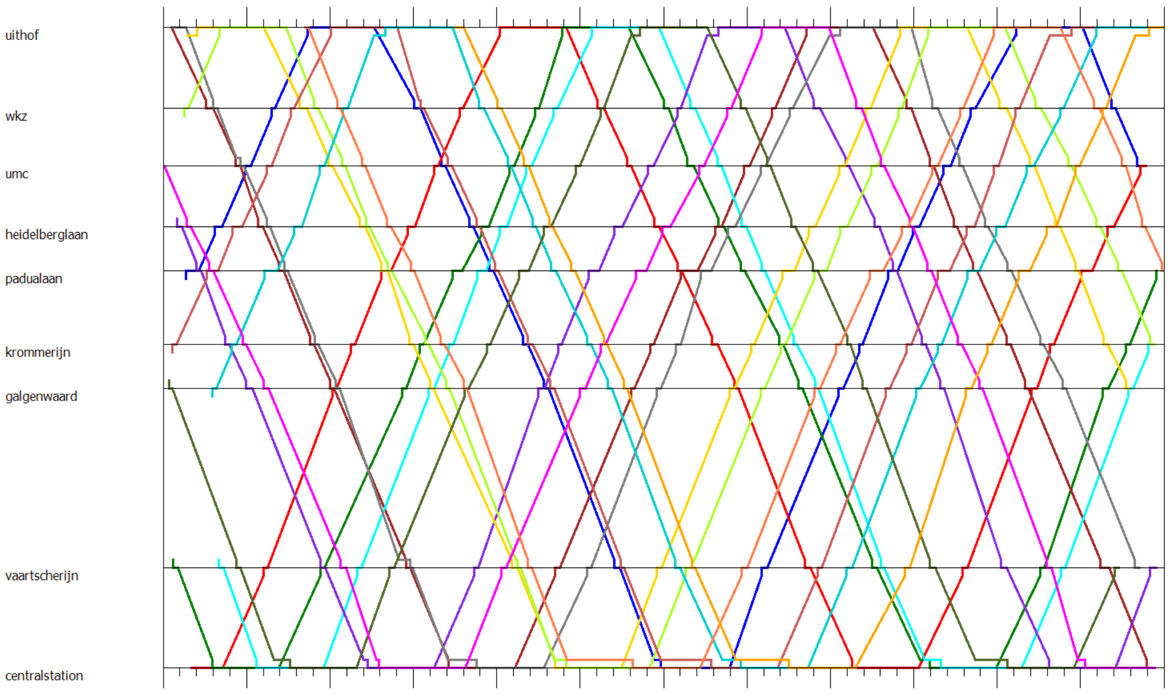

Figure 20: A timeplot of a high intensity schedule simulation with 80000 passengers **older event handler of tram recruitment**. Time shown: 08:00-09:00.

9 Appendix - Calculations

9.1 Conditional probabilities for passenger departures

These can be calculated from the probabilities $P(L_j = 1)$, which are the ones given in the forecast data. This procedure is based on the observation that for $j = 2, \ldots, 9$:

$$\begin{aligned} P(L_j = 1 | L_1 = 0) &= \frac{P(L_j = 1)P(L_1 = 0 | L_j = 1)}{P(L_1 = 0)}, \text{ where } P(L_1 = 0 | L_j = 1) = 1 \\ &= \frac{P(L_j = 1)}{1 - P(L_1 = 1)} \\ &= \frac{P(L_j = 1)}{\sum_{j=2}^{9} P(L_j = 1)} \end{aligned}$$

A similar argument can be used for $j = 3, \ldots, 9$ to find $P(L_j = 1 | L_2 = 0, L_1 = 0)$ from the $P(L_j = 1 | L_1 = 0)$'s, and so forth. See code on passenger data lines 180-190 for the computations.

9.2 Waiting times

Since we are not modelling every passenger separately, the calculation of average waiting time is not immediately apparent. Let $\bar{W}$ denote the total average waiting time, while $\bar{W}_i$, $i = 1, \ldots, k$ is the average waiting time between each tram loads in passengers, happening k times in a run. Also, n_i is the number of passengers that arrive at a station between loading number $i - 1$ and i, t_i the time between these loadings, and m_i are the number of passengers that could not fit in a tram at loading i, therefore having to wait for the next one. The expected amount of waiting done by all passengers between loading $i - 1$ and i is then

$$s_i := t_i m_{i-1} + \frac{1}{2} t_i n_i,$$

because the passengers arrive according to a Poisson process. Consequently,

$$\bar{W}_i = \frac{s_i}{m_{i-1} + n_i},$$

and

$$\bar{W} = \sum_{i=1}^{k} a_i \bar{W}_i, \quad a_i = \frac{m_{i-1} + n_i}{\sum_{i=1}^{k} n_i}$$

In this way

$$\bar{W} = \sum_{i=1}^{k} \frac{m_{i-1} + n_i}{\sum_{i=1}^{k} n_i} \frac{s_i}{m_{i-1} + n_i} = \frac{\sum_{i=1}^{k} s_i}{\sum_{i=1}^{k} n_i},$$

which is the total amount of waiting divided by the total amount of people the arrive in a day. To get per 30 minute averages, we combine the $\bar{W}_i$'s similarly for the i's that fall into each interval.

The above paragraph describes the calculations at a single station. When these calculations are combined to form averages over all stations and runs in an experiment, they are weighted by how many arrived at each station or in each day/run of an experiment.

10 Appendix - Test file results

	Waiting time		Schedule deviation		Trams waiting	
Files	4/16/4	5/20/5	4/16/4	5/20/5	4/16/4	5/20/5
1	186 (183, 188)	150 (148, 152)	1 (0, 1)	5 (4, 7)	2.6% (2.4, 2.8)	2.9% (2.7, 3.1)
2	172 (170, 174)	139 (137, 141)	1 (0, 1)	5 (4, 7)	2.6% (2.4, 2.8)	2.9% (2.7, 3.2)
3	170 (167, 173)	144 (138, 153)	1 (1, 1)	24 (12, 53)	2.1% (1.9, 2.3)	4.3% (2.7, 6.7)
4	171 (167, 174)	150 (140, 162)	1 (1, 2)	42 (18, 83)	2.0% (1.8, 2.2)	5.8% (3.2, 9.7)
5	196 (192, 201)	160 (156, 164)	1 (1, 1)	7 (5, 12)	2.5% (2.3, 2.7)	2.9% (2.6, 3.6)
6	197 (190, 205)	162 (152, 176)	1 (1, 2)	53 (24, 108)	2.0% (1.8, 2.2)	6.7% (3.7, 11.4)
25	162 (160, 164)	133 (131, 136)	0 (0, 1)	8 (5, 11)	2.4% (2.2, 2.6)	2.9% (2.6, 3.5)

Table 6: Summary of performance measures for the test input files. The cells are of the form "estimate (confidence interval)".

References

[1] Law, M. A. 2015 *Simulation Modeling and Analysis* 5. edition, McGraw-Hill Education, New York.