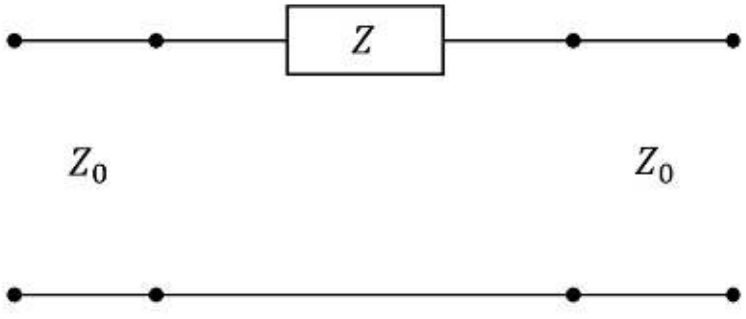
1 Practice Problems

Practice Problem 1

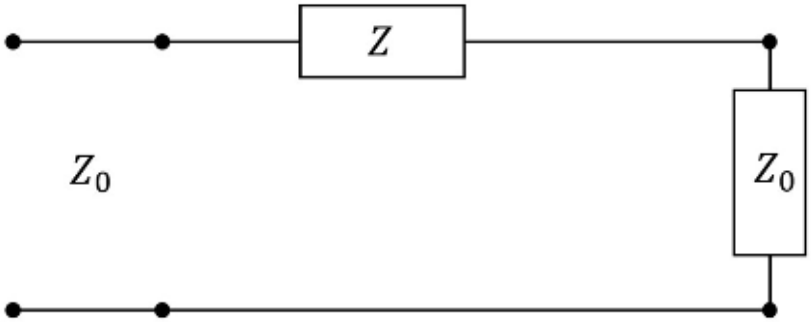
1. Find the scattering parameter matrix of the following network.



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L = S_{22}$
- $\Gamma_L = \frac{(Z+Z_0)-Z_0}{(Z+Z_0)+Z_0} = \frac{Z}{Z+2Z_0} = S_{11}$
- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L = S_{12}$

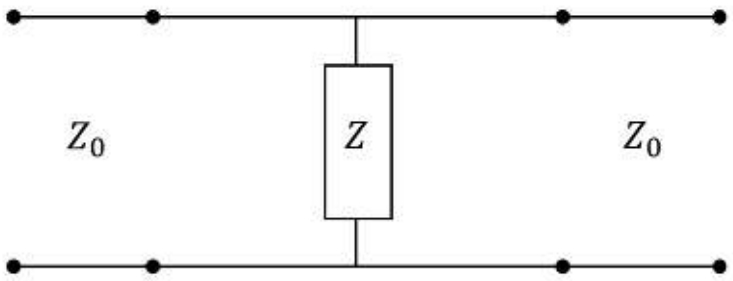


Practice Problem 1 (cont.)

- $S = \begin{bmatrix} \frac{Z}{Z+2Z_0} & 1+\frac{Z}{Z+2Z_0} \\ 1+\frac{Z}{Z+2Z_0} & \frac{Z}{Z+2Z_0} \end{bmatrix}$

Practice Problem 2

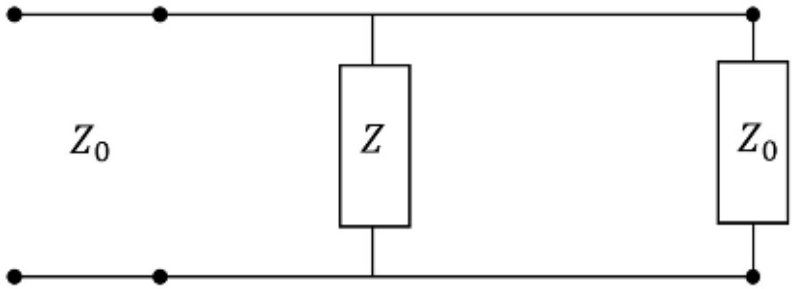
2. Find the scattering parameter matrix of the following network.



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L = S_{22}$
- $\Gamma_L = \frac{(Z||Z_0)-Z_0}{(Z||Z_0)+Z_0} = \frac{\left(\frac{ZZ_0}{Z+Z_0}\right)-Z_0}{\left(\frac{ZZ_0}{Z+Z_0}\right)+Z_0}$
- $\Gamma_L = \frac{ZZ_0-ZZ_0-Z_0^2}{ZZ_0+ZZ_0+Z_0^2} = \frac{-Z_0^2}{2ZZ_0+Z_0^2} = S_{11}$

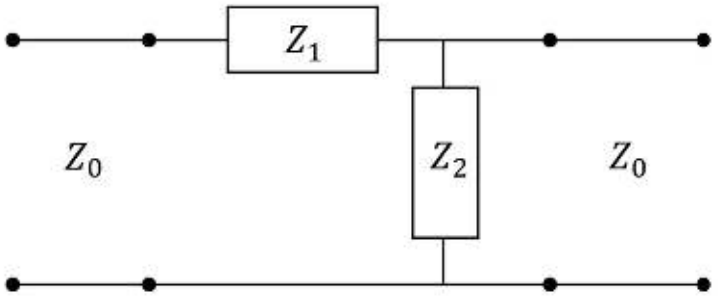


Practice Problem 2 (cont.)

- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L = S_{12}$
- $1 + \Gamma_L = 1 - \frac{Z_0^2}{2ZZ_0+Z_0^2} = S_{21}$
- $S = \begin{bmatrix} \frac{-Z_0^2}{2ZZ_0+Z_0^2} & 1 - \frac{Z_0^2}{2ZZ_0+Z_0^2} \\ 1 - \frac{Z_0^2}{2ZZ_0+Z_0^2} & \frac{-Z_0^2}{2ZZ_0+Z_0^2} \end{bmatrix}$

Practice Problem 3

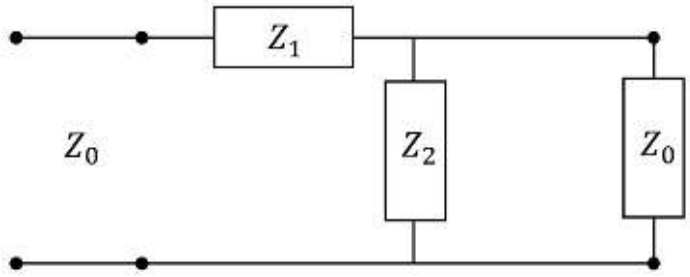
3. Find the scattering parameter matrix of the following network.



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

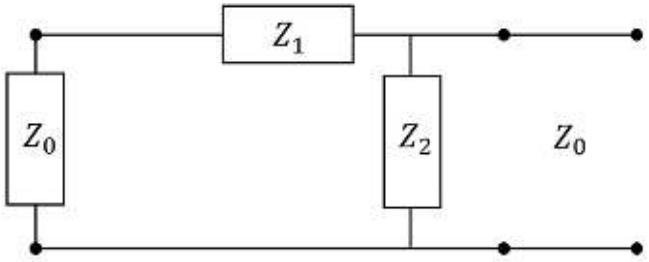
- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$



- $\Gamma_L = \frac{(Z_1 + Z_2 \| Z_0) - Z_0}{(Z_1 + Z_2 \| Z_0) + Z_0} = \frac{\left(Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}\right) - Z_0}{\left(Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}\right) + Z_0}$
- $\Gamma_L = \frac{Z_1 Z_2 + Z_1 Z_0 + Z_2 Z_0 - Z_2 Z_0 - Z_0^2}{Z_1 Z_2 + Z_1 Z_0 + Z_2 Z_0 + Z_2 Z_0 + Z_0^2} = \frac{Z_1 Z_2 + Z_1 Z_0 - Z_0^2}{Z_1 Z_2 + Z_1 Z_0 + 2 Z_2 Z_0 + Z_0^2} = S_{11}$

Practice Problem 3 (cont.)

- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - \frac{Z_1Z_2 + Z_1Z_0 - Z_0^2}{Z_1Z_2 + Z_1Z_0 + 2Z_2Z_0 + Z_0^2}$
- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$



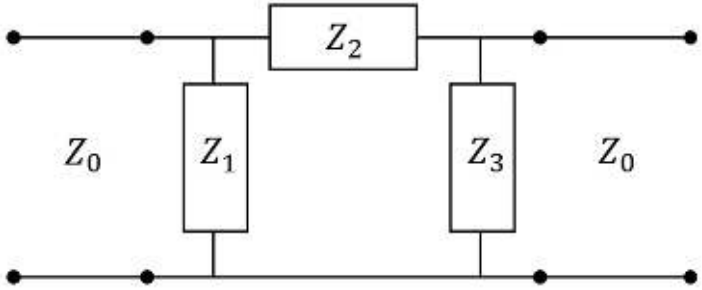
- $\Gamma_L = \frac{((Z_0+Z_1)||Z_2)-Z_0}{((Z_0+Z_1)||Z_2)+Z_0} = \frac{\left(\frac{Z_0Z_2+Z_0Z_1}{Z_0+Z_1+Z_2}\right)-Z_0}{\left(\frac{Z_0Z_2+Z_0Z_1}{Z_0+Z_1+Z_2}\right)+Z_0}$
- $\Gamma_L = \frac{Z_0Z_2+Z_0Z_1-Z_0^2-Z_0Z_1-Z_0Z_2}{Z_0Z_2+Z_0Z_1+Z_0^2+Z_0Z_1+Z_0Z_2} = \frac{-Z_0^2}{Z_0^2+2Z_0Z_1+2Z_0Z_2} = S_{22}$

Practice Problem 3 (cont.)

- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - \frac{Z_0^2}{Z_0^2+2Z_0Z_1+2Z_0Z_2}$
- $S = \begin{bmatrix} \frac{Z_1Z_2+Z_1Z_0-Z_0^2}{Z_1Z_2+Z_1Z_0+2Z_2Z_0+Z_0^2} & 1 - \frac{Z_0^2}{Z_0^2+2Z_0Z_1+2Z_0Z_2} \\ 1 - \frac{Z_1Z_2+Z_1Z_0-Z_0^2}{Z_1Z_2+Z_1Z_0+2Z_2Z_0+Z_0^2} & \frac{-Z_0^2}{Z_0^2+2Z_0Z_1+2Z_0Z_2} \end{bmatrix}$

Practice Problem 4

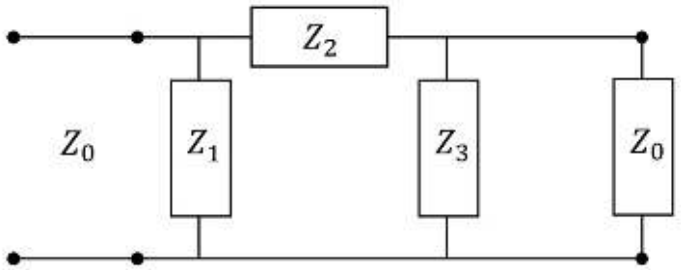
4. Find the scattering parameter matrix of the following network. Assume $Z_0 = 50\Omega, Z_1 = 25\Omega, Z_2 = 10\Omega, Z_3 = 40\Omega$



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z_1 || [Z_2 + (Z_3 || Z_0)]) - Z_0}{(Z_1 || [Z_2 + (Z_3 || Z_0)]) + Z_0}$

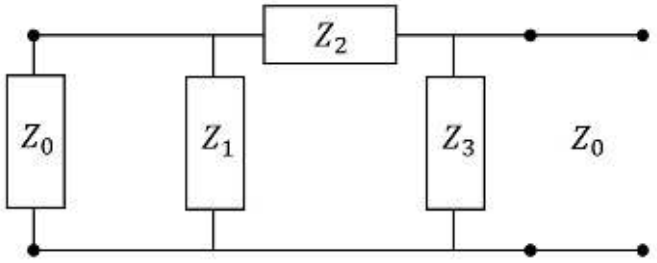


- $Z_1 || [Z_2 + (Z_3 || Z_0)] = Z_1 || \left[Z_2 + \frac{Z_3 Z_0}{Z_3 + Z_0}\right] = \frac{Z_1 Z_2 + \frac{Z_1 Z_3 Z_0}{Z_3 + Z_0}}{Z_1 + Z_2 + \frac{Z_3 Z_0}{Z_3 + Z_0}}$

Practice Problem 4 (cont.)

- $Z_1 \| [Z_2 + (Z_3 || Z_0)] = \frac{Z_1 Z_2 Z_3 + Z_1 Z_2 Z_0 + Z_1 Z_3 Z_0}{Z_1 Z_3 + Z_1 Z_0 + Z_2 Z_3 + Z_2 Z_0 + Z_3 Z_0}$
- $Z_1 \| [Z_2 + (Z_3 || Z_0)] = \frac{10000 + 12500 + 50000}{1000 + 1250 + 250 + 500 + 2000} = 14.5\Omega$
- $\Gamma_L = \frac{14.5\Omega - 50\Omega}{14.5\Omega + 50\Omega} = -0.55 = S_{11}$
- $S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{z=0} = \frac{(1+\Gamma_L) V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.55 = 0.45 = S_{21}$

Practice Problem 4 (cont.)



- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z_3 || [Z_2 + (Z_1 || Z_0)]) - Z_0}{(Z_3 || [Z_2 + (Z_1 || Z_0)]) + Z_0}$
- $Z_3 || [Z_2 + (Z_1 || Z_0)] = Z_3 || \left[Z_2 + \frac{Z_1 Z_0}{Z_1 + Z_0}\right] = \frac{Z_3 Z_2 + \frac{Z_1 Z_3 Z_0}{Z_1 + Z_0}}{Z_3 + Z_2 + \frac{Z_1 Z_0}{Z_1 + Z_0}}$
- $Z_3 || [Z_2 + (Z_1 || Z_0)] = \frac{Z_1 Z_2 Z_3 + Z_2 Z_3 Z_0 + Z_1 Z_3 Z_0}{Z_1 Z_3 + Z_3 Z_0 + Z_2 Z_1 + Z_2 Z_0 + Z_1 Z_0}$
- $Z_3 || [Z_2 + (Z_1 || Z_0)] = \frac{10000 + 20000 + 12500}{1000 + 2000 + 250 + 500 + 1250} = 8.5\Omega$
- $\Gamma_L = \frac{8.5\Omega - 50\Omega}{8.5\Omega + 50\Omega} = -0.71 = S_{22}$

Practice Problem 4 (cont.)

- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.71 = 0.29 = S_{12}$
- $S = \begin{bmatrix} -0.55 & 0.29 \\ 0.45 & -0.71 \end{bmatrix}$