Basics of S-parameters

Calculating The S-parameters

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Ansys

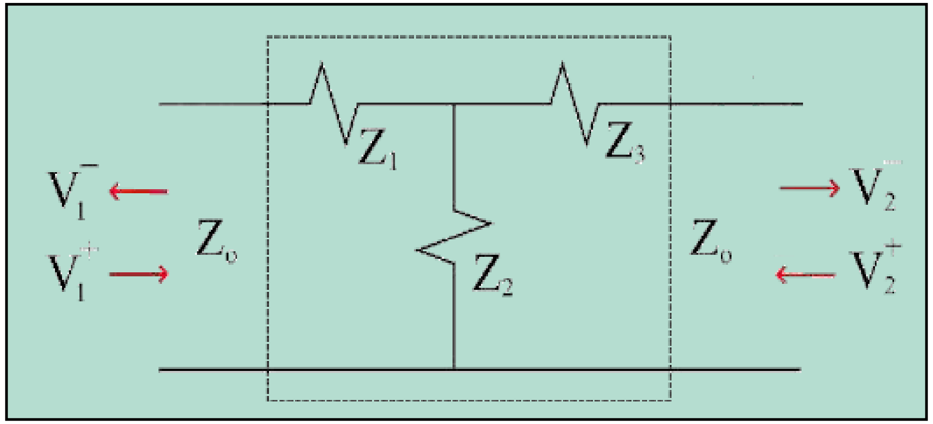
Sources

The material presented herein is from the following sources:

"Engineering Electromagnetics," by Nathan Ida, 3rd ed. (2015)
"Antenna Theory," by Constantine A. Balanis, 4th ed. (2016)
HFSS 2019 Help Files
"Elements of Electromagnetics," by Matthew N.O Sadiku, 5th ed. (2010)
"Microwave Engineering," by David Pozar, 4th ed. (2012)

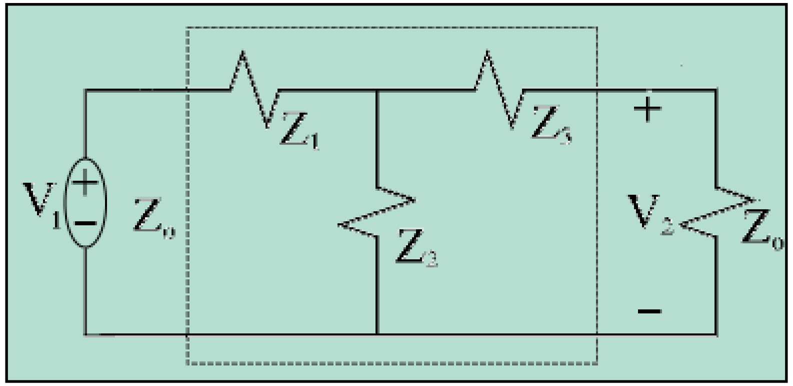
Calculating the S-Parameters (An Example)

Consider the following 2-port network:

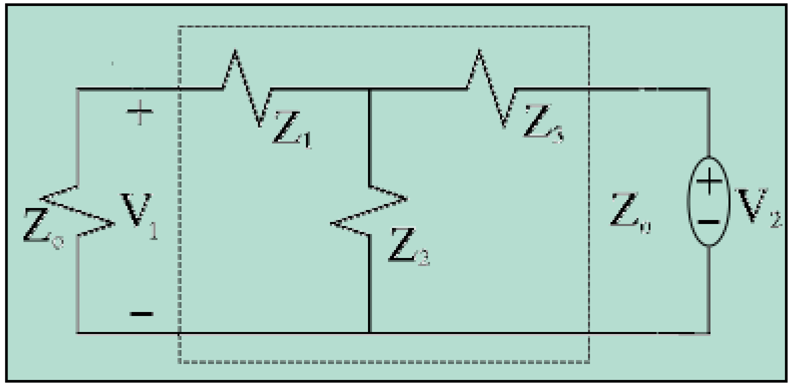


We will calculate the S-parameters of this network through a series of two measurements. For the first measurement, we will apply a test voltage at the first port, and match-terminate the second port. For the second measurement, we will apply a test voltage at the second port, and match terminate the first port.

Measurement 1:

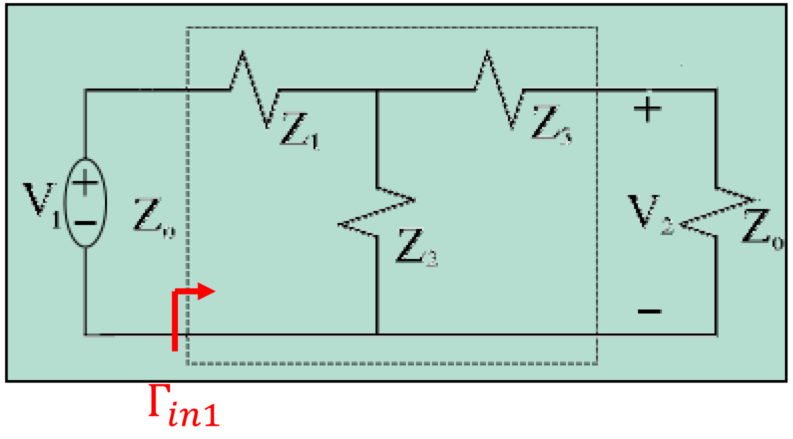


Measurement 2



Calculating the S-Parameters (An Example)

Measurement 1:



In order to calculate S_{11}, we can use the equation $V_1^- = \Gamma_{in1} V_1^+$ together with the definition $S_{11} = \frac{V_1^-}{V_1^+}$ (valid when port 2 is match-terminated) to show that, for this circuit configuration, $S_{11} = \Gamma_{in1}$

We know that $\Gamma_{in1} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o}$, and in this case,

$$Z_{in1} = Z_1 + (Z_o + Z_3)||Z_2 = Z_1 + \frac{Z_2 Z_3 + Z_2 Z_o}{Z_2 + Z_3 + Z_o}$$

So,

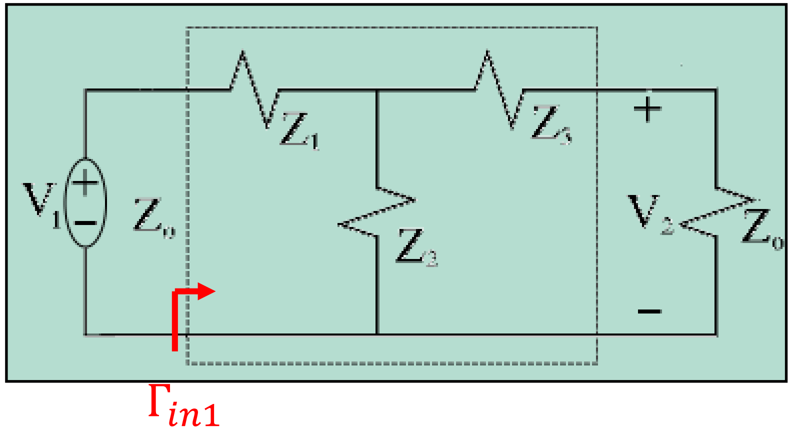
$$\Gamma_{in1} = \frac{Z_1 + (Z_o + Z_3)||Z_2 - Z_o}{Z_1 + (Z_o + Z_3)||Z_2 + Z_o}$$

And finally,

$$\boxed{S_{11} = \frac{Z_1 + (Z_o + Z_3)||Z_2 - Z_o}{Z_1 + (Z_o + Z_3)||Z_2 + Z_o}}$$

Calculating the S-Parameters (An Example)

Measurement 1:



In order to calculate S_{21}, we use circuit analysis to show that:

$$V_2 = V_1\left(\frac{Z_2||(Z_3+Z_o)}{Z_1+Z_2||(Z_3+Z_o)}\right)\left(\frac{Z_o}{Z_3+Z_o}\right)$$

And since, for this circuit configuration,

$$V_1 = V_1^+(1+S_{11}) \text{ and } V_2 = V_2^-$$

this becomes:

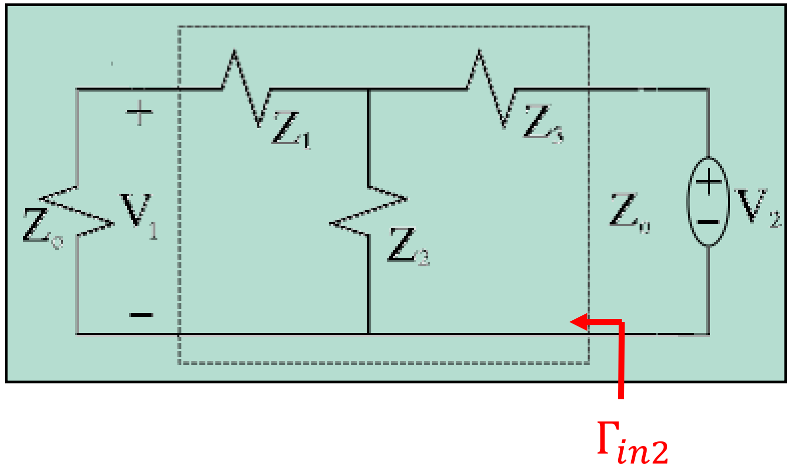
$$V_2^- = V_1^+(1+S_{11})\left(\frac{Z_2||(Z_3+Z_o)}{Z_1+Z_2||(Z_3+Z_o)}\right)\left(\frac{Z_o}{Z_3+Z_o}\right)$$

so that,

$$S_{21} = \frac{V_2^-}{V_1^+} = (1+S_{11})\left(\frac{Z_2||(Z_3+Z_o)}{Z_1+Z_2||(Z_3+Z_o)}\right)\left(\frac{Z_o}{Z_3+Z_o}\right)$$

Calculating the S-Parameters (An Example)

Measurement 2



We can similarly calculate S_{22} using the equation $V_2^- = \Gamma_{in2} V_2^+$ together with the definition $S_{22} = \frac{V_2^-}{V_2^+}$ (valid when port 1 is match-terminated) to show that, for this circuit configuration, $S_{22} = \Gamma_{in2}$

We know that $\Gamma_{in2} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o}$, and in this case,

$$Z_{in2} = Z_3 + (Z_o + Z_1)||Z_2 = Z_3 + \frac{\mathrm{Z_2 Z_1 + Z_2 Z_o}}{\mathrm{Z_2 + Z_1 + Z_o}}$$

So,

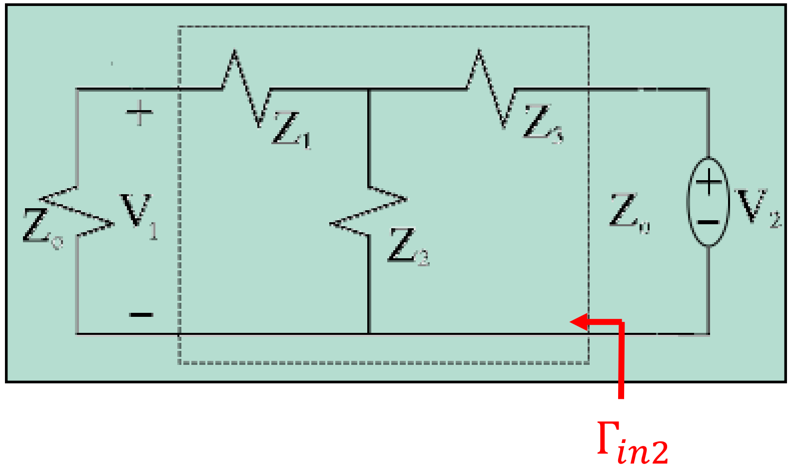
$$\Gamma_{in2} = \frac{Z_3 + (Z_o + Z_1)||Z_2 - Z_o}{Z_3 + (Z_o + Z_1)||Z_2 + Z_o}$$

And finally,

$$\boxed{\mathrm{S}_{22} = \frac{Z_3 + (Z_o + Z_1)||Z_2 - Z_o}{Z_3 + (Z_o + Z_1)||Z_2 + Z_o}}$$

Calculating the S-matrix (An Example)

Measurement 2



Finally, to calculate S_{12}, we use circuit analysis to show that:

$$V_1 = V_2\left(\frac{Z_2||(Z_1+Z_o)}{Z_3+Z_2||(Z_1+Z_o)}\right)\left(\frac{Z_o}{Z_1+Z_o}\right)$$

And since, for this circuit configuration,

$$V_2 = V_2^+(1+S_{22}) \text{ and } V_1 = V_1^-$$

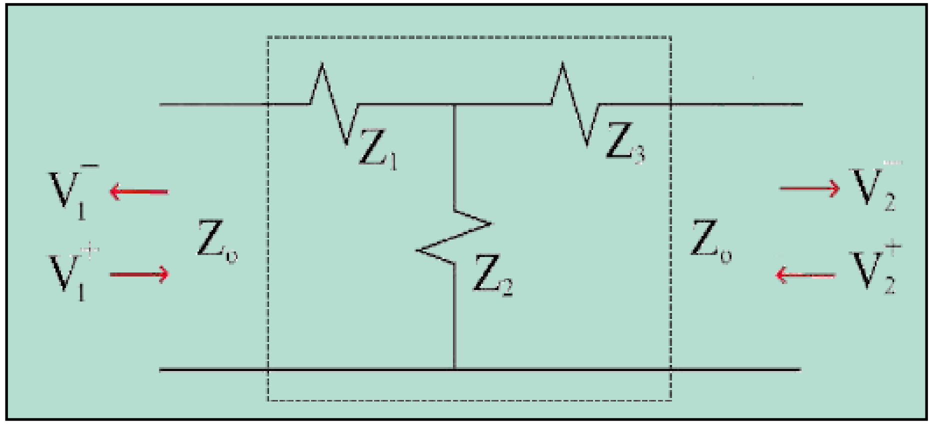
this becomes:

$$V_1^- = V_2^+(1+S_{22})\left(\frac{Z_2||(Z_1+Z_o)}{Z_3+Z_2||(Z_1+Z_o)}\right)\left(\frac{Z_o}{Z_1+Z_o}\right)$$

so that,

$$\mathrm{S}_{12} = \frac{V_1^-}{V_2^+} = (1+S_{22})\left(\frac{Z_2||(Z_1+Z_o)}{Z_3+Z_2||(Z_1+Z_o)}\right)\left(\frac{Z_o}{Z_1+Z_o}\right)$$

Calculating the S-Parameters (An Example)



So the S-matrix for the above network is:

$$[S] = \begin{bmatrix} \dfrac{Z_1 + (Z_o + Z_3)||Z_2 - Z_o}{Z_1 + (Z_o + Z_3)||Z_2 + Z_o} & (1 + S_{22})\left(\dfrac{Z_2||(Z_1 + Z_o)}{Z_3 + Z_2||(Z_1 + Z_o)}\right)\left(\dfrac{Z_o}{Z_1 + Z_o}\right) \\ (1 + S_{11})\left(\dfrac{Z_2||(Z_3 + Z_o)}{Z_1 + Z_2||(Z_3 + Z_o)}\right)\left(\dfrac{Z_o}{Z_3 + Z_o}\right) & \dfrac{Z_3 + (Z_o + Z_1)||Z_2 - Z_o}{Z_3 + (Z_o + Z_1)||Z_2 + Z_o} \end{bmatrix}$$

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