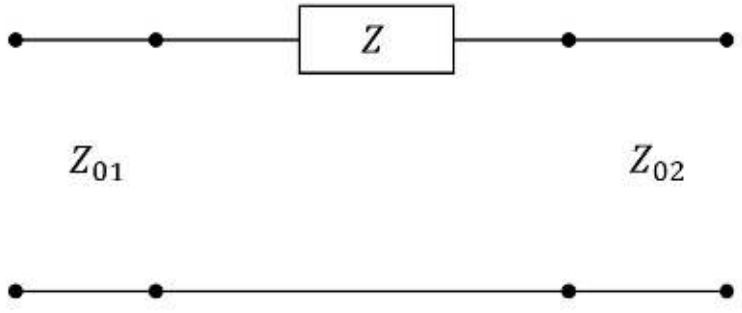
2 Homework Problems

Homework Problem 1

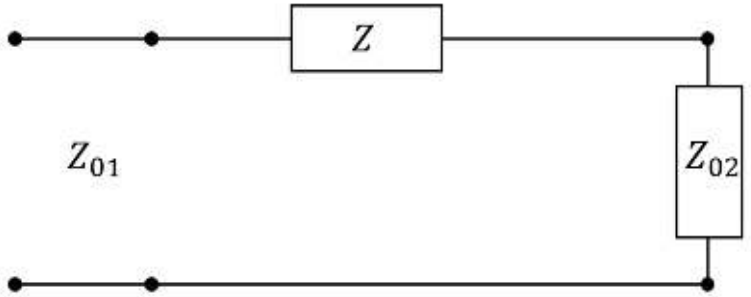
1. Find the scattering parameter matrix of the following network. Assume $Z_{01} = 50\Omega, Z_{02} = 25\Omega, Z = 10\Omega$



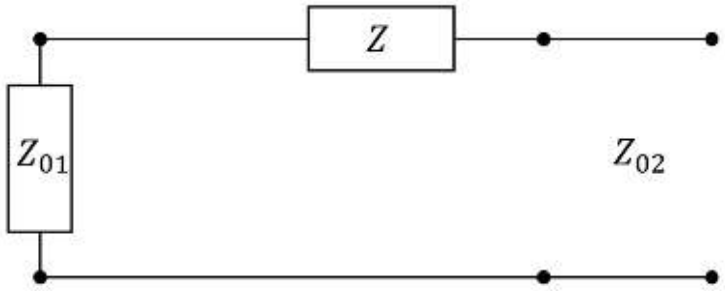
$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z+Z_{02})-Z_{01}}{(Z+Z_{02})+Z_{01}} = \frac{Z+Z_{02}-Z_{01}}{Z+Z_{02}+Z_{01}}$
- $\Gamma_L = \frac{Z+Z_{02}-Z_{01}}{Z+Z_{02}+Z_{01}} = -0.176 = S_{11}$



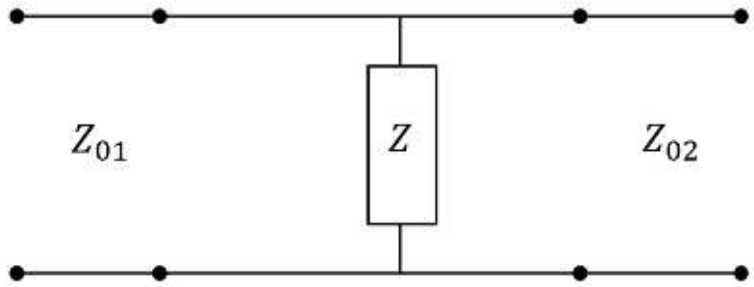
Homework 1 (cont.)



- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.176 = 0.824 = S_{21}$
- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z+Z_{01})-Z_{02}}{(Z+Z_{01})+Z_{02}} = \frac{Z+Z_{01}-Z_{02}}{Z+Z_{01}+Z_{02}} = 0.412 = S_{22}$
- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 + 0.412 = 1.412 = S_{12}$
- $S = \begin{bmatrix} -0.176 & 1.412 \\ 0.824 & 0.412 \end{bmatrix}$

Homework Problem 2

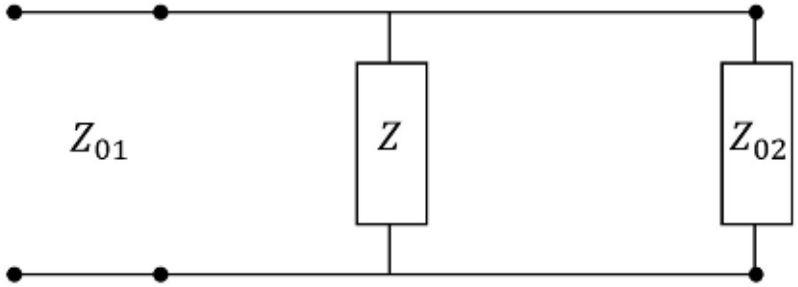
2. Find the scattering parameter matrix of the following network. Assume $Z_{01} = 50\Omega, Z_{02} = 25\Omega, Z = 10\Omega$



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

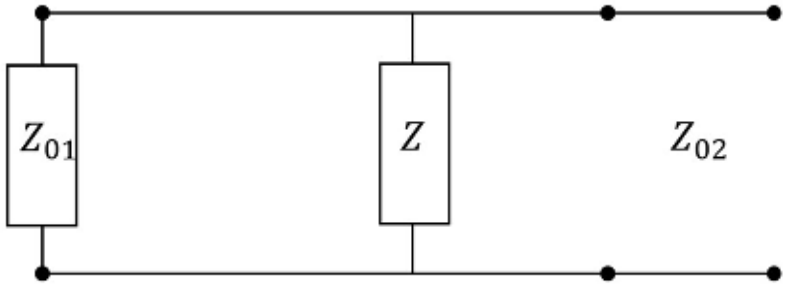
- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$



- $\Gamma_L = \frac{(Z||Z_{02}) - Z_{01}}{(Z||Z_{02}) + Z_{01}} = \frac{\left(\frac{ZZ_{02}}{Z+Z_{02}}\right) - Z_{01}}{\left(\frac{ZZ_{02}}{Z+Z_{02}}\right) + Z_{01}}$
- $\Gamma_L = \frac{ZZ_{02} - ZZ_{01} - Z_{01}Z_{02}}{ZZ_{02} + ZZ_{01} + Z_{01}Z_{02}} = \frac{250 - 500 - 1250}{250 + 500 + 1250} = -0.75 = S_{11}$

Homework Problem 2 (cont.)

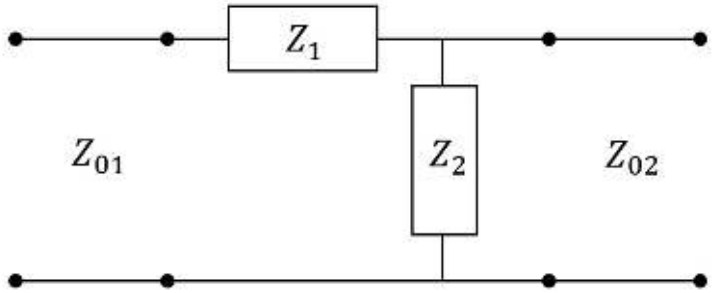
- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.75 = 0.25 = S_{21}$
- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z||Z_{01})-Z_{02}}{(Z||Z_{01})+Z_{02}} = \frac{\left(\frac{ZZ_{01}}{Z+Z_{01}}\right)-Z_{02}}{\left(\frac{ZZ_{01}}{Z+Z_{01}}\right)+Z_{02}}$



- $\Gamma_L = \frac{ZZ_{01}-ZZ_{02}-Z_{01}Z_{02}}{ZZ_{01}+ZZ_{02}+Z_{01}Z_{02}} = \frac{500-250-1250}{500+250+1250} = -0.5 = S_{22}$
- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.5 = 0.5 = S_{12}$
- $S = \begin{bmatrix} -0.75 & 0.5 \\ 0.25 & -0.5 \end{bmatrix}$

Homework Problem 3

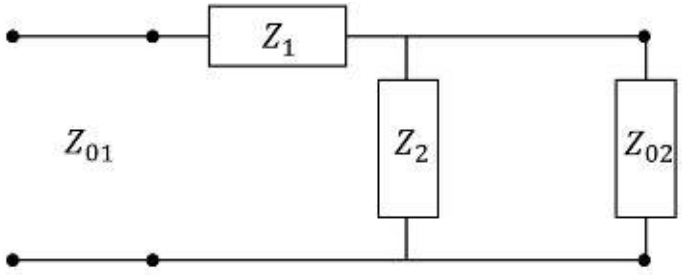
3. Find the scattering parameter matrix of the following network. Assume $Z_{01} = 50\Omega, Z_{02} = 25\Omega, Z_1 = 80\Omega, Z_2 = 120\Omega$



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

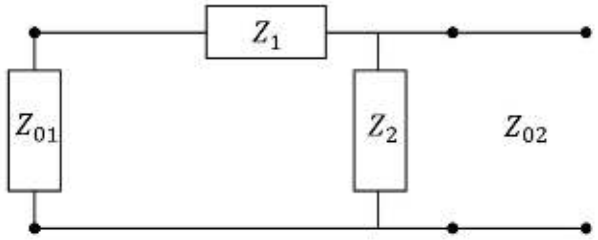
- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$



- $\Gamma_L = \frac{(Z_1 + Z_2 \| Z_{02}) - Z_{01}}{(Z_1 + Z_2 \| Z_{02}) + Z_{01}} = \frac{\left(Z_1 + \frac{Z_2 Z_{02}}{Z_2 + Z_{02}}\right) - Z_{01}}{\left(Z_1 + \frac{Z_2 Z_{02}}{Z_2 + Z_{02}}\right) + Z_{01}}$
- $\Gamma_L = \frac{Z_1 Z_2 + Z_1 Z_{02} + Z_2 Z_{02} - Z_2 Z_{01} - Z_{01} Z_{02}}{Z_1 Z_2 + Z_1 Z_{02} + Z_2 Z_{02} + Z_2 Z_{01} + Z_{01} Z_{02}} = \frac{7350}{21850} = 0.336 = S_{11}$

Homework Problem 3 (cont.)

- $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 + 0.336 = 1.336 = S_{21}$
- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$



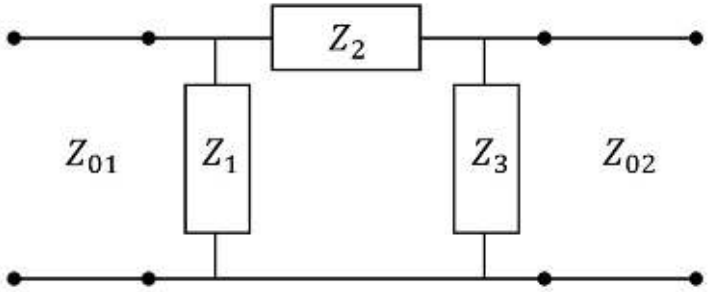
- $\Gamma_L = \frac{((Z_{01}+Z_1)||Z_2)-Z_{02}}{((Z_{01}+Z_1)||Z_2)+Z_{02}} = \frac{\left(\frac{Z_{01}Z_2+Z_{01}Z_1}{Z_{01}+Z_1+Z_2}\right)-Z_{02}}{\left(\frac{Z_{01}Z_2+Z_{01}Z_1}{Z_{01}+Z_1+Z_2}\right)+Z_{02}}$
- $\Gamma_L = \frac{Z_{01}Z_2+Z_{01}Z_1-Z_{01}Z_{02}-Z_{02}Z_1-Z_{02}Z_2}{Z_{01}Z_2+Z_{01}Z_1+Z_{01}Z_{02}+Z_{02}Z_1+Z_{02}Z_2} = \frac{-1000}{21000} = -0.048 = S_{22}$

Homework Problem 3 (cont.)

- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.048 = 0.952$
- $S = \begin{bmatrix} 0.336 & 0.952 \\ 1.336 & -0.048 \end{bmatrix}$

Homework Problem 4

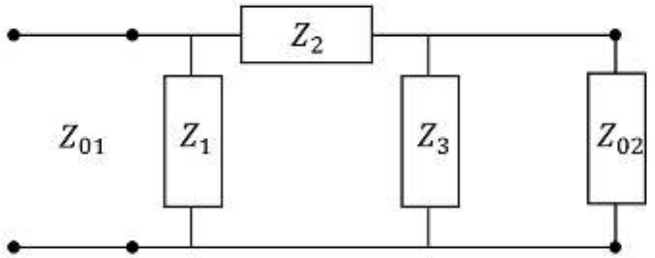
4. Find the scattering parameter matrix of the following network. Assume $Z_{01} = 50\Omega, Z_{02} = 25\Omega, Z_1 = 25\Omega, Z_2 = 10\Omega, Z_3 = 40\Omega$



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{z=0}$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0}$$

- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{z=0} = \frac{\Gamma_L V_{01}^+}{V_{01}^+} = \Gamma_L$



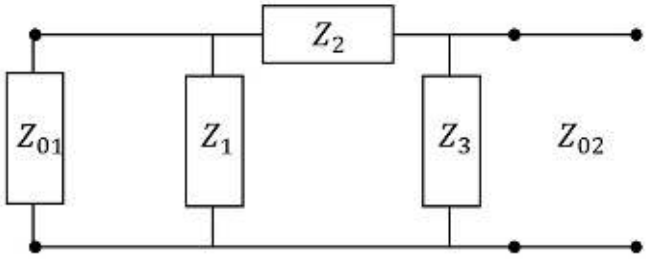
- $\Gamma_L = \frac{(Z_1||[Z_2+(Z_3||Z_{02})])-Z_{01}}{(Z_1||[Z_2+(Z_3||Z_{02})])+Z_{01}}$

- $Z_1||[Z_2 + (Z_3||Z_{02})] = Z_1||\left[Z_2 + \frac{Z_3 Z_{02}}{Z_3+Z_{02}}\right] = \frac{Z_1 Z_2 + \frac{Z_1 Z_3 Z_{02}}{Z_3+Z_{02}}}{Z_1+Z_2+\frac{Z_3 Z_{02}}{Z_3+Z_{02}}}$

Homework Problem 4 (cont.)

- $Z_1 \| [Z_2 + (Z_3 || Z_{02})] = \frac{Z_1 Z_2 Z_3 + Z_1 Z_2 Z_{02} + Z_1 Z_3 Z_{02}}{Z_1 Z_3 + Z_1 Z_{02} + Z_2 Z_3 + Z_2 Z_{02} + Z_3 Z_{02}}$
- $Z_1 \| [Z_2 + (Z_3 || Z_{02})] = \frac{10000 + 6250 + 25000}{1000 + 625 + 250 + 250 + 1000} = 13.2\Omega$
- $\Gamma_L = \frac{13.2\Omega - 50\Omega}{13.2\Omega + 50\Omega} = -0.582 = S_{11}$
- $S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{z=0} = \frac{(1+\Gamma_L) V_{01}^+}{V_{01}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.582 = 0.418 = S_{21}$

Homework Problem 4 (cont.)



- $S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{z=0} = \frac{\Gamma_L V_{02}^+}{V_{02}^+} = \Gamma_L$
- $\Gamma_L = \frac{(Z_3 || [Z_2 + (Z_1 || Z_{01})]) - Z_{02}}{(Z_3 || [Z_2 + (Z_1 || Z_{01})]) + Z_{02}}$
- $Z_3 || [Z_2 + (Z_1 || Z_{01})] = Z_3 || \left[Z_2 + \frac{Z_1 Z_{01}}{Z_1 + Z_{01}}\right] = \frac{Z_3 Z_2 + \frac{Z_1 Z_3 Z_{01}}{Z_1 + Z_{01}}}{Z_3 + Z_2 + \frac{Z_1 Z_{01}}{Z_1 + Z_{01}}}$
- $Z_3 || [Z_2 + (Z_1 || Z_{01})] = \frac{Z_1 Z_2 Z_3 + Z_2 Z_3 Z_{01} + Z_1 Z_3 Z_{01}}{Z_1 Z_3 + Z_3 Z_{01} + Z_2 Z_1 + Z_2 Z_{01} + Z_1 Z_{01}}$
- $Z_3 || [Z_2 + (Z_1 || Z_{01})] = \frac{10000 + 20000 + 12500}{1000 + 2000 + 250 + 500 + 1250} = 8.5\Omega$
- $\Gamma_L = \frac{8.5\Omega - 50\Omega}{8.5\Omega + 50\Omega} = -0.71 = S_{22}$

Homework Problem 4 (cont.)

- $S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{z=0} = \frac{(1+\Gamma_L)V_{02}^+}{V_{02}^+} = 1 + \Gamma_L$
- $1 + \Gamma_L = 1 - 0.71 = 0.29 = S_{12}$
- $S = \begin{bmatrix} -0.582 & 0.29 \\ 0.418 & -0.71 \end{bmatrix}$