Assignment 1 :

Problem 1:

I calculate while n =1 until n = 10

Absolute error for n = 1 is : 0.07786299110421091
Absolute error for n = 2 is : 0.08099564851101682
Absolute error for n = 3 is : 0.16379040865413597
Absolute error for n = 4 is : 0.4938248671067065
Absolute error for n = 5 is : 1.9808320424099009
Absolute error for n = 6 is : 9.921815357815035
Absolute error for n = 7 is : 59.604168387538266
Absolute error for n = 8 is : 417.6045473432896
Absolute error for n = 9 is : 3343.1271580516477
Absolute error for n = 10 is : 30104.381258962676

Relative error for n = 1 is : 0.07786299110421091
Relative error for n = 2 is : 0.04049782425550841
Relative error for n = 3 is : 0.027298401442355995
Relative error for n = 4 is : 0.020576036129446102
Relative error for n = 5 is : 0.016506933686749173
Relative error for n = 6 is : 0.013780299108076438
Relative error for n = 7 is : 0.011826223886416323
Relative error for n = 8 is : 0.010357255638474444
Relative error for n = 9 is : 0.009212762230080598
Relative error for n = 10 is : 0.008295960443938127

As we can find that the Absolute error grows and the relative error shrinks while the n increasing

The absolute error and relative error will change when using double precision instead of single precision. But the result that absolute error grows and the relative error shrinks will not be influenced when using double precision instead of single precision.

Test process for problem 2:

(a):

Bisection method:

initial a = 1, b = 1.1

n	a	b	c	interval	f(c)
0	1.0	1.1	1.05	0.050000000000000044	-0.009772107530853003
1	1.05	1.1	1.0750000000000002	0.02499999999999991	0.05040275009991291
2	1.05	1.0750000000000002	1.0625	0.012500000000000178	0.020021009004689727
3	1.05	1.0625	1.05625	0.006250000000000089	0.005051284917055199
4	1.05	1.05625	1.053125	0.0031249999999998224	-0.0023786513985419866
5	1.053125	1.05625	1.0546875	0.0015624999999999112	0.0013317503233913897
6	1.053125	1.0546875	1.05390625	0.0007812499999999556	-0.0005245913444569261
7	1.05390625	1.0546875	1.054296875	0.0003906250000000888	0.0004032941875151685
8	1.05390625	1.054296875	1.0541015625	0.00019531249999982236	-6.071989142752976e-05
9	1.0541015625	1.054296875	1.05419921875	9.765624999991118e-05	0.0001712693182382985
10	1.0541015625	1.05419921875	1.054150390625	4.882812499995559e-05	5.5270256149597685e-05
11	1.0541015625	1.054150390625	1.0541259765625002	2.4414062499866773e-05	-2.72593192818249e-06
12	1.0541259765625002	1.054150390625	1.05413818359375	1.2207031250044409e-05	2.6271883535322615e-05
13	1.0541259765625002	1.05413818359375	1.054132080078125	6.103515624911182e-06	1.1772906160167906e-05
14	1.0541259765625002	1.054132080078125	1.0541290283203126	3.051757812455591e-06	4.523469705031147e-06
15	1.0541259765625002	1.0541290283203126	1.0541275024414065	1.5258789061167732e-06	8.987645361280272e-07
16	1.0541259765625002	1.0541275024414065	1.0541267395019533	7.629394531694089e-07	-9.135847842678402e-07
17	1.0541267395019533	1.0541275024414065	1.05412712097168	3.8146972647368216e-07	-7.410395852502916e-09
18	1.05412712097168	1.0541275024414065	1.0541273117065433	1.9073486323684108e-07	4.4567700197006843e-07
19	1.05412712097168	1.0541273117065433	1.0541272163391118	9.536743150739824e-08	2.19133286627482e-07
20	1.05412712097168	1.0541272163391118	1.054127168655396	4.768371586472142e-08	1.0586144094659744e-07
21	1.05412712097168	1.054127168655396	1.0541271448135379	2.3841858043383013e-08	4.922552099273503e-08
22	1.05412712097168	1.0541271448135379	1.054127132892609	1.1920928910669204e-08	2.090756279216066e-08
23	1.05412712097168	1.054127132892609	1.0541271269321446	5.9604643443122995e-09	6.748583025739663e-09
24	1.05412712097168	1.0541271269321446	1.0541271239519123	2.9802322831784522e-09	-3.309064133816264e-10
25	1.0541271239519123	1.0541271269321446	1.0541271254420286	1.4901160305669237e-09	3.208838528223623e-09
26	1.0541271239519123	1.0541271254420286	1.0541271246969703	7.450582373280668e-10	1.4389662794656033e-09
27	1.0541271239519123	1.0541271246969703	1.0541271243244412	3.725291186640334e-10	5.540297109973835e-10
28	1.0541271239519123	1.0541271243244412	1.0541271241381769	1.8626433728741176e-10	1.1156187085248348e-10
29	1.0541271239519123	1.0541271241381769	1.0541271240450447	9.313216864370588e-11	-1.0967204921996654e-10

The root of the function is 1.0541271240799688

Newton method:
The initial $x_0 = 0.1$

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	error
0	0.1	-0.9946624985711805	0.11016675279762189	9.128699433470564	9.028699433470564
1	9.128699433470564	9213.736342154376	9216.984601787462	8.129051854546807	0.9996475789237564
2	8.129051854546807	3388.6199206880497	3391.853941333459	7.130005321500676	0.9990465330461316
3	7.130005321500676	1246.1344349574224	1248.2212439647474	6.131677147723322	0.9983281737773533
4	6.131677147723322	458.35827869526423	459.21880493186956	5.13355103926993	0.9981261084533921
5	5.13355103926993	168.53098585588745	169.20955013301392	4.137561240092479	0.9959897991774511
6	4.137561240092479	61.489133026690816	63.193537279494194	3.164532419839311	0.9730288202531678
7	3.164532419839311	21.70060798959475	24.677407130335872	2.28516094150769	0.8793714783316213
8	2.28516094150769	7.071758053609363	10.482405221284841	1.6105296995051441	0.6746312420025458
9	1.6105296995051441	2.0062511830455216	5.045184835189551	1.2128730727246615	0.39765662678048264
10	1.2128730727246615	0.42650692281913205	3.01280345675358	1.0713082713888615	0.14156480133579996
11	1.0713082713888615	0.04136821740603924	2.4402199014469357	1.054355611761133	0.016952659627728517
12	1.054355611761133	0.0005428660695927512	2.3763369017919924	1.054127165167898	0.0002284465932349544 2
13	1.054127165167898	9.757695007550637e-08	2.3754826661733297	1.0541271240912145	4.1076683610086206e-08
14	1.0541271240912145	3.552713678800501e-15	2.3754825125901236	1.054127124091213	1.5543122344752192e-15

The root of the function is 1.054127124091213

Secant method:
The initial $x_0 = 0.2, x_1 = 0.5$

n	x_n	x_{n+1}	x_{n+2}	error
0	0.1	0.5	2.526624133534849	2.026624133534849
1	0.5	2.526624133534849	0.6563892239014018	1.8702349096334472
2	2.526624133534849	0.6563892239014018	0.7766078528866348	0.12021862898523306
3	0.6563892239014018	0.7766078528866348	1.1834405318674779	0.4068326789808431
4	0.7766078528866348	1.1834405318674779	1.0239491721046343	0.15949135976284357
5	1.1834405318674779	1.0239491721046343	1.051200728427889	0.02725155632325471
6	1.0239491721046343	1.051200728427889	1.054197761578156	0.002997033150266892 3
7	1.051200728427889	1.054197761578156	1.0541269611844188	7.080039373708047e-05
8	1.054197761578156	1.0541269611844188	1.0541271240821573	1.6289773840938437e-07
9	1.0541269611844188	1.0541271240821573	1.054127124091213	9.055645122657552e-12

The root of the function is 1.054127124091213

(b):
Bisection method:
initial a = 1, b = 2

n	a	b	c	interval	f(c)
0	1.0	2.0	1.5	0.5	-0.15546510810816438
1	1.0	1.5	1.25	0.25	0.33935644868579024
2	1.25	1.5	1.375	0.125	0.07217126888146541
3	1.375	1.5	1.4375	0.0625	-0.046499243689368475
4	1.375	1.4375	1.40625	0.03125	0.011612475529406807
5	1.40625	1.4375	1.421875	0.015625	-0.017747907532178198
6	1.40625	1.421875	1.4140625	0.0078125	-0.0031440134399585706
7	1.40625	1.4140625	1.41015625	0.00390625	0.004215135560744043
8	1.41015625	1.4140625	1.412109375	0.001953125	0.0005307898436337499
9	1.412109375	1.4140625	1.4130859375	0.0009765625	-0.0013078042717768468
10	1.412109375	1.4130859375	1.41259765625	0.00048828125	-0.0003888053737435282
11	1.412109375	1.41259765625	1.412353515625	0.000244140625	7.091768986389191e-05
12	1.412353515625	1.41259765625	1.4124755859375	0.0001220703125	-0.0001589624775645282
13	1.412353515625	1.4124755859375	1.41241455078125	6.103515625e-05	-4.402705283718111e-05
14	1.412353515625	1.41241455078125	1.412384033203125	3.0517578125e-05	1.3444153756536625e-05
15	1.412384033203125	1.41241455078125	1.4123992919921875	1.52587890625e-05	-1.5291740728229364e-05
16	1.412384033203125	1.4123992919921875	1.4123916625976562	7.62939453125e-06	-9.238662829758049e-07
17	1.412384033203125	1.4123916625976562	1.4123878479003906	3.814697265625e-06	6.260125537449479e-06
18	1.4123878479003906	1.4123916625976562	1.4123897552490234	1.9073486328125e-06	2.66812507743186e-06
19	1.4123897552490234	1.4123916625976562	1.4123907089233398	9.5367431640625e-07	8.721282597767832e-07
20	1.4123907089233398	1.4123916625976562	1.412391185760498	4.76837158203125e-07	-2.5869295983138585e-08
21	1.4123907089233398	1.412391185760498	1.412390947341919	2.384185791015625e-07	4.231294107870376e-07
22	1.412390947341919	1.412391185760498	1.4123910665512085	1.1920928955078125e-07	1.986300396383811e-07
23	1.4123910665512085	1.412391185760498	1.4123911261558533	5.960464477539063e-08	8.638036735897359e-08
24	1.4123911261558533	1.412391185760498	1.4123911559581757	2.9802322387695312e-08	3.0255534577694476e-08
25	1.4123911559581757	1.412391185760498	1.4123911708593369	1.4901161193847656e-08	2.1931187976775846e-09
26	1.4123911708593369	1.412391185760498	1.4123911783099174	7.450580596923828e-09	-1.1838088620486076e-08
27	1.4123911708593369	1.4123911783099174	1.4123911745846272	3.725290298461914e-09	-4.822484911404246e-09
28	1.4123911708593369	1.4123911745846272	1.412391172721982	1.862645149230957e-09	-1.314683029107755e-09
29	1.4123911708593369	1.412391172721982	1.4123911717906594	9.313225746154785e-10	4.392178842849148e-10
30	1.4123911717906594	1.412391172721982	1.4123911722563207	4.656612873077393e-10	-4.377321283222102e-10
31	1.4123911717906594	1.4123911722563207	1.41239117202349	2.3283064365386963e-10	7.428502257766922e-13
32	1.41239117202349	1.4123911722563207	1.4123911721399054	1.1641532182693481e-10	-2.1849461129264114e-10
33	1.41239117202349	1.4123911721399054	1.4123911720816977	5.820766091346741e-11	-1.0887590828900784e-10

The root of the function is 1.4123911720816977

Newton method:
The initial $x_0 = 0.1$

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	error
0	0.1	5.912585092994045	-13.8	0.528448195144496	0.42844819514449606
1	0.528448195144496	2.803275215282212	-4.83543668831404	1.1081838831449158	0.5797356880004197
2	1.1081838831449158	0.6926134522100743	-2.686009551418579	1.366043521746826	0.2578596386019103
3	1.366043521746826	0.08998219495569076	-1.9999540548172212	1.4110356528105141	0.044992131063688046
4	1.4110356528105141	0.0025550619480667613	-1.8866280089626895	1.4123899536774198	0.0013543008669056888
5	1.4123899536774198	2.294436763750074e-06	-1.8832398554566376	1.4123911720228988	1.2183454789482795e-06
6	1.4123911720228988	1.8560708525683367e-12	-1.8832368080193835	1.4123911720238844	9.85656001262214e-13

The root of the function is 1.4123911720238844

Secant method:

The initial $x_0 = 0.2, x_1 = 0.5$

n	x_n	x_{n+1}	x_{n+2}	error
0	0.1	0.5	0.8964584904417008	0.3964584904417008
1	0.5	0.8964584904417008	1.2220339025140037	0.3255754120723029
2	0.8964584904417008	1.2220339025140037	1.3648853852658256	0.1428514827518219
3	1.2220339025140037	1.3648853852658256	1.4070895951407456	0.04220420987492002
4	1.3648853852658256	1.4070895951407456	1.4122287803867235	0.005139185245977895
5	1.4070895951407456	1.4122287803867235	1.4123906020574566	0.0001618216707330866
6	1.4122287803867235	1.4123906020574566	1.4123911719624231	5.699049665519595e-07
7	1.4123906020574566	1.4123911719624231	1.4123911720238844	6.146128050943389e-11

The root of the function is 1.4123911720238844