· Point mass

$\mu = a$

so $\hat{a} = \bar{x}$

· Bernoulli

$\mu = p$

so $\hat{p} = \bar{x}$

· Binomial.

$\mu = np, \quad \sigma^2 = np(1-p)$

$np = \frac{sum}{N} \Rightarrow \frac{sum}{N}(1-p) = \sigma^2$

$1-p = \frac{\sigma^2}{\frac{sum}{N}}$

$\hat{p} = 1 - \frac{\sigma^2 \cdot N}{sum}$

$\hat{n} = \frac{sum}{N \cdot \hat{p}}$

· Geometric

$\mu = \frac{1}{p}$

$\hat{p} = \frac{1}{\bar{x}}$

· Possion

$\mu = \lambda$

$\hat{\lambda} = \bar{x}$

· uniform.

$\mu = (a+b)/2 \quad \sigma^2 = (b-a)^2/12.$

$a+b/2 = \bar{x}$

$b = 2\bar{x} - a.$

$(2\bar{x} - a - a)^2/12 = \sigma^2.$

$\hat{a} = \bar{x} - \sqrt{\frac{3\sigma^2}{N}}$

$\hat{b} = \bar{x} + \sqrt{\frac{3\sigma^2}{N}}$

· Normal.

$\mu = \mu. \quad \sigma^2 = \sigma^2.$

$\mu = \bar{x}, \quad \hat{\sigma}^2 = \sigma^2.$

· Exponential.

$\beta = \mu$

$\beta = \bar{x}.$

· Gamma.

$\alpha\beta = \mu \quad \alpha\beta^2 = \sigma^2.$

$\alpha\beta = \bar{x}$

$\bar{x} \cdot \beta = \sigma^2$

$\hat{\beta} = \frac{\sigma^2}{\bar{x}}$

$\alpha = \frac{\bar{x}^2}{\sigma^2}.$

· beta.

$\alpha/(\alpha+\beta) = \mu, \quad \alpha\beta/((\alpha+\beta)^2(\alpha+\beta+1)) = \sigma^2$

$\frac{\alpha}{\alpha+\beta} = \bar{x}$

$\beta = \alpha \cdot \frac{1-\bar{x}}{\bar{x}} \quad \text{---} ①$

$\alpha\beta/((\alpha+\beta)^2(\alpha+\beta+1)) = \sigma^2 \quad \text{---} ②.$

Put ① into ② $\Downarrow$.

$\hat{\alpha} = \bar{x}\left(\frac{\bar{x} \cdot (1-\bar{x})}{\sigma^2} - 1\right) \Rightarrow \hat{\beta} = \frac{\hat{\alpha}(1-\bar{x})}{\bar{x}}$

· t v.

$$\frac{v}{v-2} = \frac{\sum_{i=0}^{n} x_i^2}{n} = t$$

$$\hat{v} = \frac{2t}{(1-t)}$$

· Chisq.

$p = \mu$.

$\hat{p} = \bar{x}$

· multinomial.

same way as normal

get each the $\hat{u}$, $\hat{\sigma}^2$

and combine.

· Multivariate Normal,

$\mu = \mu$, $V(x) = \Sigma$.

$$\begin{bmatrix} \hat{\mu}_0 \\ \vdots \\ \hat{\mu}_n \end{bmatrix} = \begin{bmatrix} \bar{x}_0 \\ \vdots \\ \bar{x}_n \end{bmatrix}$$

$\hat{V}(x) = \mathrm{cov}(X)$

Sample mean $= \hat{\alpha}_1 = E(X_i)$

$$= n_0 \frac{\hat{C}}{\hat{N}}$$ (n_0 = known)

($\hat{\alpha}_1$ & $\hat{\alpha}_2$ are calculated from data)

$$\hat{\alpha}_2 = \frac{1}{n} \sum_i X_i^2 = E(X_i^2)$$

MOM Hypergeometric Distribution

$$= Var(X_i) + (E(X_i))^2$$

$$= n_0 \left(\frac{\hat{C}}{\hat{N}}\right) \frac{(\hat{N}-\hat{C})}{\hat{N}} \left(\frac{\hat{N}-n_0}{\hat{N}-1}\right) + \left(n_0 \frac{\hat{C}}{\hat{N}}\right)^2$$

$$= \left(n_0 \frac{\hat{C}}{\hat{N}}\right) \left[\left(1 - \frac{\hat{C}}{\hat{N}}\right)\left(\frac{\hat{N}-n_0}{\hat{N}-1}\right) + n_0 \frac{\hat{C}}{\hat{N}}\right]$$

$$\Rightarrow \hat{\alpha}_2$$

$$= \hat{\alpha}_1 \left[\left(1 - \frac{\hat{\alpha}_1}{n_0}\right)\left(\frac{\hat{N}-n_0}{\hat{N}-1}\right) + \hat{\alpha}_1\right]$$

$$\Rightarrow \left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) = \left(1 - \frac{\hat{\alpha}_1}{n_0}\right) \frac{\hat{N}-n_0}{\hat{N}-1}$$

$$\Rightarrow \frac{\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) = A}{\left(1 - \frac{\hat{\alpha}_1}{n_0}\right) = B} = \frac{\hat{N}-n_0}{\hat{N}-1}$$

Scanned with CamScanner

$$A\hat{N} - A = B\hat{N} - Bn_0$$

$$A\hat{N} - B\hat{N} = A - Bn_0$$

$$\hat{N} = \frac{A - Bn_0}{A - B}$$

$$\Rightarrow \hat{N} = \frac{\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) - \left(1 - \frac{\hat{\alpha}_1}{n_0}\right) n_0}{\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) - \left(1 - \frac{\hat{\alpha}_1}{n_0}\right)}$$

$$= \frac{\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \cancel{\hat{\alpha}_1} - n_0 + \cancel{\hat{\alpha}_1}}{\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) - \left(1 - \frac{\hat{\alpha}_1}{n_0}\right)}$$

$$\hat{N} = \left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - n_0\right) \Big/ \left[\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1} - \hat{\alpha}_1\right) - \left(1 - \frac{\hat{\alpha}_1}{n_0}\right)\right]$$

$$\Rightarrow C = \frac{\hat{N}}{n_0}\hat{\alpha}_1$$