Bernoulli Distribution

$$f_X(x) = p^x (1-p)^{1-x}$$

$$\Downarrow$$

$$f_\theta(x) = \theta^x (1-\theta)^{1-x}, \quad \theta \in [0,1]$$

$H_0 : \theta \leq 0.45 \Rightarrow \theta = \theta_0$ where $\theta_0 \in (0, 0.45]$

$H_1 : \theta \geq 0.55 \Rightarrow \theta = \theta_1$ where $\theta_1 \in [0.55, 1)$

Log-likelihood ratio

$$\log \Lambda(x) = \log \left[\frac{\theta_1^x (1-\theta_1)^{1-x}}{\theta_0^x (1-\theta_0)^{1-x}} \right]$$

$$= x \log\left(\frac{\theta_1}{\theta_0}\right) + (1-x) \log\left(\frac{1-\theta_1}{1-\theta_0}\right)$$

Consecutive sum for all log-likelihoods

$$\sum_{i=1}^{n} \log \Lambda(x_i) = \sum_{i=1}^{n} \left[x_i \log\left(\frac{\theta_1}{\theta_0}\right) + (1-x_i) \log\left(\frac{1-\theta_1}{1-\theta_0}\right) \right]$$

$$= n \log\left(\frac{1-\theta_1}{1-\theta_0}\right) + \log\left[\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right] \left(\sum_{i=1}^{n} x_i\right)$$

continue if $a < \text{sum} < b$

accept H_1 if $\text{sum} \geq b$

accept H_0 if $\text{sum} \leq a$

where, $a = \log\left(\frac{\beta}{1-\alpha}\right)$ & $b = \log\left(\frac{1-\beta}{\alpha}\right)$

For $p = 0.54$

$H_0 : p \leq 0.45$ (Null hypothesis)

$H_1 : p \geq 0.55$ (Alternate hypothesis)

Since the sampling will stop when bounds are violated, the null hypothesis at that time will simply be rejected.

This is equivalent to saying that alternate hypothesis will be accepted (even if it is not true), because the test is based on rejecting or accepting the null hypothesis, irrespective of the validity of the alternate hypothesis.