

$$\text{Sample mean} = \hat{x}_1 = E(X_i)$$

$$= n_0 \frac{\hat{c}}{N} \quad (n_0 = \text{known})$$

$$\hat{x}_2 = \frac{1}{n} \sum_i X_i^2 = E(X_i^2)$$

(\hat{x}_1 & \hat{x}_2 are calculated from data)

MOM
Hypergeometric
Distribution

$$= \text{Var}(X_i) + (E(X_i))^2$$

$$= n_0 \left(\frac{\hat{c}}{N} \right) \frac{(N - \hat{c})}{N} \left(\frac{N - n_0}{N - 1} \right) + \left(n_0 \frac{\hat{c}}{N} \right)^2$$

$$= \left(n_0 \frac{\hat{c}}{N} \right) \left[\left(1 - \frac{\hat{c}}{N} \right) \left(\frac{N - n_0}{N - 1} \right) + n_0 \frac{\hat{c}}{N} \right]$$

$$\Rightarrow \hat{x}_2$$

$$= \hat{x}_1 \left[\left(1 - \frac{\hat{x}_1}{n_0} \right) \left(\frac{N - n_0}{N - 1} \right) + \hat{x}_1 \right]$$

$$\Rightarrow \left(\frac{\hat{x}_2}{\hat{x}_1} - 1 \right) = \left(1 - \frac{\hat{x}_1}{n_0} \right) \frac{N - n_0}{N - 1}$$

$$\Rightarrow \frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - 1 \right) = A}{\left(1 - \frac{\hat{x}_1}{n_0} \right) = B} = \frac{N - n_0}{N - 1}$$

$$A\hat{N} - A = B\hat{N} - Bn_0$$

$$A\hat{N} - B\hat{N} = A - Bn_0$$

$$\hat{N} = \frac{A - Bn_0}{A - B}$$

$$\Rightarrow \hat{N} = \frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right) n_0}{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)}$$

~~$$\frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right) n_0}{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)}$$~~

~~$$\frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right) n_0}{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)}$$~~

~~$$\frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right) n_0}{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)}$$~~

~~$$= \frac{\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 - n_0 + \hat{x}_1}{\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)}$$~~

~~$$\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right)$$~~

$$\hat{N} = \frac{\left(\frac{\hat{x}_2}{\hat{x}_1} - n_0 \right)}{\left[\left(\frac{\hat{x}_2}{\hat{x}_1} - \hat{x}_1 \right) - \left(1 - \frac{\hat{x}_1}{n_0} \right) \right]}$$

$$\Rightarrow \hat{C} = \frac{\hat{N}}{n_0} \hat{x}_1$$