

• Point mass

$$\mu = a$$

$$\text{so } \hat{a} = \bar{x}$$

• Bernoulli

$$\mu = p$$

$$\text{so } \hat{p} = \bar{x}$$

• Binomial.

$$\mu = np, \quad \sigma^2 = np(1-p)$$

$$np = \frac{\text{sum}}{N} \Rightarrow \frac{\text{sum}}{N}(1-p) = \sigma^2$$

$$1-p = \frac{\sigma^2}{\frac{\text{sum}}{N}}$$

$$\hat{p} = 1 - \frac{\sigma^2 \cdot N}{\text{sum}}$$

$$\hat{n} = \frac{\text{sum}}{N \cdot \hat{p}}$$

• Geometric

$$\mu = \frac{1}{p}$$

$$\hat{p} = \frac{1}{\bar{x}}$$

• Poisson

$$\mu = \lambda$$

$$\hat{\lambda} = \bar{x}$$

• uniform.

$$\mu = (a+b)/2, \quad \sigma^2 = (b-a)^2/12$$

$$(a+b)/2 = \bar{x}$$

$$b = 2\bar{x} - a$$

$$(2\bar{x} - a - a)^2/12 = \sigma^2$$

$$\hat{a} = \bar{x} - \sqrt{\frac{3\sigma^2}{N}}$$

$$\hat{b} = \bar{x} + \sqrt{\frac{3\sigma^2}{N}}$$

• Normal.

$$\mu = \mu, \quad \sigma^2 = \sigma^2$$

$$\mu = \bar{x}, \quad \hat{\sigma}^2 = \sigma^2$$

• Exponential.

$$\beta = \mu$$

$$\beta = \bar{x}$$

• Gamma.

$$\alpha\beta = \mu, \quad \alpha\beta^2 = \sigma^2$$

$$\alpha\beta = \bar{x}$$

$$\bar{x} \cdot \beta = \sigma^2$$

$$\hat{\beta} = \frac{\sigma^2}{\bar{x}}$$

$$\hat{\alpha} = \frac{\bar{x}}{\hat{\sigma}^2}$$

• beta.

$$\alpha/(\alpha+\beta) = \mu, \quad \alpha\beta/((\alpha+\beta)^2(\alpha+\beta+1)) = \sigma^2$$

$$\frac{\alpha}{\alpha+\beta} = \bar{x}$$

$$\beta = \alpha \cdot \frac{1-\bar{x}}{\bar{x}} \quad \dots \textcircled{1}$$

$$\alpha\beta/((\alpha+\beta)^2(\alpha+\beta+1)) = \sigma^2 \quad \dots \textcircled{2}$$

Put ① into ② ↓

$$\hat{\alpha} = \bar{x} \left(\frac{\bar{x} \cdot (1-\bar{x})}{\hat{\sigma}^2 - 1} \right) \Rightarrow \hat{\beta} = \frac{\hat{\alpha}(1-\bar{x})}{\bar{x}}$$

• t_v .

$$\frac{V}{\sqrt{2}} = \frac{\sum_{i=1}^n x_i^2}{n} = t$$

$$\hat{V} = \frac{2t}{(1-t)}$$

• multinomial.

Same way as normal

get each line $\hat{\mu}$, $\hat{\sigma}^2$
and combine.

• chisq.

$$p = \mu.$$

$$\hat{p} = \bar{X}$$

• Multivariate Normal,

$$\mu = \mu, \quad V(x) = \Sigma.$$

$$\begin{bmatrix} \hat{\mu}_0 \\ \vdots \\ \hat{\mu}_n \end{bmatrix} = \begin{bmatrix} \bar{x}_0 \\ \vdots \\ \bar{x}_n \end{bmatrix}$$

$$V(x) = \text{cov}(X)$$