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Modelling forest fire spread using hexagonal cellular automata

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Abstract

In this paper a new mathematical model for predicting the spread of a fire front in homogeneous and inhomogeneous environments is presented. It is based on a bidimensional cellular automata model, whose cells stand for regular hexagonal areas of the forest. The results obtained are in agreement with the fire spreading in real forests.

Keywords: Cellular automata; Fire simulation; Forest fire spreading; Mathematical modelling

1. Introduction

The study of the simulation of forest fire spreading has increased considerably in the time span of the last few years, due to its impact in forest ecosystems and in public safety and property.

Forest fire spread models are usually classified into two types: Stochastic (empirical) models and deterministic (semi-empirical or mathematical) models. The first ones aim to predict the more probable fire behavior in average conditions and accumulating data obtained from laboratory and outdoor experimental fires. On the other hand, in deterministic models, the fire behavior is usually deduced from the resolution of the physical conservation laws governing the evolution of the system formed by the flame and its environment.

Specifically, the efforts to model the growth of the fire front by means of mathematical models can be classified in two categories according to the approach: Vector models and cellular automata models. Vector models assume that the fire front spreads according to a well-defined growth law, and take a standard geometrical shape. If burning conditions are uniform, a single shape can be used to determine the fire size, the perimeter

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over time and the area by means of the use of fractals [1]. More complex vector models use wave propagation techniques based on Huygen's principle (see, for example, [2–5]), and also determine the temperature fields and the fire propagation simultaneously by performing turbulent fluid flow calculations [6].

On the other hand, several types of cellular automata models for fire spreading have been introduced in the literature: Probability-driven models (see [7]), fractal models [8], etc.

In this work we propose a new cellular automaton model based on the transfer of fractional burned area (see [9–12]). Specifically, it is an improvement model of the proposal due to Karafyllidis and Thanailakis (see [12]), and it consists of a bidimensional cellular automata with hexagonal cells. They can be very effective at simulating physical systems that interact locally (see [13]).

The rest of the paper is organized as follows: In Section 2 the basic theory of bidimensional cellular automata is introduced; the proposed model for fire spreading is described in Section 3; some graphical modellings are given in Section 4; and, finally, the conclusions are presented in Section 5.

2. Bidimensional cellular automata

Two-dimensional cellular automata (CA for short) are discrete dynamical systems formed by a finite number of identical objects called cells, which are arranged uniformly in a two-dimensional space. They are endowed with a state that changes at every discrete step of time according to a deterministic rule. More precisely, a CA can be defined as a 4-uplet $\mathscr{A} = (C, S, V, f)$, where C is the cellular space formed by a two-dimensional array of $r \times c$ cells:

$$\{(a, b), \quad 1 \leqslant a \leqslant r, \ 1 \leqslant b \leqslant c\}, \tag{1}$$

such that each of which can assume a state. In the bidimensional case, the cells are usually represented as identical square areas (see Fig. 1(a)), but in this work, the cells will be represented by means of regular hexagonal areas (see Fig. 1(b)), making a tessellation of the plane. This new representation allows us to obtain a more realistic simulation.

The state of each cell is an element of a finite or infinite state set, S. Moreover, the state of the cell (a,b) at time t is denoted by $s_{ab}^{(t)}$.

The set of indices of the CA is the ordered finite subset $V \subset \mathbb{Z} \times \mathbb{Z}$, $|V| = m$, such that for every cell (a,b), its neighborhood $V_{(a,b)}$ is the ordered set of m cells given by

$$V_{(a,b)} = \{(a + \alpha_1, b + \beta_1), \ldots, (a + \alpha_m, b + \beta_m) : (\alpha_k, \beta_k) \in V\}. \tag{2}$$

Depending on the process to be modelled, one can choose an appropriate neighborhood. In this work, the neighborhood of a cell $O = (a,b)$ is given by the set

$$V_O = \{O, N, NE, SE, S, SW, NW, NNE, E, SSE, SSW, W, NNW\}, \tag{3}$$

as it is shown in Fig. 2.

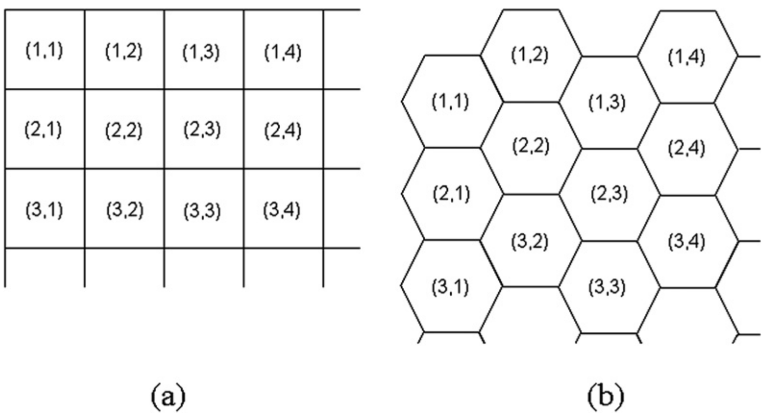

Fig. 1. (a) Square cellular space, (b) hexagonal cellular space.

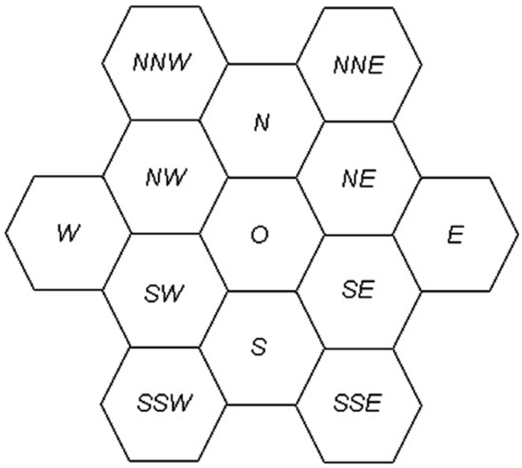

Fig. 2. Neighborhood of a cell O.

We can also distinguish two types of neighbor cells of O, depending on whether the neighbor of the cell O is an adjacent cell or not. The near neighbor cells are the set $V_n = \{N, NE, SE, S, SW, NW\}$, whereas the distant neighbor cells are given by the set $V_d = \{NNE, E, SSE, SSW, W, NNW\}$. Furthermore, each distant neighbor cell has two near neighbor cells associated, those with common sides:

Distant neighbor cell	Associated near neighbor cells
NNE	N, NE
E	NE, SE
SSE	SE, S
SSW	S, SW
W	SW, NW
NNW	NW, N

In this way, if $(\alpha, \beta) \in V_d$, then the associated neigbour cells of $(a+\alpha, b+\beta)$ are denoted by $(a+\alpha^+, b+\beta^+)$ and $(a+\alpha^-, b+\beta^-)$.

Moreover, in this case, the set defining the neighborhood, V, depends on the cell $O = (a,b)$ to be considered. In this sense, if b is odd, then

$$V^{\text{odd}} = \{(0,0), (-1,0), (0,1), (1,1), (1,0), (1,-1), (0,-1), (-1,1), (0,2), (2,1), (2,-1), (0,-2), (-1,-1)\}; \quad (4)$$

consequently, the neighborhood of the cell (a,b), with b odd, is (see Fig. 3(a)):

$$V^{\text{odd}}_{(a,b)} = \{(a,b),\ (a-1,b),\ (a,b+1),\ (a+1,b+1),\ (a+1,b),\ (a+1,b-1),\ (a,b-1),\ (a-1,b+1),\ (a,b+2),\ (a+2,b+1),\ (a+2,b-1),\ (a,b-2),\ (a-1,b-1)\}. \quad (5)$$

On the other hand, if b is even, then

$$V^{\text{even}} = \{(0,0), (-1,0), (-1,1), (0,1), (1,0), (0,-1), (-1,-1), (-2,1), (0,2), (1,1), (1,-1), (0,-2), (-2,-1)\} \quad (6)$$

and, as a consequence, the neighborhood of the cell (a,b), with b even, is (see Fig. 3(b)):

$$V^{\text{even}}_{(a,b)} = \{(a,b),\ (a-1,b),\ (a-1,b+1),\ (a,b+1),\ (a+1,b),\ (a,b-1),\ (a-1,b-1),\ (a-2,b+1),\ (a,b+2),\ (a+1,b+1),\ (a+1,b-1),\ (a,b-2),\ (a-2,b-1)\}. \quad (7)$$

As these labels do not affect to the calculus below, thereafter we will work with a generic neighbor V for the sake of simplicity.

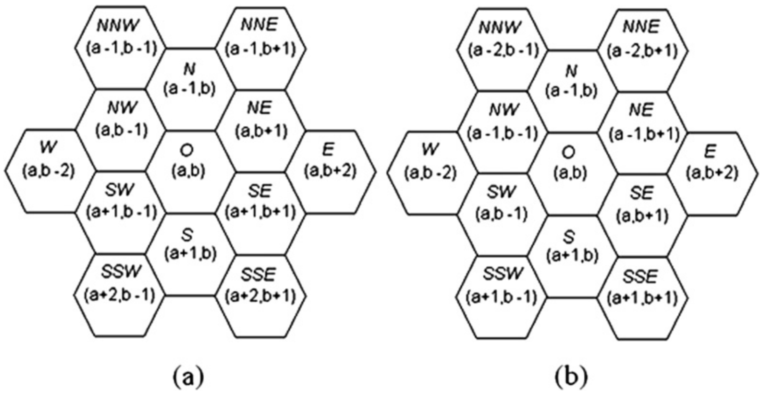

Fig. 3. The labelled neighborhoods.

As was mentioned above, the CA evolves deterministically in discrete time steps, changing the states of all cells according to a local transition function $f: S^{13} \to S$. The updated state of the cell (a, b) depends on the thirteen variables of the local transition function, which are the previous states of the cells constituting its neighborhood, that is

$$s_{ab}^{(t+1)} = f\left(s_{a+\alpha_1,b+\beta_1}^{(t)}, \ldots, s_{a+\alpha_{13},b+\beta_{13}}^{(t)}\right). \tag{8}$$

Moreover, the matrix $C^{(t)} = \left(s_{ij}^{(t)}\right)$, $1 \leqslant i \leqslant r$, $1 \leqslant j \leqslant c$, is called the configuration at time t of the CA, and $C^{(0)}$ is the initial configuration of the CA. As the number of cells of the CA is finite, boundary conditions must be considered in order to assure the well-defined dynamics of the CA. One can state several boundary conditions but in this work, we will consider null boundary conditions, that is

$$\text{if } (a, b) \notin \{(u, v),\ 1 \leqslant u \leqslant r,\ 1 \leqslant v \leqslant c\} \Rightarrow s_{ab}^{(t)} = 0. \tag{9}$$

A very important type of CA are linear CA, whose local transition function is as follows:

$$s_{ab}^{(t+1)} = g\left(\sum_{(\alpha,\beta)\in V} \mu_{\alpha\beta} s_{a+\alpha,b+\beta}^{(t)}\right), \quad \mu_{\alpha\beta} \in \mathbb{R}, \tag{10}$$

where $g : \mathbb{R} \to \mathbb{S}$ is a suitable discretization function.

3. The CA-based model for fire spreading

In this section, the model for predicting fire spreading based on two-dimensional linear cellular automata with hexagonal cellular space is proposed.

3.1. The basic model

A forest area can be interpreted as the hexagonal cellular space of a CA by simple dividing it into a two-dimensional array of identical hexagonal areas of side length L. Obviously, each one of these areas stands for a cell of the CA.

The state of a cell (a, b) at a time t, is defined as follows:

$$s_{ab}^{(t)} = \frac{\text{burned out area of(a, b)at time t}}{\text{total area of(a, b)}}, \tag{11}$$

where as a simple calculus shows, the total area of the hexagonal cell (a, b) is $3\sqrt{3}L^2/2$.

If $s_{ab}^{(t)} = 0$, then the cell (a, b) is said to be unburned at time t; if $0 < s_{ab}^{(t)} < 1$, then the cell (a, b) is partially burned out at time t, and finally, if $s_{ab}^{(t)} = 1$, the cell is said to be completely burned out at time t. In this work, we will consider the state set $S = \{0, 0.1, \ldots, 0.9, 1\}$, and consequently, each cell can assume 11 possible states.

Remark that it is possible the value $s_{ab}^{(t)}$ to be greater than 1. In this case, the state of the cell (a,b) at time t is taken to be equal to 1.

The dynamic of such automata basically supposes that the state of a cell (a,b) at a time $t+1$ linearly depends on the states of its neighbor cells at time t; specifically one has

$$s_{ab}^{(t+1)} = g\left(s_{ab}^{(t)} + \sum_{(\alpha,\beta)\in V_n} \mu_{\alpha\beta}^{(a,b)} s_{a+\alpha,b+\beta}^{(t)} + \sum_{(\alpha,\beta)\in V_d} \mu_{\alpha\beta}^{(a,b)} s_{a+\alpha,b+\beta}^{(t)}\right), \tag{12}$$

where $\mu_{\alpha\beta}^{(a,b)} \in \mathbb{R}$ are parameters involving some physical magnitudes of the cells, and the discretization function g is given by

$$\begin{aligned} g &: [0,1] \to S \\ x &\mapsto g(x) = \frac{[10x]}{10}, \end{aligned} \tag{13}$$

where $[m]$ stands for the closest integer to m.

As it is mentioned above, each cell, (a,b), represents a small hexagonal area of the forest. Then, it is endowed with three parameters: the rate of fire spread $R_{(a,b)}$, the wind speed $W_{(a,b)}$, and the height $H_{(a,b)}$ of the cell. Consequently, the expression of the parameter $\mu_{\alpha\beta}^{(a,b)}$ is as follows:

$$\mu_{\alpha\beta}^{(a,b)} = \omega_{\alpha\beta}^{(a,b)} \cdot h_{\alpha\beta}^{(a,b)} \cdot r_{\alpha\beta}^{(a,b)}, \tag{14}$$

where $\omega_{\alpha\beta}^{(a,b)}$ stands for the wind influence of the neighbor cell $(a+\alpha, b+\beta)$ on (a,b), such that

$$W_{(a,b)} = \{\omega_{\alpha\beta}^{(a,b)}, (\alpha,\beta) \in V\}; \tag{15}$$

$h_{\alpha\beta}^{(a,b)}$ represents the height influence and, as is shown below, it is a function of $H_{(a,b)} - H_{(a-\alpha,b-\beta)}$, where $H_{(a,b)}$ is the height in the central point of the hexagonal area which is represented by the cell (a,b). It is supposed that this height is the same in every point of such cell. Finally, $r_{\alpha\beta}^{(a,b)}$ is a parameter which stands for the influence of the different rates of fire spread.

3.2. The size of the discrete time step

Since cellular automata evolve in discrete time steps, it is a basic point to decide what is the size of such step of time, $\tilde{t}$. In the proposed model, this step is equal to the time needed for a one and only near neighbor cell to burn the main cell out.

The rate of fire spread of the cell (a,b), $R_{(a,b)}$, determines the time needed for this cell to be completely burned out and depends on the physical composition of the cell (see, for example [14]). Note that if the cell (a,b) stands for an incombustible hexagonal area, then $R_{(a,b)} = 0$ and $s_{ab}^{(t)} = 0$ for every t.

The importance of this parameter lies in the setting-up of the size of the time step, $\tilde{t}$. Suppose that the forest area modelled is homogeneous, i.e. the value of the rate of fire spread is the same for all cells: $R_{(a,b)} = R$, $1 \leqslant a \leqslant r$, $1 \leqslant b \leqslant c$. Then, it is easy to check that if the only burned out cell at time t in the neighborhood of O is for example N, then the time needed for O to be completely burned out is $\tilde{t} = \sqrt{3}L/R$ (see Fig. 4).

Consequently, if all cells in the neighborhood of $O = (a,b)$ are unburned at time t except only one adjacent cell, which is completely burned out, then at time $t+1$, the cell (a,b) is completely burned out: $s_{ab}^{(t+1)} = 1$.

Nevertheless, since almost all real forests are inhomogeneous, the size time step is taken to be the time needed for the cells with the larger rate spread to be completely burned out, that is

$$\tilde{t} = \sqrt{3}\frac{L}{R}, \tag{16}$$

where

$$R = \max\{R_{(a,b)},\ 1 \leqslant a \leqslant r,\ 1 \leqslant b \leqslant c\}. \tag{17}$$

It is followed that if the only completely burned out neighbor cell at time t, is a distant neighbor cell of $O = (a,b)$, say NNE, then $s_{ab}^{(t+1)} = \lambda < 1$. That value is calculated as follows. In a step time $\tilde{t}$, the near neighbor cells of the cell NNE, N and NE, and a little portion (a circular sector) of the cell O will be burned out. Specifically, as the distance covered by the fire front in $\tilde{t}$ with speed R is $\sqrt{3}L$, then the radius of the circular sector of O burned out is $\sqrt{3}L - L$ (see Fig. 5).

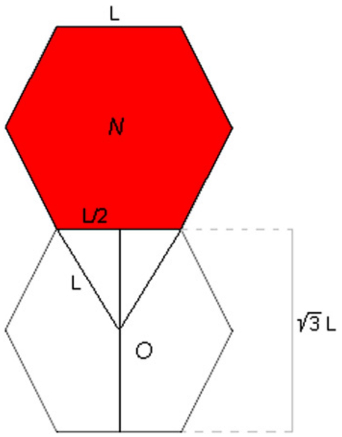

Fig. 4. Determining the size of the discrete step of time.

As a consequence, the burned out area of the cell O will be

$$\frac{\pi(\sqrt{3}-1)^2 L^2 \frac{2\pi}{3}}{2\pi} = \frac{4-2\sqrt{3}}{3}\pi L^2. \tag{18}$$

So, if all neighbor cells of (a, b) are unburned at time t, except a distant neighbor which is fully burned out, then

$$s_{ab}^{(t+1)} = \lambda = \frac{\frac{4-2\sqrt{3}}{3}\pi L^2}{\frac{3\sqrt{3}}{2}L^2} = \frac{8\sqrt{3}-12}{27}\pi \approx 0.2160. \tag{19}$$

3.3. The influence of the wind

Another factor to be incorporated to the model is the wind speed and direction, due to its important influence to the fire spreading (see [15]). As was stated above, the effect of the wind on the cell O is given by the set

$$W_{(a,b)} = \{\omega_{\alpha\beta}^{(a,b)},\ (\alpha, \beta) \in V\}, \tag{20}$$

where $\omega_{\alpha\beta}^{(a,b)} > 0$, in such a way that if no wind is blowing on $O = (a, b)$, then $\omega_{\alpha\beta}^{(a,b)} = 1$ for every $(\alpha, \beta) \in V$; if the wind is blowing from North to South, then the coefficients ω_{NW}^{O}, ω_{NNW}^{O}, ω_{N}^{O}, ω_{NNE}^{O} and ω_{NE}^{O}, must be larger than the rest of coefficients, and so on. The value of such coefficients stand for the magnitude of the wind.

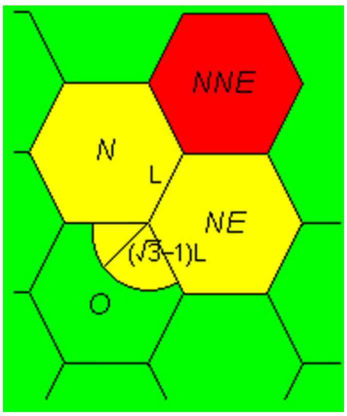

Fig. 5. The calculus of λ.

3.4. The influence of the topography

The height differences between various points in a forest also affects to the fire spreading. As is well-known, the fires show a higher rate of spread when they climb up an upward slope, whereas fires show a smaller rate of spread when they descend a downward slope.

The height influence of a near neighbor cell, $(a+\alpha, b+\beta)$, on a cell $O=(a,b)$, is given by $h_{\alpha\beta}^{(a,b)}$, which depends on the difference of height between each pair of cells considered, that is, $h_{\alpha\beta}^{(a,b)} = \phi(H_{(a,b)} - H_{(a+\alpha,b+\beta)})$. The function $y=\phi(x)$, where x stands for the height difference, must be determined according to the characteristic of the forest, and, also, it has to satisfy the following conditions:

1. If $x>0$, then $\phi(x)>1$.
2. If $x=0$, then $\phi(x)=1$.
3. If $x<0$, then $0<\phi(x)<1$.

Note that the first condition stablishes that if $H_{(a,b)} > H_{(a+\alpha,b+\beta)}$, the fire increases its rate of spread; the second condition states that when $H_{(a,b)} = H_{(a+\alpha,b+\beta)}$, the topography does not affect to the fire spread; and the third condition stablishes that if $H_{(a,b)} < H_{(a+\alpha,b+\beta)}$, the fire restrains its spreading.

Moreover, the height influence of a distant neighbor cell is affected by the influence of its associated near neighbor cells. For example, if the distant neighbor cell of O is NNE, then:

$$h_{NNE}^{O} = \frac{1}{4}[\phi(H_O - H_N) + \phi(H_N - H_{NNE}) + \phi(H_O - H_{NE}) + \phi(H_{NE} - H_{NNE})] \tag{21}$$

and so on.

Note that if the forest is flat (all cells have the same height), then $\phi(x)=1$ and consequently $h_{\alpha\beta}^{(a,b)}=1$ for every cell (a,b) and any $(\alpha,\beta)\in V$.

3.5. The rate of fire spread

Let us consider an inhomogeneous forest and set R the maximum rate of fire spread—see formula (17)—. Let $O=(a,b)$ be a cell with $R_O \leqslant R$. The purpose of this section is to determine the value of the parameter, $r_{\alpha\beta}^{(a,b)}$, which stands for the influence of the different rates of fire spread of the neighbor cells.

If all neighbor cells are unburned at time t except only one near neighbor cell of O, say for example N, then after a time step, $\tilde{t}$, the space traversed by the fire front is given by (see formula (16)):

$$r = R_O\tilde{t} = \sqrt{3}\frac{R_O}{R}L \tag{22}$$

and consequently, there are two cases to be considered: When $r \leqslant L$, and when $r > L$.

(1) If $r \leqslant L$ (see Fig. 6(a)), then $\sqrt{3}\frac{R_O}{R}L \leqslant L$, and $\frac{R_O}{R} \leqslant \frac{\sqrt{3}}{3} \approx 0.57735$. As a consequence, the burned out area of the cell O after a time step $\tilde{t}$ is given by

$$Lr + 2\frac{r^2\frac{\pi}{6}}{2} = \left(\sqrt{3} + \frac{\pi}{2}\frac{R_O}{R}\right)\frac{R_O}{R}L^2. \tag{23}$$

Consequently,

$$r_N^O = \frac{(\sqrt{3} + \frac{\pi}{2}\frac{R_O}{R})\frac{R_O}{R}L^2}{\frac{3}{2}\sqrt{3}L^2} = \frac{2\sqrt{3}}{9}\left(\sqrt{3} + \frac{\pi}{2}\frac{R_O}{R}\right)\frac{R_O}{R}. \tag{24}$$

(2) If $r > L$ (see Fig. 6(b)), then $L < \sqrt{3}\frac{R_O}{R}L$, and $0.57735 \approx \frac{\sqrt{3}}{3} < \frac{R_O}{R} \leqslant 1$. As a simple calculus shows, the burned out area of the cell $O=(a,b)$ after a time step $\tilde{t}$ is given by

$$\left[1 + \sin\left(\frac{\pi}{6} - \alpha\right) + \sqrt{3}\alpha\frac{R_O}{R}\right]\sqrt{3}\frac{R_O}{R}L^2, \tag{25}$$

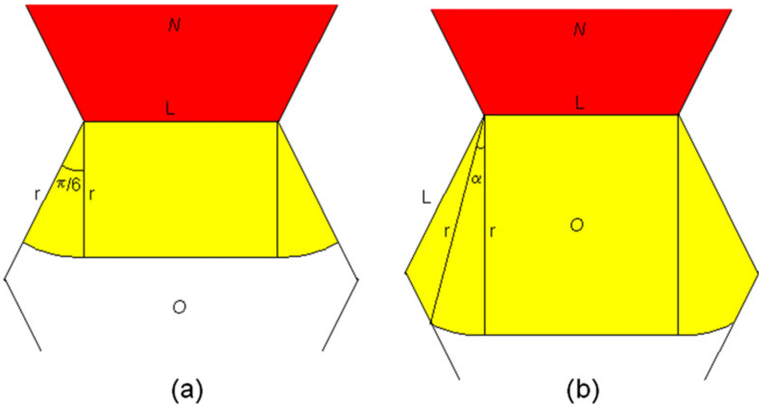

Fig. 6. The rate speed (near neighbor cell): (a) case $r \leqslant L$, (b) case $r > L$.

where

$$\alpha = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{4}\frac{R}{R_O} + \frac{\sqrt{3}}{12}\sqrt{12 - 3\frac{R^2}{R_O^2}}\right), \quad 0 \leqslant \alpha < \frac{\pi}{6}. \tag{26}$$

As a consequence:

$$r_N^O = \frac{\left[1 + \sin\left(\frac{\pi}{6} - \alpha\right) + \sqrt{3}\alpha\frac{R_O}{R}\right]\sqrt{3}\frac{R_O}{R}L^2}{\frac{3}{2}\sqrt{3}L^2} = \frac{2}{3}\left[1 + \sin\left(\frac{\pi}{6} - \alpha\right)\right]\frac{R_O}{R} + \frac{2\sqrt{3}}{3}\alpha\frac{R_O^2}{R^2}. \tag{27}$$

Note that if the forest is homogeneous, then $R_{ab} = R$ for every cell (a, b), and

$$\alpha = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{2}\right) = 0; \tag{28}$$

consequently $r_N^O = 1$, as it was expected.

On the other hand, if all neighbor cells are unburned at time t, except only one distant neighbor cell, say for example NNE, then after a time step, $\tilde{t}$, the fire front traverses the border between the near neighbor cells N and NE, and affects the main cell O. The border line between N and NE is traversed in $\tilde{t}_0 = L/\max\{R_N, R_{NE}\}$, and consequently, the space traversed along the cell O is (see Fig. 7):

$$R_O(\tilde{t} - \tilde{t}_o) = \left(\frac{\sqrt{3}}{R} - \frac{1}{\max\{R_N, R_{NE}\}}\right)R_O L. \tag{29}$$

As a consequence, the burned out area of O after a time step $\tilde{t}$ is

$$\frac{\pi\left(\frac{\sqrt{3}}{R} - \frac{1}{\max\{R_N, R_{NE}\}}\right)^2 R_O^2 L^2 \frac{2\pi}{3}}{2\pi} = \frac{\pi}{3}\left(\frac{\sqrt{3}}{R} - \frac{1}{\max\{R_N, R_{NE}\}}\right)^2 R_O^2 L^2. \tag{30}$$

Obviously, the state of the cell O is

$$r_{NNE}^O = \frac{\frac{\pi}{3}\left(\frac{\sqrt{3}}{R} - \frac{1}{\max\{R_N, R_{NE}\}}\right)^2 R_O^2 L^2}{\frac{3}{2}\sqrt{3}L^2} = \frac{2\pi\sqrt{3}}{27}\left(\sqrt{3}\frac{R_O}{R} - \frac{R_O}{\max\{R_N, R_{NE}\}}\right)^2. \tag{31}$$

Note that if the forest is homogeneous, then

$$r_{NNE}^O = \frac{2\pi\sqrt{3}}{27}(\sqrt{3} - 1)^2 = \frac{8\sqrt{3} - 12}{27} = \lambda. \tag{32}$$

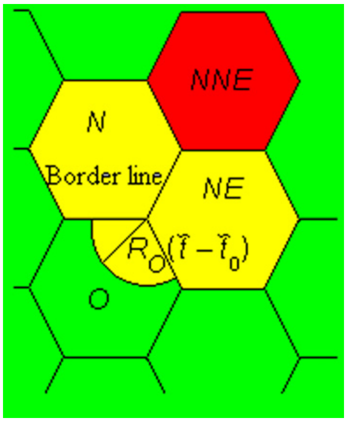

Fig. 7. The rate speed (distant neighbor cell).

3.6. The local transition function

Consequently, taking into account the results from the last subsections one can conclude that the local transition function of the hexagonal cellular automata for predicting forest fire spreading is as follows:

$$s_{ab}^{(t+1)} = \frac{1}{10}\left[10\sum_{(\alpha,\beta)\in V}\omega_{\alpha\beta}^{(a,b)}h_{\alpha\beta}^{(a,b)}r_{\alpha\beta}^{(a,b)}s_{a+\alpha,b+\beta}^{(t)}\right], \tag{33}$$

where if $(\alpha,\beta)\in V_n$,

$$h_{\alpha\beta}^{(a,b)} = \phi(H_{(a,b)} - H_{(a+\alpha,b+\beta)}) \tag{34}$$

and if $(\alpha,\beta)\in V_d$,

$$\begin{aligned} h_{\alpha\beta}^{(a,b)} = \frac{1}{4}[&\phi(H_{(a,b)} - H_{(a+\alpha^+,b+\beta^+)}) + \phi(H_{(a+\alpha^+,b+\beta^+)} - H_{(a+\alpha,b+\beta)}) + \phi(H_{(a,b)} - H_{(a+\alpha^-,b+\beta^-)}) \\ &+ \phi(H_{(a+\alpha^-,b+\beta^-)} - H_{(a+\alpha,b+\beta)})] \end{aligned} \tag{35}$$

and

$$r_{\alpha\beta}^{(a,b)} = \begin{cases} \dfrac{2\pi\sqrt{3}}{27}\left(\sqrt{3}\dfrac{R_{(a,b)}}{R} - \dfrac{R_{(a,b)}}{\max\{R_{(a+\alpha^+,b+\beta^+)}, R_{(a+\alpha^-,b+\beta^-)}\}}\right)^2, & \text{if } (\alpha,\beta)\in V_d, \\ \dfrac{2\sqrt{3}R_{(a,b)}}{9R}\left(\sqrt{3}+\dfrac{\pi}{2}\dfrac{R_{(a,b)}}{R}\right), \text{if } (\alpha,\beta)\in V_n \quad \text{and} \quad R \geqslant \sqrt{3}R_{(a,b)}, \\ \dfrac{2R_{(a,b)}}{3R}(1+\sin(\dfrac{\pi}{6}-\alpha) + \dfrac{\sqrt{3}\alpha R_{(a,b)}}{R}), \quad \text{otherwise} \end{cases} \tag{36}$$

with

$$\alpha = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{4}\frac{R}{R_{(a,b)}} + \frac{\sqrt{3}}{12}\sqrt{12 - 3\frac{R^2}{R_{(a,b)}^2}}\right). \tag{37}$$

Note that if the forest is homogeneous and there is not wind and topography conditions, then the transition function is as follows:

$$s_{ab}^{(t+1)} = g\left(s_{ab}^{(t)} + \sum_{(\alpha,\beta)\in V_n}s_{a+\alpha,b+\beta}^{(t)} + \lambda\sum_{(\alpha,\beta)\in V_d}s_{a+\alpha,b+\beta}^{(t)}\right). \tag{38}$$

4. Testing the proposed model

A good model for predicting forest fire spreading must passes some tests. In this work, we will consider eight basic tests, which are divided into two classes: homogeneous forest tests and inhomogeneous forest tests. In both types, we must consider topography conditions and weather conditions (speed and direction of the wind).

If an homogeneous flat forest without wind conditions is considered, the model must yield a circular fire front. If the are some weather conditions, the wind speed and direction must affect the forest fire front. Furthermore, if the homogeneous forest is not flat, the topography conditions must be reflected in the dynamics of fire front since, as it is mentioned above, fires show a higher rate of spread when they climb up an upward slope, whereas fires show a smaller rate of spread when they descend a downward slope.

On the other hand, if the forest is inhomogeneous, the fire front must be of circular shape. The fire advances with the same speed in all directions, in the areas whose rate of fire spread is equal to R—see formula (17)—; and this speed must decreases in the areas with another rate of fire spread.

An algorithm using the computer algebra system Mathematica has been constructed for the computational and graphical representations of the fire fronts. The hypothetical forests used are modeled by means of a bidimensional array of 100×100 cells. In the initial configuration, there is only one cell (the central cell) burned out whereas the rest of the cells are unburned, and 20 evolutions of the cellular automata are calculated. In the following figures, only the configurations at times $t = 0, 5, 10, 15, 20$ are shown. Moreover, the states of the cells are represented according to the color scheme of Fig. 8.

Note that if the state of the cell is 0, then it is unburned and it is represented by the color green; whereas if the state of the cell is 1 (the cell is completely burned out), its color is red. The rest of the states are represented by a level of colors from green to red.

4.1. Homogeneous forest tests

Let us consider an hypothetical homogeneous forest such that $R = 1$. Suppose that the forest is flat, and there is not wind conditions. Then, as it is shown in Fig. 9, the fire front is circular.

If the forest is flat and the wind is blowing from north to south (with an homogeneous linear front) according to the following formulas:

$$\begin{aligned} &\omega_N^O = \omega_{NE}^O = \omega_{NW}^O = \omega_{NNE}^O = \omega_{NNW}^O = 10, \\ &\omega_E^O = \omega_W^O = \omega_S^O = \omega_{SE}^O = \omega_{SW}^O = \omega_{SSW}^O = \omega_{SSE}^O = 1, \end{aligned} \tag{39}$$

then the evolution of the fire front is shown in Fig. 10.

Fig. 8. Colors of the states of the cells.

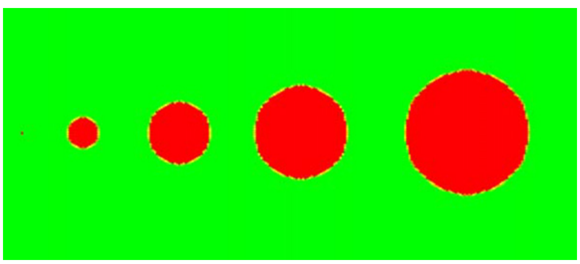

Fig. 9. Spreading of a forest fire in a homogeneous and flat forest without wind conditions.

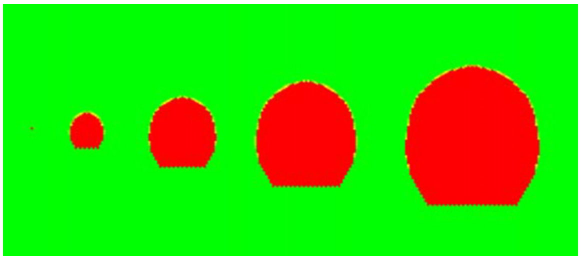

Fig. 10. Spreading of a forest fire in a homogeneous and flat forest with wind conditions (linear front).

Note that in this case the wind is blowing throughout forest in a linear front from north to south. If the wind front is defined by the function

$$v(x) = -10\exp\left(-\frac{(x-50)^2}{100}\right), \tag{40}$$

as is shown in Fig. 11, then, the evolution of the fire front is shown in Fig. 12.

Now, suppose that the forest is not flat such that

$$H_{(a,b)} = b, \quad \phi(x) = \begin{cases} e^x, & \text{if } x \leqslant 0, \\ 1 + \frac{\sqrt{x}}{10}, & \text{if } x > 0. \end{cases} \tag{41}$$

Then, the spreading of the fire forest is shown in Fig. 13.

Remark that the functions v and ϕ introduced in (40) and (41), respectively, must be calculated in an empirical way, according to the characteristics of the wind and the forest.

Finally, if the forest considers topography and wind conditions (with the above-mentioned conditions), the fire forest evolves as is shown in Fig. 14.

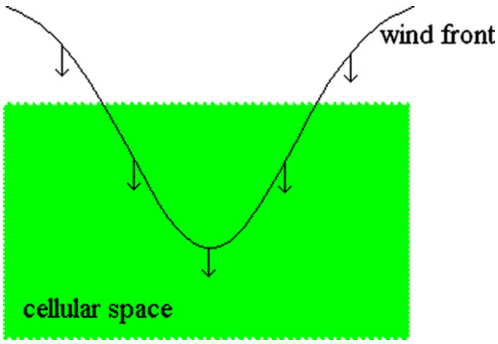

Fig. 11. Non-linear wind front.

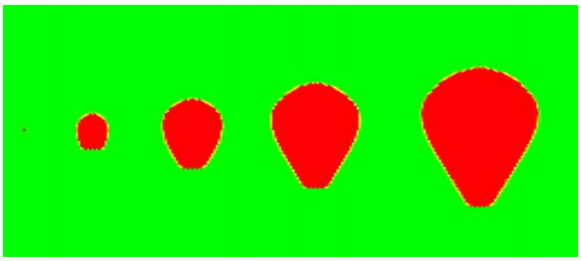

Fig. 12. Spreading of a forest fire in an homogeneous flat forest with wind conditions (non-linear front).

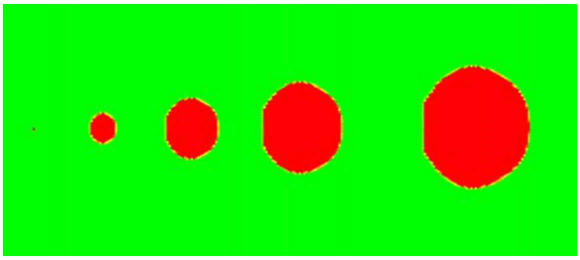

Fig. 13. Spreading of a forest fire in a homogeneous and no flat forest without wind conditions.

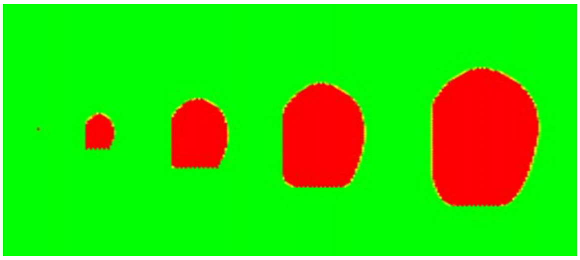

Fig. 14. Spreading of a forest fire in a non-flat forest with wind conditions (non-linear front).

4.2. Inhomogeneous forest tests

Now, suppose that the forest is inhomogeneous. For the sake of simplify we will consider the following values of the rate speed:

$$R_{(a,b)} = \begin{cases} 1, & \text{if } 1 \leqslant a \leqslant 100, \quad 1 \leqslant b \leqslant 50, \\ 3, & \text{if } 1 \leqslant a \leqslant 100, \quad 51 \leqslant b \leqslant 70, \\ 5, & \text{if } 1 \leqslant a \leqslant 100, \quad 71 \leqslant b \leqslant 90, \\ 7, & \text{if } 1 \leqslant a \leqslant 100, \quad 91 \leqslant b \leqslant 100. \end{cases} \tag{42}$$

Then, if no wind is blowing and the forest is flat, the simulation obtained is presented in Fig. 15.

Moreover, if the forest is not flat according to the formulas given in (41), the model yields the graphical representation introduced in Fig. 16.

Finally, if the wind is blowing from north to south taking into account the formula (40), the spreading of the fire front is shown in Fig. 17 (without topography conditions) and in Fig. 18 (with the topography conditions given in formula (41)).

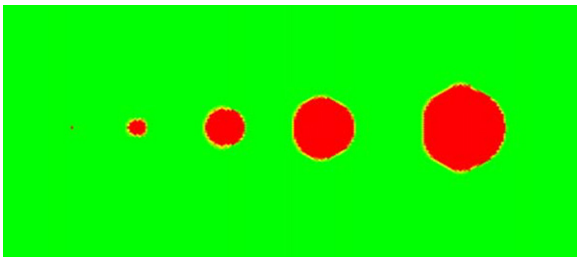

Fig. 15. Spreading of a forest fire in an inhomogeneous forest without topography and wind conditions.

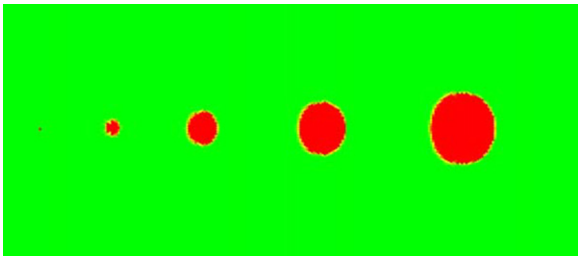

Fig. 16. Spreading of a forest fire in an inhomogeneous non-flat forest without wind conditions.

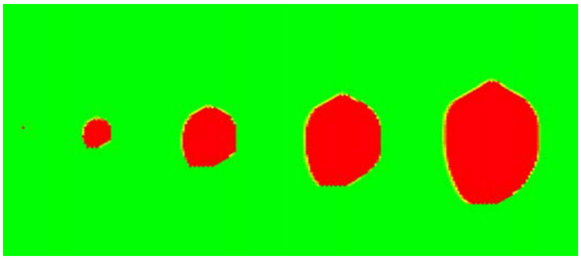

Fig. 17. Spreading of a forest fire in an inhomogeneous flat forest with wind conditions.

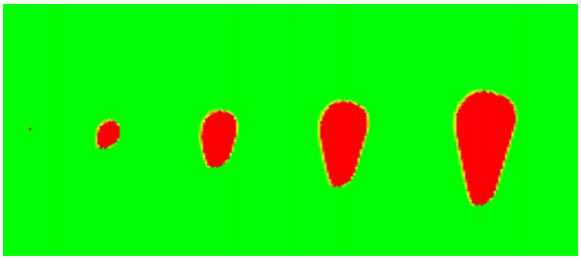

Fig. 18. Spreading of a forest fire in an inhomogeneous non-flat forest with wind conditions.

4.3. Comparison with Karafyllidis–Thanailakis model

If we compare the Karafyllidis–Thanailakis model (see [12]) with our model in the simplest case, that is, when an homogeneous and non-flat forest is considered without wind conditions, then we obtain the results shown in Figs. 19–21. In Fig. 19 the evolution of the number of cells completely burned out is given. In Fig. 20

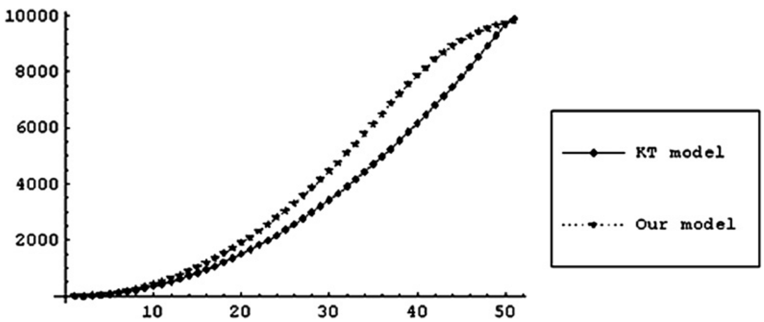

Fig. 19. Evolution of the number of completely burned out cells.

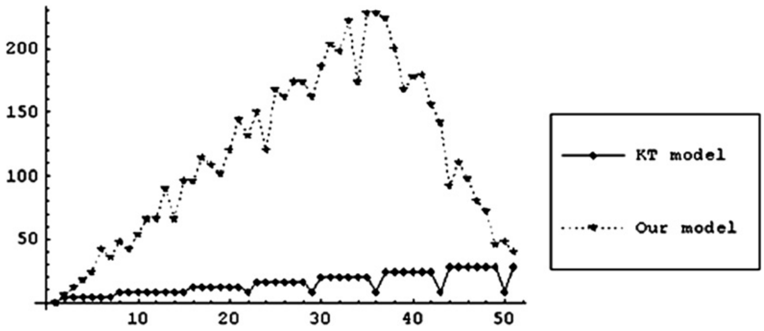

Fig. 20. Evolution of the number of partially burned out cells.

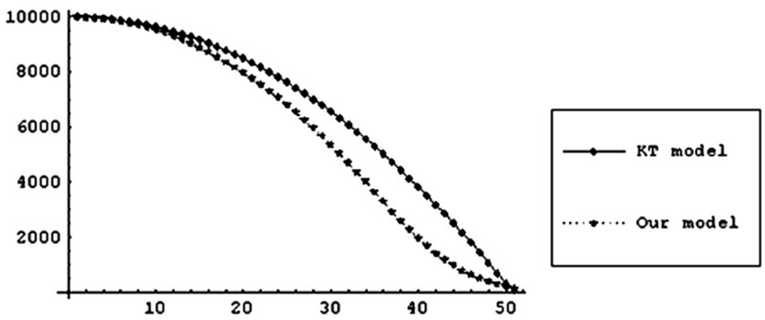

Fig. 21. Evolution of the number of unburned cells.

the evolution of the number of partially burned cells is presented and, finally, in Fig. 21, the evolution of the number of unburned cells is shown.

Note that in our model the portion of partially burned out cells is greater than in Karafyllidis–Thanailakis proposal, that is, in K–T model the probability of the partially burned out cells is slower than in our model. Consequently, we think that in this sense, our model is more accurate than K–T model. Furthermore, in our case, we obtain a more realistic simulation since the fire front is of circular shape in all of its permiter due to the use of an hexagonal array.

Moreover, in this work, the use of an hexagonal array is due to "geographical" reasons. As the cells of the cellular automata stand for regular areas of the land in which the forest fire is spreading, it is considered that all near neighbor cells must have the same border length with the central cell.

5. Conclusions

In this work, a new model for the prediction of forest fire spreading has been introduced. It is based on the use of hexagonal cellular automata and incorporates weather and land topography conditions. The states of each cell are defined by means of the transfer of fractional burned area. Also, different rates of speed are considered. The algorithm seems to be very efficient and it is easily implemented in any computer algebra system, allowing a low computational cost.

The model is applied to eight basic cases depending on the weather, topography and speed conditions. All the graphical models obtained are found to be in agreement with the experience of fire spreading in real forests.

A future work will be made on the design of a similar cellular automata model in which the states of the cells will be defined by means of the transfer of heat and energy; that is, the evolution of a burnable cell will be defined by the comparison of the energy accumulated in the cell and the energy transferred from its neighbors.

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