Assignment - 1

Ruthwika Boyapally

***Abstract*—This document contains the solution to Exercise 3.36 (a) of Oppenheim.**

Problem 1. If the input $x[n]$ to an LTI System is $x[n] = u[n]$, The output is

$$y[n] = \left(\frac{1}{2}\right)^{n-1} u[n+1] \tag{1}$$

Find $H(z)$, the z-transform of the system impulse response, and determine whether it is casual or not
Solution: We know that,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2}$$

$$X(z) = \frac{1}{1 - z^{-1}} \quad |z| > 1 \tag{3}$$

$$\sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-1} u(n+1) z^{-n} \tag{4}$$

$$= z \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-2} u(n) z^{-n} \tag{5}$$

$$= z \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n-2} z^{-n} \tag{6}$$

$$= z \frac{\frac{1}{4}}{1 - \frac{1}{2} z^{-1}} \tag{7}$$

Therefore,

$$Y(z) = \frac{1}{4(z^{-1})(1 - \frac{1}{2} z^{-1})} \quad |z| > \frac{1}{2} \tag{8}$$

Thus,

$$H(z) = \frac{1}{4(z^{-1})(1 - \frac{1}{2} z^{-1})(1 - z^{-1})} \quad |z| > 1 \tag{9}$$

$$H(z) = \frac{4z}{(1 - \frac{1}{2} z^{-1})} - \frac{4}{(1 - \frac{1}{2} z^{-1})} \quad |z| > 1 \tag{10}$$

$$y[n] = 4\left(\frac{1}{2}\right)^{n+1} u[n+1] - 4\left(\frac{1}{2}\right)^{n} u[n] \tag{11}$$

$h[n]$ starts at n=-1 which implies it is not casual