Solutions to Problem Sheet 2

NB There is more than one way to complete most of these problems. I give here only an indication of one way to do the problems and some simple steps may be missed out for brevity - these should not be regarded as model answers. Please however feel free to ask questions about any aspect of the problems. Let me know if you find any errors.

1. To use the transformation method to sample from the Weibull distribution we must invert the cdf

$$F(x) = 1 - \exp\left(-\alpha x^{\beta}\right).$$

Some straightforward manipulations show that

$$F^{-1}(u) = \left(-\frac{1}{\alpha}\log(1-u)\right)^{1/\beta}.$$

Note that since $0 \leq u \leq 1$ the quantity in parentheses is positive and so this quantity is well defined. (We assume we take only the real, positive root.)

2. Let

$$J(c) = \int_{\Omega_k} (f(x) - c)^2 p(x)dx.$$

To find the minimum we differentiate with respect to c and set the result equal to zero to find

$$\int_{\Omega_k} (f(x) - c)p(x)dx = 0.$$

Thus,

$$c = \frac{1}{\overline{p_k}} \int_{\Omega_k} f(x)p(x)dx = \overline{f_k}$$

is an extremum of $J(c)$. We know this is a minimum since $J''(c) = 2\overline{p_k} > 0$.
Hence,

$$\int_{\Omega_k} (f(x) - \overline{f_k})^2 p(x)dx \leq \int_{\Omega_k} (f(x) - \overline{f})^2 p(x)dx,$$

where $\overline{f} = E[f]$. Now

$$\sigma_s^2 = \sum_{k=1}^{M} \int_{\Omega_k} (f(x) - \overline{f_k})^2 p(x)dx \leq \sum_{k=1}^{M} \int_{\Omega_k} (f(x) - \overline{f})^2 p(x)dx = \sigma^2.$$

Hence

$$\sigma_s \leq \sigma,$$

as required.