Markov Chains problem sheet

Problem 1

A transition matrix for a 3-dimensional state space (x, y, z) is given by:

$$P = \left\{ \begin{array}{ccc} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.8 & 0.2 \end{array} \right\} \tag{1}$$

1) Why is this a proper stochastic matrix?

2) What is the probability to move from x to z?

3) What is the probability to end up in y from either x or z?

4) Determine the invariant distribution.

5) Is the Markov Chain ergodic?

Problem 2

We want to sample form a 2-dimensional target distribution $\pi = (0.1, 0.9)^T$.

1) Check that the Markov Chain with $P(x|x) = P(y|y) = 1$ and the rest zero has the correct invariant distribution. Why is this chain not of interest?

2) Find the transition distribution that has $P(x|x) = 0.1$.

3) Does detailed balance hold?

4) Repeat 2) for $P(x|x) = 0.91$.

5) Which Markov Chain would you use, that of 2) or that of 4)? Why?