

# Gibbs-sampler computer problem

To understand the Gibbs sampler better, and especially the way it can be used in data assimilation, you are going to write a Matlab program to estimate the mean of a two dimensional Gaussian distribution. As you can imagine, this Gaussian is the posterior from a linear data assimilation problem.

- 1) Define the mean and the covariance matrix of the Gaussian as  $m = (1, 3)^T$ , and

$$P = \begin{pmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{pmatrix}$$

- 2) Program the Gibbs sampler (hint: use the conditionals of the problem sheet), by drawing two samples from  $N(0, 1)$  (Matlab function randn) and transform them to the correct density by multiplying with the standard deviations of the conditionals and adding their means. Do this for each iterate.
- 3) Look at the time series of your solutions. How long is your estimate of the burn-in period?
- 4) Generate a time series of the mean  $\bar{x}_n$  for each  $n$  in the series. When has the mean converged?
- 5) Repeat items 3) and 4) for a covariance matrix with high correlations:

$$P = \begin{pmatrix} 1.0 & 0.99 \\ 0.99 & 1.0 \end{pmatrix}$$

Why do you think the results take so long to converge?

- 6) Try now to improve the Gibbs sampler by performing a co-ordinate transformation  $y = P^{-1/2}x$  first, do Gibbs on  $y$ , and transform back afterwards. For the square root of  $P$  use the Matlab sqrtm function, and use Matlab pinv for the inverse.