Problem Sheet 1

1. For a random variable with a density $p(x)$, the expectation of a function $f(x)$ is

$$I[f] = \int f(x)p(x)dx.$$

For a sequence of IID random variables $\{X_n\}_{n=1}^N \sim p$ then an empirical approximation to the integral is

$$I_N[f] = \frac{1}{N}\sum_{n=1}^{N} f(X_n).$$

Show that

$$E\left[I_N[f]\right] = I[f]$$

for any N.

2. Suppose Y is a random variable distributed with the uniform distribution on the interval $[0, 1]$. Let $g(Y) = \sin 2\pi Y$. How many samples do you need in order to estimate $E[g]$, to ensure an error of size at most 0.1 with 95% confidence?