Computing Practical 2 - Random variable generation and Variance control

Exercise 1: Transformation method Use the function `TranSample.m` to generate 1000 exponential random numbers with parameter $\lambda = 2$. Produce a histogram of your results (matlab inbuilt function `hist` may help you here). Save this plot so that you can compare it with other methods later. Make a note of the (empirical) mean of your sample. Is this close the value you would expect?

Exercise 2: Accept-reject method Use the function `AcceptReject.m` to generate 1000 exponential random numbers with parameter $\lambda = 2$. Note that we are looking to compute $Y \sim 2e^{-2y}$, so if we take $q \sim \text{unif}[0,1]$ and observe that $2e^{-2y} \leq 2$ for all y we see that we can pick $c = 2$.

Plot a histogram of your results and compute the mean of your sample. What was the empirical acceptance rate? How do these results compare with those in Exercise 1? Explain what has gone wrong and what you could do to rectify the problem.

Exercise 3: Antithetic variates We wish to estimate

$$\theta = \int_0^1 e^{-x^2} dx.$$

As usual, we may write $\theta = E[e^{-U^2}]$ where $U \sim \text{unif}[0,1]$. First compute a standard Monte Carlo estimate with 1000 samples and its (empirical) variance:

```
>> n=1000; U=rand(2*n,1); Y=exp(-U.∧ 2); [mean(Y) std(Y)∧2]
```

Now use n antithetic pairs:

```
>>u=rand(n,1); v=1-u; Z=(exp(-u.∧2) + exp(-v.∧2))/2; [mean(Z) std(Z)∧2]
```

Compare the variances of the two methods. You can compute a very good numerical approximation of the integral using the matlab erf function. How does the actual error for these realisations compare with the expected size of the error?

Exercise 4: Stratified sampling Suppose we wish to estimate the same integral as in Exercise 3 using stratified sampling. In the script `StratifiedSampling.m` we have split $[0,1]$ into 10 pieces with 100 samples per piece. Compare the results with those in Exercise 3.

Exercise 5: Importance sampling We wish to estimate the integral

$$\theta = \int_{-\infty}^{\infty} e^{-|x-a|/D} dx$$

where $a, D \in \mathbb{R}$, $D > 0$. Work out the exact value of the integral.

If we try to do this by standard Monte Carlo quadrature we immediately hit a problem - namely the range of integration is not finite so we cannot just use a uniform variable for our implied probability distribution. However since the integrand is rapidly decaying it is not a bad approximation to consider the integral over the finite range $[-L, L]$. The function `ImportSamp.m` allows you to compare this approach with Importance Sampling using the standard normal $N(0,1)$ distribution as a proposal. Try this out for $a = 0.0, D = 0.5, L = 3.0$, what happens?

Now vary the paramters a, D, L (a good idea to change one thing at a time to start with!). Think about what you expect to happen before trying it on the computer. Were you right?

For large(ish) values of X, what happens to the integrand/proposal ratio for the importance sampling? Can you explain this behaviour?