M.Sc. Course on Operational/Advanced Data Assimilation Techniques (MTMD02): Practical investigation on the B-matrix

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This practical is to investigate the effect of the background error covariance matrix on the analysis error covariance matrix. You will use a theoretical result and edit existing FORTRAN-90 code to examine the following question.

"Under what circumstances can the effect of two observations on the analysis error covariance matrix be greater than the sum of their individual effects?"

Theoretical results

There are two basic sources of information that go into a data assimilation system, namely the observations, $\mathbf{y}$, and the background state, $\mathbf{x}_B$ (which has n elements). The error statistics of each are usually described by their respective error covariance matrices, $\mathbf{R}$ and $\mathbf{B}$. The analysis, $\mathbf{x}_A$, which is formed by combining $\mathbf{y}$ and $\mathbf{x}_B$ has its own error covariance matrix, $\mathbf{P}_A$ which depends upon $\mathbf{R}$, $\mathbf{B}$ and the observation operator $\mathbf{H}$, in the following way

$$\mathbf{P}_A = (\mathbf{I} - \mathbf{KH})\mathbf{B}, \tag{1}$$

$$\text{where} \qquad \mathbf{K} = \mathbf{BH}^T(\mathbf{R} + \mathbf{HBH}^T)^{-1}.$$

$\mathbf{KHB}$ describes the reduction in the error covariance (going from background to analysis error) and is the tool used here to investigate the question above.

Single observations

For one direct observation of state vector element α, the following form of $\mathbf{H}$

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & \dots & 1 & \dots & 0 & 0 \end{pmatrix},$$

(which has zeros apart from 1 at position α) leads to the following diagonal matrix elements of $\mathbf{KHB}$

$$(\mathbf{KHB})^{(\alpha)}_{\gamma\gamma} = \frac{(\mathbf{B}_{\gamma\alpha})^2}{\mathbf{R}_{11} + \mathbf{B}_{\alpha\alpha}}, \qquad 1 \leqslant i, j \leqslant n, \tag{2}$$

where $\mathbf{R}_{11}$ is the error variance of the single observation. For another observation assimilated separately at position β, the matrix elements of $\mathbf{KHB}$ are

$$(\mathbf{KHB})^{(\beta)}_{\gamma\gamma} = \frac{(\mathbf{B}_{\gamma\beta})^2}{\mathbf{R}_{22} + \mathbf{B}_{\beta\beta}}, \tag{3}$$

where $\mathbf{R}_{22}$ is the error variance of the other single observation.

Pairs of observations

For a pair of observations (of elements α and β) assimilated together, the following form of **H**

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 & 0 & 1 & \dots & 0 & 0 \end{pmatrix},$$

(which has zeros apart from 1 at position α in the first row and position β in the second row) leads to the following diagonal matrix elements of **KHB** (for diagonal **R**)

$$(\mathbf{KHB})_{\gamma\gamma}^{(\alpha\beta)} = \frac{(\mathbf{B}_{\alpha\gamma})^2(\mathbf{R}_{22} + \mathbf{B}_{\beta\beta}) + (\mathbf{B}_{\beta\gamma})^2(\mathbf{R}_{11} + \mathbf{B}_{\alpha\alpha}) - 2\mathbf{B}_{\alpha\gamma}\mathbf{B}_{\beta\gamma}\mathbf{B}_{\alpha\beta}}{(\mathbf{R}_{11} + \mathbf{B}_{\alpha\alpha})(\mathbf{R}_{22} + \mathbf{B}_{\beta\beta}) - (\mathbf{B}_{\alpha\beta})^2}. \qquad (4)$$

Notes on these results

If there is no background error covariance between positions α and β then the combined reduction in error covariance is the sum of the individual reductions. Using $\mathbf{B}_{\alpha\beta} = 0$ in (4) we find

$$(\mathbf{KHB})_{\gamma\gamma}^{(\alpha\beta)} = (\mathbf{KHB})_{\gamma\gamma}^{(\alpha)} + (\mathbf{KHB})_{\gamma\gamma}^{(\beta)}. \qquad (5)$$

We are more interested in the general situation where $\mathbf{B}_{\alpha\beta} \neq 0$.

Your tasks

The system that the question will be studied in is the 2-D system involving *p, u* and *v* fields, where the background errors are geostrophically related to each other (you will need to see the lecture notes for some example structure functions). This is a multivariate system and so α, β and γ each represent a 2-D position and variable (e.g. γ might represent *p* in the middle of the domain).

1. Propose two observations (α and β), and a diagnostic position (γ) such that the combined reduction in error covariance, $(\mathbf{KHB})_{\gamma\gamma}^{(\alpha\beta)}$, is exactly the same as the sum of the individual reductions, $(\mathbf{KHB})_{\gamma\gamma}^{(\alpha)} + (\mathbf{KHB})_{\gamma\gamma}^{(\beta)}$ (ie that (5) is satisfied).
2. Propose two observations, and a diagnostic position such that the combined reduction in error covariance is less than the sum of the individual reductions.
3. Propose two observations, and a diagnostic position such that the combined reduction in error covariance is greater than the sum of the individual reductions.
4. The FORTRAN 90 code provided calculates $\mathbf{P}_\mathrm{A}$ and outputs some diagnostics based on a specified observation network. Use this code to do some experiments to confirm your proposals above.
5. Write a brief report (e.g. 2 pages) of your findings to answer the question above. You should discuss the implications of your results.

Hint: you may want to do a number of tests by taking pairs of observations and then taking each separately to see how the analysis error covariance matrix is affected. This may be repeated with different values of the observation error standard deviations.

How to download and compile the FORTRAN 90 code

Go to the blackboard web page for this course and download the following file from the 'Assignments' section

```
synergy.f90
```

Alternatively, the code is available via the web address

```
www.met.rdg.ac.uk/~ross/MTMD02/synergy.f90
```

You are free to use the computer system of your choice. For Meteorology Dept. UNIX, the compilation command is given in the comments at the start of the code. To compile and run the code in the PC lab follow this procedure.

- Download the code to an appropriate place (e.g. the N: drive on the PCs connects to your central university account).
- Bring up the FORTRAN package (select `Start` → `All Programs` → `Programming` → `Salford Software` → `Salford Plato IDE`).
- Load `synergy.f90`.
- To compile, select `Project` → `Compile File`.
- To run, select `Project` → `Run`.

Outline of the code

The model space of the data assimilation code involves 2-D fields of p, u and v, each on a 20×20 grid.

The code reads in some observational information from the text file "`ObsNetwork`". The first line in this file specifies the number of observations, p. The remaining p lines have the following format:

`obs type (1=`p`,2=`u`,3=`v`),` x`-pos,` y`-pos, obs error std dev`

The x and y positions are each in grid points (1 to 20) and the error standard deviation is in SI units (i.e. Pa for pressure and m/s for wind).

The code calculates the analysis error covariance matrix, $\mathbf{P}_A$ and outputs diagonal elements of the $\mathbf{B}$ and $\mathbf{P}_A$ matrices at a number of positions specified in the text file "`DiagsPos`". The first line in this file specifies the number of diagnostic positions, q. The remaining q lines have the following format:

x`-pos,` y`-pos`

Diagnostics are output at these positions for all three variables p, u and v in the file "`B_Pa_Errs.dat`", which has a self explanatory format. The input and output files may be edited/viewed with any text editor (e.g. WordPad on the PCs).

Note that the observational values do not enter into the $\mathbf{P}_A$ formula and so are not needed by this program.