Markov Chains computer problem

To understand Markov Chain better, and especially the way they are used in data assimilation, you are going to write a Matlab program to sample from the simple 2-dimensional target distribution $\pi = (0.1, 0.9)^T$ on state space $\{0, 1\}$.

Problem 1

Write a Matlab program and evaluate it by following these steps:

1) Define a transition matrix (see problem sheet)

$$P = \begin{pmatrix} 0.91 & 0.09 \\ 0.01 & 0.99 \end{pmatrix}$$

2) Define an initial probablity distribution $p(0) = 0.333$ and $p(1) = 0.667$.

3) Iterate the transition matrix on this initial probablity distribution and show that it converges to π.

4) Repeat 3) for some other choices of the initial probability distribution.

5) Repeat 1)-4) for the transition probability distribution given by

$$P = \begin{pmatrix} 0.1 & 0.9 \\ 0.1 & 0.9 \end{pmatrix}$$

Which transition distribution converges faster? Does that agree with your ideas from the problem sheet?

Problem 2

Now we are going to do the real thing. We start from some initial guess, say $x = 0$, and use the transition distributions defined above to determine the limiting distribution for x. To this end:

1) Use the previous program and run x through the transition iteration of item 1), for 100 steps. Plot the time series of x and produce a histogram.

2) Increase the number of steps until convergence (by eye) of the histogram.

3) Repeat 1) and 2) for the transition distribution from 5) above. What is your conclusion?