

Assignment 5

AI1110: Probability and Random Variables

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8 [12th CBSE Probability Exercise 13.1]:

A die is thrown three times. Events E, F are defined as follows

E : 4 on the third throw.

F : 6 on the first and 5 on the second throw.

Find the probability $\Pr(E|F)$

Also E, F are also independent events therefore from (2) and ()

$$\Pr(E \cap F) = \Pr(E) \Pr(F) = \frac{1}{6} \times \frac{1}{36} \quad (7)$$

$$\implies \Pr(E \cap F) = \frac{1}{216} \quad (8)$$

Solution: Let $X_i \in \{1, 2, 3, 4, 5, 6\}$ where $i = 1, 2, 3$ be the random variables representing the outcomes of throwing a die three times.

Since we have to find probability of E given that F has already occurred.
As,

$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} \quad (9)$$

Probability of event E = Probability of $X_3 = 4$

$$\Pr(E) = \Pr(X_3 = 4) \quad (1) \quad \text{From () and (5)}$$

Since all the outcomes are equally likely, the probability of a particular outcome is $\frac{1}{6}$

So

$$\Pr(E) = \Pr(X_3 = 4) = \frac{1}{6} \quad (2)$$

Probability of event F = Probability of $X_1 = 6, X_2 = 5$.

So

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) \quad (3)$$

Random variable X_1 depends on first throw of die and random variable X_2 depends on second throw of die, so X_1 and X_2 are independent.

So

$$\Pr(X_1 = 6, X_2 = 5) = \Pr(X_1 = 6) \Pr(X_2 = 5) \quad (4)$$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad (5)$$

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) = \frac{1}{36} \quad (6)$$

So the probability of E given that F has already happened = $\Pr(E|F) = \frac{1}{6}$

$$\implies \Pr(E|F) = \frac{\frac{1}{216}}{\frac{1}{36}} \quad (10)$$

$$\implies \Pr(E|F) = \frac{1}{6} \quad (11)$$