Assignment-1

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If the straight lines $3x-5y=7$ and $4x+ay+9=0$ are perpendicular to one another, find the value of a.

SOLUTION

1) If two lines with normals $\mathbf{n}_1, \mathbf{n}_2$ are perpendicular, then

$$\mathbf{n}_1^\top \mathbf{n}_2 = \mathbf{n}_2^\top \mathbf{n}_1 = 0 \tag{0.0.1}$$

2) The equation of a line is given by

$$\mathbf{n}^\top \mathbf{x} = c$$

where $\mathbf{n}$ is the normal vector of the line.
Vector form of 3x - 5y =7 is

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \tag{0.0.2}$$

with $\mathbf{n}_1 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ as normal vector.
Vector form of 4x+ay+9=0 is

$$\begin{pmatrix} 4 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -9 \tag{0.0.3}$$

with $\mathbf{n}_2 = \begin{pmatrix} 4 \\ a \end{pmatrix}$ as normal vector.
By result (0.0.1),

$$\mathbf{n}_1^\top \mathbf{n}_2 = 0 \tag{0.0.4}$$

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} 4 \\ a \end{pmatrix} = 0 \tag{0.0.5}$$

$$12 - 5a = 0 \tag{0.0.6}$$

$$\therefore a = \frac{12}{5} \tag{0.0.7}$$

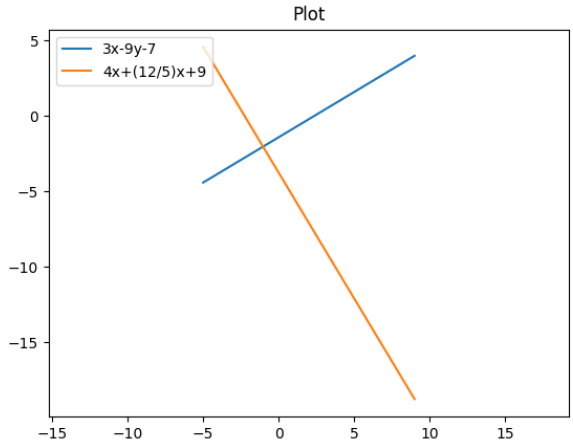

Fig. 0. Plot of two lines with a = $\frac{12}{5}$

PLOT