

A Problem On Total Probability Theorem

Chittepu Rutheesh Reddy
CS21BTECH11014

May 15, 2022

Outline

1 Total Probability Theorem

2 Question

3 Solution

Total Probability Theorem

Statement:

Let the events $E_1, E_2, E_3, \dots, E_n$ be a set of exhaustive events of a sample space S , such that $E_1, E_2, E_3, \dots, E_n$ are partitions of a sample space S , the happening of an event A from the sample space S is

$$\Pr(A) = \sum_{i=1}^n \Pr(E_i) \Pr(A|E_i) \quad (1)$$

Question

Q1 [12th CBSE Probability Exercise 13.3]:

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution

Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be the random variables representing the outcomes defined as follows.

Input Variable	Value	Description
X	0	1 st ball drawn from urn is Red
	1	2 nd ball drawn from urn is Black
Y	0	2 st ball drawn from urn is Red
	1	2 nd ball drawn from urn is Black

Table

Solution

Given data of the question, in terms of probability is presented in the table

Probability	Value
$\Pr(Y = 0 X = 0)$	$\frac{7}{12}$
$\Pr(Y = 0 X = 1)$	$\frac{5}{12}$
$\Pr(X = 0)$	$\frac{1}{2}$
$\Pr(X = 1)$	$\frac{1}{2}$

Table

Solution

The required probability is $\Pr(Y = 0)$.

By total probability theorem

$$\Pr(Y = 0) = \sum_{i=0}^1 \Pr(Y = 0|X = i) \Pr(X = i) \quad (2)$$

$$= \frac{7}{12} \cdot \frac{1}{2} + \frac{5}{12} \cdot \frac{1}{2} \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

\therefore The probability that second ball drawn is red is 0.5