

A Problem on ML Estimation

Chittepu Rutheesh Reddy
CS21BTECH11014

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Question

Q23 [Papoulis Textbook Exercise 8]:

The random variable \mathbf{x} has a Poisson distribution with mean θ . Show that the ML estimate of θ is \bar{x} .

Solution

The joint P.D.F of RVs when $\mathbf{x}_i = x_i, i \in (1, n)$ is given by

$$f(X, \theta) = e^{-n\theta} \prod \frac{\theta^{x_i}}{x_i!} \quad (1)$$

$$f(X, \theta) = e^{-n\theta} \frac{\theta^{n\bar{x}}}{\prod x_i!} \quad (2)$$

At ML estimate of θ , $f(X, \theta)$ is maximum

Solution

So,

$$\frac{df(X, \theta)}{d\theta} = -nf(X, \theta) + \frac{n\bar{x}}{\theta}f(X, \theta) \quad (3)$$

$$\text{When } \frac{df(X, \theta)}{d\theta} = 0 \quad (4)$$

$$-n + \frac{n\bar{x}}{\theta} = 0 \quad (5)$$

$$\therefore \theta = \bar{x} \quad (6)$$