Assignment 12
AI1110: Probability and Random Variables

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Q6 [Papoulis Textbook Exercise 9]:
Show that if,

$$R_v(t_1, t_2) = q(t_1)\delta(t_1 - t_2) \tag{1}$$

$w''(t) = v(t)U(t)$ and $w(0) = w'(0) = 0$, then

$$E\{w^2(t)\} = \int_0^t (t-\tau)q(\tau)d\tau \tag{2}$$

Solution: The equations,

$$w''(t) = v(t)U(t) \tag{3}$$

$$w(0) = w'(0) = 0 \tag{4}$$

specify a system with input $v(t)U(t)$ and impulse response $h(t) = tU(t)$.
And we know that

$$E\{|y(t)^2|\} = q(t) * |h(t)^2| = \int_{-\infty}^{\infty} q(t-\alpha)|h(\alpha)^2|d\alpha \tag{5}$$

So,

$$Ew(t)^2 = q(t) * t^2U(t) \tag{6}$$

$$Ew(t)^2 = \int_o^t (t-\tau)^2 q(\tau)d\tau \tag{7}$$

Hence proved.