Assignment 11
AI1110: Probability and Random Variables

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Q23 [Papoulis Textbook Exercise 8]: The random variable **x** has a Poisson distribution with mean θ. Show that the ML estimate of θ is $\bar{x}$.
Solution: The joint P.D.F of RVs when $\mathbf{x}_i = x_i$, i $\in (1, n)$ is given by

$$f(X,\theta) = e^{-n\theta}\Pi\frac{\theta^{x_i}}{x_i!} \tag{1}$$

$$f(X,\theta) = e^{-n\theta}\frac{\theta^{n\bar{x}}}{\Pi x_i!} \tag{2}$$

At ML estimate of θ, $f(X,\theta)$ is maximum,
So,

$$\frac{df(X,\theta)}{d\theta} = -nf(X,\theta) + \frac{n\bar{x}}{\theta}f(X,\theta) \tag{3}$$

$$\text{When } \frac{df(X,\theta)}{d\theta} = 0 \tag{4}$$

$$-n + \frac{n\bar{x}}{\theta} = 0 \tag{5}$$

$$\therefore \theta = \bar{x} \tag{6}$$