Assignment 6
AI1110: Probability and Random Variables

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1 [12th CBSE Probability Exercise 13.3]:
An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution: Let $X \in \{0,1\}$ and $Y \in \{0,1\}$ be the random variables representing the outcomes defined as follows.

Input Variable	Value	Description
X	0	1^{st} ball drawn from urn is Red
	1	2^{nd} ball drawn from urn is Black
Y	0	2^{st} ball drawn from urn is Red
	1	2^{nd} ball drawn from urn is Black

TABLE I

Given data of the question in terms of probability is presented in the table

Probability	**Value**
Pr(Y=0| X=0)	$\frac{7}{12}$
Pr(Y=0 |X=1)	$\frac{5}{12}$
Pr(X=0)	$\frac{1}{2}$
Pr(X=1)	$\frac{1}{2}$

TABLE II

The required probability is $\Pr(Y=0)$.
By total probability theorem

$$\Pr(Y=0) = \sum_{i=0}^{1} \Pr(Y=0|X=i)\Pr(X=i) \tag{1}$$

$$= \frac{7}{12}.\frac{1}{2} + \frac{5}{12}.\frac{1}{2} \tag{2}$$

$$= \frac{1}{2} \tag{3}$$

$\therefore$ The probability that second ball drawn is red is 0.5