A Simple Problem On Definition of Cumulative Distribution Function

Chittepu Rutheesh Reddy CS21BTECH11014

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Outline

Question

Solution

Question

Q15 [12th Papoulis Textbook Exercise 4]:

Show that, if $a \le x(\zeta) < b$ for every $\zeta \in S$, then F(x) = 1 for x > b and F(x) = 0 for x < a.

Solution

1) for x > b

$$F(x) = \Pr\left(\mathbf{x} \le x\right) \tag{1}$$

$$x(\zeta) < b, \forall \zeta$$
 (2)

$$\implies x(\zeta) < b < x, \forall \zeta \tag{3}$$

$$\implies \{\mathbf{x} \le x\} = S \tag{4}$$

$$\implies F(x) = \Pr(S) = 1 \tag{5}$$

$$\therefore F(x) = 1, \forall x > b.$$

Solution

2) for x < a

$$F(x) = \Pr\left(\mathbf{x} \le x\right) \tag{6}$$

$$a \le x(\zeta), \forall \zeta \tag{7}$$

$$\implies x < a \le x(\zeta), \forall \zeta \tag{8}$$

$$\implies \{\mathbf{x} < x\} = \phi \tag{9}$$

$$\implies F(x) = \Pr(\phi) = 0 \tag{10}$$

$$\therefore F(x) = 0, \forall x < a.$$