

# Assignment 5

## AI1110: Probability and Random Variables

Indian Institute of Technology, Hyderabad

Chittepu Rutheesh Reddy  
cs21btech11014

### 8 [12<sup>th</sup> CBSE Probability Exercise 13.1]:

A die is thrown three times. Events E, F are defined as follows

E : 4 on the third throw.

F : 6 on the first and 5 on the second throw.

Find the probability  $\Pr(E|F)$

Also E, F are also independent events therefore from (2) and (5)

$$\Pr(E \cap F) = \Pr(E) \Pr(F) = \frac{1}{6} \times \frac{1}{36} \quad (6)$$

$$\implies \Pr(E \cap F) = \frac{1}{216} \quad (7)$$

**Solution:** Let  $X_i \in \{1, 2, 3, 4, 5, 6\}$  where  $i = 1, 2, 3$  be the random variables representing the outcomes of throwing a die three times.

Since we have to find probability of E given that F has already occurred.  
As,

$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} \quad (8)$$

Probability of event E = Probability of  $X_3 = 4$

$$\Pr(E) = \Pr(X_3 = 4) \quad (1) \quad \text{From (5) and (6)}$$

Since all the outcomes are equally likely their probabilities are same

so

$$\Pr(E) = \Pr(X_3 = 4) = \frac{1}{6} \quad (2)$$

Probability of event F = Probability of  $X_1 = 6, X_2 = 5$ .

so

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) \quad (3)$$

Random variable  $X_1$  depends on first throw of die and random variable  $X_2$  depends on second throw of die so  $X_1$  and  $X_2$  are independent.

so

$$\begin{aligned} \Pr(X_1 = 6, X_2 = 5) &= \Pr(X_1 = 6) \Pr(X_2 = 5) \\ &= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \end{aligned} \quad (4)$$

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) = \frac{1}{36} \quad (5)$$

$$\implies \Pr(E|F) = \frac{\frac{1}{216}}{\frac{1}{36}} \quad (9)$$

$$\implies \Pr(E|F) = \frac{1}{6} \quad (10)$$

So the probability of E given that F has already happened =  $\Pr(E|F) = \frac{1}{6}$