Assignment

Chittepu Rutheesh Reddy
CS21BTECH11014

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Outline

Problem 1.3

Question

Find theoretical expression of $F_U(x)$

Solution

For uniform distribution , the pdf is a constant Let,

$$p_U(x) = a, 0 \leq x \leq 1 \tag{1}$$

$$\text{then, } F_U(x) = \int_0^x a dx \tag{2}$$

$$F_U(x) = ax, 0 \leq x \leq 1 \tag{3}$$

$$\text{As, } F_U(1) = 1 \implies a = 1 \tag{4}$$

$$\therefore F_U(x) = x, 0 \leq x \leq 1 \tag{5}$$

Problem 2.5

Question

$$p_X(x) = \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}), \infty < x < \infty \tag{6}$$

Calculate mean and variance of X.

Solution

Mean, $U(X) = \int_{\infty}^{\infty} xp_X(x).dx$

$$\text{So, } U(X) = \int_{-\infty}^{\infty} x\frac{1}{\sqrt{2\pi}}e^{\left(\frac{-x^2}{2}\right)}.dx \tag{7}$$

$$U(X) = \frac{-1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^t.dt, \text{where, } t = \frac{-x^2}{2} \tag{8}$$

$$U(X) = 0 \tag{9}$$

As $E(X) = 0$, $Var(X) = E(X^2)$, So

$$E(X^2) = \int_{-\infty}^{\infty} x^2\frac{1}{\sqrt{2\pi}}e^{\left(\frac{-x^2}{2}\right)}.dx \tag{10}$$

$$E(X^2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x^2e^{\left(\frac{-x^2}{2}\right)}.dx = \frac{1}{\sqrt{2\pi}}\frac{\sqrt{2\pi}}{1} \tag{11}$$

$$E(X^2) = 1 \tag{12}$$

Problem 3.2

Question

Find the theoretical expression of $F_V(X)$, where

$$V = -2ln(1 - U) \tag{13}$$

Solution

If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$X = -2ln(1 - Y) \tag{14}$$

$$ln(1 - Y) = e^{\frac{-X}{2}} \tag{15}$$

$$Y = 1 - e^{\frac{-X}{2}} \tag{16}$$

$$F_V(X) = F_U(1 - e^{\frac{-X}{2}}) \tag{17}$$

when , $0 \leq 1 - e^{\frac{-X}{2}} \leq 1$

$$0 \leq e^{\frac{-X}{2}} \leq 1 \tag{18}$$

$$X \geq 0, \text{So}, \tag{19}$$

$$F_V(X) = 1 - e^{\frac{-X}{2}}, X \geq 0 \tag{20}$$

$$F_V(X) = 0, X < 0 \tag{21}$$