

Assignment-1

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If the straight lines $3x - 5y = 7$ and $4x + ay + 9 = 0$ are perpendicular to one another, find the value of a .

SOLUTION

- 1) If two lines with normals $\mathbf{n}_1, \mathbf{n}_2$ are perpendicular, then

$$\mathbf{n}_1^\top \mathbf{n}_2 = \mathbf{n}_2^\top \mathbf{n}_1 = 0 \quad (0.0.1)$$

- 2) The equation of a line is given by

$$\mathbf{n}^\top \mathbf{x} = c$$

where \mathbf{n} is the normal vector of the line.

Vector form of $3x - 5y = 7$ is

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \quad (0.0.2)$$

with $\mathbf{n}_1 = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ as normal vector.

Vector form of $4x + ay + 9 = 0$ is

$$\begin{pmatrix} 4 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -9 \quad (0.0.3)$$

with $\mathbf{n}_2 = \begin{pmatrix} 4 \\ a \end{pmatrix}$ as normal vector.

By result (0.0.1),

$$\mathbf{n}_1^\top \mathbf{n}_2 = 0 \quad (0.0.4)$$

$$\begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} 4 \\ a \end{pmatrix} = 0 \quad (0.0.5)$$

$$12 - 5a = 0 \quad (0.0.6)$$

$$\text{So } a = \frac{12}{5} \quad (0.0.7)$$

PLOT

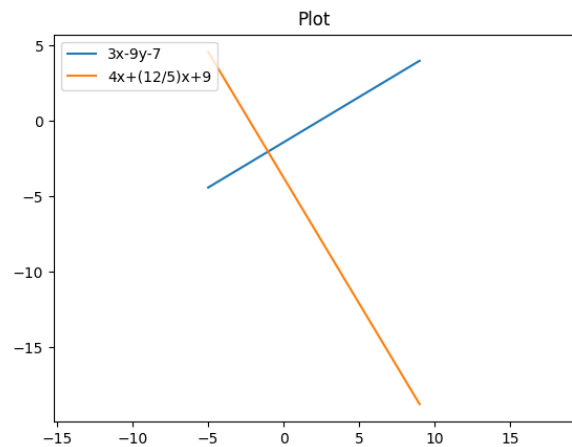


Fig. 0. Plot of two lines with $a = \frac{12}{5}$