Assignment 5
AI1110: Probability and Random Variables

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8 [12th CBSE Probability Exercise 13.1]:
A die is thrown three times. Events E,F are defined as follows
E : 4 on the third throw.
F : 6 on the first and 5 on the second throw.
Find the probability $\Pr(E|F)$

Solution: Let $X_i \in \{1,2,3,4,5,6\}$ where $i = 1,2,3$ be the random variables representing the outcomes of throwing a die three times.

Probability of event E = Probability of $X_3 = 4$

$$\Pr(E) = \Pr(X_3 = 4) \tag{1}$$

Since all the outcomes are equally likely, the probability of a particular outcome is $\frac{1}{6}$
So

$$\Pr(E) = \Pr(X_3 = 4) = \frac{1}{6} \tag{2}$$

Probability of event F = Probability of $X_1 = 6, X_2 = 5$.
So

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) \tag{3}$$

Random variable X_1 depends on first throw of die and random variable X_2 depends on second throw of die, so X_1 and X_2 are independent.
So

$$\Pr(X_1 = 6, X_2 = 5) = \Pr(X_1 = 6)\Pr(X_2 = 5) \tag{4}$$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \tag{5}$$

$$\Pr(F) = \Pr(X_1 = 6, X_2 = 5) = \frac{1}{36} \tag{6}$$

Also E,F are also independent events therefore from (2) and (6)

$$\Pr(EF) = \Pr(E)\Pr(F) = \frac{1}{6} \times \frac{1}{36} \tag{7}$$

$$\implies \Pr(EF) = \frac{1}{216} \tag{8}$$

Since we have to find probability of E given that F has already occurred.
As,

$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{9}$$

From (6) and (8)

$$\implies \Pr(E|F) = \frac{\frac{1}{216}}{\frac{1}{36}} \tag{10}$$

$$\implies \Pr(E|F) = \frac{1}{6} \tag{11}$$

So the probability of E given that F has already happened $= \Pr(E|F) = \frac{1}{6}$