

Assignment 10

AI1110: Probability and Random Variables

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Q48 [Papoulis Textbook Exercise 6]: Show that if random variables x and y are normal and independent, then

$$\Pr(xy < 0) = G\left(\frac{\eta_x}{\sigma_x}\right) + G\left(\frac{\eta_y}{\sigma_y}\right) - 2G\left(\frac{\eta_x}{\sigma_x}\right)G\left(\frac{\eta_y}{\sigma_y}\right) \quad (1)$$

Solution: See that,

$$xy < 0 = x < 0, y > 0 + x > 0, y < 0 \quad (2)$$

$$\Pr(xy < 0) = \Pr(x < 0, y > 0) + \Pr(x > 0, y < 0) \quad (3)$$

As x, y are independent

$$\Pr(xy < 0) = \Pr(x < 0) \Pr(y > 0) + \Pr(x > 0) \Pr(y < 0) \quad (4)$$

$$\Pr(xy < 0) = F(x)(1 - F(y)) + (1 - F(x))F(y) \quad (5)$$

As,

$$F(x) = 1 - G\left(\frac{\eta_x}{\sigma_x}\right), F(y) = 1 - G\left(\frac{\eta_y}{\sigma_y}\right) \quad (6)$$

Upon simplifying (5),

$$\Pr(xy < 0) = G\left(\frac{\eta_x}{\sigma_x}\right) + G\left(\frac{\eta_y}{\sigma_y}\right) - 2G\left(\frac{\eta_x}{\sigma_x}\right)G\left(\frac{\eta_y}{\sigma_y}\right) \quad (7)$$

Hence, proved.