

A Problem on Auto Correlation

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Question

Q6 [Papoulis Textbook Exercise 9]:

Show that if,

$$R_v(t_1, t_2) = q(t_1)\delta(t_1 - t_2) \quad (1)$$

$w''(t) = v(t)U(t)$ and $w(0) = w'(0) = 0$, then

$$E\{w^2(t)\} = \int_0^t (t - \tau)q(\tau)d\tau \quad (2)$$

Solution

The equations,

$$w''(t) = v(t)U(t) \quad (3)$$

$$w(0) = w'(0) = 0 \quad (4)$$

specify a system with input $v(t)U(t)$ and impulse response $h(t) = tU(t)$.
And we know that

$$E\{|y(t)^2|\} = q(t) * |h(t)^2| = \int_{-\infty}^{\infty} q(t - \alpha) |h(\alpha)^2| d\alpha \quad (5)$$

Solution

So,

$$Ew(t)^2 = q(t) * t^2 U(t) \quad (6)$$

$$Ew(t)^2 = \int_0^t (t - \tau)^2 q(\tau) d\tau \quad (7)$$

Hence proved.