A Problem on Two Independent Normal Random Variables

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Outline

Question

Q48 [12^{th} Papoulis Textbook Exercise 6]:

Show that if random variables **x** and **y** are normal and independent, then

$$\Pr(xy < 0) = G\left(\frac{\eta_x}{\sigma_x}\right) + G\left(\frac{\eta_y}{\sigma_y}\right) - 2G\left(\frac{\eta_x}{\sigma_x}\right)G\left(\frac{\eta_y}{\sigma_y}\right) \tag{1}$$

Solution

See that,

$$xy < 0 = x < 0, y > 0 + x > 0, y < 0 \tag{2}$$

$$\Pr(xy < 0) = \Pr(x < 0, y > 0) + \Pr(x < 0, y > 0) \tag{3}$$

As x,y are independent

$$\Pr(xy < 0) = \Pr(x < 0)\Pr(y > 0) + \Pr(x < 0)\Pr(y > 0) \tag{4}$$

$$\Pr(xy < 0) = F(x)(1 - F(y)) + (1 - F(x))F(y) \tag{5}$$

Solution

As,

$$F(x) = 1 - G\left(\frac{\eta_x}{\sigma_x}\right), F(y) = 1 - G\left(\frac{\eta_y}{\sigma_y}\right) \tag{6}$$

Upon simplifying (5),

$$\Pr(xy < 0) = G\left(\frac{\eta_x}{\sigma_x}\right) + G\left(\frac{\eta_y}{\sigma_y}\right) - 2G\left(\frac{\eta_x}{\sigma_x}\right)G\left(\frac{\eta_y}{\sigma_y}\right) \tag{7}$$

Hence, proved.