Q6

For documented code, please see Q6.py, Q6p2.py and Q6p3.py

Image of mean and first 4 Principle modes of variation:

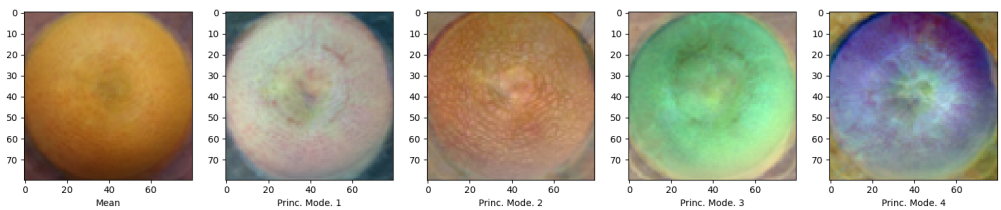

Figure 1: Images of Mean and Principle Modes of Variation

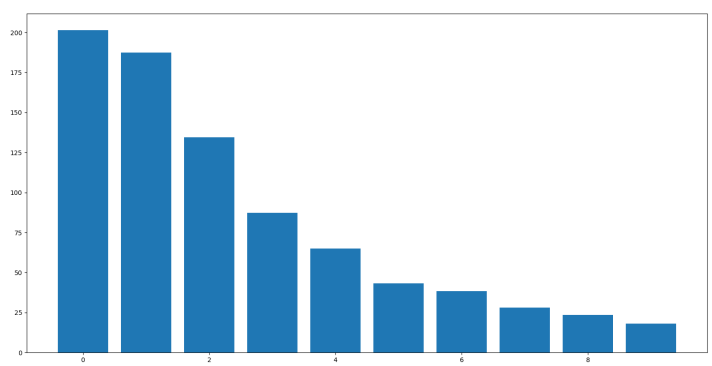

Figure 2: Bar Graph of Top 10 Eigenvalues

The algorithm used to generate fruit images as a linear combination of the top 4 eigenvectors is identical to the idea used in Q3. This is a result of Frobenius norm of a matrix being equivalent to Euclidean norm or a vector. Hence, just as Euclidean norms of the difference of two vectors are minimized by projecting one onto the other, Frobenius norm can be minimized by projecting one matrix onto another. In Q3, we had to reduce number of independent variables to 1, here, we have to reduce the number of independent variables to 4. PCA can be employed in this. We use the matrices below (N is the sample size, = 16; M is the total number of pixels in the images, $= 80 \times 80 \times 3 = 19200$):

$$\text{The sample matrix: } S_{N\times M} = \begin{bmatrix} x_{11} & \dots & x_{1M} \\ \vdots & \ddots & \vdots \\ x_{N1} & \dots & x_{NM} \end{bmatrix}$$

We follow PCA and calculate unit eigenvectors E_1, E_2, E_3, E_4 and eigenvalues v_1, v_2, v_3, v_4 of the covariance matrix of $C_{M\times M}$. The vectors are labelled E_1, E_2, E_3, E_4 in descending order of their eigenvalues. We combine the four eigenvectors into a matrix denoted by $E_{R\times M}$ where R = 4 is the number of vectors, and the ith row of this matrix is E_i. We procure the projection matrix $P_{N\times R}$:

$$P_{N\times R} = S_{N\times M} \times (E^T)_{M\times R}$$

which projects the original sample's coordinates onto the span of E_1, E_2, E_3, E_4. The projection matrix holds only the scalar measure of these projections. To give them direction, we multiply the projection matrix with $E_{R\times M}$, and to factor in the intercepts, we shift by the mean matrix $\mu_{N\times M}$ which is obtained by replicating the mean row vector $\mu_{1\times M} = \begin{bmatrix} \mu_{x_1} & \dots & \mu_{x_M} \end{bmatrix}$ across N rows.

$$V_{N\times M} = P_{N\times R} \times E_{R\times M} + \mu_{N\times M}$$

Following this, we reshape and rescale $V_{N\times M}$ to read it as an image.
The term "Reconstructed" means
For generating random fruit, we have used the formula below:

- Projected mean matrix 1x19200 onto the span defined by the four Principle modes of variation. ($Newmean = mean_{1\times 19200} \times E_{19200\times 4}$)
- Projected instances of original dataset onto the same span. ($P_{16\times 4} = S_{16\times 19200} \times E_{19200\times 4}$)
- Computed new covariance matrix $C'_{4\times 4}$ for $P_{16\times 4}$
- Drawn samples from the multivariate gaussian defined by Newmean and the new covariance matrix.

The matrices are scaled properly to allow being represented as images.
Note: The images presented here show up after sampling a lot of images from the multivariate gaussian, as most of them are close to the mean.

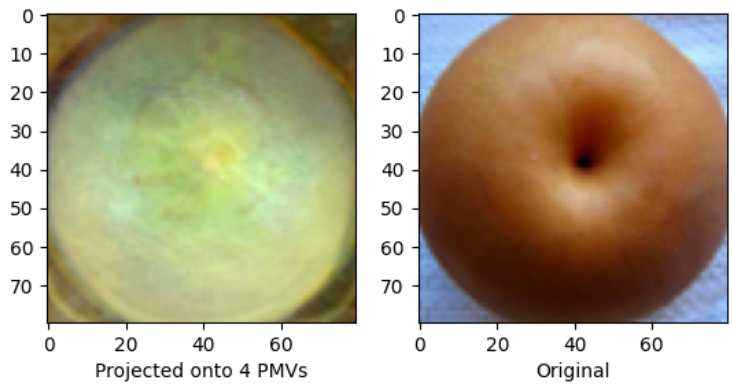

Figure 3: Original and Reconstructed images for fruit 1

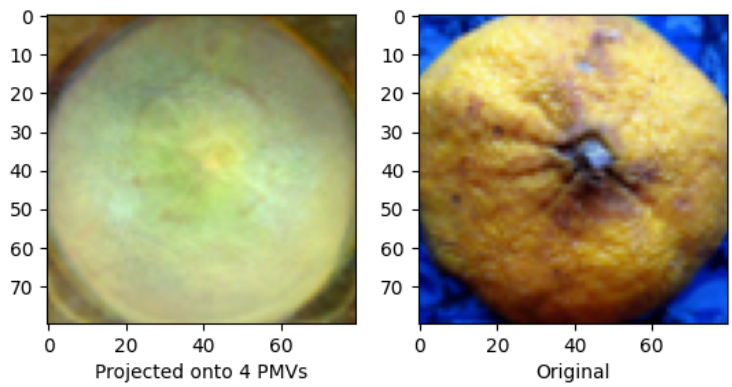

Figure 4: Original and Reconstructed images for fruit 2

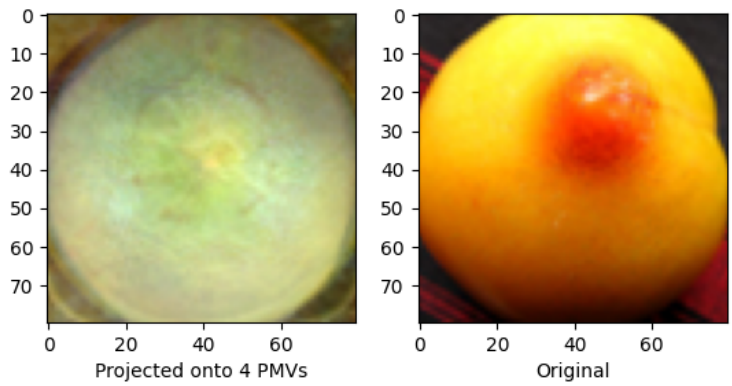

Figure 5: Original and Reconstructed images for fruit 3

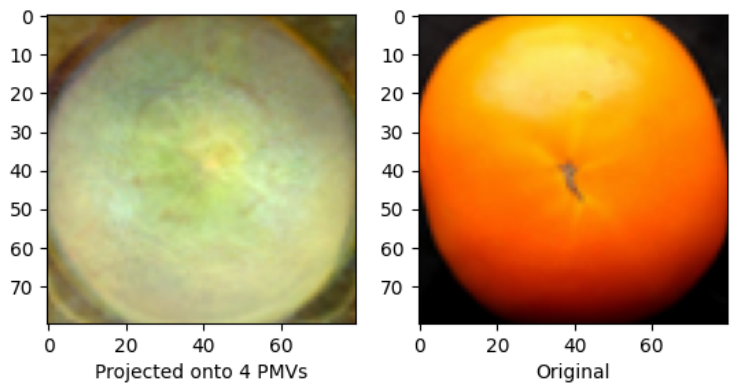

Figure 6: Original and Reconstructed images for fruit 4

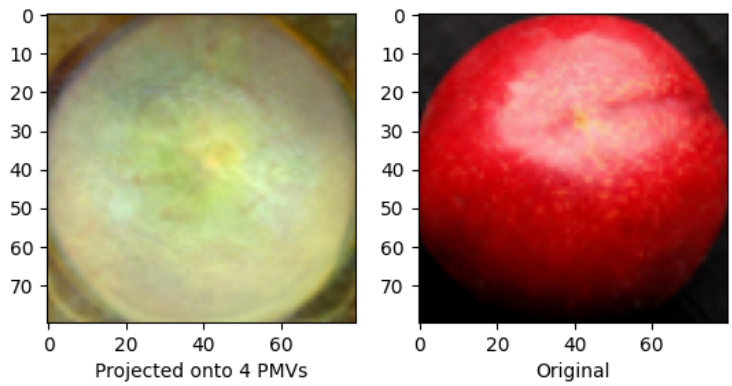

Figure 7: Original and Reconstructed images for fruit 5

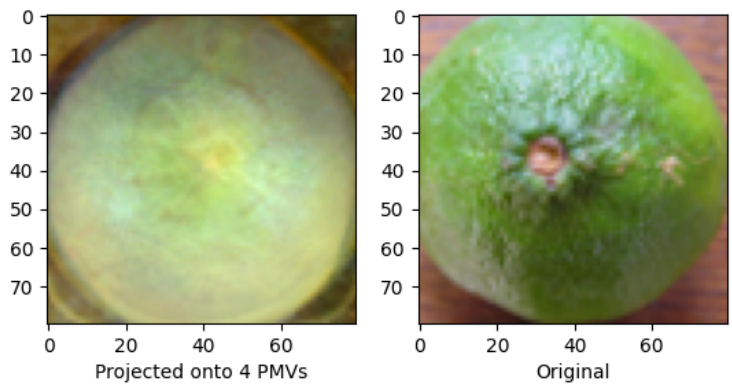

Figure 8: Original and Reconstructed images for fruit 6

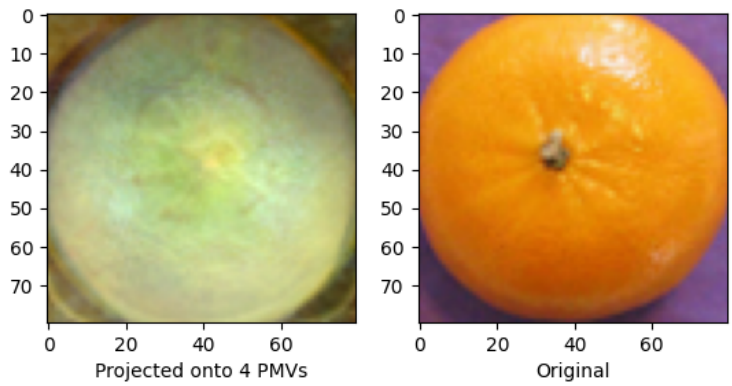

Figure 9: Original and Reconstructed images for fruit 7

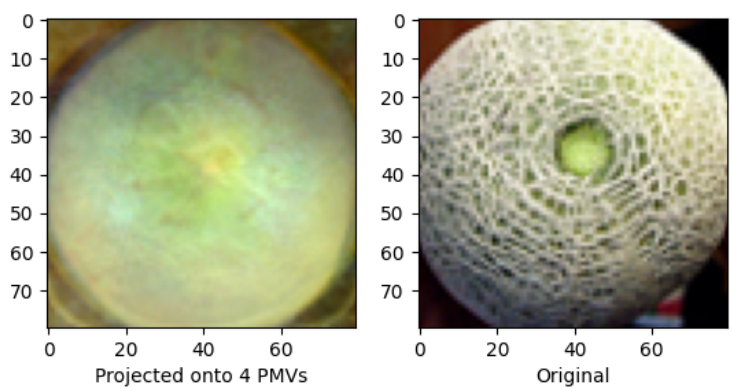

Figure 10: Original and Reconstructed images for fruit 8

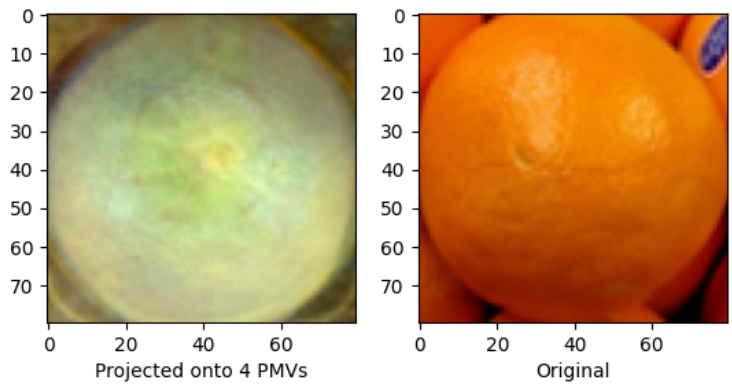

Figure 11: Original and Reconstructed images for fruit 9

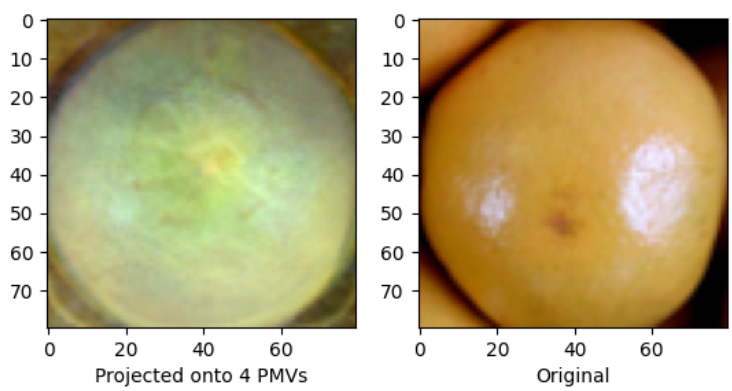

Figure 12: Original and Reconstructed images for fruit 10

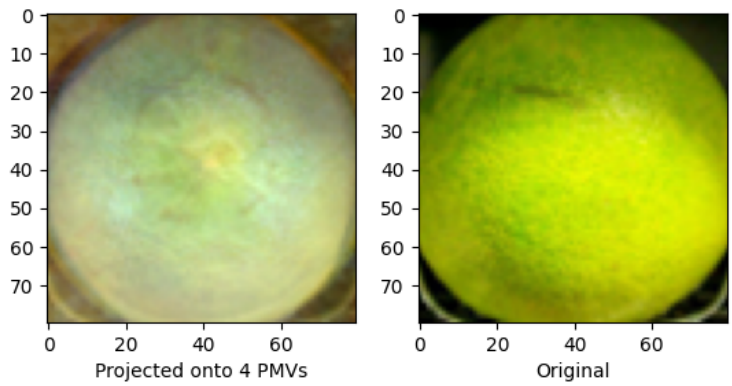

Figure 13: Original and Reconstructed images for fruit 11

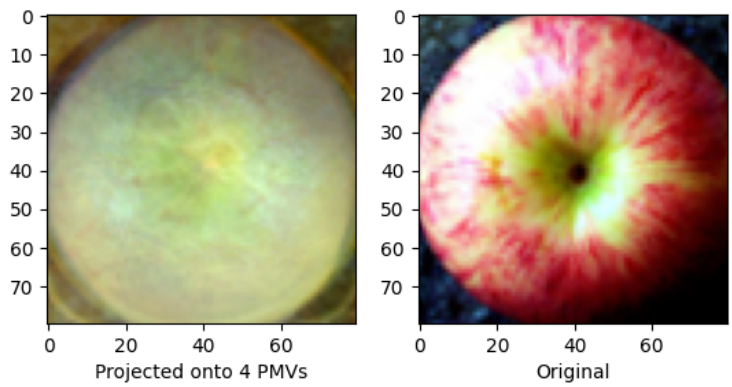

Figure 14: Original and Reconstructed images for fruit 12

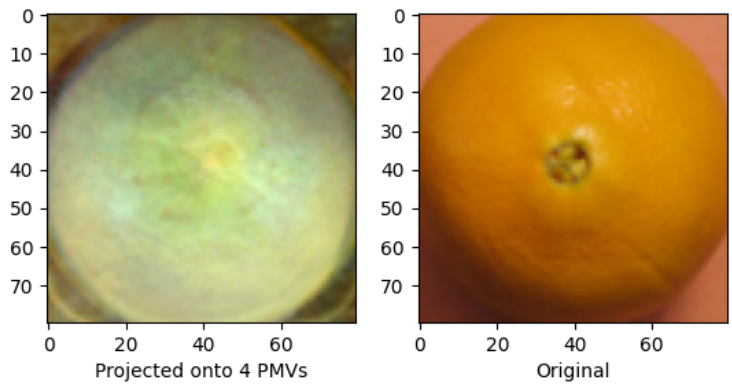

Figure 15: Original and Reconstructed images for fruit 13

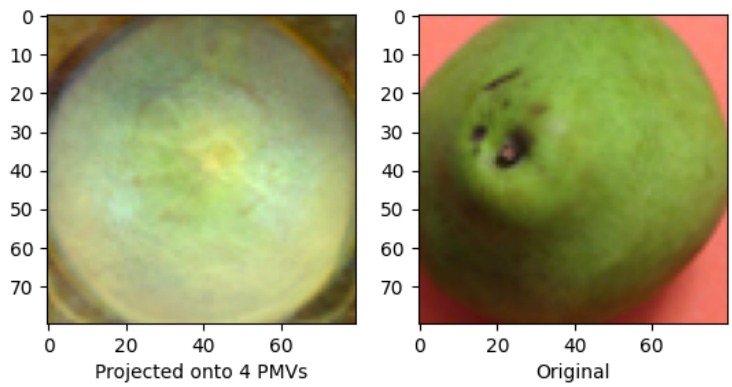

Figure 16: Original and Reconstructed images for fruit 14

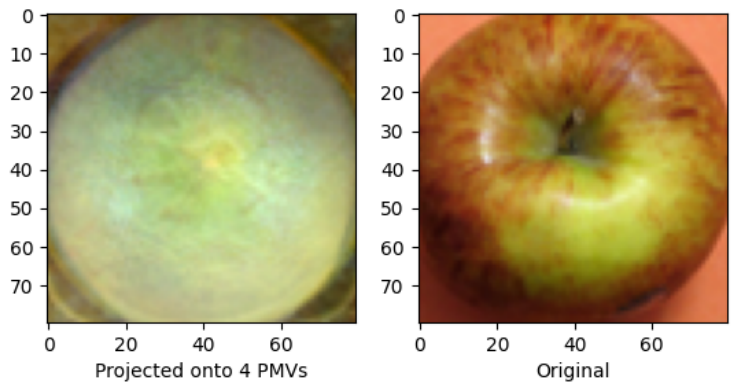

Figure 17: Original and Reconstructed images for fruit 15

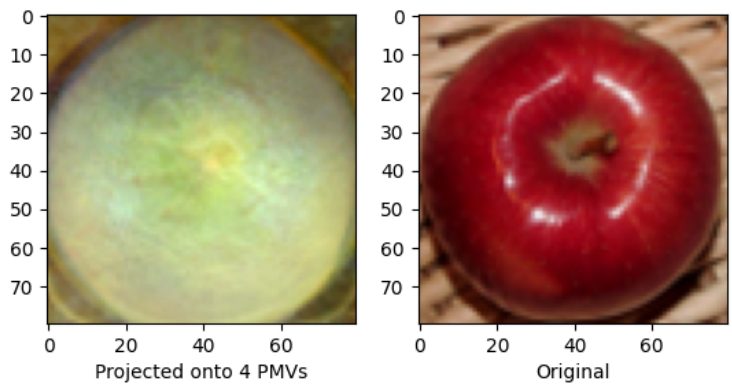

Figure 18: Original and Reconstructed images for fruit 16

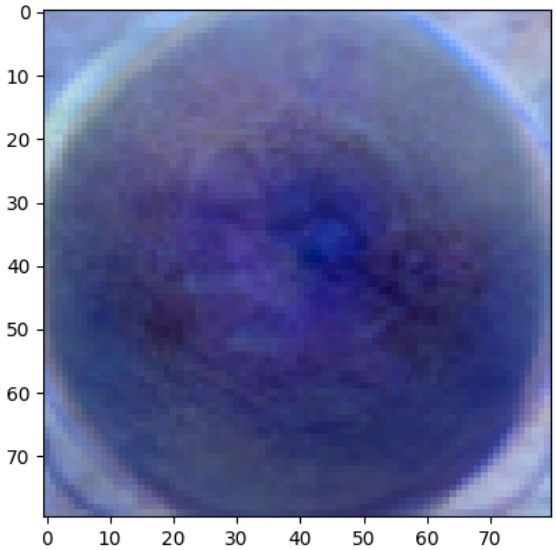

Figure 19: New Fruit 1

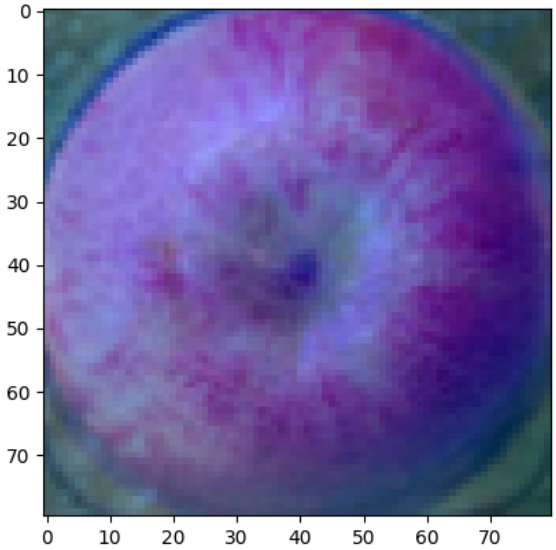

Figure 20: New Fruit 2

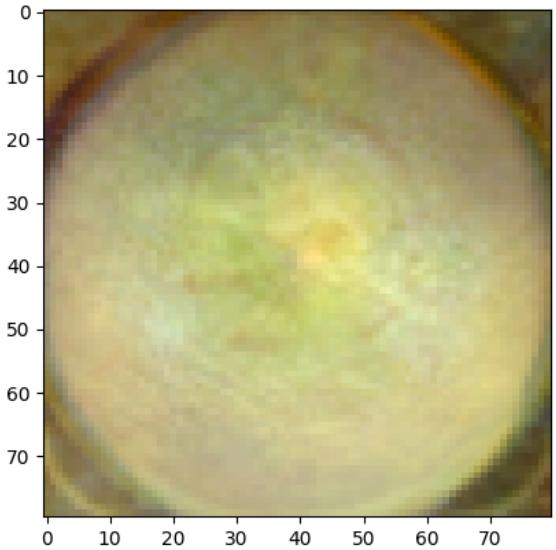

Figure 21: New Fruit 3

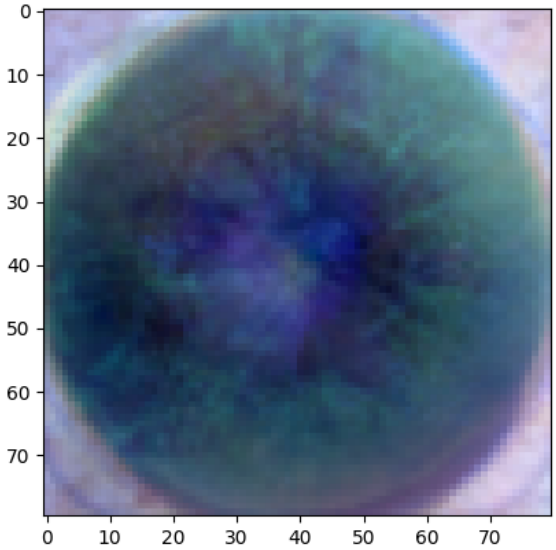

Figure 22: New Fruit 4

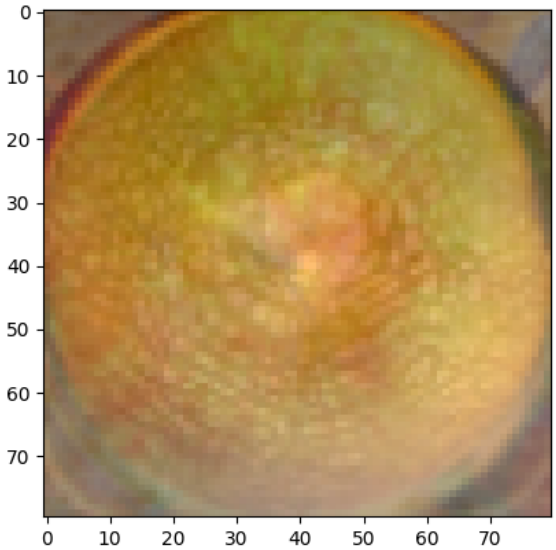

Figure 23: New Fruit 5

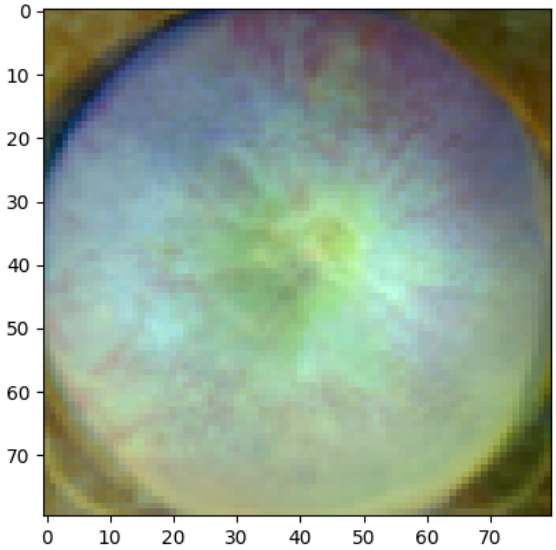

Figure 24: New Fruit 6

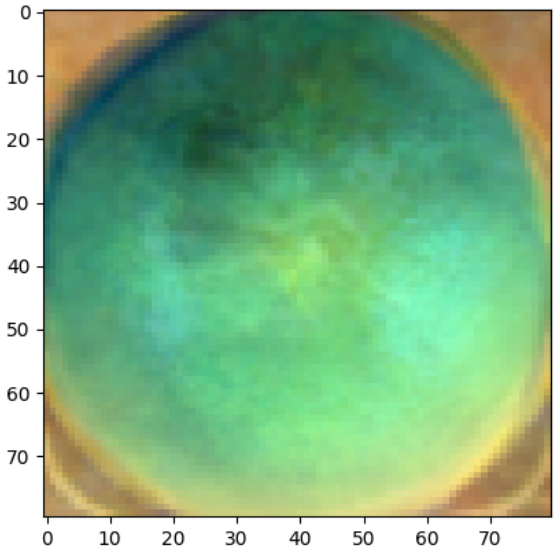

Figure 25: New Fruit 7

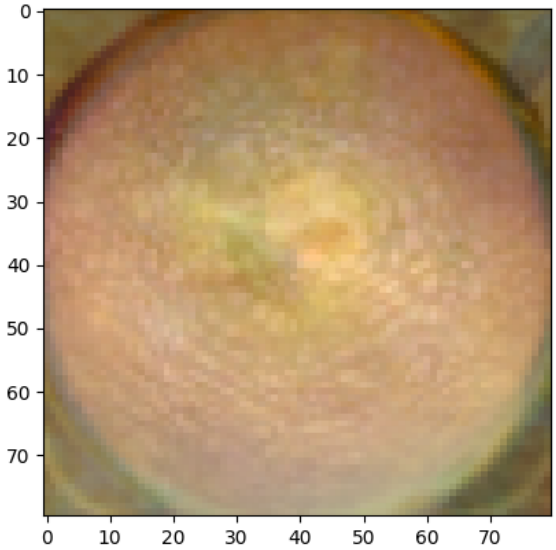

Figure 26: New Fruit 8

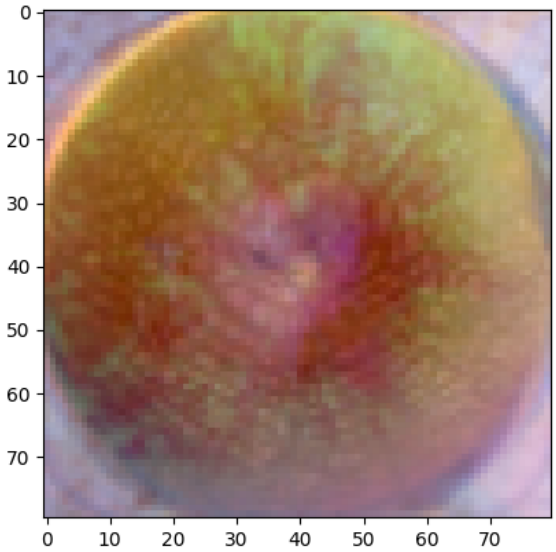

Figure 27: New Fruit 9