**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: B.0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:** False.

Explanation: The age range of 38 to 44 is within one standard deviation of the mean (38 ± 6). As per the empirical rule for a normal distribution, the majority of values (about 68%) fall within one standard deviation of the mean. Since 44 is more than one standard deviation above the mean, there are fewer employees older than 44 compared to those between 38 and 44.

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1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** False.

Explanation: Since the mean age is 38 and the standard deviation is 6, attracting employees under the age of 30 (which is more than one standard deviation below the mean) would target a smaller portion of the distribution. According to the empirical rule for a normal distribution, attracting about 36 employees under the age of 30 would cover less than 34% of the distribution. Therefore, the statement is not expected to be true.

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1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

1. Distribution of 2*X*1​:

Mean: *E*[2*X*1​] = 2⋅*μ*

Variance: *Var*(2*X*1​)=(2)^2⋅*Var*(*X*1​) = 4⋅*σ*2

1. Distribution of *X*1​+*X*2​:

Mean: *E*[*X*1​+*X*2​] = *μ*+*μ* = 2⋅*μ*

Variance: *Var*(*X*1​+*X*2​) = *Var*(*X*1​)+*Var*(*X*2​) = 2⋅*σ*2

* Both 2*X*1​ and *X*1​+*X*2​ have the same mean 2⋅*μ*, but their variances differ.
* 2*X*1​ has a larger variance (4⋅*σ*2) compared to *X*1​+*X*2​ (2⋅*σ*2).

In summary, while both random variables have the same mean, the variability (or spread) of 2*X*1​ is greater than that of *X*1​+*X*2​. Top of Form

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** To find two values, *a* and *b*, symmetric about the mean (*μ*) such that the probability of the random variable *X* taking a value between them is 0.99, we need to find the values that correspond to the central 99% of the normal distribution.

For a normal distribution, approximately 99% of the data lies within ±2.576 standard deviations from the mean. So, we can use this value to find *a* and *b*:

*a*=*μ*−2.576⋅*σ* and *b*=*μ*+2.576⋅*σ*

Given: *μ*=100 , *σ*2=202

*a*=100−2.576\*202​ = 100−51.52 = 48.48

*b*=100+2.576\*202​ = 100+51.52 = 151.52

**D. 48.5, 151.5**

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1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:** To find a Rupee range centered on the mean such that it contains 95% probability for the annual profit of the company, we need to consider the distribution of the total profit.

Given:

Profit1 ~ *N*(5,32) (in $ Million)

Profit2 ~ *N*(7,42) (in $ Million)

Let *X* be the total profit of the company in $ Million, then *X*=Profit1+Profit2.

The mean of *X* is the sum of the means of Profit1 and Profit2:

Mean(Total Profit)=Mean(Profit1)+Mean(Profit2) = 5+7 = 12

The variance of *X* is the sum of the variances of Profit1 and Profit2:

Var(Total Profit)=Var(Profit1)+Var(Profit2) = 32+42=9+16 = 25

S.D.(X) = sqrt(25) ​= 5

Now, we want to find a Rupee range centered on the mean such that it contains 95% probability. In a normal distribution, approximately 95% of the data falls within ±1.96 standard deviations from the mean.

The Rupee range will be:

Range = Mean±1.96 \*S.D = =12±1.96\*5

Lower Limit=12−1.96×5 = 2.2

Upper Limit=12+1.96×5 = 21.8

Therefore, the Rupee range (centered on the mean) containing 95% probability for the annual profit of the company is approximately Rs.2.2 Million *to Rs.* 21.8 Million

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:** from scipy import stats

mean\_profit = 5+7

std\_dev\_profit = (3\*\*2 + 4\*\*2)\*\*0.5

fifth\_percentile = stats.norm.ppf(0.05, loc=mean\_profit, scale=std\_dev\_profit)

# Convert the result to Rupees (assuming 1$ = Rs. 45)

fifth\_percentile\_rupees = fifth\_percentile \* 45

print("5th percentile of profit for the company (in rupees):", fifth\_percentile\_rupees)

5th percentile of profit for the company (in rupees): 169.9079

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** from scipy.stats import norm

mean\_profit1 = 5

std\_dev\_profit1 = 3

probability\_loss\_profit1 = norm.cdf(0, loc=mean\_profit1, scale=std\_dev\_profit1)

print("Probability of making a loss for Profit1:", probability\_loss\_profit1)

mean\_profit2 = 7

std\_dev\_profit2 = 4

probability\_loss\_profit2 = norm.cdf(0, loc=mean\_profit2, scale=std\_dev\_profit2)

print("Probability of making a loss for Profit2:", probability\_loss\_profit2)

**Probability of making a loss for Profit1: 0.047790352**

**Probability of making a loss for Profit2: 0.040059156**

The division with a higher probability of making a loss has a higher likelihood of having negative annual profits .