**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer**: B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer: Probability that the employee will be greater than the age pf 44 =**15.86%**.So Probability of no of employees between 38-44 years of age =**34.1345%**. Hence the statement that “ More Employees at the processing center are older than 44 than between 38 and 44" is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: The number of employees with the probability 0.912 of them being under age 30 = 0.912\*400=36.48.

Hence here the statement is True .

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2. µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer: Here from given details we have to exclude 0.005 of the left and right tails.

Hence, we want to find the 0.5th and 99.5th percentiles Z score values.

Z value at 0.5th percentile Z(0.5)= -2.576

Z value at 99.5th percentile Z(99.5)= 2.576

Z=(X-100)/20 =>X=20z+100

a = (20\* (-2.576))+100 = 48.5

B= (20\*2.576)+100=151.5

D.So, two values symmetric about the mean for given standard normal distribution are 48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: The Mean Profit of both division: 12 Million$

The Mean Profit of both division: 54.0 Crore Rupees

The Standard Deviation of both division: 5.0 Million$

The Standard Deviation of both division: 22.5 Crore Rupees

Rupee Ranges from 2.2 to 21.8 Million$ in Annual profit of the Company 95% of the time

Rupee Ranges from 9.900000000000002 to 98.1 Crore Rupees in Annual profit of the Company 95% of the time

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The 5th percentile of Profit for the company is 3.78 Million$

The 5th percentile of Profit for the company is 17.0 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Probability of Division #1 making a loss is 4.78 %

The Probability of Division #2 making a loss is 4.01 %

The Division 1 has a larger Probability of making a loss