

## UNIT - V

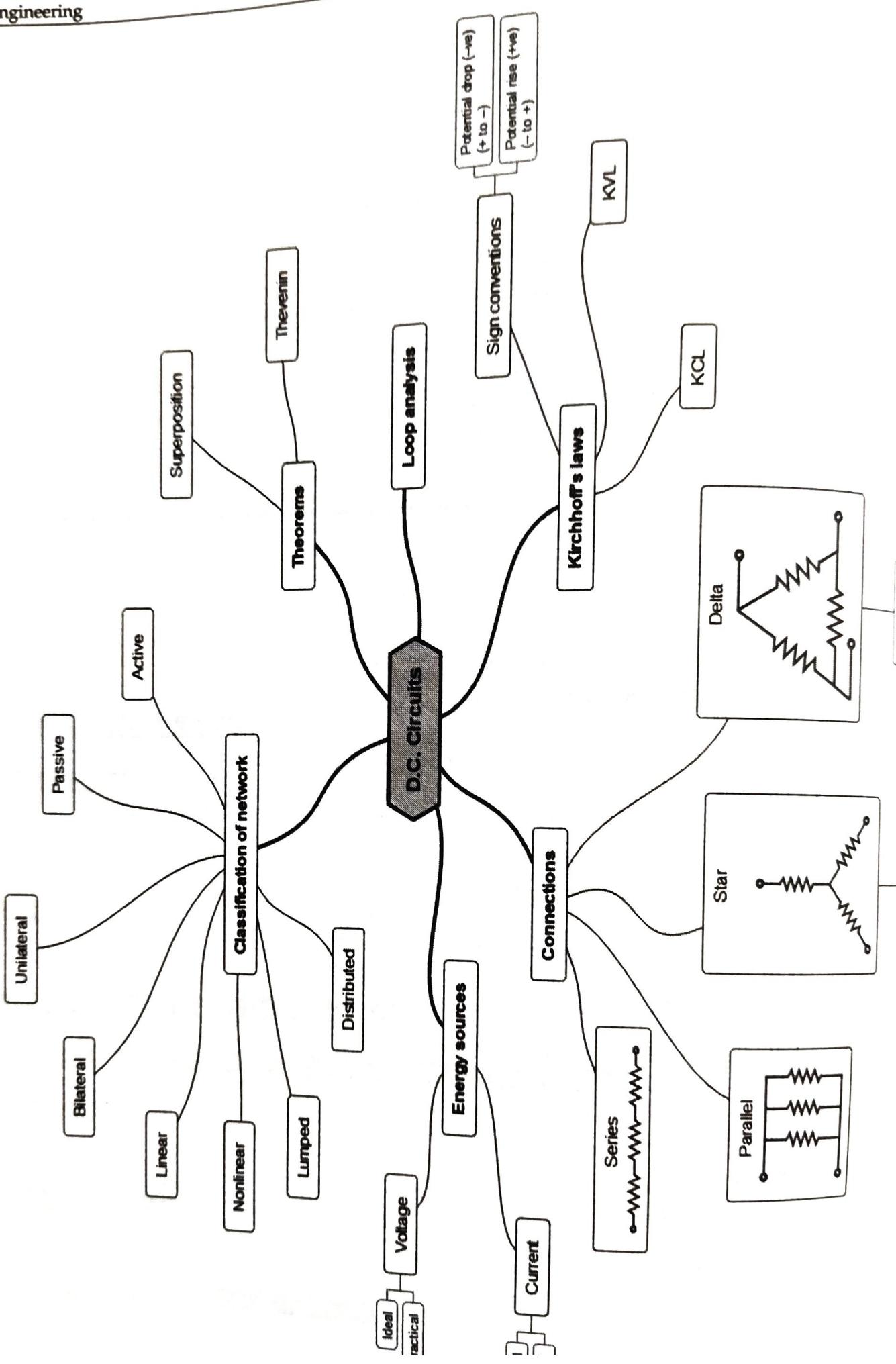
# D.C. Circuits

### Syllabus

*Classification of electrical networks, Energy sources – ideal and practical voltage and current sources, Simplifications of networks using series and parallel combinations and star-delta conversions, Kirchhoff's laws and their applications for network solutions using loop analysis, Superposition theorem, Thevenin's theorem.*

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**Mind Map - D. C. Circuits**

### 1. Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical components, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy gives rise to complicated electrical circuits, generally referred as **networks**. The terms circuit and network are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit. This chapter includes various techniques of analysing d.c. circuits.

### 2. Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

#### 2.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network**. Such a network is shown in the Fig. 7.2.1.

#### 2.2 Network Element

- Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element**.
- Network elements can be either active elements or passive elements.
- Active elements are the elements which supply power or energy to the network.
- Voltage source and current source are the examples of active elements.
- Passive elements are the elements which either store energy or dissipate energy in the form of heat.
- Resistor, inductor and capacitor are the three basic passive elements.
- Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

#### 7.2.3 Branch

- A part of the network which connects the various points of the network with one another is called a **branch**.
- In the Fig. 7.2.1, AB, BC, CD, DA, DE, CF and EF are the various branches.
- A branch may consist more than one element.

#### 7.2.4 Junction Point

- A point where three or more branches meet is called a **junction point**.
- Point D and C are the junction points in the network shown in the Fig. 7.2.1.

#### 7.2.5 Node

- A point at which two or more elements are joined together is called **node**. The junction points are also the nodes of the network.
- In the network shown in the Fig. 7.2.1, A, B, C, D, E and F are the nodes of the network.

#### 7.2.6 Mesh (or Loop)

- Mesh (or Loop)** is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path.
- A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice.
- In the Fig. 7.2.1 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

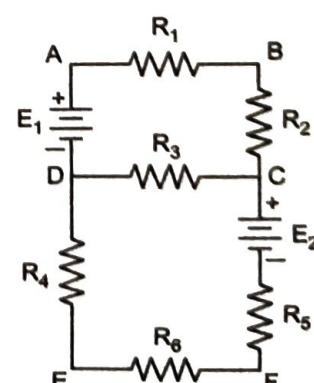


Fig. 7.2.1 An electrical network

### 7.3 : Classification of Electrical Networks

SPPU : May-04, 08, 12, Dec.-03, 08, 09, 11

#### i) Linear Network :

A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network**.

- The Ohm's law can be applied to such network.
- The mathematical equations of such network can be obtained by using the law of superposition.
- The response of the various network elements is linear with respect to the excitation applied to them.

#### ii) Non linear Network :

- A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network**.
- The Ohm's law may not be applied to such network.
- Such network does not follow the law of superposition.
- The response of the various elements is not linear with respect to their excitation.
- The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

#### iii) Bilateral Network :

- A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network**.
- Network consisting only resistances is good example of bilateral network.

#### iv) Unilateral Network :

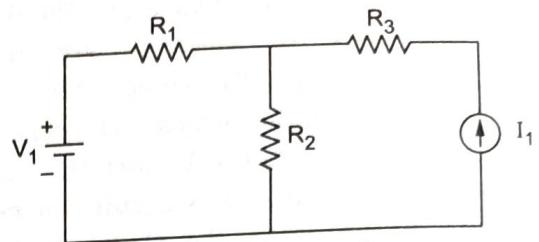
- A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network**.
- Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

#### v) Active Network :

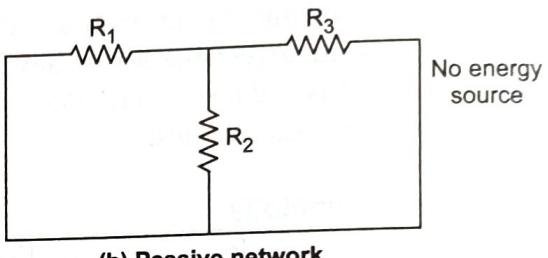
- A circuit which contains at least one source of energy is called **active**. An energy source may be a voltage or current source.

#### vi) Passive Network :

- A circuit which contains no energy source is called **passive circuit**. This is shown in the Fig. 7.3.1.



(a) Active network



(b) Passive network

Fig. 7.3.1

#### vii) Lumped Network :

- A network in which all the network elements are physically separable is known as **lumped network**.
- Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

#### viii) Distributed Network :

- A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network**.
- The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, anywhere in the circuit.
- The classification of networks can be shown as,

## Electrical circuits or networks

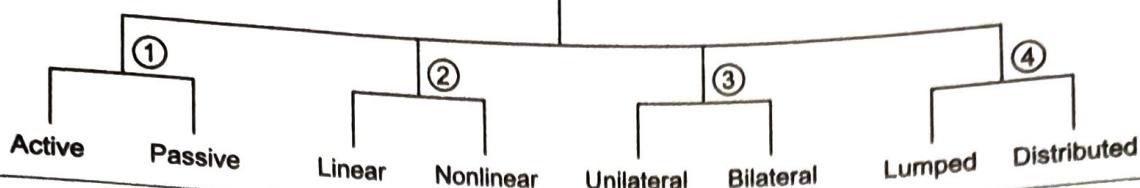


Fig. 7.3.2 Classification of networks

**Expected Questions****1. Explain linear and nonlinear networks.**

SPPU : May-04, 12, Dec.-03, 09, 11, Marks 4

**2. Explain active and passive networks.****3. Explain unilateral and bilateral networks.**

SPPU : May-04, 12, Dec.-03, 09, 11, Marks 4

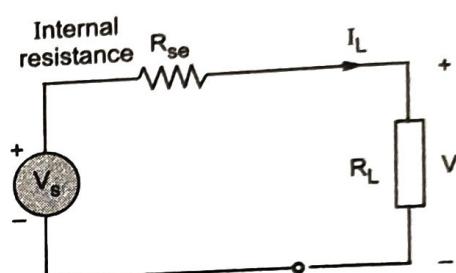
**4. Classify the electrical networks.**

SPPU : May-08, Dec.-08, Marks 4

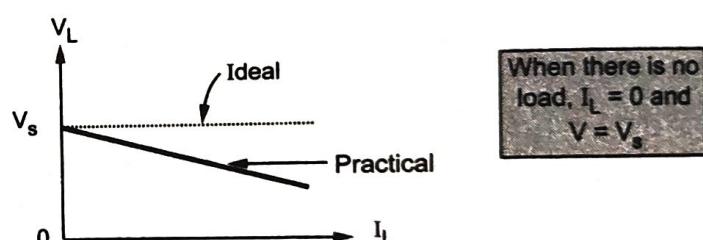
**5. Explain lumped and distributed networks.**

SPPU : Dec.-11, Marks 2

- The symbol for ideal voltage source is shown in the Fig. 7.4.1 (a). This is connected to the load as shown in Fig. 7.4.1 (b).
- At any time the value of voltage at load terminals remains same. This is indicated by V-I characteristics shown in the Fig. 7.4.1 (c).



(a) Circuit



(b) Characteristics

Fig. 7.4.2 Practical voltage source

**Practical voltage source :**

- But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by  $R_{se}$  as shown in the Fig. 7.4.2.
- Because of the  $R_{se}$ , voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = -(R_{se}) I_L + V_s = V_s - I_L R_{se}$$

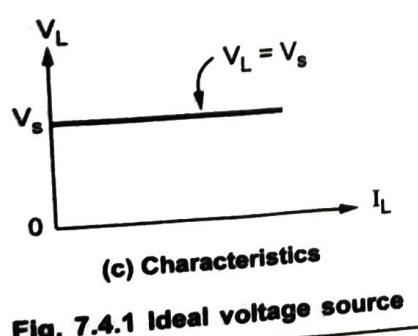
**Key Point** For ideal voltage source,  $R_{se} = 0$ .

Fig. 7.4.1 Ideal voltage source

- Voltage sources are further classified as follows,

### i) Time Invariant Sources :

- The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources**.
- These are denoted by capital letters. Such a source is represented in the Fig. 7.4.3 (a).

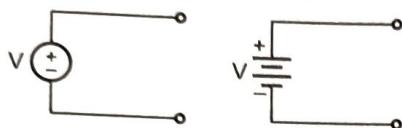


Fig. 7.4.3 (a) D.C. source

### ii) Time Variant Sources :

- The sources in which voltage is varying with time are known as **time variant voltage sources** or **A.C. sources**.
- These are denoted by small letters. This is shown in the Fig. 7.4.3 (b).

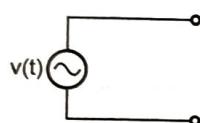
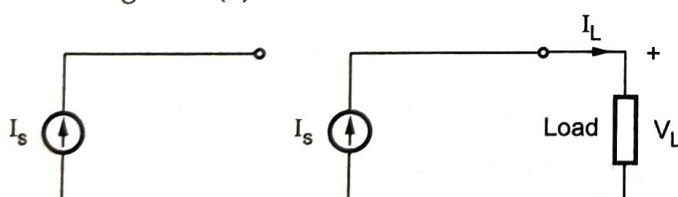


Fig. 7.4.3 (b) A.C. source

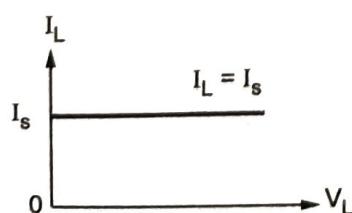
## 7.4.2 Current Source

- Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals.
- The symbol for ideal current source is shown in the Fig. 7.4.4 (a). This is connected to the load as shown in the Fig. 7.4.4 (b).



(a) Symbol

(b) Circuit



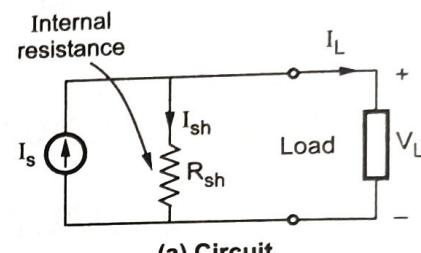
(c) Characteristics

Fig. 7.4.4 Ideal current source

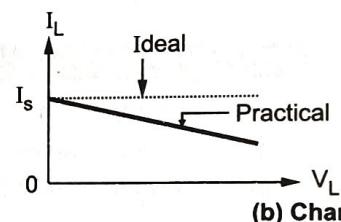
- At any time, the value of the current flowing through load  $I_L$  is same i.e. is irrespective voltage appearing across its terminals. This is explained by V-I characteristics shown in Fig. 7.4.4 (c).

### Practical current source :

- But practically, every current source has internal resistance, shown in parallel with current source and it is represented by  $R_{sh}$ . This is shown in the Fig. 7.4.5.



(a) Circuit



$$I_L + I_{sh} = I_s \\ \text{Thus as } I_{sh} \text{ increases, } I_L \text{ decreases} \\ I_L < I_s$$

Fig. 7.4.5 Practical current source

- Because of  $R_{sh}$ , current through its terminals decreases slightly with increase in voltage at the terminals.

**Key Point** For ideal current source,  $R_{sh} = \infty$ .

- Similar to voltage sources, current sources are classified as follows :

### i) Time Invariant Sources :

- The sources in which current is not varying with time are known as **time invariant current source** or **D.C. sources**. These are denoted by capital letters.
- Such a current source is represented in the Fig. 7.4.6 (a).

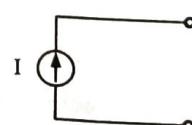


Fig. 7.4.6 (a) D.C. source

### Time Variant Sources :

The sources in which current is varying with time are known as time variant current sources or A.C. sources. These are denoted by small letters. Such a source is represented in the Fig. 7.4.6 (b).

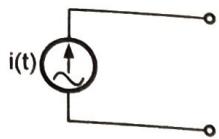


Fig. 7.4.6 (b) A.C. source

### 4.3 Dependent Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 7.4.7 and further classified as,

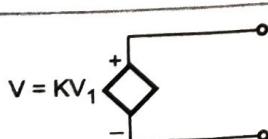
**i) Voltage Dependent Voltage Source :** It produces a voltage as a function of voltages elsewhere in the given circuit. This is called VDVS. It is shown in the Fig. 7.4.7 (a).

**ii) Current Dependent Current Source :** It produces a current as a function of currents elsewhere in the given circuit. This is called CDCS. It is shown in the Fig. 7.4.7 (b).

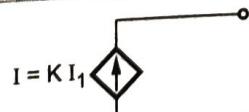
**iii) Current Dependent Voltage Source :** It produces a voltage as a function of current elsewhere in the given circuit. This is called CDVS. It is shown in the Fig. 7.4.7 (c).

**iv) Voltage Dependent Current Source :** It produces a current as a function of voltage elsewhere in the given circuit. This is called VDCS. It is shown in the Fig. 7.4.7 (d).

K is constant and  $V_1$  and  $I_1$  are the voltage and current respectively, present elsewhere in the given circuit.



(a)



(b)

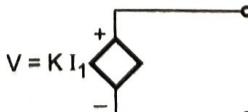


Fig. 7.4.7



(d)

- The dependent sources are also known as controlled sources.

### Expected Questions

1. Explain ideal and practical voltage sources.

SPPU : May-06, Dec.-03, 05, Marks 4

2. Explain ideal and practical current sources.

SPPU : May-06, Marks 4

### 7.5 : Series Circuit

A series circuit is one in which several resistances are connected one after the other. Such connection is also called end to end connection or cascade connection. There is only one path for the flow of current.

- Consider the resistances shown in the Fig. 7.5.1.

Current same  
voltage division

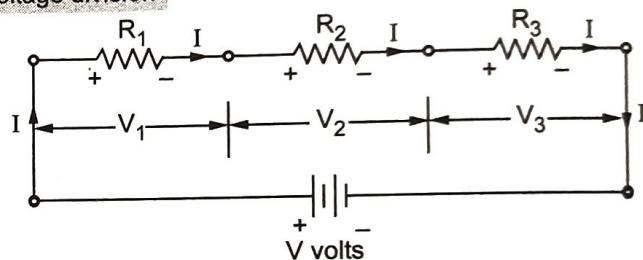


Fig. 7.5.1 A series circuit

- The resistance  $R_1$ ,  $R_2$  and  $R_3$  are said to be in series. The combination is connected across a source of voltage  $V$  volts.
- Naturally the current flowing through all of them is same indicated as  $I$  amperes.
- Let  $V_1$ ,  $V_2$  and  $V_3$  be the voltages across the terminals of resistances  $R_1$ ,  $R_2$  and  $R_3$  respectively then,  $V = V_1 + V_2 + V_3$ .
- Now according to Ohm's law,  $V_1 = IR_1$ ,  $V_2 = IR_2$ ,  $V_3 = IR_3$ .

- Current through all of them is same i.e.  $I$ .

$$\therefore V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

- Applying Ohm's law to overall circuit,

$$V = I R_{eq}$$

where  $R_{eq}$  = Equivalent resistance of the circuit.

- By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

- Thus the total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

**For  $n$  resistances in series,**

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$$

### 7.5.1 Characteristics of Series Circuit

- 1) The same current flows through each resistance.
- 2) The supply voltage  $V$  is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \dots + V_n$$

- 3) The equivalent resistance is equal to the sum of the individual resistances.
- 4) The equivalent resistance is the largest of all the individual resistances.

i.e  $R > R_1, R > R_2, \dots, R > R_n$

#### Expected Question

1. Derive an expression for an equivalent resistance of  $n$  resistances connected in series.

## 7.6 : Parallel Circuit

- The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

Consider a parallel circuit shown in the Fig. 7.6.1.

In the parallel connection shown, the three resistances  $R_1, R_2$  and  $R_3$  are connected in parallel and combination is connected across a source of voltage 'V'.

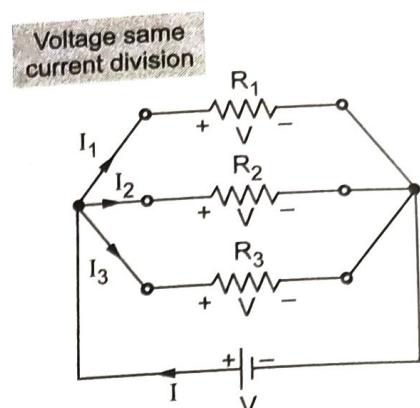


Fig. 7.6.1 A parallel circuit

- In parallel circuit current passing through each resistance is different. Let total current drawn is 'I' as shown. There are 3 paths for this current, one through  $R_1$ , second through  $R_2$  and third through  $R_3$ . These individual currents are shown as  $I_1, I_2$  and  $I_3$ .
- The voltage across the two ends of each resistor  $R_1, R_2$  and  $R_3$  is the same and equals the supply voltage  $V$ . Hence apply Ohm's law to each resistance,

$$V = I_1 R_1, V = I_2 R_2, V = I_3 R_3$$

$$\text{i.e. } I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

$$\text{But } I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \dots \right] \quad \dots (7.6.1)$$

- For overall circuit the Ohm's law can be applied

$$V = I R_{eq} \quad \text{and} \quad I = \frac{V}{R_{eq}} \quad \dots (7.6.2)$$

where  $R_{eq}$  = Total or equivalent resistance of the circuit

- Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

## Characteristics of Parallel Circuit

- 1) The voltage or potential difference across all the resistances in parallel is always same.
- 2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.
- 3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
- 4) The equivalent resistance is the smallest of all the resistances.

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

- 3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
- 4) The equivalent resistance is the smallest of all the resistances.

$$R < R_1, R < R_2, \dots, R < R_n$$

- 5) The equivalent conductance is the arithmetic addition of the individual conductances.

### Expected Question

1. Derive an expression for an equivalent resistance of 'n' resistances connected in parallel.

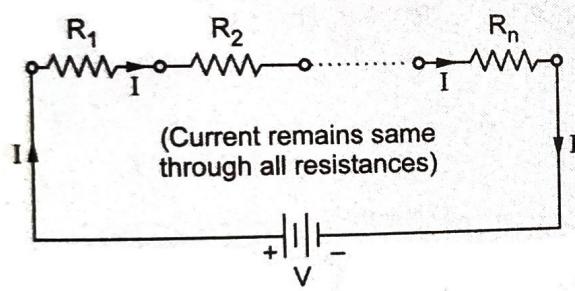
SPPU : May-17, 19

## Comparison of Series and Parallel Circuits

No.

### Series Circuit

The connection is as shown,



The same current flows through each resistance.

The voltage across each resistance is different.

The sum of the voltages across all the resistances is the supply voltage.  $V = V_1 + V_2 + V_3 + \dots + V_n$

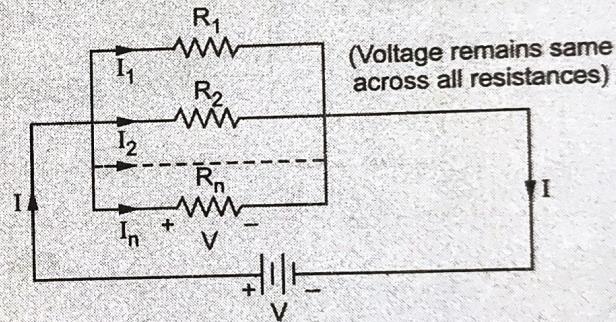
The equivalent resistance is,

$$R_{eq} = R_1 + R_2 + \dots + R_n$$

The equivalent resistance is the largest than each of the resistances in series.  $R_{eq} > R_1, R_{eq} > R_2, \dots, R_{eq} > R_n$

### Parallel Circuit

The connection is as shown,



The same voltage exists across all the resistances in parallel.

The current through each resistance is different.

The sum of the currents through all the resistances is the supply current.  $I = I_1 + I_2 + \dots + I_n$

$$\text{The equivalent resistance is, } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

**Ex. 7.7.1** Find equivalent resistance between AB for the circuit shown in Fig. 7.7.1.

SPPU : May-19, Marks 7

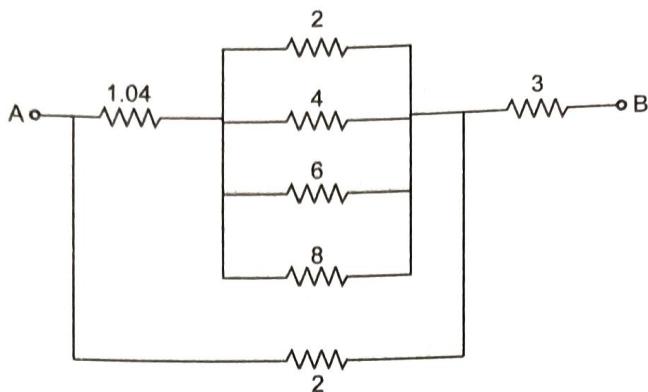


Fig. 7.7.1

**Sol.** :  $2 \Omega$ ,  $4 \Omega$ ,  $6 \Omega$  and  $8 \Omega$  are in parallel.

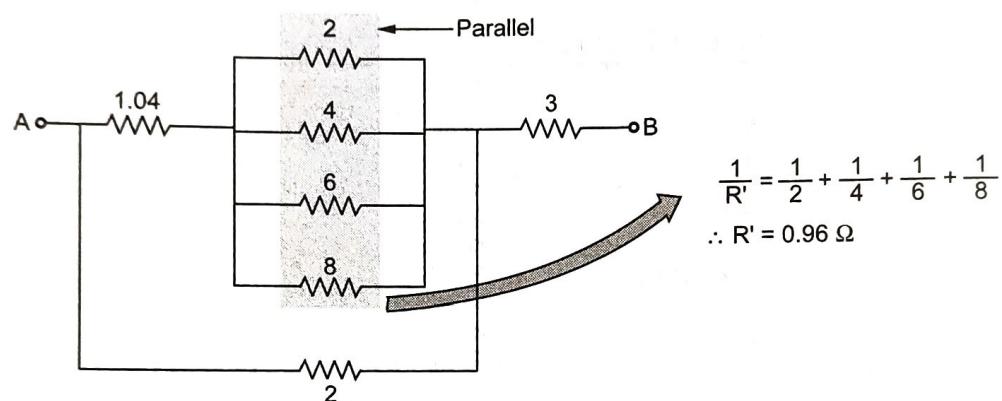


Fig. 7.7.1 (a)

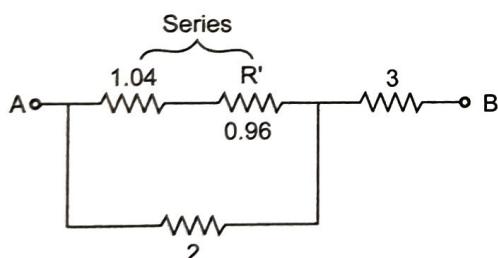


Fig. 7.7.1 (b)

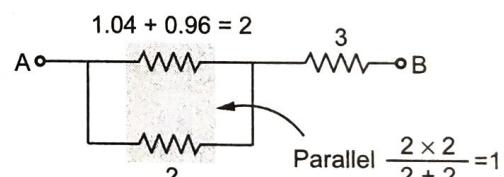


Fig. 7.7.1 (c)

$$\begin{aligned}\therefore R_{AB} &= 1 + 3 \\ &= 4 \Omega\end{aligned}$$



Fig. 7.7.1 (d)

**Ex. 7.7.2** Find the resistance of the circuit shown ( $R_{AD}$ ).

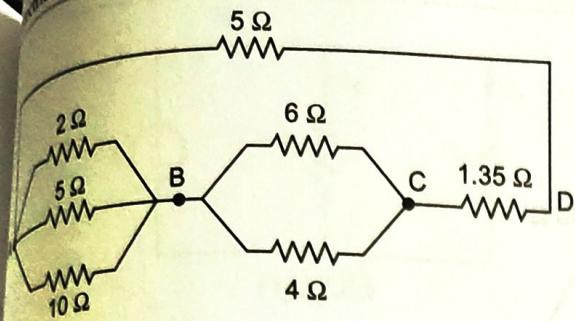


Fig. 7.7.2

The resistances  $2\ \Omega$ ,  $5\ \Omega$ ,  $10\ \Omega$  are in parallel

$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} \quad \text{i.e. } R' = 1.25\ \Omega$$

resistances of  $6\ \Omega$  and  $4\ \Omega$  are in parallel giving,

$$R'' = \frac{6 \times 4}{6+4} = 2.4\ \Omega$$

the circuit becomes as shown in the Fig. 7.7.2 (a).

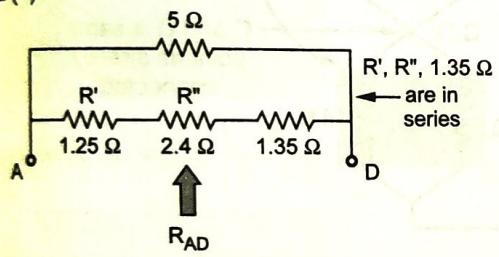


Fig. 7.7.2 (a)

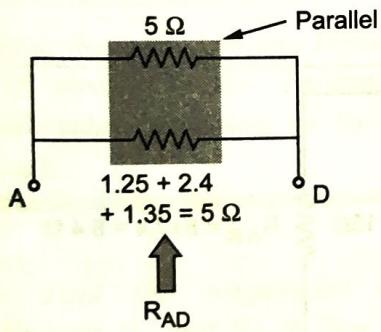


Fig. 7.7.2 (b)

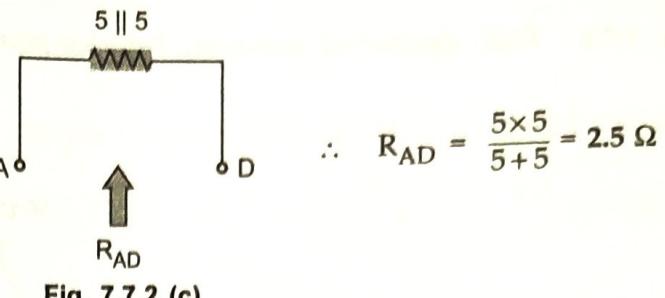


Fig. 7.7.2 (c)

Ex. 7.7.3 Find equivalent resistance between A and B for the circuit shown in Fig. 7.7.3.

SPPU : May-17, Marks 7

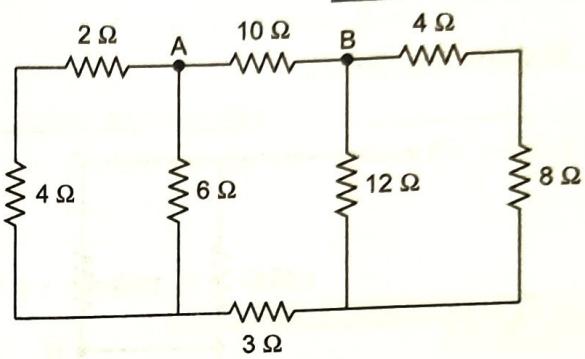


Fig. 7.7.3

Sol. :  $2\ \Omega$  and  $4\ \Omega$  are in series while  $4\ \Omega$  and  $8\ \Omega$  are in series as shown in the Fig. 7.7.3 (a).

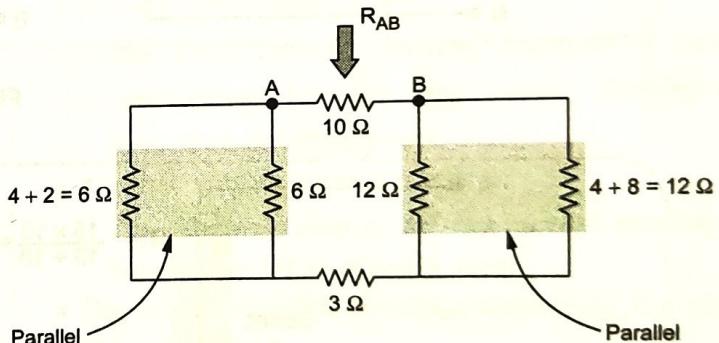


Fig. 7.7.3 (a)

$$\therefore R_{AB} = 5.4545\ \Omega \quad \dots \text{Refer Fig. 7.7.3 (d)}$$

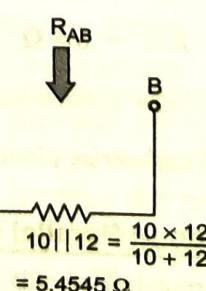
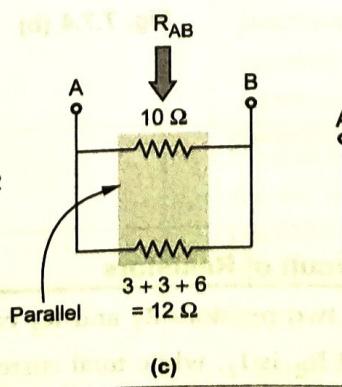
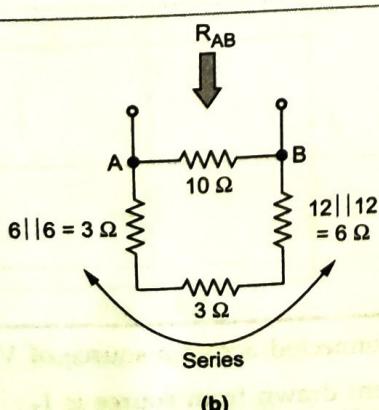


Fig. 7.7.3

**Ex. 7.7.4** Find equivalent resistance between points A-B.

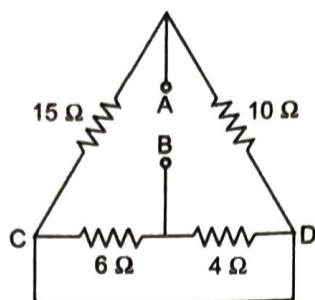


Fig. 7.7.4

**Sol.** : Redraw the circuit,

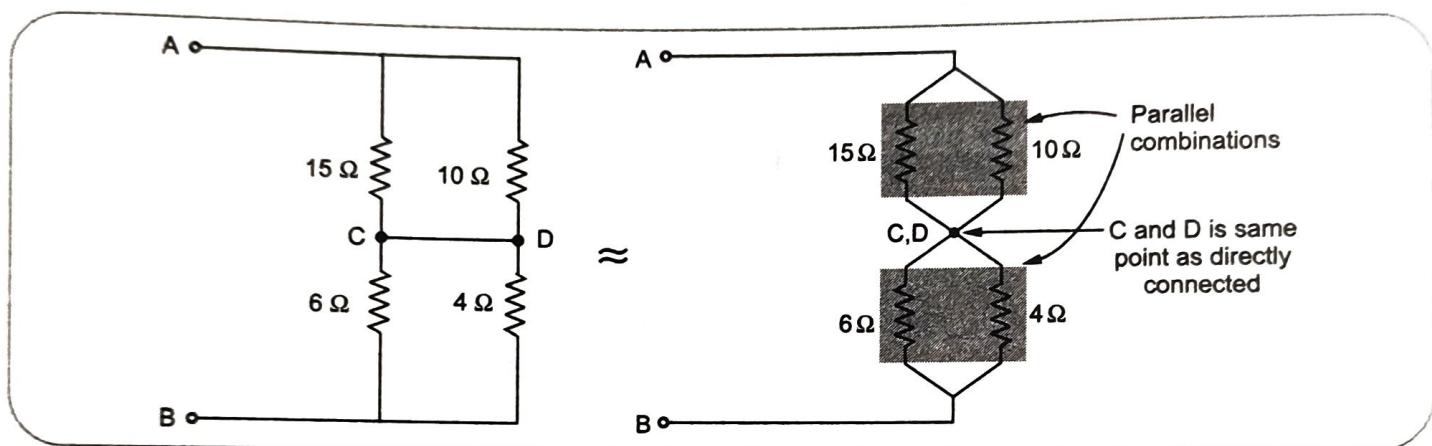


Fig. 7.7.4 (a)

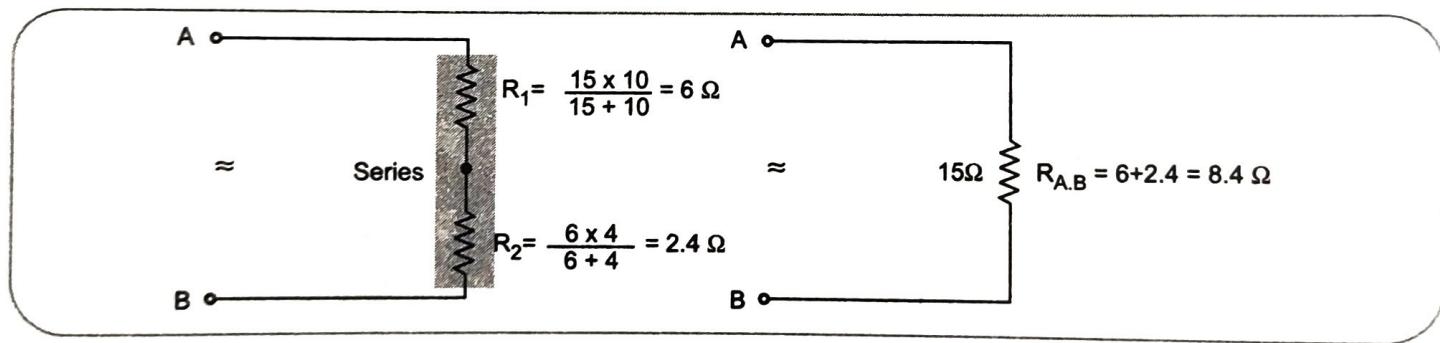


Fig. 7.7.4 (b)

∴

$$R_{AB} = 8.4 \Omega$$

#### Expected Question

1. Compare parallel and series circuits.

#### 7.8 : Current Division in Parallel Circuit of Resistors

- Consider a parallel circuit of two resistors  $R_1$  and  $R_2$  connected across a source of  $V$  volts.
- Current through  $R_1$  is  $I_1$  and  $R_2$  is  $I_2$ , while total current drawn from source is  $I_T$ .

∴

$$I_T = I_1 + I_2$$

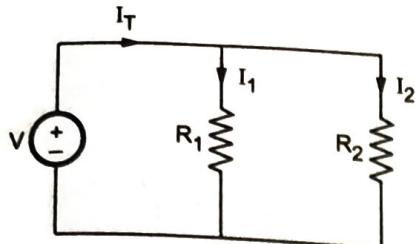


Fig. 7.8.1

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2} \quad \text{i.e.}$$

$$V = I_1 R_1 = I_2 R_2$$

$$I_1 = I_2 \left( \frac{R_2}{R_1} \right)$$

Substituting value of  $I_1$  in  $I_T$ ,

$$I_T = I_2 \left( \frac{R_2}{R_1} \right) + I_2 = I_2 \left[ \frac{R_2}{R_1} + 1 \right] = I_2 \left[ \frac{R_1 + R_2}{R_1} \right]$$

$$I_2 = \left[ \frac{R_1}{R_1 + R_2} \right] I_T$$

$$\text{Now } I_1 = I_T - I_2 = I_T - \left[ \frac{R_1}{R_1 + R_2} \right] I_T = \left[ \frac{R_1 + R_2 - R_1}{R_1 + R_2} \right] I_T$$

$$I_1 = \left[ \frac{R_2}{R_1 + R_2} \right] I_T$$

In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.

**Ex. 7.8.1** Find the magnitudes of total current, current through  $R_1$  and  $R_2$  if,  $R_1 = 10 \Omega$ ,  $R_2 = 20 \Omega$ , and  $V = 50 \text{ V}$ .

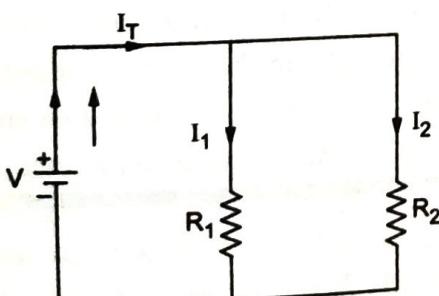


Fig. 7.8.2

Sol.: The equivalent resistance of two is,

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10 + 20} = 6.67 \Omega$$

$$\therefore I_T = \frac{V}{R_{\text{eq}}} = \frac{50}{6.67} = 7.5 \text{ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T \left( \frac{R_2}{R_1 + R_2} \right) = 7.5 \times \left( \frac{20}{10 + 20} \right)$$

$$= 5 \text{ A}$$

$$\text{and } I_2 = I_T \left( \frac{R_1}{R_1 + R_2} \right) = 7.5 \times \left( \frac{10}{10 + 20} \right) \\ = 2.5 \text{ A}$$

It can be verified that  $I_T = I_1 + I_2$

#### Expected Question

1. Explain the current division in parallel circuit of resistors.

#### 7.9 : Kirchhoff's Laws

SPPU : May-04, 05, 06, 07, 11, 12, 13, 14, 16, 17,  
Dec.-07, 10, 13, 14, 17

- There are two Kirchhoff's laws.

##### 1. Kirchhoff's Current Law (KCL)

- The law can be stated as,

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

- Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

- The word algebraic means considering the signs of various currents.

$$\sum I \text{ at junction point} = 0$$

**Sign convention :** Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

- Consider a junction point in a complex network as shown in the Fig. 7.9.1. The currents  $I_1$  and  $I_2$  are positive as entering the junction while  $I_3$  and  $I_4$  are negative as leaving the junction.

- Applying KCL,  $\sum I \text{ at junction O} = 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \quad \text{i.e.} \quad I_1 + I_2 = I_3 + I_4$$

- The law is very helpful in network simplification.

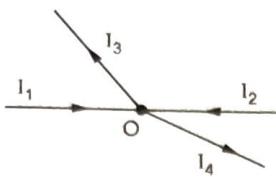


Fig. 7.9.1 Junction point

## 2. Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f.s in the path"

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

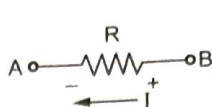
$$\text{Around a closed path } \sum V = 0$$

- The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.
- Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

### Sign conventions for KVL can be stated as :

- When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.
- In the Fig. 7.9.2 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.
- In the Fig. 7.9.2 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Now while tracing a closed path for KVL, if we go from - ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise**. For example, if the branch AB is traced from A to B in the Fig. 7.9.2 (a) then the drop across it must be considered as rise and must be taken as  $+ IR$ .



(a)

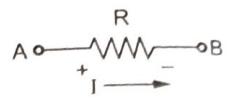


Fig. 7.9.2 (b)

- While tracing a closed path, if we go from +ve marked terminal to - ve marked terminal, that marked terminal to - ve marked terminal, that voltage must be taken as negative. This is called **potential drop**. For example, in the Fig. 7.9.2 (a) only, if the branch is traced from B to A then it should be taken as negative, as  $- IR$  while in the Fig. 7.9.2 (b) if the branch is traced from A to B then it should be taken as negative, as  $- IR$ .

**Step 1 :** Draw the circuit diagram from the given information and insert all the values of sources with appropriate polarities and all the resistances.

**Step 2 :** Mark all the branch currents with some assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents minimum as far as possible to limit the mathematical calculations required to solve them later on.

- A particular current leaving a particular source has some magnitude, then same magnitude of current should enter that source after travelling through various branches of the network.

**Step 3 :** Mark all the polarities of voltage drops and rises as per the directions of the assumed branch currents flowing through various branch resistances of the network. This is necessary for application of KVL to various closed loops.

**Step 4 :** Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any previous equations.

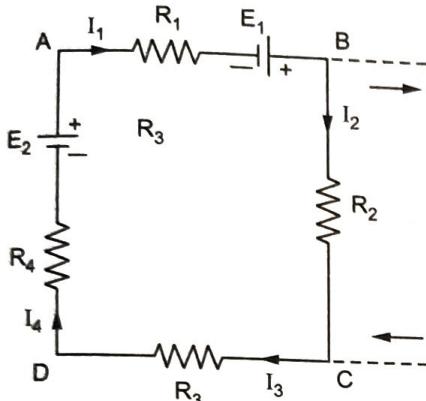
KVL must be applied to sufficient number of loops such that each element of the network is included at least once in any of the equations.

**Step 5 :** Solve the simultaneous equations for the unknown currents. From these currents, unknown voltages and power consumption in different resistances can be calculated.

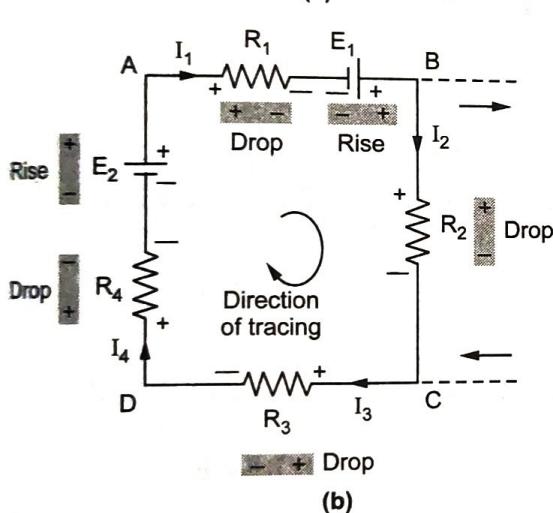
### Application of KVL to a Closed Path

Consider a closed path of a complex network with various branch currents assumed as shown in the Fig. 7.9.3 (a).

Due to these currents the various voltage drops taken place across various resistances are marked as shown in the Fig. 7.9.3 (b).



(a)



(b)

Fig. 7.9.3 (a), (b) Closed loop of a complex network

The polarity of voltage drop along the current direction is to be marked as positive (+) to negative (-).

Let us trace this closed path in clockwise direction A-B-C-D-A.

Across  $R_1$  there is voltage drop  $I_1 R_1$  and as getting from +ve to -ve, it is drop and must be taken as negative while applying KVL.

Every  $E_1$  is getting traced from negative to positive i.e. it is a rise hence must be considered as positive.

Similarly considering rise or drop across all the elements of loop, KVL equation is obtained as,

$$-I_1 R_1 + E_1 - I_2 R_2 - I_3 R_3 - I_4 R_4 + E_2 = 0$$

... Required KVL equation

$$\text{i.e. } E_1 + E_2 = I_1 R_1 + I_2 R_2 + I_3 R_3 + I_4 R_4$$

- If we trace the closed loop in opposite direction i.e. along A-D-C-B-A and follow the same sign convention, the resulting equation will be same as what we have obtained above.

**Key Point** So while applying KVL, direction in which loop is to be traced is not important but following the sign convention is most important.

- The same sign convention is followed in this book to solve the problems.

### What to do if current source exists ?

**Key Point** If there is current source in the network then complete the current distribution considering the current source. But while applying KVL, the loops should not be considered involving current source. The loop equations must be written to those loops which do not include any current source. This is because drop across current source is unknown.

- For example, consider the circuit shown in the Fig. 7.9.4

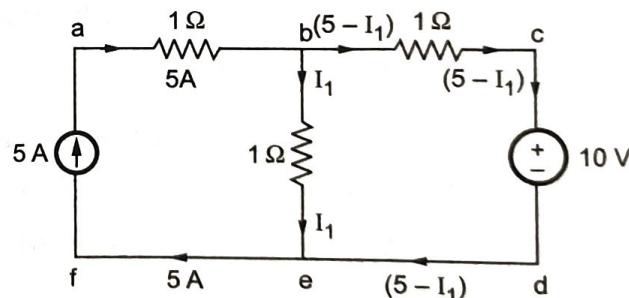


Fig. 7.9.4

- The current distribution is completed in terms of current source value.
- Then KVL must be applied to the loop bcdeb, which does not include current source.
- The loop abefa should not be used for KVL application, as it includes current source. Its effect is already considered at the time of current distribution.

**Ex. 7.9.1** Find the  $V_{CE}$  and  $V_{AG}$  for the circuit shown in Fig. 7.9.5.

SPPU : May-06, Marks 8

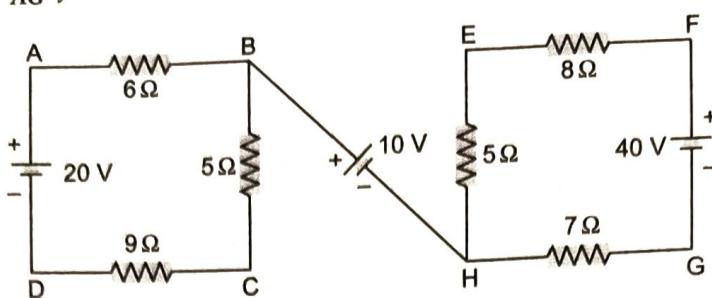


Fig. 7.9.5

**Sol. :** Assume the two currents as shown in the Fig. 7.9.5 (a)

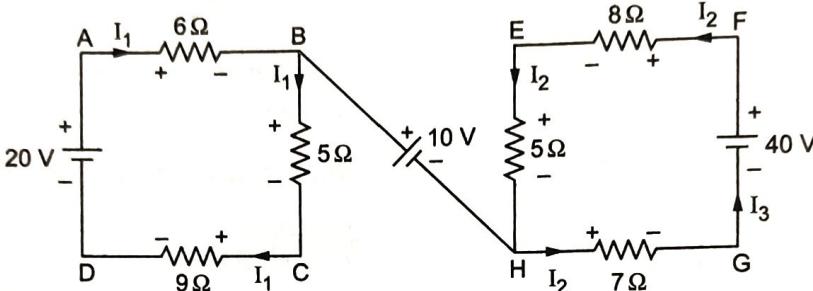


Fig. 7.9.5 (a)

Applying KVL to the two loops,

$$-6I_1 - 5I_1 - 9I_1 + 20 = 0$$

$$\text{and } -8I_2 - 5I_2 - 7I_2 + 40 = 0$$

$$\therefore I_1 = 1 \text{ A}$$

$$\text{and } I_2 = 2 \text{ A}$$

i) Trace the path C-E, [Refer Fig. 7.9.5 (b)].

$$\therefore V_{CE} = -5 \text{ V}$$

= 5 V with C negative

ii) Trace the path A-G, [Refer Fig. 7.9.5 (c)].

$$\therefore V_{AG} = 30 \text{ V with A positive}$$

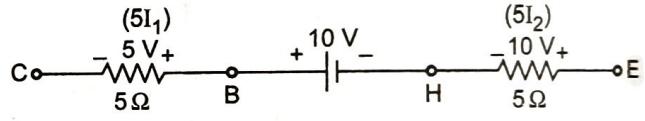


Fig. 7.9.5 (b)

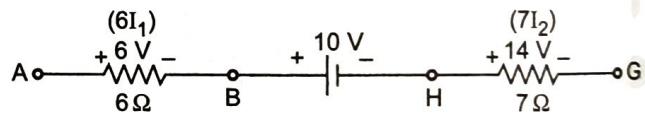


Fig. 7.9.5 (c)

**Ex. 7.9.2** Use Kirchhoff's laws to find current supplied by the battery for the circuit shown in Fig. 7.9.6.

SPPU : May-06, Marks 6

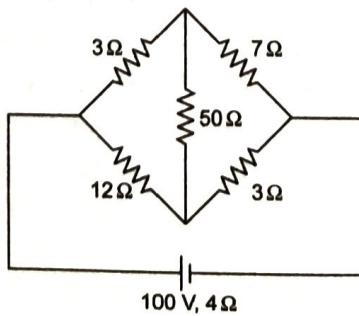


Fig. 7.9.6

The branch currents using KCL are shown in Fig. 7.9.6 (a).

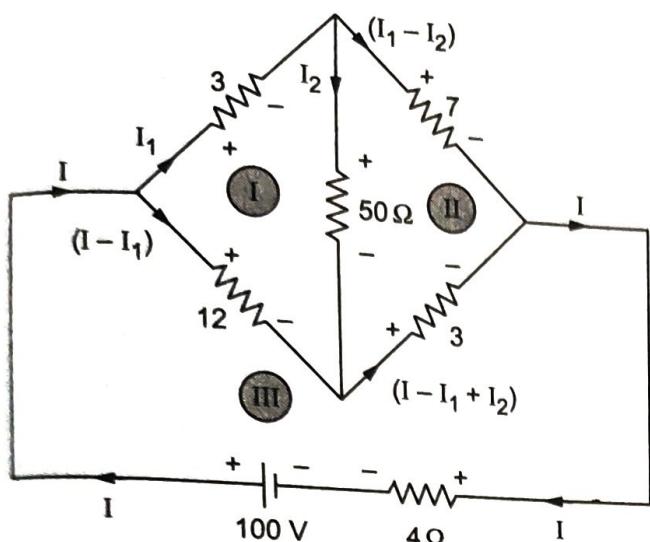


Fig. 7.9.6 (a)

Apply KVL to the three loops I, II and III.

$$-3I_1 - 50I_2 + 12(I - I_1) = 0$$

$$12I - 15I_1 - 50I_2 = 0 \quad \dots (1)$$

$$-7(I_1 - I_2) + 3(I - I_1 + I_2) + 50I_2 = 0$$

$$3I - 10I_1 + 60I_2 = 0 \quad \dots (2)$$

$$-12(I - I_1) - 3(I - I_1 + I_2) - 4I + 100 = 0$$

$$-19I + 15I_1 - 3I_2 = -100 \quad \dots (3)$$

Solving equations (1), (2) and (3),

$I = 10.1634 \text{ A}$  ... Current supplied by battery

### 7.9.3 Find value of R using KCL and KVL.

SPPU : May-12, Marks 4

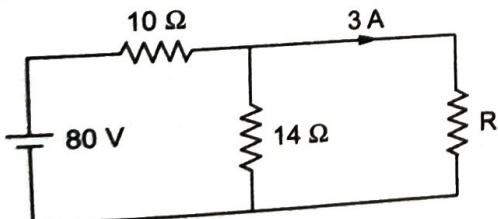


Fig. 7.9.7

Sol.: Show the branch currents as shown in the

Fig. 7.9.7 (a), using KCL.

Using KVL to two loops,

$$-10I_1 - 14(I_1 - 3) + 80 = 0$$

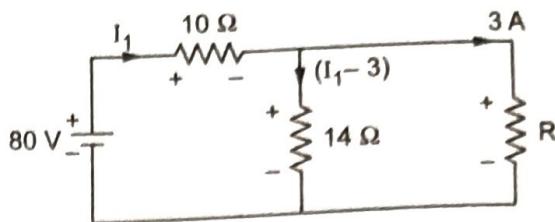


Fig. 7.9.7 (a)

$$I_1 = 5.0833 \text{ A}$$

$$-3R + 14(I_1 - 3) = 0$$

$$\text{i.e.} \quad -3R + 14(5.0833 - 3) = 0$$

$$\therefore R = 9.7222 \Omega$$

### Expected Questions

- Explain Kirchhoff's laws as applied to d.c. circuits, with suitable examples.

SPPU : May-04, 05, 07, 11, 12, 13, 14, 16, 17, Dec.-07, 10, 13, 14, 17. Marks 6

- Explain the sign convention used for the application of KVL.
- Explain the steps to apply Kirchhoff's laws for solving the circuit.

### 7.10 : Star and Delta Connection of Resistances

SPPU : May-99, 03, 06, 09, 10, 11, 12, 13, 14, 15, 18, 19, Dec.-99, 2000, 04, 05, 06, 07, 09, 10, 11, 12, 14, 15, 16

- If the three resistances are connected in such manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.
- The Fig. 7.10.1 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 7.10.1 (a) and (b) are exactly identical. The Fig. 7.10.1 (b) can be redrawn as Fig. 7.10.1 (a) or vice-versa, in the circuit from simplification point of view.
- If the three resistances are connected in such manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.
- The Fig. 7.10.2 (a) and (b) show delta connection of three resistances. The Fig. 7.10.2 (a) and (b) are exactly identical.

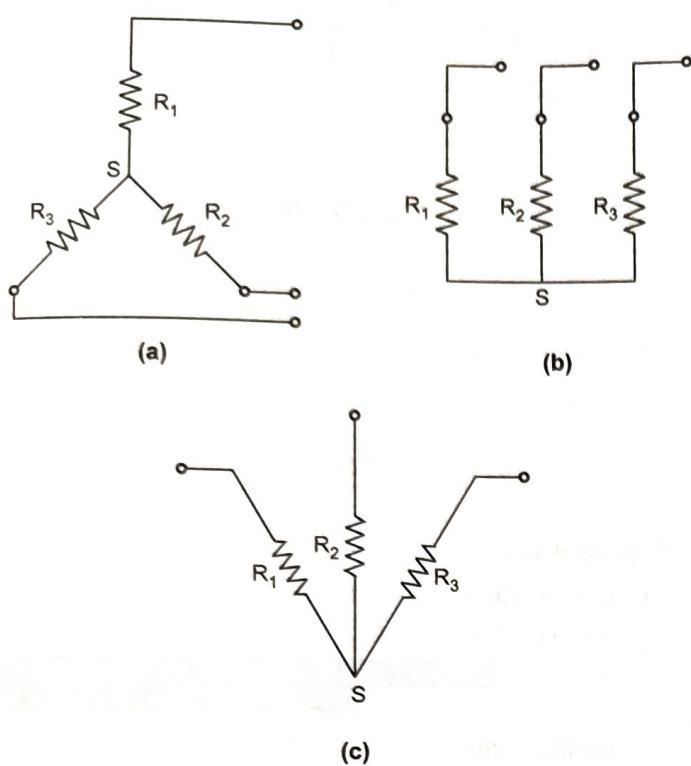


Fig. 7.10.1 Star connection of three resistances

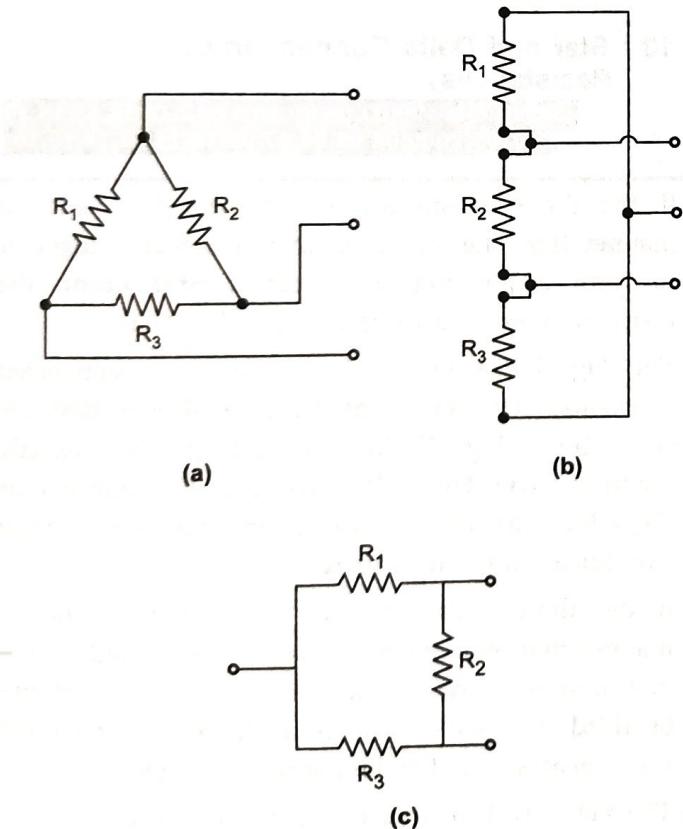


Fig. 7.10.2 Delta connection of three resistances

**Key Point** Delta connection always forms a loop, closed path.

### 7.10.1 Delta-Star Transformation

- Consider the three resistances  $R_{12}, R_{23}, R_{31}$  connected in Delta as shown in the Fig. 7.10.3.
- The terminals between which these are connected in Delta are named as 1, 2 and 3.
- It is always possible to replace these Delta connected resistances by three equivalent Star connected resistances  $R_1, R_2, R_3$  between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 7.10.3 which is called equivalent Star of Delta connected resistances.

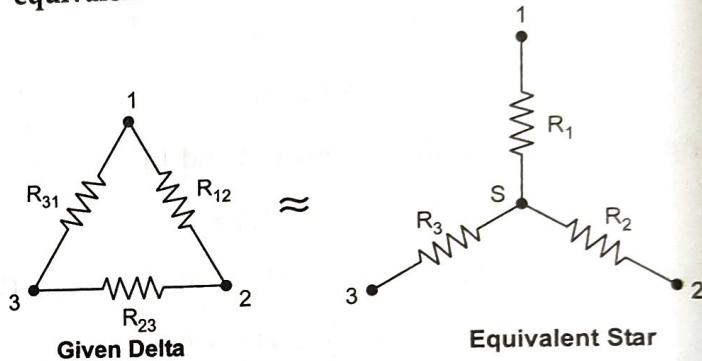


Fig. 7.10.3

**Key Point** To call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections. Thus if  $R_{12}$  is the resistance measured between the terminals 1 and 2 of delta then it must be equal to resistance  $R_{12}$  measured between the terminals 1 and 2 of equivalent star network.

- Let us analyse Delta connection first, shown in the Fig. 7.10.3 (a).
- Consider the terminals 1 and 2. Let us find equivalent resistance between 1 and 2. We can redraw the network as viewed from the terminals (1) and (2), without considering terminal 3. This is shown in the Fig. 7.10.3(b).
- Now terminal '3' is not considered, so between terminals (1) and (2) we get the combination as,  $R_{12}$  parallel with  $(R_{31} + R_{23})$  as  $R_{31}$  and  $R_{23}$  are in series.

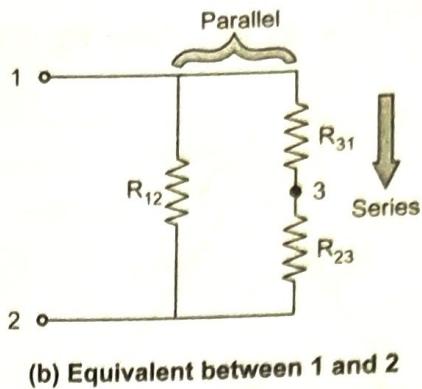
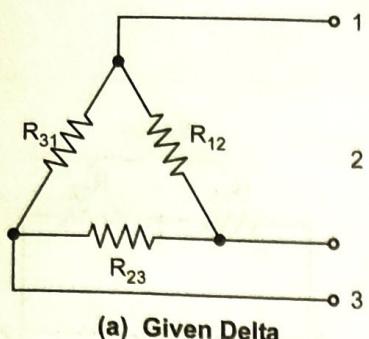


Fig. 7.10.3

$$\therefore \text{Between (1) and (2) the resistance is, } = \frac{R_{12}(R_{31} + R_{23})}{R_{12} + (R_{31} + R_{23})} \quad \dots(7.10.1)$$

[using  $\frac{R_1 R_2}{R_1 + R_2}$  for parallel combination]

- Consider the same two terminals of equivalent Star connection shown in the Fig. 7.10.4.

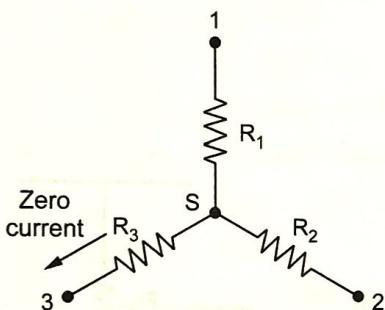


Fig. 7.10.4 Star connection

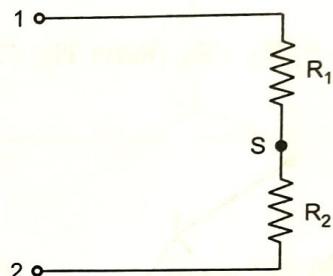


Fig. 7.10.5 Equivalent between 1 and 2

- Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).
- And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 7.10.5.

$$\therefore \text{Between (1) and (2) the resistance is } = R_1 + R_2 \quad \dots(7.10.2)$$

- This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.
- To call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (7.10.1) and (7.10.2),

$$\frac{R_{12}(R_{31} + R_{23})}{R_{12} + (R_{23} + R_{31})} = R_1 + R_2 \quad \dots(7.10.3)$$

- Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31} + R_{12})}{R_{12} + (R_{23} + R_{31})} = R_2 + R_3 \text{ (Refer Fig. 7.10.6)} \quad \dots(7.10.4)$$

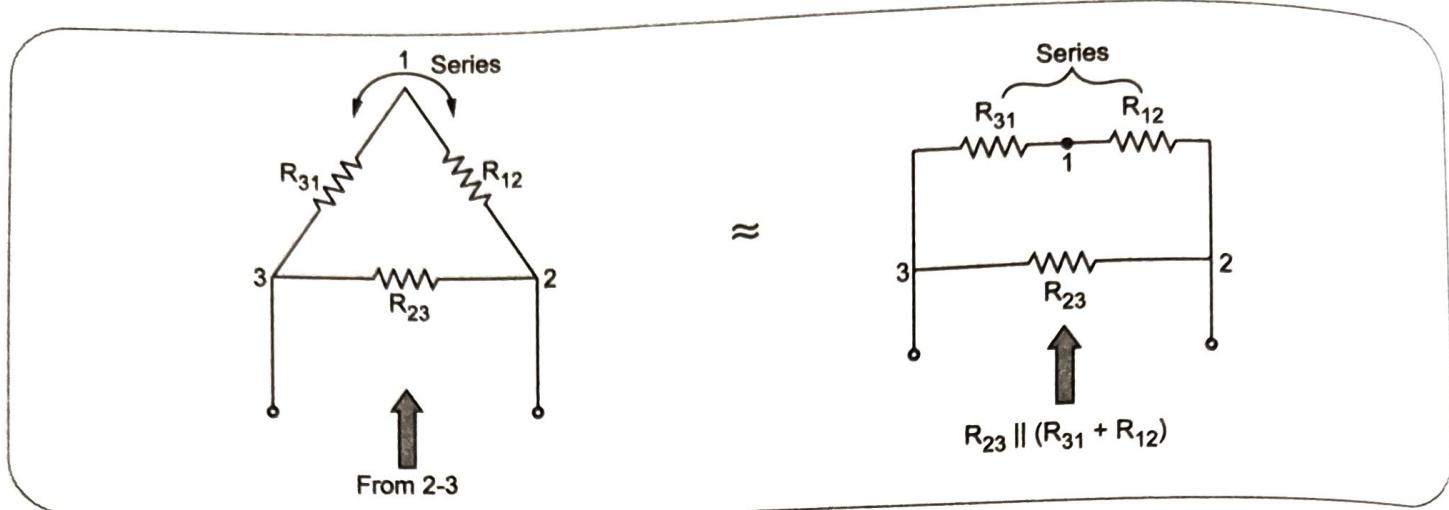


Fig. 7.10.6

- Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12} + R_{23})}{R_{12} + (R_{23} + R_{31})} = R_3 + R_1 \text{ (Refer Fig. 7.10.7)} \quad \dots(7.10.5)$$

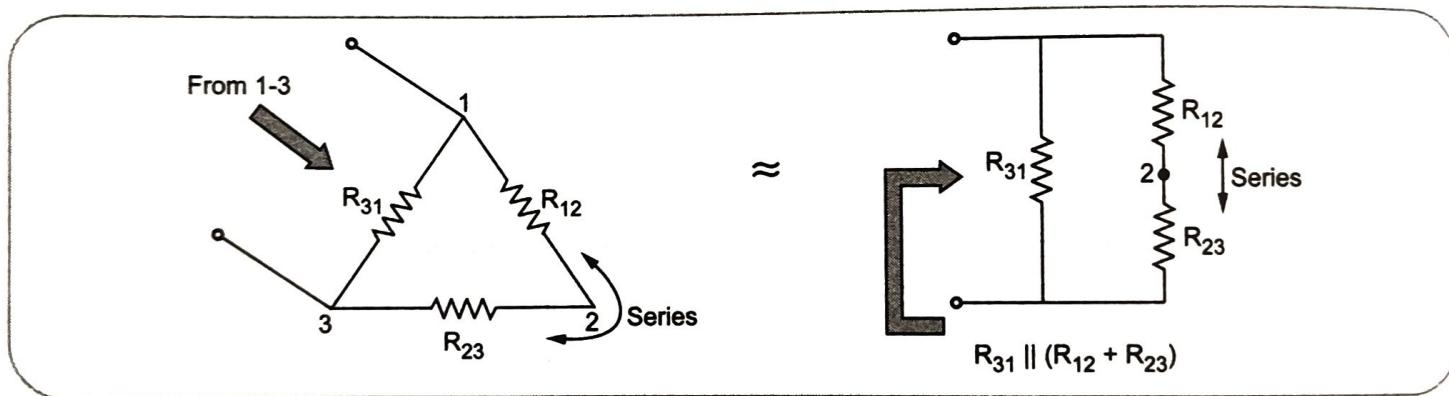


Fig. 7.10.7

- We are interested in calculating what are the values of  $R_1, R_2, R_3$  in terms of known values  $R_{12}, R_{23}$ , and  $R_{31}$ .

- Subtracting (7.10.4) from (7.10.3),

$$\frac{R_{12}(R_{31} + R_{23}) - R_{23}(R_{31} + R_{12})}{(R_{12} + R_{23} + R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \dots(7.10.6)$$

- Adding (7.10.6) and (7.10.5),

$$\frac{R_{12} R_{31} - R_{23} R_{31} + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\frac{R_{12} R_{31} - R_{23}}{R_{12} + R_{23} + R_{31}} \cdot \frac{R_{31} + R_{12} + R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1 \quad \text{i.e.} \quad 2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

Similarly by using another combinations of subtraction and addition with equations (7.10.3), (7.10.4) and (7.10.5) we can get,

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

and

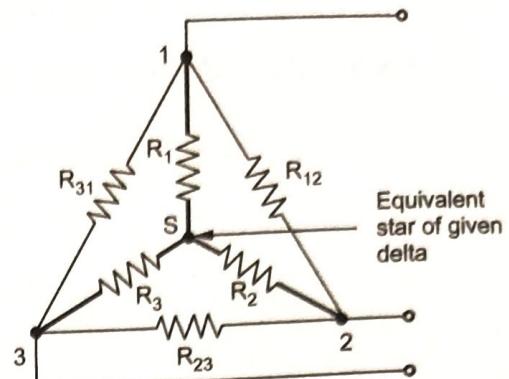


Fig. 7.10.8 Delta and equivalent Star

### Easy way of remembering the result :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

### Star-Delta Transformation

- Consider the three resistances  $R_1, R_2$  and  $R_3$  connected in Star as shown in Fig. 7.10.9.
- Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances  $R_{12}, R_{23}$  and  $R_{31}$ , between the same terminals. This is called **equivalent Delta of the given star**.
- We are interested in finding out values of  $R_{12}, R_{23}$  and  $R_{31}$  in terms of  $R_1, R_2$  and  $R_3$ .
- For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \dots(7.10.7)$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \quad \dots(7.10.8)$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \dots(7.10.9)$$

- Now multiply (7.10.7) and (7.10.8), (7.10.8) and (7.10.9), (7.10.9) and (7.10.7) to get following three equations.

$$R_1 R_2 = \frac{R_{12}^2 R_{31} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots(7.10.10)$$

$$R_2 R_3 = \frac{R_{23}^2 R_{12} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots(7.10.11)$$

$$R_3 R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \quad \dots(7.10.12)$$

- Now add (7.10.10), (7.10.11) and (7.10.12)

$$\therefore R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{31} R_{23} (R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

But  $\frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} = R_1$

From equation (7.10.7)

- Substituting in above in R.H.S. we get,

$$\therefore R_1 R_2 + R_2 R_3 + R_3 R_1 = R_1 R_{23}$$

$$\therefore R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

- Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$$

and

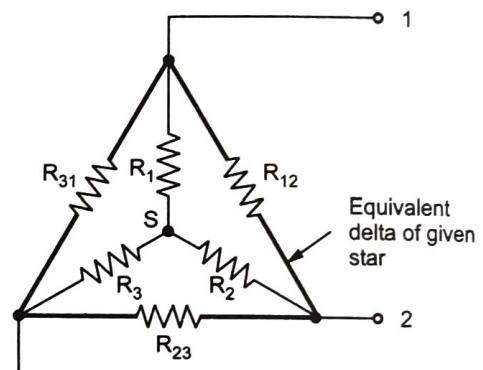


Fig. 7.10.10 Star and equivalent Delta

#### Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.

- Result for equal resistances in star and delta :

- If all resistances in a Delta connection have same magnitude say R, then its equivalent Star will contain,

$$R_1 = R_2 = R_3 = \frac{R \times R}{R + R + R} = \frac{R}{3}$$

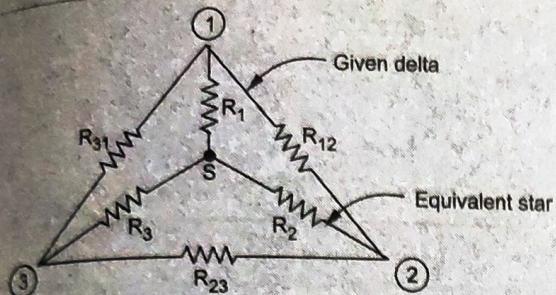
i.e. equivalent Star contains three equal resistances, each of magnitude one third the magnitude of the resistances connected in Delta.

If all three resistances in a Star connection are of same magnitude say R, then its equivalent Delta contains all resistances of same magnitude of,

$$R_{12} = R_{31} = R_{23} = R + R + \frac{R \times R}{R} = 3R$$

i.e. equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.

Delta-Star

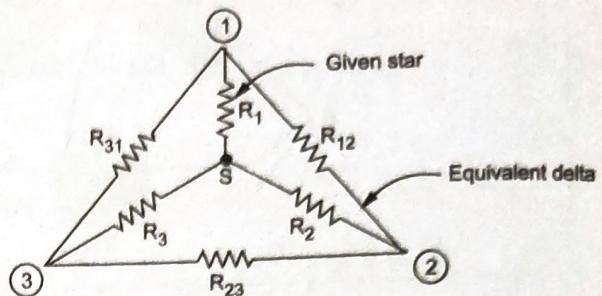


$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

Star-Delta



$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_1R_3}{R_2}$$

Table 7.10.1 Star-Delta and Delta-Star Transformations

**Ex. 7.10.1** Find the equivalent resistance between terminals B and C of the circuit shown in the Fig. 7.10.11.

SPPU : May-99, 11, Marks 7

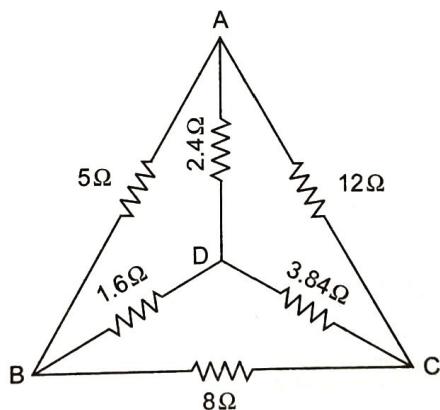


Fig. 7.10.11

**Sol.:** Solution is also possible by converting Delta to Star which gives solution in less steps.

Converting star ADCB to delta ACB.

$$R_{AB} = 2.4 + 1.6 + \frac{2.4 \times 1.6}{3.84} = 5 \Omega$$

$$R_{AC} = 2.4 + 3.84 + \frac{2.4 \times 3.84}{1.6} = 12 \Omega$$

$$R_{BC} = 1.6 + 3.84 + \frac{1.6 \times 3.84}{2.4} = 8 \Omega$$

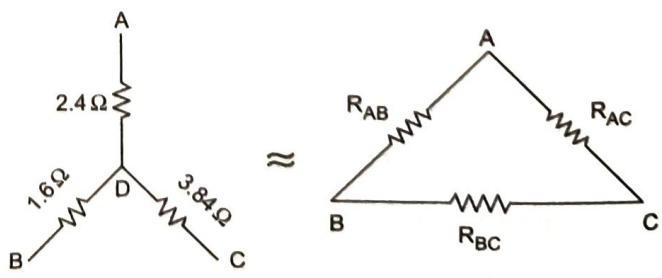
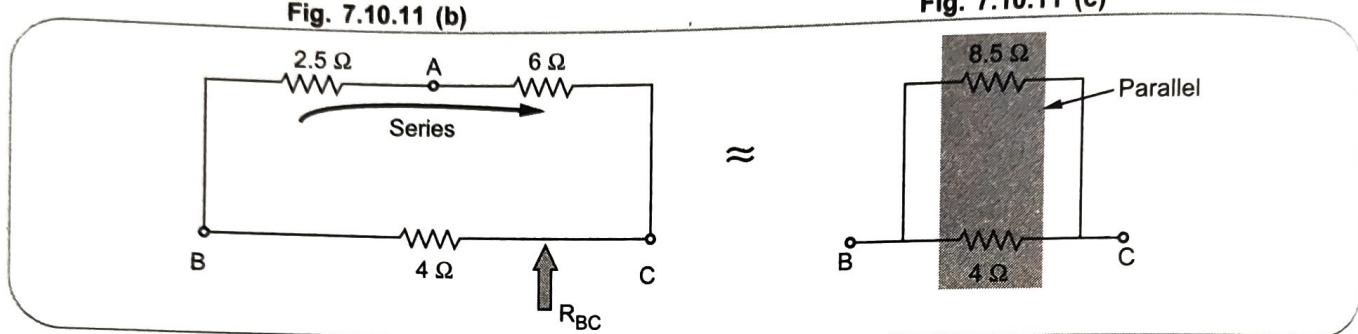
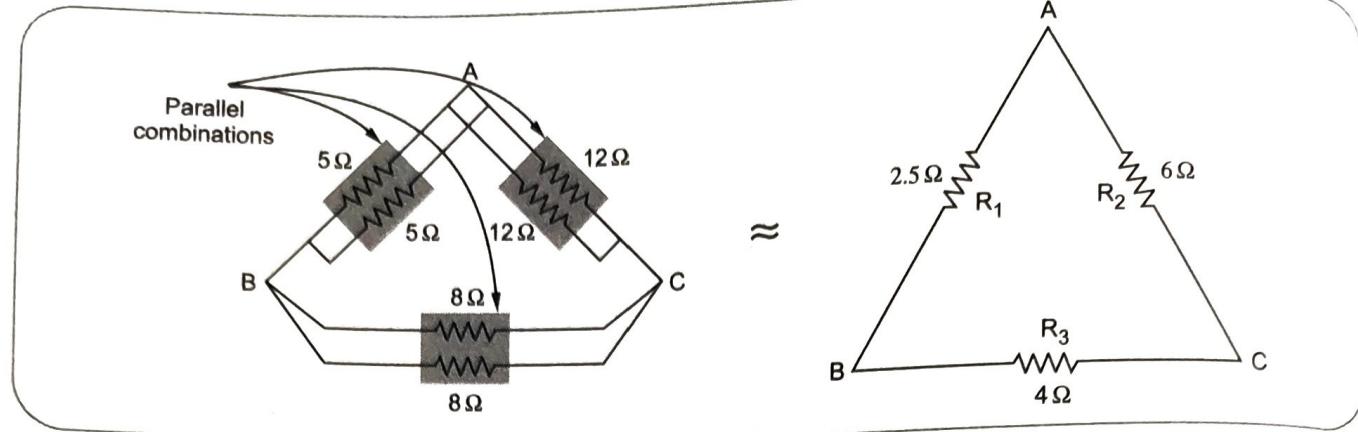


Fig. 7.10.11 (a)

**Fig. 7.10.11 (d)**

$$R_1 = \frac{5 \times 5}{5+5} = 2.5 \Omega,$$

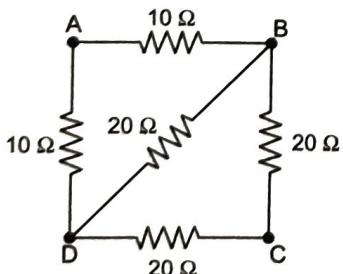
$$R_2 = \frac{12 \times 12}{12+12} = 6 \Omega,$$

$$R_3 = \frac{8 \times 8}{8+8} = 4 \Omega$$

$$R_{BC} = \frac{4 \times 8.5}{4+8.5} = 2.72 \Omega \text{ [Fig. 7.10.11(e)]}$$

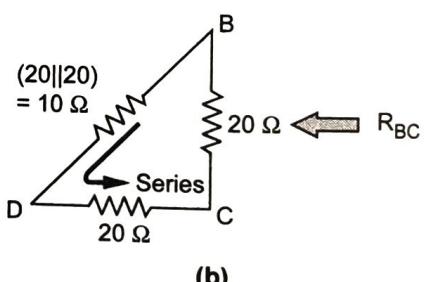
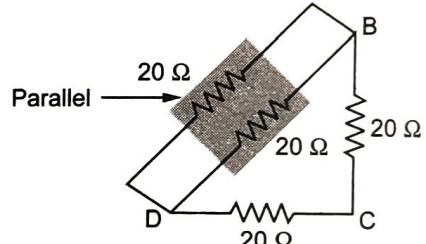
**Ex. 7.10.2** Find the resistance between (1) B and C and (2) A and C in the network shown in the Fig. 7.10.12.

SPPU : Dec.-99, 2000

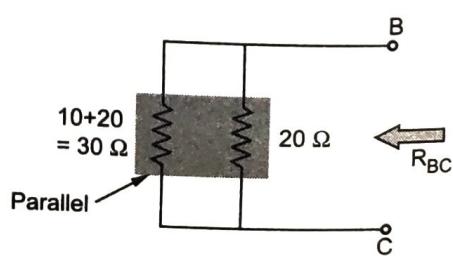
**Fig. 7.10.12**

**Sol. : i) Between B and C**

As looking through B and C, 10 Ω and 10 Ω are in series, as both carry same current.



Again, 10 Ω and 20 Ω are in series.

**Fig. 7.10.12 (c)**

Electrical Engg.

$$R_{BC} = 20 \parallel 30 = \frac{20 \times 30}{20 + 30} = 12 \Omega$$

Between A and C

Converting delta BCD to equivalent star,

$$R_{BS} = R_{CS} = R_{DS} = \frac{20 \times 20}{(20 + 20 + 20)} = 6.67 \Omega$$

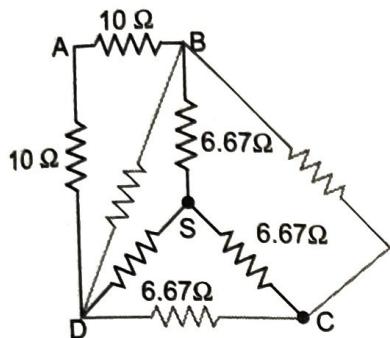


Fig. 7.10.12 (d)

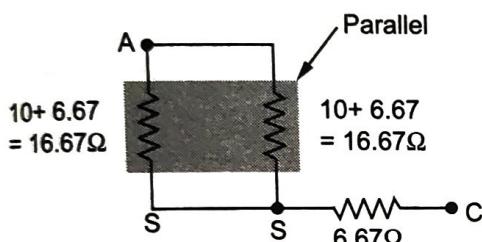


Fig. 7.10.12 (e)

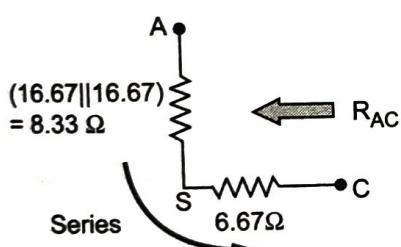


Fig. 7.10.12 (f)

$$R_{AC} = 8.33 + 6.67 = 15 \Omega$$

Ex. 7.10.3 Calculate the resistance between terminals A-B.  
SPPU : May-12, Marks 4

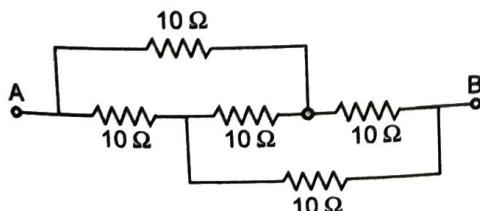


Fig. 7.10.13

Sol. : Refer Fig. 7.10.13 (a),

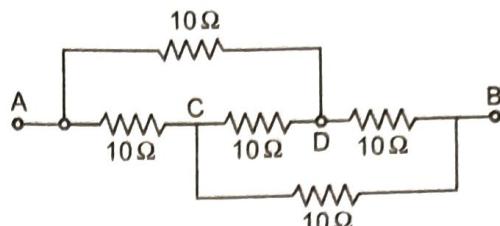


Fig. 7.10.13 (a)

Loop A-C-D forms  $\Delta$  converting to Star,

$$R_{AS} = \frac{10 \times 10}{10 + 10 + 10} = 3.33 \Omega$$

$$= R_{CS} = R_{DS}$$

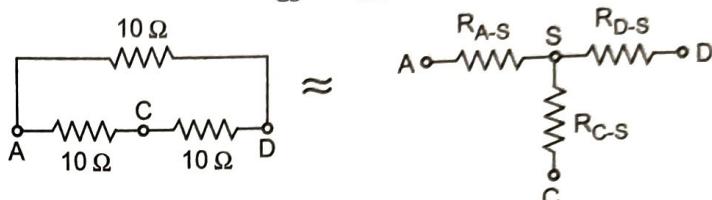


Fig. 7.10.13 (b)

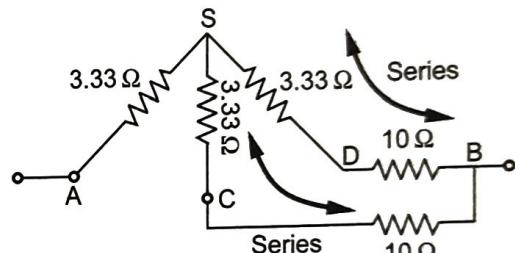


Fig. 7.10.13 (c)

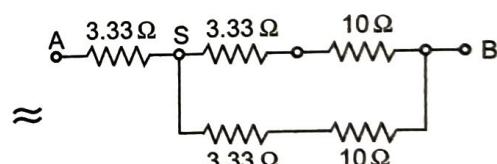


Fig. 7.10.13 (d)

$$R_{AB} = 3.333 + 6.666 = 10 \Omega$$

Ex. 7.10.4 Determine the resistance between the terminals X and Y for the circuit shown in Fig. 7.10.14.  
SPPU : Dec.-06, Marks 9

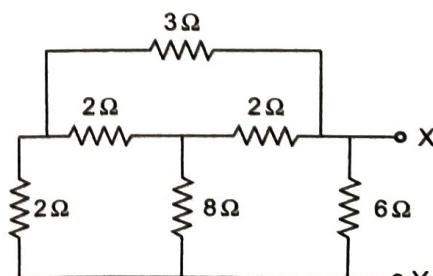


Fig. 7.10.14

**Sol.** : Convert delta of  $3\ \Omega$ ,  $2\ \Omega$ ,  $2\ \Omega$  to star.

$$R_1 = \frac{3 \times 2}{3+2+2} = 0.8571\ \Omega$$

$$R_2 = \frac{2 \times 2}{3+2+2} = 0.5714\ \Omega$$

$$R_3 = \frac{3 \times 2}{3+2+2} = 0.8571\ \Omega$$

$$2.8571 \parallel 8.5714 = \frac{2.8571 \times 8.5714}{2.8571 + 8.5714}$$

$$= 2.1428\ \Omega$$

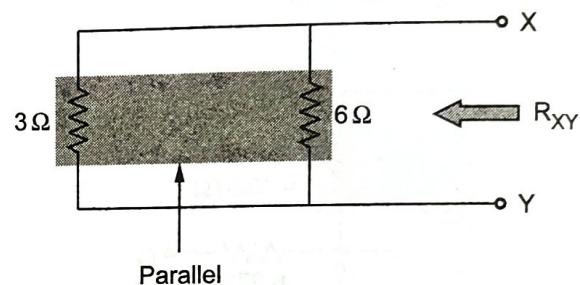
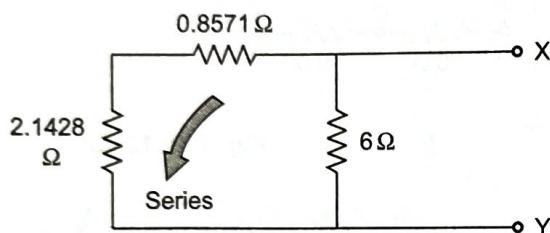
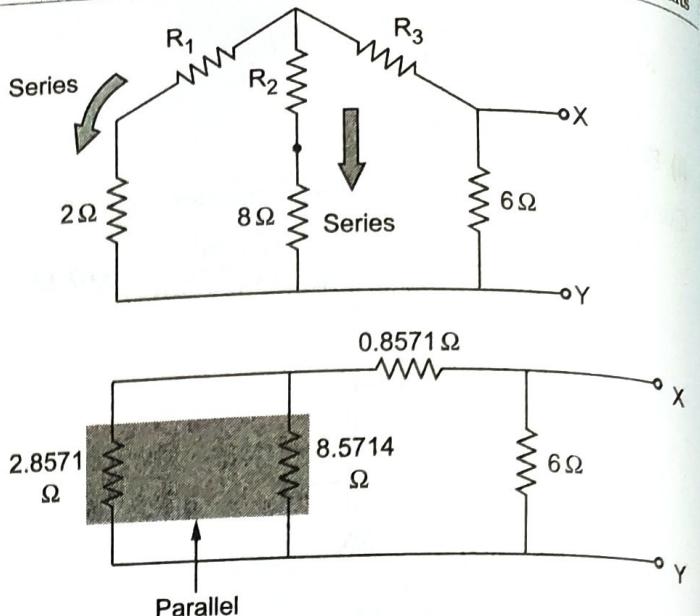


Fig. 7.10.14 (a)

$$\therefore R_{XY} = 3 \parallel 6 = \frac{6 \times 3}{6+3} = 2\ \Omega$$

**Ex. 7.10.5** Determine the resistance between the terminals X and Y for the circuit shown in Fig. 7.10.15.

SPPU : Dec.-06, 10, Marks 6

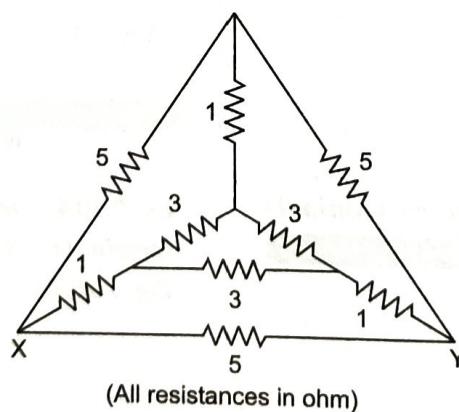


Fig. 7.10.15

**Sol.** : Converting inner delta to star.

$$\text{Each resistance} = \frac{3 \times 3}{3+3+3} = 1\ \Omega \text{ in equivalent star}$$

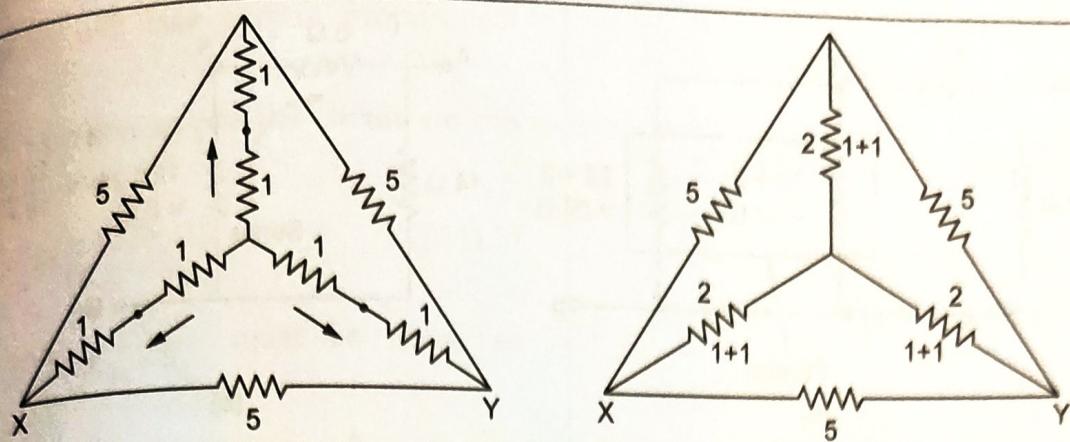


Fig. 7.10.15 (a)

Converting inner star to delta.

$$\text{Each resistance} = 2 + 2 + \frac{2 \times 2}{2} = 6 \Omega \text{ in equivalent delta}$$

All three parallel combinations,

$$5 \parallel 6 = \frac{5 \times 6}{5+6} = 2.7272 \Omega$$

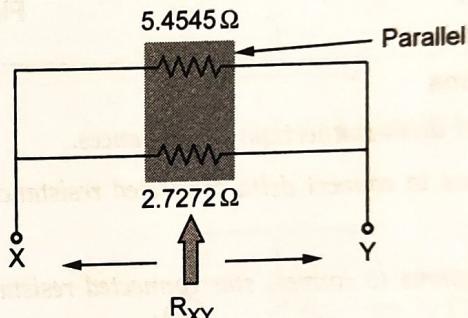
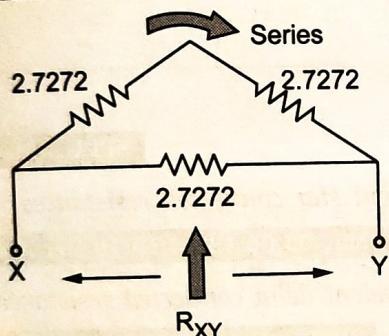
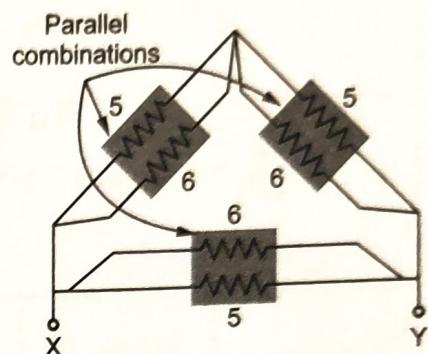


Fig. 7.10.15 (b)

$$R_{XY} = 5.4545 \parallel 2.7272 = 1.8181 \Omega$$

Ex. 7.10.6 Determine effective resistance between A and B.

SPPU : May-18, Marks 6

Sol.: Convert 6 ohm delta to star. Each resistance in star is,

$$R' = \frac{6 \times 6}{6+6+6} = 2 \Omega$$

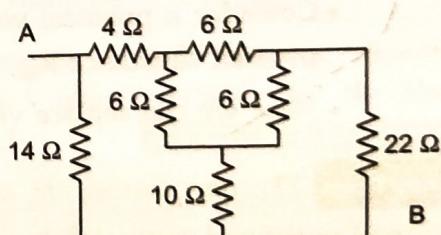
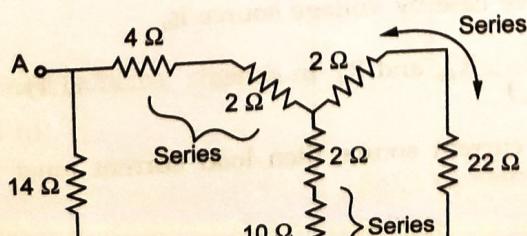
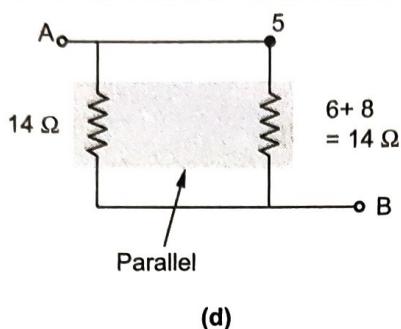
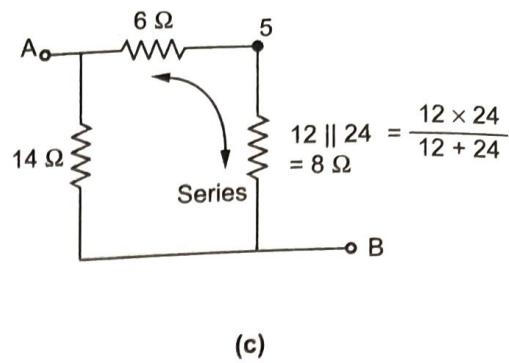
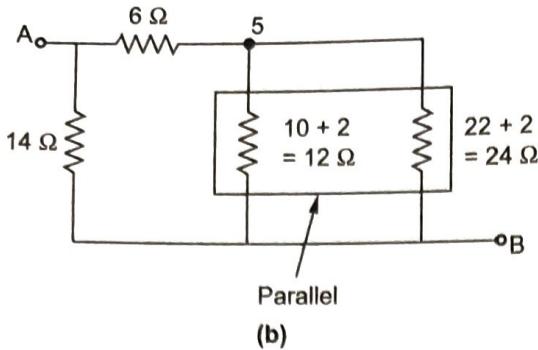


Fig. 7.10.16





$$\begin{aligned} & 14 \parallel 14 \\ & \frac{14 \times 14}{14 + 14} = 7 \Omega \\ & R_{AB} = 7 \Omega \end{aligned}$$

(e)

Fig. 7.10.16

**Expected Questions**

1. Define star and delta connection of resistances.

SPPU : May-06, Dec.-09, Marks 3

2. Derive equations to convert delta connected resistances to equivalent star connected resistances.

SPPU : May-03, 06, 13, 14, 15, 19, Dec.-04, 05, 11, 14, 15, Marks 6

3. Derive an equations to convert star connected resistances to equivalent delta connected resistances.

SPPU : Dec.-07, 09, 12, 16, May-09, 10, 12, 18, Marks 6

**7.11 : Source Transformation**

SPPU : May-06, Dec.-06

- Consider a practical voltage source shown in the Fig. 7.11.1 (a) having internal resistance  $R_{se}$ , connected to the load having resistance  $R_L$ .
- Now we can replace voltage source by equivalent current source.

**Key Point** The two sources are said to be equivalent, if they supply equal load current to the load, with same load connected across its terminals

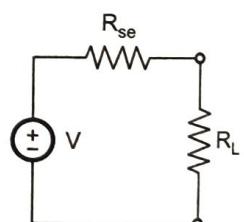


Fig. 7.11.1 (a) Voltage source

- The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se} + R_L)}, R_{se} \text{ and } R_L \text{ in series} \quad \dots(7.11.1)$$

- If it is to be replaced by a current source then load current must be  $\frac{V}{(R_{se} + R_L)}$

- Consider an equivalent current source shown in the Fig. 7.11.1 (b).
- The total current is I.

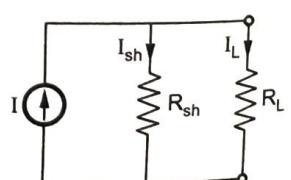


Fig. 7.11.1 (b) Current source

resistances will take current proportional values.

In current division in parallel circuit we can

$$I_L = I \times \frac{R_{sh}}{(R_{sh} + R_L)} \quad \dots(7.11.2)$$

Thus  $I_L$  and  $\frac{V}{R_{se} + R_L}$  must be same, so

(1) and (2),

$$\frac{V}{R_{se} + R_L} = \frac{I \times R_{sh}}{R_{sh} + R_L}$$

internal resistance be,

$$R_{se} = R_{sh} = R \text{ say.}$$

$$V = I \times R_{sh} = I \times R \text{ or } I = \frac{V}{R_{sh}}$$

$$I = \frac{V}{R} = \frac{V}{R_{se}}$$

If voltage source is converted to current then current source  $I = \frac{V}{R_{se}}$  with parallel resistance equal to  $R_{se}$ .

If current source is converted to voltage then voltage source  $V = I R_{sh}$  with series resistance equal to  $R_{sh}$ .

direction of current of equivalent current source always from -ve to +ve, internal to the source.

In converting current source to voltage source, polarity of voltage is always as +ve terminal at top arrow and -ve terminal at bottom of arrow, as direction of current is from -ve to +ve, internal to source. This ensures that current flows from positive to negative terminal in the external circuit.

Note the directions of transformed sources, shown in Fig. 7.11.2 (a), (b), (c) and (d).

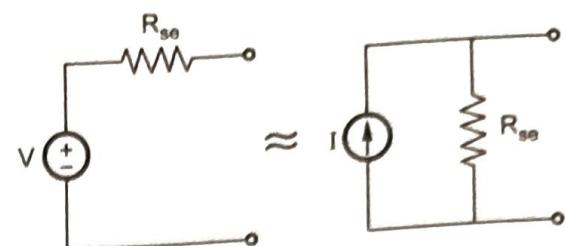


Fig. 7.11.2 (a)  $I = \frac{V}{R_{se}}$

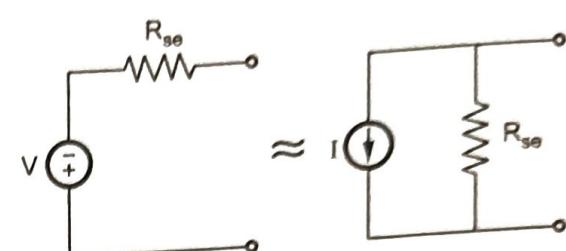


Fig. 7.11.2 (b)  $I = \frac{V}{R_{se}}$

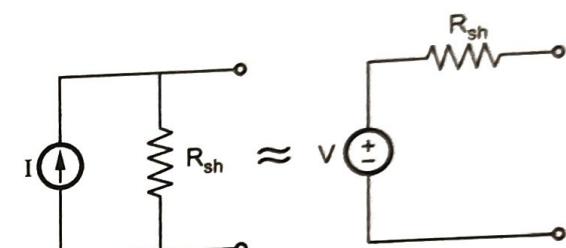


Fig. 7.11.2(c)  $V = I \times R_{sh}$

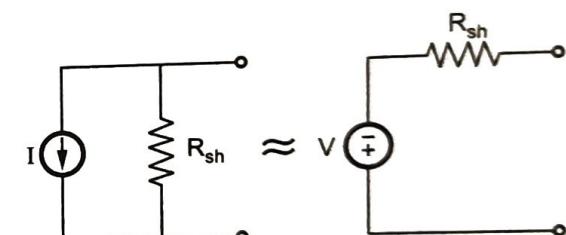


Fig. 7.11.2 (d)  $V = I \times R_{sh}$

Ex. 7.11.1 Using source transformations, determine the voltage across 5 ohm resistance for the circuit shown in Fig. 7.11.3.

SPPU : Dec.-06, Marks 6

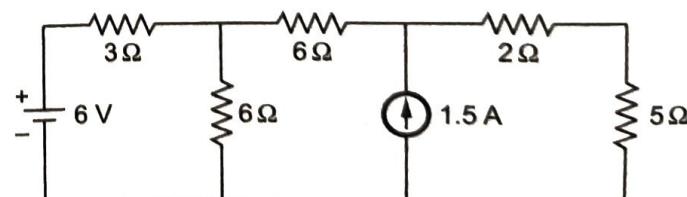


Fig. 7.11.3

Sol. : Converting 6 V voltage source to current source.

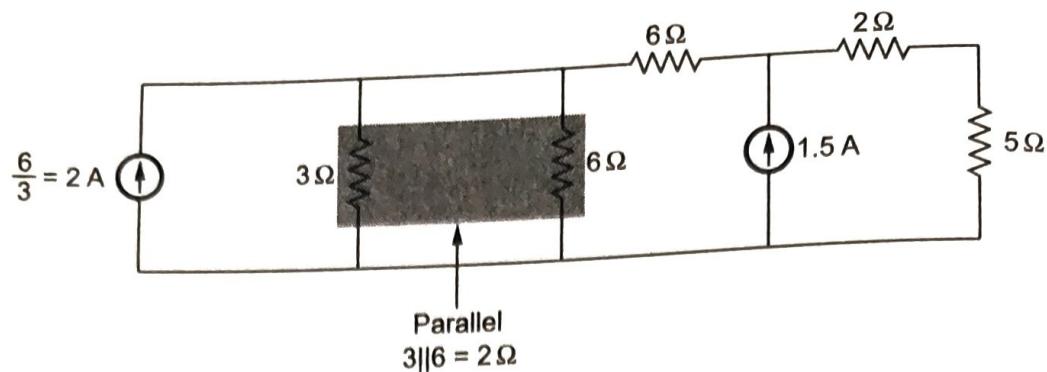
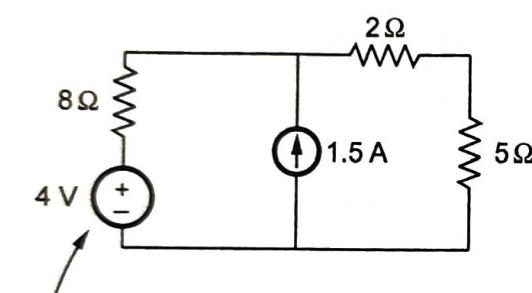
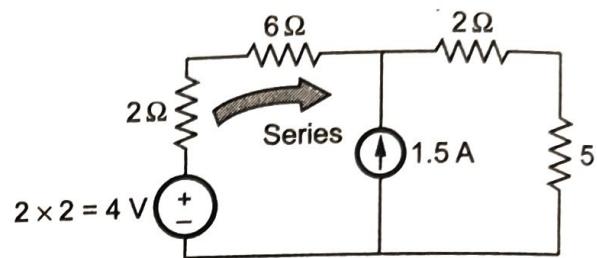
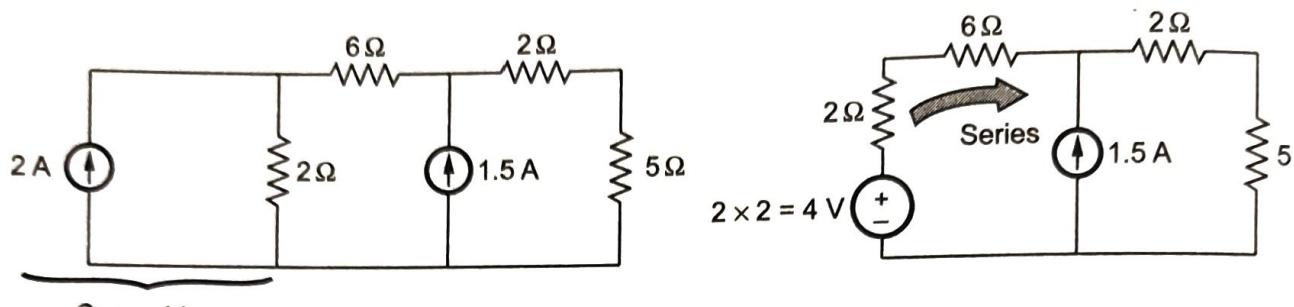
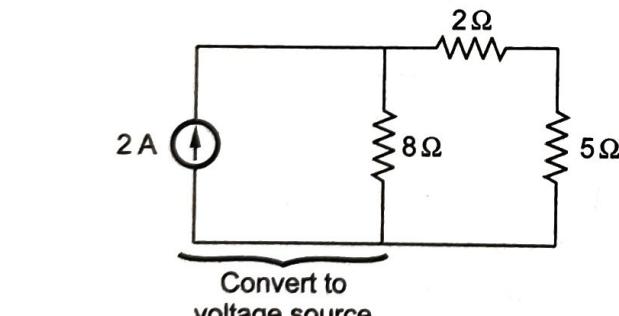
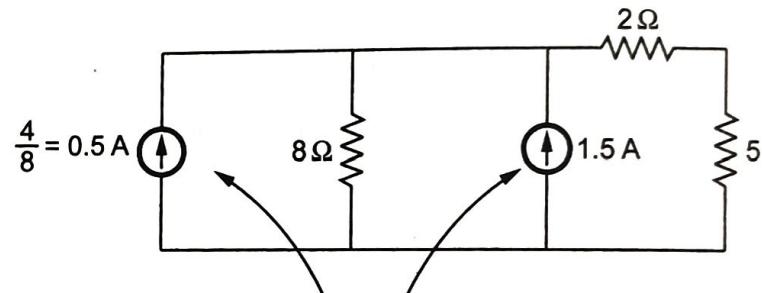


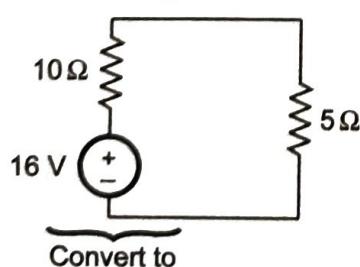
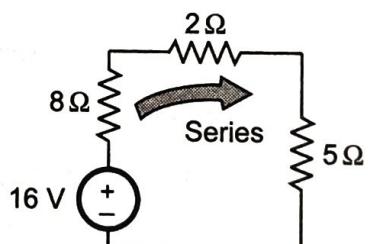
Fig. 7.11.3 (a)



Convert to current source



Convert to voltage source



Convert to current source

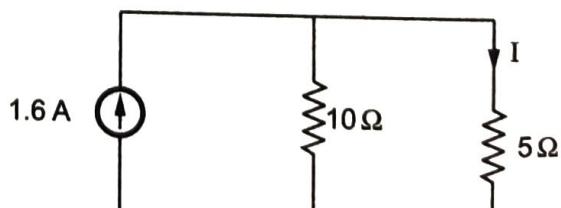


Fig. 7.11.3 (b)

Using current division rule,

$$I = 1.6 \times \frac{10}{15} = 1.0667\text{ A}$$

**Expected Question**

1. Explain how voltage source can be converted to current source.

SPPU : May-06, Marks 4

**7.12 : Concept of Loop Current**

A loop current is that current which simultaneously links with all the branches, defining a particular loop.

The Fig. 7.12.1 shows a network. In this,  $I_1$  is the loop current for the loop ABFEA and simultaneously links with the branches AB, BF, FE and EA. Similarly  $I_2$  is the second loop current for the loop BCGFB and  $I_3$  is the third loop current for the loop CDFGC.

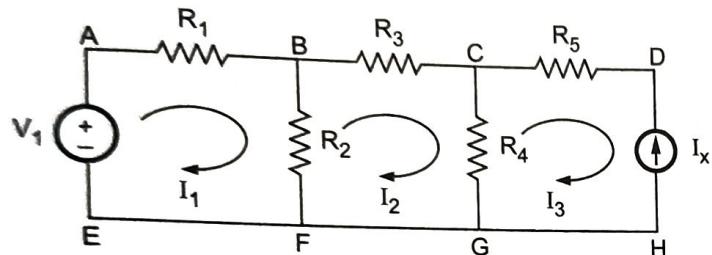


Fig. 7.12.1 Concept of loop current

**Observe :**

- For the common branches of the various loops, multiple loop currents get associated. For example to the branch BF, both  $I_1$  and  $I_2$  are associated.
- The branch current is always unique hence a branch current can be expressed in terms of associated loop currents.

**Key Point** The total branch current is the algebraic sum of all the loop currents associated with that branch.

$$I_{BF} = I_1 - I_2 \text{ from B to F}$$

$$I_{CG} = I_2 - I_3 \text{ from C to G}$$

- The branches consisting current sources, directly decide the values of the loop currents flowing through them.

- The branch DH consists current source of  $I_x$  amperes and only the loop current  $I_3$  is associated with the branch DH in opposite direction. Hence  $I_3 = -I_x$ .

- Assuming such loop currents and assigning the polarities for the drops across the various branches due to the assumed loop currents, the Kirchhoff's voltage law can be applied to the

loops. Solving these equations, the various loop currents can be obtained. Once the loop currents are obtained, any branch current can be calculated.

**7.13 : Loop Analysis or Mesh Analysis**

SPPU : Dec.-15, 17, May-14, 15, 17, 19

- This method of analysis is specially useful for the circuits that have many nodes and loops.
- The difference between application of Kirchhoff's laws and loop analysis is, in loop analysis instead of branch currents, the loop currents are considered for writing the equations.
- Another difference is, in this method, each branch of the network may carry more than one current.
- The total branch current must be decided by the algebraic sum of all currents through that branch. While in analysis using Kirchhoff's laws, each branch carries only one current.
- The advantage of this method is that for complex networks the number of unknowns reduces which greatly simplifies calculation work.
- Consider following network shown in the Fig. 7.13.1. There are two loops. So assuming two loop currents as  $I_1$  and  $I_2$ .

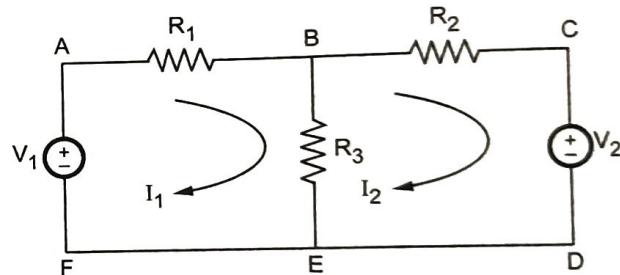


Fig. 7.13.1

**Key Point** While assume loop currents, consider the loops such that each element of the network will be included atleast once in any of the loops.

- Now branch B-E carries two currents;  $I_1$  from B to E and  $I_2$  from E to B. So net current through branch B-E will be  $(I_1 - I_2)$  and corresponding drop across  $R_3$  must be as shown below in the Fig. 7.13.2.



Fig. 7.13.2

er loop A - B - E - F - A,

ch B-E, polarities of voltage drops will be B -ve for current  $I_1$  while E +ve, B -ve for 2 flowing through  $R_3$ .

while writing loop equations assume main current as positive and remaining loop current treated as negative for common branches.

loop equations for the network shown in 7.13.3.

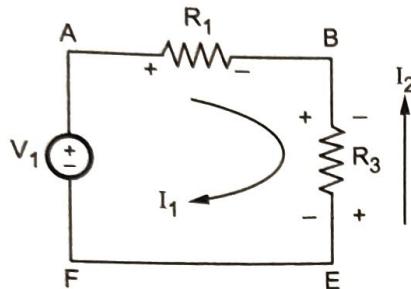


Fig. 7.13.3

A - B - E - F - A,

$$-I_1 R_1 - I_1 R_3 + I_2 R_3 + V_1 = 0$$

B - C - D - E - B

$$-I_2 R_2 - V_2 - I_2 R_3 + I_1 R_3 = 0$$

ving above simultaneous equations any branch current can be determined.

### Points to Remember for Loop Analysis

assuming loop currents make sure that st one loop current links with every element.

two loops should be identical.

use minimum number of loop currents.

ert current sources if present, into their valent voltage sources for loop analysis, never possible.

urrent in a particular branch is required, then o choose loop current in such a way that one loop current links with that branch.

### Supermesh

If there exists a current source in any of the of the network then a loop cannot be defined the current source as drop across the current unknown, from KVL point of view.

ample, consider the network shown in the 3.4. In this circuit, branch B-E consists of a

current source. So loop A-B-E-F-A cannot be defined as loop from KVL point of view, as drop across the current source is not known.

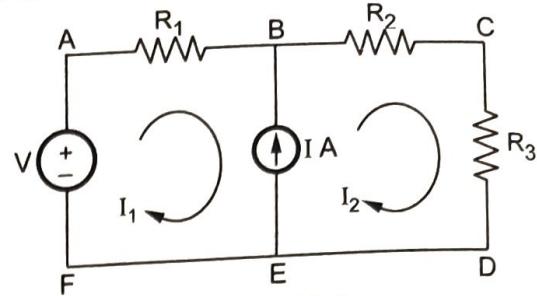


Fig. 7.13.4

- In such case, to get the required equation interms of loop currents, analyse the branch consisting of a current source independently. Express the current source interms of the assumed loop currents.
- For example, in the Fig. 7.13.4 analyse the branch BE. The current source is of IA in the direction of loop current  $I_2$ . So  $I_2$  is more than  $I_1$  and we can write an equation,

$$I = I_2 - I_1$$

- So all such branches, consisting current sources must be analysed independently. Get the equations for current sources interms of loop currents. Then apply KVL to the remaining loops which are existing without involving the branches consisting of current sources.
- The loop existing, around a current source which is common to the two loops is called supermesh. In the Fig. 7.13.4, the loop A-B-C-D-E-F-A is supermesh.

### 7.13.3 Steps for the Loop Analysis

**Step 1 :** Choose the various loops.

**Step 2 :** Show the various loop currents and the polarities of associated voltage drops.

**Step 3 :** Before applying KVL, look for any current source. Analyse the branch consisting current source independently and express the current source value interms of assumed loop currents. Repeat this for all the current sources.

**Step 4 :** After the step 3, apply KVL to those loops, which do not include any current source. A loop cannot be defined through current source from KVL point of view. Follow the sign convention.

**Step 5 :** Solve the equations obtained in step 3 and step 4 simultaneously, to obtain required unknowns

Ex. 7.13.1 Apply  
on by  $4\Omega$  resistance for the circuit.

SPPU : May-14, Marks 7

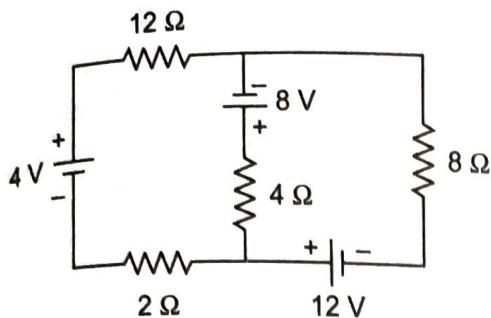


Fig. 7.13.5

Ans. : Step 1 : Shows the loop currents.

Step 2 : Mark the polarities of voltage drops due to the loop currents as shown in in the Fig. 7.13.5 (a).

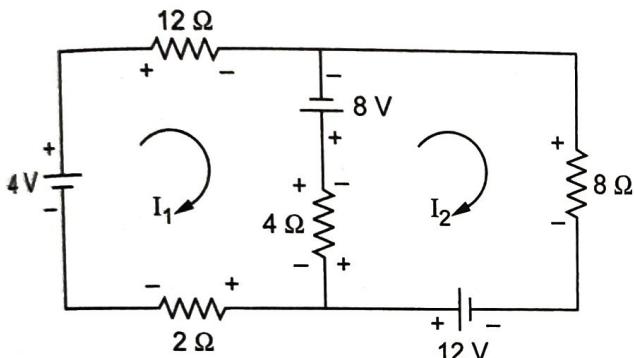


Fig. 7.13.5 (a)

Step 3 : Applying KVL to the two loops.

$$-12I_1 + 8 - 4I_1 + 4I_2 - 2I_1 + 4 = 0 \text{ i.e. } 18I_1 - 4I_2 = 12 \quad \dots(1)$$

$$-8I_2 + 12 - 4I_2 + 4I_1 - 8 = 0 \text{ i.e. } -4I_1 + 12I_2 = 4 \quad \dots(2)$$

Step 4 : Solving equation (1) and (2),

$$I_1 = 0.8 \text{ A}, I_2 = 0.6 \text{ A}$$

$$\therefore I_{4\Omega} = I_1 - I_2 = 0.8 - 0.6 = 0.2 \text{ A} \downarrow$$

Ex. 7.13.2 Using Kirchhoff's law, determine the current flowing through  $6\Omega$  resistance.

SPPU : May-15, Marks 7

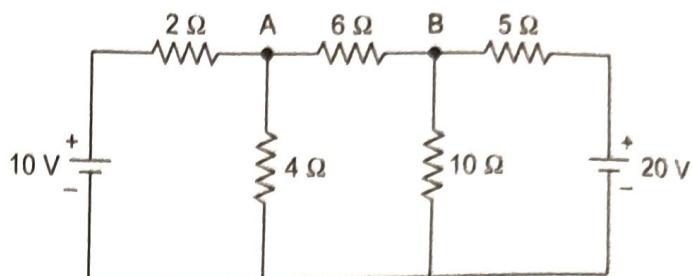


Fig. 7.13.6

Ans. : Step 1 : Shows the loop currents.

Step 2 : Mark the polarities of voltage drops due to the loop currents as shown in in the Fig. 7.13.6 (a).

Step 3 : Applying KVL to the three loops,

$$-2I_1 - 4I_1 + 4I_2 + 10 = 0 \text{ i.e. } 6I_1 - 4I_2 = 10 \quad \dots(1)$$

$$-6I_2 - 10I_2 + 10I_3 - 4I_2 + 4I_1 \text{ i.e. } 4I_1 - 20I_2 + 10I_3 = 0 \quad \dots(2)$$

$$-5I_3 - 20 - 10I_3 + 10I_2 = 0 \text{ i.e. } 10I_2 - 15I_3 = 20 \quad \dots(3)$$

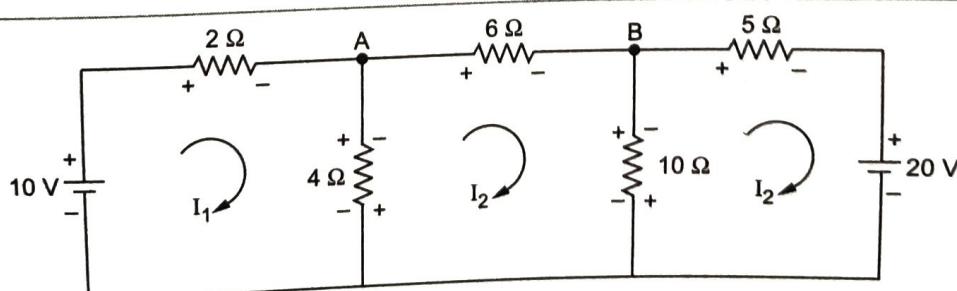
Step 4 : Solving equation (1), (2) and (3),

$$I_1 = 1.25 \text{ A}, I_2 = -0.625 \text{ A}, I_3 = -1.75 \text{ A}$$

$$\therefore I_{6\Omega} = I_2 = -0.625 \text{ A} \rightarrow$$

Negative sign indicates that the direction is opposite to that assumed.

$$\therefore I_{6\Omega} = 0.625 \text{ A} \leftarrow \text{i.e. B to A}$$



**Ex. 7.13.3 Find current flowing through AB using Kirchhoff's laws for the circuit shown in Fig. 7.13.7.**

SPPU : May-17, Marks 7

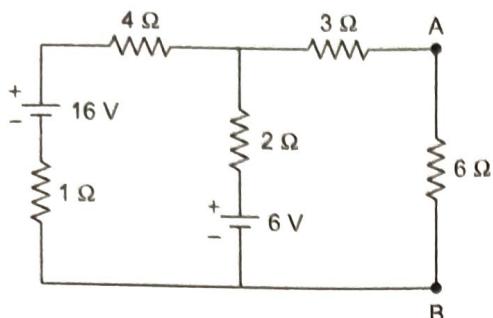


Fig. 7.13.7

**Sol. : Step 1 :** Show the loop currents.

**Step 2 :** Mark the polarities of voltage drops due to the loop currents as shown in in the Fig. 7.13.7 (a).

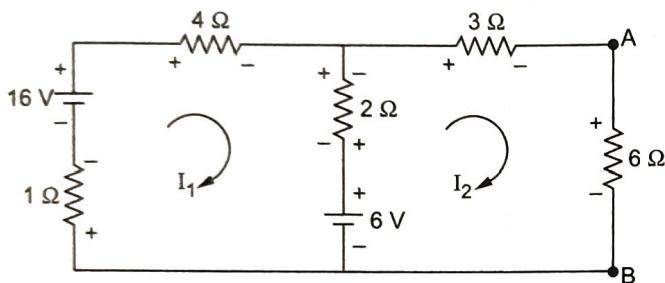


Fig. 7.13.7 (a)

**Step 3 :** Apply KVL to the two loops.

$$-4I_1 - 2I_1 + 2I_2 - 6 - I_1 + 16 = 0 \text{ i.e. } 7I_1 - 2I_2 = 10 \quad \dots(1)$$

$$-3I_2 - 6I_2 + 6 - 2I_2 + 2I_1 = 0 \text{ i.e. } -2I_1 + 11I_2 = 6 \quad \dots(2)$$

**Step 4 :** Solving equation (1) and (2),

$$I_1 = 1.6712 \text{ A}, I_2 = 0.8493 \text{ A}$$

$$\therefore I_{6\Omega} = I_{AB} = I_2 = 0.8493 \text{ A} \downarrow$$

**Ex. 7.13.4 For the circuit shown in Fig. 7.13.8. Find the current flowing through PQ using Kirchhoff's laws.**

SPPU : May-19, Marks 6

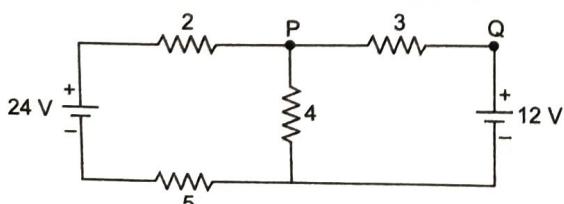


Fig. 7.13.8

**Sol. : Step 1 :** Show the loop currents.

**Step 2 :** Mark the polarities of voltage drops due to the loop currents as shown in in the Fig. 7.13.8 (a).

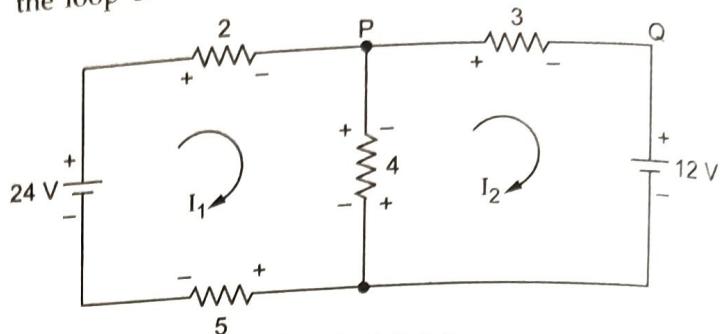


Fig. 7.13.8 (a)

**Step 3 :** Applying KVL to the two loops,

$$-2I_1 - 4I_1 + 4I_2 - 5I_1 + 24 = 0 \text{ i.e. } 11I_1 - 4I_2 = 24$$

... (1)

$$-3I_2 - 12 - 4I_2 + 4I_1 = 0 \text{ i.e. } +4I_1 - 7I_2 = 12 \quad \dots(2)$$

**Step 4 :** Solving equation (1) and (2),

$$I_1 = 1.9672 \text{ A}, I_2 = -0.5901 \text{ A} \rightarrow$$

Negative sign indicates that the direction of  $I_2$  is opposite to that assumed.

$$\therefore I_{PQ} = I_2 = +0.5901 \text{ A i.e. Q to P}$$

**Ex. 7.13.5 Using Mesh/Loop analysis, calculate current flowing through  $R_{BC}$  for Fig. 7.13.9.**

SPPU : Dec.-15, Marks 6

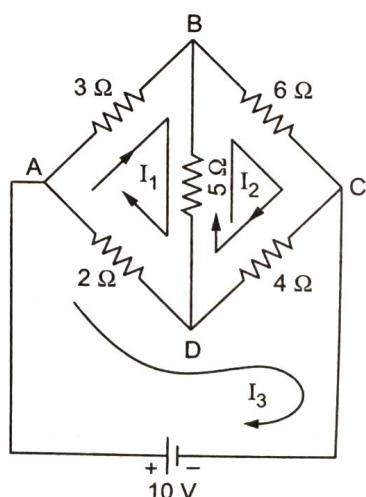


Fig. 7.13.9

The various  
shown in the Fig. 7.13.9 (a).

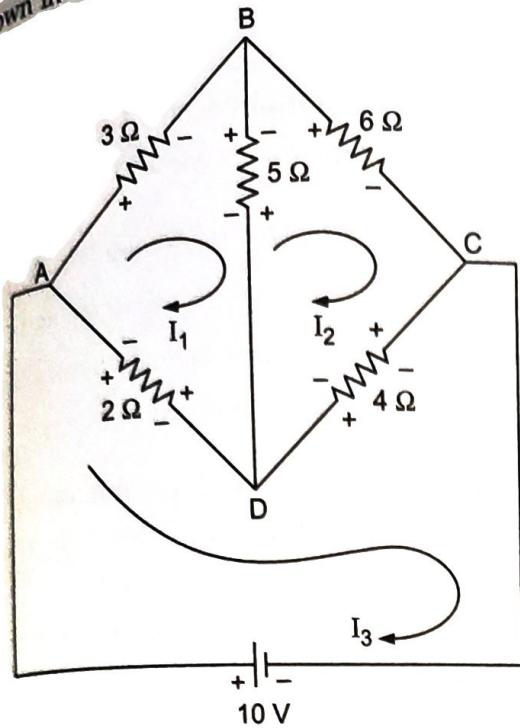


Fig. 7.13.9 (a)

Applying KVL to the three loops,

$$-3I_1 - 5I_1 + 5I_2 - 2I_1 + 2I_3 = 0$$

$$\text{i.e. } -10I_1 + 5I_2 + 2I_3 = 0 \quad \dots(1)$$

$$-6I_2 - 4I_2 + 4I_3 - 5I_2 + 5I_1 = 0$$

$$\text{i.e. } 5I_1 - 15I_2 + 4I_3 = 0 \quad \dots(2)$$

$$-2I_3 + 2I_1 - 4I_3 + 4I_2 + 10 = 0$$

$$\text{i.e. } 2I_1 + 4I_2 - 6I_3 = -10 \quad \dots(3)$$

Solving I<sub>1</sub> = 1.111 A, I<sub>2</sub> = 1.111 A, I<sub>3</sub> = 2.777 A

$I_{BC} = I_2 = 1.111 \text{ A from B to C}$

Ex. 7.13.6 Determine all loop currents using KVL. All resistances are in ohm.

SPPU : Dec.-17, Marks 7

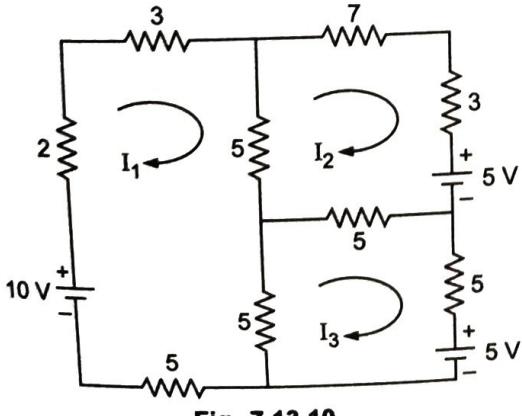


Fig. 7.13.10

Sol. : Show the polarities of voltage drops as shown in the Fig. 7.13.10 (a).

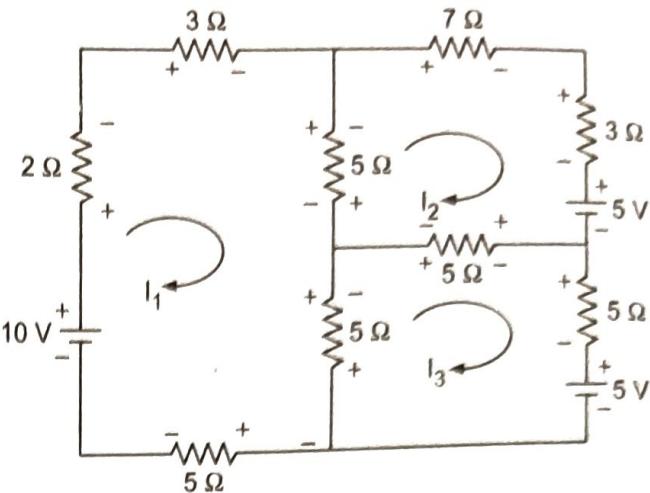


Fig. 7.13.10 (a)

Applying KVL to the three loops,

$$-3I_1 - 5I_1 + 5I_2 - 5I_1 + 5I_3 - 5I_1 + 10 - 2I_1 = 0$$

$$\therefore 2I_1 - 5I_2 - 5I_3 = 10 \quad \dots(1)$$

$$-7I_2 - 3I_2 - 5 - 5I_2 + 5I_3 - 5I_2 + 5I_1 = 0$$

$$\therefore 5I_1 - 20I_2 + 5I_3 = 5 \quad \dots(2)$$

$$-5I_3 + 5I_2 - 5I_3 - 5 - 5I_3 + 5I_1 = 0$$

$$\text{i.e. } 5I_1 + 5I_2 - 15I_3 = 5 \quad \dots(3)$$

Solving, I<sub>1</sub> = 0.3714 A,

$$I_2 = -0.2285 \text{ A},$$

$$I_3 = -0.2857 \text{ A}$$

- Negative sign indicates that the actual directions of I<sub>2</sub> and I<sub>3</sub> are opposite to those assumed in the given circuit.

## 7.14 : Short and Open Circuits

- In the network simplification, short circuit or open circuit existing in the network plays an important role.

### 7.14.1 Short Circuit

- When any two points in a network are joined directly to each other with a thick metallic conducting wire, the two points are said to be short circuited. The resistance of such short circuit is zero.
- The part of the network, which is short circuited is shown in the Fig. 7.14.1. The points A and B are

short circuited. The resistance of the branch AB is  $R_{sc} = 0 \Omega$ .

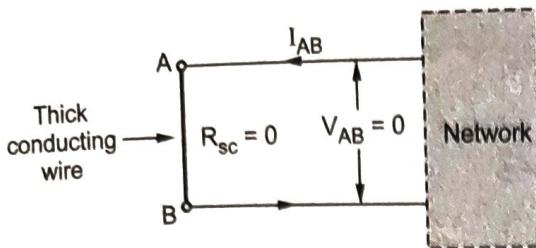


Fig. 7.14.1

- The current  $I_{AB}$  is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0 \text{ V}$$

**Key Point** Thus, voltage across short circuit is always zero though current flows through the short circuited path.

#### 7.14.2 Open Circuit

- When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.
- As there is no direct connection in an open circuit, the resistance of the open circuit is  $\infty$ .
- The part of the network which is open circuited is shown in the Fig. 7.14.2. The points A and B are said to be open circuited. The resistance of the branch AB is  $R_{oc} = \infty \Omega$ .

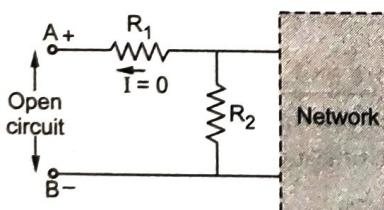


Fig. 7.14.2

There exists a voltage across the points AB called open circuit voltage,  $V_{AB}$  but  $R_{oc} = \infty \Omega$ . According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0 \text{ A}$$

**Key Point** Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.

#### 7.14.3 Redundant Branches and Combinations

- The redundant means excessive and unwanted.

**Key Point** If in a circuit there are branches or combinations of elements which do not carry any current then such branches and combinations are called redundant from circuit point of view.

- The redundant branches and combinations can be removed and these branches do not affect the performance of the circuit.

The two important situations of redundancy which may exist in practical circuits are,

**Situation 1 :** Any branch or combination across which there exists a short circuit, becomes redundant as it does not carry any current.

If in a network, there exists a direct short circuit across a resistance or the combination of resistances then that resistance or the entire combination of resistances becomes **inactive** from the circuit point of view. Such a combination is redundant from circuit point of view.

To understand this, consider the combination of resistances and a short circuit as shown in the Fig. 7.14.3 (a) and (b).

In Fig. 7.14.3 (a), there is short circuit across  $R_3$ . The current always prefers low resistance path hence entire current  $I$  passes through short circuit and hence resistance  $R_3$  becomes redundant from the circuit point of view.

In Fig. 7.14.3 (b), there is short circuit across combination of  $R_3$  and  $R_4$ . The entire current flows through short circuit across  $R_3$  and  $R_4$  and no current can flow through combination of  $R_3$  and  $R_4$ . Thus that combination becomes meaningless from the circuit point of view. Such combinations can be eliminated while analysing the circuit.

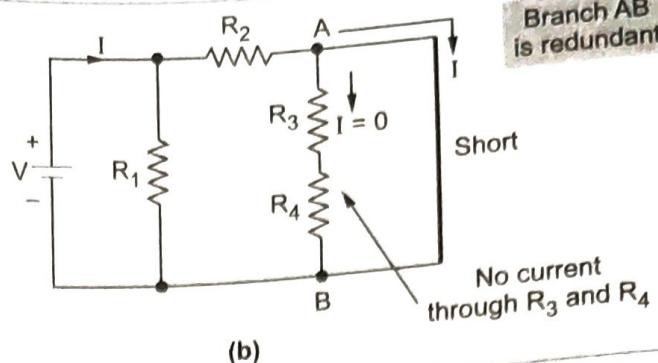
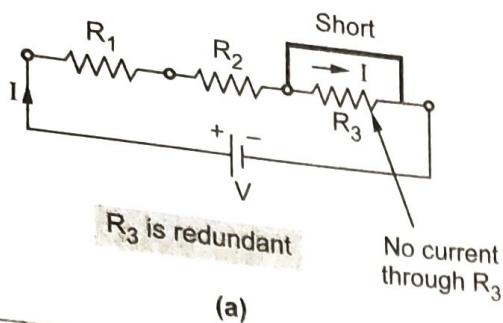
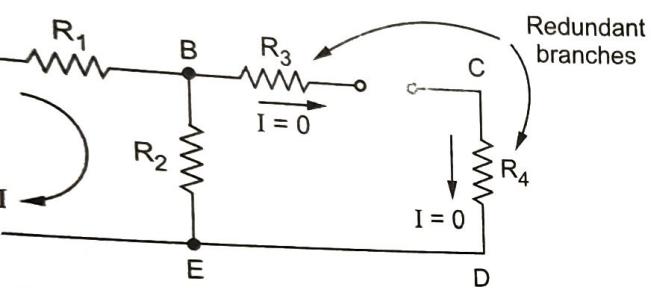


Fig. 7.14.3 Redundant branches

**n 2 :** If there is open circuit in a branch or section, it can not carry any current and is redundant.

7.14.4 as there exists open circuit in branch BC and CD can not carry any current become redundant from circuit point of view.



#### 7.14.4 Redundant branches due to open circuit

#### Superposition Theorem

SPPU : May-05, 06, 07, 09, 10, 12, 15, 18,  
Dec.-04, 05, 06, 10, 13, 14, 17

Theorem is applicable for linear and bilateral networks.

**ment :** In any multisource complex network consisting of linear bilateral elements, the voltage across current through any given element of the network is equal to the algebraic sum of the individual voltages or currents, produced independently across or in that element by each source acting independently, when all other remaining sources are replaced by their respective internal resistances.

**oint** If the internal resistances of the sources are zero then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by an open circuit.

#### 7.15.1 Explanation of Superposition Theorem

- Consider a network, shown in the Fig. 7.15.1, having two voltage sources  $V_1$  and  $V_2$ .

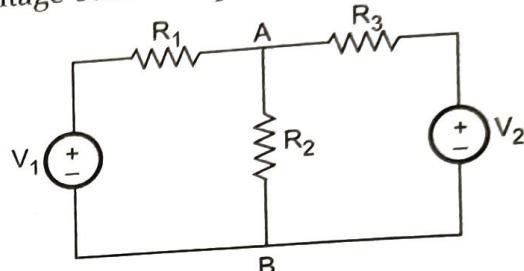


Fig. 7.15.1

- Let us calculate, the current in branch A-B of the network, using superposition theorem.

**Step 1 :** According to Superposition theorem, consider each source independently.

- Let source  $V_1$  volts is acting independently. At this time, other sources must be replaced by internal impedances.

- But as internal impedance of  $V_2$  is not given, the source  $V_2$  must be replaced by short circuit. Hence circuit becomes, as shown in the Fig. 7.15.1 (a).

- Using any of the network reduction techniques discussed earlier, obtain the current through branch A-B i.e.  $I_{AB}$  due to source  $V_1$  alone.

**Step 2 :** Now consider source  $V_2$  volts alone, with  $V_1$  replaced by a short circuit, to obtain the current through branch A-B.

- The corresponding circuit is shown in the Fig. 7.15.1 (b).
- Obtain  $I_{AB}$  due to  $V_2$  alone, by using any of the network reduction techniques discussed earlier.

**Step 3 :** According to the Superposition theorem, the total current through branch A-B is the sum of

the currents through branch A-B produced by each source acting independently.

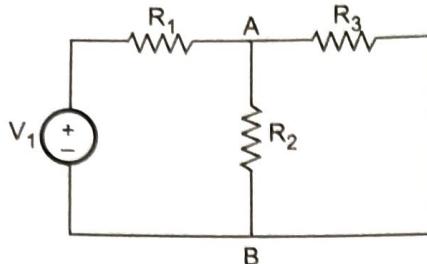


Fig. 7.15.1 (a)

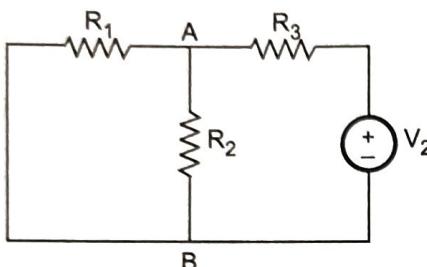


Fig. 7.15.1 (b)

$\therefore$  Total  $I_{AB} = I_{AB}$  due to  $V_1 + I_{AB}$  due to  $V_2$

### 7.15.2 Steps to Apply Superposition Theorem

**Step 1 :** Select a single source acting alone. Short the other voltage sources and open the current sources, if internal resistances are not known. If known, replace them by their internal resistances.

**Step 2 :** Find the current through or the voltage across the required element, due to the source under consideration, using a suitable network simplification technique.

**Step 3 :** Repeat the above two steps for all the sources

**Step 4 :** Add the individual effects produced by individual sources, to obtain the total current in or voltage across the element.

**Ex. 7.15.1** Apply superposition theorem to calculate current flowing in  $3\Omega$  resistance for the network.

SPPU : May-14, Marks 7

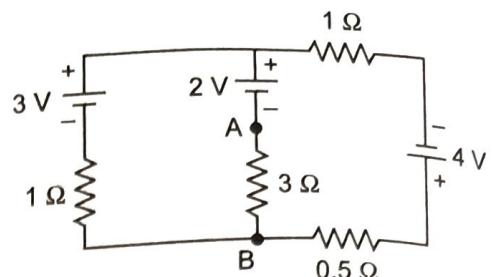


Fig. 7.15.2

**Sol. : Step 1 :** Consider 3 V alone, short other sources.

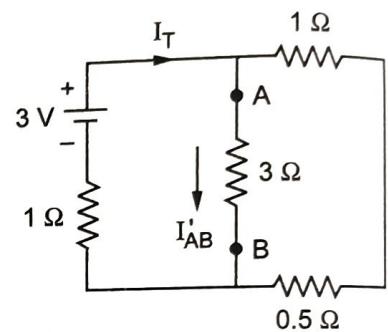


Fig. 7.15.2 (a)

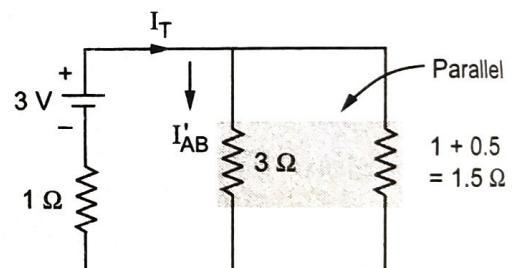


Fig. 7.15.2 (b)

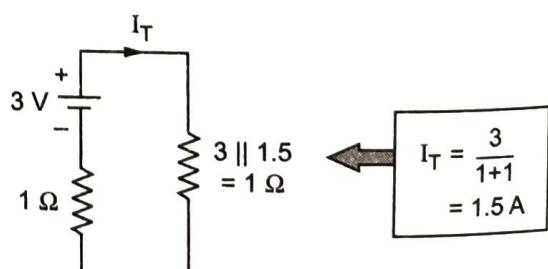


Fig. 7.15.2 (c)

From Fig. 7.15.2 (c),  $I_T = \frac{3}{1+1} = 1.5 \text{ A}$

Using current division rule to the Fig. 7.15.2 (b),

$$I'_{AB} = \frac{I_T \times 1.5}{3+1.5} = \frac{1.5 \times 1.5}{4} = 0.5 \text{ A}$$

Consider 2 V source alone, short other

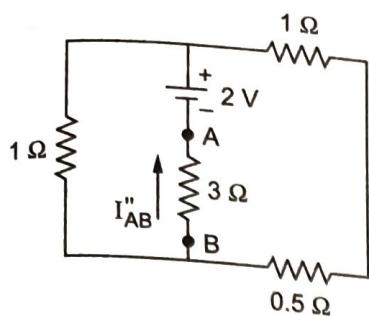


Fig. 7.15.2 (d)

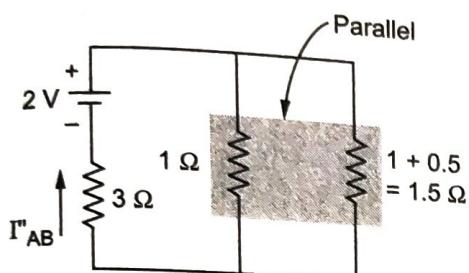


Fig. 7.15.2 (e)

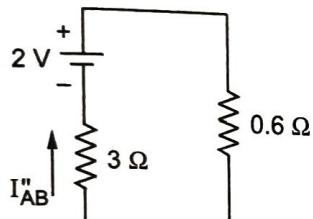


Fig. 7.15.2 (f)

$$I''_{AB} = \frac{2}{3+0.6} = 0.555 \text{ A} \uparrow$$

3 : Consider 4 V source alone, short other

$$I_T = \frac{4}{1+0.5+0.75} = 1.777 \text{ A}$$

Using current division rule to the Fig. 7.15.2 (g),

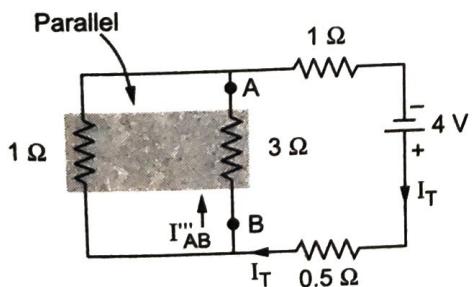


Fig. 7.15.2 (g)

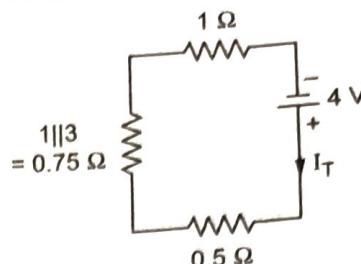


Fig. 7.15.2 (h)

$$I'''_{AB} = \frac{I_T \times 1}{1+3} = 0.444 \text{ A} \uparrow$$

$$\begin{aligned} \text{Step 4 : } I_{AB} &= I'_{{AB}} + I''_{{AB}} + I'''_{{AB}} \\ &= (0.5 \text{ A} \downarrow) + (0.555 \text{ A} \uparrow) + (0.444 \text{ A} \uparrow) \\ &\therefore = (0.5 \text{ A} \downarrow) + (1 \text{ A} \uparrow) = 0.5 \text{ A} \uparrow = I_{3\Omega} \end{aligned}$$

Ex. 7.15.2 Apply superposition theorem, to the circuit shown in Fig. 7.15.3 to calculate current flowing in 10Ω resistance. SPPU : Dec.-14. Marks 6

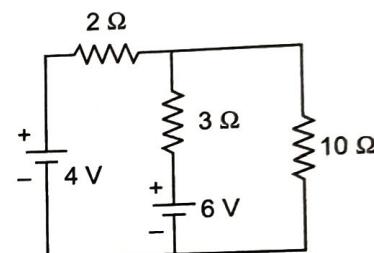


Fig. 7.15.3

Sol. : Step 1 : Consider 4 V source, short 6 V.

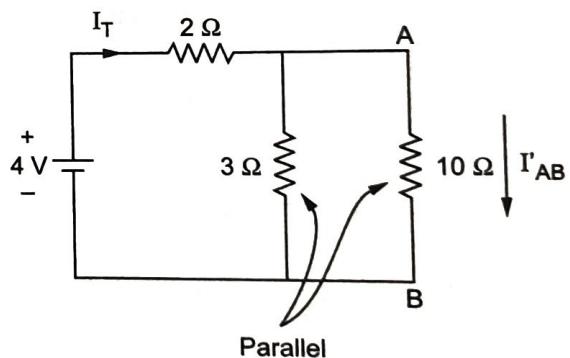


Fig. 7.15.3 (a)

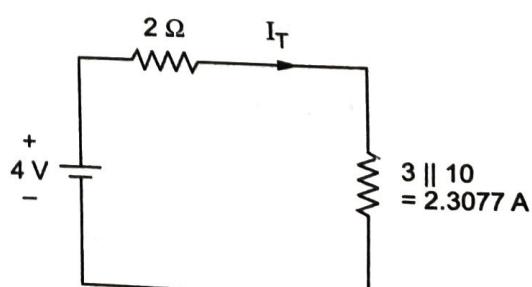


Fig. 7.15.3 (b)

$$\therefore I_T = \frac{4}{2+2.3077} = 0.92857 \text{ A}$$

Using current distribution rule to the Fig. 7.15.3 (a),

$$I'_{AB} = \frac{I_T \times 3}{(3+10)} = \frac{0.92857 \times 3}{13} \\ = 0.21428 \text{ A} \downarrow \quad \dots \text{ Due to } 4 \text{ V}$$

**Step 2 :** Consider 6 V source, short 4 V.

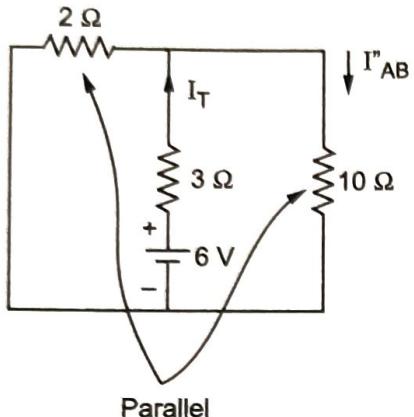


Fig. 7.15.3 (c)

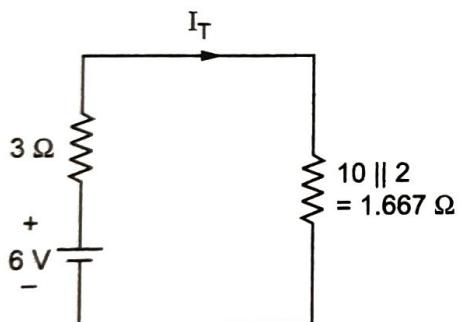


Fig. 7.15.3 (d)

$$I_T = \frac{6}{3+1.667} = 1.28571 \text{ A}$$

Using current distribution rule to the Fig. 7.15.3 (c),

$$I''_{AB} = \frac{I_T \times 2}{(2+10)} = \frac{1.2857 \times 2}{12} \\ = 0.21428 \text{ A} \downarrow \quad \dots \text{ Due to } 6 \text{ V}$$

**Step 3 :**  $I_{10\Omega} = I'_{AB} + I''_{AB}$

$$= 0.21428 \text{ A} \downarrow + 0.21428 \text{ A} \downarrow \\ = 0.42857 \text{ A} \downarrow$$

**Ex. 7.15.3 Calculate the current through resistance using superposition principle.**

SPPU : Dec.-17, May-18, Ma

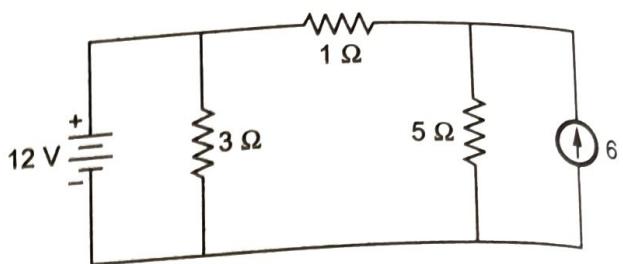


Fig. 7.15.4

**Sol. : Step 1 :** Consider 12 V alone, open 6 A

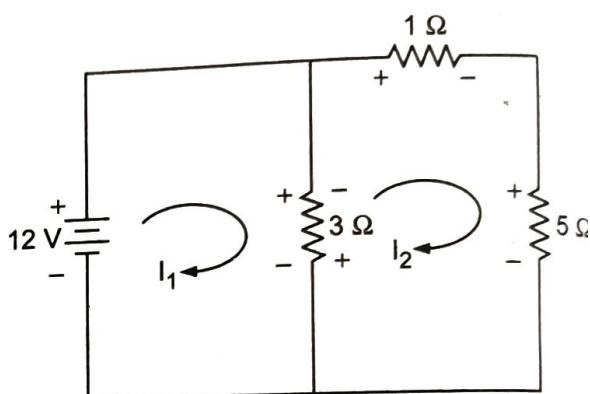


Fig. 7.15.4 (a)

$$-3I_1 + 3I_2 + 12 = 0$$

$$3I_1 - 9I_2 = 0$$

$$\text{Solving, } I_1 = 6 \text{ A}, I_2 = 2 \text{ A}$$

$$\therefore I'_{3\Omega} = I_1 - I_2 = 4 \text{ A} \downarrow$$

$$I'_{1\Omega} = I_2 = 2 \text{ A} \rightarrow, I'_{5\Omega} = I_2 = 2 \text{ A} \downarrow$$

**Step 2 :** Consider 6 A alone, short 12 V [Fig. 7.15.4 (b), (c) on next page]

Using current distribution rule,

$$I''_{1\Omega} = \frac{6 \times 5}{5+1} = 5 \text{ A} \leftarrow,$$

$$I''_{5\Omega} = \frac{6 \times 1}{5+1} = 1 \text{ A} \downarrow$$

$$I''_{3\Omega} = 0 \text{ A}$$

$$\text{Step 3 : } I_{3\Omega} = I'_{3\Omega} + I''_{3\Omega} = 4 \text{ A} \downarrow$$

$$I_{5\Omega} = I'_{5\Omega} + I''_{5\Omega} = 2 \downarrow + 1 \downarrow$$

$$= 3 \text{ A} \downarrow$$

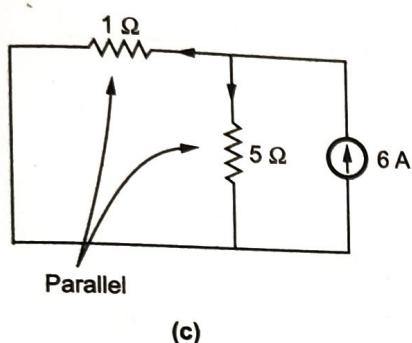
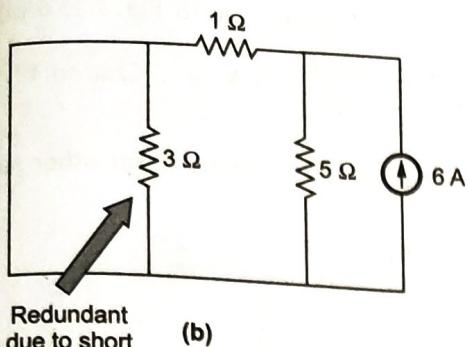


Fig. 7.15.4

$$\begin{aligned} I_{1\Omega} &= I'_{1\Omega} + I''_{1\Omega} = 2 \text{ A} \rightarrow + 5 \text{ A} \leftarrow \\ &= (5 - 2) \leftarrow = 3 \text{ A} \leftarrow \end{aligned}$$

**Ex. 7.15.4** Find current flowing through  $3\Omega$  resistance by Superposition Theorem for the circuit shown in the Fig. 7.15.5. SPPU : May-06.09, Marks 8

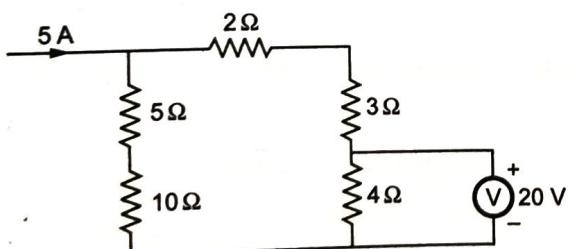


Fig. 7.15.5

**Sol. : Step 1 :** Consider 5 A alone, short 20 V source.

Using current division rule to the Fig. 7.15.5 (b),

$$I'_3 = 5 \times \frac{15}{15+2+3} = 3.75 \text{ A} \downarrow \quad \dots \text{ Due to } 5 \text{ A alone}$$

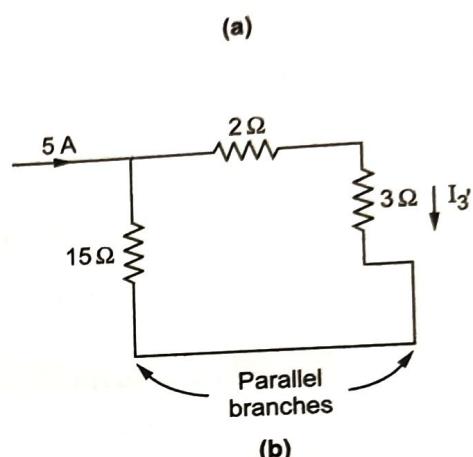
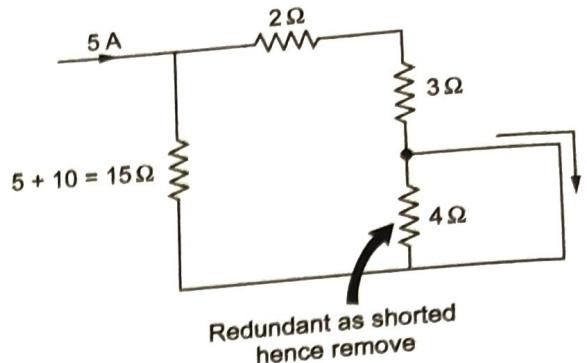


Fig. 7.15.5

**Step 2 :** Consider 20 V alone, open 5 A i.e. treat it zero.

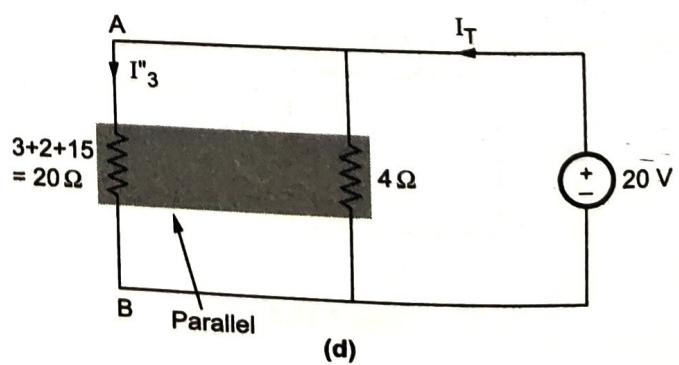
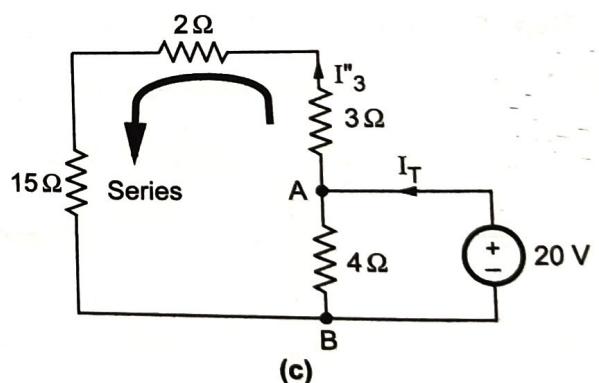


Fig. 7.15.5

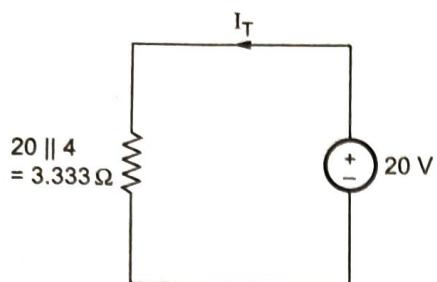


Fig. 7.15.5 (e)

$$\therefore I_T = \frac{20}{3.333} = 6 \text{ A}$$

Using current division rule,

$$I''_3 = 6 \times \frac{4}{20+4} = 1 \text{ A} \uparrow \quad \dots \text{ Due to } 20 \text{ V alone}$$

$$\therefore I_{3\Omega} = 3.75 \text{ A} \downarrow + 1 \text{ A} \uparrow = 2.75 \text{ A} \downarrow$$

**Ex. 7.15.5** Find current in 2-ohm resistance by using Superposition theorem.

SPPU : May-09, 12, Dec.-04, Marks 6

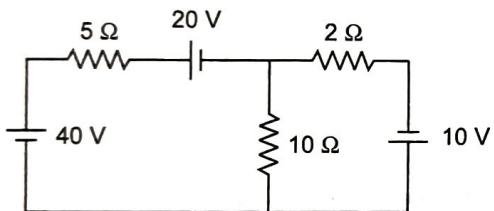
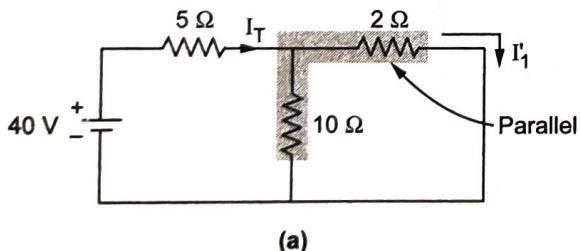
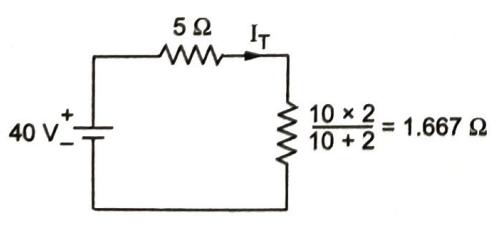


Fig. 7.15.6

**Sol. : Step 1 :** Consider 40 V alone, short other sources.



(a)



(b)

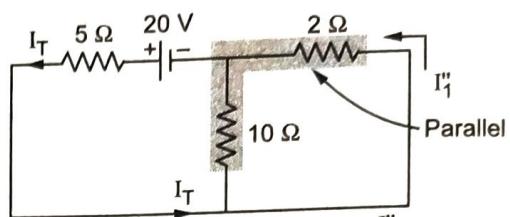
Fig. 7.15.6

$$\therefore I_T = \frac{40}{5+1.667} = 6 \text{ A}$$

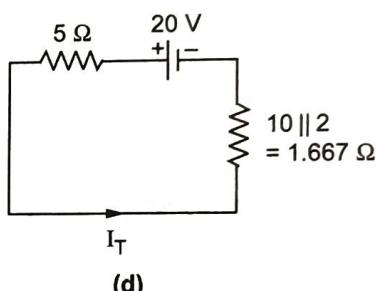
$\therefore$  Using current division rule to Fig. 7.15.6 (a),

$$I'_1 = I_T \times \frac{10}{10+2} = \frac{6 \times 10}{12} = 5 \text{ A} \rightarrow \dots \text{ Due to } 40 \text{ V source}$$

**Step 2 :** Consider 20 V source short other sources.



(c)



(d)

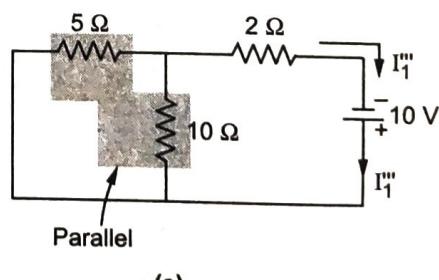
Fig. 7.15.6

$$\therefore I_T = \frac{20}{5+1.667} = 3 \text{ A}$$

$$\therefore I''_1 = I_T \times \frac{10}{2+10} = \frac{3 \times 10}{12} = 2.5 \text{ A} \leftarrow$$

$\dots$  Due to 20 V source

**Step 3 :** Consider 10 V source, short other sources.



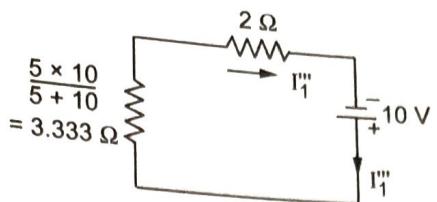
Parallel

(e)

From Fig. 7.15.6 (f),

$$\therefore I'''_1 = \frac{10}{2+3.333}$$

$= 1.875 \text{ A} \rightarrow \dots \text{ Due to } 10 \text{ V source}$



(f)

Fig. 7.15.6

**Step 4 :** The current through  $2\Omega$  is,

$$\begin{aligned} I_{2\Omega} &= I'_1 + I''_1 + I'''_1 \\ &= (5 \text{ A } \rightarrow) + (2.5 \text{ A } \leftarrow) + (1.875 \text{ A } \rightarrow) = 4.375 \text{ A } \rightarrow \end{aligned}$$

### Expected Question

1. State and explain Superposition theorem.

SPPU : Dec.-05, 06, 10, 13, May-05, 07, 09, 10, 15, Marks 6

## 6 : Thevenin's Theorem

SPPU : Dec.-05, 06, 09, 10, 12, 14, 15, 16, 17, May-04, 05, 08, 10, 11, 14, 15, 16, 19

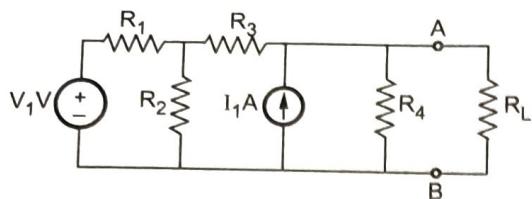
Let us see the statement of the theorem.

**Statement :** Any combination of linear bilateral circuit elements and active sources, regardless of the connection or complexity, connected to a given load  $R_L$ , may be replaced by a simple two terminal network consisting of a single voltage source of  $V_{TH}$  volts and a single resistance  $R_{eq}$  in series with the voltage source, across the two terminals of the load  $R_L$ . The voltage  $V_{TH}$  is the open circuit voltage measured at the two terminals of interest, with load resistance  $R_L$  removed. This voltage is also called **Thevenin's equivalent voltage**. The  $R_{eq}$  is the **equivalent resistance** of the given network as viewed through the terminals where  $R_L$  is connected, but with  $R_L$  removed and all the active sources are replaced by their internal resistances.

**Point** If the internal resistances are not known then independent voltage sources are to be replaced by the short circuit while the independent current sources must be replaced by the open circuit.

### 6.1 Explanation of Thevenin's Theorem

- The concept of Thevenin's equivalent across the terminals of interest can be explained by considering the circuit shown in the Fig. 7.16.1(a).
- The terminals A-B are the terminals of interest across which  $R_L$  is connected. Then Thevenin's equivalent across the load terminals A-B can be obtained as shown in the Fig. 7.16.1 (b).



(a)

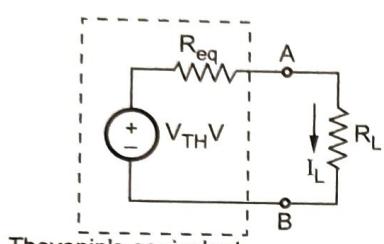


Fig. 7.16.1

- The voltage  $V_{TH}$  is obtained across the terminals A-B with  $R_L$  removed. Hence  $V_{TH}$  is also called open circuit Thevenin's voltage.
- The circuit to be used to calculate  $V_{TH}$  is shown in the Fig. 7.16.2 (a), for the network considered above.
- While  $R_{eq}$  is the equivalent resistance obtained as viewed through the terminals A-B with  $R_L$  removed, voltage sources replaced by short circuit and current sources by open circuit. This is shown in the Fig. 7.16.2 (b).

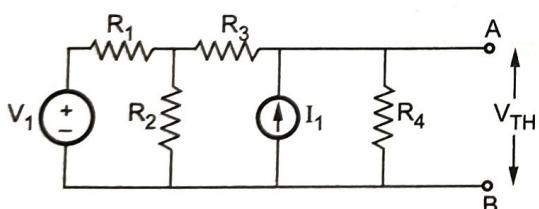
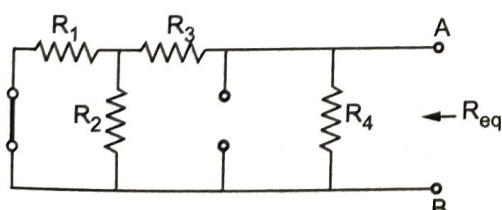
(a) Calculation of  $V_{TH}$ (b) Calculation of  $R_{eq}$ 

Fig. 7.16.2

While obtaining  $V_{TH}$ , any of the network simplification techniques can be used.

- When the circuit is replaced by Thevenin's equivalent across the load resistance, then the load current can be obtained as,

$$I_L = \frac{V_{TH}}{R_L + R_{eq}}$$

- By using this theorem, current through any branch of the circuit can be obtained, treating that branch resistance as the load resistance and obtaining Thevenin's equivalent across the two terminals of that branch.

### 7.16.2 Steps to Apply Thevenin's Theorem

**Step 1 :** Remove the branch resistance through which current is to be calculated.

**Step 2 :** Calculate the voltage across these open circuited terminals, by using any of the network simplification techniques. This is  $V_{TH}$ .

**Step 3 :** Calculate  $R_{eq}$  as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all independent sources by their internal resistances. If the internal resistances are not known then replace independent voltage sources by short circuits and independent current sources by open circuits.

**Step 4 :** Draw the Thevenin's equivalent showing source  $V_{TH}$  with the resistance  $R_{eq}$  in series with it, across the terminals of branch of interest.

**Step 5 :** Reconnect the branch resistance. Let it be  $R_L$ . The required current through the branch is given by,

$$I = \frac{V_{TH}}{R_{eq} + R_L}$$

### 7.16.3 Limitations of Thevenin's Theorem

The limitations of Thevenin's theorem are,

- Not applicable to the circuits consisting of nonlinear elements.
- Not applicable to unilateral networks.

3. There should not be magnetic coupling between the load and circuit to be replaced by Thevenin's theorem.

4. In the load side, there should not be controlled sources, controlled from some other part of the circuit.

**Ex. 7.16.1** Apply Thevenin's theorem to calculate current flowing in  $5\ \Omega$  resistance for the network.

SPPU : Dec.-16, Marks 7

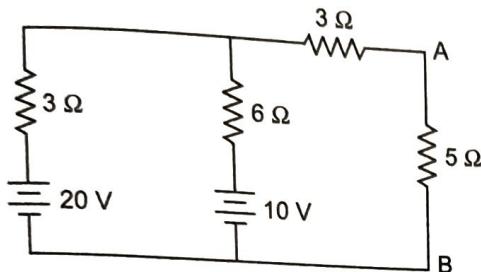


Fig. 7.16.3

**Sol. :** Step 1 : Remove  $5\ \Omega$  resistance.

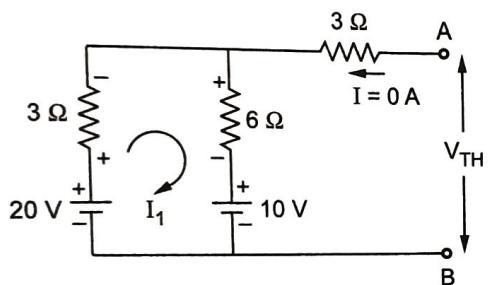


Fig. 7.16.3 (a)

**Step 2 :** Find open circuit voltage,  $V_{AB} = V_{TH}$   
Apply KVL to the loop,  $-6I_1 - 10 + 20 - 3I_1 = 0$

$$\text{i.e. } I_1 = 1.111\ A$$

Tracing path AB as shown in the Fig. 7.16.3 (b).

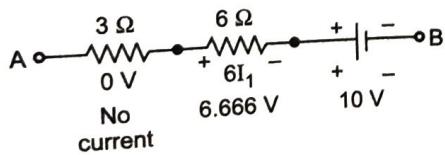
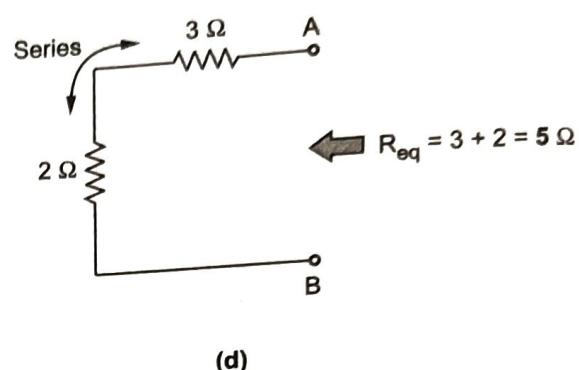
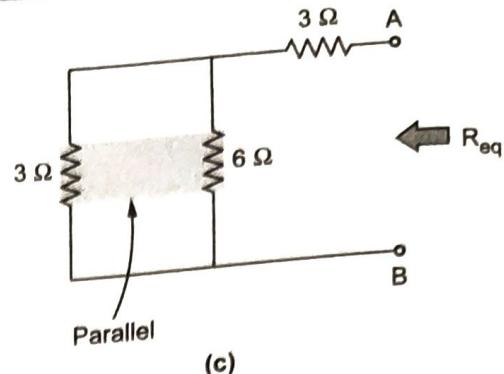


Fig. 7.16.3 (b)

$$V_{AB} = V_{TH} = 6.666 + 10 = 16.666\ V$$

**Step 3 :** Find  $R_{eq}$ , shorting voltage sources as shown in the Fig. 7.16.3 (c)



(d)

Fig. 7.16.3

**Step 4 :** Thevenin's equivalent is shown in the Fig. 7.16.3 (e).

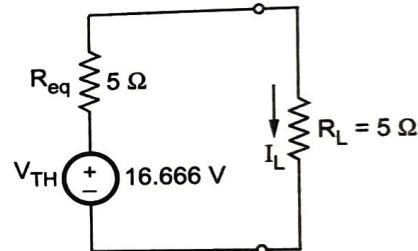


Fig. 7.16.3 (e)

$$\text{Step 5 : } I_L = \frac{V_{TH}}{R_{eq} + R_L} = \frac{10.666}{5+5} = 1.6666\ A \downarrow$$

**Ex. 7.16.2** Use Thevenin's theorem to find the current in the branch BD of the network shown in Fig. 7.16.4.

SPPU : May-04, 05, 10, Dec.-06, Marks 8

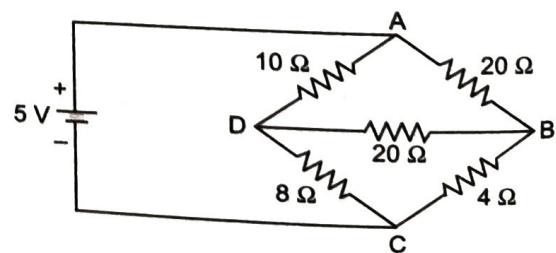


Fig. 7.16.4

**Sol. : Step 1 :** Remove branch BD.

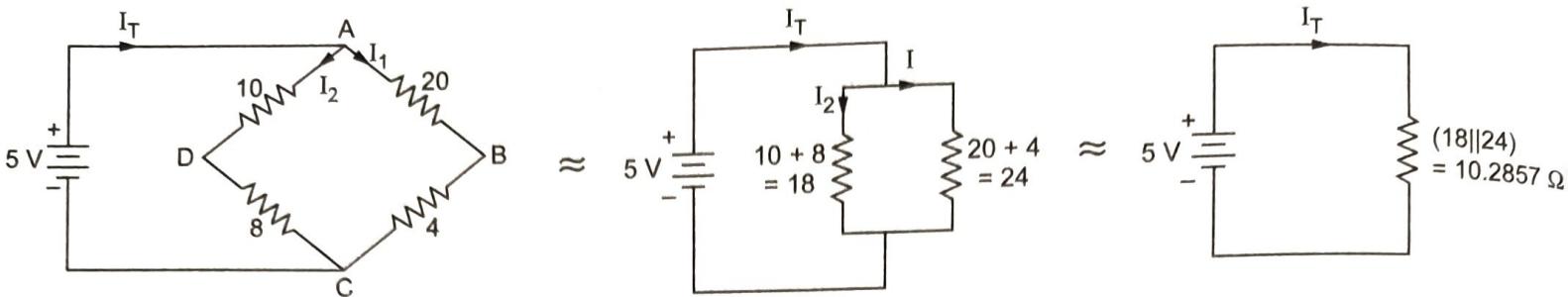


Fig. 7.16.4 (a)

**Step 2 :** Find the open circuit voltage,  $V_{TH} = V_{BD}$

$$I_T = \frac{5}{10.2857} = 0.48611 \text{ A}$$

Using current distribution,

$$I_1 = I_T \times \frac{24}{24+18} = 0.2778 \text{ A} \quad \text{and} \quad I_2 = I_T - I_1 = 0.20833 \text{ A}$$

The various drops due to  $I_1$  and  $I_2$  are as shown in the Fig. 7.16.4 (b). To find  $V_{BD}$ , trace the path BCD as shown as shown in the Fig. 7.16.4 (c).

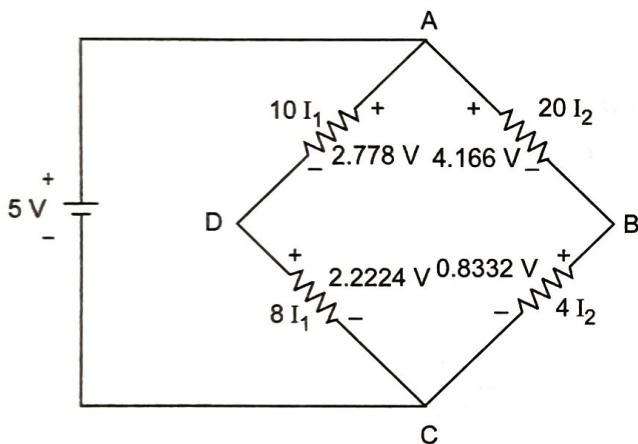


Fig. 7.16.4 (b)

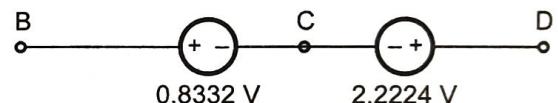


Fig. 7.16.4 (c)

$$\therefore V_{BD} = 2.2224 - 0.8332 = 1.3892 \text{ V with B negative w.r.t. D.}$$

$$\therefore V_{TH} = 1.3892 \text{ V with B negative}$$

**Step 3 :** Find  $R_{eq}$ , removing BD and shorting voltage source.

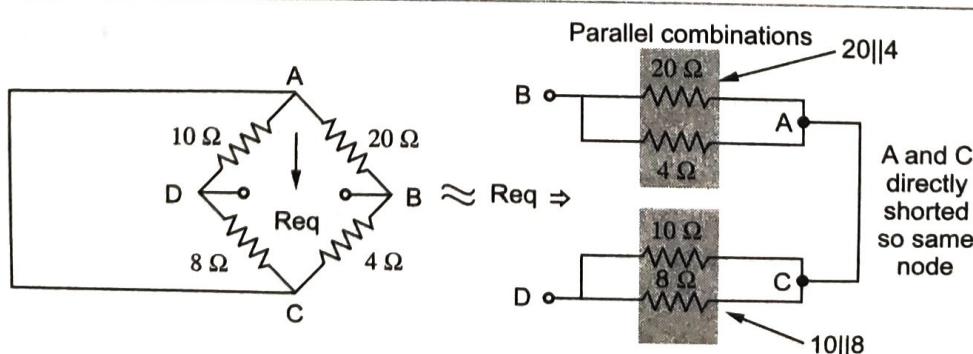


Fig. 7.16.4 (d)

$$\therefore R_{eq} = (20 \parallel 4) + (10 \parallel 8) = 3.333 + 4.444 = 7.7777 \Omega$$

**Step 4 :** Thevenin's equivalent is shown.

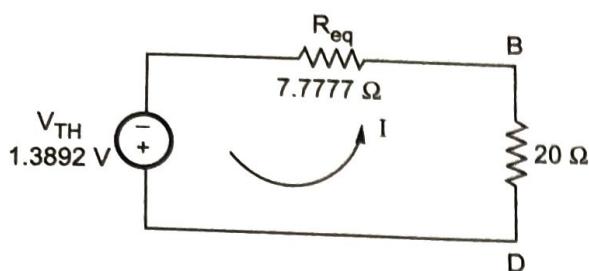


Fig. 7.16.4 (e)

$$\text{Step 5 : } I = \frac{V_{TH}}{R_{eq} + 20} = \frac{1.3892}{27.7777} = 0.05 \text{ A from D to B}$$

**Ex. 7.16.3** Using Thevenin's theorem find the current flowing through 8 ohm resistance for the network shown in Fig. 7.16.5 : SPPU : Dec.-09, Marks 10

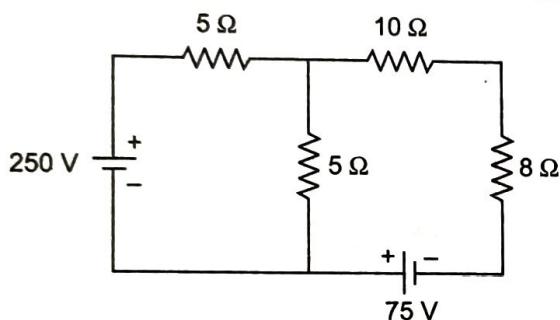
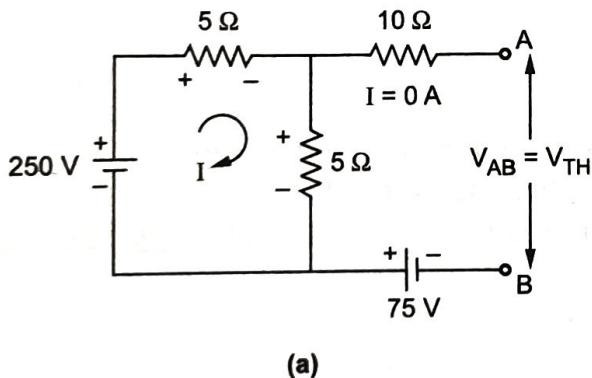
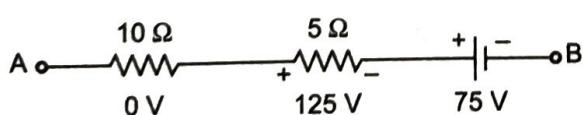


Fig. 7.16.5

**Sol. :** **Step 1 :** Remove the 8 Ω resistance.



(a)



(b)

Fig. 7.16.5

**Step 2 :** Calculate the open circuit voltage  $V_{TH}$ .

$$-5I - 5I + 250 = 0$$

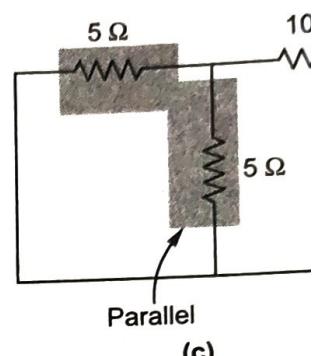
$$\therefore I = 25 \text{ A}$$

$$\text{Drop across } 5 \Omega = 5 \times 25 = 125$$

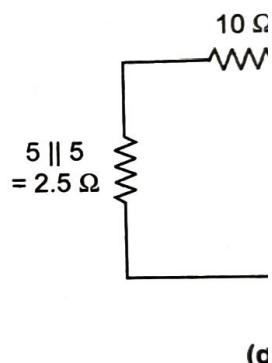
Tracing the path A to B  
Fig. 7.16.5 (b).

$$\therefore V_{TH} = V_{AB} = 125 \\ = 200 \text{ V with}}$$

**Step 3 :** Calculate  $R_{eq}$  by shorting all voltage sources.



(c)



(d)

Fig. 7.1

$$\therefore R_{eq} = 10 + 2.5 =$$

**Step 4 :** Thevenin's equivalent for Fig. 7.16.5 (e).

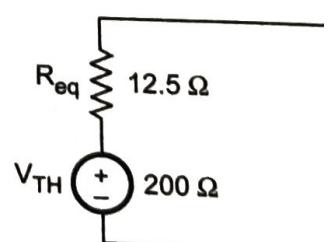


Fig. 7.1

**Sol.** **Step 5 :** The current  $I_L$  through  $8\ \Omega$  is,

$$I_L = \frac{V_{TH}}{R_{eq} + R_L} = \frac{200}{12.5 + 8} = 9.7561\ A$$

**Ex. 7.16.4** Apply Thevenin's theorem to the circuit shown in Fig. 7.16.6 to calculate current in  $1\ \Omega$  resistance :

**SPPU : Dec.-10, Marks 6**

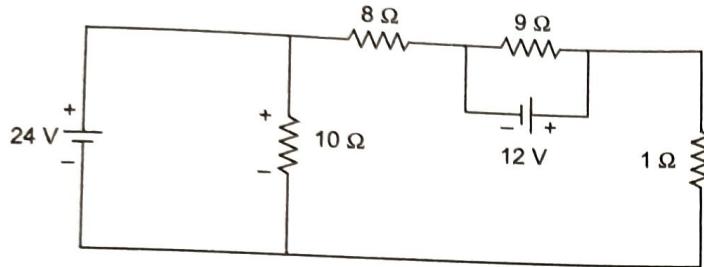


Fig. 7.16.6

**Sol. : Step 1 :** Remove  $1\ \Omega$  resistance.

**Step 2 :** Find the open circuit voltage  $V_{TH}$ .

The two loop currents are shown in the Fig. 7.16.6 (a).

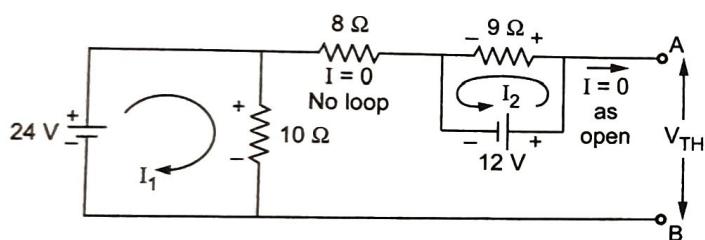


Fig. 7.16.6 (a)

Applying KVL,  $-10 I_1 + 24 = 0$  i.e.  $I_1 = 2.4\ A$

$-9 I_2 + 12 = 0$  i.e.  $I_2 = 1.333\ A$

$\therefore$  Drop across  $9\ \Omega = 9 I_2 = 12\ V$

$\therefore$  Drop across  $10\ \Omega = 10 I_1 = 24\ V$

Tracing the path A to B as shown in the Fig. 7.16.6 (b).

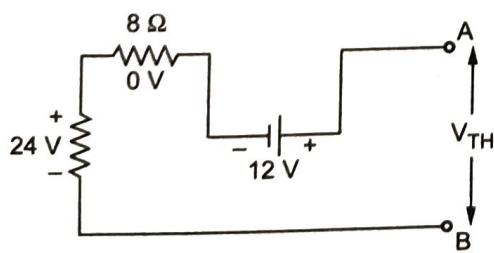


Fig. 7.16.6 (b)

$\therefore V_{TH} = 24 + 12 = 36\ V$  with A positive.

**Step 3 :** To find  $R_{eq}$ , short the voltage sources as shown in the Fig. 7.16.6 (c). As  $9\ \Omega$  and  $10\ \Omega$  are redundant due to shorts across them,

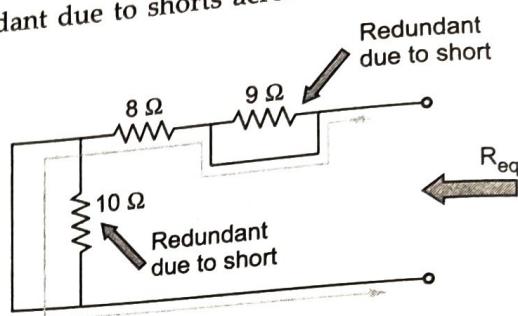


Fig. 7.16.6 (c)

$$R_{eq} = 8\ \Omega$$

**Step 4 :** Thevenin's equivalent is shown in the

Fig. 7.16.6 (d).

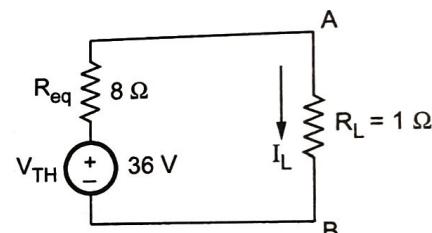


Fig. 7.16.6 (d)

$$\begin{aligned} \text{Step 5 : } I_L &= \frac{V_{TH}}{R_L + R_{eq}} = \frac{36}{8+1} \\ &= 4\ A \downarrow \quad \dots \text{ Current through } 1\ \Omega \end{aligned}$$

**Ex. 7.16.5** Apply Thevenin's theorem, to calculate current flowing in  $10\ \Omega$  resistance, for the circuit shown in Fig. 7.16.7.

**SPPU : Dec.-14, Marks 7**

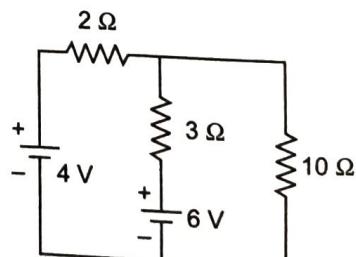


Fig. 7.16.7

**Sol. : Step 1 :** Remove  $10\ \Omega$  resistor

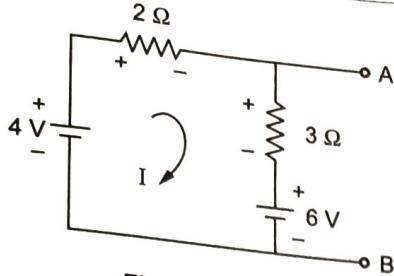


Fig. 7.16.7 (a)

**Step 2 :** Find  $V_{TH} = V_{AB}$  shown in the Fig. 7.16.7 (a).  
Applying KVL,  $-2I - 3I - 6 + 4 = 0$

$$\therefore I = -0.4 \text{ A}$$

Trace the path A - B as shown in the Fig. 7.16.7 (b)

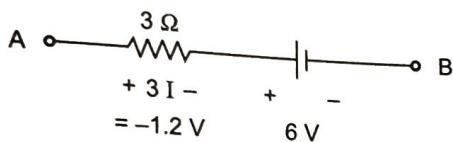


Fig. 7.16.7 (b)

$$\therefore V_{TH} = V_{AB} = -1.2 + 6$$

$$= 4.8 \text{ V with A positive}$$

**Step 3 :** Find  $R_{eq}$ , shorting voltage sources.

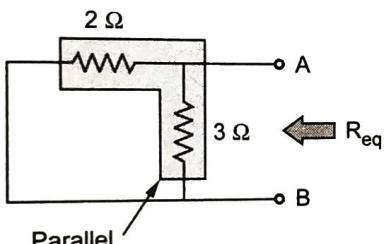


Fig. 7.16.7 (c)

$$\therefore R_{eq} = 3\Omega \parallel 2\Omega = \frac{3 \times 2}{3+2} = 1.2 \Omega$$

**Step 4 :** Thevenin's equivalent is shown in the Fig. 7.16.7 (d).

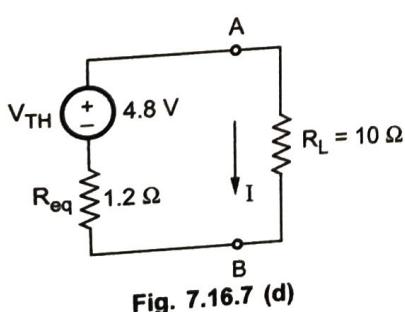


Fig. 7.16.7 (d)

$$\therefore I = \frac{V_{TH}}{R_{eq} + R_L} = \frac{4.8}{12 + 10} \\ = 0.42857 \text{ A } \downarrow = I_{10\Omega}$$

**Ex. 7.16.6** Using Thevenin's theorem, determine the value of current flowing through  $6\Omega$  resistance.  
**SPPU : May-15. Marks 7**

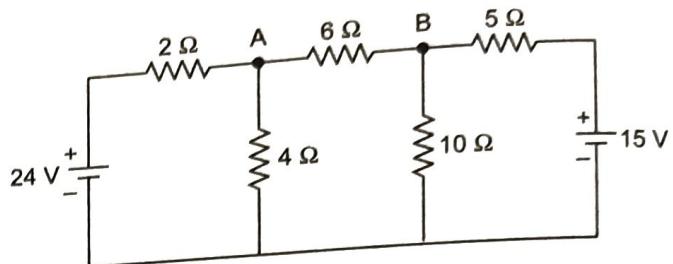


Fig. 7.16.8

**Sol. :** **Step 1 :** Remove branch AB.

**Step 2 :** Find open circuit voltage  $V_{TH} = V_{AB}$   
Applying KVL to the two loops,

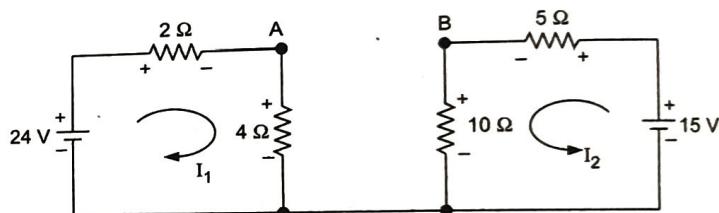


Fig. 7.16.8 (a)

$$-2I_1 - 4I_1 + 24 = 0$$

$$\text{i.e. } I_1 = 4 \text{ A}$$

$$-5I_2 - 10I_2 + 15 = 0$$

$$\text{i.e. } I_2 = 1 \text{ A}$$

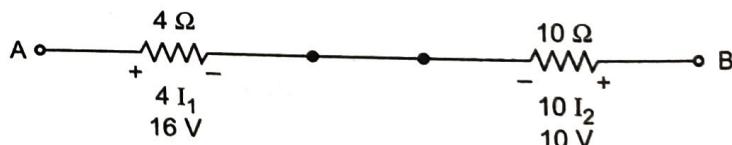


Fig. 7.16.8 (b)

From Fig. 7.16.8 (b),

$$V_{TH} = V_{AB} = 16 - 10 = 6 \text{ V with A +ve}$$

**Step 3 :** Find  $R_{eq} = R_{TH}$  replacing all voltage sources by short circuit.

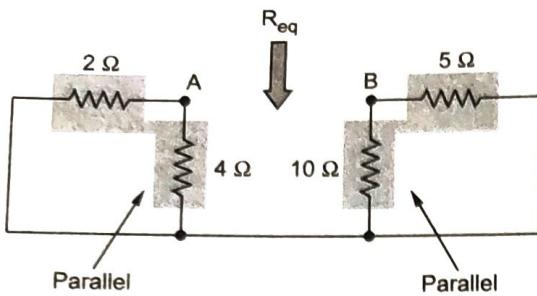


Fig. 7.16.8 (c)

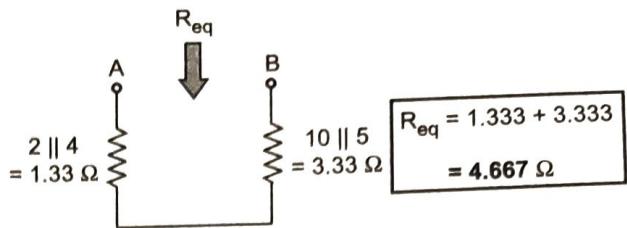


Fig. 7.16.8 (d)

**Step 4 :** The Thevenin's equivalent is shown in the Fig. 7.16.8 (e).

**Step 5 :**  $I = \frac{V_{TH}}{R_L + R_{eq}} = \frac{6}{6 + 4.667} = 0.5625 \text{ A} \downarrow$

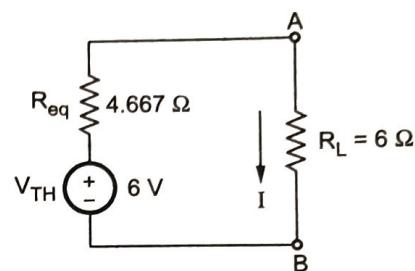


Fig. 7.16.8 (e)

### Expected Question

- State and explain Thevenin's theorem.

SPPU : Dec.-05, 12, 15, 17, May-08, 11, 14, 16, 19, Marks 6

### Formulae at a Glance

- For n resistances in series,  $R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$

- In general if 'n' resistances are connected in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

- Current Division in Parallel Circuit of Two Resistors :**

$$I_1 = \left[ \frac{R_2}{R_1 + R_2} \right] I_T \quad \text{and} \quad I_2 = \left[ \frac{R_1}{R_1 + R_2} \right] I_T$$

- Kirchhoff's Current Law (KCL) :

$$\sum I \text{ at junction point} = 0$$

- Sign convention for KCL :** Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

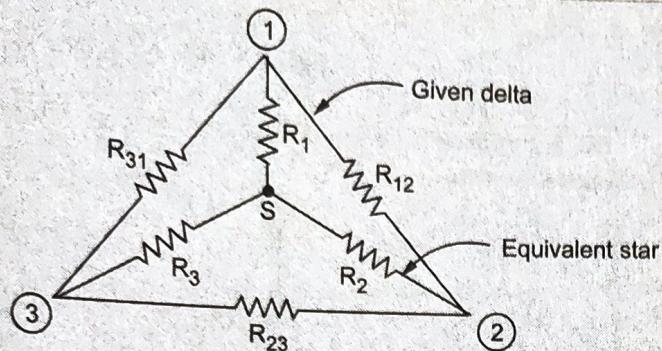
- Kirchhoff's Voltage Law (KVL) : Around a closed path  $\sum V = 0$

- Sign convention for KVL :**

While tracing a closed path for KVL, if we go from - ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise**.

- While tracing a closed path, if we go from +ve marked terminal to - ve marked terminal, that voltage must be taken as negative. This is called potential drop.

Delta-Star

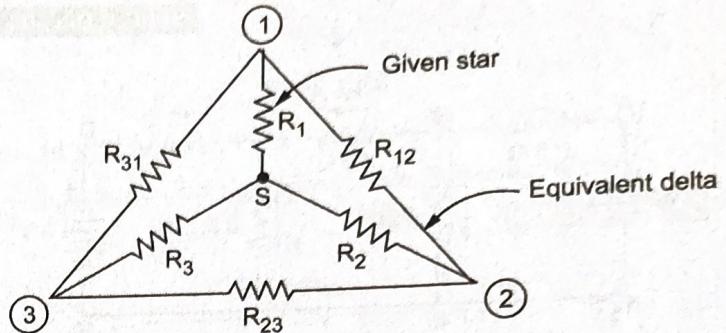


$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

Star-Delta



$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_1R_3}{R_2}$$

### Result for equal resistances in star and delta :

- If all resistances in a Delta connection have same magnitude say  $R$ , then its equivalent Star will contain,

$$\therefore R_1 = R_2 = R_3 = \frac{R \times R}{R + R + R} = \frac{R}{3}$$

- If all three resistances in a Star connection are of same magnitude say  $R$ , then its equivalent Delta contains all resistances of same magnitude of,

$$\therefore R_{12} = R_{31} = R_{23} = R + R + \frac{R \times R}{R} = 3R$$

### Examples for Practice

**Ex. 1 :** Find the value of ' $R$ ' so that 1 A would flow in it, for the network in the Fig. 7.1.

SPPU : May-01

[Ans. : 5.388 Ω]

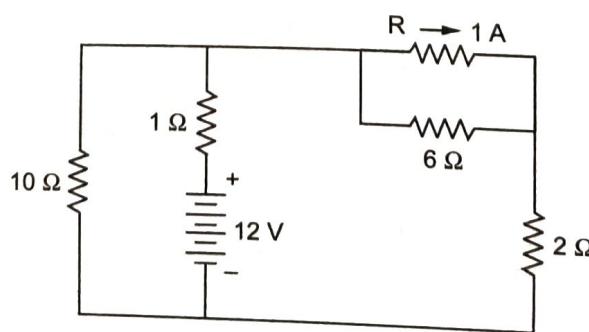


Fig. 7.1

**Ex. 2 :** For the d.c. circuit shown in Fig. 7.2, write the Kirchhoff's law equations in the branch currents  $I_1, I_2$  and  $I_3$  as shown for loops ABGHA, BCFGB and CDEFc.

Solve these equations to find current  $I_2$ .

SPPU : Dec.-03

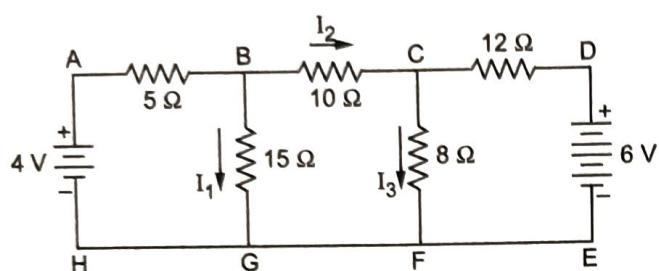


Fig. 7.2

[Ans. : 32.345 mA]

**Ex. 3 :** For Fig. 7.3 shows a d.c. two-source network; the branch currents  $I_1$  and  $I_2$  are as marked in it. Write, using Kirchhoff's laws, two independent simultaneous equations in  $I_1$  and  $I_2$ . Solve these to find  $I_1$ .

SPPU : May-04

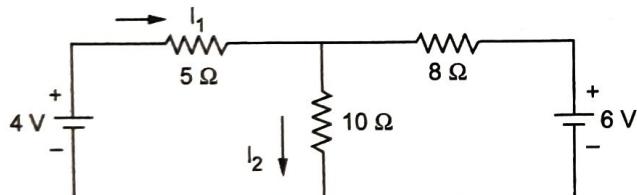


Fig. 7.3

[Ans. : 0.0705 A →]

**Ex. 4 :** Convert the given Delta in the Fig. 7.4 into equivalent Star.

[Ans. : 1.67 Ω, 5 Ω, 2.5 Ω]

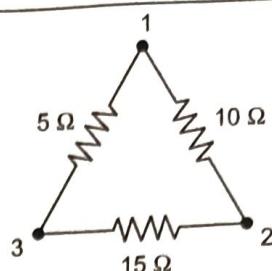


Fig. 7.4

**Ex. 5 :** Convert the given star in the Fig. 7.5 into an equivalent delta.

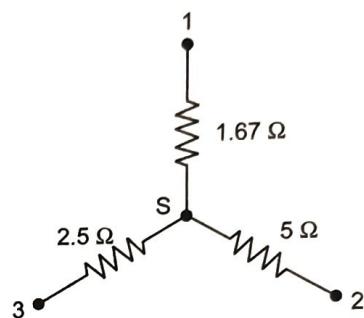


Fig. 7.5

[Ans. : 10 Ω, 15 Ω, 5 Ω]

**Ex. 6 :** Find equivalent resistance between points A-B.

[Ans. : 8.4 Ω]

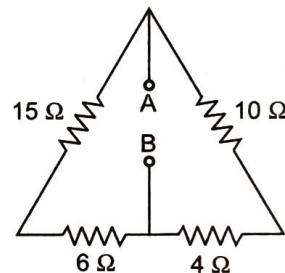


Fig. 7.6

**Ex. 7 :** Calculate the equivalent resistance between the terminals (X) and (Y) for the circuit shown in Fig. 7.7.

SPPU : Dec.-07

[Ans. : 10.31818 Ω]

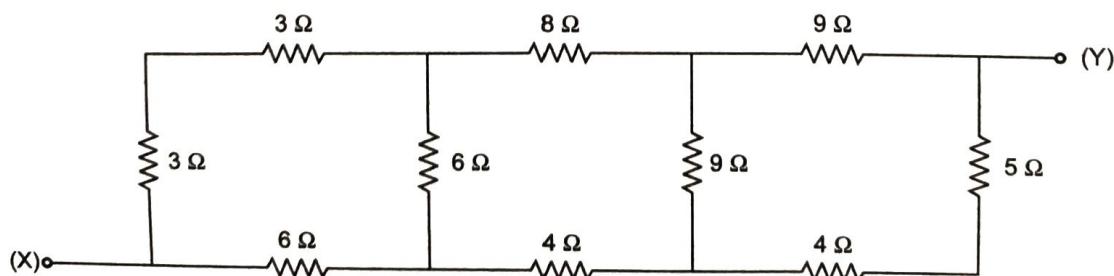


Fig. 7.7

**Ex. 8 :** A circuit is shown in the Fig. 7.8 (a). Using delta-star analysis, reduce it to the circuit as shown in the Fig. 7.8 (b). Find the values of  $R_a$ ,  $R_b$  and  $R_c$  in the equivalent form of the circuit.

[Ans. : 17.1428  $\Omega$ ]

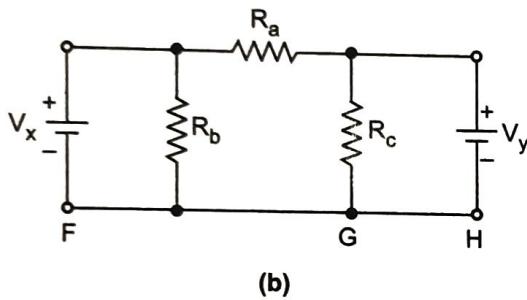
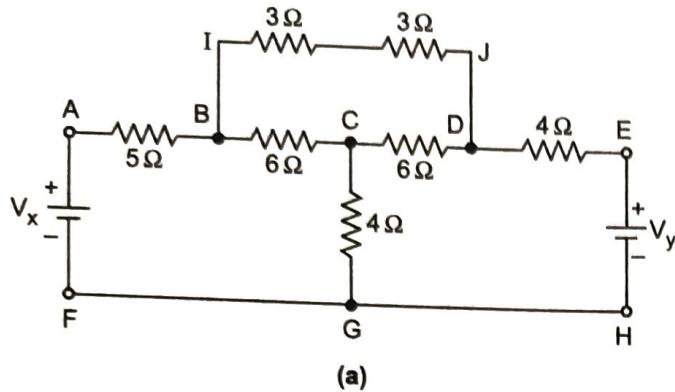
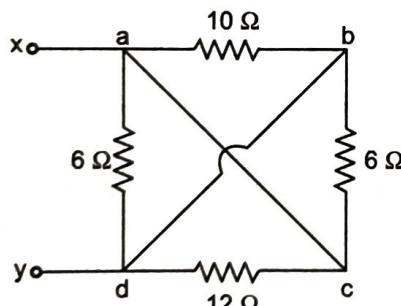


Fig. 7.8

**Ex. 9 :** Find the equivalent resistance across terminals X and Y.



[Ans. : 2.1053  $\Omega$ ]

**Ex. 10 :** Find the equivalent resistance across the terminals A and B shown in the Fig. 7.10. All resistances are in ohms.

[Ans. : 2.417  $\Omega$ ]

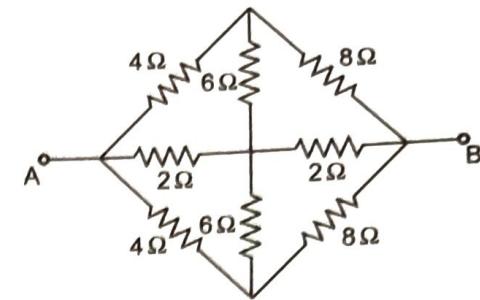


Fig. 7.10

**Ex. 11 :** For a given circuit shown in Fig. 7.11, find out the equivalent resistance between terminals X and Y.

SPPU : May-03

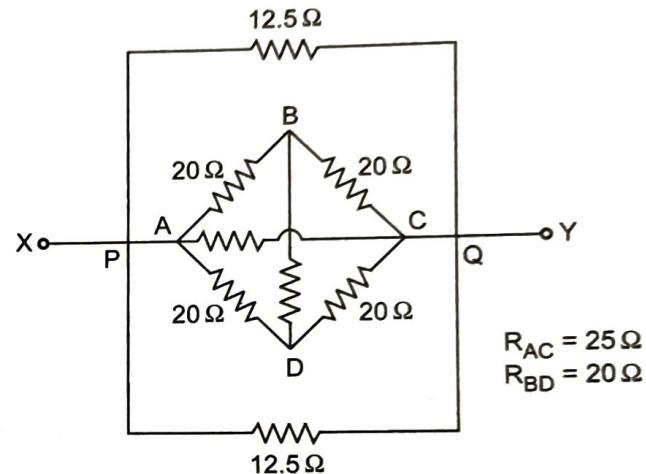


Fig. 7.11

[Ans. : 4  $\Omega$ ]

**Ex. 12 :** Find the effective resistance across terminals M-N of the resistive network shown in Fig. 7.12.

SPPU : Dec-03

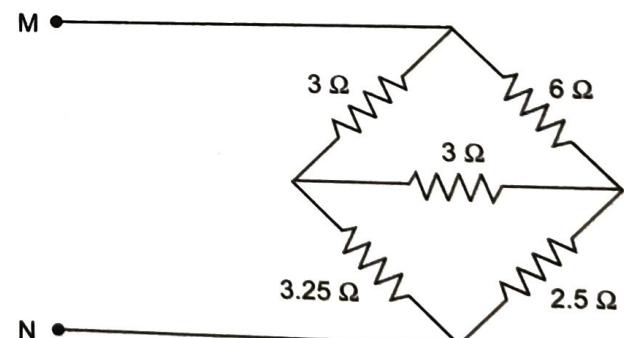


Fig. 7.12

[Ans. : 3.5  $\Omega$ ]

**Ex. 13 :** Using loop analysis calculate the current delivered by the battery shown in Fig. 7.13.

SPPU : May-99

[Ans. : 1.3852 A]

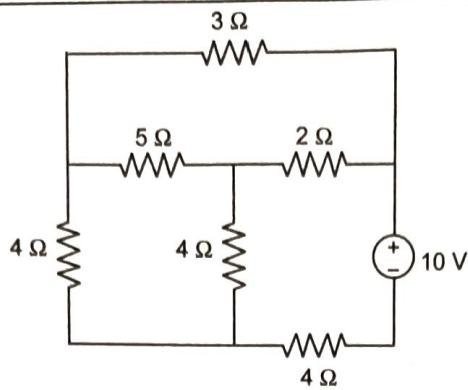


Fig. 7.13

**Ex. 14 :** Determine the current supplied by  $30\text{ V}$  battery in the circuit shown in the Fig. 7.14 by using loop analysis. [SPPU : May-98]

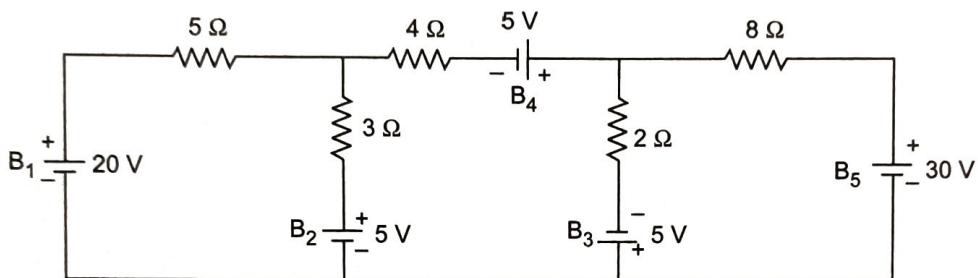


Fig. 7.14

[Ans. :  $3.1358\text{ A}$ ]

**Ex. 15 :** In a bridge circuit the resistance of branch  $AB = 30\ \Omega$ ,  $BC = 41\ \Omega$ ,  $AD = 6\ \Omega$ , while a  $4\text{ V}$  battery is connected between points  $A$  and  $C$ . An ammeter with internal resistance of  $10\ \Omega$ , is connected between points  $B$  and  $D$ . The resistance of branch  $CD$  is ' $R$ ' ohms. If ammeter is showing a reading of  $15\text{ mA}$  ( $\downarrow$ ), determine value of  $R$ . [Ans. :  $5.205\ \Omega$ ]

**Ex. 16 :** Two batteries  $A$  and  $B$  having e.m.f.s of  $209\text{ V}$  and  $211\text{ V}$  having internal resistance  $0.3\ \Omega$  and  $0.8\ \Omega$  respectively are to be charged from a d.c. source of  $225\text{ V}$ . If for that purpose they were connected in parallel and resistance of  $4\ \Omega$  was connected between the supply and batteries to limit charging current, find

- Magnitude and direction of current through each battery.
- Power delivered by source.

[Ans. :  $3.663\text{ A} \uparrow$ ,  $4.482\text{ A} \downarrow$ ,  $0.8183\text{ A} \uparrow$ ,  $824.175\text{ watts}$ ]

**Ex. 17 :** In the circuit shown in Fig. 7.15, find the source current by the loop analysis method.

[SPPU : May-04]

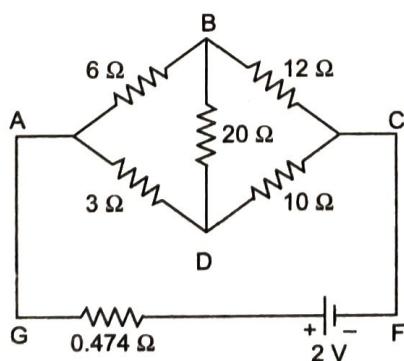


Fig. 7.15

[Ans. :  $0.25\text{ A}$ ]

**Ex. 18 :** Write the loop equations for the circuit shown in the Fig. 7.16 and hence find current flowing through  $4\Omega$  resistance.

SPPU : Dec.-04

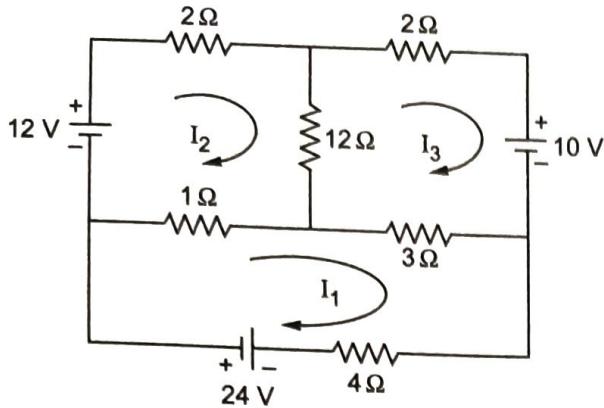


Fig. 7.16

[Ans. :  $4.111\text{ A} \leftrightarrow$ ]

**Ex. 19 :** For the circuit shown in the Fig. 7.17, write the Kirchhoff's law equations for loops BCDB, CEDC and ABDEFA in terms of the branch currents  $I_1$ ,  $I_2$  and  $I_3$  as shown. Find current  $I_1$  by solving these equations.

SPPU : May-05

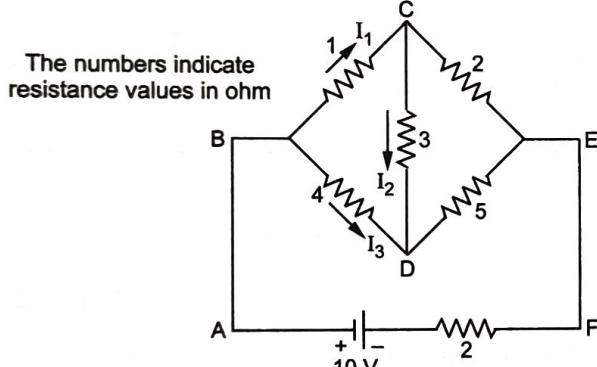


Fig. 7.17

[Ans. :  $1.8272\text{ A}$ ]

**Ex. 20 :** Determine the current supplied by each battery in the circuit shown in Fig. 7.18 by using loop analysis.

SPPU : Dec.-05

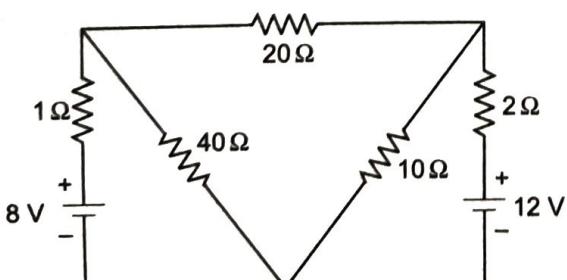


Fig. 7.18

[Ans. :  $0.1005\text{ A}$ ,  $1.0807\text{ A}$ ]

**Ex. 21 :** Calculate current through the  $15\Omega$  resistance using Kirchhoff's law and verify your answer using Superposition theorem as well. The circuit is shown in the Fig. 7.19.

SPPU : Dec.-98

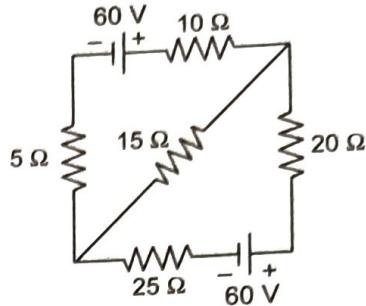


Fig. 7.19

[Ans. :  $2.2857\text{ A}$ ,  $1.7142\text{ A} \downarrow$ ,  $0.5714\text{ A} \downarrow$ ,  $2.2857\text{ A} \downarrow$ ]

**Ex. 22 :** Use Superposition theorem to find the current through the  $20\text{ ohm}$  resistance shown in the Fig. 7.20.

SPPU : May-99

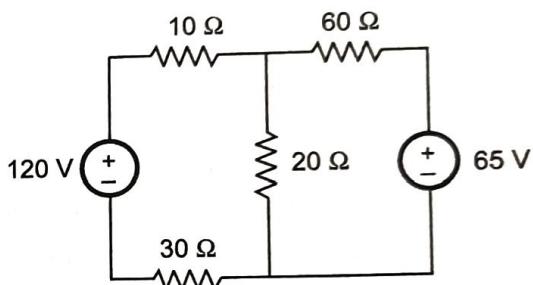


Fig. 7.20

[Ans. :  $1.6363\text{ A}$ ,  $2.2272\text{ A} \downarrow$ ]

**Ex. 23 :** Find the current through branch AB, using Superposition theorem.

SPPU : May-99

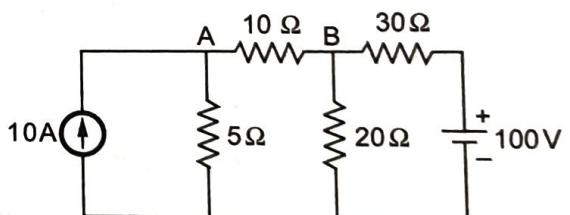


Fig. 7.21

[Ans. :  $1.85185\text{ A} \rightarrow$ ,  $1.48148\text{ A}$ ,  $0.3703\text{ A}$ ]

**Ex. 24 :** For the network shown in Fig. 7.22, find the current in the  $2\text{-ohm}$  resistance by using Superposition theorem.

SPPU : Dec.-05

[Ans. :  $1.1764\text{ A} \downarrow$ ,  $-2.9411\text{ A} \downarrow$ ,  $3.5286\text{ A}$ ]

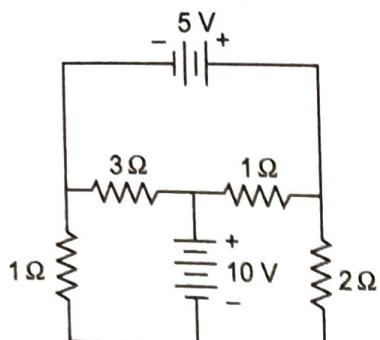


Fig. 7.22

**Ex. 25 :** Find, by Superposition theorem, the current  $I_3$  in the 8 ohm resistance in the circuit shown in Fig. 7.23.

SPPU : Dec.-03

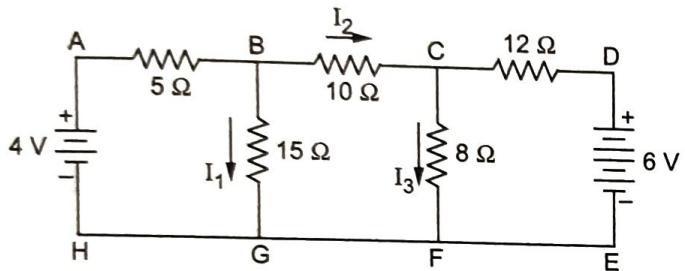


Fig. 7.23

[Ans. : 0.31993 A ↓]

**Ex. 26 :** Using Superposition theorem, calculate the current flowing in 1Ω resistance for the network shown in Fig. 7.24.

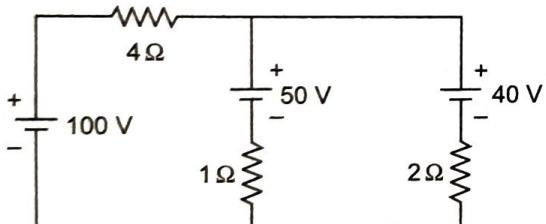


Fig. 7.24

[Ans. : 14.2857 A ↓, 21.4316 A ↑, 11.4285 A ↓, 4.2826 A ↓]

**Ex. 27 :** Find the current  $I_2$ , in Fig. 7.25, by application of Thevenin's theorem.

SPPU : Dec.-03

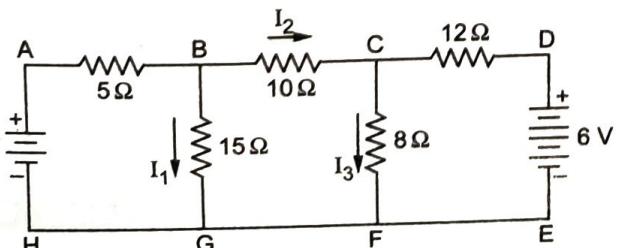


Fig. 7.25

[Ans. : 8.55 W, 32.345 mA]

**Ex. 28 :** For the circuit shown in the Fig. 7.26, find the current through 20Ω using Thevenin's theorem.

SPPU : Nov. - 87

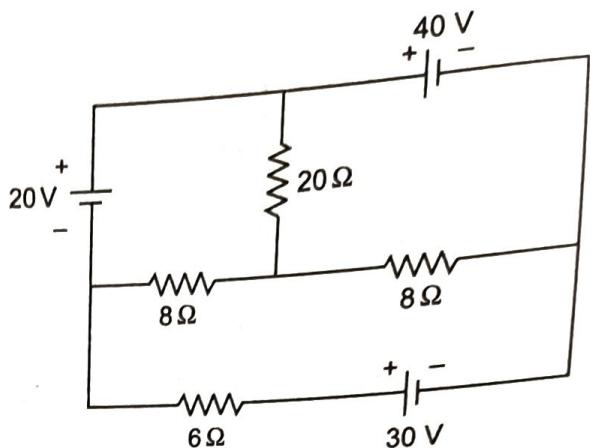


Fig. 7.26

[Ans. : 1.25 A ↓]

**Ex. 29 :** Find the current in 4 Ω resistance by Thevenin's theorem.

SPPU : Nov.-87

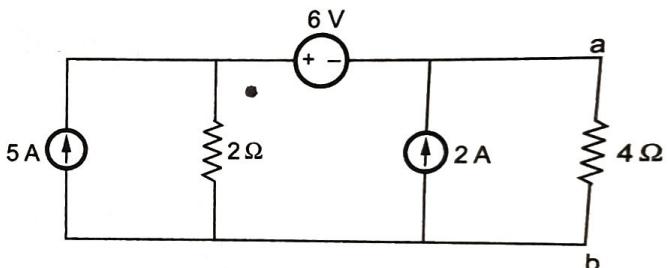


Fig. 7.27

[Ans. : 1.333 A ↓]

**Ex. 30 :** The network has following configuration, Arm AB = 10 Ω, Arm CD = 20 Ω, Arm BC = 30 Ω, Arm DA = 20 Ω, Arm DE = 5 Ω, Arm EC = 10 Ω and a galvanometer of 40 Ω is connected between B and E. Find by Thevenin's theorem, the current in the galvanometer if 2 V source is connected between A and C.

SPPU : May-86

[Ans. : 20.56 mA]

