Facility Location Problem w/ Capacity Constraint

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Problem Description:

A business wants to decide where to open its facilities among different locations to best meet customers demand. The optimum number and location of facilities should minimize the total cost of delivering shipments. This cost includes building cost for a facility and supply costs from the facility to a customer center.

Additionally, **two constraints** must be satisfied, each customer center should be supplied by at least one facility i.e. all customer demand should be met. Secondly the amount of shipment made from a particular facility should be lower or equal than the capacity of that facility.

Important Note:

We did not copy the trnloc1.mod uploaded by you on the IESEG website because it already had the solution to our problem. Instead we understood the objectives and prepared our methodology and ampl model files to provide a different way of solving the same problem.

Methodology

Below we define variables and parameters to mathematically model our facility location problem.

I = set of facilities, indexed by i.
J = set of customer centers, indexed by j.
S[ij] = supply cost from i to j.
F[i] = fixed cost of building a facility.
C[i] = capacity of facility i.

Decision Variables:

X[ij] = binary variable, 1 if a customer center j is supplied by facility i, 0, otherwise.
Y[i] = binary variable, 1 if a facility is built in location, 0, otherwise.

Objective Function: **Minimize**

$$\sum_i \sum_j S_{ij} X_{ij} + f_i Y_i$$

Subject to:

Constraint # 1- each customer center should be served by at least one facility. i.e. all customer demand should be met.

$$\sum_i X_{ij} \geq 1 \quad , \quad \text{for } j = 1..n$$

Constraint # 2 supply to customer centers should be lower than or equal to capacity of facility i

$$\sum_j x_{ij} \leq C_i Y_i$$

Software Implementation:

In ampl this problem was solved in below manner with a model and data file.

FL.mod (AMPL Model File)

```
param N;                    # number of facility / customer centers

param S {1..N, 1..N};       # supply cost from facility i to customer center j

param F {1..N};             #fixed cost of building facility i

param C {1..N};             #capacity for facility i

var x {1..N, 1..N} binary;          #binary variable, 1 is customer center j is supplied by facility i, 0, otherwise

var y {1..N} binary;                #binary variable, 1 if facility is built, 0, otherwise.

minimize Total_Cost: sum {i in 1..N, j in 1..N} S[i,j]*x[i,j] + sum {i in 1..N} F[i]*y[i];

subject to Demand {j in 1..N} :

sum {i in 1..N} x[i,j] = 1;

subject to Capacity {i in 1..N} :

sum {j in 1..N} x[i,j] <= C[i]*y[i];
```

FL.dat (AMPL data file):

We randomly generated data to consider combination of 10 facilities supplying goods to 10 customer centers.

```
param N := 10;

param S: # supply cost from facility i to customer j
      1    2    3    4    5    6    7    8    9    10:=
  1   11   9    4    7    14   15   12   9    12   10
  2   10   8    9    3    10   8    9    12   8    9
  3   13   9    10   6    7    6    10   11   7    10
```

```
4	8	11	5	9	14	8	6	9	15	12
5	10	14	10	8	11	8	7	6	10	8
6	7	8	10	5	14	10	8	4	5	7
7	10	5	12	15	6	8	5	10	9	9
8	9	8	6	12	5	10	11	14	20	16
9	8	6	4	10	8	12	15	6	8	9
10	10	8	12	15	10	14	13	5	7	17;

param F := # fixed cost of building a facility i
	1	55
	2	50
	3	20
	4	40
	5	20
	6	40
	7	38
	8	45
	9	65
	10	55;

Param C:= # capacity of facility I, measured in # of customer centers facility can supply to
	1	3
	2	3
	3	2
	4	5
	5	2
	6	3
	7	2
	8	3
	9	4
	10	3;
```

<u>Results in AMPL & Interpretation:</u>

Our business objective was to decide which facilities to open out of total 10 possible facilities to meet demand of 10 customer centers. Our constraints included meeting all customer demand (i.e. each customer center should be supplied by at least one facility) as well as supply from a facility should be lower / equal to capacity of the facility.

Below screenshot shows results obtained in AMPL.

```
ampl: reset;
ampl: model FL.mod;
ampl: data FL.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.8.0.0: optimal integer solution; objective 163
56 MIP simplex iterations
0 branch-and-bound nodes
ampl: display y;
y [*] :=
 1  0
 2  0
 3  0
 4  1
 5  1
 6  0
 7  0
 8  1
 9  0
10  0
;

ampl: display x;
x [*,*]
:    1   2   3   4   5   6   7   8   9  10    :=
1    0   0   0   1   0   0   0   0   0   0
2    0   0   0   1   0   0   0   0   0   0
3    0   0   0   1   0   0   0   0   0   0
4    0   0   0   1   0   0   0   0   0   0
5    0   0   0   0   0   0   0   1   0   0
6    0   0   0   1   0   0   0   0   0   0
7    0   0   0   0   1   0   0   0   0   0
8    0   0   0   0   1   0   0   0   0   0
9    0   0   0   0   0   0   0   1   0   0
10   0   0   0   0   0   0   0   1   0   0
;
```

Cplex solver was used and it ran 56 iterations.

Variable y shows us that only facility 4, 5 and 8 should be constructed. These two facilities minimize the cost function. Meet all demand and fulfil the supply constraint of each facility.

Variable x shows us that all customer demand centers are supplied from facility 4, 5 & 8. No shipment is made from other facilities (columns) to customer centers (rows) because they are not built.