50. A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

Short-sleeved

	Pattern		
Size	**Pl**	**Pr**	**St**
S	.04	.02	.05
M	.08	.07	.12
L	.03	.07	.08

Long-sleeved

	Pattern		
Size	Pl	Pr	St
S	.03	.02	.03
M	.10	.05	.07
L	.04	.02	.08

a. What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?

b. What is the probability that the next shirt sold is a medium print shirt?

c. What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?

d. What is the probability that the size of the next shirt sold is medium? That the pattern of the next shirt sold is a print?

e. Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium?

f. Given that the shirt just sold was a medium plaid, what is the probability that it was short-sleeved? Long-sleeved?

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(a) P(the next shirt sold is a medium, long-sleeved, Print shirt) = 0.05

(b) P(the next shirt sold is a medium print shirt)

$= 0.07 + 0.05 = 0.12$

(c) P (next shirt sold is short-sleeved shirt)

$= 0.04 + 0.02 + 0.05 + 0.08 + 0.07 + 0.12 + 0.03 + 0.07 + 0.08$

$= 0.56$

P(next shirt sold is long-sleeved shirt)

$= 0.44$

d) P(size of next shirt sold is medium)

$= (0.08 + 0.07 + 0.12) + (0.10 + 0.05 + 0.07) = 0.49$

e) P(next shirt sold is print) = 0.25

e) B → event that shirt just sold was a short-sleeved plaid.

$P(B) = 0.04 + 0.08 + 0.03 = 0.15$

Now; A → Its size is medium

P(A|B) ? ⇒ $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.15} = \frac{8}{15} = 0.533$

(f) B → Shirt just sold was a medium plaid

$P(B) = 0.08 + 0.10 = 0.18$

A → It was short-sleeved.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.18} = \frac{8}{18} = \frac{4}{9} = 0.44$$

Next;

$$P(A|B) = \frac{0.10}{0.18} = \frac{10}{18} = \frac{5}{9} = 0.55$$

Q: If a publisher of non-technical books takes great pains to assure that its books are free from typographical errors, so that the probability of any given page containing such errors is 0.005 and errors are independent from page to page. What is the prob. that one of its 400 page novels will contains ① exactly 1 page with error ② Atmost 3 page errors?

Soln

$n = 400$

X: no of pages containing errors.

$p = 0.005$

Note ÷ In statistical theory, if $n > 30$ then we consider it as large trial or sample.

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Here, $n \to$ very large (400)

$p \to$ very small (0.005)

$n.p = 400 \times 0.005 = 2 \quad (=\lambda)$

① $$p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \; ; \; x = 0, 1, 2 \ldots$$

In our question

$$p_X(x) = \frac{e^{-2}\,2^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$\Rightarrow p_X(1) = \frac{e^{-2}\,2^1}{1!} = 0.13$$

② $$p(X \leq 3) = p_X(0) + p_X(1) + p_X(2) + p_X(3)$$

$$= e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right]$$

$$= 0.85$$

Q: There are 50 telephone lines in an exchange. The probability of them being busy is 0.10, What is the prob. that all lines are busy?

Sol^n: $n = 50$; $p = 0.1$; X = No of lines which are busy.

$\lambda = 50 \times 0.1 = 5$

$$P(X=50) = e^{-5}\left(\frac{5^{50}}{50!}\right) = 1.97 \times 10^{-32}$$ Ans

Q: If the prob. that an individual suffers a bad reaction from an injection of a given serum, is 0.001. Determine the prob. that out of 2000 individuals, exactly 3 individuals suffers bad reaction?

Solⁿ: X : No of individuals suffers the bad reaction

$n = 2000, \ p = 0.001$

$\lambda = n.p = 2000 \times 0.001 = 2$

$$p_X(x) = \frac{e^{-2} 2^x}{x!} \ ; \ x = 0, 1, 2 \text{---}$$

$$p_X(3) = \frac{e^{-2} 2^3}{3!} = 0.18$$

Application of Poisson distribution :-

Poisson Process :- Occurance of event over time interval is called poisson process. This is one of the most widely used counting process. It is usually used in the scenarios where we are counting the occurance of certain events that appears to happen at a certain rate(α) but completely at random.

Examples :-

① No of earthquake in a year

② No of car accident in a year

③ No of ~~printing mistake~~ breakdown in electronic computer.

The parameter α is specified as rate of the process.

The no of events occurred during a time interval of length t is a poission r.v with parameter $\lambda = \alpha t$

Thus ;

$$P_x(t) = \frac{e^{-(\alpha t)} \cdot (\alpha t)^x}{x!}.$$

So the expected no of events during this time interval is αt.

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Q.X(17) The number of request for assistance recived by a towing service, is a poission distⁿ with rate $\alpha = 4/hrs$.

① Compute prob that exactly 10 ~~recived~~ request are recived during a particular 2 hrs period?

② How many calls would you expect during this time interval?

Soln/

Here $\alpha = 4/h$, $t = 2$ hrs

$\lambda = 4 \times 2 = 8$

$$P_X(10) = \frac{e^{-8}\, 8^{10}}{10!} = 0.099$$

$E(X) = 2 \times 4 = 8$, $V(X) = 2 \times 4 = 8$