Tutorial - 6

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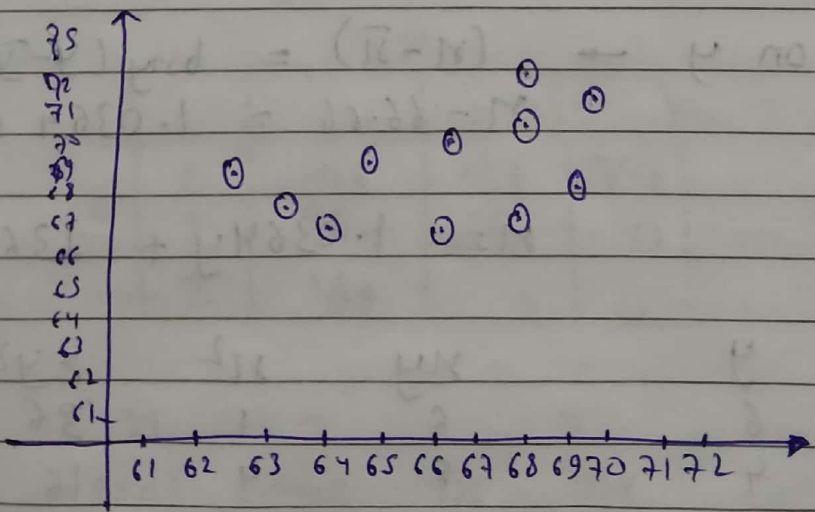
Q-1

x	y	$dx = (x - x_A)$	$dy = y - y_A$	dx^2	dy^2	$dxdy$
65	68	0	3	0	9	0
63	66	-2	1	4	1	-2
67	68	+2	3	4	9	6
64	65	-1	0	1	0	0
68	69	3	4	9	16	12
62	66	-3	1	9	1	-3
70	68	5	3	25	9	15
66	65	1	0	1	0	0
68	71	3	6	9	36	18
67	67	2	2	4	4	4
69	68	4	3	16	9	12
71	70	6	5	36	25	30
		$\Sigma dx = 20$	$\Sigma dy = 31$	$\Sigma dx^2 = 118$	$\Sigma dy^2 = 119$	$\Sigma dxdy = 92$



$$b_{xy} = \frac{\Sigma dxdy - \frac{1}{n}\Sigma dx \cdot \Sigma dy}{\Sigma dy^2 - \frac{1}{n}(\Sigma dy)^2}$$

$$= \frac{92 - \frac{1}{12} 20.31}{11.9 - \frac{(31)^2}{12}} = 1.0364$$

$$b_{yx} = \frac{92 - \frac{1}{12} 20.31}{11.8 - \frac{(20)^2}{12}}$$

$$= \frac{0.4763}{\sqrt{b_{xy} - b_{yx}}}$$

$$= 0.6979$$

(i) y on $x \rightarrow (y - 67.66) = b_{yx}(x - \bar{x})$

$\therefore y = 67.66 + 0.4763 - 31.75$

$= 0.4763x + 35.909$

x on $y \rightarrow (x - \bar{x}) = b_{xy}(y - \bar{y})$

$x - 66.66 = 1.0364 (y - \bar{y})$

$x = 1.0364y + 136.782$

Sol-2

x	y	xy	x^2	Σy^2
1	6	6	1	36
2	4	8	4	16
3	3	9	9	9
4	5	20	16	25
5	4	20	25	16
6	2	12	36	4
$\Sigma x = 21$	$\Sigma y = 24$	$\Sigma xy = 75$	$\Sigma x^2 = 91$	$\Sigma y^2 = 106$

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x \cdot \Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

$$= \frac{75-84}{106-96} = -0.9$$

x on y $= x - \frac{21}{6} = -0.9\left(y - \frac{24}{6}\right)$

$= x + 0.9y = 7.1$

y on x $\longrightarrow$ $y - 4 = -0.3829(x - 3.5)$

$y + 0.3829x = 5.34015$

$x,\ y = 4 \longrightarrow x = 3.5$

$y,\ x = 4 \longrightarrow y = 3.80855$

Q-3 $f(x, y)$

		x		
		0	1	2
y	0	3/28	9/28	3/28
	1	3/14	3/14	0
	2	1/28	0	0

x	0	1	2
$p(x)$	10/28	15/28	3/28

$E(x) = 0 + \frac{15}{28} + \frac{6}{28} = 0.75$

y	0	1	2
$p(y)$	$\frac{15}{28}$	$\frac{12}{28}$	$\frac{1}{28}$

$$E(y) = 0 + \frac{12}{28} + \frac{2}{28} = 0.5$$

$$E(xy) = 0 + 0 + 0 + \frac{1 \times 3}{14} \times 1 + 0 + 0 + 0 = \frac{3}{14}$$

$$\text{cov}(x,y) = E(xy) - E(x) \cdot E(y)$$

$$= \frac{3}{14} - \frac{21}{28} \times \frac{14}{28}$$

$$= -\frac{9}{56} = -0.1604$$

$$V(x) = \sigma^2(x) = E(x^2) - (E(x))^2$$
$$= 0 + 1^2 \times \frac{15}{28} + 2^2 \times \frac{3}{28} - \frac{21 \times 21}{28 \times 28} = 0.4017$$

$$V(y) = \sigma^2 y = E(y^2) - (E(y))^2$$

$$= 0 + \frac{12}{28} + 4 \times \frac{1}{28} - \frac{14 \times 14}{28 \times 28}$$

$$= 0.3214$$

$$\text{corr}(x,y) = \frac{\text{cov}(x,y)}{\sigma x \, \sigma y} = \frac{-9/56}{0.6337 \times 0.5669}$$

$$= -0.44736$$

Q-4 $f(x,y) = \begin{cases} 8xy & 0 \le y \le x < 1 \\ 0 & \text{else} \end{cases}$

$$f_x(x) = \int_{y=0}^{x} 8xy\, dy$$

$$= 8x \int_0^x y \cdot dy = 8x \left(\frac{y^2}{2}\right)_0^x$$

$$= 4x^3$$

$$f_x(y) = \int_{x=y}^{1} 8xy\, dx$$

$$= 8y \cdot \left(\frac{x^2}{2}\right)_y^1$$

$$= 8y \left[\frac{1}{2} - \frac{y^2}{2}\right]$$

$$= 4y\,[1 - y^2]$$

$$E(x) = \int_0^1 4x^4 dx = \frac{4}{5}(1-0) = \frac{4}{5}$$

$$E(y) = \int_0^1 4y^2(1-y^2)\,dx = \int_0^1 4y^2 dy - \int_0^1 4y^4 dy$$

$$= \frac{8}{15}$$

$$E(xy) = \int_0^1 \int_y^1 8x^2y^2\, dx\, dy$$

$$8\int_0^1 y^2 \int_y^1 x^2\, dx\, dy$$

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$$= 8\int_0^1 y^2 \left[\frac{x^3}{3}\right]_y^1 dy$$

$$= \frac{8}{3}\left[\int_0^1 y^2\,dy - \int_0^1 y^5\,dy\right.$$

$$= \frac{8}{3}\left[\frac{1}{3} - \frac{1}{6}\right] = \frac{4}{9}$$

$$\text{cov}(x,y) = E(xy) - E(x)\cdot E(y) = \frac{4}{9} - \frac{4}{5}\times\frac{8}{15}$$

$$= \frac{4}{225}$$

$$E(x^2) = \int_0^1 x^2 \cdot 4x^3\,dx = 4\left(\frac{x^6}{6}\right)_0^1 = \frac{4}{6} = \frac{2}{3}$$

$$E(y^2) = \int_0^1 4y^3(1-y^2)\,dy = 4\left(\frac{y^4}{4}\right)_0^1 - 4\left(\frac{y^6}{6}\right)_0^1$$

$$= 1 - \frac{4}{6} = \frac{1}{3}$$

$$\sigma^2 x = \frac{2}{3} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}, \quad \sigma^2 y = \frac{1}{3} - \left(\frac{8}{15}\right)^2 = \frac{11}{225}$$

$$r = \frac{\text{cov}(x,y)}{\sigma x\,\sigma y} = \frac{4/225}{\sqrt{\frac{2}{75}\cdot\frac{11}{225}}} = 0.4922$$

Q-5

$y = a + bx$

$E(x) = E(\lambda)$

$E(x^2) = E(\lambda^2)$

$E(y) = E(a + bx)$

$= E(a) + bE(x)$

$Cov(x, y) = E(xy) - E(x).E(y)$

$$y = \frac{Cov(x, y)}{\sigma x . \sigma y}$$

$E(x).E(y) = aE(x) + b[(E(x)).E(x)]$

$= aE(x) + b[E(x)]^2$

$E(y^2) = E((a + bx)^2)$

$= E(a^2) + 2abE(x) + b^2E(x^2)$

$\sigma^2 x = E(x^2) - [E(x)]^2$

$\sigma^2 y = E(y^2) - [E(y)]^2$

$= a^2 + 2abE(x) + b^2E(x^2)$
$-[a^2 + 2abE(x) + b^2E(x^2)]$

$= b^2[E(x^2) - [E(x)]^2]$

$$Corr(x, y) = \frac{E(xy) - E(x)E(y)}{\sigma x \, \sigma y}$$

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$$= \frac{a\, E(x) + b\, E(x^2) - [a\, E(x) + b[E(x)]^2]}{\sqrt{b^2 [E(x^2) - [E(x)]^2]^2}}$$

$$= b/|b|$$

Now, $\text{corr}(x, y) = 1$ if $b > 0$
$\text{corr}(x, y) = -1$ if $b < 0$

Q-6 $f(x, y) = 2$ for $0 < x \le y < 1$

$$f_x(x) = \int_{y=x}^{1} 2\, dy = 2y \int_{x}^{1} 2(1-x)$$

$$f_y(y) = \int_0^y 2\, dy = 2y$$

$$E(x) = \int_0^1 2y^2\, dy - 2\left[\frac{y^3}{3}\right]_0^1 = \frac{2}{3}$$

$$E(xy) = \int_0^1 \int_0^y 2yx\, dx\, dy$$

$$= \int_0^1 \int_0^y 2xy\, dx\, dy$$

$$= \int_0^1 2y\left(\frac{x^2}{2}\right)\Big|_0^y dy$$

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$= \int_0^1 y^3 dy = \left[\frac{y^4}{4}\right]_0^1 = \frac{1}{4}$

$Cov(x, y) = E(xy - Ex - Ey)$

$= \frac{1}{4} - \frac{2}{9}$

$= \frac{1}{36}$

$\sigma^2 x = E(x^2) - (E(x))^2$

$E(X^2) = \int_0^1 2x^2 (1-x) dx = 2\left[\left(\frac{x^3}{3}\right)_0^1 - \left(\frac{x^4}{4}\right)_0^1\right]$

$= 2\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{1}{6}$

$E(y^2) = \int_0^1 2y^3 dy = \frac{1}{2}$

$\sigma^2 x = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$

$\sigma^2 y = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

$\gamma = \frac{1/36}{\sqrt{\frac{1}{18} \times \frac{1}{18}}} = \frac{1/36}{1/18} = \frac{1}{2}$

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Q-7

x	y	$dx = x-15$	$dy = y-70$	$dxdy$	dx^2	dy^2
12	73	-3	3	-9	9	9
16	67	1	-3	-3	1	9
13	74	-2	4	-8	4	16
18	63	3	-7	-21	9	49
19	73	4	3	12	16	9
12	84	-3	14	-42	9	196
18	60	3	-10	-30	9	100
19	62	4	-8	-32	16	64
12	76	-3	6	-18	9	36
14	71	-1	1	-1	1	1
		$\Sigma dx = 3$	$\Sigma dy = 3$	$\Sigma dx\,dy = -152$	$\Sigma dx^2 = 83$	$\Sigma dy^2 = 489$

$$b_{yx} = \frac{\Sigma dxdy - \frac{1}{10}\Sigma dx \Sigma dy}{\Sigma dx^2 - \frac{1}{10}\Sigma(dx)^2}$$

$$= \frac{-152 - \frac{1}{16} 3\times 3}{83 - \frac{1}{10}\times 9} = -1.86$$

$$b_{xy} = \frac{-152 - \frac{1}{10}\times 9}{489 - 0.9} = -0.313$$

$$\bar{x} = \overline{dx} + 15 = 0.3 + 15 = 15.3$$

$$\bar{y} = \overline{dy} + 70 = 0.3 + 70 = 70.3$$

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if on x

$$y - 70.3 = 6yx\,(x - 15.3)$$
$$y + 1.86x = 1.56 \times 15.3 + 70.3$$
$$y + 1.86x = 98.758$$
$$y = \sqrt{bxy - byx}$$
$$= \sqrt{1.86 \times 0.313} = 0.763$$

Q-8

x	y	R_1	R_2	$d_i^2 = (R_i - R_2)^2$
6	8	3.5	7	12.25
5	7	2	4.5	6.25
8	5	7.5	4.5	9
8	10	7.5	9.5	4
7	5	5.5	1	20.25
6	8	3.5	7	12.25
10	10	10	9.5	0.25
4	6	1	2.5	2.25
9	8	9	7	4
7	6	5.5	2.5	9
				$\Sigma d_i^2 = 79.5$

$$y = 1 - 6\left\{79.5 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(2^3 - 2)\right.$$
$$\left. - \frac{1}{12}\frac{(2^3-2) + \frac{1}{12}(2^3-2)}{10 \times 99} + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(3^3 - 3)\right)$$

$$= 1 - 6\left\{\frac{79.5 + \frac{1}{12} \times 6}{990} + \frac{1}{12} \times 29\right\} = \underline{0.487}$$