Tutorial - 4

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Sol-1

$N =$ Total tablets $= 12 + 4 = 16$
$m =$ narcotic drugs $= 4$
$n =$ Total samples $= 3$

let x be the number of narcotic tablet selected in the sample

since selections are made without replacement

x Hypergeometric $(N = 16, m = 4, n = 3)$

$$P(X = x) = \frac{({}^{4}C_{x})({}^{12}C_{3-x})}{{}^{16}C_{3}} \qquad x = 0, 1, 2, 3$$

Now, P (traveller arrested)

$= 1 -$ P (traveller not arrested)

$= 1 - P(X = 0)$

$$= 1 - \frac{({}^{4}C_{0})({}^{12}C_{3})}{{}^{16}C_{3}}$$

$$= 1 - \frac{(1)\left(\frac{12 \times 11 \times 10}{6}\right)}{\frac{16 \times 15 \times 14}{6}} = \frac{68}{112}$$

Page No.:
Date: / /

Sol-2 All 4 will fire

$N =$ total no. of missiles $= 10$
$m =$ Defective missiles $= 3$
$n =$ Total samples $= 4$

→ let x be the no. of non-defective missiles

$$P(X=x) = \frac{({}^7C_x)({}^3C_{4-x})}{{}^{10}C_4}$$

① All 4 will fire $= P(X=4)$

$$= \frac{({}^7C_4)({}^3C_0)}{{}^{10}C_4} = \frac{1}{6}$$

② At most 2 will not fire $= 1 - P(3 \text{ missile will fire})$

$$= 1 - \frac{{}^7C_1\,{}^3C_3}{{}^{10}C_4}$$

$$= \frac{29}{30}$$

③ How many defective missiles we expect to be included among that are selected

No. of defective missile
$X_i = 1$ if 1^{st} missile is choosen
$X_i = 0$ if its not

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Page No.:
Date: / /

$E(X_1+X_2+X_3) = E(X_1)+E(X_2)+E(X_3) = P(X_1 \text{ is choosen}) + P(X_2 \text{ is choosen}) + P(X_3 \text{ is choosen})$

→ The no. of ~~defective~~ choices such that i^{th} defective missile is choosen.

$$E[X_1+X_2+X_3] = 3 \times \left({}^{7}C_4\right)\left({}^{10}C_4\right) = \frac{6}{5}$$

Sol-3 $X \sim G(p)$

$$P(X=k) = \begin{cases} p(1-p)^{k-1} & \forall\, k = 1,2,3,\ldots \\ 0 & \text{else} \end{cases}$$

(i) $P(x \text{ is even}) = P(X=2) + P(X=4) + P(X=6) + \ldots$

$= p(1-p) + p(1-p)^3 + p(1-p)^5 p \ldots$

$p<1$ $\quad 1-p<1 \longrightarrow \frac{a}{1-r}$

$$= \frac{p(1-p)}{1-(1-p)^2}$$

$$= \frac{p(1-p)}{2p-p^2} = \frac{1-p}{2-p}$$

(ii) $P(x \text{ is odd}) = 1 - P(X \text{ is even})$

$$= 1 - \left(\frac{1-p}{2-p}\right)$$

$$= \frac{1}{2-p}$$

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(iii) $P(2 \le x \le 9 \mid X \ge 4) = \dfrac{P(4 \le X \le 9)}{P(X \ge 4)}$

$$= \frac{p(1-p)^3 + p(1-p)^4 + \cdots + p(1-p)^9}{p(1-p)^3 + p(1-p)^4 + \cdots}$$

$$= \frac{\left(\dfrac{p(1-p)^3\,(1-(1-p)^6)}{1-(1-p)}\right)}{\left(\dfrac{p(1-p)^3\,(1-p)}{1-(1-p)}\right)}$$

$$= 6p - 15p^2 + 20p^3 - 15p^4 + 6p^5 - p^6$$

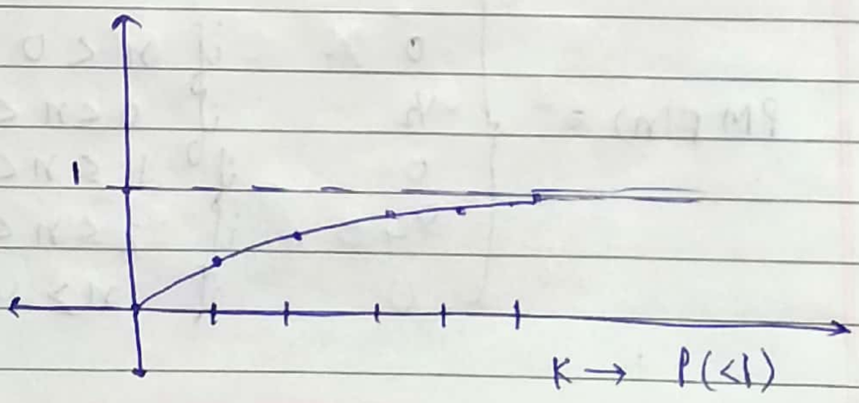
(iv) CDF of X

$$F_X(k) = P(X \le k) = p + p(1-p) + p(1-p)^2 + \cdots p(1-p)^{k-1}$$

$$= \frac{p(1-(1-p)^k)}{1-(1-p)}$$

$$= 1 - (1-p)^k$$

* Graphs



Page No.:
Date: / /

Sol-4 $X =$ amount of time (in min)
X is exponential
$X \sim \exp(\lambda)$
avg amount of time = 4 mins

$$\mu = 4 \rightarrow \frac{1}{\lambda} = 4 \Rightarrow \lambda = \frac{1}{4}$$

$$P(3 < x < 5) = P(x < 5) - P(x < 3)$$
$$= (1 - e^{-5\lambda}) - (1 - e^{-3\lambda})$$
$$= e^{-3\lambda} - e^{-5\lambda}$$

$$P(3 < x < 5) = e^{-3/4} - e^{-5/4} = \underline{\underline{0.1858}}$$

Sol-5 (a)
$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x/2 & \text{if } 0 \le x < 1 \\ 1/2 & \text{if } 1 \le x < 2 \\ x/4 & \text{if } 2 \le x \le 4 \\ 1 & \text{if } x \ge 4 \end{cases}$$

It is CDF bcz $\lim_{x \to -\infty} F(x) = 0$

$\lim_{x \to \infty} F(x) = 1$

Non-decreasing continuous

$$PMF(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1/2 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 \le x < 2 \\ 1/4 & \text{if } 2 \le x < 4 \\ 0 & \text{if } x \ge 4 \end{cases}$$

Now $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$$= 0 + \int_0^1 \frac{x}{2} dx + 0 + \int_4^2 \frac{x}{4} dx + 0$$

$$= \left(\frac{x^2}{4}\right)_0^1 + \left(\frac{x^2}{8}\right)_4^2$$

$$= \frac{7}{4} \qquad \therefore \quad E(x) = 1.75$$

(ii) $$F(x) = \begin{cases} 0 & \forall\ x < 1 \\ \frac{(x-1)^2}{8} & \forall\ 1 \le x < 3 \\ 1 & \forall\ x \ge 3 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = \frac{(1-1)^2}{8} = 0 = f(0)$$

also if $\lim_{x \to \infty} f(x) = 0$ $\left\{ \lim_{x \to \infty} f(x) = 1 \right.$

it is increasing valid CDF

$$\text{PDF} = \begin{cases} 0 & \forall\ x < 1 \\ \frac{x-1}{4} & \forall\ 1 \le x < 3 \\ 1/2 & x = 3 \\ 0 & \forall\ x > 3 \end{cases}$$

Page No.:
Date: / /

$$F(x) = 0 + \int_1^3 x\left(\frac{x-1}{4}\right)dx + 3\times\frac{1}{2} + 0$$

$$= \frac{3}{2} + \frac{1}{4}\int (x^2 - x)\,dx = \frac{3}{2} + \frac{1}{4}\left(\frac{27}{3} - 4\right)$$

$$= \frac{8}{3} = 2.66$$

Sol-6 (i) $\Gamma 9/2$

$$= \Gamma n+1 = n\Gamma n$$

$$\Gamma\frac{9}{2} = \frac{7}{2}\Gamma\frac{7}{2} = \frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma\frac{1}{2}$$

$$= \frac{105\sqrt{\pi}}{16}$$

(ii) $A = \int_0^\infty x^7 e^{-6x}\,dx$ let $6x = X$, $6\,dx = dX$

$$\int_0^\infty \frac{X^7}{6^7} e^{-X} \frac{dX}{6}$$

$$= \frac{1}{6^8}\int_0^\infty X^7 e^{-X}\,dX$$

$$= \frac{1}{6^8}\int_0^\infty X^{8-1} e^{-X}\,dX$$

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$$= \frac{1}{6^8} \Gamma 8 = \frac{7!}{6^8} = 0.003$$

(iii) $B(5,9) = \frac{\Gamma 5 \, \Gamma 9}{\Gamma 5+9} = \frac{4! \times 8!}{13!}$

$$= 0.000155$$

(iv) $A = \int_0^1 x^7 (1-x)^9 \, dx$

$$A = B(8,10) = \frac{\Gamma 8 \, \Gamma 10}{\Gamma 18}$$

$$= \frac{7! \, 9!}{17!}$$

$$= 0.\underline{00000}514$$