30. An individual who has automobile insurance from a certain company is randomly selected. Let Y be the number of moving violations for which the individual was cited during the last 3 years. The pmf of Y is

y	0	1	2	3
$p(y)$	.60	.25	.10	.05

a. Compute $E(Y)$.

b. Suppose an individual with Y violations incurs a surcharge of $\$100Y^2$. Calculate the expected amount of the surcharge.

31. Refer to Exercise 12 and calculate $V(Y)$ and σ_Y. Then determine the probability that Y is within 1 standard deviation of its mean value.

Sol^n 30

y	0	1	2	3
$P(y)$	0.60	0.25	0.10	0.05

(c) $V(Y)$

(d) Determine the prob. that Y is within 1 standard deviation of its mean value?

a) $E(Y) = \sum_{y \in R_y} y \cdot p_y(y)$

$= \sum_{y=0}^{3} y \cdot p_y(y) = 0 \times 0.60 + 1 \times 0.25 + 2 \times 0.10 + 3 \times 0.05$

$= 0 + 0.25 + 0.20 + 0.15$

$= 0.60$

b) $E(100Y^2) = 100\, E(Y^2)$ $\quad (\because E(aX) = a\,E(X))$

$= 100 \left[\sum_{y=0}^{3} y^2 \cdot p_y(y) \right]$

$= 100 \left[0^2 \times 0.60 + 1^2 \times 0.25 + 2^2 \times 0.10 + 3^2 \times 0.05 \right]$

$= 100 [0 + 0.25 + 0.40 + 0.45]$

$= 100 [1.10]$

$= 110$

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(c) $V(Y) = E(Y^2) - (E(Y))^2$

$= 1.10 - (0.60)^2$

$= 1.10 - 0.36$

$\boxed{V(Y) = 0.74}$

Standard deviation $\sigma_Y = \sqrt{0.74}$

$= 0.8602$

(d) $P(|Y - E(Y)| \leq 1 \times \sigma_y) = ?$

$= P(-\sigma_y < Y - E(Y) < \sigma_y)$ $\boxed{(\because |X| < 1 \Rightarrow -1 < X < 1)}$

$= P(-0.8602 < Y - 0.60 < 0.8602)$

$= P(-0.8602 + 0.6 < Y < 0.6 + 0.8602)$

$= P(-0.2602 < Y < 1.4602)$

$= P(0 \leq Y \leq 1)$

$= P(Y=0) + P(Y=1)$

$= 0.60 + 0.25$

$= 0.85$ Ans

Q: 33 ~~Let X~~ Let X be a Bernoulli r.v then
a) Compute $E(X^2)$ (b) find $V(X)$ (c) compute $E(X^{78})$
(d) $E(X^{79})$

Sol^n

$$p_X(x) = p^x q^{(1-x)} \; ; \; x = 0, 1$$
$$= p^x (1-p)^{1-x} \; ; \; x = 0, 1$$

$$E(X^2) = \sum_{x=0}^{1} x^2 \cdot p_X(x)$$
$$= 0^2 \times p^0 (1-p)^{1-0} + 1^2 \times p^1 (1-p)^{1-1}$$
$$= 0 + 1 \times p(1-p)^0$$
$$= p(1-p)^0$$
$$= p \cdot 1$$
$$= p$$

(b) $V(X) = E(X^2) - (E(X))^2$

$$E(X) = \sum_{x=0}^{1} x \cdot p_X(x)$$
$$= 0 \cdot p_X(0) + 1 \cdot p_X(1)$$
$$= 0 + 1 \cdot p^1 (1-p)^{1-1}$$
$$= p(1-p)^0$$
$$= p$$

$$V(X) = p - p^2$$
$$= p(1-p) = p \cdot q$$

(c) $E(X^{78}) = \sum_{x=0}^{1} x^{78} \cdot p_X(x)$

$= 0^{78} \times p_X(0) + 1^{78} \times p_X(1)$ No.- 48

$= 0 + 1 \cdot p^{1} (1-p)^{1-1}$

$= 0 + p(1-p)^{0}$

$\boxed{E(X^{78}) = p}$

$E(X^{79}) = \sum_{x=0}^{1} x^{79} p_X(x)$

$= 0^{79} \times p_X(0) + 1^{79} \times p_X(1)$

$= 0 + 1 \cdot p^{1} (1-p)^{1-1}$

$\boxed{E(X^{79}) = p}$

37. The n candidates for a job have been ranked 1, 2, 3, ..., n. Let X = the rank of a randomly selected candidate, so that X has pmf

$$p(x) = \begin{cases} 1/n & x = 1, 2, 3, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

(this is called the *discrete uniform distribution*). Compute $E(X)$ and $V(X)$ using the shortcut formula. [*Hint:* The sum of the first n positive integers is $n(n + 1)/2$, whereas the sum of their squares is $n(n + 1)(2n + 1)/6$.]

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$$p(x) = \begin{cases} \frac{1}{n} ; & x = 1, 2, 3, \ldots, n \\ 0 ; & \text{otherwise} \end{cases}$$

$$E(X) = \sum_{x \in R_X} x \cdot p_X(x)$$

$$= \sum_{x=1}^{n} x \cdot p(x)$$

$$= 1 \cdot p(1) + 2 \cdot p(2) + 3 \cdot p(3) + \cdots + n \cdot p(n)$$

$$= 1 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n} + 3 \cdot \frac{1}{n} + 4 \cdot \frac{1}{n} + \cdots + n \cdot \frac{1}{n}$$

$$= \frac{1}{n} [1 + 2 + 3 + 4 + \cdots n]$$

$$= \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2}$$

$$\boxed{E(X) = \frac{n+1}{2}}$$ #

$$E(X^2) = \sum_{x=1}^{n} x^2 \cdot p_X(x)$$

$$= 1^2 \cdot p(1) + 2^2 \cdot p(2) + 3^2 \cdot p(3) + \cdots + n^2 \cdot p(n)$$

$$= 1^2 \cdot \frac{1}{n} + 2^2 \cdot \frac{1}{n} + 3^2 \cdot \frac{1}{n} + \cdots + n^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n} [1^2 + 2^2 + 3^2 + \cdots + n^2]$$

$$= \frac{1}{\cancel{n}} \left[\frac{\cancel{n}}{6} (n+1)(2n+1) \right]$$

$$= \frac{(n+1)(2n+1)}{6}$$

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$$V(X) = E(X^2) - (E(X))^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{(n+1)}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)}{2}\left[\frac{n-1}{6}\right]$$

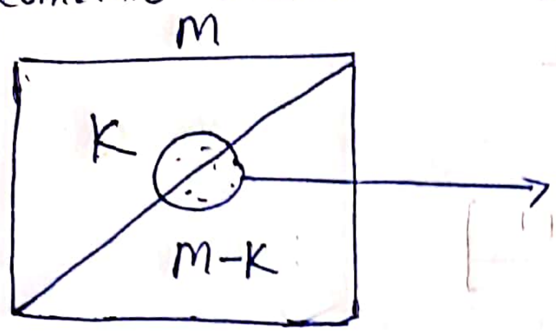
$$= \frac{(n+1)(n-1)}{12}$$

$$\boxed{V(X) = \frac{n^2-1}{12}}$$

Note → $p_X(x) = \begin{cases} \frac{1}{n} ; & x = 1, 2, 3 \ldots, n \\ 0 ; & \text{otherwise} \end{cases}$

This is called discrete uniform distribution.

Hypergeometric Distribution :-



drawn sample of size n without replacement.

$M \rightarrow$ Total no of objects

$K \rightarrow$ Objects of a type-I (e.g success or defective)

$M-K \rightarrow$ Objects of other type or type-II (failure or non-defective)

$n \rightarrow$ size of sample which is drawn

X: no of success in drawn sample

$$p_X(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}} \quad ; \quad x = 0, 1, 2, \ldots, n$$

$$E(X) = \frac{n.K}{M} \quad ; \quad V(X) = n.\frac{K}{M}.\left(1-\frac{K}{M}\right).\left(\frac{M-n}{M-1}\right)$$