

Tutorial-2

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U20CS110

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Sol-(1)

given

$$P(A \cup B) = 3/4$$

$$P(A \cap B) = 1/4$$

$$P(\bar{A}) = 2/3$$

$$\begin{aligned} P(A) &= 1 - P(\bar{A}) \\ &= 1 - 2/3 \end{aligned}$$

$$\boxed{P(A) = 1/3}$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(B) = \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \frac{2}{3}$$

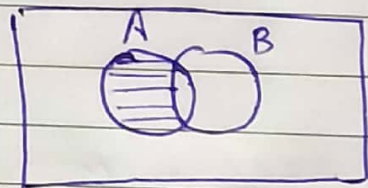
$$P(A \cap \bar{B})$$

→ from Venn diagram

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12}$$



$$\boxed{P(A \cap \bar{B}) = \frac{1}{12}}$$

Ques-2) Equation is $ax^2 + bx + c = 0$
 $\Rightarrow a, b, c \in \{1, 2, 3, \dots, 6\}$

for real roots $b^2 - 4ac > 0$
 $b^2 \geq 4ac$

Now for $b=1$ no a & c possible

$$b=2; \quad a=c=1$$

$$b=3; \quad \{a, c\} = \{1, 1\}, \{1, 2\}, \{2, 1\}$$

$$b=4; \quad \{a, c\} = \{1, 1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 1\}, \{2, 2\}, \{3, 1\}, \{4, 1\}$$

$$b=5; \quad \{a, c\} = \{1, 1\}, \{2, 1\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 1\}, \{2, 2\}, \{2, 3\}, \{3, 1\}, \{3, 2\}, \{4, 1\}, \{5, 1\}, \{6, 1\}$$

$$b=6 \quad \{a, c\} = \{1, 1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 1\}, \{2, 2\}, \{2, 3\}, \{2, 4\}, \{3, 1\}, \{3, 2\}, \{3, 3\}, \{4, 1\}, \{4, 2\}, \{5, 1\}, \{6, 1\}$$

Hence total favourable events = 43
 and total possible events are $(6 \times 6 \times 6) = 216$

$$P(A) = \text{probability of eqn having real root} = \frac{43}{216}$$

\rightarrow for complex roots $b^2 - 4ac < 0$
 or we have to find $P(\bar{A})$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{43}{216}$$

$$P(\bar{A}) = \frac{173}{216}$$

Sol-3

Acc to question

$$P(\text{no six}) < \frac{1}{2}$$

$$\text{now } P(6) = \frac{1}{6} \quad P(\bar{6}) = \frac{5}{6}$$

$$P(\text{no six}) = \left(\frac{5}{6}\right)^n$$

n is no. of throws

$$\left(\frac{5}{6}\right)^n < \frac{1}{2}$$

applying log on both side

$$n (\log 5 - \log 6) > \log 1 - \log 2$$

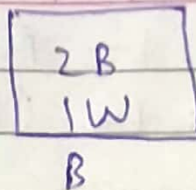
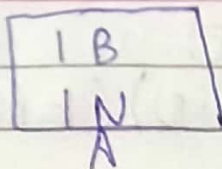
$$n > \frac{-0.301}{-0.070}$$

$$n > 3.81$$

$$\boxed{n=4} \text{ minimum}$$

Hence minimum 4 trials are required

Sol-4



Both boxes are equivalent

\therefore selecting one from both side is equally likely for A & B

$$\rightarrow P(A) = P(B) = \frac{1}{2}$$

$$P(\text{black}) \text{ for A} = \frac{1}{2}$$

$$P(\text{black}) \text{ for B} = \frac{2}{3}$$

$$P(\text{marble is black}) = P(A) \times \frac{1}{2} + P(B) \times \frac{2}{3}$$

$$= \left(\frac{1}{2} \times \frac{1}{2} \right) + \left(\frac{1}{2} \times \frac{2}{3} \right)$$

$$= \frac{1}{4} + \frac{1}{3}$$

$$P(\text{marble is black}) = \frac{7}{12}$$

Sol-5

$$P(A) = \frac{1}{2} \quad P(B) = \frac{3}{4} \quad P(C) = \frac{1}{4}$$

probability of question solved = $P(A)$

also $P(\theta) = 1 - \text{probability of question not solved}$

$$P(\theta) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$\begin{aligned}
 P(0) &= 1 - P(\bar{A})P(\bar{B})P(\bar{C}) \\
 &= 1 - (1 - P(A))(1 - P(B))(1 - P(C)) \\
 &= 1 - \left[1 - \frac{1}{2}\right]\left[1 - \frac{3}{4}\right]\left[1 - \frac{1}{4}\right] \\
 &= 1 - \left[\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right]
 \end{aligned}$$

$$P(0) = 1 - \frac{3}{32} = \frac{29}{32}$$

$$P(\text{Question solved}) = \frac{29}{32}$$

Sol-6 Here $n=20$, $p=0.3$ (very small compared to n)

we can apply poisson distribution

$$\lambda = np = 20 \times 0.3 = 6$$

$$PMF = \frac{\lambda^n}{n!} e^{-\lambda} = \frac{6^n}{n!} e^{-6}$$

from binomial distribution

$$PMF = \binom{20}{n} p^n q^{20-n} = \binom{20}{n} (0.3)^n (0.7)^{20-n}$$

$$i) \quad P(X=7) = \binom{20}{7} (0.3)^7 (0.7)^{13} \\ = 0.164$$

from poisson distribution

$$P(X=7) = \frac{6^7}{7!} e^{-6} = 55.543 \times 0.0024 = 0.137$$

$$ii) \quad P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) \\ = \binom{20}{0} (0.3)^0 (0.7)^{20} + \binom{20}{1} (0.3)^1 (0.7)^{19} \\ + \binom{20}{2} (0.3)^2 (0.7)^{18} \\ = (0.7)^{18} [0.49 + 4.2 + 17.1] \\ = 0.035$$

from poisson distribution

$$P(X \leq 2) = e^{-6} \left[\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} \right] \\ = e^{-6} [1 + 6 + 18]$$