Conditional Probability :- Let A and B be two events associated with sample space S then the conditional prob. of A given that B has already occured is defined by

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$\Rightarrow \boxed{P(A \cap B) = P(B) \cdot P(A|B)} \rightarrow$$ This is called Multiplication formula for two events

$$\because P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B|A)}$$

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Multiplication Rule for 3 Events :-

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$$

Proof: $P(A) \cdot P(B|A) \cdot P(C|A \cap B)$

$$= \cancel{P(A)} \cdot \frac{\cancel{P(B \cap A)}}{\cancel{P(A)}} \cdot \frac{P(C \cap A \cap B)}{\cancel{P(A \cap B)}}$$

$$= P(C \cap A \cap B) \quad \#$$

For n events :-

$$P(A_1 \cap A_2 \ldots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdot \ldots P(A_n|A_1 \cap A_2 \ldots A_{n-1})$$

From: R.H.S

$$P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \ldots P(A_n|A_1 \cap A_2 \ldots A_{n-1})$$

$$= \cancel{P(A_1)} \cdot \frac{\cancel{P(A_2 \cap A_1)}}{\cancel{P(A_1)}} \cdot \frac{P(A_3 \cap A_1 \cap A_2)}{\cancel{P(A_1 \cap A_2)}} \ldots \frac{P(A_n \cap A_1 \cap A_2 \ldots A_{n-1})}{\cancel{P(A_1 \cap A_2 \ldots A_{n-1})}}$$

$$= P(A_1 \cap A_2 \cap A_3 \ldots \cap A_n) \quad \#$$

$$= \text{L.H.S.}$$

Independent Events :- Let A and B be two events associated with sample space then A and B are said to be independent events if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

This can be also defined as

$$\boxed{P(A|B) = P(A)}$$

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i.e $\dfrac{P(A\cap B)}{P(B)} = P(A)$

$\Rightarrow P(A\cap B) = P(A).P(B)$ #

Q: Roll a fair dice twice. Let A be the event of getting 6 on first roll and B is 4 on second roll. Verify that events A and B are independent.

Sol^n $|S| = 36$

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	--	(1,6)
2	(2,1)	(2,2)			-	(2,6)
3						
4						
5						
6	(6,1)	(6,2)	(6,3)	..		(6,6)

$A = \{(6,1),(6,2) \cdots (6,6)\}$

$|A| = 6$

$B = \{(1,4),(2,4),(3,4) .. (6,4)\}$

$|B| = 6$

$P(A) = \frac{6}{36} = 1/6$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$A\cap B = \{(6,4)\}$

$|A\cap B| = 1$

$P(A\cap B) = \frac{1}{36} = P(A).P(B)$

$\Rightarrow$ A and B are independent events #

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Result : If A and B are Independent then A^c and B^c are also independent.

Proof: Given : $P(A \cap B) = P(A) \cdot P(B)$

Claim : $P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$

$$P(A^c \cap B^c) = P((A \cup B)^c)$$

$$= 1 - P(A \cup B) \quad [\because P(A^c) = 1 - P(A)]$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= P(A^c) - P(B)[1 - P(A)]$$

$$= P(A^c) - P(B) \cdot P(A^c)$$

$$= P(A^c)[1 - P(B)]$$

$$= P(A^c)\, P(B^c)$$

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