Q: A die is thrown 9 times. Getting 3 or 6 is considered to be success. Find the probability of getting atleast two success? Also find the probability of getting exactly one success?

Soln

$S = \{1, 2, 3, 4, 5, 6\}$

3 or 6 → Considered as success

1, 2, 4, 5 → Considered as failure

$n = 9$ (No of trials)

$p = \frac{2}{6} = \frac{1}{3}$ (probability of success)

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$ (prob. of failure)

X: No of success in 9 trials.

Here all the conditions of Binomial distn satisfies.

$$p_X(x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0, 1, 2, \ldots n.$$

$$p_X(x) = \binom{9}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{9-x} ; \quad x = 0, 1, 2, \ldots 9.$$

(i) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X = 0) + P(X = 1)]$

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$$= 1 - \left[\binom{9}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{9-0} + \binom{9}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{9-1} \right]$$

$$= 1 - [0.026 + 0.11]$$

$$= 0.85$$

(ii) $P(X=1) = \binom{9}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^8$

$$= 0.11$$

$$E(X) = np = 9 \times \frac{1}{3} = 3$$

$$V(X) = npq = 9 \times \frac{1}{3} \times \frac{2}{3} = 2$$

Note: Standard deviation is positive square root of variance.
i.e Standard deviation $= \sqrt{\text{Variance}}$.

Midsem 2020

Q: A boiler has five relief valves. The probability that any particular valve will open on demand is 0.95. Assuming independent operation of the valves, calculate P(at least one valve opens) and P(atleast one valve fails to open)?

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Sol^n $n = 5$; $p = 0.95$ (valve will open)

$q = 1 - 0.95 = 0.05$ (valve will not open)

X : no of valve will open out of 5 valves.

$\Rightarrow P_X(x) = \binom{5}{x} (0.95)^x (0.05)^{5-x}$; $x = 0, 1, 2, 3, 4, 5$

a) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \binom{5}{0} (0.95)^0 (0.05)^{5-0}$

$= 1 - 1 \times 1 \times (0.05)^5$

$= 0.999$ Ans

b)

X : No of valves fails to open out of 5 valves

$n = 5$

$p = 0.05$

$q = 0.95$

$P_X(x) = \binom{5}{x} (0.05)^x (0.95)^{5-x}$; $x = 0, 1, 2, 3, 4, 5$.

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \binom{5}{0} (0.05)^0 (0.95)^{5-0}$

$= 1 - 1 \times 1 \times (0.95)^5$

$= 0.2262$ Ans

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3.2 (mid Sem 2020)

Q.13 A mail-order computer business has six telephone lines. Let X denote the number of lines No.- 39 in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
$P_X(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

Calculate the prob. of each of the following events

a) {at most 3 lines are in use}

b) {fewer than 3 lines are in use}

c) {Atleast 3 lines are in use}

d) {between two and five lines, inclusive are in use}

e) {between two and four lines, inclusive, are not in use}

f) {at least 4 lines are not in use}

Soln a) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.10 + 0.15 + 0.20 + 0.25$

$= 0.70$

b) $P(X < 3) = P(X=0) + P(X=1) + P(X=2) = 0.45$

c) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= 0.55$

(d) $P(2 \leq X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5) = 0.71$

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(e) X: No of lines are in use

6-X: no of lines not in use

$$P(2 \leq 6-X \leq 4) = P(2-6 \leq 6-X-6 \leq 4-6)$$

$$= P(-4 \leq -X \leq -2)$$

$$= P(2 \leq X \leq 4)$$

$$= P(X=2) + P(X=3) + P(X=4)$$

$$= 0.65$$

(f) $P(6-X \geq 4) = P(6-X-6 \geq 4-6)$

$$= P(-X \geq -2)$$

$$= P(X \leq 2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= 0.45$$

Alternative! (e) $P(2 \leq 6-X \leq 4) = P(2$ lines are not in use) + P(3 lines are not in use) + P(4 lines are not in use)

$$= P(4 \text{ lines are in use}) + P(3 \text{ lines are in use}) + P(2 \text{ lines are in use})$$

(f) $P(6-X \geq 4) = P(4$ lines are not in use) + P(5 lines are not in use) + P(6 lines are not in use)

$$= P(2 \text{ lines are in use}) + P(1 \text{ lines are in use}) + P(0 \text{ lines are in use})$$

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Q: The prob. of a player hitting a target at a shooting range is 0.25. If he shoots 10 times, what is the prob. that he hit the target atleast once? What is the average no of hitting the target?

No.- 40

Soln

$n = 10; \quad p = 0.25 ; \quad q = 0.75$

X : He hit the target

$$P(X = x) = \binom{10}{x} (0.25)^x (0.75)^{10-x} ; \quad x = 0, 1, 2 \cdots 10$$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \binom{10}{0} (0.25)^0 (0.75)^{10-0}$$

$$= 0.94 \quad \text{Ans}$$

$$E(X) = n \cdot p$$

$$= 10 \times 0.25$$

$$= 2.5$$

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