Negative Binomial Distribution :-
(Type - I)

$r \rightarrow$ Number of success. (fixed)

$X :\rightarrow$ Number of trials to get fixed no of success.

$p \rightarrow$ prob. of success

$q \rightarrow$ prob. of failure.

A r.v X is said to have Negative Binomial (NB) of type-I if its p.m.f is given as

$$p_X(x) = \binom{x-1}{r-1} p^{r-1} . p . q^{x-r}$$

$$\boxed{p_X(x) = \binom{x-1}{r-1} p^{r} q^{x-r} \; ; \; x = r, r+1, r+2, \ldots}$$

$$E(X) = \frac{r}{p} \; ; \quad V(X) = \frac{r \cdot q}{p^2}$$

Type-II

X : No. of failure before r^{th} success.

$x \longrightarrow x + r$ (in type-I)

$$p_X(x) = \binom{x+r-1}{r-1} p^{r} . q^{x+\cancel{r}-\cancel{r}}$$

$$\boxed{p_X(x) = \binom{x+r-1}{r-1} . p^{r} . q^{x} \; ; \; x = 0, 1, 2, 3, \ldots}$$

$$E(X) = \frac{r.q}{p} \; ; \; V(X) = \frac{r.q}{p^2}$$

3.5

Q. 75

No.- 45

Suppose that p = prob. of male birth $= 0.5$.

A couple wishes to have exactly 2 female birth in their family. They will have a children untill this condition is fulfilled.

i a) What is the prob. that family has x male children?

ii) What is the prob. that the family has 4 children?

iii) What is the prob. that atmost 4 children?

iv) How many male children would you expect this family to have? How many children would you expect this family to have?

V. Imp
End sem 2019
Sche. 3.5
Sol. 75

(I) $r = 2$ (Two female children)

X: no of male children (failure before 2 female children)

$p = 0.5 = q$; So we can use type-II NB.

$$p_X(x) = \binom{x+r-1}{r-1} p^r q^x \; ; \; x = 0, 1, 2 \ldots$$

$$= \binom{x+2-1}{2-1} (0.5)^2 (0.5)^x \; ; \; x = 0, 1, 2 \ldots$$

$$= (x+1)(0.5)^{2+x} \; ; \; x = 0, 1, 2 \ldots$$

(ii) X: no of children (No of trials to get fixed no of success)

$r = 2$; $p = q = 0.5 \Rightarrow$ We can use type-I NB

$$p_X(x) = \binom{x-1}{r-1} p^r q^{x-r} \; ; \; x = r, r+1, r+2 \ldots$$

$$p_X(4) = \binom{4-1}{2-1}(0.5)^2 \cdot (0.5)^{4-2}$$

$$= \binom{3}{1}(0.5)^2 \cdot (0.5)^2$$

$$= 3 \times (0.5)^4$$

$$= 0.1875$$

(iii) $P(X \leq 4) = P(2 \leq X \leq 4)$

$$= P(X=2) + P(X=3) + P(X=4)$$

$$= \binom{2-1}{2-1}(0.5)^2(0.5)^{2-2} + \binom{3-1}{2-1}(0.5)^2(0.5)^{3-2} + \binom{4-1}{2-1}(0.5)^2(0.5)^{4-2}$$

$$= 0.6875$$

(iv) $E(X) = E(\text{No of failure before success})$

$$= \frac{r \cdot q}{p} = \frac{2 \times \cancel{0.5}}{\cancel{0.5}} = 2$$

$E(X) = E(\text{No of trials to get fixed success})$

$$= \frac{r}{p} = \frac{2}{0.5} = \frac{2 \times 2}{1} = 4$$

Q: Raghu is making cold sales calls, the probability of a sale on each call is 0.4. No.- 45

The call may be considered as inverse Binomial trials.

(a) What is the prob. that he has exactly 5 failed calls before his second successfull calls?

(b) What is the prob. that he has fewer than 5 calls before 2nd successfull sale calls?

Sol^n (a) X: no of failed calls before 2nd successfull calls
(No of failure before fixed success)

Type-II

$$p_X(x) = \binom{x+r-1}{r-1} p^r q^x \; ; \; x = 0, 1, 2, \ldots$$

$$p_X(5) = \binom{5+2-1}{2-1} (0.4)^2 (0.6)^5 = 0.074$$

(b) X: No of calls before 2nd successfull calls
(Type-I)

$$p_X(x) = \binom{x-1}{r-1} p^r q^{x-r} \; ; \; x = r, r+1, r+3 \ldots$$

$$P(2 \leq X \leq 5) = P(2 \leq X \leq 5)$$

$$= P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= \binom{2-1}{2-1}(0.4)^{2} \cdot (0.6)^{2-2} + \binom{3-1}{2-1}(0.4)^{2}(0.6)^{3-2}$$

$$+ \binom{4-1}{2-1}(0.4)^{2}(0.6)^{4-2} + \binom{5-1}{2-1}(0.4)^{2}(0.6)^{5-2}$$

$$= \quad \#$$