Continuous Random Variable

Def^n: A r.v X is said to be continuous if range space of X is continuous.

eg: $x \in (0,1)$; $x \in [0,1]$; $x \in \mathbb{R}$ i.e $x \in (-\infty, \infty)$; $1 \leq x \leq 2$. ; $x \geq 0$

→ A r.v X is said to be continuous if its CDF is given as

$$\boxed{F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\, dt}$$

The function $f_X(x)$ is known as probability density function (pdf).

Moreover if X is cont. r.v then

$$\boxed{\frac{d}{dx} F_X(x) = f_X(x)}$$

Here $f_X(x)$ should satisfy the following two properties

(i) $f_X(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

Q1 Let X be r.v with pdf

$$f_X(x) = \begin{cases} 2K & ; \quad 0 < x < 1/2 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

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Find the value K?

Soln $\because f_X(x)$ is pdf

then $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$\boxed{7/// Fb}$

Imp

Note : Probability at a single point in case of continuous random variable is 0.

$$P(X=a) = \int_a^a f_X(x)\,dx \qquad (\text{if } X \sim f_X(x))$$

$$= 0 \ \#$$

Now, $\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{0} f_X(x)\,dx + \int_0^{1/2} f_X(x)\,dx + \int_{1/2}^{\infty} f_X(x)\,dx = 1$

$$\Rightarrow 0 + \int_0^{1/2} 2K\,dx + 0 = 1$$

$$\Rightarrow 2K \cdot \left[\tfrac{1}{2}\right] = 1 \Rightarrow \boxed{K = 1}$$

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Ex: Let X be a continuous r.v with p.d.f

$$f_X(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ a & ; \quad 1 \le x \le 2 \\ -ax+3a & ; \quad 2 \le x \le 3 \\ 0 & ; \quad x > 3 \end{cases}$$

(i) find the value of a

(ii) $P(X \le 1.5)$

Sol^n $\because f_X(x)$ is pdf

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\Rightarrow \left(\cancel{\int_{-\infty}^{0} f(x)dx} + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \cancel{\int_{3}^{\infty} f(x)dx} \right) = 1$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$\Rightarrow a\left[\frac{x^2}{2}\right]_0^1 + a[x]_0^1 + \left(-\frac{ax^2}{2} + 3ax\right)_2^3 = 1$$

$$\Rightarrow a\cdot\frac{1}{2} + a\cdot 1 + \left[\left(-\frac{a\cdot 9}{2} + 3a\cdot 3\right) - \left(-\frac{a\cdot 4}{2} + 3a\cdot 2\right)\right] = 1$$

$$\Rightarrow \boxed{a = \frac{1}{2}}$$

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(ii) $P(X \leq 1.5) = \int_0^{1.5} f_X(x)\,dx$

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$= \int_0^1 f_X(x)\,dx + \int_1^{1.5} f_X(x)\,dx$

$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx$

$= a\left[\frac{x^2}{2}\right]_0^1 + a\left[x\right]_1^{1.5}$

$= a.\frac{1}{2} + a.(1.5-1)$

$= a.\frac{1}{2} + a.0.5$

$= \frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4}$

$P(X \leq 1.5) = 1/2$ #

Expected Value: Let X be cont. r.v then

$$E(X) = \int_{-\infty}^{\infty} x.f_X(x)\,dx.$$

Variance:

$$V(X) = EX^2 - (EX)^2$$