Tutorial - 2

U20CS110
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Prove by Mathematical induction

Ques-1 Let $S(n) = 1+2+ \dots n, n \in N$
Prove by M.I $P(n): S(n) = \frac{n(n+1)}{2}$

Sol- $S(n) = 1+2+ \dots n, \; n \in N$

let $n=1$

$S(n) = 1$

$S(n) = \frac{n(n+1)}{2}$

$S(1) = \frac{1(2)}{2} = 1$

L.H.S = R.H.S

true for $n=1$

let it true for $n=K$

$S(K) = 1+2+ \dots K = \frac{K(K+1)}{2}$

Now will prove for $n = K+1$

$S(K+1) = 1+2+ \dots +K+K+1$
$= \frac{K(K+1)}{2} + K+1$
$= (K+1)\left[\frac{K+2}{2}\right]$

L.H.S $= (K+1)\frac{(K+2)}{2}$

R.H.S = $S(K+1) = (K+1)\frac{(K+2)}{2}$

L.H.S = R.H.S Hence proved

$n+n+1$
$2n+1$

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(2) $S(n) = 1+3+5+ \ldots (2n-1)$

prove by MI $S(n) = n^2$

~~Let S n~~

to prove)

$S(n) = 1+3+5+ \ldots (2n-1) = n^2$

$S(1) = 1 = 1$

true for $n=1$

Let it true for $n=K$

$S(K) = 1+3+5+ \ldots (2K-1) = K^2 \quad -①$

Now will prove it for $n = K+1$

L.H.S $= S(K+1) = 1+3+5+ \ldots (2K-1) + (2K+1)$

$= K^2 + (2K+1)$

$= (K+1)^2$

$=$ R.H.S

Hence proved

(3) Prove by MI that

$(n+1)^2 + (n+2)^2 + \ldots (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$

for all $n \in N$

to prove

$(n+1)^2 + (n+2)^2 + \dots (n+n)^2 = \frac{n(2n+1)(7n+1)}{6}$

for $n=1$

$L.H.S = 2^2 = 4$

$R.H.S = \frac{1(2+1)(7+1)}{6} = 4$

$L.H.S = R.H.S$

Hence true for $n=1$

let it true for $n=K$

$(K+1)^2 + (K+2)^2 \dots (2K)^2 = \frac{K(2K+1)(7K+1)}{6}$

Now will check for $n=K+1$

$(K+1)^2 + (K+2)^2 + \dots (2K)^2 + (2K+2)^2$

$= \frac{(K+1)(2K+3)(7K+8)}{6}$

$L.H.S = \underbrace{(K+1)^2 + (K+2)^2 + \dots (2K)^2}$ $+ (2K+2)^2$

$= \frac{K(2K+1)(7K+1)}{6} + (2K+2)^2$

$= \frac{1}{6}\left[K[14K^2+9K+1] + 24[K^2+1+2K]\right]$

$= \frac{1}{6}[14K^3 + 33K^2 + 49K + 24]$

$= \frac{1}{6}(k+1)(2k+3)(7k+8)$

$= R.H.S$

Hence proved

Ques-4 Prove by MI,

$1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \ldots (n-1) \cdot 2 + n \cdot 1 = \frac{1}{6} n(n+1)(n+2)$ for $\forall n \in N$

Sol → let $n = 1$

$1 = \frac{1}{6} 1 (2)(3)$

$1 = 1$

true for $n = 1$

→ let it true for $n = k$

$1 \cdot k + 2 \cdot (k-1) + 3(k-2) + \ldots (k-1)2 + k \cdot 1 = \frac{1}{6} k(k+1)(k+2)$

→ Now will prove for $n = k+1$

$1 \cdot (k+1) + 2 \cdot (k) + 3(k-1) + \ldots k \cdot 2 + (k+1)1 + = \frac{1}{6}(k+1)(k+2)(k+3)$

$S(k+1) - S(k) = \frac{n(n+1)}{2} + (n+1)$

$= \frac{(n+1)(n+2)}{2}$

$$S(K+1) = \frac{(K+1)(K+2)}{2} + S(K)$$

$$= \frac{(K+1)(K+2)}{2} + \frac{K(K+1)(K+2)}{6}$$

$$= \frac{(K+1)(K+2)}{6}\left[3 + K\right]$$

$$= \frac{(K+1)(K+2)(K+3)}{6} = R.H.S$$

Hence proved

(5) Prove by M.I $n(n+1)(n+2)(n+3)$ is divisibly by 24.

Sol- $P(n) = n(n+1)(n+2)(n+3)$

for $n = 1$

$P(1) = 1(2)(3)(4)$
$= 24$

Clearly $P(1)$ divisible by 24

→ let $P(K)$ is divisibly by 24

$P(K) = K(K+1)(K+2)(K+3) = 24t$

Now will prove $P(K+1)$ is divisibly

$P(K+1) = (K+1)(K+2)(K+3)(K+4)$

$= K(K+1)(K+2)(K+3)$
$+ 4(K+1)(K+2)(K+3)$

$= 24t + 4 \cdot 6t_1$

$= 24(t + t_1)$

$P(K+1) = 24T$

clearly $P(K+1)$ is divisibly by 24

Hence proved

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