Tutorial-5

U20CS110
Krishna Pandey

1. On a laboratory assignment, if equipment is working, the density function of the observed outcome, X if

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Calculate $P\left(x \leq \frac{1}{3}\right)$

(b) what is the probability that x will exceed 0.5

Sol—

ⓐ $$P\left(x \leq \frac{1}{3}\right) = \int_0^{1/3} f(x)\,dx$$

$$= \int_0^{1/3} 2(1-x)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_0^{1/3}$$

$$= \frac{2}{3} - \frac{1}{9}$$

$$= \cancel{\frac{11}{18}} \quad \frac{5}{9}$$

(ii) $$P(x \geq 0.5) = 1 - P(x \leq 0.5)$$

$$= 1 - \int_0^{1/2} 2(1-x)\,dx$$

Page No.:
Date: / /

$$= 1 - \left[2x - x^2\right]_0^{1/2}$$

$$= 1 - \left[1 - \frac{1}{4}\right] = \frac{1}{4}$$

2. Let X denote the amount of time in hours that a battery on a solar calculator will operate adequately between exposures to light sufficient to recharge the battery. The density function for X is given by $F(x) = \left(\frac{50}{6}\right)x^{-3} \quad 2 < x < 10$

(a) Find the average time that a battery will last before needing to be recharged

(b) Find $E[X^2]$, and use it to find the variance of X.

Sol

$$f(x) = \frac{50}{6} x^{-3} \qquad 2 < x < 10$$

(a)

$$E(X) = \int_2^{10} x \cdot f(x)\, dx$$

$$= \int_2^{10} x \cdot \frac{50}{6} x^{-3}\, dx$$

$$= \frac{50}{6}\left[\frac{x^{-1}}{-1}\right]_2^{10}$$

$$= -\frac{50}{6}\left[\frac{1}{10} - \frac{1}{2}\right] = \frac{10}{3}$$

(b)

$$E(X^2) = \int_2^{10} x^2 f(x)\, dx$$

$\frac{\cancel{8}\,4}{1\cancel{0}} \times \frac{50}{6}$

$$= \int_2^{10} \frac{50}{6} x^{-1} \, dx$$

$$= \frac{50}{6} \ln \frac{10}{2}$$

$$= \frac{50}{6} \ln 5$$

Ques-3- Define a random variable having Normal distribution. State necessary and sufficient condition for a f^n to be a continious density F^n. Verify these condition for normal ~~condition~~. Distribution

Sol- $f(x) > 0 \quad \forall \, x \in R_n$

$$f(x) = \frac{1}{\sigma 2\pi} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad e^{-t} > 0, \; \frac{(x-\mu^2)}{2\sigma^2} > 0$$

$$f(x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2.\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx$$

$$\frac{x-\mu}{\sigma} = t$$

$$dx = \sigma \, dt$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{t^2}{2}} \sigma \, dt$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} \, dt$$

Page No.:
Date: / /

$$\frac{2}{\sqrt{2\cdot\pi}} \int_0^\infty e^{-t^2/2}\, dt$$

$\frac{t^2}{2} = u \qquad t\,dt = du \qquad \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-u} (2u)^{-1/2}\, du$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\sqrt{2}} \cdot \Gamma\frac{1}{2}$$

$$= \frac{1}{\sqrt{\pi}} \sqrt{\pi} = 1$$

It's continious density f^n

Sol-4 $\quad \mu = 3, \quad \sigma^2 = 16, \quad \sigma = 4$

$Z : \left(\frac{x-\mu}{\sigma}\right) \quad \& \quad F_Z(z) = \int_{-\infty}^{z} f_Z t\, dt$

(a) $P(x<11) = P\left(\frac{x-3}{4} < \frac{11-3}{4}\right)$

$$P\left(\frac{x-3}{4} < 2\right)$$

$$F_Z(2)$$

$$\phi(2)$$

(b) $P(x>-1) : P\left(\frac{x-3}{4} > \frac{-1-3}{4}\right)$

$$= P\left(\frac{x-3}{4} > -1\right)$$

Page No.:
Date: / /

$= P(Z > -1) = P(Z < 1)$

$= \phi(1)$

(c) $P(2 < x < 7) = P\left(\frac{2-3}{7} < \frac{x-3}{4} < \frac{7-3}{4}\right)$

$P(-4 < Z < 1)$

$\phi(1) - \phi(-1/4)$

$\phi(1) + \phi(1/4) - 1$

Q-5- $\mu = 500$, $\sigma = 100$ $x \sim N(500, 100^2)$

(a) $P(x < 600)$ - for single student

$P(x < 600) = P\left[\left(\frac{x-500}{100}\right) < \frac{600-500}{100}\right)$

$= P(Z < 1) = \phi(1)$

for 5 students $(\phi(1))^5$

(b) $P(x > 640) = 1 - P\left(\frac{x-500}{100} < \frac{140}{100}\right)$

$= 1 - P\left(Z - \frac{14}{10}\right)$

$= 1 - P(Z < 1.4) = 1 - \phi(1.4)$

for 3 students $(1 - \phi(1.4))^3$

Page No.:
Date: / /

(c) $P(x < 640) = \phi(1.4) =$ for 2 students, $(\phi(1.4))^2$

Total $= [1 - \phi(1.4)]^3 [\phi(1.4)]^2$

Sol-6 $\mu = 40, \sigma = 4, x \sim N(40, 16)$

$P(x > 50) = 1 - P(x \cancel{<} 50)$

$= 1 - P\left(\frac{x-40}{4} < \frac{50-40}{4}\right)$

$= 1 - P(z > 2.5) = 1 - \phi(2.5)$

$P(x < 50) = \phi(2.5)$

So, out of 4, 2 will exceed 50 and 2 will not

Sol-7 $w = rv^2, r = 3$

$v \sim N(6, 1)$

(a) $E(w) = E(rv^2)$ $\quad E(v) = 6$

$= r[(v^2)$ $\quad E^2(v) = 36$

$= r[V(v) + E^2(v)]$ $\quad V(v) = 1$

$= 3[1+36] = 111$

(b) $P(w > 120) = P(3 \cdot v^2 > 120)$

$= P(v^2 > 40)$

$= 1 - P(v < \sqrt{40})$

$= 1 - P\left(\frac{v-6}{1} < \frac{\sqrt{40}-6}{1}\right) = 1 - P(z < 0.3245)$

$= 1 - \phi(0.3245)$

Page No.:
Date: / /

Sol-8 Joint PMF, let x and y be d.r.v

$$P_{x,y}(x_i, y_i) = P(x = x_i, y = y_i)$$

where $(x_i, y_i) \in R_{x+y} \leq R^2$

(i) $P_{x,y}(x_i, y_i) \geq 0$

$$\sum \sum P_{x,y}(x_0, y_i) = 1$$

$(x_i, y_i) \in R_{xy}$

Joint Probability Density function

let x & y be C.R.V $f_{x,y}(x_i; y_i) = F(x(x)\ y(y))$

where $(x_i, y_i) \in \{-\infty, \infty\}$

(i) $f_{x,y}(x_i, y_i) \geq 0$

(ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x,y}(x, y)\, dx dy = 1$

Sol-9 $x = 0, 1, 2, 3$ let $x = 3$

$y = 0, 1, 2, 3$

$R_{x,y} = \{(0,0), (0,1), ((0,2) \ldots (3,3)\}$

$$R_{x,y}(x_i, y_i) = \frac{{}^3C_x \; {}^4C_4 \; {}^5C_3}{{}^{12}C_3} \cdot (x_i, y_i)$$

	0	1	2	3	Sum
0	1/22	2/11	3/22	1/55	84/220
1	3/22			0	108/220
2	3/44	3/55	0	0	27/220
3	1/220	0	0	0	7/220

Total sum = 1

Sol-10 $\quad f(x, y) = \begin{cases} 2 e^{-x} e^{-2y} & 0 < x < \infty, \ 0 < y < \infty \\ & \text{otherwise} \end{cases}$

(a) $P\{x > 1, y < 1\} = \int_0^1 \int_1^\infty 2 e^{-x} e^{-2y} \, dx\, dy$

$$2 \left[\frac{e^{-2y}}{2} \right]_0^1 \left[\frac{e^{-x}}{-1} \right]_1^\infty$$

$$(e^{-2} - e^{-0}) \ (e^{-\infty} - e^{-1})$$

$$(1 - e^{-2}) \ (e^{-1})$$

(b) $P(x < y) = \int_0^\infty \int_0^y 2 e^{-x} e^{-2y} \, dx\, dy$

$$= \int_0^\infty 2 e^{-2y} (1 - e^{-y}) \, dy$$

$$= 2 \int_0^\infty (e^{-2y} - e^{-3y}) \, dy$$

$$= 2 \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{2 \cdot 1}{6}$$

$$= 1/3$$

Page No.:
Date: / /

(c) $P(x<a) = \int_0^a \int_0^\infty 2 \cdot e^{-x} e^{-2y} \, dx \, dy$

$$= \int_0^a 2e^{-x} \left(\frac{e^{-\infty} - e^0}{2} \right) dx$$

$$= \int_0^a e^{-x} dx = \left(\frac{e^{-x}}{-1} \right)_0^a$$

$$= (1 - e^{-a})$$