Tutorial: 7
Parameter Estimation

1. A random sample of size 9 yields the following observation on the random variable X, the coal consumption in millions of tons by electric utilities for a given year:
   406 395 400 450 390 410 415 401 408
   What is the average number of tons of coal used by electric utilities across the country per year? Also find median and mode of this data. By stating the differences in formula calculate the sample standard deviation and variance.
2. Let X be the height of a randomly chosen individual from population. In order to estimate the mean and variance of X, we observe a random sample $x_1, x_2, \dots, x_7$ .Thus x_i 's are independent and individual distribution having the same distribution as X. We obtain the following values (in centimeters):
   166.8 , 171.4 , 169.1 , 178.5 , 168 , 157.9 , 170.1
   Find the values of the sample mean, sample variance and sample standard deviation for the observed sample. $(Ans : \bar{x} = 168.8 , s^2 = 37.7 \;\&\; s = 6.1)$

3. Prove the following:
   a) If $\widehat{\Theta}_1$ is an unbiased estimator for Θ and W is a zero mean random variable, then $\widehat{\Theta}_2 = \widehat{\Theta}_1 + W$ is also an unbiased estimator of Θ .
   b) If $\widehat{\Theta}_1$ is an estimator for Θ such that $\mathrm{E}(\widehat{\Theta}_1) = \mathrm{a}\Theta + \mathrm{b}$, where $a \neq 0$, then $\widehat{\Theta}_2 = \frac{\widehat{\Theta}_1 - \mathrm{b}}{\mathrm{a}}$.

4. Let $x_1, x_2, \dots, x_n$ be a *iid* random sample from a distribution with mean $\mathrm{E}(x_i) = \Theta$,and variance $Var(x_i) = \sigma^2$.Consider the following two estimators for Θ :
   a) $\widehat{\Theta}_1 = x_1$
   b) $\widehat{\Theta}_2 = \frac{x_1 + x_2 + \dots + x_n}{n}$, then find $Var(\widehat{\Theta}_1)$ and $Var(\widehat{\Theta}_2)$.
   Also for n>1 show that $\widehat{\Theta}_1$ is minimum variance estimator.
   [**HINT:** $\mathrm{Var}(\alpha X + \beta Y) = \alpha^2 \mathrm{Var}(X) + \beta^2 \mathrm{Var}(Y)$] $(Ans : Var(\widehat{\Theta}_1) = \sigma^2 , Var(\widehat{\Theta}_2) = \sigma^2/n)$

5. For the following *iid* random samples, find the likelihood function :
   a) $x_i \sim$ Binomial (3, $\widehat{\Theta}$) and we have observed $(x_1, x_2, x_3, x_4) = (1,3,2,2)$. $(Ans : 27(\widehat{\Theta})^2(1 - \widehat{\Theta})^4)$
   b) $x_i \sim$ Bernoulli ($\widehat{\Theta}/3$) and we have observed $(x_1, x_2, x_3, x_4) = (1,0,1,1)$. $(Ans : \left(\widehat{\Theta}/3\right)^3 \left(1 - \widehat{\Theta}/3\right))$

6. Find the maximum likelihood of $\widehat{\Theta}$ based on this random sampleas mentioned in above example 4. $(Ans\ 1: \ ^1/_{3n} \sum_{i=1}^{n} x_i \ and \ 2: \widehat{\Theta} \approx 2)$

7. Let $x_1, x_2, \dots, x_n$ be a *iid* random sample from a geometric ($\widehat{\Theta}$) distribution, where $\widehat{\Theta}$ is unknown. Find the maximum likelihood of $\widehat{\Theta}$ based on this random sample. $(Ans : \mathrm{n}/\sum_{\mathrm{i}=1}^{\mathrm{n}} x_i)$

8. Let $x_1, x_2, \dots, x_n$ be a random sample from a uniform $(0, \widehat{\Theta})$ distribution, where $\widehat{\Theta}$ is unknown. Find the maximum likelihood of $\widehat{\Theta}$ based on this random sample. $(Ans : \max(x_1, x_2, \dots, x_n))$

9. Find Maximum likelihood function for the normal (n, σ) Population, for all three cases: mean unknown, variance unknown and, mean and variance both unknown.