**Q)*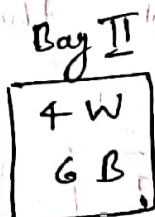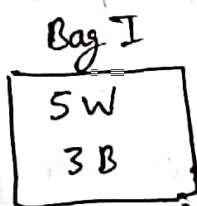* Let there are two identical bags. Bag I contains 5 white and 3 black balls. Bag II contains 4 white balls and 6 black balls. A bag is selected at random and 2 balls are drawn, each found to be black. Find the probability that bag I was selected?

Soln:

Bag I

| 5 W |
| 3 B |

Bag II

| 4 W |
| 6 B |

$A_1 \rightarrow$ Event of selecting Bag I

$A_2 \rightarrow$ " " " Bag II

$B \rightarrow$ Event of drawing black ball

$B|A_1 \rightarrow$ " " " " from bag I

$B|A_2 \rightarrow$ " " " " bag II

$P(A_1|B) = ?$

By Bayes Theorem

$$P(A_1|B) = \frac{P(B|A_1) \cdot P(A_1)}{P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2)}$$

$P(A_1) = \frac{1}{2} \quad ; \quad P(A_2) = \frac{1}{2}$

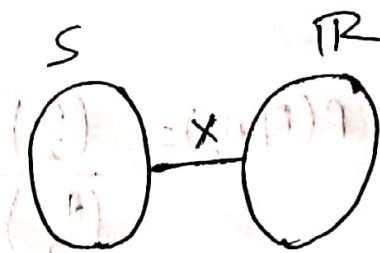$$P(B|A_1) = \frac{\binom{3}{2}}{\binom{8}{2}} \quad ; \quad P(B|A_2) = \frac{\binom{6}{2}}{\binom{10}{2}}$$

$$\therefore \quad P(A_1|B) = \frac{\dfrac{\binom{3}{2}}{\binom{8}{2}} \times \dfrac{1}{2}}{\dfrac{\binom{3}{2}}{\binom{8}{2}} \times \dfrac{1}{2} + \dfrac{\binom{6}{2}}{\binom{10}{2}} \times \dfrac{1}{2}} = 0.241$$

Random Variable :

Random variable is
a function from sample space to real no.

(Ex) Consider a random experiment as tossing a coin
3 times.

$S = \{$ HHH, HHT, HTH, THH, THT, TTT, TTH, HTT $\}$

$\qquad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow$

$\qquad s_1 \qquad\quad s_2 \qquad\quad s_3 \qquad\quad s_4 \qquad\quad s_5 \qquad\quad s_6 \qquad\quad s_7 \qquad\quad s_8$

$X:$ no of Heads

$X(s_1) = 3$ $\qquad\qquad X(s_5) = 1$

$X(s_2) = 2$ $\qquad\qquad X(s_6) = 0$

$X(s_3) = 2$ $\qquad\qquad X(s_7) = 1$

$X(s_4) = 2$ $\qquad\qquad X(s_8) = 1$

$R_X = \{0, 1, 2, 3\}$

Note: Random variable follows the law of algebra.

1-e $\qquad$ 9f X and Y are R.V then X+Y, X-Y, X·Y

and $\frac{X}{Y}; Y \neq 0$ are also random variable.

Define Y: no of tails

$Y(s_1) = 0$ $Y(s_5) = 2$

$Y(s_2) = 1$ $Y(s_6) = 3$

$Y(s_3) = 1$ $Y(s_7) = 2$

$Y(s_4) = 2$ $Y(s_8) = 2$

Define $R_{x-y} = \{ 3-0, \ 2-1, \ 2-1, \ 2-1, \ 1-2, \ 0-3, \ 1-2$

$1-2 \}$

$= \{ -3, \ -1, \ 1, \ 3 \}$