Tutorial-3
Subject: Discrete Mathematics (MA-221)
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Do all questions.

1. Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.
2. Use the truth table to show that the following De Morgan law is true:
$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$
3. Using truth table prove that the following statement is a tautology:
$$[(p \rightarrow q) \wedge p] \rightarrow q$$
4. Translate the following statements into English, where C(x) is "x is a comedian" and F(x) is "x is funny" and domain is all people.
(i) $\exists x\,(C(x) \rightarrow F(x))$ and (ii) $\exists x\,(C(x) \wedge F(x))$
(be careful, (i) and (ii) are not equivalent)
5. Determine the truth value of each of these statements if the domain is set of all integers:
(i) $\forall n\,(n+1 > n)$ (ii) $\exists n\,(2n = 3n)$
(iii) $\exists n\,(n = -n)$ (iv) $\forall n\,(n^2 \geq n)$
6. Express each of these system specifications using predicates, quantifiers, and logical connectives:
(i) At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 KBs of free space.
(ii) Each participant on the conference call whom the host of the call did not put on a special list was billed.
7. Determine the truth value of the statement : $\exists x \forall y\,(x \leq y^2)$
(i) if the domain is the positive real numbers
(ii) the non-zero integers
(iii) the nonzero real numbers
8. Use rules of inference to show that if
$\forall x\,(P(x) \vee Q(x))$ and $\forall x\,((\sim P(x) \rightarrow Q(x)) \rightarrow R(x))$ are true, then $\forall x\,(\sim R(x) \rightarrow P(x))$ is also true.
9. If $\forall x \exists y\, P(x,y)$ is true, does it necessarily follow that $\exists x \forall y\, P(x,y)$ is true? If not, give an example.
10. Let F(x,y) be the statement "x can fool y", where the domain is all people. Use quantifiers to express these statements:
(i) Everyone can fool Rohan. (ii) Rohit can fool everyone.
(iii) Everybody can fool somebody. (iv) There is no one who can fool everybody.
(v) Everyone can be fooled by someone. (vi) No one can fool both Rohan and Rohit.
(vii) Nancy can fool exactly two people. (viii) Noone can fool himself/herself.
(ix) There is someone who can fool exactly one person besides himself/herself.

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Prepared by: Dr. Amit Sharma, AMHD, SVNIT.