$$E(X) = \sum_{x \in R_X} x \cdot p_X(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\boxed{= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\cancel{x} \lambda^x}{\cancel{x}(x-1)!}}$$

$\because \; x! = x(x-1)!$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

Let $x-1 = t \Rightarrow x = t+1$

When; $x = 1$; $t = 0$

When; $x \to \infty$; $t \to \infty$

$$= e^{-\lambda} \sum_{t=0}^{\infty} \frac{\lambda^{t+1}}{t!}$$

$$= e^{-\lambda} \lambda \sum_{t=0}^{\infty} \frac{\lambda^t}{t!}$$

$$= e^{-\lambda} \cdot \lambda \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$= e^{-\lambda} \lambda \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$= e^{-\lambda} \lambda \cdot e^{\lambda}$$

$$= \lambda e^{-\lambda + \lambda}$$

$$= \lambda e^0$$

$$\boxed{E(X) = \lambda}$$

Scanned with CamScanner

$$V(X) = E(X^2) - (E(X))^2$$

$$= E(X(X-1)+X) - (E(X))^2$$

$$= E(X(X-1)) + E(X) - (E(X))^2$$

Now, $E(X(X-1)) = \sum_{x \in R_X} x(x-1)\, p_X(x)$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!}$$

Put $x-2 = t \Rightarrow x = t+2$

When $x = 2$; $t = 0$

When $x \to \infty$; $t \to \infty$

$$\Rightarrow E(X(X-1)) = e^{-\lambda} \sum_{t=0}^{\infty} \frac{\lambda^{t+2}}{t!}$$

$$= e^{-\lambda} \cdot \lambda^2 \sum_{t=0}^{\infty} \frac{\lambda^t}{t!}$$

$$= e^{-\lambda} \lambda^2 \cdot e^{\lambda}$$

$$= \lambda^2$$

Now; $V(X) = \cancel{\lambda^2} + \lambda - \cancel{(\lambda)^2}$

$$\boxed{V(X) = \lambda}$$

$$M_X(t) = E(e^{tX}) = \sum_{x \in R_X} e^{tx} \cdot p_X(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx} \cdot \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} \left[\frac{(\lambda e^t)^0}{0!} + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots \right]$$

$$= e^{-\lambda} \left[1 + \frac{(\lambda e^t)}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots \right]$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$\boxed{M_X(t) = e^{-\lambda(1-e^t)}}$$