Imp

Distribution Function or Cumulative Distribution Function (CDF) ÷

Let X be a discrete r.v then CDF of X is defined as

$F_X(x) = P(X \le x)$ where X is r.v and x is observed value.

Example:

R.E ÷ Tossing a coin trice.

X: no of Heads. Find the CDF of X?

sol^n: $S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

$R_X = \{0, 1, 2, 3\}$; Also $p_X(0) = \frac{1}{8}$; $p_X(1) = \frac{3}{8}$; $p_X(2) = \frac{3}{8}$

$p_X(3) = \frac{1}{8}$

$F_X(0) = P(X \le 0)$

$= \frac{1}{8}$

$F_X(1) = P(X \le 1) = p_X(0) + p_X(1)$

$= \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

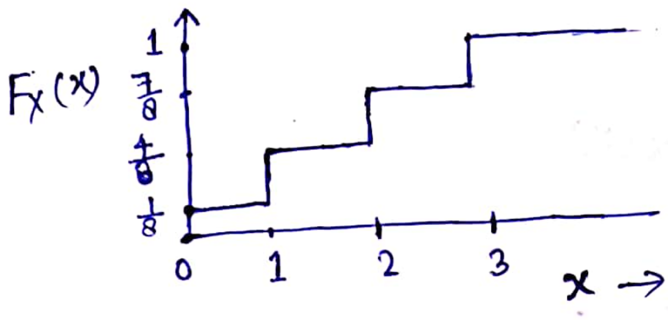
$F_X(2) = P(X \le 2) = p_X(0) + p_X(1) + p_X(2)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

$F_X(3) = P(X \le 3) = p_X(0) + p_X(1) + p_X(2) + p_X(3)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8}$

Plot of CDF :-



$$F_X(x) = \begin{cases} 0 & ; \quad x < 0 \\ \frac{1}{8} & ; \quad 0 \le x < 1 \\ \frac{4}{8} & ; \quad 1 \le x < 2 \\ \frac{7}{8} & ; \quad 2 \le x < 3 \\ 1 & ; \quad x \geq 3 \end{cases}$$

Properties of CDF :-

(i) $F_X(\infty) = 1$

e.g. $P(X \le x) = P(X \le \infty) = 1$

(ii) $F_X(-\infty) = 0$

e.g. $P(X < -\infty) = 0$

(iii) $0 \le F_X(x) \le 1$

(iv) CDF is non-decreasing function i.e either increasing or remain constant.

(v) CDF is right continuous.

i.e

$$\lim_{h \to 0} F_X(x+h) = F_X(x)$$

Imp

Interval properties of CDF :-

(i) $P(a < X \leq b) = F_X(b) - F_X(a)$ (Original form)

(ii) $P(a < X < b) = \underbrace{P(a < X < b) + P(X = b)} - P(X = b)$

$= P(a < X \leq b) - P(X = b)$

$= F_X(b) - F_X(a) - P(X = b)$

(iii) $P(a \leq X < b) = P(a < X < b) + P(X = a)$

$= \underbrace{P(a < X < b) + P(X = b)} - P(X = b) + P(X = a)$

$= P(a < X \leq b) - P(X = b) + P(X = a)$

$= F_X(b) - F_X(a) - P(X = b) + P(X = a)$

(iv) $P_X(x_i) = F_X(x_i) - F_X(x_i^-)$

No.- 33

Q: $F_X(x) = \begin{cases} 0 & ; \; x < 0 \\ \frac{1}{8} & ; \; 0 \le x < 1 \\ \frac{4}{8} & ; \; 1 \le x < 2 \\ \frac{7}{8} & ; \; 2 \le x < 3 \\ 1 & ; \; x \geq 3 \end{cases}$

Find PMF of X for given C.D.F ?

Soln $\because$ $p_X(x_i) = F_X(x_i) - F_X(x_i^-)$

$p_X(0) = F_X(0) - F_X(0^-)$

$\Rightarrow p_X(0) = \frac{1}{8} - 0 = \frac{1}{8}$

$p_X(1) = F_X(1) - F_X(1^-)$

$p_X(1) = \frac{4}{8} - \frac{1}{8} = \frac{3}{8}$

$p_X(2) = F_X(2) - F_X(2^-)$

$p_X(2) = \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$

$p_X(3) = F_X(3) - F_X(3^-)$

$p_X(3) = 1 - \frac{7}{8} = \frac{1}{8}$

Thus PMF of X is

$X = x_i$	0	1	2	3
$p_X(x_i)$	1/8	3/8	3/8	1/8

3.2

Q: 23 Let X be r.v whose CDF is given as

$$F_X(x) = \begin{cases} 0 & ; \quad x < 0 \\ 0.06 & ; \quad 0 \le x < 1 \\ 0.19 & ; \quad 1 \le x < 2 \\ 0.39 & ; \quad 2 \le x < 3 \\ 0.67 & ; \quad 3 \le x < 4 \\ 0.92 & ; \quad 4 \le x < 5 \\ 0.97 & ; \quad 5 \le x < 6 \\ 1 & ; \quad x \geq 6 \end{cases}$$

Find (a) $P(X=2)$ (b) $P(X>3)$ (c) $P(2 \le X \le 5)$ (d) $P(2 < X < 5)$

Soln

(a) $P(X=2) = F_X(2) - F_X(\bar{2})$

$= 0.39 - 0.19 = 0.20$

(b) $P(X>3) = 1 - P(X \le 3)$

$= 1 - F_X(3)$

$= 1 - 0.67 = 0.33$

(c) $P(2 \le X \le 5) = P(2 < X \le 5) + P(X=2)$

$= F_X(5) - F_X(2) + P(X=2)$

$= 0.97 - 0.39 + 0.20$

$= 0.78$

(d) $P(2<X<5) = \underbrace{P(2<X<5) + P(X=5)} - P(X=5)$

$= P(2<X\le 5) - P(X=5)$

$= F_X(5) - F_X(2) - [F_X(5) - F_X(5^-)]$

$= -F_X(2) + F_X(5^-)$

$= -0.39 + 0.92$

$= 0.53$

Sec 3.2

Q.24 The CDF of X is a follows :

$$F_X(x) = \begin{cases} 0 & ; x<1 \\ 0.30 & ; 1\le x<3 \\ 0.40 & ; 3\le x<4 \\ 0.45 & ; 4\le x<6 \\ 0.60 & ; 6\le x<12 \\ 1 & ; x\geq 12 \end{cases}$$

a) What is pmf of X?

b) Using just cdf, compute $P(3\le X\le 6)$ and $P(4\le X)$

$\underline{Sol^n}$: (a) $p_X(1) = F_X(1) - F_X(\bar{1}) = 0.30 - 0 = 0.30$

$p_X(3) = F_X(3) - F_X(\bar{3}) = 0.40 - 0.30 = 0.10$

$p_X(4) = F_X(4) - F_X(\bar{4}) = 0.45 - 0.40 = 0.05$

$p_X(6) = F_X(6) - F_X(\bar{6}) = 0.60 - 0.45 = 0.15$

$p_X(12) = F_X(12) - F_X(\bar{12}) = 1 - 0.60 = 0.40$

Thus pmf of X is given as

$X = x_i$	1	3	4	6	12
$p_X(x_i)$	0.30	0.10	0.05	0.15	0.40

(b) $P(3 \le X \le 6) = P(3 < X \le 6) + P(X=3)$

$= F_X(6) - \cancel{F_X(3)} + \cancel{F_X(3)} - F_X(\bar{3})$

$= 0.60 - 0.30$

$= 0.30$

$$P(4 \le X) = P(X \geq 4) = 1 - P(X < 4)$$

$$= 1 - [P(X<4) + P(X=4) - P(X=4)]$$

$$= 1 - [P(X \le 4) - P(X=4)]$$

$$= 1 - [F_X(4) - \{F_X(4) - F_X(\bar{4})\}]$$

$$= 1 - [\cancel{F_X(4)} - \cancel{F_X(4)} + F_X(\bar{4})]$$

$$= 1 - F_X(\bar{4})$$

$$= 1 - 0.40$$

$$= 0.60$$ Ans