Ex Tossing a coin is also a Bernoullian trial as either we will get Head or Tail.

Ex Rolling a dice can be considered as Bernoullian trial if we consider 6 as success and rest are failure

Binomial Distribution :- A discrete r.v X is said to have Binomial dist^n if its p.m.f is given as

$$p_X(x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0, 1, 2, \ldots n.$$

where $n \rightarrow$ No of trials (fixed)

$X \rightarrow$ No of success in fixed no of 'n' trials (Random Variable)

$p \rightarrow$ prob. of success

$1-p = q \rightarrow$ prob. of failure, s.t $p+q=1$

Note :- All the trials are independent.
It is denoted as $Bin(n, p)$.

$$\underset{1}{\overset{p}{\nearrow}} \quad \underset{2}{\overset{p}{\nearrow}} \quad \underset{3}{\_} \quad \overset{q}{\nearrow} \quad \nearrow \quad \_ \quad \_ \quad \_ \quad \underset{n}{\overset{p}{\nearrow}}$$

$$\binom{n}{x} p^{x} q^{(n-x)}$$

$x \rightarrow$ no of success

$(n-x) \rightarrow$ no of failure

$$\underbrace{p \cdot p \cdot p \cdots p}_{x \text{ times}} \quad \underbrace{q \, q \, q \cdots q}_{(n-x) \text{ times}}$$

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Q: (1) Verify that it is proper p.m.f.

2) Find Mean and Variance for Binomial distⁿ.

Solⁿ: (1) To verify that it is p.m.f, we need to check two conditions

$$p_X(x) \geq 0 \quad \text{and} \quad \sum_{x \in R_X} p_X(x) = 1$$

$$p_X(x) = \binom{n}{x} p^x q^{n-x} \geq 0 \quad \forall x \quad (\text{Trivially true})$$

Now,
$$\sum_{x=0}^{n} p_X(x) = \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}$$
$$= (p+q)^n$$
$$= 1^n \quad (\because p+q=1)$$
$$= 1$$

$$(p+q)^n = \binom{n}{0} p^0 q^n + \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \cdots \binom{n}{n} p^n q^0$$

$\Rightarrow$ $p_X(x)$ is proper p.m.f.

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$$E(X) = \sum_{x \in R_X} x \cdot p_X(x)$$

$$= \sum_{x=0}^{n} x \cdot \binom{n}{x} p^{x} q^{n-x} \quad ; \quad q = 1-p$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!\,(n-x)!} p^{x} (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!\,(n-x)!} p^{x} q^{n-x}$$

Let $x-1 = t \Rightarrow x = t+1$

When $x = 1$; $t = 0$

When $x = n$; $t = n-1$

$$= \sum_{t=0}^{n-1} \frac{n!}{t!\,(n-t-1)!} p^{t+1} q^{n-t-1}$$

$$= \sum_{t=0}^{n-1} \frac{n\,(n-1)!\, p^{t+1} q^{n-t-1}}{t!\,(n-t-1)!}$$

$$= np \sum_{t=0}^{n-1} \frac{(n-1)!\, p^{t} q^{n-t-1}}{t!\,(n-t-1)!}$$

$$= np \sum_{t=0}^{(n-1)} \binom{n-1}{t} p^{t} q^{n-t-1}$$

Binomial expansion of $(p+q)^{n-1}$

$$= np\,(p+q)^{n-1}$$

$\because (p+q) = 1$

$$= np \cdot 1$$

$$\boxed{E(X) = np}$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= E(X(X-1)+X) - (E(X))^2$$
$$= E(X(X-1)) + E(X) - (E(X))^2$$

Now, $E(X(X-1)) = \sum_{x \in R_X} x(x-1)\, p_X(x)$

$$= \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!\; p^x \cdot q^{n-x}}{x!\,(n-x)!}$$

$$= \sum_{x=2}^{n} \frac{n!\; p^x q^{n-x}}{(x-2)!\,(n-x)!}$$

$$= \sum_{x=2}^{n} \frac{n!\; p^x (1-p)^{n-x}}{(x-2)!\,(n-x)!}$$

Let $x-2 = t \Rightarrow x = t+2$

when $x = 2$; $t = 0$

when $x = n$; $t = n-2$

$$= \sum_{t=0}^{n-2} \frac{n(n-1)(n-2)!\; p^{t+2} (1-p)^{n-t-2}}{t!\,(n-t-2)!}$$

$$= n(n-1)p^2 \sum_{t=0}^{n-2} \frac{(n-2)!}{t!\,(n-t-2)!} p^t (1-p)^{n-t-2}$$

$$= n(n-1)p^2 \sum_{t=0}^{n-2} \binom{n-2}{t} p^t q^{n-t-2}$$

$$= n(n-1)\,p^2 \cdot (p+q)^{n-2}$$

$$= n(n-1)p^2 \cdot 1 \qquad (\because p+q=1)$$

$$= n(n-1)p^2$$

Now;

$$V(X) = n(n-1)p^2 + np - (np)^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p) \qquad \because 1-p = q$$

$$\boxed{V(X) = npq}$$

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