Binomial Approximation of Hypergeometric Distribution :-

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When $M \longrightarrow \infty$ (very large)

and $\frac{K}{M} \longrightarrow p$ (where $0 < p < 1$)

then $HG(n, K, M) \longrightarrow Bin(n, p)$

So, $E(X) = n.\frac{K}{M} = n.p$ $\left(\text{as } \frac{K}{M} \rightarrow p\right)$

$$V(X) = n.\frac{K}{M}\left(1 - \frac{K}{M}\right)\left(\frac{M-n}{M-1}\right)$$

$$= n.p(1-p)\lim_{M\to\infty}\left(\frac{M-n}{M-1}\right) \quad \left(\text{as } \frac{K}{M}\to p,\ M\to\infty\right)$$

$$= n.p(1-p)\left(\frac{1}{1}\right)$$

$$V(X) = n.p(1-p)$$

$$\boxed{V(X) = npq}$$

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69. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 6 examined that have a defective compressor.

a. Calculate $P(X = 4)$ and $P(X \leq 4)$

b. Determine the probability that X exceeds its mean value by more than 1 standard deviation.

c. Consider a large shipment of 400 refrigerators, of which 40 have defective compressors. If X is the number among 15 randomly selected refrigerators that have defective compressors, describe a less tedious way to calculate (at least approximately) $P(X \leq 5)$ than to use the hypergeometric pmf.

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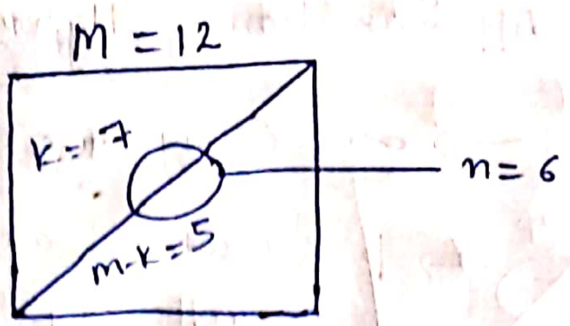

$K = 7$ the refrigerators who are defective

$M-K = 5$ the refrigerator who are less defective.

X: the no among the first 6 examined that have a defective compressor

$$p_X(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}} \; ; \; x = 0, 1, 2, \cdots, n$$

a) $$P(X=4) = \frac{\binom{7}{4}\binom{12-7}{6-4}}{\binom{12}{6}} = \frac{\binom{7}{4}\binom{5}{2}}{\binom{12}{6}}$$

$$= 0.3787$$

$$P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= 1 - P(X > 4)$$

$$= 1 - [P(X=5) + P(X=6)]$$

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$$= 1 - \left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}} \right]$$

$= 1 - 0.1212121$

$= 0.8787879$

(b) $P(X - \mu > 1 \cdot \sigma_X)$?

$\mu = E(X) = n \cdot \frac{K}{m} = \cancel{6} \times \frac{7}{\cancel{12}_2} = 3.5$

$V(X) = n \cdot \frac{K}{m} \left(1 - \frac{K}{m}\right) \left(\frac{m-n}{m-1}\right)$

$= 6 \times \frac{7}{12} \left(1 - \frac{7}{12}\right) \left(\frac{12-6}{12-1}\right)$

$= 6 \times \frac{7}{12} \times \frac{5}{12} \times \frac{6}{11} = 0.7954545$

$\sigma_X = 0.8918826 = \sqrt{V(X)}$

So, $P(X - 3.5 > 1 \times 0.8918) = P(X > 3.5 + 0.8918)$

$P(X > 4.3918) = P(X \geq 5)$

$= P(X = 5) + P(X = 6)$

$= 0.1212$

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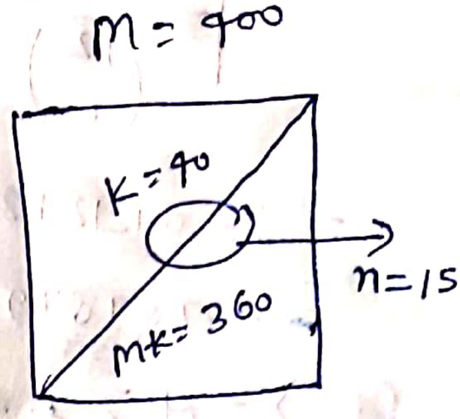
(c) $M = 400$ (very large) ; $K = 40$

$$\frac{K}{M} = \frac{40}{400} = 0.1 \; (= p)$$



$\because$ $M \to \infty$; $\frac{K}{M} \to p$ then

$$HG(n, K, M) \longrightarrow Bin(n, p)$$

__Note:__ Using Binomial distn is less tedious way as we have to deal with two parameters only.

X : no of defective compressor among the randomly selected refrigerator.

$$p_X(x) = \binom{n}{x} p^x q^{n-x}$$

Here $n = 15$; $p = 0.10$; $q = 0.90$

$$P(X \le 5) = p_X(0) + p_X(1) + p_X(2) + \cdots + p_X(5)$$

$$= \binom{15}{0}(0.1)^0(0.9)^{15-0} + \binom{15}{1}(0.1)^1(0.5)^{15-1} + \cdots + \binom{15}{5}(0.1)^5(0.9)^{15-5}$$

$$= 0.9978$$