Chebyshev's Inequality :-

Let X be a r.v with $E(X) = \mu$ and variance $\sigma^2 = V(X)$, then

① $$\boxed{P[\,|X-\mu| \geq \epsilon\,] \leq \frac{\sigma^2}{\epsilon^2}}$$; where $\epsilon > 0$ a small positive real no.

Also, this can be written as

② $$\boxed{P[\,|X-\mu| < \epsilon\,] \geq 1 - \frac{\sigma^2}{\epsilon^2}}$$

Note: If we put $\epsilon = k\sigma$ then above can be written as

$$P[\,|X-\mu| < k\sigma\,] \geq 1 - \frac{\sigma^2}{k^2\sigma^2}$$

③ $$\Rightarrow \boxed{P[\,|X-\mu| < k\sigma\,] \geq 1 - \frac{1}{k^2}}$$

Also,

④ $$\boxed{P[\,|X-\mu| \geq k\sigma\,] \leq \frac{1}{k^2}}$$

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Ex: The r.v Y has pmf given in the accompanying table

y	45	46	47	48	49	50	51	52	53	~~55~~ 54	55
$p_Y(y)$	0.05	0.10	0.12	0.14	0.25	0.17	0.06	0.05	0.03	0.02	0.01

find $E(Y)$, $SD(Y)$ and $P(|Y-\mu| \geq t)$ for $t = 2\sigma, 3\sigma$

Soln

$$E(Y) = \sum_{y=45}^{55} y \cdot p_Y(y) = 48.84$$

$$V(Y) = EY^2 - (EY)^2$$

$$= 2389.84 - (48.84)^2$$

$$= 4.4944$$

$$\Rightarrow \sigma = \sqrt{V(X)} = 2.12$$

By chebyshav inequality

$$P[|Y-\mu| \geq t) \leq \frac{\sigma^2}{t^2} = \frac{\sigma^2}{(2\sigma)^2} = \frac{1}{4}$$

$$\Rightarrow P(|Y-\mu| \geq t) \leq \frac{1}{4}$$

Similarly for $t = 3\sigma$

$$P(|Y-\mu| \geq t) \leq \frac{1}{9}$$