Classification of Random Variable :-

(1) Descrete Random Variable (2) Continuous Random Variable

Descrete Random Variable :- If the range space of random variable (r.v) is countable (either finite or countably infinite) then the r.v is said to be descrete r.v.

e.g $R_X = \{0, 1, 2, 3\}$

$R_{X-Y} = \{-3, -1, 1, 3\}$ } finite

$R_Z = \{1, 2, 3, 4, \ldots\}$ ⟶ Countably infinite

Section 2.4

Q.58 Show that for any 3 events A, B and C with $P(C) > 0$,

$$P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$$

Proof: From L.H.S

$$P(A \cup B \mid C) = \frac{P((A \cup B) \cap C)}{P(C)}$$

$$= \frac{P((A \cap C) \cup (B \cap C))}{P(C)}$$

$$= \frac{P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))}{P(C)}$$

$$= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)}$$

$$= P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$$

$$= \text{R.H.S}$$

#

Probability Mass Function (PMF) ÷

Let X be descrete r.v associated with sample space S. Let $R_X = \{x_1, x_2, \dots x_n\}$ be the range space of X and each x_i is associated with a number $p_X(x_i)$ or $P(X=x_i)$ then $p_X(x_i)$ is said to be PMF of X if it satisfies the following two properties

i) $p_X(x_i) \geq 0$

ii) $\sum_{x_i \in R_X} p_X(x_i) = 1$

$p_X(x_i)$

$X \to$ r.v

$x_i \to$ observed value of X

Ex: Consider a random experiment, Tossing a coin 3 times

X: no of Heads (Random Variable)

$S = \{ \underset{s_1}{HHH}, \underset{s_2}{HHT}, \underset{s_3}{HTH}, \underset{s_4}{HTT}, \underset{s_5}{TTT}, \underset{s_6}{TTH}, \underset{s_7}{THT}, \underset{s_8}{THH} \}$ → sample space

$X(s_1) = 3$

$X(s_2) = 2$

$X(s_3) = 2$

$X(s_4) = 1$

$X(s_5) = 0$

$X(s_6) = 1$

$X(s_7) = 1$

$X(s_8) = 2$

$R_X = \{0, 1, 2, 3\}$

Now calculate $p_X(x_i)$ where $x_i \in R_X$

$p_X(0) = \frac{1}{8}$ {Only TTT favourable to this case among all 8 outcomes}

$p_X(1) = \frac{3}{8}$

$p_X(2) = \frac{3}{8}$

$p_X(3) = \frac{1}{8}$

$\Rightarrow$

$X = x_i$	0	1	2	3
$p_X(x_i)$	$1/8$	$3/8$	$3/8$	$\frac{1}{8}$

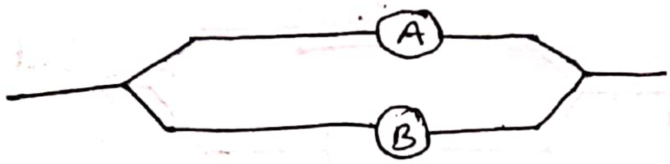
$\because$ $p_X(x_i) \geq 0$ and $\sum_{x_i \in R_X} p_X(x_i) = 1$

$\Rightarrow p_X(x_i)$ is PMF of X. #

Some More Important Problems

Ex 1 ~~The event E_1~~

A system consists of two components A and B connected parallely in the following way:



The event E_1 denotes that the component A works with prob 0.9 and the event E_2 denotes that the component B works with prob. 0.8. Find the prob. that the system works?

Soln: For functioning the system atleast one of these two component should work.

i.e. ~~$P(A \cup B)$~~ $P(E_1 \cup E_2) = ?$

$$P(E_1) = 0.9 \; ; \; P(E_2) = 0.8$$

Also, components A and B works independently, so

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= P(E_1) + P(E_2) - P(E_1) \cdot P(E_2) \qquad (\because \text{A and B are independent})$$

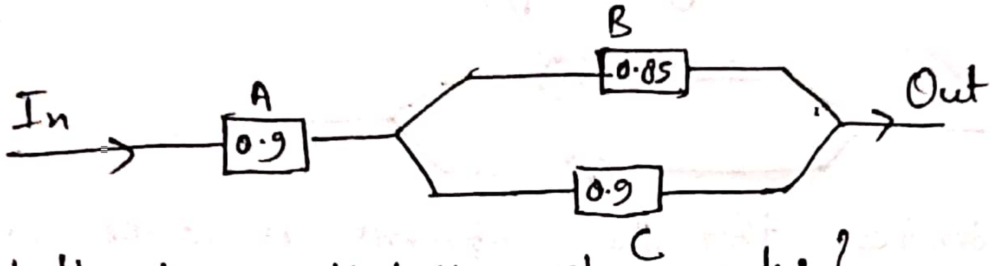
$$= 0.9 + 0.8 - 0.9 \times 0.8$$

$$= 1.70 - 0.72 = 0.98 \text{ Ans}$$

Ex-2 : In the following system A, B and C are the components works with probabilities

$P(\{A \text{ works}\}) = 0.9$, $P(\{B \text{ works}\}) = 0.85$

$P(\{C \text{ works}\}) = 0.9$



Find the prob. that the system works?

Sol^n Required prob.

$$P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$$

$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

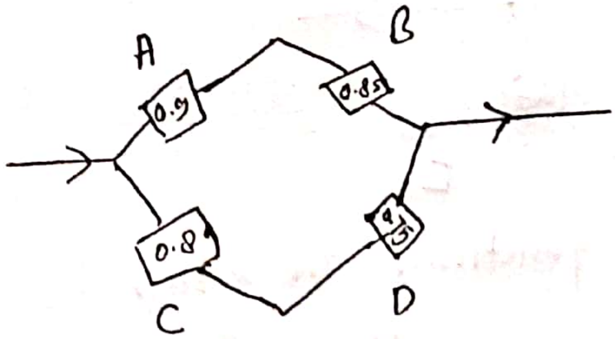
$\because$ A, B and C works independently

$$\Rightarrow P(A \cap (B \cup C)) = P(A)P(B) + P(A) \cdot P(C) - P(A) \cdot P(B) \cdot P(C)$$

$$= 0.9 \times 0.85 + 0.9 \times 0.9 - 0.9 \times 0.85 \times 0.9$$

$$= 0.8895$$

Ex. 3 :



In the system: $P(\{A \text{ works}\}) = 0.9$; $P(\{B \text{ works}\}) = 0.85$
$P(\{C \text{ works}\}) = 0.8$; $P(\{D \text{ works}\}) = 0.75$

Find the prob. that the system works?

Sol^n ∵ Components A and B are in series so they must functions simultaneously
similarly theo events C and D must work simultaneously

So; $P(\text{system works}) = P((A \cap B) \cup (C \cap D))$

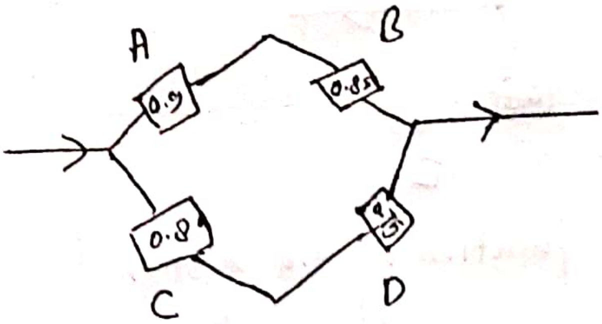
$$= P(A \cap B) + P(C \cap D) - P(A \cap B \cap C \cap D)$$

$$= P(A) \cdot P(B) + P(C) \cdot P(D) - P(A) \cdot P(B) P(C) P(D)$$

$$= 0.9 \times 0.85 + 0.8 \times 0.75 - 0.9 \times 0.85 \times 0.8 \times 0.75$$

$$= \underline{\underline{0.906}}$$

Ex. 3 :



In the system : $P(\{A \text{ works}\}) = 0.9$; $P(\{B \text{ works}\}) = 0.85$
$P(\{C \text{ works}\}) = 0.8$; $P(\{D \text{ works}\}) = 0.75$

Find the prob. that the system works?

Soln $\because$ Components A and B are in series so they must functions simultaneously
similarly the events C and D must work ~~...~~ simultaneously $\therefore$

So; $P(\text{system works}) = P((A \cap B) \cup (C \cap D))$

$$= P(A \cap B) + P(C \cap D) - P(A \cap B \cap C \cap D)$$

$$= P(A) \cdot P(B) + P(C) \cdot P(D) - P(A) \cdot P(B) P(C) P(D)$$

$$= 0.9 \times 0.85 + 0.8 \times 0.75 - 0.9 \times 0.85 \times 0.8 \times 0.75$$

$$= \underline{\underline{0.906}}$$