Tutorial-1

U20CS110
B110

Ques 1 Four universities - 1, 2, 3, 4 are palying basketball tournament. (1 will play with 2 and 3 with 4)

- Then two winners will play for the championship and 2 loosers will also play. 1324 means (1 beat 2, 3 beat 4 and 1 beat 3 and 2 beat 4}
- List all outcomes in S

In 1st round (1-2 and 3-4 will play)

So total outcomes when 1 wins :
{1324, 1342, 1423, 1432}

When 2 wins
{2314, 2341, 2413, 2431}

When 3 wins
{3124, 3142, 3214, 32,41}

When 4 wins
{4123, 4132, 4213, 4231}

So total 16 outcomes

- Let A denotes the event that 1 wins the tournam
list outcomes in A.

A = {1324, 1342, 1423, 1432}

→ Let B denote the event that 2 gets into the championship game. List outcomes in B

B = {2314, 2341, 2413, 2431, 4213, 4231
3214, 3241}

→ What are outcomes in $A \cup B$ and $A \cap B$.
What are outcomes in A'.

$A \cup B$ = {1324, 1342, 1423, 1432, 2314, 2341,
2413, 2431, 4213, 4231, 3214, 3241}

$A \cap B = \phi$

A' = when A not win
= {2413, 2431, 2314, 2341, 3214, 3241
3124, 3142, 4123, 4132, 4231, 4213}

2. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of 3 successive vehicles.
→ List all outcomes in the event A that all 3 vehicles go in the same direction

A = {LLL, RRR, SSS}

→ List all outcomes in the event B that all 3 vehicles takes different direction

B = {LRS, RSL, RLS, SRL, SLR
LSR}

$3 \times 3 \times 3$

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→ List all outcomes in the event C that exactly two of the 3 vehicles turn right.

$C = \{LRR, SRR, RLR, RRL, RSR, RRS$

→ List all outcomes in the event D that exactly two vehicles go in the same direction.

$D = \{LRR, RLR, RRL, SRR, RSR, RRS,$
$LSS, SLS, SSL, SSR, SRS, RSS$
$RLL, LRL, LLR, SLL, LSL, LLS \}$

→ List outcomes in D', $C \cup D$ and $C \cap D$

$D' = \{LLL, RRR, SSS, LRS, RLS, SLR, SRL$
$LSR, RSL \}$
$= \{$all in same dirc^n or all in different.

$C \cup D = D$

$C \cap D = C$

3. A certain system can experience 3 different types of defects. Let A_i $(i = 1, 2, 3)$ denote the event that the system has a defect of type i. Suppose that
$P(A_1) = 0.12$, $P(A_2) = 0.07$, $P(A_3) = 0.05$
$P(A_1 \cup A_2) = 0.13$, $P(A_1 \cup A_3) = 0.14$, $P(A_2 \cup A_3)$
$= 0.10$, $P(A_1 \cup A_2 \cup A_3) = 0.01$

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→ What is the probability that system does not have type 1 defect?

Sol-

$P(A_1) = 0.12 \quad P(A_2) = 0.07 \quad P(A_3) = 0.05$

$P(A_1 \cup A_2) = 0.13 \quad P(A_2 \cup A_3) = 0.10 \quad P(A_1 \cup A_3) = 0.14$

$$P(A_1 \cap A_2 \cap A_3) = 0.01$$

$$P(A_1') = 1 - P(A_1)$$
$$= 1 - 0.12 = 0.88$$

→ Probability that system has both type 1 and type 2 defects?

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$$
$$= 0.12 + 0.07 - 0.13$$
$$= 0.06$$

→ Probability that system has both type 1 and type 2 defects but not type 3 defect?

Sol-

$$P(A_1 \cap A_2 \cap \bar{A}_3) = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$$
$$= 0.06 - 0.01$$
$$= 0.05$$

→ What is the probability that system has at most two of these defects?

$$P(\text{at most } 2) = 1 - P(\text{all three})$$
$$= 1 - 0.01$$
$$= 0.99$$

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4. Consider randomly selecting a single individual and having that person test drive 3 different vehicles. Define events A_1, A_2 and A_3 by
A_1 = likes vehicle #1, A_2 = likes vehicle #2
A_3 = likes vehicles #3.
Suppose that $P(A_1) = 0.55$, $P(A_2) = 0.65$,
$P(A_3) = 0.70$, $P(A_1 \cup A_2) = 0.80$, $P(A_2 \cap A_3) = 0.40$
$P(A_1 \cup A_2 \cup A_3) = 0.88$.

→ What is the probability that the individual likes both vehicles #1 and #2?

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$$
$$= 0.55 + 0.65 - 0.80$$
$$= 0.40$$

→ Determine and interpret $P(A_2/A_3)$

$$P(A_2/A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{0.40}{0.70}$$
$$= 0.5714$$

means probability of A_2 when A_3 has been already occured.

→ Are A_2 and A_3 independent events. Answer in 2 different ways.

Sol
$$P(A_2 \cap A_3) = 0.40$$

$$P(A_2) \cdot P(A_3) = 0.65 \times 0.70$$

$$P(A_2 \cap A_3) \neq P(A_2) \cdot P(A_3)$$ ∴ are dependent event

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Now $P\left(\frac{A_2}{A_3}\right) \neq 0$ $\therefore$ A_2 and A_3 are dependent events

→ If you learn that individuals did not like vehicle #1, what now is the probability that he/she liked at least one of the other 2 vehicles.

Sol – $P[(A_2 \cup A_3) / \bar{A_1}) = \frac{P[(A_2 \cup A_3) \cap \bar{A_1})]}{P(\bar{A_1})}$

$\cancel{\frac{P[(A_2 \cup A_3)' \cup A_1)']}{P(\bar{A_1}}}$

$= \frac{P(A_2 \cup A_3 \cup A_1) - P(A_1)}{P(\bar{A_1})}$

$= \frac{0.88 - .55}{1 - 0.55}$

$= \frac{0.33}{0.45} = 0.7333$