Uniform Distribution: Let X be a continuous random variable then X is said to have a uniform distⁿ if its pdf is given as

$$f_X(x) = \begin{cases} \frac{1}{b-a} & ; a \le x \le b \\ 0 & ; \text{otherwise} \end{cases}$$

Note:
It is denoted as $X \sim U[a, b]$

Q: Verify that it is proper pdf and also find $E(X)$ and $V(X)$.

Solⁿ

$$\int_a^b f_X(x)\,dx = 1 \quad (\text{Claim})$$

$$\Rightarrow \int_a^b \frac{1}{b-a}\,dx = \frac{1}{b-a}\int_a^b dx$$

$$= \frac{1}{b-a}[x]_a^b$$

$$= \frac{1}{b-a}[b-a]$$

$$= 1$$

$$E(X) = \int_a^b x \cdot f_X(x)\,dx = \int_a^b x \cdot \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a} \frac{[b^2-a^2]}{2}$$

$$= \frac{1}{\cancel{(b-a)}} \frac{\cancel{(b-a)}\,(b+a)}{2}$$

$$\boxed{E(X) = \frac{a+b}{2}}$$

Next, $V(X) = E(X^2) - (E(X))^2$

So, $E(X^2) = \int_a^b x^2 \cdot f_X(x)\,dx$

$$= \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a} \int_a^b x^2\,dx = \frac{1}{b-a} \left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a} \left(\frac{b^3-a^3}{3}\right)$$

$$= \frac{1}{\cancel{(b-a)}} \frac{(\cancel{b-a})(b^2+a^2+ab)}{3}$$

$$= \frac{(b^2+a^2+ab)}{3}$$

Now, $V(X) = \frac{b^2+a^2+ab}{3} - \left[\frac{(a+b)}{2}\right]^2$

$$= \frac{b^2+a^2+ab}{3} - \frac{a^2+b^2+2ab}{4} = \frac{4b^2+4a^2+4ab-3a^2-3b^2-6ab}{12}$$

$$\boxed{V(X) = \frac{(b-a)^2}{12}}$$ #

2. Suppose the reaction temperature X (in °C) in a certain chemical process has a uniform distribution with $A = -5$ and $B = 5$.

a. Compute $P(X < 0)$.

b. Compute $P(-2.5 < X < 2.5)$.

c. Compute $P(-2 \le X \le 3)$.

d. For k satisfying $-5 < k < k + 4 < 5$, compute $P(k < X < k + 4)$.

Sec 4.1
Q.2
Sol^n

$$f_X(x) = \frac{1}{B-A} \quad ; \quad A \le x \le B \quad \text{(According to quest.)}$$

$$= \frac{1}{5-(-5)} \qquad (\because A = -5 \, ; \, B = 5)$$

$$f_X(x) = \begin{cases} \frac{1}{10} & ; \; -5 \le x \le 5 \\ 0 & ; \; \text{otherwise} \end{cases}$$

a) $$P(X < 0) = \int_{-5}^{0} f_X(x)\,dx$$

$$= \int_{-5}^{0} \frac{1}{10}\,dx = \frac{1}{10}[x]_{-5}^{0}$$

$$= \frac{1}{10}[0-(-5)]$$

$$= \frac{5}{10} = \frac{1}{2}$$

b) $$P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} f_X(x)\,dx$$

$$= \int_{-2.5}^{2.5} \frac{1}{10}\,dx = \frac{1}{10}[x]_{-2.5}^{2.5}$$

$$= \frac{1}{10}[2.5-(-2.5)] = \frac{5}{10}$$

$$= \frac{1}{2}$$

(c) $P(-2 \leq X \leq 3)$

$$= \int_{-2}^{3} \frac{1}{10}\, dx$$

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$$= \frac{1}{10} [x]_{-2}^{3}$$

$$= \frac{1}{10} [3-(-2)] = \frac{5}{10} = \frac{1}{2}$$

(d) $P(k < X < k+4) = \int_{k}^{k+4} \frac{1}{10}\, dx$

$$= \frac{1}{10} [x]_{k}^{k+4}$$

$$= \frac{1}{10} [\cancel{k}+4-\cancel{k}]$$

$$= \frac{4}{10}$$

$$= 0.4$$

3. The error involved in making a certain measurement is a continuous rv X with pdf

$$f(x) = \begin{cases} .09375(4 - x^2) & -2 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Sketch the graph of $f(x)$.

b. Compute $P(X > 0)$.

c. Compute $P(-1 < X < 1)$.

d. Compute $P(X < -.5 \text{ or } X > .5)$.

4.4

Sol^n 3

$X \sim f_X(x)$

$$f_X(x) = \begin{cases} 0.09375(4-x^2) & ; -2 \le x \le 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

b) $P(X>0) = \int_0^2 f_X(x)\, dx$

$$= \int_0^2 0.09375(4-x^2)\,dx$$

$$= 0.09375 \int_0^2 (4-x^2)\,dx$$

$$= 0.09375 \left(4x - \frac{x^3}{3}\right)_0^2 = 0.09375\left[\left(8-\frac{8}{3}\right)-(0-0)\right]$$

$$= 0.09375 \times \frac{16}{3} = 0.5$$

c) $P(-1 < X < 1) = \int_{-1}^{1} f_X(x)\,dx$

d) $P(X < -.5 \text{ or } X > 0.5) = P(-2 < X < -0.5 \text{ or } 0.5 < X < 2)$

$$= \int_{-2}^{-0.5} f_X(x)\,dx + \int_{0.5}^{2} f_X(x)\,dx$$

$$= \int_{-2}^{-0.5} 0.09375(4-x^2)\,dx + \int_{0.5}^{2} 0.09375(4-x^2)\,dx$$

$$=$$

8. In commuting to work, a professor must first get on a bus near her house and then transfer to a second bus. If the waiting time (in minutes) at each stop has a uniform distribution with $A = 0$ and $B = 5$, then it can be shown that the total waiting time Y has the pdf

$$f(y) = \begin{cases} \frac{1}{25}y & 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25}y & 5 \le y \le 10 \\ 0 & y < 0 \text{ or } y > 10 \end{cases}$$

a. Sketch a graph of the pdf of Y.

b. Verify that $\int_{-\infty}^{\infty} f(y)\,dy = 1$.

c. What is the probability that total waiting time is at most 3 min?

d. What is the probability that total waiting time is at most 8 min?

e. What is the probability that total waiting time is between 3 and 8 min?

f. What is the probability that total waiting time is either less than 2 min or more than 6 min?

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$$f_Y(y) = \begin{cases} \frac{1}{25} y & ; \; 0 \le y < 5 \\ \frac{2}{5} - \frac{1}{25} y & ; \; 5 \le y \le 10 \\ 0 & ; \; y < 0 \text{ or } y > 10 \end{cases}$$

a) Sketch a graph of Y

b) $\int_{-\infty}^{\infty} f_Y(y)\, dy = 1$

$$\Rightarrow \left(\cancel{\int_{-\infty}^{0}} + \int_{0}^{5} + \int_{5}^{10} + \cancel{\int_{10}^{\infty}} \right) f_Y(y)\, dy$$

$$\Rightarrow \int_0^5 \frac{1}{25} y\, dy + \int_5^{10} \left(\frac{2}{5} - \frac{1}{25} y \right) dy$$

$$= \frac{1}{25} \left[\frac{y^2}{2} \right]_0^5 + \left[\frac{2}{5} y - \frac{1}{25 \cdot 2} y^2 \right]_5^{10}$$

$$= \frac{1}{25 \times 2} [25 - 0] + \frac{2}{5}(10-5) - \frac{1}{50}(100-25)$$

$$= \frac{1}{2} + 2 - \frac{\cancel{75}\,3}{\cancel{50}\,2}$$

$$= \frac{1}{2} + 2 - \frac{3}{2}$$

$$= 1$$

c) $P(Y \le 3) = \int_0^3 \frac{1}{25} y \, dy$

$= \frac{1}{25}\left[\frac{y^2}{2}\right]_0^3 = \frac{1}{25 \times 2}[9-0]$

$= \frac{9}{50}$

d) $P(Y \le 8) = \int_0^5 f_Y(y)\,dy + \int_5^8 f_Y(y)\,dy$

$= \int_0^5 \frac{1}{25} y\,dy + \int_5^8 \left(\frac{2}{5} - \frac{1}{25} y\right) dy =$ ___

e) $P(3 \le Y \le 8) = \int_3^5 f_Y(y)\,dy + \int_5^8 f_Y(y)\,dy$

f) $P(Y \le 2 \text{ or } Y \geq 6) = P(Y \le 2) + P(Y \geq 6)$

$= \int_0^2 f_Y(y)\,dy + \int_6^{10} f_Y(y)\,dy$

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