

Continuous Random Variable

Defⁿ: A r.v X is said to be continuous if range space of X is continuous.

eg: $x \in (0, 1)$; $x \in [0, 1]$; $x \in \mathbb{R}$ i.e. $x \in (-\infty, \infty)$
; $1 \leq x \leq 2$; $x \geq 0$

→ A r.v X is ~~known~~ said to be continuous if its CDF is given as

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(t) dt$$

The function $f_X(x)$ is known as probability density function (pdf).

Moreover if X is cont. r.v then

$$\frac{d}{dx} F_X(x) = f_X(x)$$

Here $f_X(x)$ should satisfy the following two properties

(i) $f_X(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f_X(x) dx = 1$

Q1 Let X be r.v with pdf

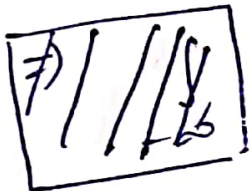
$$f_X(x) = \begin{cases} 2k & ; 0 < x < \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

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Find the value k ?

Soln $\because f_X(x)$ is pdf

then $\int_{-\infty}^{\infty} f_X(x) dx = 1$



Note: Probability at a single point in case of continuous random variable is 0.

$$P(X=a) = \int_a^a f_X(x) dx = 0 \quad \left(\text{if } X \sim f_X(x) \right)$$

Now, $\int_{-\infty}^{\infty} f_X(x) dx = \int_{-\infty}^0 f_X(x) dx + \int_0^{\frac{1}{2}} f_X(x) dx + \int_{\frac{1}{2}}^{\infty} f_X(x) dx = 1$

$$\Rightarrow 0 + \int_0^{\frac{1}{2}} 2k dx + 0 = 1$$

$$\Rightarrow 2k \cdot \left[\frac{x}{2} \right]_0^{\frac{1}{2}} = 1 \Rightarrow \boxed{k=1}$$

Ex): Let X be a continuous r.v with p.d.f

$$f_X(x) = \begin{cases} ax & ; 0 \leq x \leq 1 \\ a & ; 1 \leq x \leq 2 \\ -ax + 3a & ; 2 \leq x \leq 3 \\ 0 & ; x > 3 \end{cases}$$

(i) find the value of a

(ii) $P(X \leq 1.5)$

Soln $\therefore f_X(x)$ is pdf

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$\Rightarrow \left(\int_{-\infty}^0 f(x) dx + \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^3 f(x) dx + \int_3^{\infty} f(x) dx \right) = 1$$

$$\Rightarrow \int_0^1 ax dx + \int_1^2 a dx + \int_2^3 (-ax + 3a) dx = 1$$

$$\Rightarrow a \left[\frac{x^2}{2} \right]_0^1 + a [x]_0^1 + \left(-\frac{ax^2}{2} + 3ax \right) \Big|_2^3 = 1$$

$$\Rightarrow a \cdot \frac{1}{2} + a \cdot 1 + \left[\left(-\frac{a \cdot 9}{2} + 3a \cdot 3 \right) - \left(-\frac{a \cdot 4}{2} + 3a \cdot 2 \right) \right] = 1$$

$$\Rightarrow \boxed{a = \frac{1}{2}}$$

$$(ii) P(X \leq 1.5) = \int_0^{1.5} f_X(x) dx$$

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$$= \int_0^1 f_X(x) dx + \int_1^{1.5} f_X(x) dx$$

$$= \int_0^1 \frac{1}{2} dx + \int_1^{1.5} \frac{1}{2} dx$$

$$= \frac{1}{2} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{2} [x]_1^{1.5}$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot (1.5 - 1)$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 0.5$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X \leq 1.5) = \frac{1}{2}$$

Expected Value: let x be cont. r.v then

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x) dx.$$

Variance:

$$V(X) = E X^2 - (E X)^2$$