Applied Mathematics & Humanities Department

SVNIT Surat

Discrete Mathematics (Code: MA-211)

Tutorial -2
Aug 20, 2021

Last Date: Within 10 days | Platform: Google Classroom/MSTeams

Do all the questions.

1. Write the definitions of the following terms with examples.
   1. Hamming distance 2. Hamming weight 3. Minimum distance of a set (or code) 4. Minimum weight of a set (or code) 5. Generator matrix 6. Parity check matrix 7. Codewords 8. Code 9. Group code 10. Encoding function 11. Message 12.Undetectable error.

2. Given the generator matrix:
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$
   Find the corresponding code C and also find the corresponding parity check matrix H to decode the following received words. Also check if all the words decoded uniquely?
$$(i)\ 1\ 1\ 0\ 1\ 0\ 1 \qquad (ii)\ 0\ 0\ 1\ 1\ 1\ 1 \qquad (iii)\ 1\ 1\ 0\ 0\ 0\ 1 \qquad (iv)\ 1\ 1\ 1\ 1\ 1\ 1$$
   Hint: You can think about coset (syndrome) decoding.

3. Find the code for which the parity check matrix is as follows:
$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$
   Also find the minimum distance of the code and hence find the error detecting and error correcting capacity of the code.

4. Prove that the encoding function $e : B^2 \to B^5$ defined as: $e(00) = 00000, e(01) = 01110, e(10) = 10101, e(11) = 11011$ is a group code. (Hint: Make the composition table)

5. Find all the left cosets of the subgroup $H = \{0, 3, 6, 9, 12\}$ in the group $G = \{0, 1, 2, \ldots, 3, 14\}$.

6. Check whether the determinant map from set of all real square matrices of order 2 to the set of real numbers is a homomorphism?

7. Let G and G' are two groups under multiplications such that a map ϕ from the group G (set of all nonzero real numbers) to the group $G' = \{1, -1\}$ is defined as: positive numbers go to 1 and negative numbers go to -1. Prove that ϕ is a homomorphism.

8. Let $\phi : G \to G'$ be a homomorphism. Define a set $Im(\phi) = \{\phi(x) : x \in G\} \subseteq G'$, the image of G under ϕ. Check if $Im(\phi)$ is a subgroup of G'.

9. Let $G = \mathbb{R}^2$ be a group under component-wise addition. Let H be the subgroup of G consisting of the points on the line $y = 3x$, i.e, $H = \{(x, 3x) : x \in \mathbb{R}\}$. Show that the coset $(3, 7) + H$ is a straight line parallel to $y = 3x$. Draw the graph to understand the scenario.

10. Let $C = \{0000000, 1110100, 0111010, 0011101, 1001110, 0100111, 1010011, 1101001\}$. What is the error-correcting and error detecting capacity of C?