midsem 2020

Q: find the value of K, for which the function $f(x) = \frac{K\mu^x}{x!}$, $(\mu > 0)$, $x = 0, 1, 2, 3, \ldots$ represents pmf.

Sol^n $\because$ $f_X(x)$ is pmf

$$\Rightarrow \sum_{x=0}^{\infty} f_X(x) = 1$$

$$\Rightarrow \sum_{x=0}^{\infty} K \cdot \frac{\mu^x}{x!} = 1$$

$$\Rightarrow K\left[\frac{\mu^0}{0!} + \frac{\mu^1}{1!} + \frac{\mu^2}{2!} + \cdots\right] = 1$$

$$\Rightarrow K\left[1 + \frac{\mu}{1!} + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \cdots\right] = 1$$

$$\Rightarrow K \cdot e^{\mu} = 1$$

$$\Rightarrow K = \frac{1}{e^{\mu}}$$

$$\Rightarrow \boxed{K = e^{-\mu}}$$ #

Ch 3.3

Q) 34 Give an example where $E(X)$ does not exist?

Soln

Note :- ① $\sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6}$

② $\sum_{x=1}^{\infty} \frac{1}{x^p}$; is convergent if; $p > 1$

; divergent if; $0 \leq p \leq 1$

③ Convergent of series : A infinite series is said to be convergent if its nth partial sum is bounded

i.e $\lim_{n \to \infty} S_n = K$ (fixed constant)

where $S_n = u_1 + u_2 + u_3 + \cdots + u_n$

where u_i is the ith term of the series.

Q1 Let X be discrete r.v with p.m.f

$$p_X(x) = \frac{K}{x^2} \; ; \; x = 1, 2, 3, \ldots$$

Find $E(X)$?

Sol^n $E(X) = \sum_{x=1}^{\infty} x \cdot \frac{K}{x^2} = K \cdot \sum_{x=1}^{\infty} \frac{x}{x^2}$

$$= K \sum_{x=1}^{\infty} \frac{1}{x}$$

$= K.$ (divergent series)

$\rightarrow \infty$

$\Rightarrow$ $E(X)$ does not exist.

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10. The number of pumps in use at both a six-pum station and a four-pump station will be determined. Give the possible values for each of the following random variables:

a. T = the total number of pumps in use

b. X = the difference between the numbers in use at stations 1 and 2

c. U = the maximum number of pumps in use at either station

d. Z = the number of stations having exactly two pumps in use

3.2

Q.10

a) $T = \{0, 1, 2, 3, \cdots, 10\}$ No.- 54

b) $X_1 = \{0, 1, 2, 3, 4, 5, 6\}$

$X_2 = \{0, 1, 2, 3, 4\}$

$X = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

min value = 0 pumps used on station 1 and 4 pumps used at station 2 then it will be $0 - 4 = -4$

max value = 6 pumps used at station 1 and 0 on 2nd so max value will be $6 - 0 = 6$

(c) $U = \{0, 1, 2, 3, 4, 5, 6\}$

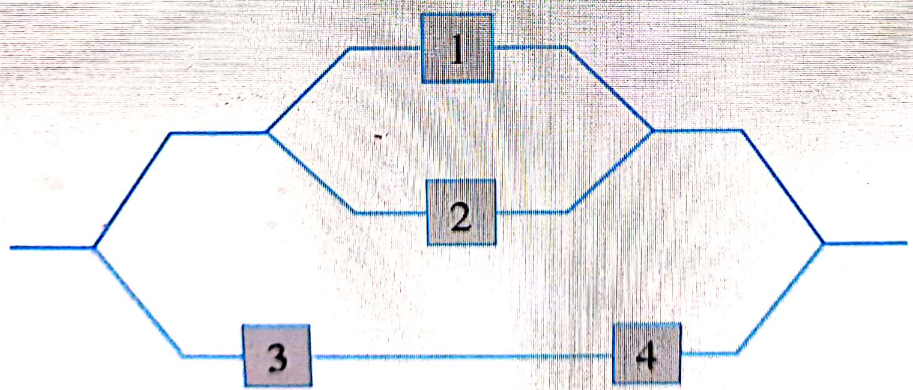
d) $Z = \{0, 1, 2\}$

no station having exactly two pumps inus

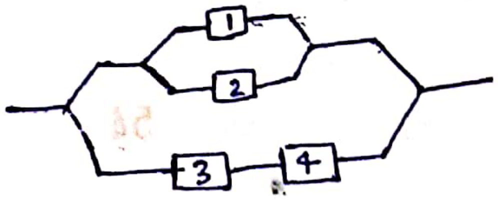
one " " " " " "

both ~~too~~ " " " " " "

80. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and $P(\text{component } i \text{ works}) = .9$ for $i = 1,2$ and $= .8$ for $i = 3,4$, calculate $P(\text{system works})$.



2.3
Q.80



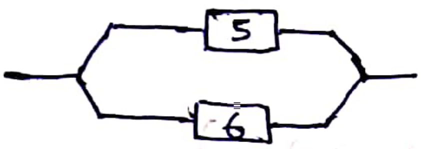
$A_1 \rightarrow$ component 1st works | $P(A_1) = P(A_2) = 0.9$

$A_2 \rightarrow$ " 2 " | $P(A_3) = P(A_4) = 0.8$

$A_3 \rightarrow$ " 3 "

$A_4 \rightarrow$ " 4 "

~~$A_5 \rightarrow$ " 5 "~~



$A_5 \rightarrow$ Component 5 work

$A_6 \rightarrow$ " 6 "

$$P(A_5) = P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= P(A_1) + P(A_2) - P(A_1) \cdot P(A_2)$$

(∵ components are indep.)

$$= 0.9 + 0.9 - (0.9)^2$$

$$= 1.80 - .81$$

$$= 0.99$$

$$P(A_6) = P(A_3 \cap A_4) = P(A_3) \cdot P(A_4)$$

$$= 0.8 \times 0.8$$

$$= 0.64$$

$$P(\text{system works}) = P(A_5 \cup A_6) = P(A_5) + P(A_6) - P(A_5 \cap A_6)$$

$$= P(A_5) + P(A_6) - P(A_5) \cdot P(A_6)$$

$$= 0.99 + 0.64 - 0.99 \times 0.64$$

$$= 0.9964$$

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