Q: $p_X(x) = pq^{x-1}$; $x = 1, 2, \ldots$

a) find the C.D.F of ~~uniform~~ geometric dist^n

b) Sketch the graph of $F(x; p)$ for $x = 1, 2, 3, 4, 5$ and $p = 0.75$

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Sol

9)

$$F_X(x) = P(X \le x)$$

$$= \sum_{t=1}^{x} p q^{t-1}$$

$$= p \sum_{t=1}^{x} q^{t-1}$$

$$= p[q^0 + q^1 + q^2 + \cdots q^{x-1}]$$

$$= p[1 + q + q^2 + \cdots + q^{x-1}]$$

$$= p\left[\frac{1 - q^x}{1 - q}\right] \qquad \left(\because 1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}\right)$$

$$= \frac{\cancel{p}(1 - q^x)}{\cancel{p}} \qquad (\because 1 - q = p)$$

$$F_X(x) = 1 - q^x \; ; \quad x = 1, 2, 3, \ldots$$

$$= 1 - (1-p)^x \; ; \quad x = 1, 2, 3, \ldots$$

(Corrected one)

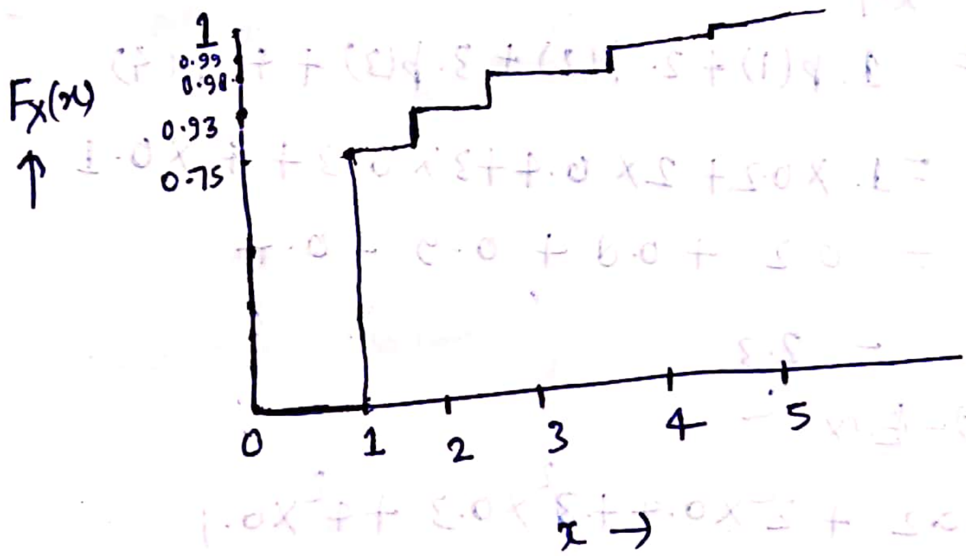
(b) Sketch Diagram:

$$F_X(x) = 1-(1-p)^x \quad ; x = 1, 2, 3, \ldots$$

In our case

$$F_X(x) = 1-(1-0.75)^x \quad ; x = 1, 2, 3, 4, 5$$

$$= 1-(0.25)^x \quad ; x = 1, 2, 3, 4, 5.$$



$$F_X(x) = \begin{cases} 0 & ; x < 1 \\ 0.75 & ; 1 \leq x < 2 \\ 0.93 & ; 2 \leq x < 3 \\ 0.98 & ; 3 \leq x < 4 \\ 0.99 & ; 4 \leq x < 5 \\ \simeq 1 & x \geq 5 \end{cases}$$

C.D.F for Binomial Distribution :-

$$F_X(x) = P(X \le x)$$

$$= \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$$

Note :- Here n and p are parameters.

midsem 2017

Q: We draw cards repeatedly without replacement from a packet of 100 cards, 60 of which refer to male and 40 to female person. What is the prob. of obtaining second female card before the third male card?

Sol^n The combinations of drawing second female card before third male card are

$\{FF, FMF, FMMF, MFF, MFMF, MMFF\}$

where F → female card ; $P(F) = \frac{40}{100} = 0.4$

M → Male card ; $P(M) = \frac{60}{100} = 0.6$

$P(FF) = \frac{40}{100} \times \frac{39}{99}$; $P(FMF) = \frac{40}{100} \times \frac{60}{99} \times \frac{39}{98}$

$$P(MFF) = \frac{60}{100} \times \frac{40}{99} \times \frac{39}{98}$$

$$P(MFMF) = \frac{60}{100} \times \frac{40}{99} \times \frac{59}{98} \times \frac{39}{97}$$

$$P(MMFF) = \frac{60}{100} \times \frac{59}{99} \times \frac{40}{98} \times \frac{39}{97}$$

so required prob

$= P(FF) + P(FMF) + P(FMMF) + P(MFF) + P(MFMF) + P(MMFF)$

= ______

With Replacement: (Hint!)

$$P(FF) = \frac{40}{100} \times \frac{40}{100}$$

$$P(FMF) = \frac{40}{100} \times \frac{60}{100} \times \frac{40}{100}$$

mid sem 2013

Q1) Find the mean and variance of r.v

$Y = 3X+5$. Given that the p.m.f No.- 52

corresponds to r.v X is

$$f_X(x) = x \; ; \; x = 0, 1$$

Sol^n

Mean of Y

$E(Y) = E(3X+5)$ $\quad (\because E(ax+b) = aE(x)+b)$

$= 3 \cdot E(X) + 5$

Variance of Y

$V(Y) = V(3X+5)$ $\quad (\because V(ax+b) = a^2V(x))$

$= 3^2 \cdot V(X)$

$= 9V(X)$

Now; $E(X) = \sum_{x=0}^{1} x \cdot f_X(x) = 0 \cdot f(0) + 1 \, f(1)$

$= 0 + 1 \cdot 1$

$= 1$

$V(X) = E(X^2) - (E(X))^2$

$E(X^2) = \sum_{x=0}^{1} x^2 \cdot f_X(x) = 0^2 \cdot f(0) + 1^2 \cdot f(1)$

$= 0 + 1 \cdot 1 = 1$

Thus $V(X) = 1 - 1 = 0$

$\Rightarrow E(Y) = 3 \cdot 1 + 5 = 8$

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$V(Y) = 9 \cdot 0 = 0$

(b) A chemical supply company currently has in stock 100 lb of a certain chemical, which it sells to customers in 5-lb batches. Let $X =$ the number of batches ordered by a randomly chosen customer, and suppose that X has pmf

x	1	2	3	4
$p(x)$	.2	.4	.3	.1

Compute $E(X)$ and $V(X)$. Then compute the expected number of pounds left after the next customer's order is shipped and the variance of the number of pounds left. [3]

mid sem 2020

Solⁿ: $E(X) = \sum_{x=1}^{4} x \cdot p_X(x)$

$= 1 \cdot p(1) + 2 \cdot p(2) + 3 \cdot p(3) + 4 \cdot p(4)$

$= 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.1$

$= 0.2 + 0.8 + 0.9 + 0.4$

$= 2.3$

$V(X) = E(X^2) - (E(X))^2$

$E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.3 + 4^2 \times 0.1$

$= 0.2 + 1.6 + 2.7 + 1.6$

$= 6.1$

$V(X) = 6.1 - (2.3)^2$

$V(X) = 0.81$

(b) The no of pounds left after the next customer's order shipped is

$Y = 100 - 5 \cdot X$

$E(Y) = 100 - 5\,E(X) = 100 - 5 \times 2.3 = 88.5$

$V(Y) = V(100 - 5X) = (-5)^2\, V(X) = 25 \times 0.81$

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