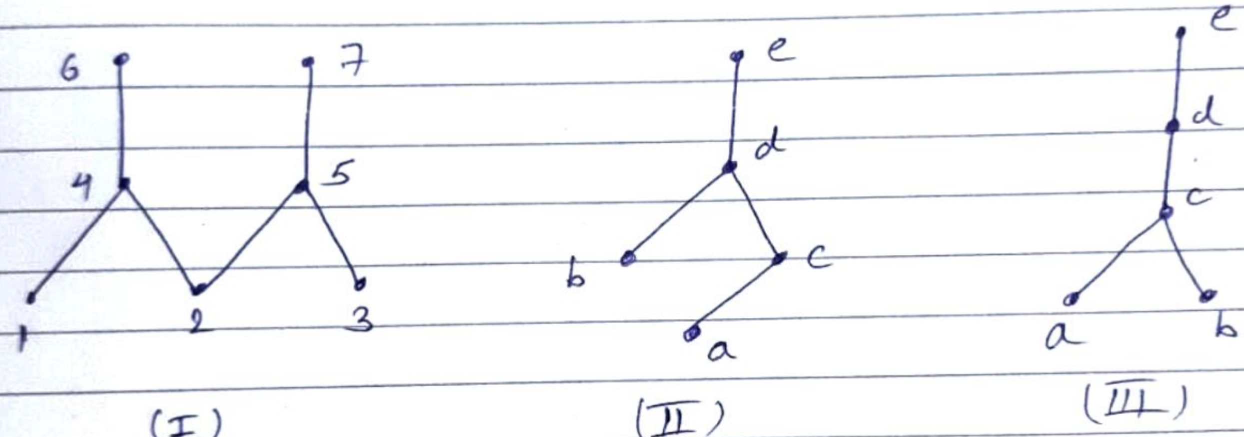
MA-221 COED

Tutorial

Lattices & Boolean Algebra

Q.1 Prove that for a bounded distributive Lattice L, the complements are unique if they exist.

Q.2 Determine whether the posets shown in fig. are lattice or not?



Q3. Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and the relation divides (|) be a partial ordering relation on D_{50}.

(a) Draw the Hasse diagram of D_{50} with relation divides.

(b) Determine all upper bounds and lower bounds of 5 and 10.

(c) Determine g.l.b. and l.u.b. of 5 and 10.

(d) Determine the greatest and least element of D_{50}.

Q.4. Show that the following Boolean expressions are equivalent

(i) $x \wedge (y \vee (y' \wedge (y \vee y')))$; x

(ii) $(z' \vee x) \wedge ((x \wedge y) \vee z) \wedge (z' \vee y)$; $x \wedge y$

Q.5 Define complemented Lattice and give an example.

Q.6 If $(L, \leq)$ is a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively then show that -

$$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b \quad \forall a, b, c \in L$$

Q.7 For any $a, b \in B$, Prove that -

(i) $a + a \cdot b = a$ (ii) $a \cdot (a + b) = a$ (absorption Laws)

Q.8 for any $a, b \in B$, Prove De Morgan's Laws -

(i) $(a+b)' = a' * b'$ (ii) $(a * b)' = a' + b'$

Q.9 Prove that the complement of every element on a Boolean algebra B is unique.

Q.10 Define Modular Lattice and give an example.