Tutorial - 6

(TUT-1 Indra mam)

Page No.:
Date: / /

U20CS110
Krishna Pandey

Q1) Find the rank of the following matrix:

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{pmatrix}$ $\rightarrow R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - 2R_1$

$\rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ $\rightarrow R_3 \rightarrow R_3 - R_2$

$- \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = -2-6 = -8 \neq 0.$

- So, Rank of matrix is 2.

(b) $\begin{pmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{pmatrix}$

$- \begin{pmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{pmatrix}$ $R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - R_1$

$\begin{pmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $R_3 \rightarrow R_3 + \frac{R_2}{3}$

$\rightarrow$ Since, Two non-zero rows,
Rank of matrix = 2.

Scanned with CamScanner

Page No.:
Date: / /

2) $\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & -3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 3/2 & -1/2 & -1/2 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & -3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$ $R_1 \rightarrow \frac{R_1}{2}$

$\rightarrow \begin{pmatrix} 1 & 3/2 & -1/2 & -1/2 \\ 0 & -5/2 & -3/2 & -7/2 \\ 0 & -7/2 & 1/2 & -1/2 \\ 0 & -6 & 3 & -4 \end{pmatrix}$ $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 3R_1$, $R_4 \rightarrow R_4 - 6R_1$

$\rightarrow \begin{pmatrix} 1 & 3/2 & -1/2 & -1/2 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & -7/2 & \frac{1}{2} & -1/2 \\ 0 & -6 & 3 & -4 \end{pmatrix}$ $R_2 \rightarrow \frac{R_2}{-2.5}$

$\rightarrow \begin{pmatrix} 1 & 3/2 & -1/2 & -1/2 \\ 0 & 1 & 0.6 & 1.4 \\ 0 & 0 & -6.6 & -4.4 \\ 0 & 0 & -6.6 & -4.4 \end{pmatrix}$ $R_3 \rightarrow R_3 + 3.5R_2$, $R_4 \rightarrow R_4 + 6R_2$

$\rightarrow \begin{pmatrix} 1 & 3/2 & -1/2 & -1/2 \\ 0 & 1 & 0.6 & 1.4 \\ 0 & 0 & 6.6 & 4.4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\Rightarrow$ Since, There is 3 non-zero rows
Rank of matrix is 3.

Page No.: Date: / /

d) $\begin{pmatrix} 2 & -1 & 0 & 5 \\ 0 & 3 & 1 & 4 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{5}{2} \\ 0 & 3 & 1 & 4 \end{pmatrix} \quad R_2 \rightarrow \frac{R_1}{2}$

- since, 2 non-zero rows, Rank of matrix is 2.

Q2) Reduce the following matrix to triangular form.

$$\begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix}$$

$\rightarrow \begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix} \quad \begin{matrix} R_2 \rightarrow R_2 + 3R_1 \\ R_3 \rightarrow R_3 + \frac{5}{3}R_1 \end{matrix}$

$\rightarrow \begin{pmatrix} 3 & -4 & -5 \\ 0 & -11 & -11 \\ 0 & -\frac{11}{3} & -\frac{22}{3} \end{pmatrix}$

$\rightarrow \begin{pmatrix} 3 & -4 & -5 \\ 0 & -11 & -11 \\ 0 & 0 & -\frac{11}{3} \end{pmatrix} \quad R_2 \rightarrow R_2 \times_{M}, \; R_3 \rightarrow R_3 - \frac{R_2}{3}$

$\rightarrow \begin{pmatrix} 3 & -4 & -5 \\ 0 & 1 & 1 \\ 0 & 0 & -11 \end{pmatrix} \quad R_2 \rightarrow R_2/_{-11}, \; R_3 \rightarrow 3R_3$

Page No.:
Date: / /

Q3) For the matrix, $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}$

→ $A = PAQ$ Normal form.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & -1/2 \\ -1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & -1/2 \\ -1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore P = \begin{bmatrix} 3/2 & -1/2 & -1/2 \\ -1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q5) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

(i) find A^{-1}

Q4) Find the inverse of $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ by elementary row operation.

→ Adjoining with I_3

$$\rightarrow \left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \quad R_1 \to R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right) \quad R_3 \to R_3 - 3R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right) \quad R_3 \to R_3 + 3R_2$$

Page No.:
Date: / /

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/2 & 3/2 & 1/2 \end{array}\right) \quad R_3 \to R_3/2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & 5/2 & -3/2 & 1/2 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -\frac{15}{2} & 11/2 & -3/2 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & 5/2 & -3/2 & 1/2 \end{array}\right) \quad R_1 \to R_1 - 3R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & 5/2 & -3/2 & 1/2 \end{array}\right) \quad R_1 \to R_1 - 2R_2$$

$\therefore I_3 \mid A^{-1}$

$$\therefore A^{-1} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{pmatrix}$$

Q5) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

1) find A^{-1}

- $A^{-1} = \frac{1}{|A|} \text{adj} A^T$

$|A| = 3(-3+4) + 3(2) + 4(-2)$

Scanned with CamScanner

Page No.:
Date: / /

$= 3+6-8$
$= 1$

$$\text{adj}A = \begin{pmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -4 \end{pmatrix}$$

$$\text{adj}A^T = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{pmatrix}$$

Page No.:
Date: / /

$$A^2 \cdot A = \begin{pmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{pmatrix} \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{pmatrix}$$

$\therefore$ LHS = RHS

$A^3 = A^{-1}$

Scanned with CamScanner