

Linear Discriminant Analysis

LDA

Linear Discriminant Analysis (LDA) is one of the commonly used dimensionality reduction techniques in machine learning to solve more than two-class classification problems. It is also known as Normal Discriminant Analysis (NDA) or Discriminant Function Analysis (DFA).



Linear Discriminant Analysis (LDA)

Linear Discriminant Analysis or **Normal Discriminant Analysis** or **Discriminant Function Analysis** is a dimensionality reduction technique which is commonly used for the classification problems

For example, we have two classes and we need to separate them efficiently. Classes can have multiple features. Using only a single feature to classify them may result in some overlapping. So, we will keep on increasing the number of features for proper classification

Linear Discriminant Analysis (LDA), is a supervised Learning technique!



Linear Discriminant Analysis (LDA)

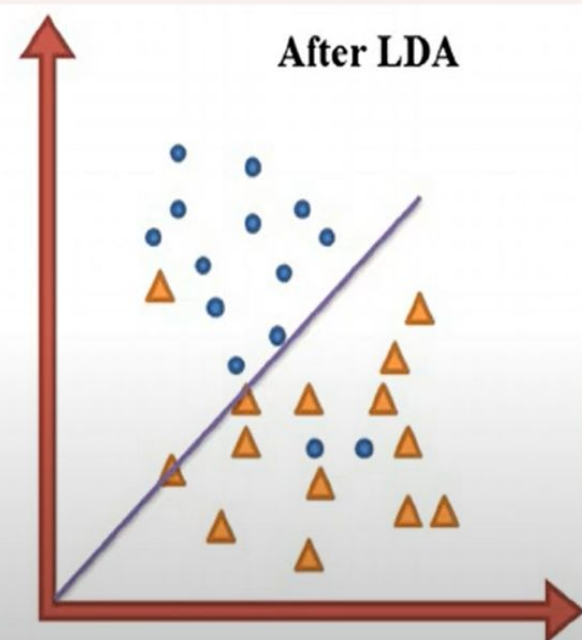
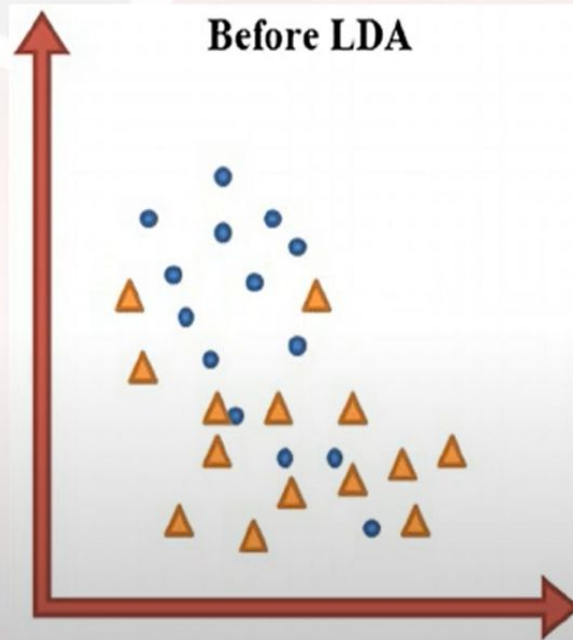
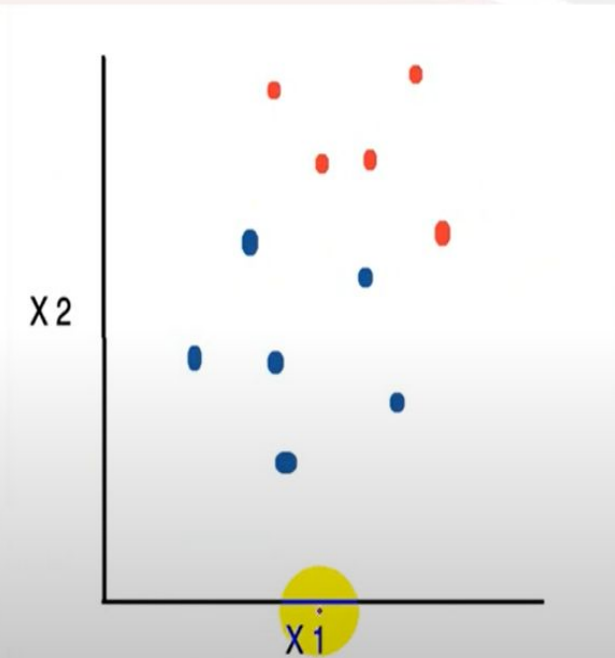
LDA can be achieved in three steps :

The first step is to calculate the separability between different classes (i.e the distance between the mean of different classes) also called as between-class variance

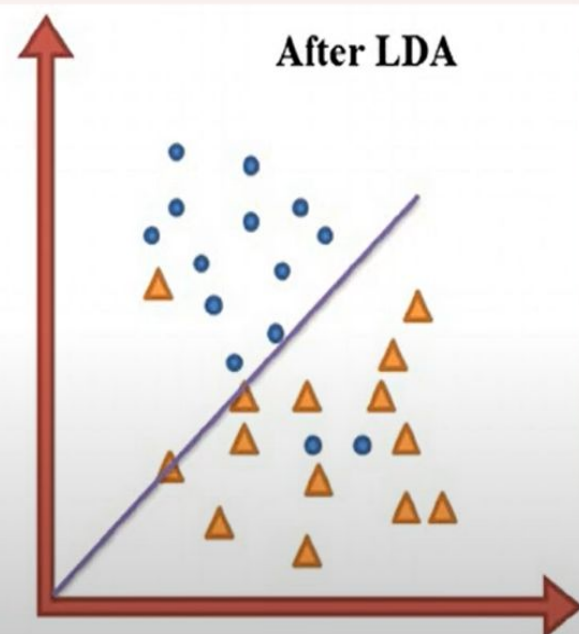
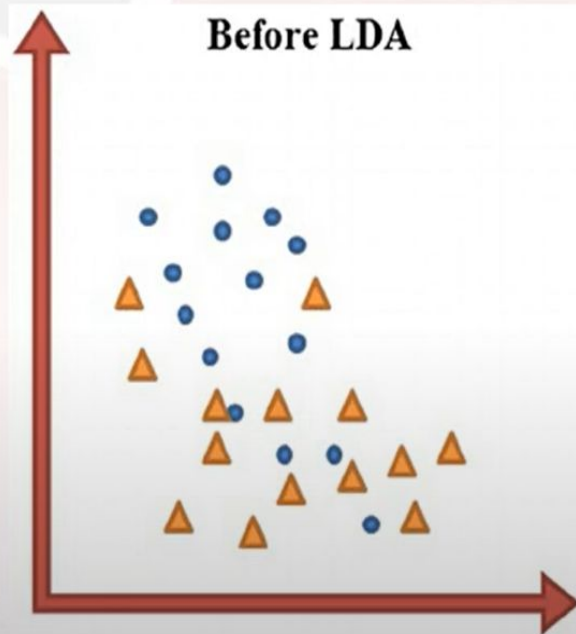
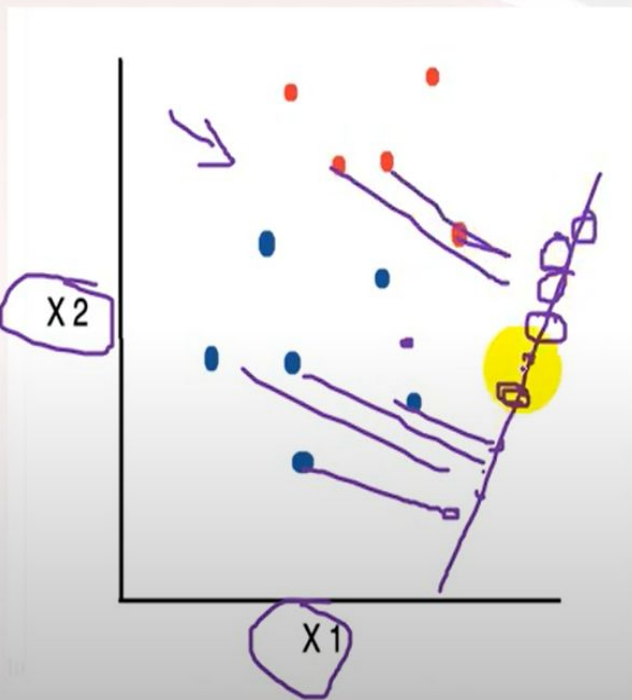
Second Step is to calculate the distance between the mean and sample of each class, which is called the within class variance

The third step is to construct the lower dimensional space which maximizes between class variance and minimizes the within class variance

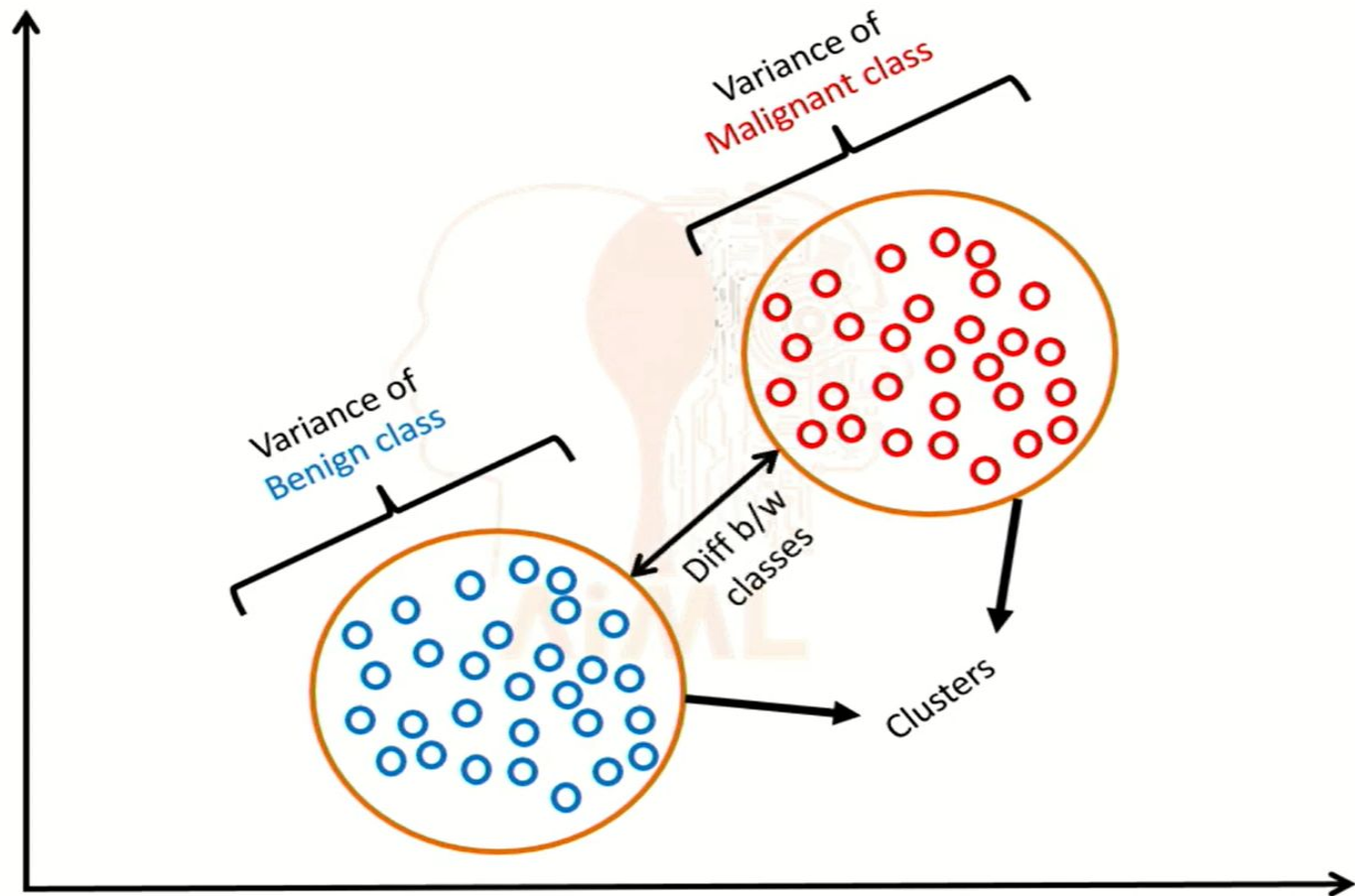
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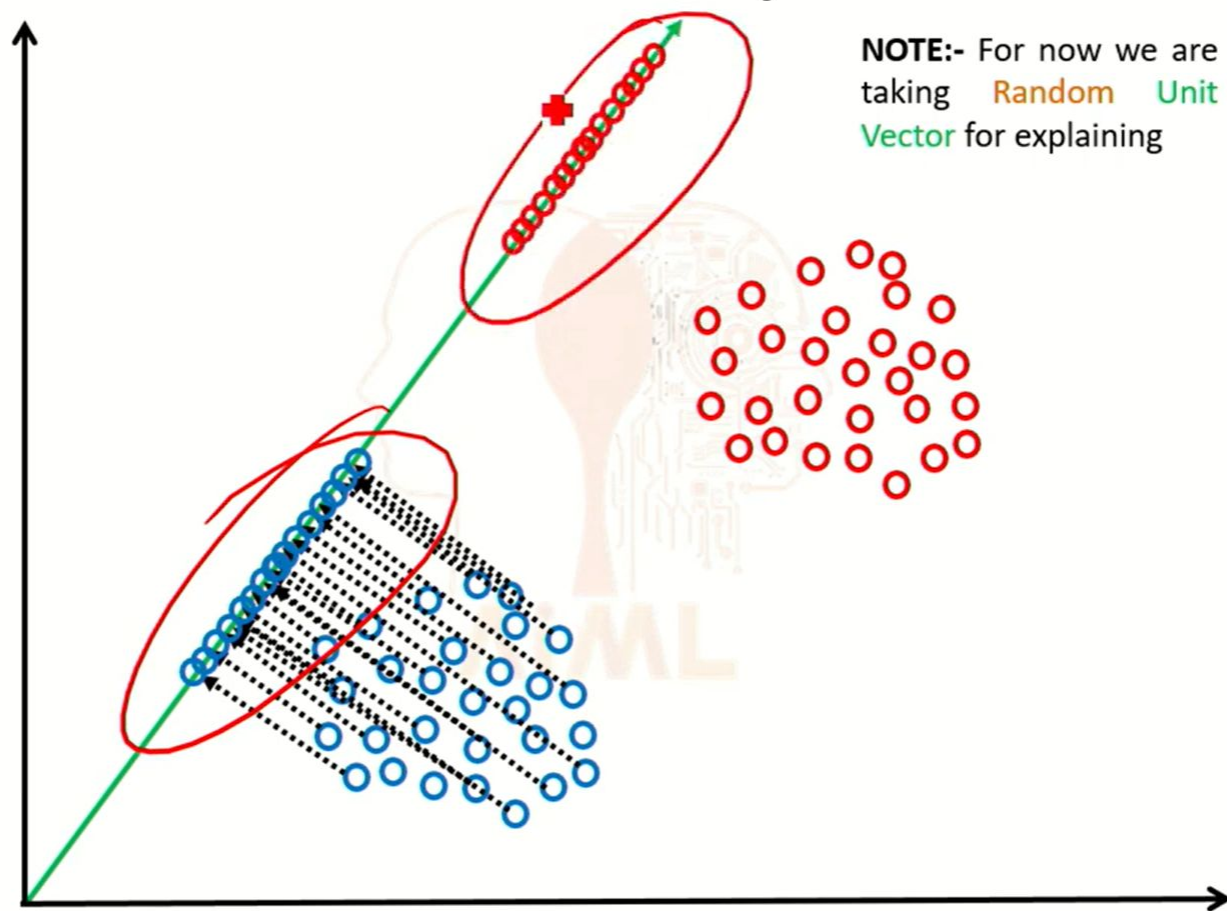
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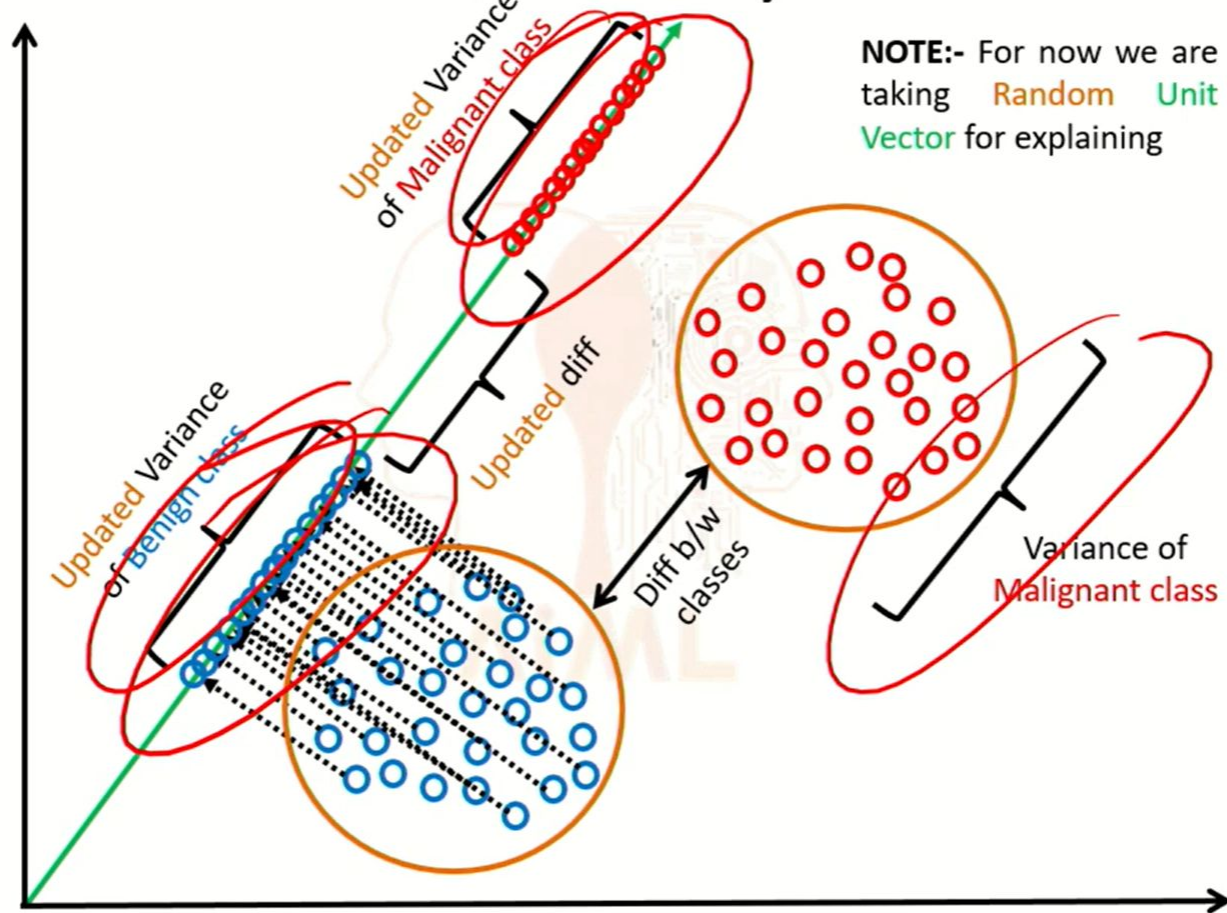
LDA as Dimensionality Reduction



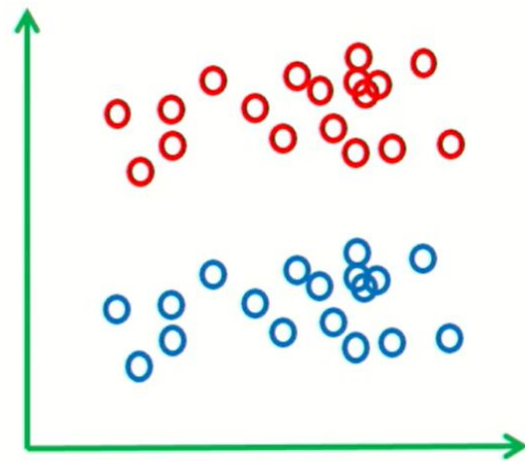
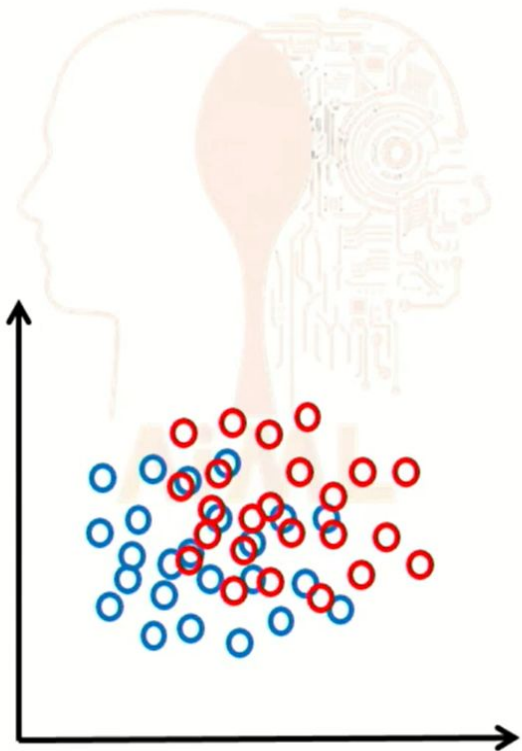
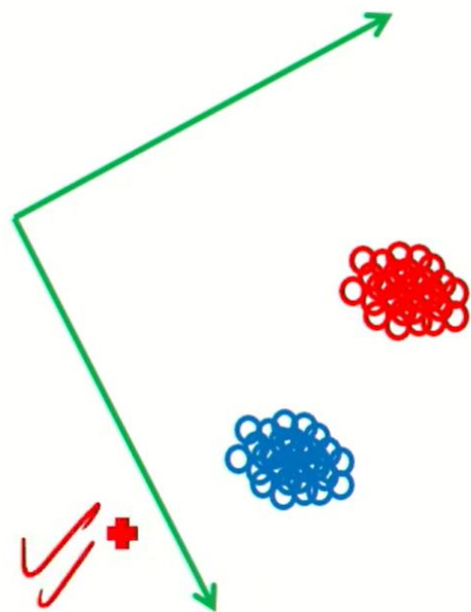
LDA as Dimensionality Reduction



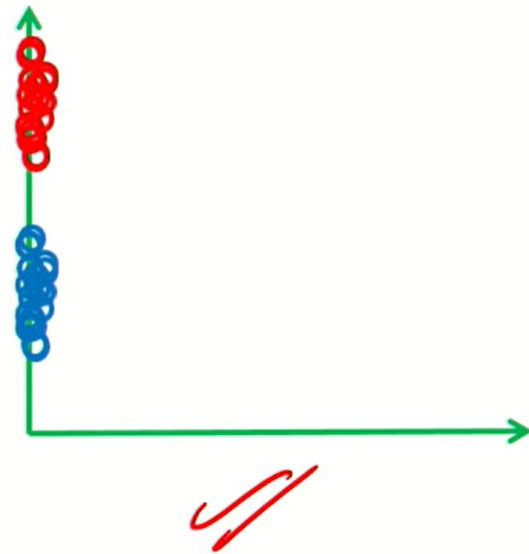
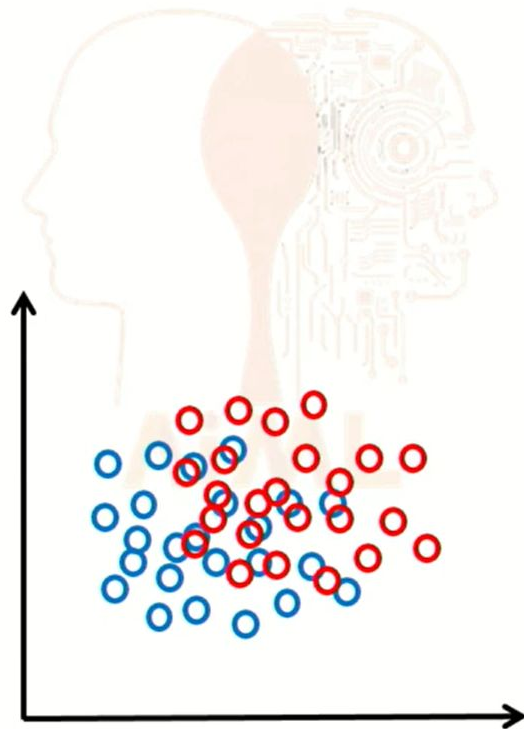
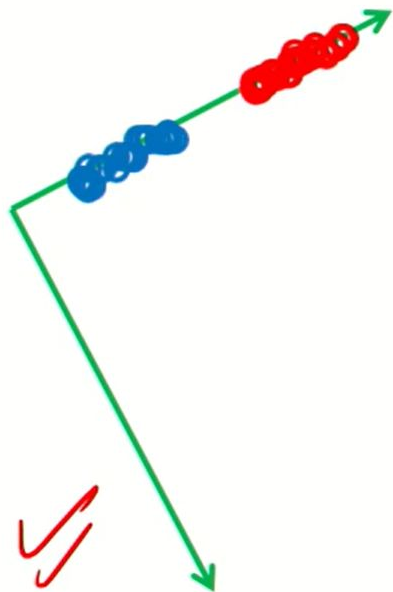
LDA as Dimensionality Reduction



LDA as Dimensionality Reduction



LDA as Dimensionality Reduction



Why LDA?

Logistic Regression is one of the most popular classification algorithms that perform well for binary classification but falls short in the case of multiple classification problems with well-separated classes. At the same time, LDA handles these quite efficiently.

LDA can also be used in data pre-processing to reduce the number of features, just as PCA, which reduces the computing cost significantly.

LDA is also used in face detection algorithms. In Fisherfaces, LDA is used to extract useful data from different faces. Coupled with eigenfaces, it produces effective results.

Drawbacks of Linear Discriminant Analysis (LDA)

Although, LDA is specifically used to solve supervised classification problems for two or more classes which are not possible using logistic regression in machine learning. But LDA also fails in some cases where the Mean of the distributions is shared. In this case, LDA fails to create a new axis that makes both the classes linearly separable.

Extension to Linear Discriminant Analysis (LDA)

Linear Discriminant analysis is one of the most simple and effective methods to solve classification problems in machine learning. It has so many extensions and variations as follows:

1. **Quadratic Discriminant Analysis (QDA):** For multiple input variables, each class deploys its own estimate of variance.
2. **Flexible Discriminant Analysis (FDA):** it is used when there are non-linear groups of inputs are used, such as splines.
3. **Flexible Discriminant Analysis (FDA):** This uses regularization in the estimate of the variance (actually covariance) and hence moderates the influence of different variables on LDA.

Real-world Applications of LDA

Some of the common real-world applications of Linear discriminant Analysis are given below:

Face Recognition

Face recognition is the popular application of computer vision, where each face is represented as the combination of a number of pixel values. In this case, LDA is used to minimize the number of features to a manageable number before going through the classification process. It generates a new template in which each dimension consists of a linear combination of pixel values. If a linear combination is generated using Fisher's linear discriminant, then it is called Fisher's face.

Medical

In the medical field, LDA has a great application in classifying the patient disease on the basis of various parameters of patient health and the medical treatment which is going on. On such parameters, it classifies disease as mild, moderate, or severe. This classification helps the doctors in either increasing or decreasing the pace of the treatment.

Continues...

Customer Identification

In customer identification, LDA is currently being applied. It means with the help of LDA; we can easily identify and select the features that can specify the group of customers who are likely to purchase a specific product in a shopping mall. This can be helpful when we want to identify a group of customers who mostly purchase a product in a shopping mall.

For Predictions

LDA can also be used for making predictions and so in decision making. For example, "will you buy this product" will give a predicted result of either one or two possible classes as a buying or not.

In Learning

Nowadays, robots are being trained for learning and talking to simulate human work, and it can also be considered a classification problem. In this case, LDA builds similar groups on the basis of different parameters, including pitches, frequencies, sound, tunes, etc.

Difference between Linear Discriminant Analysis and PCA

Below are some basic differences between LDA and PCA:

PCA is an unsupervised algorithm that does not care about classes and labels and only aims to find the principal components to maximize the variance in the given dataset. At the same time, LDA is a supervised algorithm that aims to find the linear discriminants to represent the axes that maximize separation between different classes of data.

LDA is much more suitable for multi-class classification tasks compared to PCA. However, PCA is assumed to be an as good performer for a comparatively small sample size.

Both LDA and PCA are used as dimensionality reduction techniques, where PCA is first followed by LDA.

How to Prepare Data for LDA

Classification Problems. This might go without saying, but LDA is intended for classification problems where the output variable is categorical. LDA supports both binary and multi-class classification.

Gaussian Distribution. The standard implementation of the model assumes a Gaussian distribution of the input variables. Consider reviewing the univariate distributions of each attribute and using transforms to make them more Gaussian-looking (e.g. log and root for exponential distributions and Box-Cox for skewed distributions).

Remove Outliers. Consider removing outliers from your data. These can skew the basic statistics used to separate classes in LDA such the mean and the standard deviation.

Same Variance. LDA assumes that each input variable has the same variance. It is almost always a good idea to standardize your data before using LDA so that it has a mean of 0 and a standard deviation of 1.

Summarizing the LDA approach in 5 steps

Listed below are the 5 general steps for performing a linear discriminant analysis

1. Compute the d -dimensional mean vectors for the different classes from the dataset.
2. Compute the scatter matrices (in-between-class and within-class scatter matrix).
3. Compute the eigenvectors ($\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_d$) and corresponding eigenvalues ($\lambda_1, \lambda_2, \dots, \lambda_d$) for the scatter matrices.
4. Sort the eigenvectors by decreasing eigenvalues and choose k eigenvectors with the largest eigenvalues to form a $d \times k$ dimensional matrix \mathbf{W} (where every column represents an eigenvector).
5. Use this $d \times k$ eigenvector matrix to transform the samples onto the new subspace. This can be summarized by the matrix multiplication: $\mathbf{Y} = \mathbf{X} \times \mathbf{W}$ (where \mathbf{X} is a $n \times d$ -dimensional matrix representing the n samples, and \mathbf{y} are the transformed $n \times k$ -dimensional samples in the new subspace).