83. Components arriving at a distributor are checked for defects by two different inspectors (each component is checked by both inspectors). The first inspector detects 90% of all defectives that are present, and the second inspector does likewise. At least one inspector does not detect a defect on 20% of all defective components. What is the probability that the following occur?

a. A defective component will be detected only by the first inspector? By exactly one of the two inspectors?

b. All three defective components in a batch escape detection by both inspectors (assuming inspections of different components are independent of one another)?

2.5
Secc

Q.83

$A \rightarrow$ Event that first inspector detects

$B \rightarrow$ Event that second inspector detects

$P(A) = 0.9 = P(B)$

A' = Event that first inspector does not detects

B' = " " second " " " "

$P(A' \cup B') = 0.20 \Rightarrow 1 - P(A \cap B) = 0.20$

$\Rightarrow P(A \cap B) = 0.80$

9) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.9 - 0.8 = 0.10$

b)

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$P(A \cap B^c) + P(A^c \cap B)$

$= 0.10 + P(B) - P(A \cap B)$

$= 0.10 + 0.9 - 0.8$

$= 0.10 + 0.10$

$= 0.20$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (defects is inspected by either inspector)

$= 0.9 + 0.9 - 0.80$

$= 1.80 - 0.80$

$= 1$

Hence the prob. of both inspectors missing a defect

$= P(A \cup B)^c$

$= 1 - P(A \cup B)$

$= 1 - 1$

$= 0$

Thus prob. that all 3 defective components escape detection

$= 0 \times 0 \times 0$ (All independent)

$= 0$

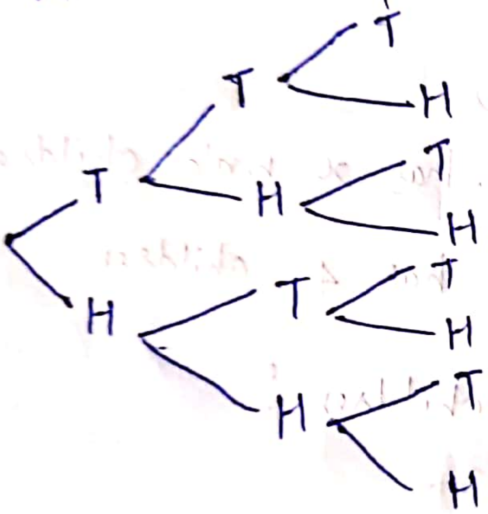
Q: Draw a tree diagram to show results of tossing a coin three times?

a) What is the prob. of getting 2 Heads?

b) What is the prob. of getting at least 2 Heads

c) What is the prob. of getting at most 2 Heads?

Sol



TTT HTT

TTH HTH

THT HHT

THH HHH

$P(2 \text{ Heads}) = \frac{3}{8}$

$P(\text{atleast } 2 \text{ Heads}) = \frac{4}{8}$

$P(\text{at most } 2 \text{ Heads}) = \frac{7}{8}$

Geometric Distribution :-

X: no of trials to get 1^{st} success. No.- 46

Note :- This is the special case of Negative Binomial type-I.

Here $r=1$,

So. the p.m.f of r.v X is said to have geometric distn if it is given as

$$p_X(x) = \binom{x-1}{1-1} p^1 \cdot q^{x-1} \; ; \; x = 1, 2, 3, \ldots$$

$$= \binom{x-1}{0} p \cdot q^{x-1} \; ; \; x = 1, 2, 3 \ldots$$

$$\boxed{p_X(x) = p\, q^{x-1} \; ; \; x = 1, 2, 3 \ldots \infty}$$

It is denoted as Geo(p).

① Verify that it is proper p.m.f.

② Find the E(X) and V(X) ?

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Proof!

① Claim: $\sum_{x=1}^{\infty} p \cdot q^{x-1} = 1$

Now, $\sum_{x=1}^{\infty} p q^{x-1} = p \sum_{x=1}^{\infty} q^{x-1}$

$= p [q^0 + q^1 + q^2 + q^3 + \cdots]$

$= p [1 + q + q^2 + q^3 + \cdots]$

$= p \cdot \frac{1}{(1-q)}$ $(\because 1-q = p)$

$= \cancel{p} \cdot \frac{1}{\cancel{p}}$

$= 1$ #

② $E(X) = \sum_{x=1}^{\infty} x \cdot p_X(x) = \sum_{x=1} x \cdot p q^{x-1}$

$= p \sum_{x=1}^{\infty} x q^{x-1}$

$= p [1 \cdot q^0 + 2 \cdot q + 3 q^2 + 4 q^3 + \cdots]$

$= p [1 + 2q + 3q^2 + 4q^3 + \cdots]$

$= p \cdot S$

Let $S = 1 + 2q + 3q^2 + 4q^3 + \cdots$

$- q \cdot S = \quad -q \pm 2q^2 \pm 3q^3 + \cdots$

$S - S \cdot q = 1 + q + q^2 + q^3 + \cdots$

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$\Rightarrow S(1-q) = 1 + q + q^2 + q^3 + \cdots$

$\Rightarrow S(1-q) = \dfrac{1}{1-q}$

$\Rightarrow S = \dfrac{1}{(1-q)^2} = \dfrac{1}{p^2} \quad (\because 1-q = p)$

$\therefore E(X) = p \cdot S$

$= \cancel{p} \cdot \dfrac{1}{p^{\cancel{2}}}$

$$\boxed{E(X) = \frac{1}{p}}$$

$V(X) = E(X^2) - (E(X))^2$

$= E(X(X-1) + X) - (E(X))^2$

$= E(X(X-1)) + E(X) - (E(X))^2$

$E(X(X-1)) = \sum_{x=1}^{\infty} x(x-1) \cdot p \cdot q^{x-1}$

$= p \sum_{x=1}^{\infty} x(x-1) q^{x-1}$

$= p[0 + 2 \cdot 1 \cdot q + 3 \cdot 2 \cdot q^2 + 4 \cdot 3 \cdot q^3 + 5 \cdot 4 \cdot q^4 + \cdots]$

$= p \cdot 2 \cdot q[1 + 3q + 6q^2 + 10q^3 + \cdots]$

$= 2pq(1-q)^{-3}$ Binomial expansion

$= \dfrac{2\cancel{p}q}{p^3} = \dfrac{2q}{p^2}$

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$$\because (1-x)^{-n} = 1 + (-n)(-x) + \frac{(-n)(-n-1).(-x)^2}{2!} + \frac{(-n)(-n-1)(-n-2)(-x)^3}{3!} + \cdots$$

Verification

$$(1-q)^{-3} = 1 + (-3)(-q) + \frac{(-3)(-3-1)(-q)^2}{2!} + \frac{(-3)(-3-1)(-3-2)(-q)^3}{3!} + \cdots$$

$$= 1 + 3q + \frac{3.4q^2}{2} + \frac{(-3).(-4).(-5)(-q^3)}{6} + \cdots$$

$$= 1 + 3q + 6q^2 + 10q^3 + \cdots$$

Thus

$$V(X) = E(X(X-1)) + E(X) - (E(X))^2$$

$$= \frac{2q}{p^2} + \frac{1}{p} - \frac{1}{p^2}$$

$$= \frac{2q + p - 1}{p^2}$$

$$= \frac{2q - (1-p)}{p^2} = \frac{2q - q}{p^2}$$

$$\boxed{V(X) = \frac{q}{p^2}}$$

#

Q1 You play a game of chance that you can either win or ~~lose~~ lose (there are no other possibilities) and you play this game untill you lose. Your prob. of losing is 0.57. What is the prob. that it takes 5 game untill you lose?

Soln

$$p_X(x) = p \cdot q^{x-1} \; ; \quad x = 1, 2, 3 \ldots$$

$p = 0.57 \; ; \; q = 0.43$

X: no of game untill you lose

$$p_X(5) = 0.57 \times (0.43)^{5-1}$$

$$= 0.57 \times (0.43)^4$$

$$= 0.0194$$