**Tutorial-5**

1. On a laboratory assignment, if the equipment is working, the density function of the observed outcome, $X$ is $f(X)=\begin{cases}2(1-x), & 0<x<1\\ 0, & otherwise\end{cases}$

   (a) Calculate $P\left(X\le\frac{1}{3}\right)$.

   (b) What is the probability that $X$ will exceed $0.5$?

2. Let X denote the amount of time in hours that a battery on a solar calculator will operate adequately between exposures to light sufficient to recharge the battery. The density function for X is given by $f(x)=(50/6)x^{-3}\quad 2<x<10$.

   (a) Find the average time that a battery will last before needing to be recharged.

   (b) Find $E[X^2]$, and use it to find the variance of X.

3. Define a random variable having Normal distribution. State necessary and sufficient conditions for a function, to be a continuous density function. Verify these conditions for normal distribution.

4. If X is a normal random variable with mean $\mu = 3$ and variance $\sigma_2 = 16$, find (a) $P\{X < 11\}$; (b) $P\{X > -1\}$; (c) $P\{2 < X < 7\}$

5. The Scholastic Aptitude Test mathematics test scores across the population of high school seniors follow a normal distribution with mean 500 and standard deviation 100. If five seniors are randomly chosen, find the probability that (a) all scored below 600 and (b) exactly three of them scored above 640.

6. The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$, $\sigma = 4$. What is the probability that in 2 of the next 4 years the rainfall will exceed 50 inches? Assume that the rainfalls in different years are independent.

7. The power W dissipated in a resistor is proportional to the square of the voltage V. That is, $W = rV^2$ where r is a constant. If r = 3, and V can be assumed (to a very good approximation) to be a normal random variable with mean 6 and standard deviation 1, find (a) E [W]; (b) $P\{W > 120\}$.

8. Define Joint probability mass function and Joint probability density function for marginal distribution.

9. Suppose that 3 batteries are randomly chosen from a group of 3 new, 4 used but still working, and 5 defective batteries. If we let X and Y denote, respectively, the number of new and used but still working batteries that are chosen. Find the joint probability mass function of X and Y , $p(i, j) = P\{X = i, Y = j\}$.Also, find the tabular representation of probabilities for $0 \le i, j \le 3$.

10. The joint density function of X and Y is given by $f(x,y)=\begin{cases}2e^{-x}e^{-2y} & 0<x<\infty, 0<y<\infty\\ 0 & otherwise\end{cases}$ Compute (a) $P\{X > 1, Y < 1\}$; (b) $P\{X < Y\}$; and (c) $P\{X < a\}$.