Spring Semester, 2022
B.Tech-II, Semester-(IV)
Tutorial Sheet-4
MA- 212

Full Marks: 10
Answer all of the following questions. All notations have their usual meanings.

1. To avoid detection at customs, a traveler places 4 narcotic tablets in a bottle containing 12 tablets that are similar in narcotic tablets. If the customs official selects 3 of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics?

2. From a lot of 10 missiles, 4 are selected at random and fired. If a lot contains 3 defective missiles that will not fire, what is the probability that :

(i) All 4 will fire.

(ii) At most 2 will not fire.

(iii) How many defective missiles might we expect to be included among the 4 that are selected.

3. Let $X \sim G(p)$. Determine:

(i) P(X is even).

(ii) P(X is odd).

(iii) $P(2 \leq X \leq 9 | X \geq 4)$.

(iv) CDF of X.

4. Let X = amount of time (in minutes) a postal clerk spends with his or her customer. The time is known to have an exponential distribution with the average amount of time equal to four minutes. Find the probability that a clerk spends three to five minutes with a randomly selected customer.

5. Are the given functions distribution functions? If so, find the corresponding PDF/ PMF:

(i)

$$F(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \frac{x}{2}, & \text{if } 0 \leq x < 1 \\ \frac{1}{2}, & \text{if } 1 \leq x < 2 \\ \frac{x}{4}, & \text{if } 2 \leq x < 4 \\ 1, & \text{if } x \geq 4. \end{cases}$$

(ii)

$$F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{(x-1)^2}{8}, & \text{if } 1 \leq x < 3 \\ 1, & \text{if } x \geq 3. \end{cases}$$

Moreover, evaluate $E(X)$?

5. Answer the following:

(i) $\Gamma(\frac{9}{2})$.

(ii) $\int_0^\infty x^7 e^{-6x} dx$.

(iii) B(5, 9).

(iii) $\int_0^1 x^7 (1-x)^9 dx$.