Beta Distribution :- A r.v X is said to have Beta dist^n if

$$f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \; ; \quad 0 \le x \le 1$$

$$\alpha > 0 \; ; \; \beta > 0$$

where $B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$

Note! $B(\alpha,\beta) = \dfrac{\Gamma\alpha\,\Gamma\beta}{\Gamma\alpha+\beta}$

Q: Verify that it is proper p.m.f then find $E(X)$ and $Var(X)$

Sol^n $\int_0^1 f_X(x)\,dx = 1$

$$\Rightarrow \int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta-1}\,dx}{B(\alpha,\beta)} = \frac{1}{B(\alpha,\beta)}\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

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$$= \frac{1}{\cancel{B(\alpha,\beta)}} \cdot \cancel{B(\alpha,\beta)}$$

$$= 1 \quad \#$$

$$E(X) = \int_0^1 x \cdot f_X(x)\,dx = \int_0^1 x \cdot \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}\,dx$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{\alpha}(1-x)^{\beta-1}\,dx$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{(\alpha+1)-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{1}{B(\alpha,\beta)} \quad B(\alpha+1,\beta)$$

$$= \frac{1}{\frac{\Gamma\alpha\,\Gamma\beta}{\Gamma\alpha+\beta}} \cdot \frac{\Gamma\alpha+1 \cdot \Gamma\beta}{\Gamma\alpha+\beta+1} = \frac{\cancel{\Gamma\alpha+\beta}}{\cancel{\Gamma\alpha}\,\cancel{\Gamma\beta}} \cdot \frac{\alpha\,\cancel{\Gamma\alpha} \cdot \cancel{\Gamma\beta}}{(\alpha+\beta)\,\cancel{\Gamma\alpha+\beta}}$$

$$\boxed{E(X) = \frac{\alpha}{\alpha+\beta}}$$

$$V(X) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Note: If we take $\alpha = \beta = 1$ then

$$X \sim U(1,1)$$

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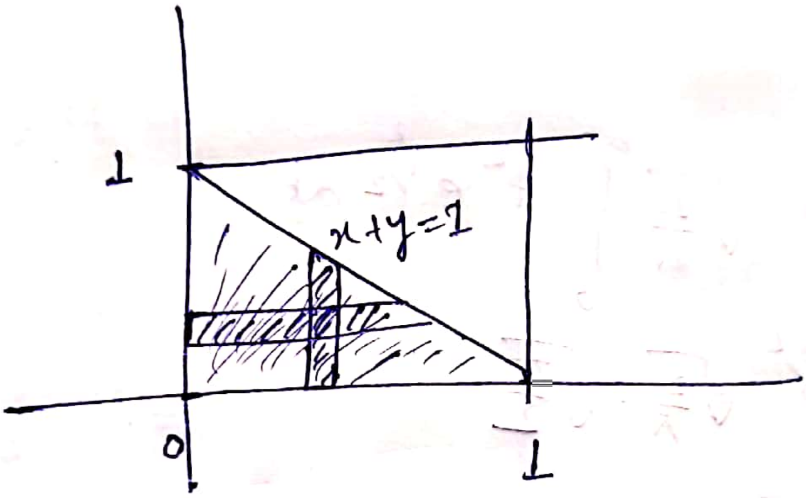
$$f(x,y) = \begin{cases} 24xy & ; 0 \le x \le 1, 0 \le y \le 1 \\ & ; x+y \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

Soln



$$f_X(x) = \int_{y=0}^{1-x} 24xy \, dy$$

$$= \begin{cases} 12x(1-x)^2 & ; 0 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

$$f_Y(y) = \int_{x=0}^{1-y} 24xy \, dx = 24y \left[\frac{x^2}{2}\right]_0^{1-y}$$

$$= 12 (1-y)^2$$

Thus

$$f_Y(y) = \begin{cases} 12(1-y)^2 \ ; & 0 \le y \le 1 \\ 0 \ ; & \text{otherwise} \end{cases}$$

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$$E(XY) = \iint xy \, f_{XY}(x,y) \, dy \, dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{1-x} xy \cdot 24xy \, dy \, dx$$

$$= \int_{x=0}^{1} 24x^2 \left[\frac{y^3}{3}\right]_{y=0}^{1-x} dx$$

$$= \int_{x=0}^{1} 8x^2 \cdot (1-x)^3 \, dx$$

$$= \frac{2}{15}$$

$$E(X) = \int_0^1 x \cdot f_X(x) \, dx = \frac{2}{5}$$

$$E(Y) = \frac{2}{5}$$

$$\text{cov}(X,Y) = E(XY) - E(X) \cdot E(Y)$$

$$= \frac{2}{15} - \frac{2}{5} \cdot \frac{2}{5} = \frac{-2}{75}$$

Signature of Supervisor