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Maths Tutorial-3

Sol- 1) RHS = $p \rightarrow (q \wedge r)$

$\bar{p} \vee (q \wedge r)$

using distributive law

$(\bar{p} \vee q) \wedge (\bar{p} \vee r)$

$(p \rightarrow q) \wedge (p \rightarrow r)$

= LHS

Hence proved

Sol- 2) $\overline{p \wedge q} = \bar{p} \vee \bar{q}$

p	q	$p \wedge q$	$\overline{p \wedge q}$	$\bar{p}$	$\bar{q}$	$(\bar{p} \vee \bar{q})$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Truth tables of $\overline{p \wedge q}$ & $\bar{p} \vee \bar{q}$ are same, hence $\overline{p \wedge q} = \bar{p} \vee \bar{q}$.

Sol- 3) $[(p \rightarrow q) \wedge p] \rightarrow q$

$[(\bar{p} \vee q) \wedge p] \rightarrow q$

$\overline{[(\bar{p} \vee q) \wedge p]} \vee q$

$[\overline{(\bar{p} \vee q)} \vee \bar{p}] \vee q$ [De-morgan's law]

$[(p \wedge \bar{q}) \vee \bar{p}] \vee q$

p	q	$\bar{p}$	$\bar{q}$	$(p \wedge \bar{q})$	$[(p \wedge \bar{q}) \vee \bar{p}]$	$[(p \wedge \bar{q}) \vee \bar{p}] \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

All true values

Hence proved that $[(p \to q) \wedge p] \to q$ is a tautology.

Sol-4) i) $\exists x (C(x) \longrightarrow F(x))$

if x is a comedian, then x is funny

ii) $\exists x (C(x) \wedge F(x))$

x is a commedian and x is a funny.

Sol-5) i) $\forall n (n+1 > n)$

True

ii) $\exists n \ (2n = 3n)$

at $n = 0$, true

iii) $\exists n \ (n = -n)$

at $n = 0$, true.

iv) $\forall n \ (n^2 \geq n)$

True

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(6) (a) Let, $p(x)$ be a disk
x has more than 10 KB of free space
$q(y)$ be a message
y can be saved

~~$\exists x\ p(x) \rightarrow \exists y\ q$~~

$$\exists x\ p(x) \longrightarrow \exists y\ q(y)$$

(b) Let, $p(x)$ be, "x participated on the conference call".
$q(x)$ be, "x was not put on the special list".
$r(x)$ be, "x was billed".

$$\forall x\ [\{p(x) \wedge q(x)\} \longrightarrow r(x)]$$

(7) $\exists x\ \forall y\ (x \le y^2)$
i) True
ii) True
iii) True

(8) If $\forall x\ (p(x) \vee Q(x))$ and $\forall x((P(x) \rightarrow Q(x)) \rightarrow R(x))$ are True,
then, $\forall x\ (\bar{R}(x) \rightarrow p(x))$ is also True.

Given $p(x) \vee Q(x)$ is True

$$[\bar{P}(x) \rightarrow Q(x)] \longrightarrow R(x) \text{ is True.}$$

Now; $[\bar{P}(x) \to Q(x)] \to R(x)$ is True

$[\bar{\bar{P}}(x) \vee Q(x)] \to R(x)$ is True

$[P(x) \vee Q(x)] \to R(x)$ is True

$$\underbrace{\left(\overline{\underbrace{P(x) \vee Q(x)}_{\text{True}}}\right)}_{\text{False}} \vee R(x) \text{ is True}$$

False $\vee$ $R(x)$ is True

$\therefore$ $R(x)$ is True.

Now, for $\bar{R}(x) \to P(x)$

$\bar{\bar{R}}(x) \to P(x)$

$\underbrace{R(x)}_{\text{always True}} \to P(x)$

$\therefore$ $\forall x\ [\bar{R}(x) \to P(x)]$ has to be true

Hence proved.

9) $\forall x\ \exists y\ (P(x, y)) \to T$

$\exists x\ \forall y\ (P(x, y)) \to T$

Yes, it is also true

using Conditional

$(\forall x)[P(x) \vee Q(x)] \to \exists x\ P(x) \vee (\forall x) Q(x)$

Q-10 $F(x,y)$ - " x can fool y "

(i) $\forall x \; F(x, \text{Rohan})$

(ii) $\forall y \; F(\text{Everyone}, y)$

(iii) $\forall x \; \exists y \; F(x,y)$

(iv) $\forall y \; \exists x \; \bar{F}(x,y)$

(v) $\forall y \; \exists x \; F(x,y)$

(vi) $\forall x \; \overline{(F(x, \text{Rohan}) \wedge F(x, \text{Rohit}))}$

(vii) $\exists x \, \exists y \, [(x != y) \wedge F(\text{Nancy}, x) \wedge F[\text{Nancy}, y) \wedge \forall w [F(\text{Nancy}, w) \rightarrow (w = x \forall w = y)]]$

(viii) $\forall x \; \bar{F}(x,x)$

(ix) $\forall x \; \exists! y \, [F(x,y) \wedge (x != y) \wedge \forall w \, (F(x,w) \rightarrow w = y)]$

END