Lecture - 1 P.N. 1

Normal Distribution :-

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; \quad x \in \mathbb{R}, \ \mu \in \mathbb{R}, \ \sigma > 0 \qquad -(1)$$

$$E(X) = \mu$$

$$V(X) = \sigma^2$$

Denote as $X \sim N(\mu, \sigma^2)$.

Now consider

$$Z = \frac{X-\mu}{\sigma} \quad ; \quad \text{Given that } E(X) = \mu ; V(X) = \sigma^2$$

$$E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = E\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma}E(X) - E\left(\frac{\mu}{\sigma}\right) \quad (\because E(aX+b) = aEX+b)$$

$$= \frac{1}{\sigma}\cdot\mu - \frac{\mu}{\sigma} = 0$$

$$V(Z) = V\left(\frac{X-\mu}{\sigma}\right) = V\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right)$$

$$\because V(aX+b) = a^2 V(X)$$

$$\Rightarrow V(Z) = \frac{1}{\sigma^2} V(X)$$

$$= \frac{1}{\cancel{\sigma^2}}\cancel{\sigma^2} = 1$$

Thus $Z \sim N(0,1) \rightarrow$ Called standard normal distributi

Density is given as

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad ; \quad z \in \mathbb{R}$$

(By putting $\mu = 0$ and $\sigma^2 = 1$ in equation (1))

CDF :-

$$F_Z(z) = \int_{-\infty}^{z} f_Z(t)\,dt = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$

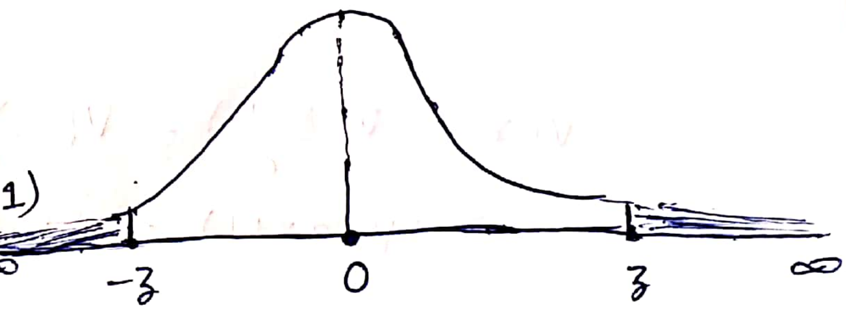
which is not integrable. So CDF of standard normal dist^n is not in close form. But we can evaluate the CDF of it by using the standard normal table. It is denoted by

$$\boxed{\phi(z) = F_Z(z) = \int_{-\infty}^{z} f_Z(t)\,dt}$$

Result : Prove that $\phi(-z) = 1 - \phi(z)$

Proof: Since standard normal dist^n is symmetric about its mean (0).

Also we have;



$$1 = \int_{-\infty}^{\infty} f_Z(z)\,dz \quad (\because \text{Whole area under curve is } 1)$$

$$= \int_{-\infty}^{-z} f_Z(t)\,dt + \int_{-z}^{\infty} f_Z(t)\,dt$$

$$1 = \phi(-z) + \int_{-z}^{\infty} f_Z(t)\,dt$$

$$\Rightarrow \quad \phi(-z) = 1 - \int_{-z}^{\infty} f_Z(t)\,dt = 1 - \int_{-z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$

Now put $t = -u$

$dt = -du$

when $t = -z$; $u = z$

when $t \to \infty$; $u \to -\infty$

P.N. 3

$$\Rightarrow \quad \phi(-3) = 1 - \int_{-\infty}^{3} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

(We remove the -ve sign of du as we interchang the limit; upper as lower and lower as upper)

$$= 1 - \phi(3)$$

Thus $\boxed{\phi(-3) = 1 - \phi(3)}$ #