55. Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that 16% of all ticks in a certain location carry Lyme disease, 10% carry HGE, and 10% of the ticks that carry at least one of these diseases in fact carry both of them. If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease?

2.4 section

55 $A \to$ Event that deer tick carries Lyme disease

$B \to$ " " " " " HGE disease

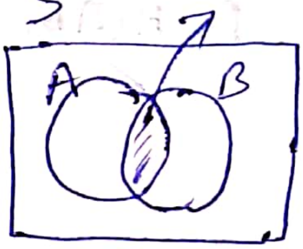
$P(A) = 0.16$; $P(B) = 0.10$

$P(A \cap B \mid A \cup B) = 0.10$

$P(A \mid B) = ?$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

S



$$\because P(A \cap B \mid A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = 0.1$$

$$= \frac{P(A \cap B)}{P(A \cup B)} = 0.1$$

$$\Rightarrow P(A \cap B) = 0.1 \cdot P(A \cup B)$$

$$\Rightarrow P(A \cap B) = 0.1\,[P(A) + P(B) - P(A \cap B)]$$

$$\Rightarrow P(A \cap B) = 0.1 \cdot P(A) + 0.1\, P(B) - 0.1 \cdot P(A \cap B)$$

$$\cancel{\Rightarrow P(A \cap B) = 0.1 \cdot P(A)}$$

$$\Rightarrow P(A \cap B) + 0.1 \cdot P(A \cap B) = 0.1 \cdot P(A) + 0.1 \cdot P(B)$$

$$\Rightarrow 1.1 \cdot P(A \cap B) = 0.1 \times 0.16 + 0.1 \times 0.16$$

$$\Rightarrow P(A \cap B) = \frac{0.1 \times 0.16 + 0.1 \times 0.16}{1.1} = 0.0236$$

$$\Rightarrow P(A \mid B) = \frac{0.0236}{0.10} = 0.236$$

59. At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2), and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

a. What is the probability that the next customer will request plus gas and fill the tank ($A_2 \cap B$)?

b. What is the probability that the next customer fills the tank?

c. If the next customer fills the tank, what is the probability that regular gas is requested? Plus? Premium?

Sedan 2.4 (59)

$A_1 \rightarrow$ regular gas

$A_2 \rightarrow$ plus gas

$A_3 \rightarrow$ premium gas

$P(A_1) = 0.40$; $P(A_2) = 0.35$; $P(A_3) = 0.25$

$B \rightarrow$ Event of fill their tank

$P(B|A_1) = 0.30$; $P(B|A_2) = 0.60$; $P(B|A_3) = 0.50$

(a) $P(A_2 \cap B) = P(B|A_2) \cdot P(A_2)$ (By multiplication formula)

$= 0.60 \times 0.35$

$= 0.21$

(Total probability)

(b) $P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \, P(A_3)$

$= 0.30 \times 0.40 + 0.60 \times 0.35 + 0.50 \times 0.25$

$= 0.455$

(c) $$P(A_1|B) = \frac{P(B|A_1) \cdot P(A_1)}{P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) P(A_3)}$$

$$= \frac{0.30 \times 0.40}{0.455} = 0.2637$$

87. Consider randomly selecting a single individual and having that person test drive 3 different vehicles. Define events A_1, A_2, and A_3 by

A_1 = likes vehicle #1 A_2 = likes vehicle #2
A_3 = likes vehicle #3

Suppose that $P(A_1) = .55$, $P(A_2) = .65$, $P(A_3) = .70$, $P(A_1 \cup A_2) = .80$, $P(A_2 \cap A_3) = .40$, and $P(A_1 \cup A_2 \cup A_3) = .88$.

a. What is the probability that the individual likes both vehicle #1 and vehicle #2?

b. Determine and interpret $P(A_2 | A_3)$.

c. Are A_2 and A_3 independent events? Answer in two different ways.

d. If you learn that the individual did not like vehicle #1, what now is the probability that he/she liked at least one of the other two vehicles?

Section 2.5

87 $P(A_1) = 0.55$; $P(A_2) = 0.65$;

$P(A_3) = 0.70$; $P(A_1 \cup A_2) = 0.80$;

$P(A_2 \cap A_3) = 0.40$; $P(A_1 \cup A_2 \cup A_3) = 0.88$

(a) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$

$= 0.55 + 0.65 - 0.80$

$= 0.40$

(b) $P(A_2 | A_3) = \dfrac{P(A_2 \cap A_3)}{P(A_3)} = \dfrac{0.40}{0.70} = 0.5714$

(c) if A_2 and A_3 are independent then

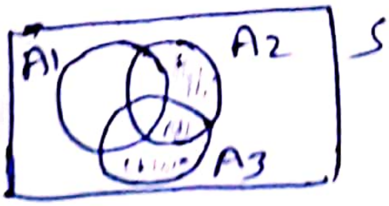
$P(A_2 \cap A_3) = P(A_2) \cdot P(A_3)$

$P(A_2 | A_3) = P(A_2)$

Here $P(A_2 \cap A_3) = 0.40 \neq P(A_2) \cdot P(A_3)$

$\Rightarrow$ A_2 and A_3 are not independent.

d) $P(A_2 \cup A_3 | A_1^c) = \dfrac{P((A_2 \cup A_3) \cap A_1^c)}{P(A_1^c)} = \dfrac{P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)}{1 - P(A_1)}$



$= \dfrac{P(A_1 \cup A_2 \cup A_3) - P(A_1)}{1 - P(A_1)} = \dfrac{0.88 - 0.55}{1 - 0.55} = \dfrac{11}{15}$ Ans midsem 2019

Q: Let $f_X(x)$ be the probability mass function of random variable X, with

$$f(-3) = f(3) = \frac{3}{16} \quad \text{and} \quad f(-2) = f(2) = \frac{5}{16}.$$

Can ~~further~~ $f_X(x)$ have any further positive values?

Sol^n: $\because$ $f_X(x)$ is p.m.f.

$$\Rightarrow \sum_{x \in R_X} f_X(x) = 1$$

Here; $f(-3) + f(3) + f(-2) + f(2) = \frac{3}{16} + \frac{3}{16} + \frac{5}{16} + \frac{5}{16} = 1$

$\Rightarrow$ $f_X(x)$ can not take any further positive values.

Moment:

a) Non-Central Moment: Let X be a r.v then the n^{th} non-central moment is defined as

$$E(X^n) = \sum_{x \in R_X} x^n \, p_X(x).$$

Note: The first order non-central moment is mean or average of the r.v X.

$$\because E(X^1) = E(X) = \sum_{x \in R_X} x \cdot p_X(x) \rightarrow \text{Mean of } X$$

(b) Central Moment ÷ The n^{th} order central moment of r.v X is defined as

$$E(X-\mu)^n = \sum_{x \in R_X} (x-\mu)^n p_X(x)$$ where μ is mean or average of r.v X.

Note: The second central moment of r.v X is its variance.

$$E(X-\mu)^2 = \sum_{x \in R_X} (x-\mu)^2 \cdot p_X(x)$$

$$E(X-\mu)^2 = E(X-E(X))^2 \quad (\because \mu = E(X))$$

$$= \text{Variance of } X$$

$$= V(X)$$

Bernoullian Trial ÷ In statistical experiment, a trial is said to be Bernoullian trial if it has only two possible outcomes, say success & failure.

In general prob. of success is denoted as p and prob. of failure is denoted by $q\ (=1-p)$ s.t $p+q=1$