Axiomatic Defination of Probability (Kolmogrov 1933) :-

Let S be the sample space and $\mathcal{B}$ be the collection of all possible subsets (power set) of S and consider an event $A \in \mathcal{B}$ then the probability of event A must satisfies the following properties

(i) $P(A) \geq 0 \quad \forall\ A \in \mathcal{B}$

(ii) $P(S) = 1$

(iii) If $A_1, A_2 \cdots A_n$ are pairwise disjoint or mutually exclusive then

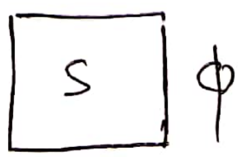
$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

i/e $P(A_1 \cup A_2 \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

Some Important Results :

(1) $P(\phi) = 0$

<u>Proof</u>:



Here $S \cup \phi = S$

$P(S \cup \phi) = P(S)$

Since S and ϕ are mutually exclusive events

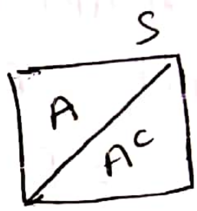
So, $P(S) + P(\phi) = P(S)$ (By (iii) property)

$\Rightarrow \cancel{1} + P(\phi) = \cancel{1}$ (By (ii) property)

$\Rightarrow \boxed{P(\phi) = 0}$ #

(2) $P(A^c) = 1 - P(A)$

Proof:



We can write

$A \cup A^c = S$

$\Rightarrow P(A \cup A^c) = P(S)$

$\because$ A and A^c are mutually exclusive events

$\Rightarrow P(A) + P(A^c) = P(S)$ (By (iii) Property)

$\Rightarrow P(A) + P(A^c) = 1$ (By (ii) property)

$\Rightarrow \boxed{P(A^c) = 1 - P(A)}$ #

3) $0 \leq P(A) \leq 1$

Proof: $\because P(A) \geq 0$ (By (i) property)

Also $P(A) = 1 - P(A^c) \leq 1$ ($\because P(A^c) \geq 0$)

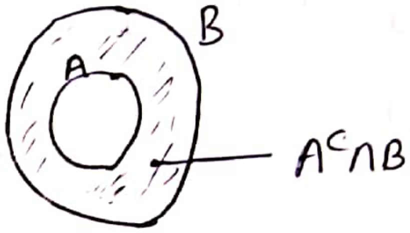
$\Rightarrow P(A) \leq 1$

$\Rightarrow \boxed{0 \leq P(A) \leq 1}$ #

(4) If $A \subseteq B \Rightarrow P(A) \leq P(B)$

Proof:

We can write



$$B = A \cup (A^c \cap B)$$

$\Rightarrow P(B) = P(A \cup (A^c \cap B))$

$\because$ A and $(A^c \cap B)$ are mutually exclusive

$\Rightarrow P(B) = P(A) + P(A^c \cap B)$ (By (iii) property)

$\Rightarrow P(B) - P(A) = P(A^c \cap B) \geq 0$ ($\because$ Prob. of any event is greater than or equal to zero)

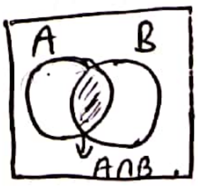
$\Rightarrow P(B) - P(A) \geq 0$

$\Rightarrow P(B) \geq P(A)$

$\Rightarrow \boxed{P(A) \leq P(B)}$ #

(5) $P(A \cap B^c) = P(A) - P(A \cap B)$

Proof:



$\because A = (A \cap B) \cup (A \cap B^c)$

$\Rightarrow P(A) = P[(A \cap B) \cup (A \cap B^c)]$

$\because$ $A \cap B$ and $A \cap B^c$ are mutually exclusive events

$\Rightarrow P(A) = P(A \cap B) + P(A \cap B^c)$ (By (iii) property)

$\Rightarrow \boxed{P(A \cap B^c) = P(A) - P(A \cap B)}$ #

~~imp~~ Addition Formula :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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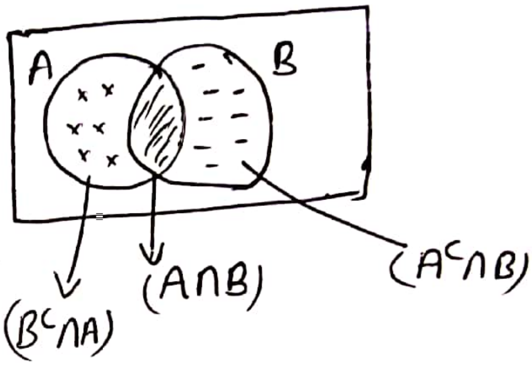
Proof:



$\because$ $(A \cup B) = (A \cap B) \cup (B^c \cap A) \cup (A^c \cap B)$

$\because$ $(A \cap B)$, $(B^c \cap A)$ and $(A^c \cap B)$ are pairwise mutually exclusive

$$\Rightarrow P(A \cup B) = P[(A \cap B) \cup (B^c \cap A) \cup (A^c \cap B)]$$

$$= P(A \cap B) + P(A \cap B^c) + P(A^c \cap B) \quad (\because \text{By (iii) property})$$

$$= P(\cancel{A \cap B}) + P(A) - P(\cancel{A} \cap B) + P(B) - P(A \cap B)$$

$$\Rightarrow \boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)} \quad \#$$

Addition formula for 3 events :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Proof: Let $B \cup C = D$

then $P(A \cup D) = P(A) + P(D) - P(A \cap D)$ (By Addition formula for 2 events)

$$= P(A) + P(B \cup C) - P(A \cap (B \cup C))$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - [P((A \cap B) \cup (A \cap C))]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))]$$

$$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \quad \#$$