Tutorial-7 (Tut-2 Indira mam)

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U20CS110

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Q1) $x - 2y + 3t = 2$

$2x + y + z + t = -4$

$4x - 3y + z + 7t = 8$

→ step-1, Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & 8 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & -1 & -5 & -8 \\ 0 & 5 & -1 & -5 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & -1 & -5 & -8 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right] \quad R_3 \to R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 0 & 1 & -1/5 & -1 & -8/5 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2/5 \\ R_3 \to R_3/8 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 0 \\ 0 & 1 & -1/5 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Here, Rank of A : B is = 3

while, Rank of A is = 2.

$\rho(A:B) \neq \rho(A)$

Hence, no solution.

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Q2) ① $x + 2y + 3z = 0$

$3x + 4y + 4z = 0$

$7x + 10y + 12z = 0$

- Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right] \quad R_2 \to -R_2/2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_3 \to R_3 + 4R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2 \to R_3 - 5/2 R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 3R_3 \\ R_1 \to R_1 - 2R_2 \end{array}$$

$\rho(A:B) = 3$
$\rho(A) = 3$

Here, $\rho(A:B) = \rho(A) = 3$ (no. of unknowns)
so, unique solution.

$\rightarrow$ $1x + 0 + 0 = 0$
$0 + y + 0 = 0$
$0 + 0 + z = 0$

$\therefore$ $x = y = z = 0$, only unique solution.

(2) $4x + 2y + z + 3w = 0$
$6x + 3y + 4z + 7w = 0$
$2x + y + w = 0$

$\rightarrow$ Augmented matrix

$$\left[\begin{array}{cccc|c} 4 & 2 & 1 & 3 & 0 \\ 6 & 3 & 4 & 7 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} -1 & 1/2 & 1/4 & 3/4 & 0 \\ 6 & 3 & 4 & 7 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{array}\right] \quad R_1 \rightarrow R_1/4$$

$$\left[\begin{array}{cccc|c} 1 & 1/2 & 1/4 & 3/4 & 0 \\ 0 & 0 & 5/2 & 5/2 & 0 \\ 0 & 0 & -1/2 & -1/2 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 - 6R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{array}$$

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$$\left[\begin{array}{cccc|c} 1 & 1/2 & 1/4 & 3/4 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to 2R_2/5 \\ R_3 \to -2R_3 \\ R_3 \to R_3 - R_2 \end{array}$$

$\rightarrow \rho(A:B) = 2$

$\rho(A) = 2$

$\rho(A:B) = \rho(A) < 4$ (no. of unknowns)

$\therefore$ This system of equations has infinite many solution.

Q3) $x + y + z = 1$

$2x + y + 4z = k$

$4x + y + 10z = k^2$

$$\rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 20 + 4 + 4 - 2 - 16 - 10 = 0$$

for this equation having sol,

$\Delta_1, \Delta_2, \Delta_3 = 0$.

$$\therefore \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ k^2 & 1 & 4 \\ k^2 & 1 & 10 \end{vmatrix}$$

$= 6 + 4k^2 - 10k + k - k^2$

$= 3k^2 - 9k + 6$

$= 3(k^2 - 3k + 2)$

$= 3(k-1)(k-2)$

so, $k = 1$ and $k = 2$.

→ for. $K = 1$ & $K = 2$. This system will have infinite sol.

or,
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & K \\ 4 & 1 & 10 & K^2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & K-2 \\ 0 & -3 & 6 & K^2-4 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & K-2 \\ 0 & 0 & 0 & K^2-4-3K+6 \end{array}\right] \quad R_3 \to R_3 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & K-2 \\ 0 & 0 & 0 & K^2-3K+2 \end{array}\right]$$

Now, $\rho(A) = 2$

So, for having solution.

$$\rho(A:B) = 2$$
$$K^2 - 3K + 2 = 0$$
$$K = 1 \text{ or } K = 2$$

and, if $K = 1$ or $K \neq 2$ then, This system won't have any solution.

$\rho(A:B) = \rho(A) \leq r$ (No. of Variables)

so, system will have infinite solutions.

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Q4) ① $2x - 3y + 7z = 5$
$3x + y - 3z = 13$
$2x + 19y - 47z = 32$

→ Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right] \quad R_1 \to R_1/2$$

$$\left[\begin{array}{cccc} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 22 & -54 & 27 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 \to 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 1 & -27/11 & 1 \\ 0 & 0 & 0 & 5 \end{array}\right] \quad \begin{array}{l} R_2 \to 2R_2 \\ R_3 \to R_3 - 22R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 1 & -27/11 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad R_3 \to R_3/5$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 0 \\ 0 & 1 & -27/11 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_3 \\ R_1 \to R_1 - 5R_3/2, \end{array}$$

Now, $\rho(A:B) = 2$, $\rho(A) = 2$

So, soln doesn't exist for this system of equation.

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(11) $x + 2y + z = 3$
$2x + 7y + 2z = 5$
$3x - 5y + 5z = 2$
$3x + 9y - z = 4$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \\ 3 & 9 & -1 & 4 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -2 & 0 & -1 \\ 0 & -11 & 2 & -7 \\ 0 & 3 & -5 & -5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 2 & -3/2 \\ 0 & 3 & -5 & -5 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & -41/4 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\rho(A:B) \neq \rho(A) \rightarrow$ so, eq^n is Inconsistent.

Q5) $x + ay + z = 3$
$x + 2y + 2z = 6$
$x + 5y + 3z = 9$

$\rightarrow$ $\Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = -4 - a + 2 - (a+1)$

$\Delta_1 = \begin{vmatrix} 3 & a & 1 \\ 6 & 2 & 3 \\ 6 & 5 & 2 \end{vmatrix} = 2(9-b)(1-a)$

$\Delta 2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} = (9-b)$

$\Delta 3 = \begin{vmatrix} 1 & a & 3 \\ 1 & 2 & b \\ 1 & 5 & b \end{vmatrix} = b(2-a) - 3(7-2a)$

for unique solution :-

$\Delta \neq 0$ so, $a \neq -1$

and, also . $\Delta_1, \Delta_2, \Delta_3$.

$- b \neq 9$.

$\therefore$ $a \neq -1$, $b \neq 9$ for unique solution.

Q6) $(\lambda-1)x + (3\lambda-1)y + 2\lambda z = 0$

$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$

$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$.

→ $\Delta = 0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & -\lambda+3 & \lambda-3 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0 \qquad R_1 \to R_1 - R_2$$

$$\begin{vmatrix} 0 & -\lambda+3 & \lambda-3 \\ \lambda-1 & 5\lambda-5 & 6 \\ 2 & 4\lambda-2 & 2\lambda \end{vmatrix} = 0 \qquad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{vmatrix} 0 & 0 & \lambda-3 \\ \lambda-1 & 5\lambda+1 & 6 \\ 2 & 6\lambda-2 & 2\lambda \end{vmatrix} = 0 \qquad C_2 \to C_2 + C_3.$$

$\therefore (\lambda-3)\left((\lambda-1)(6\lambda-2) - 2(5\lambda+1)\right) = 0.$

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$\therefore \lambda - 3 = 0$ or $6\lambda^2 - 8\lambda + 2 - 10\lambda - 2 = 0$

$\therefore \lambda = 0$ or $\lambda = 3$.

for $\lambda = 0$,

$-x + y = 0$
$-x - 2y + 3z = 0$
$2x + y - 3z = 0$

$\therefore x = y$ $\therefore 3z = 3x$
$x = z$

$\therefore x : y : z = 1 : 1 : 1$

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