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Maths Tutorial-3

U20CS110
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Sol-1 Random variable is a function from which assigns value to each of experiment outcome. Random variable are often designated by letters and can be classified as discreate as well as continious.

① T:- The turnaround time for a computer

→ here value of T is continious so RV will be continious

② M:- The number of meteroites hiting a satellite per day.

→ here values of M will be discreate so RV will be Discreate.

③ X:- The no. of power failures per month in SVNIT HOSTEL

→ here values of X will be discreate so RV will Discrete

Sol-2 Total throws $n = 3$

success $p = \frac{2}{6} = \frac{1}{3}$

Failure $q = \frac{4}{6} = \frac{2}{3}$

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$P_X(X) = ({}^{n}C_{X})\,(p)^{X}\,(q)^{n-X}$ $\quad X \to$ success trials

$R_X \to \{0,1,2,3\}$

for $X = 0 \to$ PMF $({}^{3}C_{0})\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{3} = \frac{8}{27}$

$x = 1 \to ({}^{3}C_{1})\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{2} = \frac{12}{27}$

$x = 2 \to \frac{6}{27}$

$x = 3 \to \frac{1}{27}$

$E(X) = np = 3 \times \frac{1}{3} = 1$

$V(X) = npq = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3} = \sigma^{2}(x)$

$\therefore \sigma(x) = \sqrt{\frac{2}{3}}$

Sol-3

X	0	1	2	3	4	5
F(x)	0.7	0.2	0.05	0.03	0.01	(1)

(a) $\sum_{x \in R} F(x) = 1$

$0.7 + 0.2 + 0.05 + 0.03 + 0.01 + F_5 = 1$

$\boxed{F_5 = 0.01}$

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(b) Now $E(X) = \sum_{x \in R} x \cdot f(x)$

$= 0 \times 0.7 + 1 \times 0.2 + 2 \times 0.05 + 3 \times 0.03$
$+ 4 \times 0.01 + 5 \times 0.01$

$= 0.48$

(c) $E(X^2) = \sum_{x \in R} x^2 f(x)$

$= 0^2 \times 0.7 + 1^2 \times 0.2 + 2^2 \times 0.05 + 3^2 \times 0.03 + 4^2 \times 0.01$
$+ 5^2 \times 0.01$

$= 1.08$

(d) $Var(X) = E(X^2) - [E(X)]^2$

$= 1.08 - (0.48)^2$
$= 0.8496$

(e) $\sigma_x^2 = Var\,x = 0.8496$

(f) $\sigma_x = \sqrt{0.8496} = 0.9217$

(g) σ_x is measured in the same units as original data. That is, for instance data are in feet, then sample variance will be sq feet and standard deviation in units of feet.

Sol-4 $F_x(x) = P(x \leq x)$

$F_x(0) = P(x \leq 0) = 0.7$
$F_x(1) = P(x \leq 1) = 0.7 + 0.2 = 0.9$

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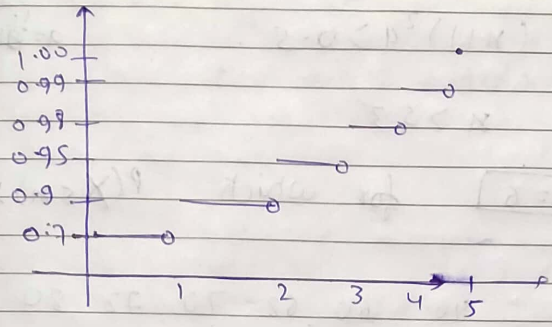
$F_x(2) = P(x \le 2) = 0.9 + 0.05 = 0.95$
$F_x(3) = P(x \le 3) = 0.98 + 0.03 = 0.98$
$F_x(4) = P(x \le 4) = 0.98 + 0.01 = 0.99$
$F_x(5) = P(x \le 5) = 0.99 + 0.01 = 1$



0	$x < 0$
0.7	$x = 0$
0.9	$x \in [0,1)$
0.95	$x \in [1,2)$
0.98	$x \in [2,3)$
0.99	$x \in [3,4)$
1	$x \in 5$

$P_x(3) = F(3) - F(\bar{3})$
$= 0.98 - 0.95$
$= 0.03$

Hence using CDF $P_x(3)$ 0.03 is verified.

Sol-5

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) $\sum_{x \in R} P(x) = 1$

$(1 + 3 + 5 + \cdots 17)\, a = 1$

$9^2 a = 1$

$a = \frac{1}{81}$

(ii) $P(x < 3)$
$P(0) + P(1) + P(2)$
$(1+3+5)\, a = 9 \times \frac{1}{81}$

$= \frac{1}{9}$

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$$P(X \le x) > 0.5$$

$$P_X(0) + P_X(1) + \cdots P(x) > 0.5$$

$$a + 3a + 5a + \cdots (2x+1)a > 0.5$$

$$(x+1)^2 a > 0.5 \qquad a = \frac{1}{81}$$

$$x > 5.3$$

so $\boxed{x = 6}$ for which $P(X \le x) > 0.5$

Sol-6

X/Y	40	60	68	70	72	80	100
$f(x)$	0.01	0.04	0.05	0.80	0.05	0.04	0.01
$f(y)$	0.4	0.05	0.04	0.02	0.04	0.05	0.4

(a) $E(X) = \sum_{x \in R} x f_X(x)$

$= 40 \times 0.01 + 60 \times 0.04 + 68 \times 0.05 + 70 \times 0.8 + 72 \times 0.05$
$+ 80 \times 0.04 + 100 \times 0.01$

$= 70$

$E(Y) = \sum_{y \in R} y f_Y(y)$

$= 40 \times 0.4 + 60 \times 0.05 + 68 \times 0.04 + 70 \times 0.02 + 72 \times 0.04$
$+ 80 \times 0.05 + 100 \times 0.4$

$= 70$

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(b) $\mu_x = E(x) = 70$
$\mu_y = E(y) = 70$

(c) $E(x^2) = \sum_{x \in R} x^2 f(x)$

$= (40)^2 \times 0.01 + (60)^2 \times 0.04 + (68)^2 \times 0.05 + (70)^2 \times 0.8 + (72)^2 \times 0.05 + (80)^2 \times 0.04 + (100)^2 (0.01)$

$= 4926.4$

similarly
$E(y^2) = \sum_{y \in R} y^2 f(y)$

$= 5630.32$

(d) $Var(x) = E(x^2) - [E(x)]^2$
$= 4926.4 - (70)^2$
$= 26.4$

$Var(y) = E(y^2) - [E(y)]^2$

$= 5630.32 - 4900$
$= 730.32$

(e) $\sigma x : \sqrt{var(x)} = 5.138$
$\sigma y : \sqrt{var(y)} = 27.824$

(f) units of standard deviation is same as that of x and y, heart beat of patient

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Drug x is more efficient as
$var(x) < Var(y)$
and $\sigma(x) < \sigma(y)$

Sol-7 A Random variable is a variable whose value is unknown or a function that assigns values to each of an experiments outcomes. Random variable is often designated by letters and classified in Discreate and continious values.

suppose Random variable
$X(P, n)$
succes → Trial

$P(x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$

$M_x(t) = \Sigma\, [e^{tx}]$

$= \sum_{i=0}^{n} e^{tx} P(x)$

$= \sum_{i=0}^{n} ({}^{n}C_{x}) (e^{t}p)^{x} (1-p)^{n-x}$

$M_x(t) = (e^{t}p + 1 - p)^{n}$

$M_x'(0) = E(x) = M_x$
$M_x(t) = n\,[e^{t}p + 1-p]^{n-1}\, pe^{t}$
$M_x(0) = n\,[p+1-p]^{n-1} p = np$ mean

$E(x^2) = M''_x(0)$

$M''_x(t) = n(n-1)p^2 + np \cdot E(x^2)$

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$var(x) = E(x^2) - [E(x)]^2 = np(1-p)$

Sol-8 $p = 1/3$ $q = 2/3$ $n = 7$

(a) $P_r(X=3) = {}^7C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^4$

$= \frac{560}{2187}$

(b) $P(X \geq 1) = 1 - P_r(x < 1)$

$= 1 - P_r(x=0)$

$= 1 - {}^7C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^7$

$= \frac{2059}{2187}$

Sol-9 $p = 1/6$ $q = 5/6$ $n = 10$

Now $P_r(x<2) = P(x=0) + P(x=1)$

$= {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9$

$= 0.2422$

Sol-10 A Discrete random variable X which has following Prob mass f^n (PMF)

$P(X) = \frac{\lambda^x e^{-\lambda}}{x}$ $x = 0, 1, 2, \ldots$

is called Poission Random variable
& Distribution is called poission distributn

$$n \to \infty,\ p \to 0,\ np = \lambda$$

riv

$$\lim_{n \to \infty} P(x)$$

$$\lim_{n \to \infty} {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$\lim_{n \to \infty} \frac{n!}{(n-x)!\,x!}\, p^{x} (1-p)^{n-x}$$

$$\lim_{n \to \infty} \frac{n(n-1)\cdots[n-(x-1)]}{x!\ (n-x)!} \left(\frac{\lambda}{n}\right)^{x} \left[1-\frac{\lambda}{n}\right]^{n-x}$$

$$\lim_{n \to \infty} \frac{n^{x}\left(\left(1-\frac{1}{n}\right)\left(1- \cdots \left(1-\frac{x-1}{n}\right)\right)\right)}{x!} \frac{\lambda^{x}}{n^{x}} \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$\lim_{n \to \infty} \frac{1}{x!} \lambda^{x} \left[1-\frac{\lambda}{n}\right]^{n} \left[1-\frac{\lambda}{n}\right]^{-x}$$

$$\frac{\lambda^{x}}{x!} e^{-\lambda} (1)$$

$$\frac{\lambda^{x} e^{-\lambda}}{x!}$$ proved

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Sol-11) $P = 0.3 \quad q = 0.7$

(a) probability that 8th child is 3rd

$= {}^{8-1}C_{3-1} \; (0.3)^3 \; (0.7)^{8-3}$

$= {}^{7}C_{2} \; (0.3)^3 \; (0.7)^5$

$= 0.09529$

(b) $P = 0.3 \quad q = 0.7 \quad r = 3$

$E(x) = \frac{r}{p}$

$= \frac{3}{0.3} = 10$