

Properties of Variance \div

(1) $V(a) = 0$ where a is constant

Proof: $V(a) = E(a^2) - (E(a))^2$ (By defⁿ)
 $= a^2 - (a)^2$ ($\because E(a) = a$ and $E(a^2) = a^2$)
 $= a^2 - a^2 = 0 \quad \#$

(2) $V(aX + b) = a^2 V(X)$ ♦

Proof: $V(aX + b) = E((aX + b)^2) - (E(aX + b))^2$
 $= E(a^2 X^2 + b^2 + 2abX) - (aE(X) + b)^2$ ($\because E(aX + b) = aE(X) + b$)
 $= E(a^2 X^2) + E(b^2) + E(2abX)$
 $\quad - (a^2 (E(X))^2 + b^2 + 2abE(X))$
 $= a^2 E(X^2) + b^2 + 2abE(X) - a^2 (E(X))^2 - b^2 - 2abE(X)$
 $= a^2 (E(X^2) - (E(X))^2) = a^2 V(X) \quad \#$

Q: If Mean of X is 2 then find the $E(-2X+3)$?

Soln

$$\begin{aligned} E(-2X+3) &= -2 E(X) + 3 \quad (\because E(aX+b) = aE(X)+b) \\ &= -2 \cdot 2 + 3 \quad (\because E(X) = 2 \text{ given}) \\ &= -4 + 3 \\ &= -1 \end{aligned}$$

Q: If variance of X is $\sqrt{2}$ then find

(i) $V(-\sqrt{2}X+6)$ (ii) $V(3X)$ (iii) $V(6-4X)$

Soln

$$\begin{aligned} \text{(i)} \quad V(-\sqrt{2}X+6) &= (-\sqrt{2})^2 V(X) \\ &= 2 V(X) \\ &= 2\sqrt{2} \quad \# \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad V(3X) &= 3^2 V(X) \\ &= 9 \times \sqrt{2} = 9\sqrt{2} \quad \# \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad V(6-4X) &= (-4)^2 V(X) \\ &= 16 \times \sqrt{2} \\ &= 16\sqrt{2} \quad \# \end{aligned}$$