47. Return to the credit card scenario of Exercise 12 (Section 2.2), where $A = \{\text{Visa}\}$, $B = \{\text{MasterCard}\}$, $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$. Calculate and interpret each of the following probabilities (a Venn diagram might help).

a. $P(B \mid A)$ **b.** $P(B' \mid A)$

c. $P(A \mid B)$ **d.** $P(A' \mid B)$

e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?

Section 2.4

47 $A = \{\text{Visa}\}$; $P(A) = 0.50$

$B = \{\text{MasterCard}\}$; $P(B) = 0.40$

and $P(A \cap B) = 0.25$

(a) $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.25}{0.50} = \dfrac{25}{50} = 0.5$

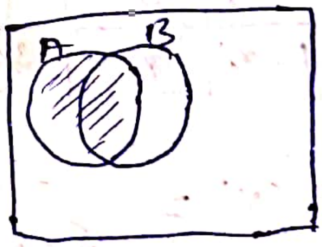
(b) $P(B' \mid A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{P(A) - P(A \cap B)}{P(A)}$

$= \dfrac{0.50 - 0.25}{0.50} = \dfrac{0.25}{0.50} = 0.5$

(c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.25}{0.40} = \dfrac{25}{40} = \dfrac{5}{8} = 0.625$

(d) $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{0.40 - 0.25}{0.40}$

$= \dfrac{0.15}{0.40} = \dfrac{15}{40} = \dfrac{3}{8} = 0.375$



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(e) $P(A \mid A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)}$

$= \dfrac{P(A)}{P(A) + P(B) - P(A \cap B)}$

$= \dfrac{0.50}{0.50 + 0.40 - 0.25} = \dfrac{0.50}{0.65} = 0.769$

Mathematical Expectation :- Let X be descrete r.v with pmf $p_X(x)$ then expected value of X is denoted by $E(X)$ and is defined as

$$E(X) = \sum_{x \in R_X} x \cdot p_X(x).$$

It is also called average or mean value of r.v X.

Ex: Find the expected value of X in previous example.

soln: R.E :- Tossing a coin Thrice

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X: No of Heads

$R_X = \{0, 1, 2, 3\}$

$X = x_i$	0	1	2	3
$p_X(x_i)$	$1/8$	$3/8$	$3/8$	$1/8$

$$E(X) = \sum_{x \in R_X} x \cdot p_X(x)$$

$$= 0 \cdot p_X(0) + 1 \cdot p_X(1) + 2 \cdot p_X(2) + 3 \cdot p_X(3)$$

$$= 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = \frac{12}{8} = \frac{3}{2} = 1.5$$

On average we will get 1.5 times of Heads when a coin is tossed thrice.

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Properties of Expectation :-

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(1) $E(a) = a$

<u>Proof:</u> $E(a) = \sum_{x \in R_X} a \cdot p_X(x)$

$= a \cdot \sum_{x \in R_X} p_X(x)$

$= a \cdot 1$ $\quad \left(\because \sum_{x \in R_X} p_X(x) = 1 \text{ as it is pmf} \right)$

$= a$ #

(2) $E(aX+b) = aE(X) + b$

Proof) $E(aX+b) = \sum_{x \in R_X} (ax+b)\, p_X(x)$

$= \sum_{x \in R_X} [a.x\, p_X(x) + b\, p_X(x)]$

$= a \sum_{x \in R_X} x \cdot p_X(x) + b \sum_{x \in R_X} p_X(x)$

$= a \cdot E(X) + b \cdot 1$ $\quad \left(\sum_{x \in R_X} p_X(x) = 1 \text{ as it is pmf} \right)$

$= aE(X) + b$ #

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Variance :- Let X be a descrete r.v then variance of X is defined as

$$V(X) = E(X - E(X))^2$$

$$= E(X^2 + (E(X))^2 - 2X\,E(X))$$

constant

$$= E(X^2) + (E(X))^2 - 2\,E(X).\,E(X)$$

$$= E(X^2) + (E(X))^2 - 2(E(X))^2$$

$$\boxed{V(X) = E(X^2) - (E(X))^2}$$

Note:
Expected value of r.v is always Constant
$\because E(X) = \sum_{x \in R_X} x.\,p_X(x)$
= constant value

Q: Find the variance of X for previous example?

Soln R.E : Tossing a coin thrice

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

X: No of Heads

$R_X = \{0, 1, 2, 3\}$

$$V(X) = E(X^2) - (E(X))^2$$

$$E(x^2) = \sum_{x=0}^{3} x^2.\,p_X(x)$$

$$= 0^2.\,p_X(0) + 1^2.\,p_X(1) + 2^2.\,p_X(2) + 3^2.\,p_X(3)$$

$$= 0 + 1.\,\frac{3}{8} + 4.\,\frac{3}{8} + 9.\,\frac{1}{8}$$

$$= \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$$

$$= \frac{24}{8} = 3$$

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Now; $E(X) = \sum_{x=0}^{3} x \cdot p_X(x)$

$= 0 \cdot p_X(0) + 1 \cdot p_X(1) + 2 \cdot p_X(2) + 3 \cdot p_X(3)$

$= 0 + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8}$

$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$

$V(X) = 3 - (1.5)^2$

$= 3 - 2.25$

$= 0.75$