Result: $P(A|B) + P(A^c|B) = 1$

Proof: L.H.S $=$

$$P(A|B) + P(A^c|B)$$

$$= \frac{P(A\cap B)}{P(B)} + \frac{P(A^c \cap B)}{P(B)}$$

$$= \frac{P(A\cap B) + P(A^c\cap B)}{P(B)} = \frac{P(A\cap \cancel{B}) + P(B) - P(\cancel{A}\cap B)}{P(B)}$$

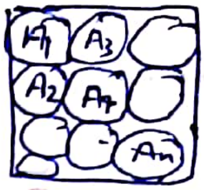
$$= 1 \quad \text{R.H.S}$$

No.- 087

Partition of ~~Sample~~ Sample Space :

Let $A_1, A_2 \ldots A_n$ are events in S then $A_1, A_2 \ldots A_n$ are said to be partition of set S if

S



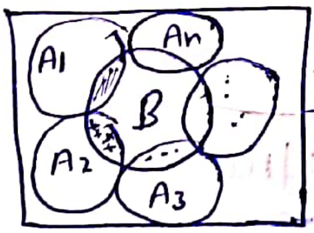
$$\bigcup_{i=1}^{n} A_i = S \text{ and}$$

$$(A_i \cap A_j) = \phi \quad \forall\, i,j \ (i \neq j)$$

Total probability : Let $A_1, A_2, \ldots A_n$ be the n events such that they form partition for set S and B be any event s.t $B \subseteq S$ then the total probability of B is given as

$$P(B) = \sum_{i=1}^{n} P(A_i) \cdot P(B \mid A_i)$$

Proof :



S

$\because \bigcup_{i=1}^{n} A_i = S$ ($\because A_i$ are partition of S) — (1)

We can write ; $B = S \cap B = B \cap S$

$\Rightarrow B = B \cap \left(\bigcup_{i=1}^{n} A_i \right)$ from (1)

$\Rightarrow B = (B \cap A_1) \cup (B \cap A_2) \cup \ldots (B \cap A_n)$

$$\Rightarrow P(B) = P[(B\cap A_1) \cup (B\cap A_2) \cdots \cup (B\cap A_n)]$$

$\because$ $B\cap A_i$; $i=1,2\cdots n$ are pairwise mutually exclusive

$$\Rightarrow P(B) = P(B\cap A_1) + P(B\cap A_2) + \cdots + P(B\cap A_n)$$

$$\Rightarrow P(B) = P(A_1)\cdot P(B|A_1) + P(A_2)\cdot P(B|A_2) + \cdots + P(A_n)\cdot P(B|A_n)$$

$$\Rightarrow \boxed{P(B) = \sum_{i=1}^{n} P(A_i)\cdot P(B|A_i)} \quad \#$$

Bayes Theorem ÷ (Thomas Bayes (1702))

Let $A_1, A_2, \cdots A_n$ are n mutually exhaustive events with non-zero probability of a random experiment, form a partition of S. If B is any event from the sample space then

$$P(A_i|B) = \frac{P(A_i)\cdot P(B|A_i)}{\sum_{i=1}^{n} P(A_i)\cdot P(B|A_i)}$$

Proof: $P(A_i|B) = \dfrac{P(A_i\cap B)}{P(B)} = \dfrac{P(A_i)\cdot P(B|A_i)}{P(B)}$ (By multiplication formula)

$$= \frac{P(A_i)\cdot P(B|A_i)}{\sum_{i=1}^{n} P(A_i)\, P(B|A_i)} \longrightarrow \text{By total probability}$$