

CSCI 3070U Assignment 1 Report

Part 1

1) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$$= 2^2 T\left(\frac{n}{4}\right) + \frac{n}{\log \frac{n}{2}} + \frac{n}{\log n} \quad (\text{Sub in } T\left(\frac{n}{2}\right))$$

$$= 2^3 T\left(\frac{n}{8}\right) + \frac{n}{\log \frac{n}{4}} + \frac{n}{\log \frac{n}{2}} + \frac{n}{\log n} \quad (\text{Sub in } T\left(\frac{n}{4}\right))$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{2^k}\right) + \dots + n \sum_{i=0}^{k-1} \frac{1}{\log \frac{n}{2^i}}$$

Sub in $T(1) = 1$
Base: $n = 2^k$
 $k = \log n$
 $\log k = \log(\log n)$

$$= 2^{\log n} T(1) + n \sum_{i=0}^{\log n - 1} \frac{1}{k - i}$$

$$= n + n \log k \quad (\text{by harmonic summation})$$

$\therefore T(n) = O(n \log \log n)$ for $c > 0$

2) $T(n) = 7T\left(\frac{n}{2}\right) + n^2$

$T(n) \rightarrow$

$7 \times T\left(\frac{n}{2}\right) \quad T\left(\frac{n}{2}\right)$

$7 \times \frac{n^2}{4} \quad \frac{n^2}{4}$

$7^2 \times T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right) \quad T\left(\frac{n}{4}\right)$

\rightarrow

$7 \times \frac{n^2}{4} \quad \frac{n^2}{4}$

$\log_2 n$

$7^2 \times \frac{n^2}{16} \quad \frac{n^2}{16} \quad \frac{n^2}{16} \quad \frac{n^2}{16}$

$7^k \times T\left(\frac{n}{2^k}\right)$

$7^k \left(\frac{n}{2^k}\right)^2$

Base:
 $T(1) = 1$
 $\frac{n}{2^k} = 1$
 $n = 2^k$
 $\log_2 n = k$

$$\rightarrow T(n) = \sum_{i=0}^{k=\log_2 n} n^2 + 7^k \left(\frac{n}{2^k}\right)^2$$

(sub in k) $= n^2 \log_2 n + 7^{\log_2 n}$

$\therefore T(n) = O(n^2 \log_2 n)$
for some $c > 0$

Part 1 Continued

$$3.) T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$$

Guess: $T(k) = O(ck)$ for some $c > 0$

$$\begin{aligned} T(k) &= T(k/2) + T(k/4) + T(k/8) + k \\ &\leq ck/2 + ck/4 + ck/8 + k \\ &= k(c/2 + c/4 + c/8 + 1) \\ &= \frac{7}{8}ck + k \end{aligned}$$

$$\therefore \underline{T(n) = O(cn)} \quad \text{for } 1 \leq c/8$$

$$c \geq 8$$

$$4.) T(n) = 2T(n/4) + \sqrt{n}$$

$$a=2 \quad b=4 \quad f(n) = \sqrt{n}$$

$$\log_b a = 0.5$$

$$\sqrt{n} = O(n^{0.5-\epsilon}) \quad \text{for } \epsilon > 0$$

$$\sqrt{n} = O(n^{0.5})$$

$$T(n) = O(\sqrt{n} \log_4 n)$$

$$\therefore T(n) = O(c\sqrt{n} \log_4 n)$$

where $c > 1$

Part 2

	Category	Function
log	\log_{10}	$\log_{10} n$
	\log_2	$\log_2 n$
linear	n	$2^{\log_2 n}$, $18n$
	$n^{1.5}$	$\sqrt{n^3}$
poly	n^2	n^2 , $3n^2 + 7n + 15$
	n^3	$n^3 - \log n$, n^3
	n^4	$\frac{3}{4}n^4$
	2^n	2^n
exp	4^n	4^n
	5^n	$n^{11} + 5^n + 17n$
	$n!$	$n!$