# CSCI 3010U Assignment 2 Report

## Question 1

n point masses $m_i$, where $i \in [1, n]$

Uniform force of $F_g = mg$ at these point masses

$$F_{net} = \sum_{i=1}^{n} F_i = g \sum_{i=1}^{n} m_i$$

$$Let\ d_{ik} = \begin{cases} 0, & i = k \\ distance\ from\ m_i \text{ to } m_k, & otherwise \end{cases}$$

where $i, k \in [1, n]$

$$\tau_{net} = \sum_{i=1}^{n} (F_i \sum_{k=1}^{n} d_{ik})$$

## Question 2

We care about random processes because sometimes certain simulations may be too costly or certain approximations would be too hard to describe deterministically. Random processes also help us estimate errors and determine a range of results.

## Question 3

***Generating Random Initial Variables***

The requirements asked for a radius in [0.1, 02]m, velocity magnitude in (0, 10]m/s, positions constrained by the box dimensions of 5x5m, and mass in [1, 5] kg.

Since these dimensions are too small to display in python directly I multiplied all (except mass) by **100** to display it and lowered the time step to **0.01** since the velocity would be high. More details on how I generated the random numbers are shown below.

***Box Width = 5***

***Box Height = 5***

***Radius in [0.1, 0.2]***

***Position in [0, 5]:***

1. Position X-Component: Random uniform in [0 + ***Radius***, ***Box Width*** – ***Radius***]
2. Position Y-Component: Random uniform in [0 + ***Radius***, ***Box Height*** – ***Radius***]

***Velocity in (0, 10]:***

1. Velocity Magnitude:

$$\text{Random uniform in } (0, 10]$$

2. Velocity Component Sign:

$$xComponentSign = \ (\text{Random choice between} -1 \text{ and } 1)$$
$$yComponentSign = \ (\text{Random choice between} -1 \text{ and } 1)$$

3. Velocity X-Component:

$$xComponentSign \times (\text{Random uniform value in } [0, 1])$$

4. Velocity Y-Component:

$$yComponentSign \times \sqrt{(Velocity\ Magnitude)^2 - \ (Velocity\ xComponent)^2}$$

***Disk-Wall Collision:***

1. For each disk, a collision is checked between all 4 walls at each time step. The wall positions to check for collision were set as follows:
   - Left Wall = (0, y-position of disk)
   - Right Wall = (***Box Width,*** y-position of disk)
   - Top Wall = (x-position of disk, ***Box Height***)
   - Bottom Wall = (x-position of disk, 0)
2. Get the distance between the disk and the wall and check if its less than or equal to the radius of the disk. If so we get the direction vector of the displacement. If the distance between the wall is greater than the radius of the disk, then no collision.
3. Get the relative velocity which would just be the velocity of the disk since the walls are not moving.
4. Get the dot product of the relative velocity and the displacement direction vector. If this value is greater than or equal to 0 we know the disk is moving parallel to the wall or moving away from it.
5. Calculate impulse using the coefficient of restitution 1 making it fully elastic collisions and we calculate the velocity after collision using impulse.
6. The above steps are repeated for each wall.

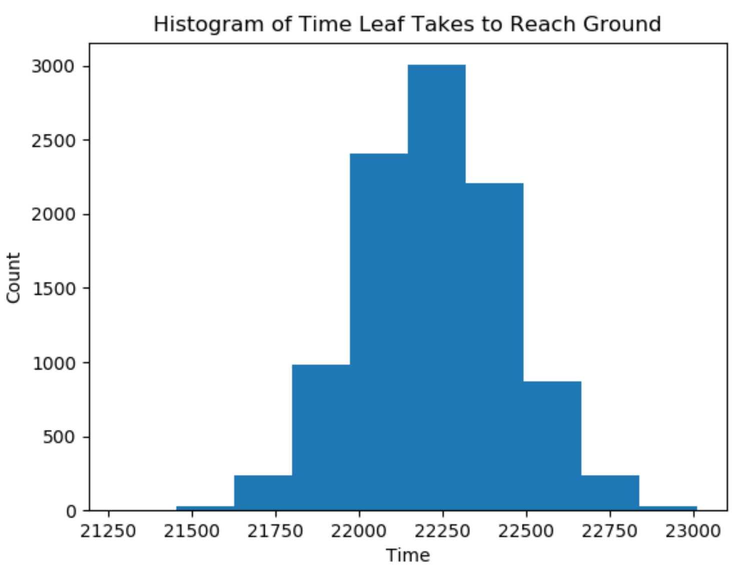
***Disk-Disk Collision:***

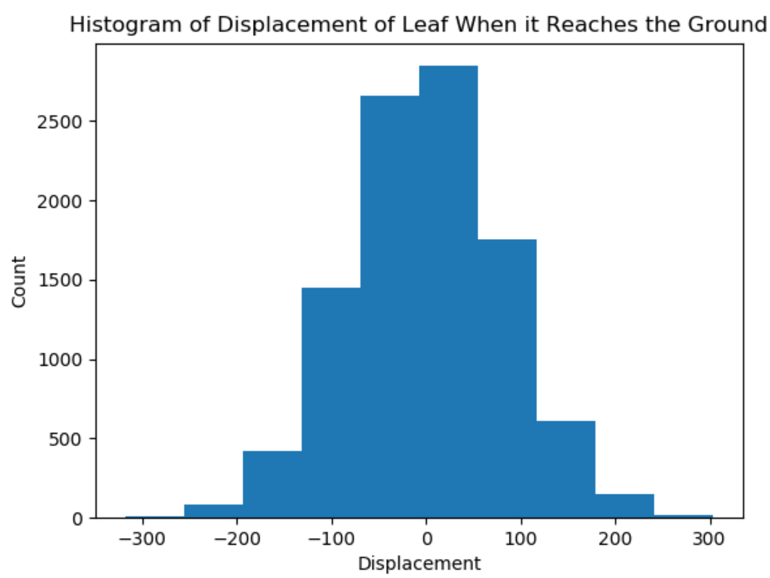
1. For each disk, a collision is checked between it and every other disk. Only two disks are checked at once. This is done after checking wall collisions for the first disk.
2. The steps are the same as the Disk-Wall Collision except the relative velocity will be the difference between the two disks velocity vectors and the resulting velocity for the second disk is also calculated along with the first disk's.
3. The above steps are repeated for every disk.

**Question 4**

| Simulation Ran 10000 times | | | | |
|---|---|---|---|---|
| **Height** | **Time** | | **Displacement** | |
| | **μ** | **σ** | **μ** | **σ** |
| **10** | 22.1416 | 49.60574944 | 0.0365 | 6.45316775 |
| **100** | 222.1463 | 493.89029631 | -0.0444 | 66.74362864 |
| **1000** | 2223.5223 | 4962.85130271 | 0.2082 | 669.80705276 |
| **10000** | 22223.6677 | 49291.3878767 | 0.666 | 6745.305044 |

*The below histograms are for height 10000 where the simulation was run 10000 times.*



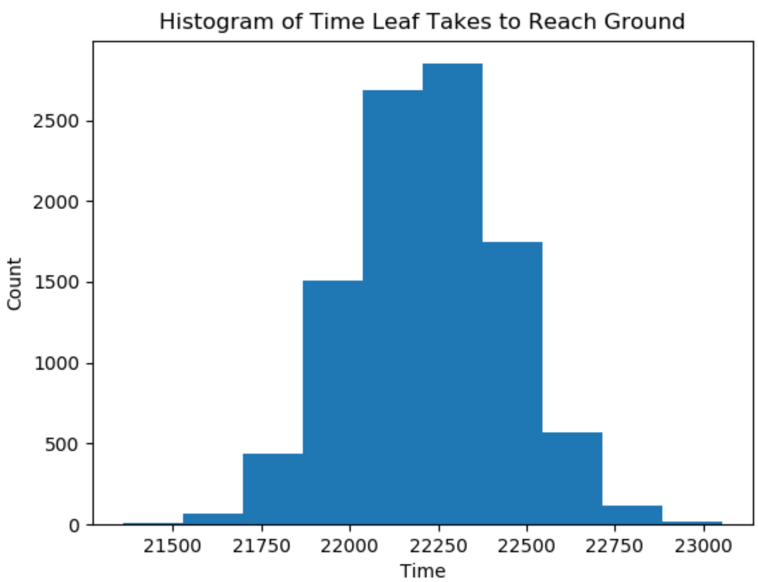

From my observations we get the following correlations:

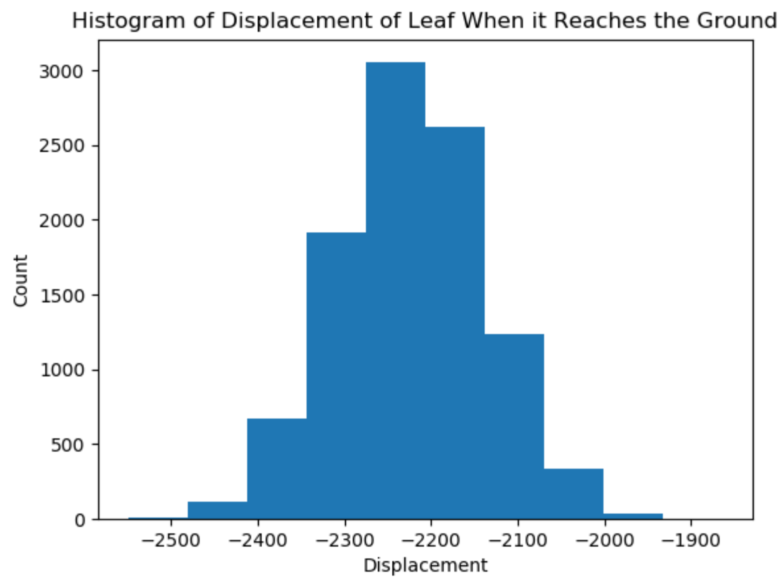
- $time(\mu) \approx 2.22 \times \text{height}$
- $time(\sigma) \approx 4.95 \times \text{height}$
- $displacement(\mu)$ has no correlation to height, but from the histogram and mean value, we see the displacement is mostly near 0. This makes sense because the left and right probabilities are equal.
- $displacement(\sigma) \approx 0.67 \times \text{height}$

We also observe from running the simulation a greater number of times, the correlation of height with time and displacement becomes more accurate.

| Simulation Ran 10000 times with left P(0.2) and right P(0.1) | | | | |
|---|---|---|---|---|
| **Height** | **Time** | | **Displacement** | |
| | **μ** | **σ** | **μ** | **σ** |
| **10** | 22.2103 | 49.23207391 | -2.2645 | 7.50173975 |
| **100** | 222.4407 | 490.17748351 | -22.1786 | 76.06370204 |
| **1000** | 2222.5526 | 4915.37323324 | -222.3862 | 741.77544956 |
| **10000** | 22223.7591 | 48349.2538672 | -2222.6953 | 7413.05105791 |

*The below histograms are for height 10000 where the simulation was run 10000 times with left and right probabilities of 0.2 and 0.1, respectively.*




From my observations we get the following correlations which appear to be the similar as left and right probabilities being equal, except for the displacement:

- $time(\mu) \approx 2.22 \times \text{height}$
- $time(\sigma) \approx 4.9 \times \text{height}$
- $displacement(\mu) \approx -0.22 \times \text{height}$
- $displacement(\sigma) \approx 0.75 \times \text{height}$

Increasing left probability to 0.20 and lowering the right one to 0.1 has increased the displacement variance factor by 0.1 and gives a correlation to the mean displacement.