AER 1216 – Fundamentals of UAVs

Lecture 13 - Quadrotor Dynamics and Control III

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Quadrotor Dynamics and Control II

1. How to linearly control a planar (2D) quadrotor?
    a. Linearize dynamics and apply PD controller
2. What other linear controllers can we use?
    a. Pole placement and LQR
3. When should we use nonlinear controllers?
    a. When system behavior deviates from linearization
4. How to nonlinearly control a planar (2D) quadrotor?
    a. Nested architecture, nonlinear eq's and measurements
5. How to control the planar quadrotor in simulation?

Quadrotor Dynamics and Control Lectures

Lecture 1	Lecture 2	Lecture 3
Introduction to quadrotor control Controlling the up-down motion of the quadrotor	Controlling the quadrotor in 2D (y-z plane) Introduction to nonlinear controllers	Controlling the quadrotor in 3D Setting up a commercially available quadrotor
Installing simulator and initial controller exploration	Applying a 2D controller to the simulated quadrotor	How to set up a commercially available quadrotor

Inspired by Vijay Kumar’s Aerial Robotics Coursera course
https://www.coursera.org/learn/robotics-flight

1. How to linearly control a quadrotor?
    a. Quadrotor model
    b. Linearization
    c. Nested control loop
2. How to nonlinearly control a quadrotor?
3. How to set up a commercially available drone?
4. What steps to fly a commercially available drone?

Quadrotor Model (3D)

$$q = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$: $\boldsymbol{r}$

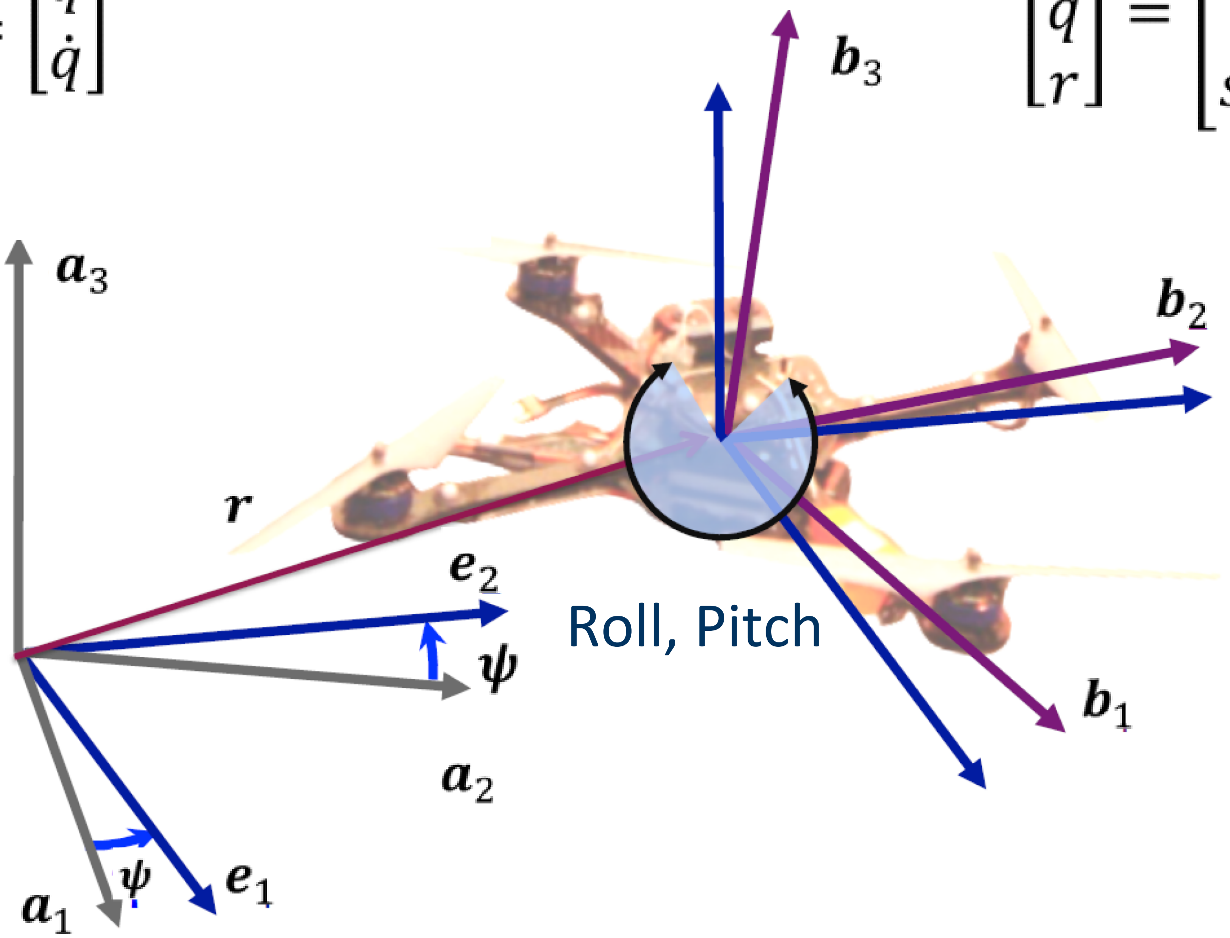
ϕ Roll

θ Pitch

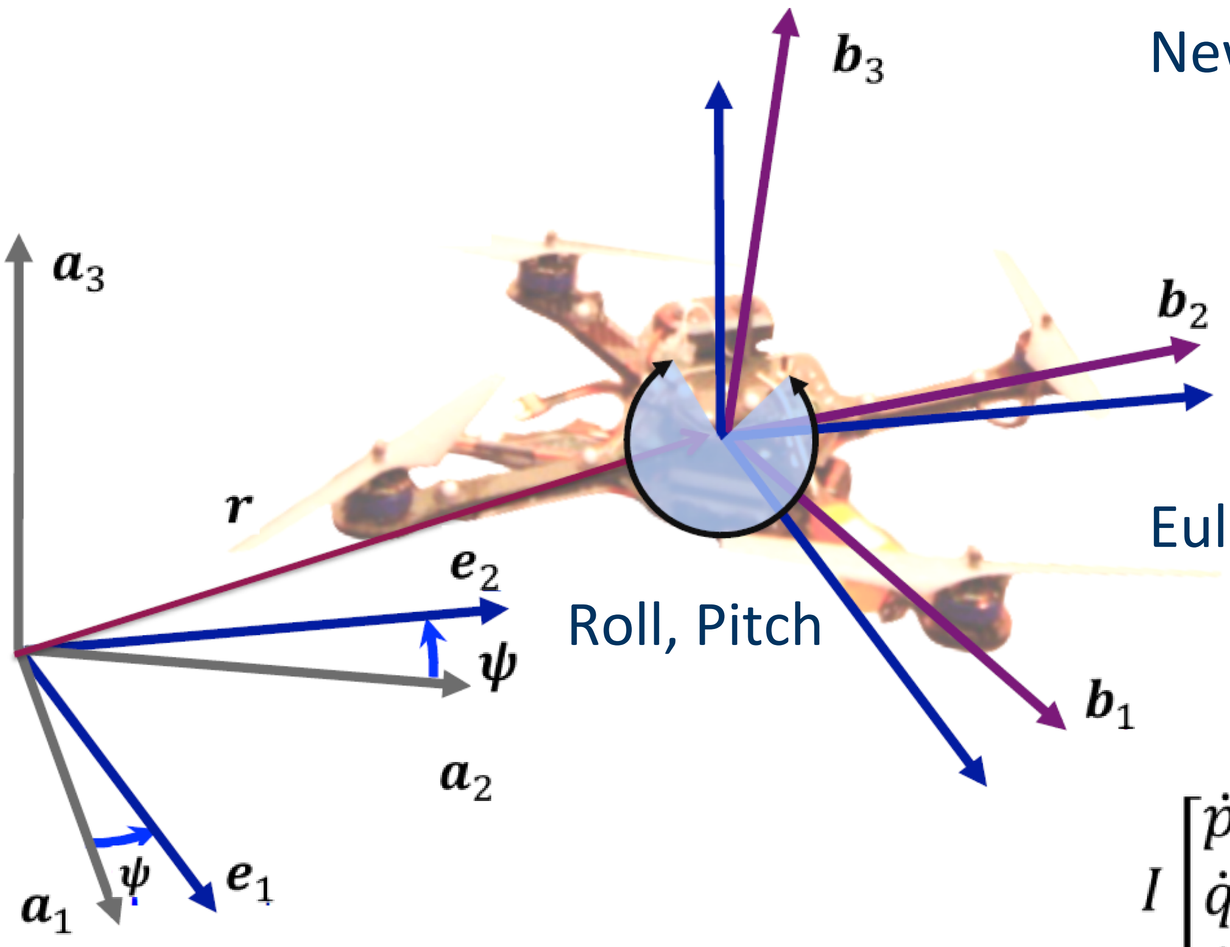
ψ Yaw

Angular velocities components in b

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$



Quadrotor Model (3D)



Newton's equations of motion: $\boldsymbol{F} = m\boldsymbol{a}$

$$m\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

Euler's equations of motion:

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) = \boldsymbol{M}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

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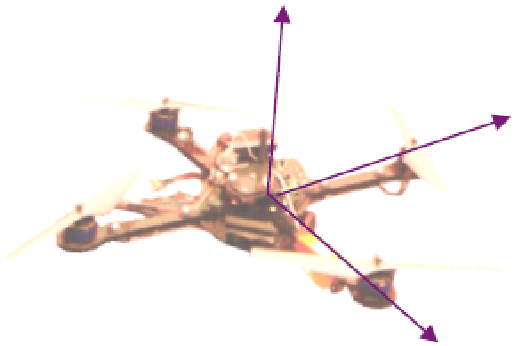
Linearized Dynamics Model



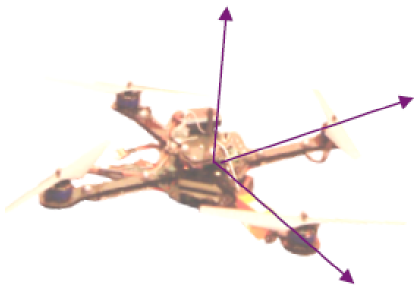
Linearize the dynamics
at the hover configuration

$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$
$$(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0)$$

$$m\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Linearized Dynamics Model



$$R = \begin{bmatrix} . & . & \cos\psi\sin\theta + \cos\theta\sin\phi\sin\psi \\ . & . & \sin\psi\sin\phi - \cos\psi\cos\theta\sin\phi \\ . & . & \cos\theta\cos\phi \end{bmatrix}$$

$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \boxed{F_1 + F_2 + F_3 + F_4} \end{bmatrix}$$

$$u_1$$

Linearization

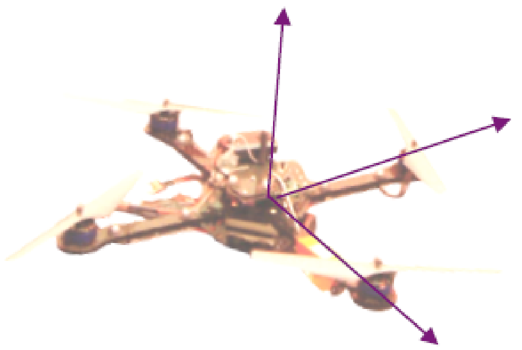
$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$\cos\phi \sim 1$$
$$\sin\phi \sim \phi$$

$$\ddot{r}_1 = \ddot{x} = g(\theta\cos\psi + \phi\sin\psi)$$
$$\ddot{r}_2 = \ddot{y} = g(\theta\sin\psi - \phi\cos\psi)$$

$$\cos\theta \sim 1$$
$$\sin\theta \sim \theta$$

Linearized Dynamics Model



Assuming that the quadrotor is symmetric,
the linearized equations of
the angular accelerations are decoupled

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

$$(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0)$$

$$I\underbrace{\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{\boldsymbol{\omega}}} = \underbrace{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}_{\boldsymbol{u}_2} - \cancelto{0}{\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}}$$

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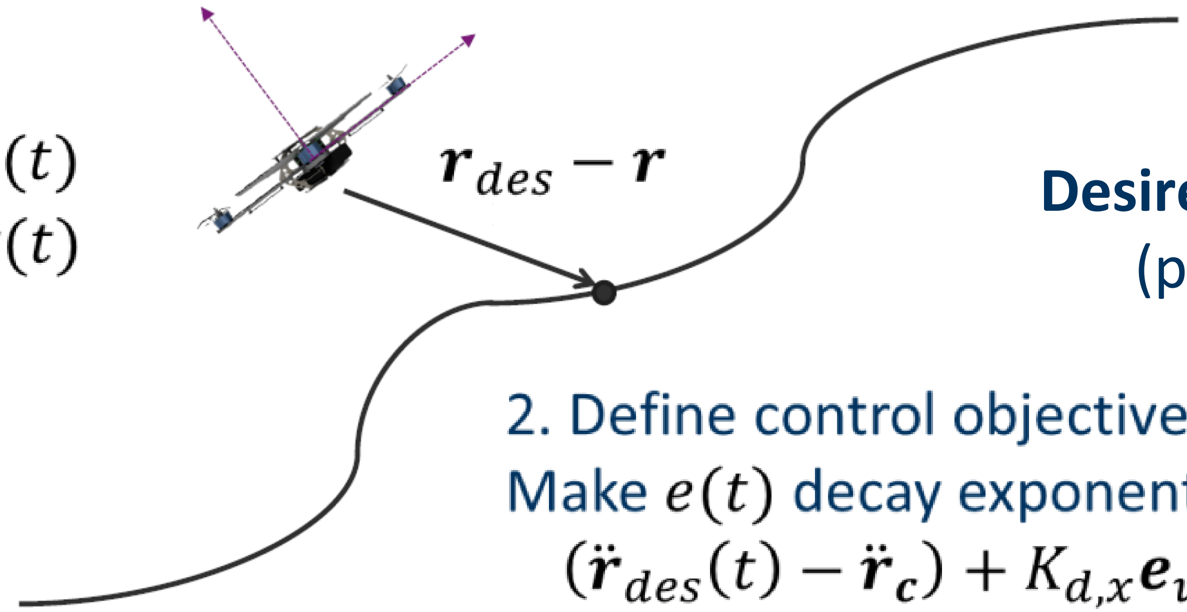
Trajectory Tracking Problem Statement

- Given the quadrotor, design a controller to follow the desired differentiable trajectory $\boldsymbol{r}_{des}(t), \dot{\boldsymbol{r}}_{des}(t), \ddot{\boldsymbol{r}}_{des}(t)$

1. Define error:

$$\boldsymbol{e}_p = \boldsymbol{r}_{des}(t) - \boldsymbol{r}(t)$$
$$\boldsymbol{e}_v = \dot{\boldsymbol{r}}_{des}(t) - \dot{\boldsymbol{r}}(t)$$



$$\boldsymbol{r}_{des}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \psi(t) \end{bmatrix}$$

Desired trajectory
(pos., yaw)

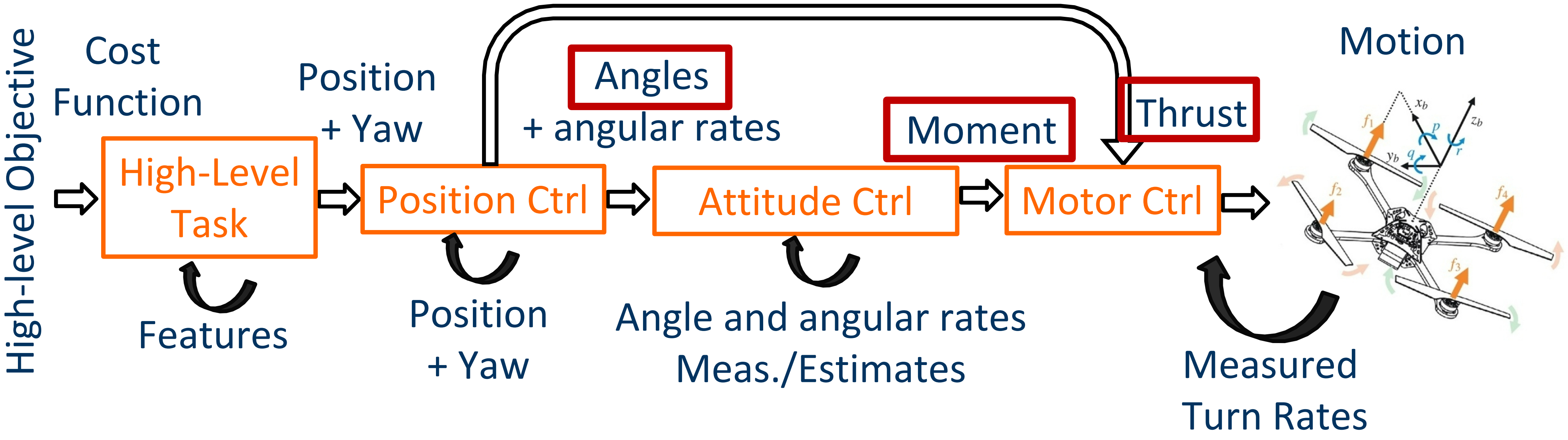
2. Define control objective:

Make $e(t)$ decay exponentially to 0

$$(\ddot{\boldsymbol{r}}_{des}(t) - \ddot{\boldsymbol{r}}_c) + K_{d,x}\boldsymbol{e}_v + K_{p,x}\,\boldsymbol{e}_p = 0$$

3. Choose appropriate controller.

Nested Control Architecture



We focus on calculating thrust and moment (motor control is typically proportional)

Nested Control Architecture

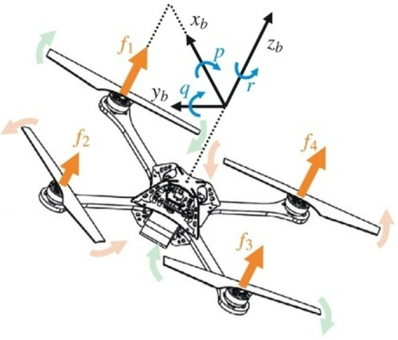
Angles
+ angular rates

Moment

Attitude Ctrl ⇨

Angle and angular rates
Meas./Estimates

Motion

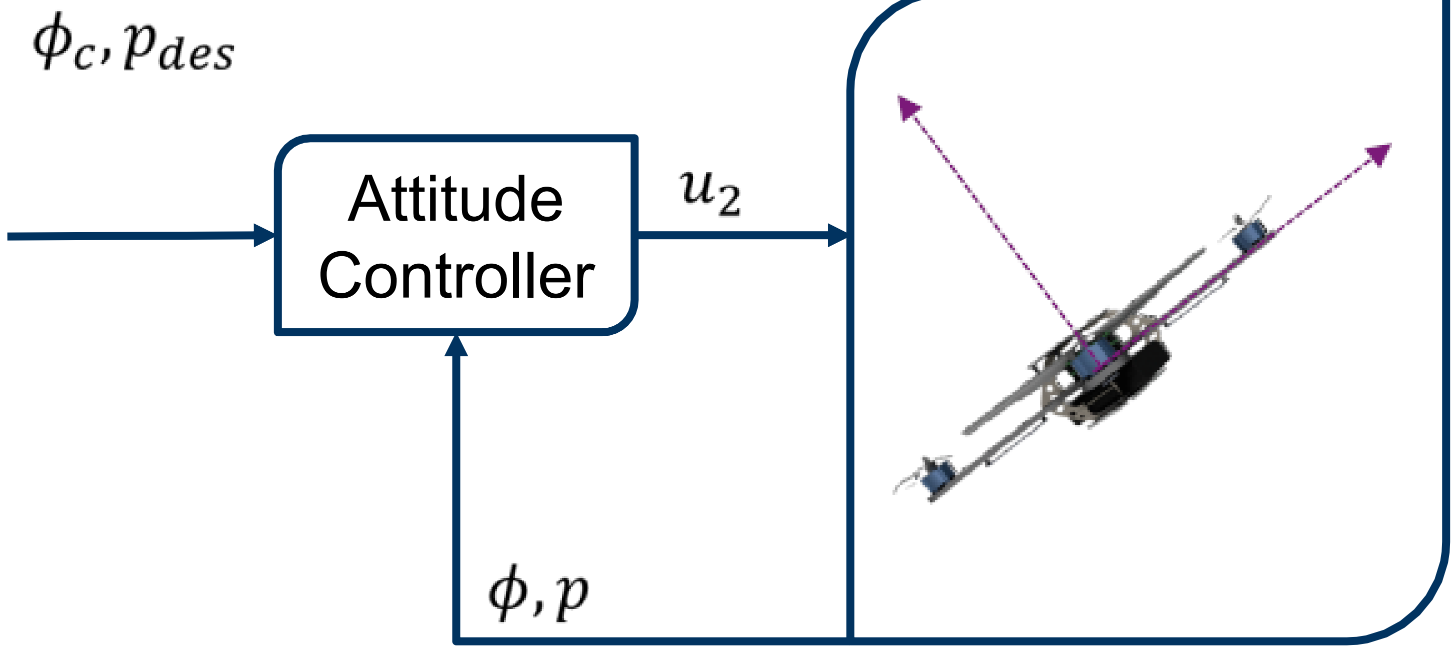


Attitude Controller

Make $e = \phi_c - \phi$ decay exponentially to 0: (Simplify using $\ddot{\phi}_c$=0)

$$(\ddot{\phi}_c - \ddot{\phi}) + k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_{des} - p) = 0$$

$$\ddot{\phi} = k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_{des} - p)$$



Linearized dynamics:

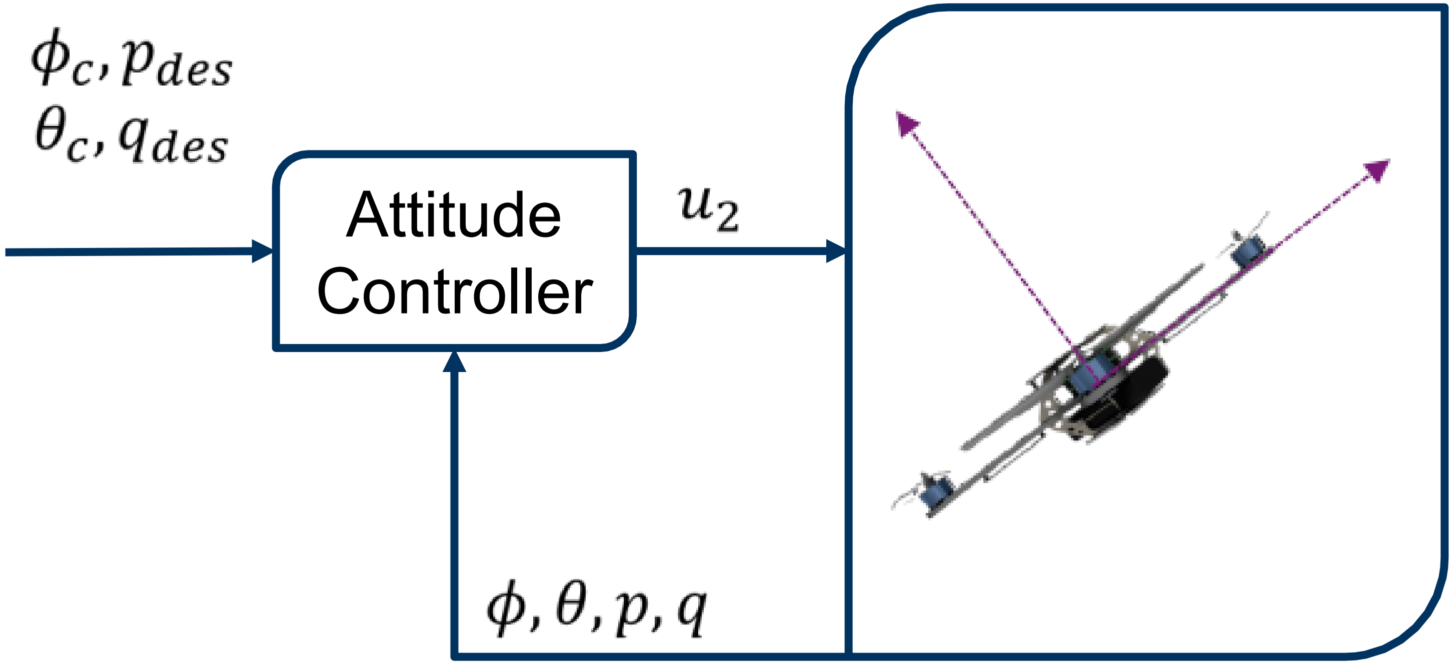
$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Attitude Controller

Make $e = \theta_c - \theta$ decay exponentially to 0: (Simplify using $\ddot{\theta}_c$=0)

$$(\ddot{\theta}_c - \ddot{\theta}) + k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_{des} - q) = 0$$

$$\ddot{\theta} = k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_{des} - q)$$



Linearized dynamics:

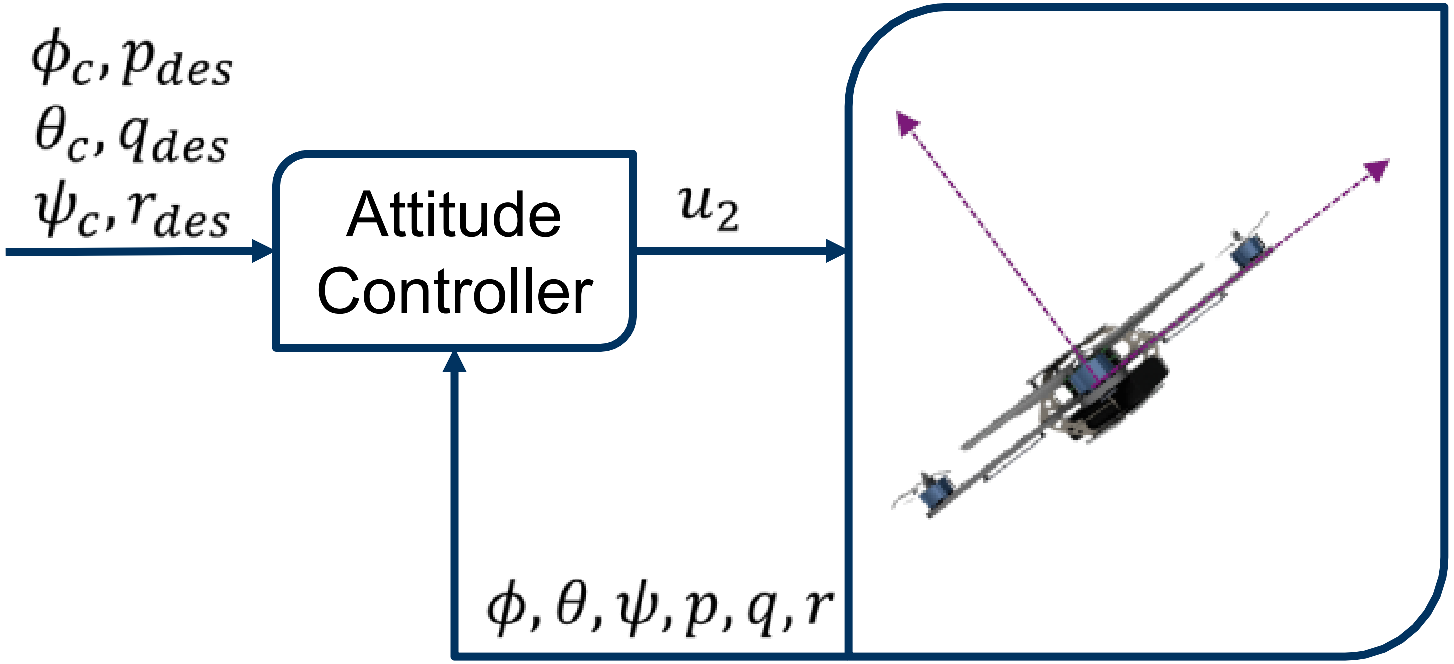
$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Attitude Controller

Make $e = \psi_c - \psi$ decay exponentially to 0: (Simplify using $\ddot{\psi}_c$=0)

$$(\ddot{\psi}_c - \ddot{\psi}) + k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_{des} - r) = 0$$

$$\ddot{\psi} = k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_{des} - r)$$



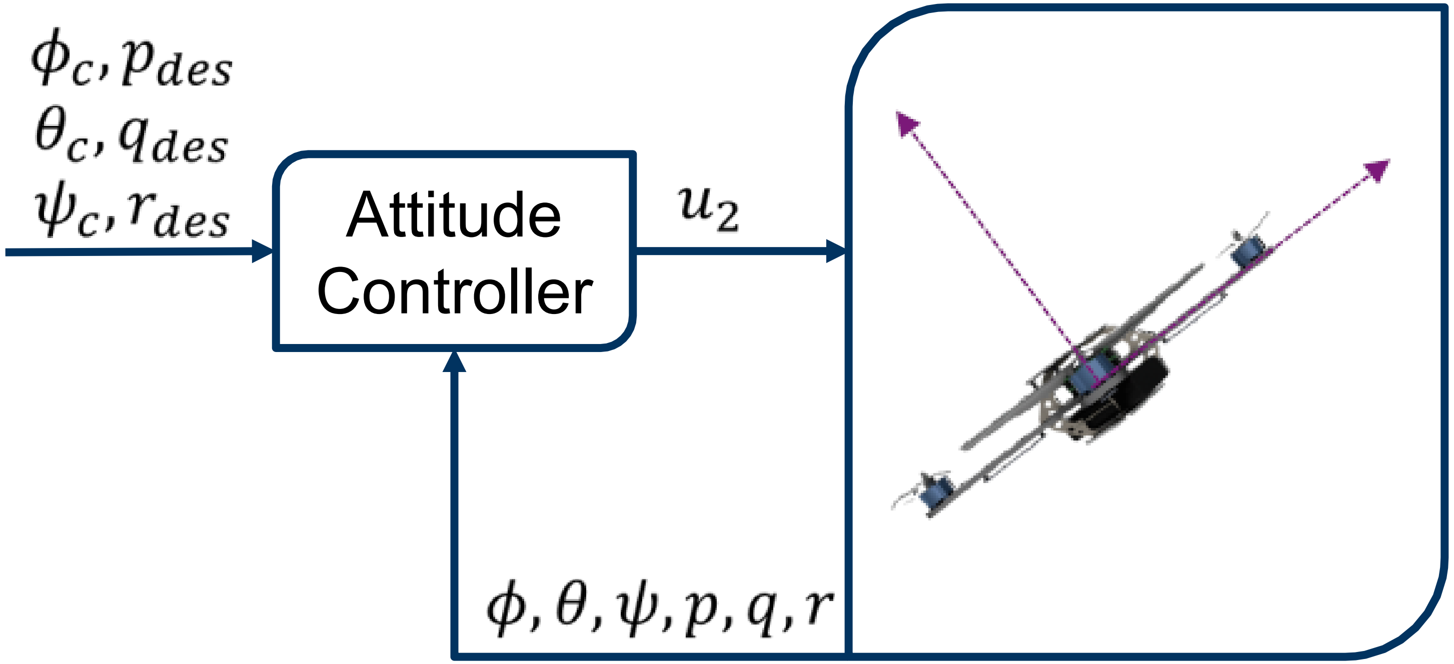
Linearized dynamics:

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Attitude Controller

Summarizing:

$$\boxed{\boldsymbol{u}_2 = \begin{bmatrix} I_{xx}(k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p)) \\ I_{yy}(k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q)) \\ I_{zz}(k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r)) \end{bmatrix}}$$



Linearized dynamics:

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Nested Control Architecture

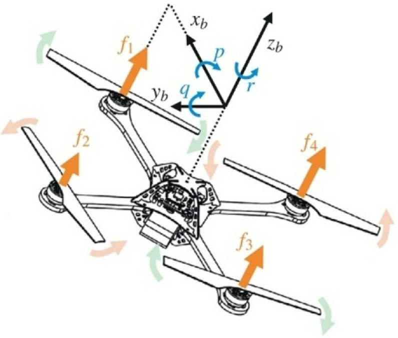
Position
+ Yaw

Angles
+ angular rates

Moment

Motion

Position Ctrl ⇨ Attitude Ctrl ⇨



Position
+ Yaw

Angle and angular rates
Meas./Estimates

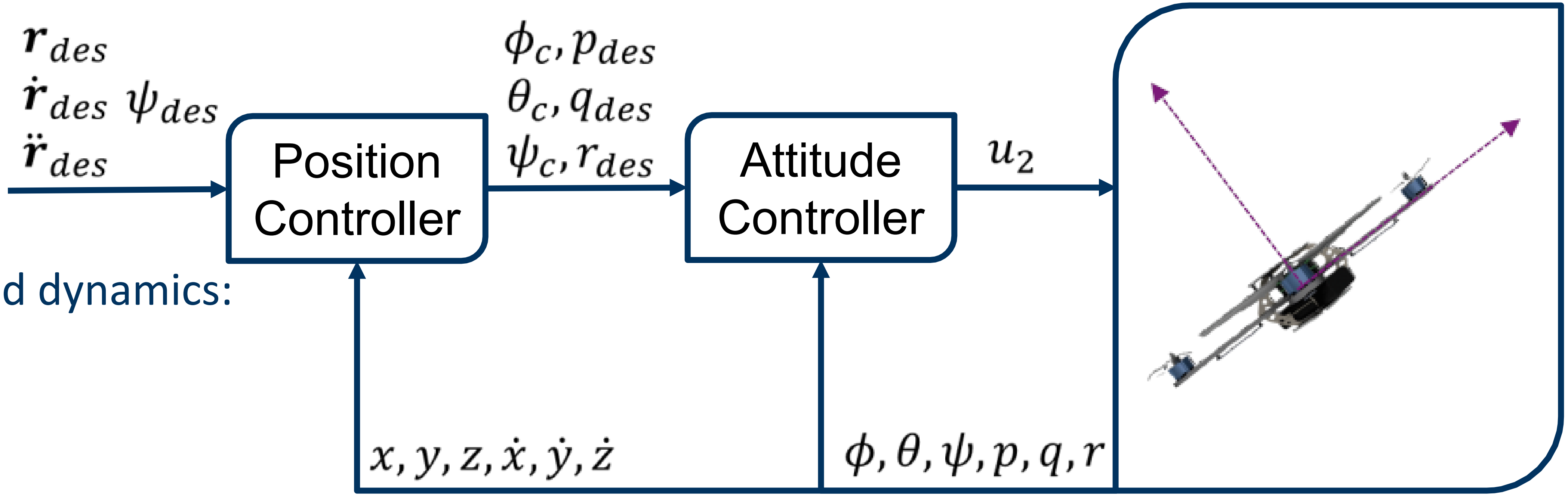
Position Controller

Make $e = x_{des} - x$ decay exponentially to 0:

$$(\ddot{x}_{des} - \ddot{x}) + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x}) = 0$$

For hover $\dot{x}_{des}, \ddot{x}_{des} = 0$

$$\ddot{x}_c = \ddot{x}_{des} + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x})$$



Linearized dynamics:

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Position Controller

Make $e = y_{des} - y$ decay exponentially to 0:

$$(\ddot{y}_{des} - \ddot{y}) + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y}) = 0$$

For hover $\dot{y}_{des}, \ddot{y}_{des} = 0$

$$\ddot{y}_c = \ddot{y}_{des} + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y})$$

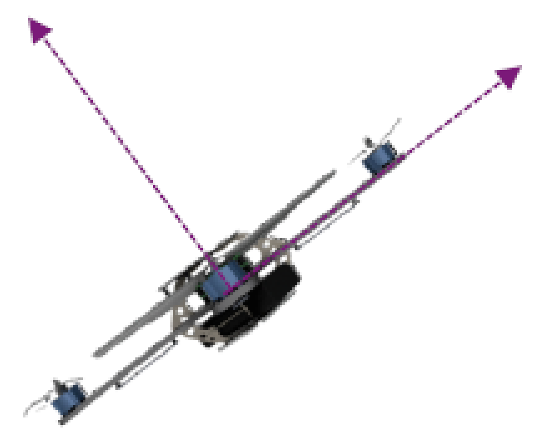
$\boldsymbol{r}_{des}$
$\dot{\boldsymbol{r}}_{des}$ ψ_{des}
$\ddot{\boldsymbol{r}}_{des}$

Position Controller

ϕ_c, p_{des}
θ_c, q_{des}
ψ_c, r_{des}

Attitude Controller

u_2



$x, y, z, \dot{x}, \dot{y}, \dot{z}$

$\phi, \theta, \psi, p, q, r$

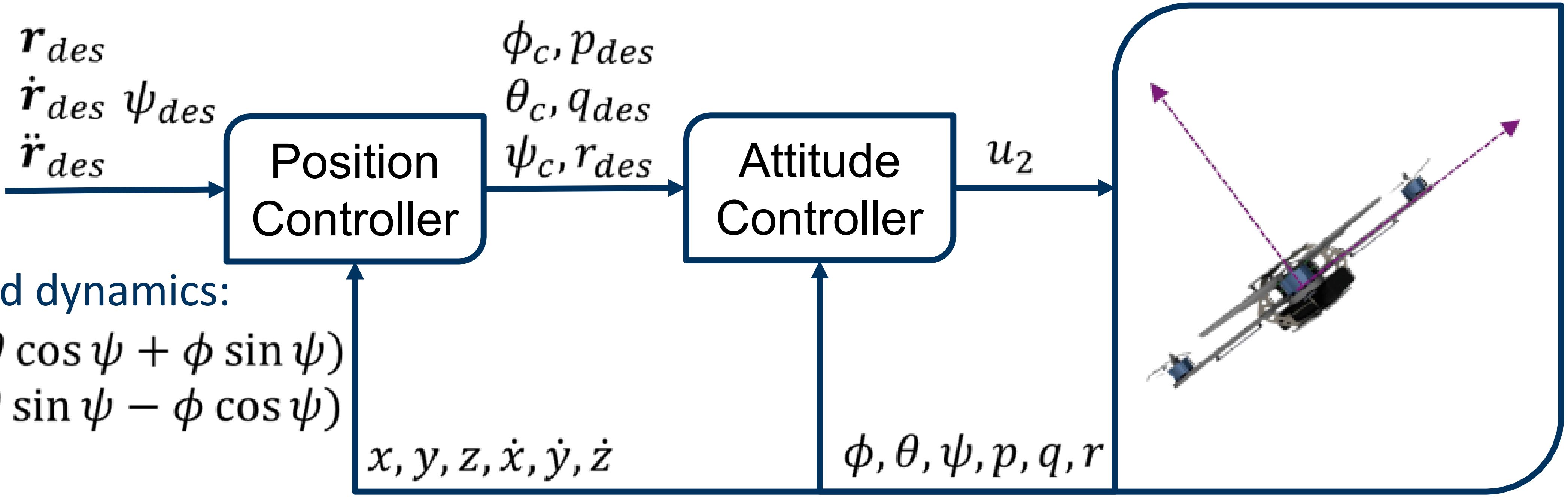
Linearized dynamics:

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Position Controller

Using the linearized dynamics:

$$\phi_c = (\ddot{x}_c \sin\psi_{des} - \ddot{y}_c \cos\psi_{des})/g$$
$$\theta_c = (\ddot{x}_c \cos\psi_{des} - \ddot{y}_c \sin\psi_{des})/g$$
$$\psi_c = \psi^{des}$$



Linearized dynamics:

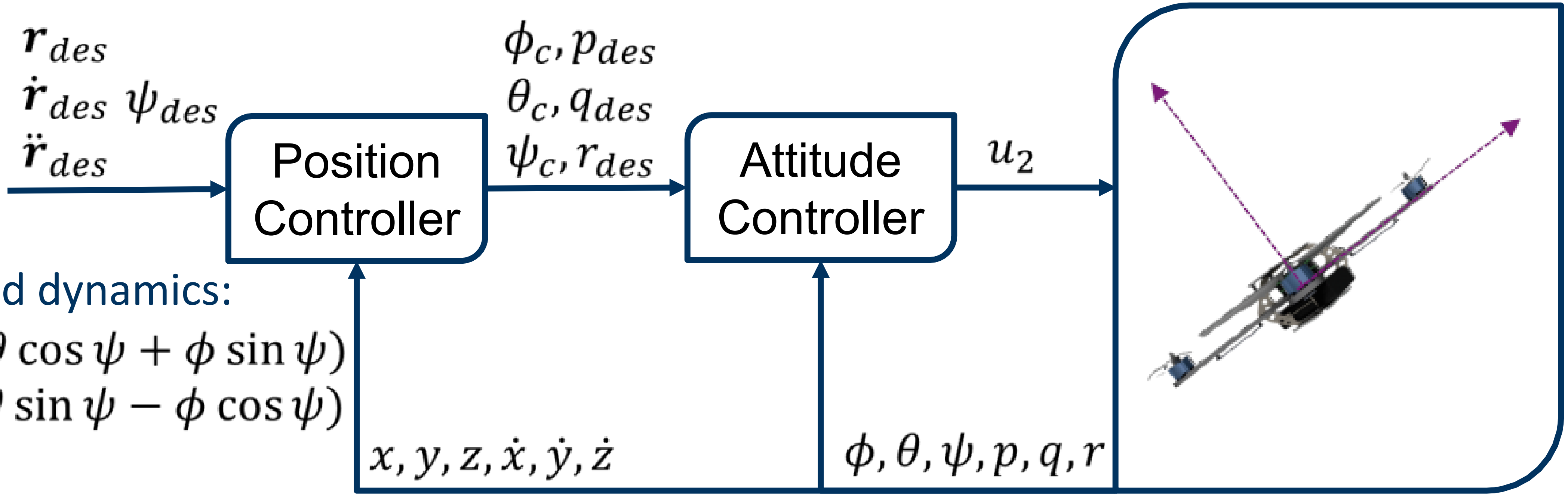
$$\ddot{x} = g(\theta\cos\psi + \phi\sin\psi)$$
$$\ddot{y} = g(\theta\sin\psi - \phi\cos\psi)$$

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Position Controller

Desired pitch and roll velocities are set to 0 and desired yaw velocity is set according to its desired trajectory

$\boxed{p_c = 0}$ $\boxed{q_c = 0}$ $\boxed{r_c = r_{des}}$



Linearized dynamics:

$$\ddot{x} = g(\theta \cos\psi + \phi \sin\psi)$$
$$\ddot{y} = g(\theta \sin\psi - \phi \cos\psi)$$

$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Nested Control Architecture

Thrust

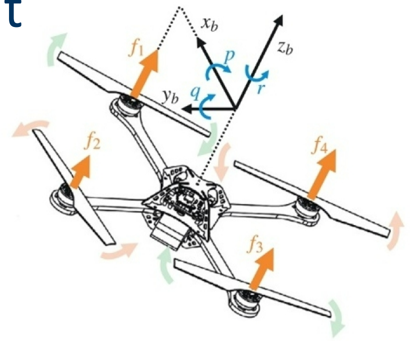
Position
+ Yaw

Angles
+ angular rates

Moment

Motion

Position Ctrl → Attitude Ctrl →



Position
+ Yaw

Angle and angular rates
Meas./Estimates

Position Controller

Make $e = z_{des} - z$ decay exponentially to 0:

$$(\ddot{z}_{des} - \ddot{z}) + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z}) = 0$$

For hover $\dot{z}_{des}, \ddot{z}_{des} = 0$

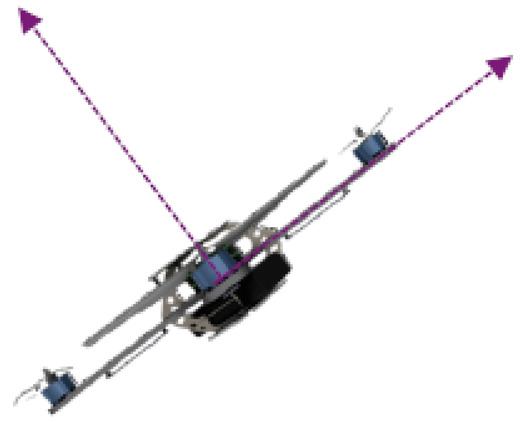
$$\ddot{z}_c = \ddot{z}_{des} + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z})$$

$\boldsymbol{r}_{des}$, $\dot{\boldsymbol{r}}_{des}$, ψ_{des}, $\ddot{\boldsymbol{r}}_{des}$ → Position Controller

Position Controller → ϕ_c, p_{des}; θ_c, q_{des}; ψ_c, r_{des} → Attitude Controller

Position Controller → u_1 →

Attitude Controller → u_2 →



$x, y, z, \dot{x}, \dot{y}, \dot{z}$ → Position Controller

$\phi, \theta, \psi, p, q, r$ → Attitude Controller

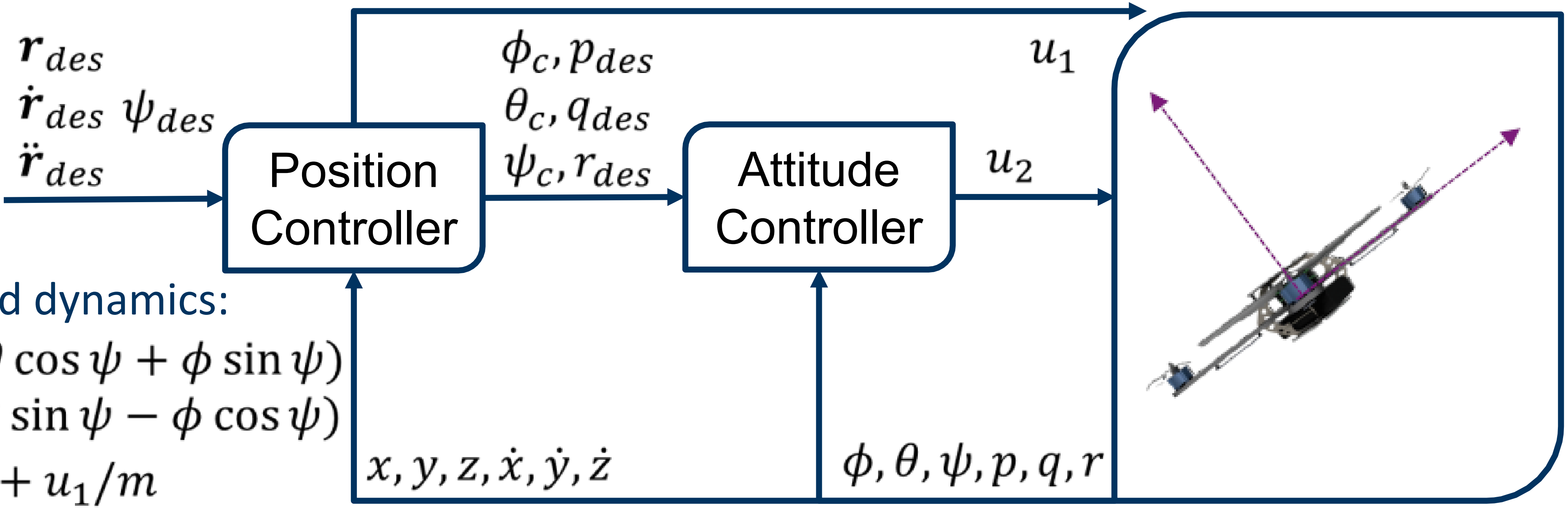
Linearized dynamics:

$$\ddot{x} = g(\theta \cos\psi + \phi \sin\psi)$$
$$\ddot{y} = g(\theta \sin\psi - \phi \cos\psi)$$
$$\ddot{z} = -g + u_1/m$$
$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Position Controller

Using the linearized dynamics:

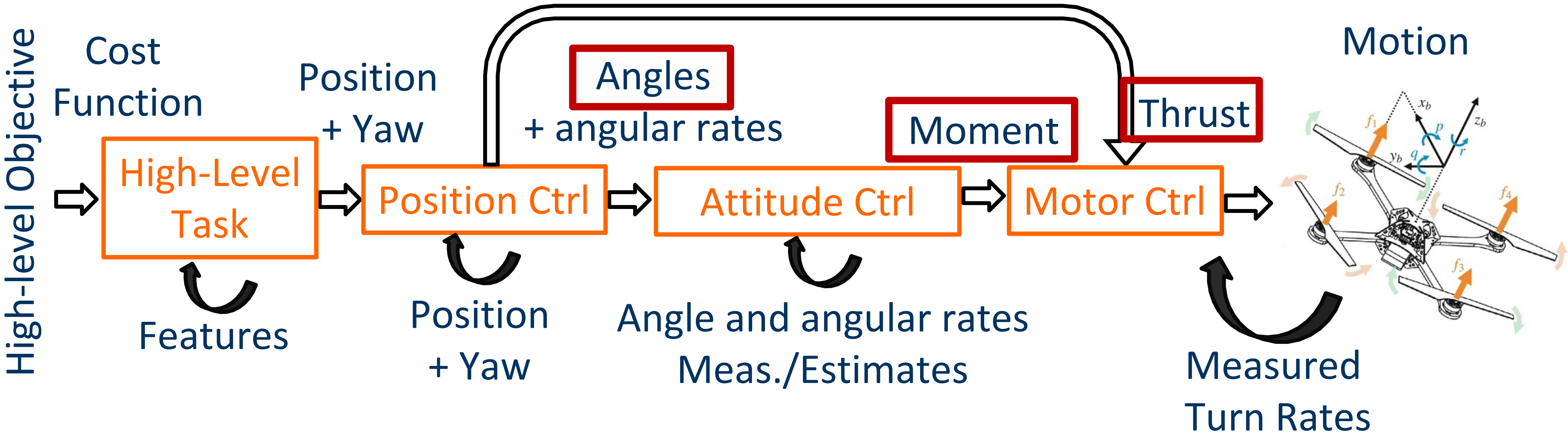
$$\boxed{u_1 = m(\ddot{z}_c + g)}$$



Linearized dynamics:

$$\ddot{x} = g(\theta \cos\psi + \phi \sin\psi)$$
$$\ddot{y} = g(\theta \sin\psi - \phi \cos\psi)$$
$$\ddot{z} = -g + u_1/m$$
$$I\dot{\boldsymbol{\omega}} = \boldsymbol{u}_2$$

Nested Control Architecture



Control Equations

$$\boxed{u_1 = m(\ddot{z}_c + g)}$$

$$\boxed{\boldsymbol{u}_2 = \begin{bmatrix} I_{xx}(k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p)) \\ I_{yy}(k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q)) \\ I_{zz}(k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r)) \end{bmatrix}}$$

$$\boxed{\begin{gathered} \phi_c = (\ddot{x}_c \sin\psi_{des} - \ddot{y}_c \cos\psi_{des})/g \\ \theta_c = (\ddot{x}_c \cos\psi_{des} - \ddot{y}_c \sin\psi_{des})/g \\ \psi_c = \psi^{des} \end{gathered}}$$

$$\boxed{p_c = 0}$$

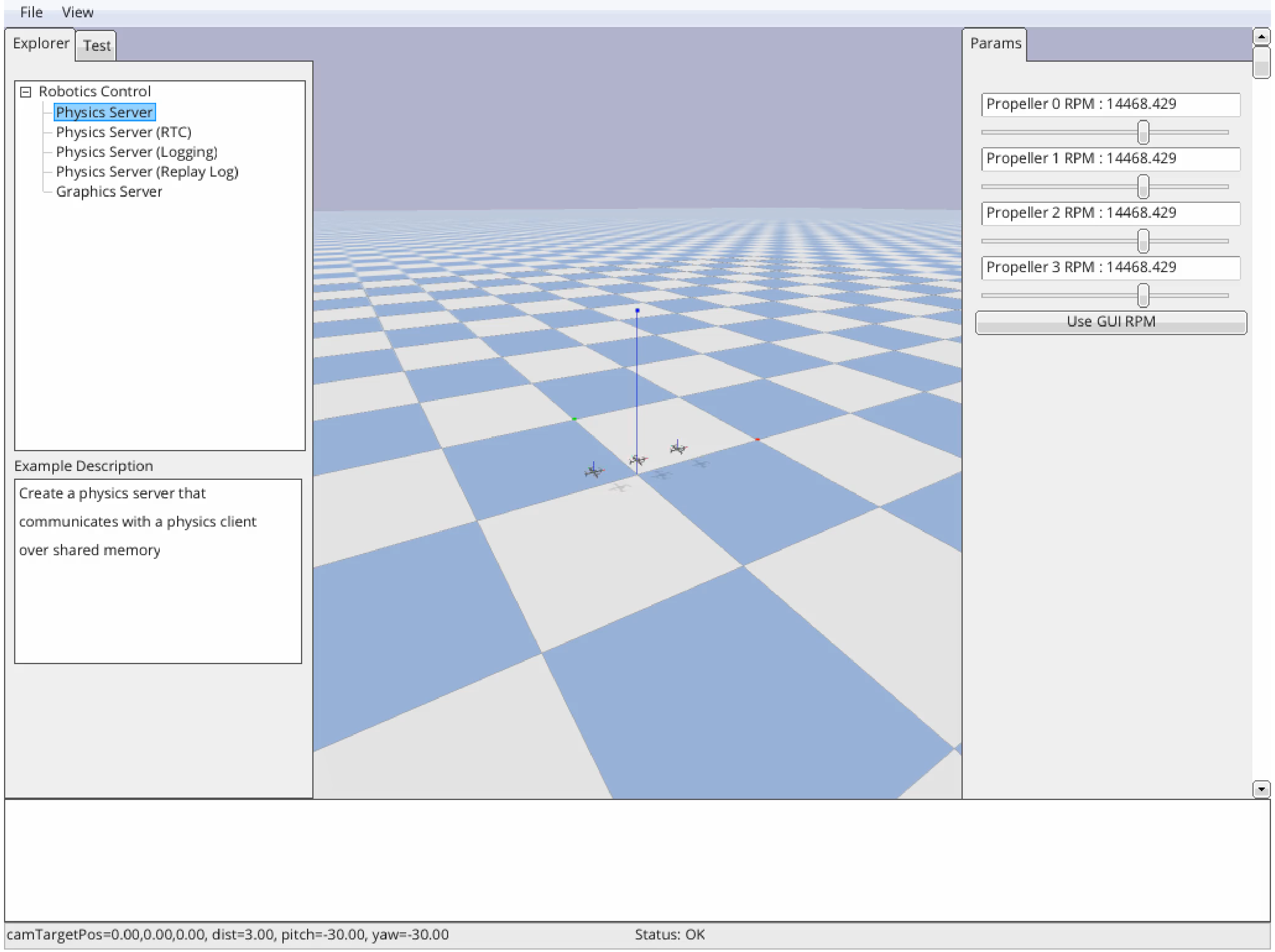
$$\boxed{q_c = 0}$$

$$\boxed{r_c = r_{des}}$$

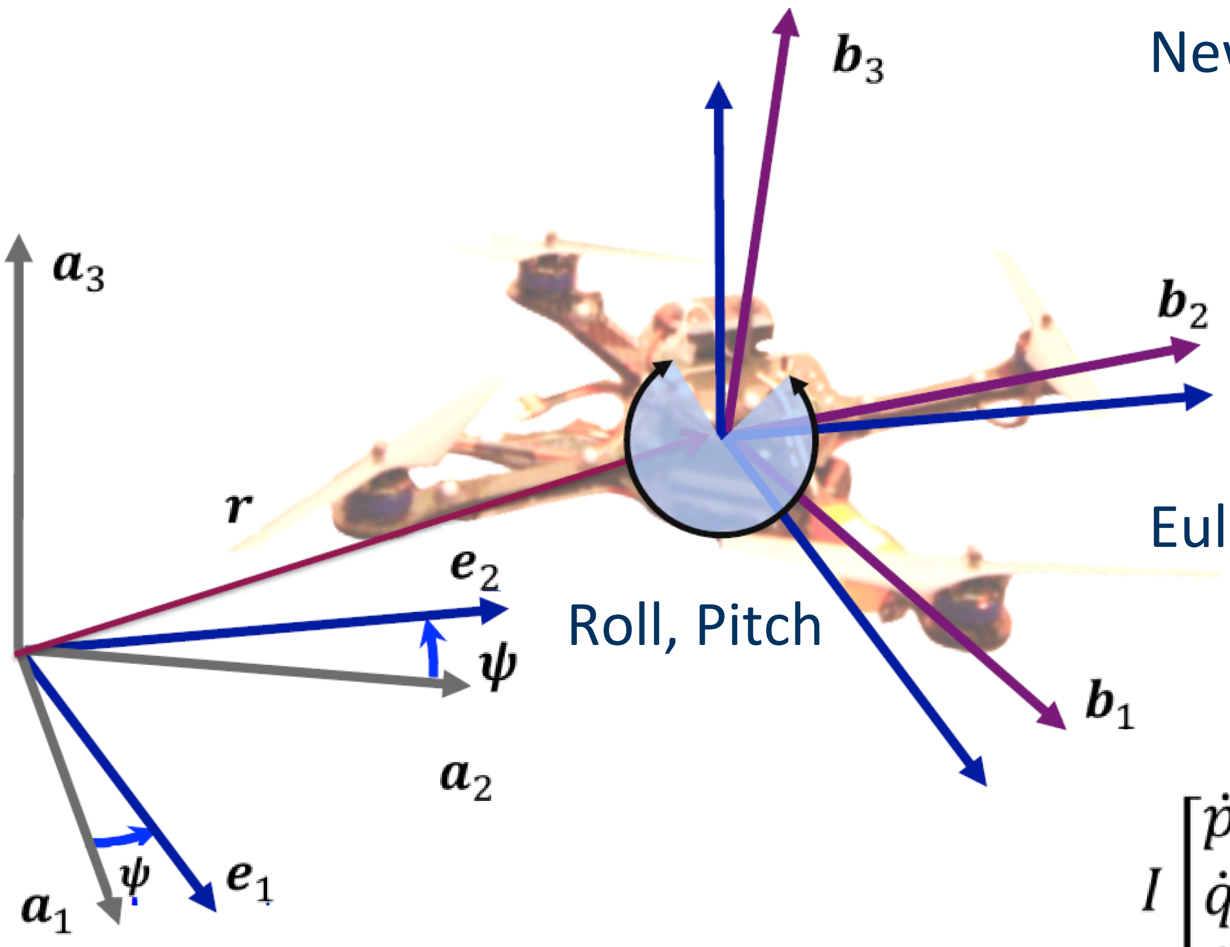
1. How to linearly control a quadrotor?
2. How to nonlinearly control a quadrotor?
3. How to set up a commercially available drone?
4. What steps to fly a commercially available drone?

When should we use nonlinear controllers?

When we have aggressive flight



Quadrotor Model (3D)



Newton's equations of motion: $c = \dfrac{\sum_{i=1}^{4} F_i}{m}$

$$\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

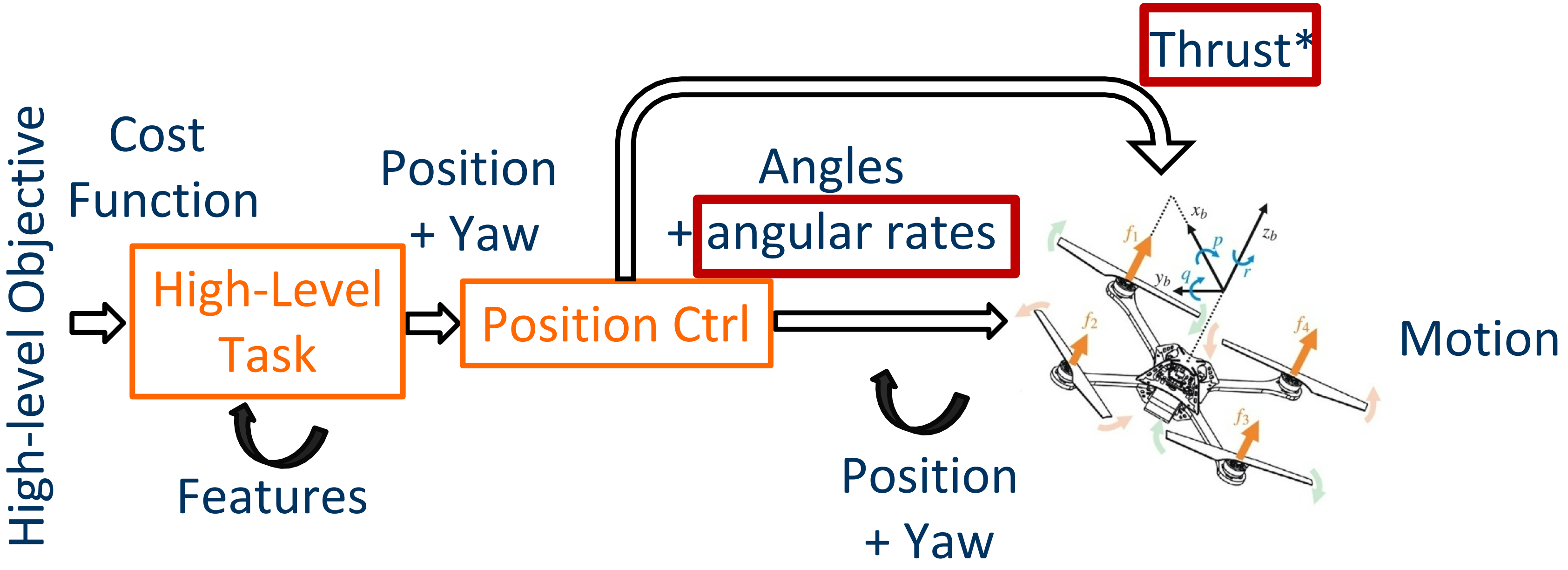
is mass-normalized.

Euler's equations of motion:

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) = \boldsymbol{M}$$

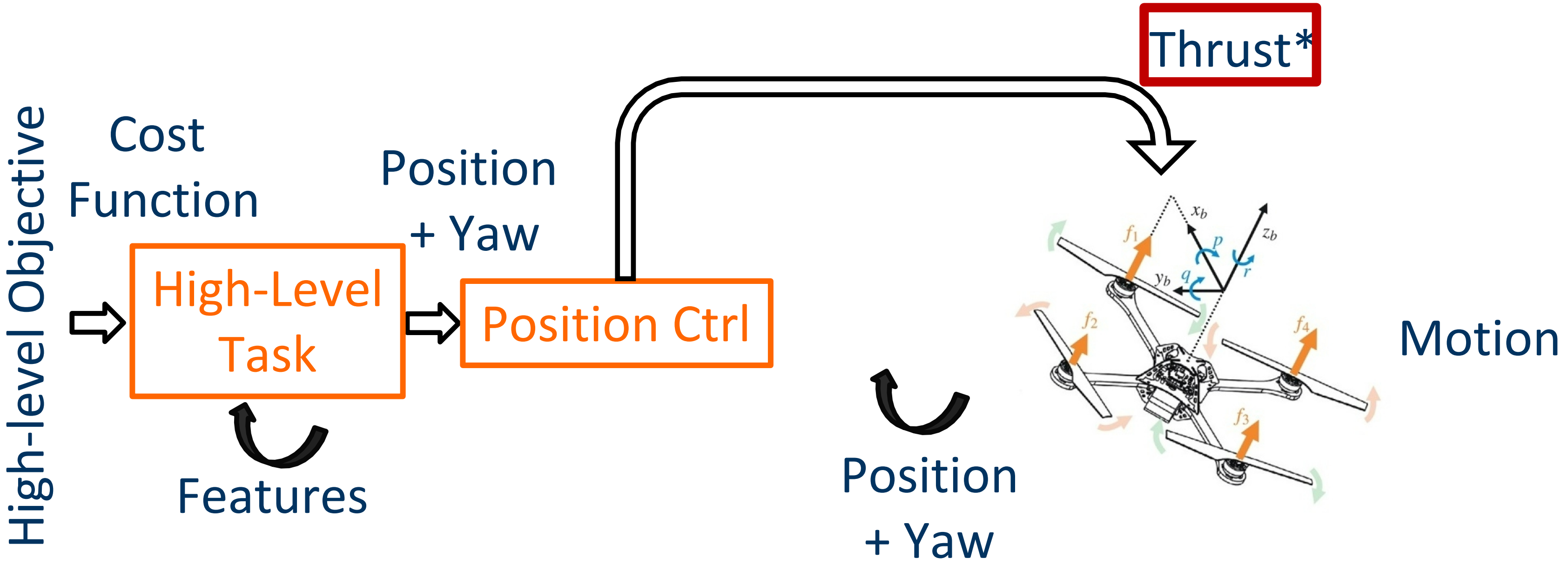
$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Nonlinear Controller 1



Assume quadrotor onboard controller requires thrust and angular rates

Nonlinear Controller 1



Nonlinear Controller 1

Make $e = z_{des} - z$ decay exponentially to 0:

$$(\ddot{z}_{des} - \ddot{z}) + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z}) = 0$$

$$\ddot{z}_c = \ddot{z}_{des} + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z})$$

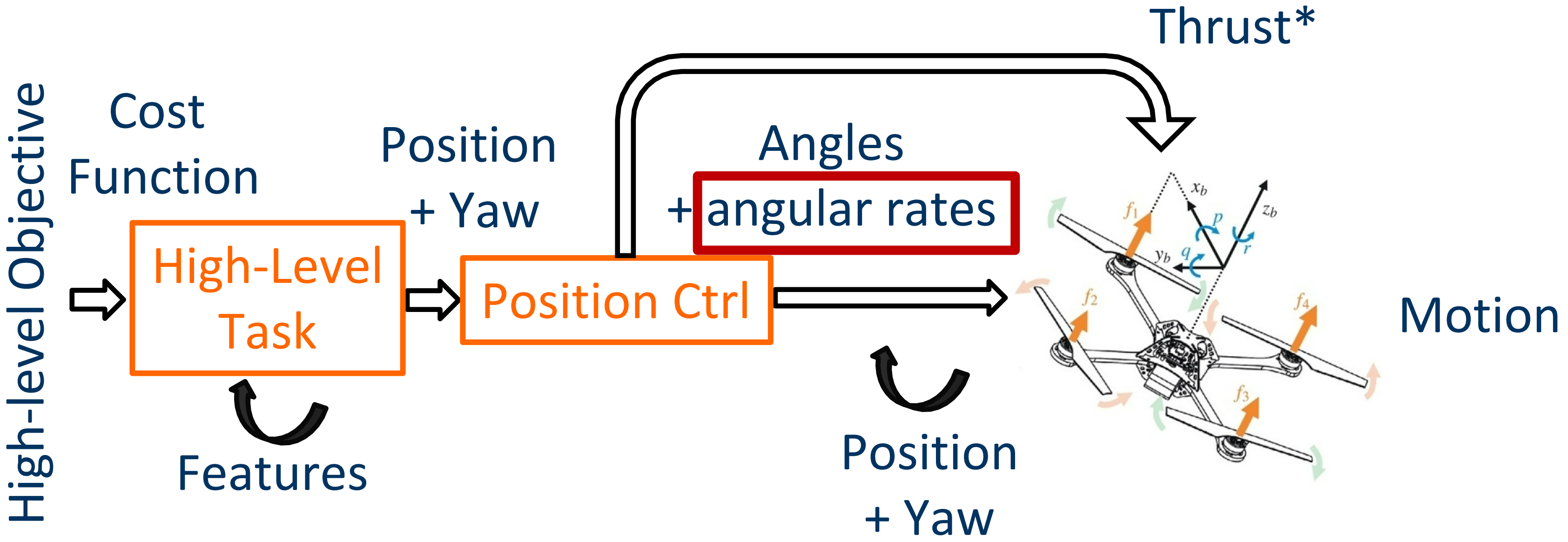
Recall the equation of motion: $\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$

Solve for c : $c = \frac{1}{R_{33}}(\ddot{z} + g)$

Calculate desired thrust

$$\boxed{c_d = \frac{1}{R_{33}}(\ddot{z}_{des} + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z}) + g)}$$

Nonlinear Controller 1



Nonlinear Controller 1

Make $e = x_{des} - x$ decay exponentially to 0:

$$(\ddot{x}_{des} - \ddot{x}) + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x}) = 0$$

$$\ddot{x}_c = \ddot{x}_{des} + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x})$$

Make $e = y_{des} - y$ decay exponentially to 0:

$$(\ddot{y}_{des} - \ddot{y}) + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y}) = 0$$

$$\ddot{y}_c = \ddot{y}_{des} + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y})$$

Rotational kinematics:

$$\dot{\boldsymbol{R}} = \boldsymbol{R}\begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}$$

From the equations of motion:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} R_{13,d} \\ R_{23,d} \end{bmatrix} c_d$$

Shape reaction of matrix entries as a first-order system:

$$\dot{R}_{13,d} = \frac{1}{\tau_{13,d}}(R_{13,d} - R_{13}) \qquad \dot{R}_{23,d} = \frac{1}{\tau_{23,d}}(R_{23,d} - R_{23})$$

Use rotational kinematics and rate of change calculated above:

$$\boxed{\begin{bmatrix} p_d \\ q_d \end{bmatrix} = \frac{1}{R_{33}}\begin{bmatrix} R_{21} & -R_{11} \\ R_{22} & -R_{12} \end{bmatrix}\begin{bmatrix} \dot{R}_{13,d} \\ \dot{R}_{23,d} \end{bmatrix}}$$

Nonlinear Controller 1

Motor rate controller:

shaping $\dot{\boldsymbol{\omega}}$ as first-order system:

$$\boldsymbol{I}\begin{bmatrix}\frac{1}{\tau_p}(p_d - p)\\ \frac{1}{\tau_q}(q_d - q)\\ \frac{1}{\tau_r}(r_d - r)\end{bmatrix} + \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \boldsymbol{I}\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}l(F_{2,d} - F_{4,d})\\ l(F_{3,d} - F_{1,d})\\ \kappa(F_{1,d} - F_{2,d} + F_{3,d} - F_{4,d})\end{bmatrix}$$

$$\left(F_{1,d} + F_{2,d} + F_{3,d} + F_{4,d}\right) = mc_d$$

Nonlinear Controller 1

Motor rate controller:

$$\begin{bmatrix} p_d \\ q_d \end{bmatrix} = \frac{1}{R_{33}} \begin{bmatrix} R_{21} & -R_{11} \\ R_{22} & -R_{12} \end{bmatrix} \begin{bmatrix} \dot{R}_{13,d} \\ \dot{R}_{23,d} \end{bmatrix}$$

shaping $\dot{\boldsymbol{\omega}}$ as first-order system:

$$\boldsymbol{I} \begin{bmatrix} \frac{1}{\tau_p}(p_d - p) \\ \frac{1}{\tau_q}(q_d - q) \\ \frac{1}{\tau_r}(r_d - r) \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \boldsymbol{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} l(F_{2,d} - F_{4,d}) \\ l(F_{3,d} - F_{1,d}) \\ \kappa(F_{1,d} - F_{2,d} + F_{3,d} - F_{4,d}) \end{bmatrix}$$

$$(F_{1,d} + F_{2,d} + F_{3,d} + F_{4,d}) = mc_d$$

$$c_d = \frac{1}{R_{33}}(\ddot{z}_{des} + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z}) + g)$$

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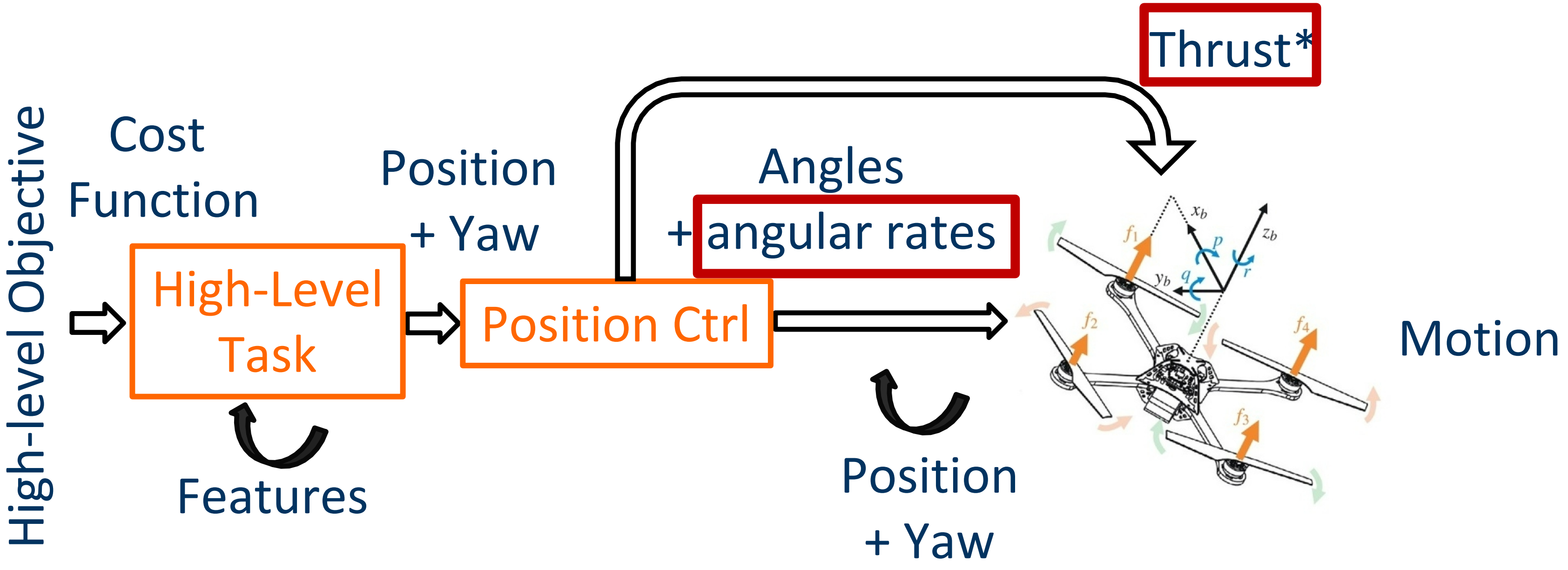
Nonlinear Controller 2



Assume quadrotor onboard controller requires thrust and angles

Nonlinear Controller 2

Newton's equations of motion

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -g\end{bmatrix} + R\begin{bmatrix}0\\ 0\\ f\end{bmatrix}$$

$$f = F_1 + F_2 + F_3 + F_4$$

Euler's equations of motion

$$\boldsymbol{I}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}L(F_2 - F_4)\\ L(F_3 - F_1)\\ \kappa(F_1 - F_2 + F_3 - F_4)\end{bmatrix} - \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \boldsymbol{I}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

Using $\psi = 0$ to simplify the rotation matrix:

$$R = \begin{bmatrix}. & . & \cos\phi\sin\theta\\ . & . & \sin\phi\\ . & . & \cos\theta\cos\phi\end{bmatrix}$$

Hence:

$$\ddot{x} = f\cos\phi\sin\theta$$

$$\ddot{y} = -f\sin\phi$$

$$\ddot{z} = f\cos\theta\cos\phi - g$$

Nonlinear Controller 2

Make $e = x_{des} - x$ decay exponentially to 0:

$$(\ddot{x}_{des} - \ddot{x}) + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x}) = 0$$

$$\ddot{x}_c = \ddot{x}_{des} + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x})$$

Make $e = y_{des} - y$ decay exponentially to 0:

$$(\ddot{y}_{des} - \ddot{y}) + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y}) = 0$$

$$\ddot{y}_c = \ddot{y}_{des} + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y})$$

Use measurements to calculate the current mass-normalized force: $\boxed{f = \dfrac{\ddot{z} + g}{\cos\theta \cos\phi}}$

Calculate commanded roll and pitch:

$$\boxed{\begin{aligned} \phi_c &= \sin^{-1}\left(\frac{-\ddot{y}_C}{f}\right) \\ \theta_c &= \sin^{-1}\left(\frac{-\ddot{x}_C}{f\cos\phi_c}\right) \end{aligned}}$$

Nonlinear Controller 2

$$\boxed{\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\0\\-g\end{bmatrix} + R\begin{bmatrix}0\\0\\f\end{bmatrix}}$$

$$\boxed{\boldsymbol{I}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}L(F_2 - F_4)\\ L(F_3 - F_1)\\ \kappa(F_1 - F_2 + F_3 - F_4)\end{bmatrix} - \begin{bmatrix}p\\q\\r\end{bmatrix} \times \boldsymbol{I}\begin{bmatrix}p\\q\\r\end{bmatrix}}$$

$$\boxed{\begin{aligned}\phi_c &= \sin^{-1}\left(\frac{-\ddot{y}_C}{f}\right)\\ \theta_c &= \sin^{-1}\left(\frac{-\ddot{x}_C}{f\cos\phi_c}\right)\end{aligned}}$$

$$\boxed{f = \frac{\ddot{z} + g}{\cos\theta\cos\phi}}$$

$$\boxed{f = F_1 + F_2 + F_3 + F_4}$$

Q&A

Break

1. How to linearly control a quadrotor?
2. How to nonlinearly control a quadrotor?
3. How to set up a commercially available drone?
4. What steps to fly a commercially available drone?

How to set up a commercially available drone?

Flying safely and legally:

- Knowledge and training
- Equipment choice and setup
- Planning
- Pilot mindset

Knowledge and training

Governing bodies and legislation*

- Transport Canada
    1. Aeronautics Act
    2. Canadian Aviation Regulations (250g – 25kg)
- Criminal Code
- Trespass to Property Act (Ontario)
- Laws related to voyeurism and privacy

Drone safety

Flying your drone safely and legally, drone registration, drone pilot certification, where and where not to fly, reporting a drone incident.

National Drone Safety Awareness Day – November 13

Services and information

Flying your drone safely and legally
Rules and regulations, fines and penalties, safety tips.

Privacy guidelines for drone users
Privacy guidelines for recreational, commercial and government drone operators.

Find your category of drone operation
Basic and advanced operations, special flight operations.

Choosing the right drone
Safety assurance ratings and requirements.

Registering your drone
How to register, benefits, penalties, managing your registration.

Where to fly your drone
Search the interactive map, sharing airspace safely, prohibited areas.

Most requested

- Drone Management Portal login
- New rules for drones in Canada
- *Canadian Aviation Regulations* - Part IX
- Standard 921 – Small Remotely Piloted Aircraft in Visual Line-Of-Sight (VLOS)
- Standard 922 – Remotely Piloted Aircraft Systems Safety Assurance

Contributors

- Transport Canada
- National Research Council Canada
- Parks Canada

tc.Canada.ca/en/aviation/drone-safety

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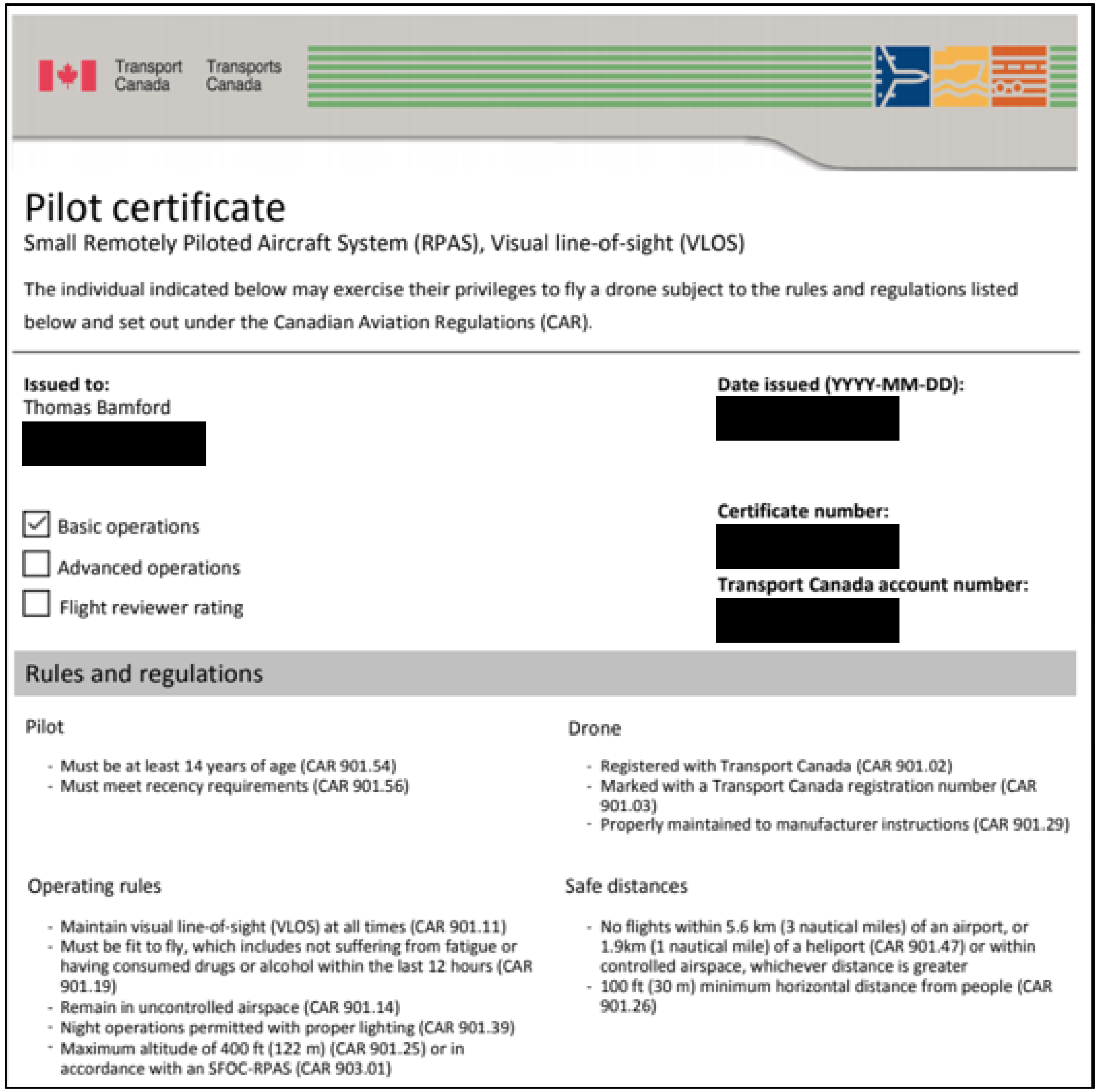
DISCLAIMER: Use the following information at your own risk.
Regulations and legislation change frequently.

Knowledge and training

Before you fly:

- Drone pilot licence (250g – 25kg)
  1. Knowledge requirements
  2. Take the exam (and flight review)



Transport Canada Transports Canada

Pilot certificate

Small Remotely Piloted Aircraft System (RPAS), Visual line-of-sight (VLOS)

The individual indicated below may exercise their privileges to fly a drone subject to the rules and regulations listed below and set out under the Canadian Aviation Regulations (CAR).

Issued to:
Thomas Bamford
[redacted]

Date issued (YYYY-MM-DD):
[redacted]

- [x] Basic operations
- [ ] Advanced operations
- [ ] Flight reviewer rating

Certificate number:
[redacted]

Transport Canada account number:
[redacted]

Rules and regulations

Pilot

- Must be at least 14 years of age (CAR 901.54)
- Must meet recency requirements (CAR 901.56)

Drone

- Registered with Transport Canada (CAR 901.02)
- Marked with a Transport Canada registration number (CAR 901.03)
- Properly maintained to manufacturer instructions (CAR 901.29)

Operating rules

- Maintain visual line-of-sight (VLOS) at all times (CAR 901.11)
- Must be fit to fly, which includes not suffering from fatigue or having consumed drugs or alcohol within the last 12 hours (CAR 901.19)
- Remain in uncontrolled airspace (CAR 901.14)
- Night operations permitted with proper lighting (CAR 901.39)
- Maximum altitude of 400 ft (122 m) (CAR 901.25) or in accordance with an SFOC-RPAS (CAR 903.01)

Safe distances

- No flights within 5.6 km (3 nautical miles) of an airport, or 1.9km (1 nautical mile) of a heliport (CAR 901.47) or within controlled airspace, whichever distance is greater
- 100 ft (30 m) minimum horizontal distance from people (CAR 901.26)

Knowledge and training

Before you fly:

- Drone pilot licence (250g – 25kg)
    1. Knowledge requirements
    2. Take the exam (and flight review)
- Drone
    1. Choose the right drone
    2. Register
    3. Follow manufacturer's instructions

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Knowledge and training

Before you fly:

- Drone pilot licence (250g – 25kg)
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  2. Take the exam (and flight review)
- Drone
  1. Choose the right drone
  2. Register
  3. Follow manufacturer's instructions
- Survey the area

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Knowledge and training

Before you fly:

- Drone pilot licence (250g – 25kg)
    1. Knowledge requirements
    2. Take the exam (and flight review)
- Drone
    1. Choose the right drone
    2. Register
    3. Follow manufacturer’s instructions
- Survey the area
    1. Obstacles

Knowledge and training

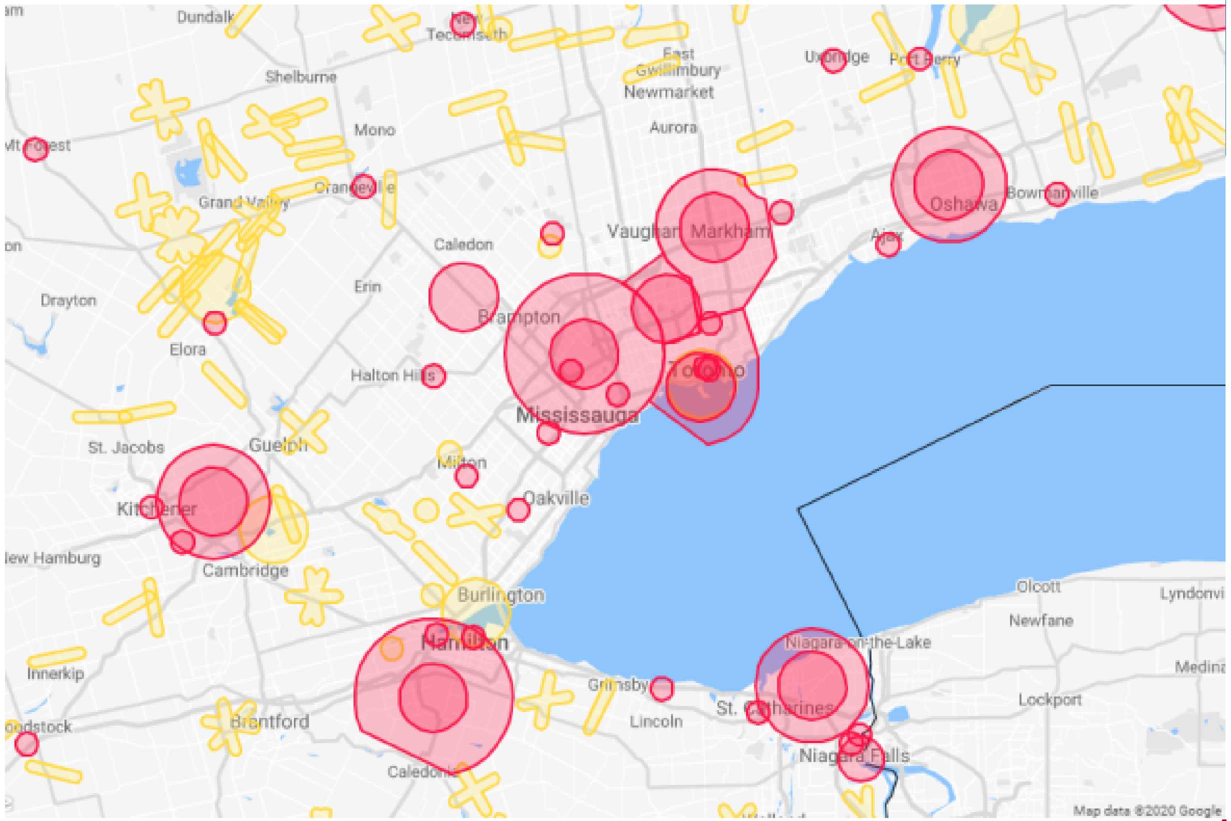
Before you fly:

- Drone pilot licence (250g – 25kg)
  1. Knowledge requirements
  2. Take the exam (and flight review)
- Drone
  1. Choose the right drone
  2. Register
  3. Follow manufacturer's instructions
- Survey the area
  1. Obstacles
  2. Weather

Knowledge and training

Before you fly:

- Drone pilot licence (250g – 25kg)
    1. Knowledge requirements
    2. Take the exam (and flight review)
- Drone
    1. Choose the right drone
    2. Register
    3. Follow manufacturer's instructions
- Survey the area
    1. Obstacles
    2. Weather
    3. Airspace (and op. category)
    - ➢ NOTAMs, Canada Flight Supplement, Designated Airspace Handbook, charts, flight authorization (advanced only)



National Research Council interactive map *

DISCLAIMER: For convenience only.

Knowledge and training

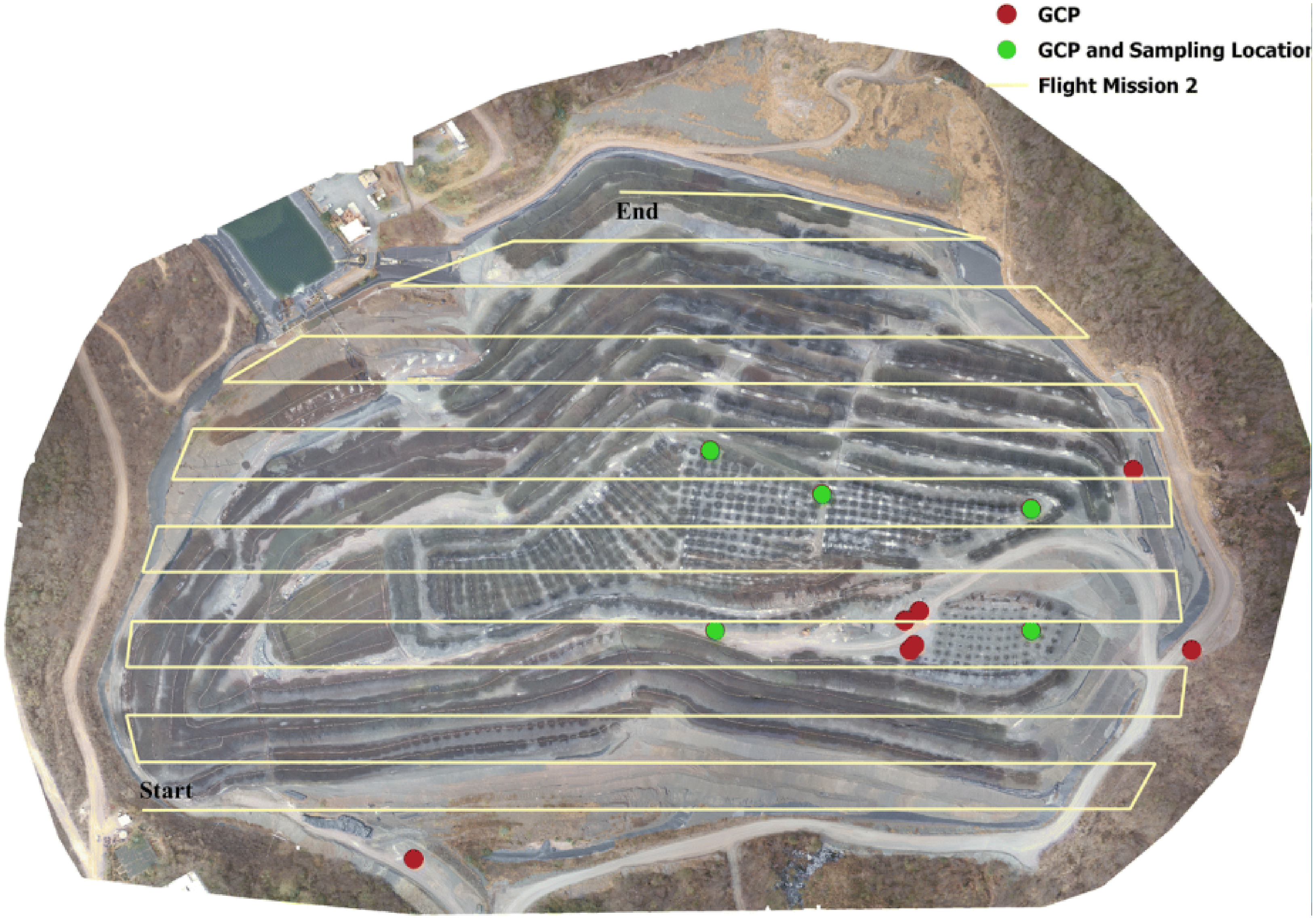
While flying, fly your drone:

- Where you can see it at all times (VLOS)
- Below 122m
- Away from bystanders (>30m horizontal)
- Away from advertised events
  - ex. concerts, parades
- Away from emergency ops.
  - ex. forest fires
- Away from airports and heliports
  - 5.6km from airports
  - 1.9km from heliports
- Outside controlled airspace (Basic only)
- Far away from other aircraft

Equipment choice and setup

Surveying in a quarry:

- Goal setting
- Drone selection
- Flight mission design



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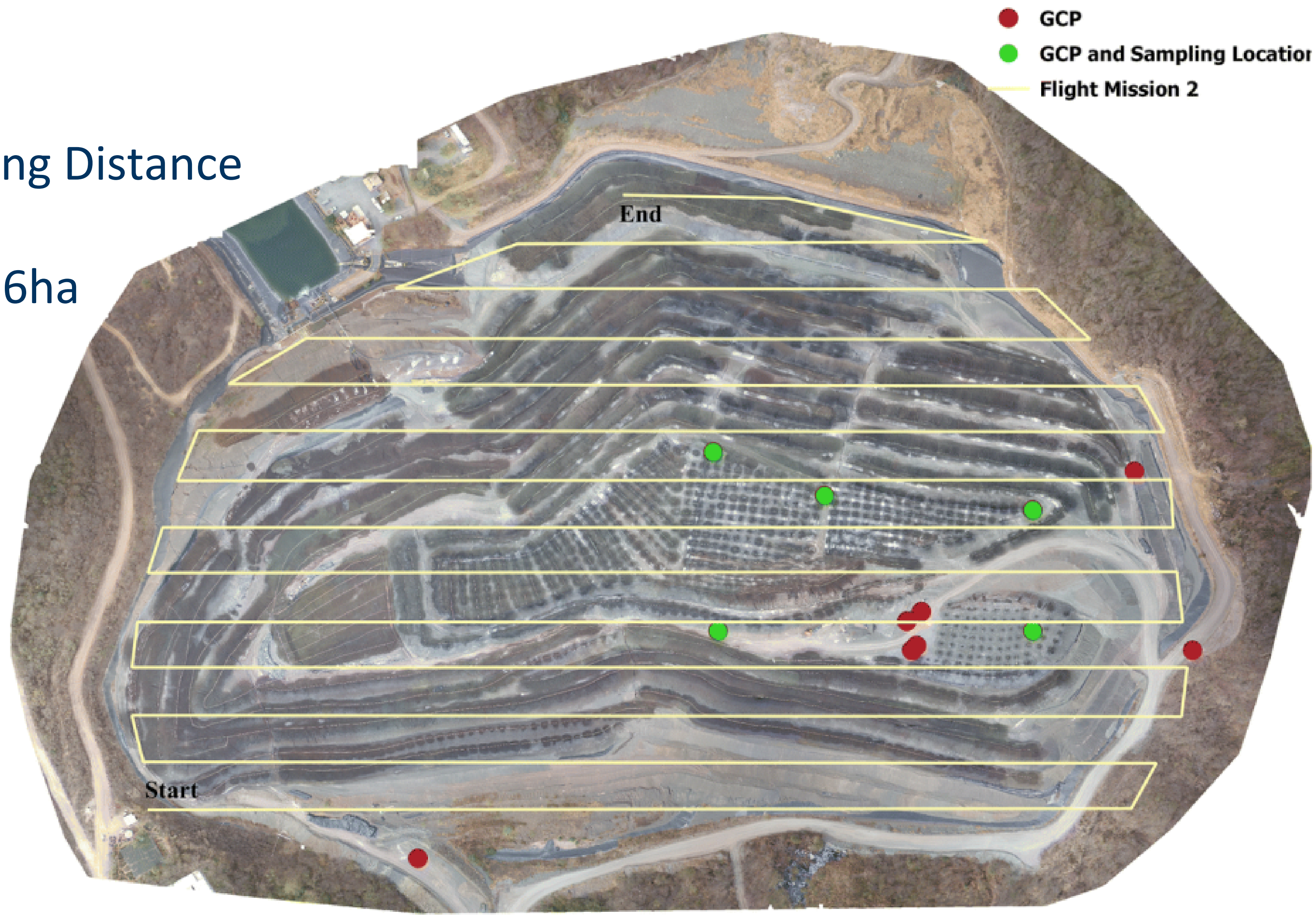
Equipment choice and setup

Goal setting:

1. Orthomosaic
2. <2cm/px Ground Sampling Distance
3. <5cm relative accuracy
4. <20min flight time for 0.6ha



Equipment choice and setup

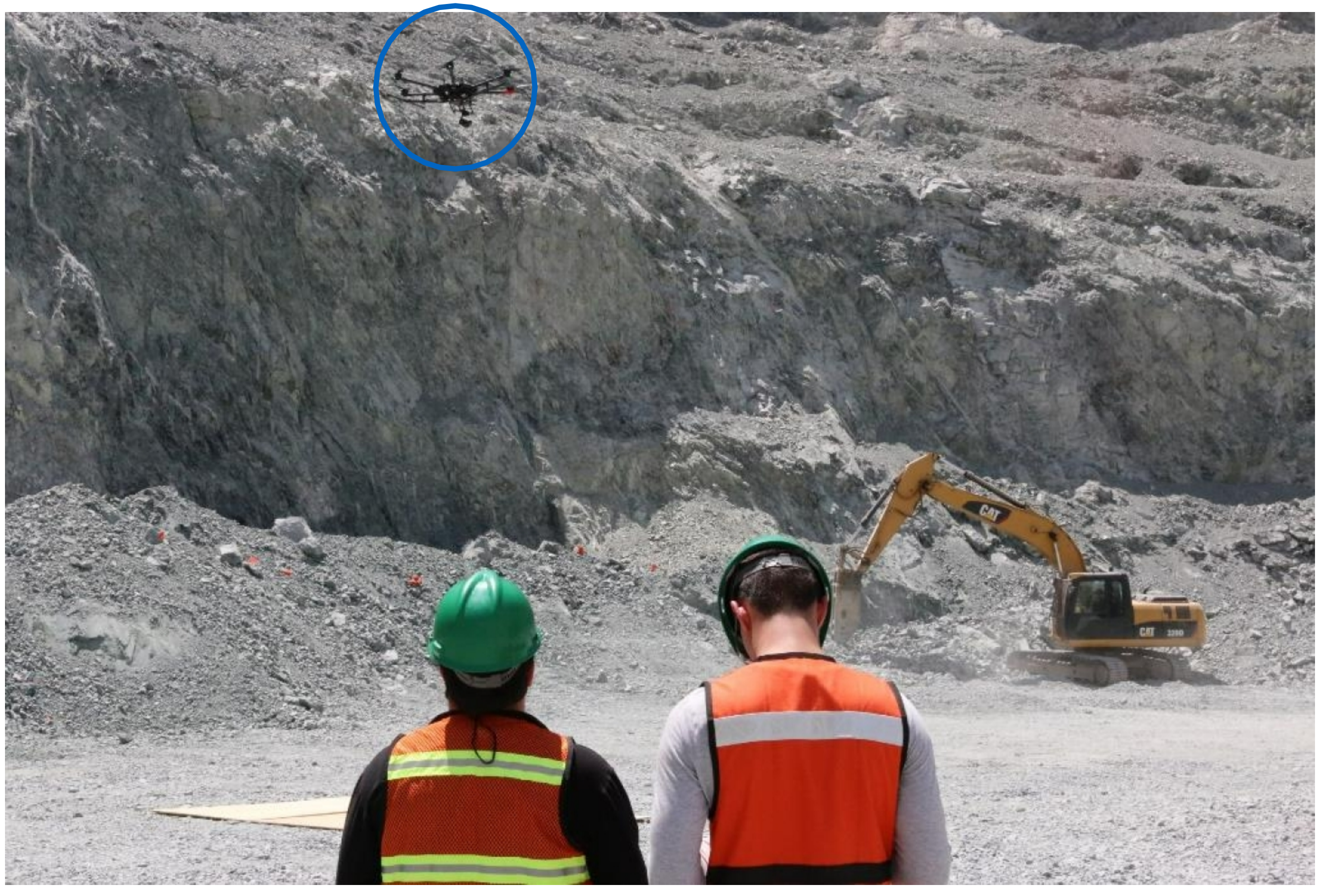
Drone selection:

- Payload
  1. Camera and lens
  2. Flight height
- Positioning
  a. GPS and/or Ground Control Points
  b. Real-time kinematic (RTK)
  c. Post-processed kinematic (PPK)
- Flight time
  a. Multirotor
  b. Fixed-wing

Equipment choice and setup

Drone selection:

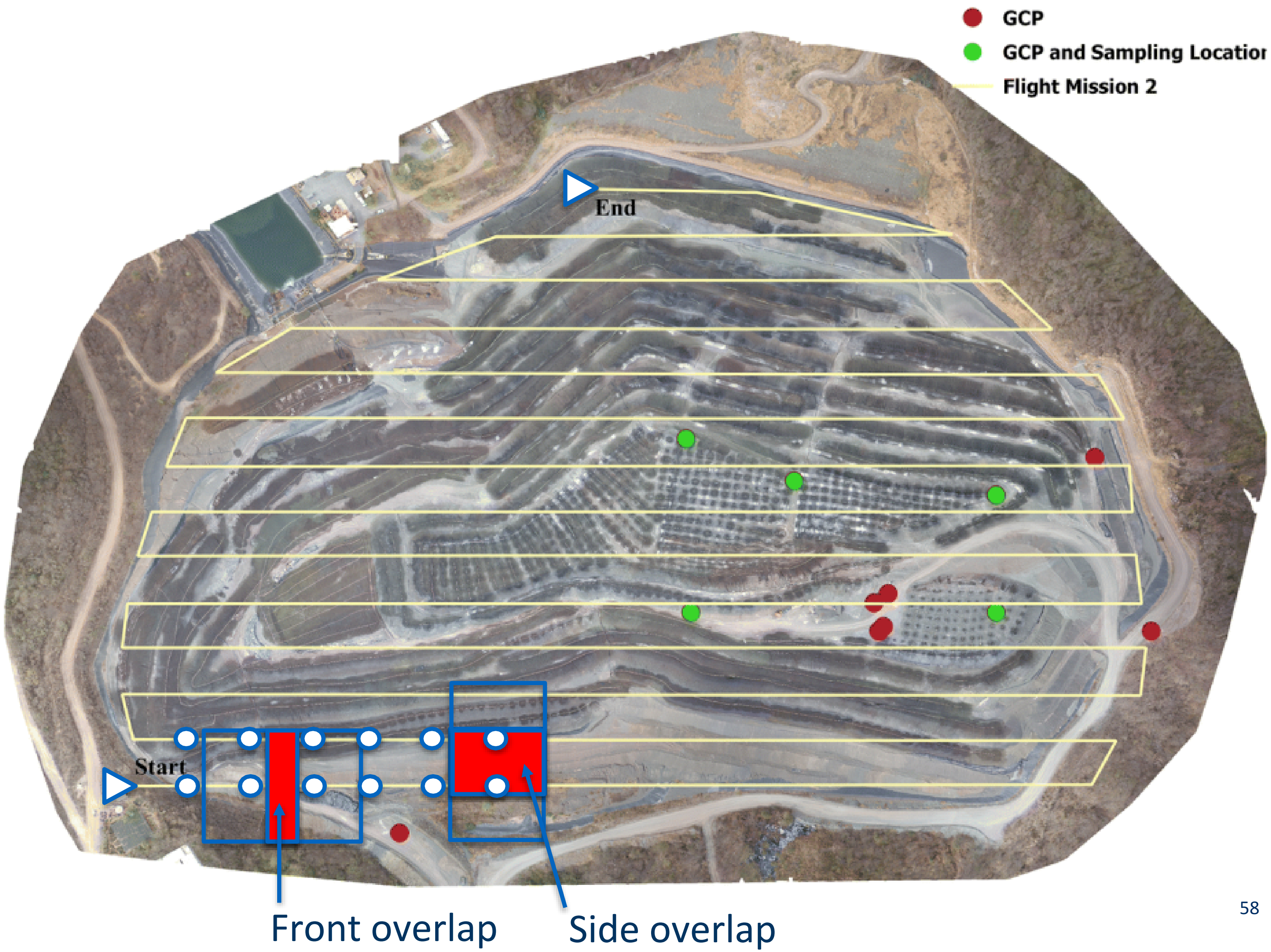
1. Multirotor (DJI Matrice 600 Pro)
2. Camera (DJI Zenmuse X5 with 15mm lens)
3. GPS



Equipment choice and setup

Flight mission design:

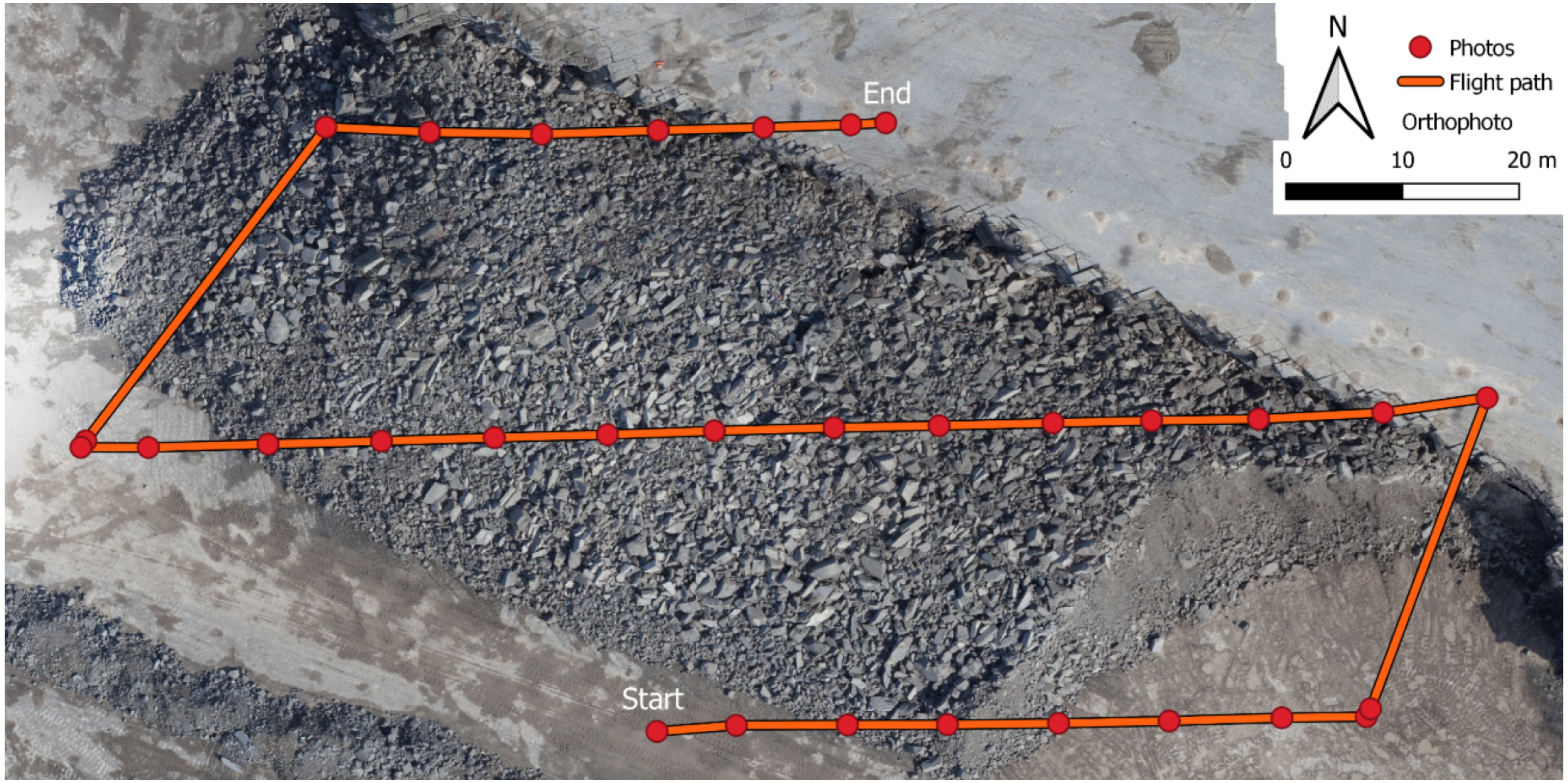
- Grid type
- Flight height
- Front and side overlap
- Camera tilt
- Heading
- Flight speed



Equipment choice and setup

Flight mission design:

1. Single grid
   - ➢ 370m flight length
2. 75m flight height
3. 85% front overlap
4. 70% side overlap
5. -90° camera tilt
6. 90° heading (East)
7. 4.9m/s flight speed
   - ➢ 2min, 32 images



Planning

Surveying in a quarry:

- Operation category
- Pilot knowledge and training
- Drone
- Weather
- Area survey
- Equipment for operation
- Navigation and UI

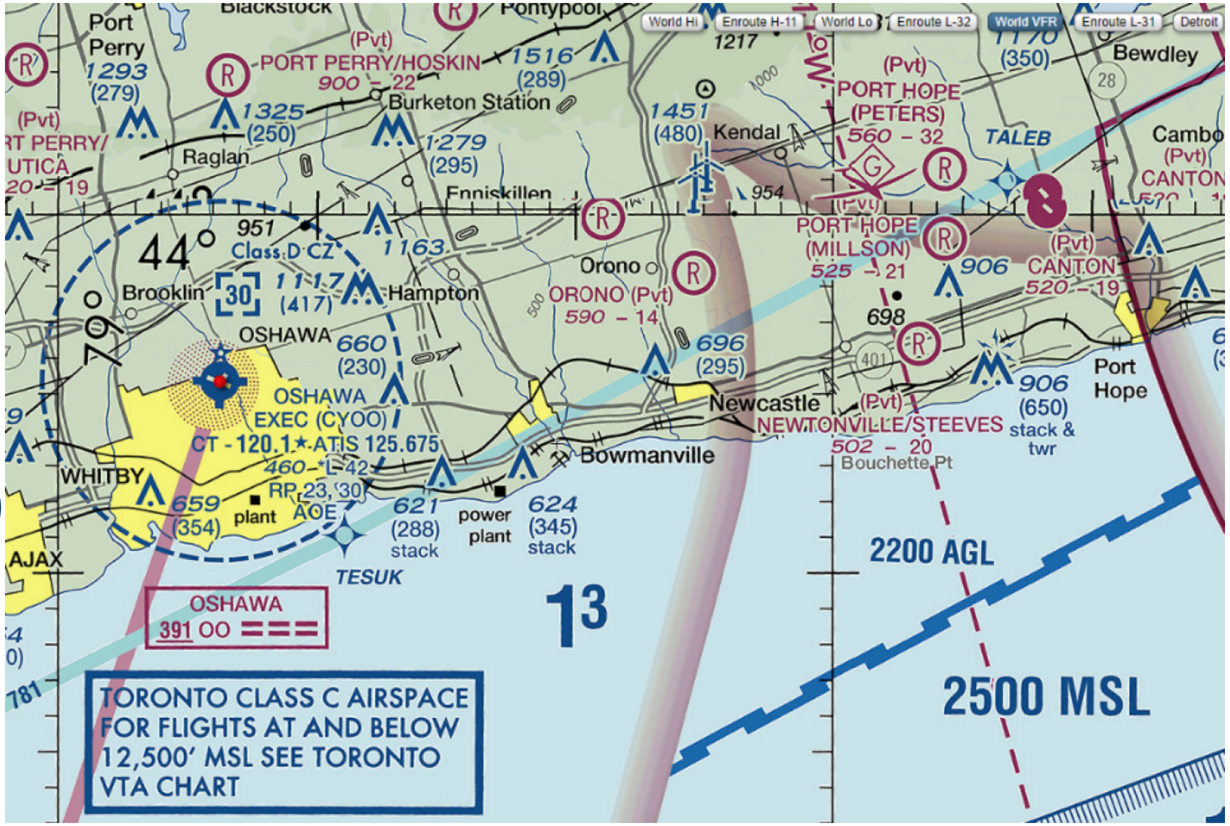
Planning

Operation category:

- Airspace
    1. NOTAMs
    2. Canada Flight Supplement
    3. Designated Airspace Handbook
    4. Charts
    5. Flight authorization (Advanced only)

✓ Basic Operation

➢ Prior to June 2019, SFOC



SkyVector Aeronautical Charts

Planning

Pilot knowledge and training:

✓ Small RPAS, VLOS with Basic Operations

✓ Experienced with DJI Matrice 600 Pro

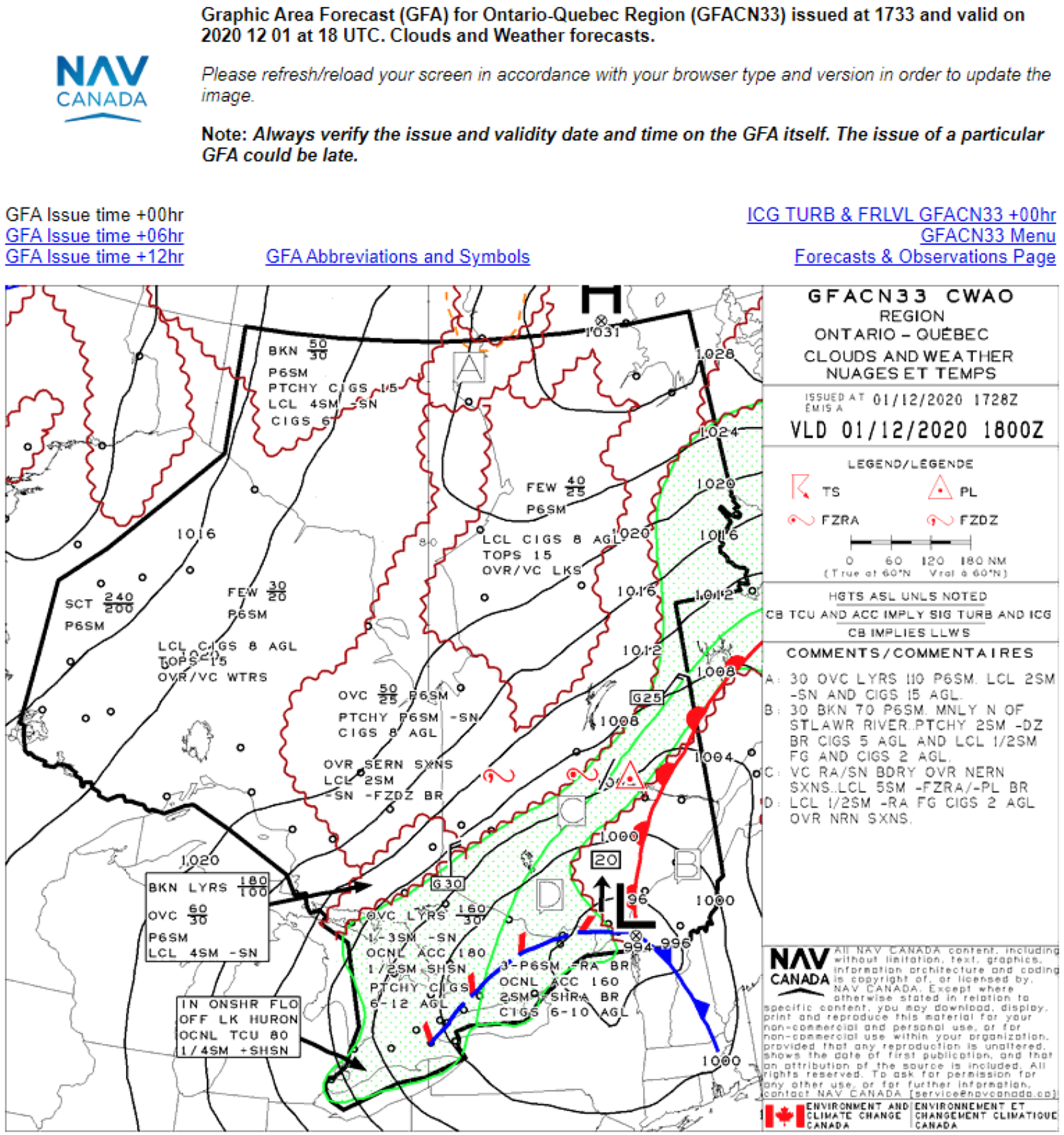
Drone:

1. Registration
2. Checklists
3. Maintenance (Including firmware)
4. Limitations
    - Max takeoff weight 15.5kg
    - Max wind resistance 8m/s
    - Max service ceiling 2500m
    - Flight time (No payload) 32min
    - Operating temperature -10° to 40°C

Planning

Weather:

- Rain, snow, fog
- Wind <8m/s
- Temperature -10° to 40°C



NAV CANADA Aviation Weather

Planning

Area survey:

- ✓ Boundaries of the area
- ✓ Type of airspace and regulatory requirements
- ✓ Altitudes and routes to be used
- ✓ Proximity of manned aircraft operations
- ✓ Proximity of aerodromes, airports and heliports
- ✓ Location and height of obstacles
- ✓ Weather and environmental conditions
- ✓ Horizontal distance from persons not involved

Planning

Area survey:

- ✓ Sufficient amount of energy for safe completion
- ✓ Each crew member instructed
  - Duties they are to perform
  - Location and use of emergency equipment
- ✓ Maximum distance from pilot aircraft can travel without endangering aviation safety
- ✓ Aircraft is serviceable in accordance with manufacturer's instructions

Planning

Equipment for operation:

- Drone
- Batteries
- Remote(s)
- Camera (Lens and storage)
- Landing pad
- Checklists and logbook
- Tools
- Spare parts
- Personal Protective Equipment
- Fire extinguisher
- First aid kit

Planning

Navigation and UI:

- ✓ Experienced with controller
- ✓ Experienced with app
- ✓ Familiar with drone behaviours
  - ➢ ex. return-to-home (RTH)

DJI GS Pro

Planning

Navigation and UI:

- ✓ Experienced with controller
- ✓ Experienced with app
- ✓ Familiar with drone behaviours
  - ➢ ex. return-to-home (RTH)
- ✓ Uploaded flight boundaries to app
- ✓ Designed flight plan in app
- ✓ BONUS: Simulator
  - Test drone behaviours
  - Test flight plan

DJI GS Pro

Pilot mindset

"Drones are aircraft – which makes you a pilot. When you fly your drone, you're sharing the skies with other drones and aircraft" – Transport Canada

Tips for first-time pilots:

- Make sure it is safe to fly
- Fly with someone who has flown before
- Fly in an open space and away from people
- Fly close to the ground and at low speed
- Fly during daylight and in good weather

Q&A

Break

1. How to linearly control a quadrotor?
2. How to nonlinearly control a quadrotor?
3. How to set up a commercially available drone?
4. What steps to fly a commercially available drone?

Q&A

Break

Quadrotor Dynamics and Control Lectures

Lecture 1	Lecture 2	Lecture 3
Introduction to quadrotor control Controlling the up-down motion of the quadrotor	Controlling the quadrotor in 2D (y-z plane) Introduction to nonlinear controllers	Controlling the quadrotor in 3D Setting up a commercially available quadrotor
Installing simulator and initial controller exploration	Applying a 2D controller to the simulated quadrotor	How to set up a commercially available quadrotor

Inspired by Vijay Kumar's Aerial Robotics Coursera course
https://www.coursera.org/learn/robotics-flight

1. Motivation
    a. Quadrotors require controllers to work well
2. How does a quadrotor move?
    a. Varying motor speeds
3. What is the quadrotor control architecture?
    a. Nested control architecture
4. How to control a 1D quadrotor?
    a. PD controller for a 1D quadrotor
5. Simulator and initial control exploration

1. How to linearly control a planar (2D) quadrotor?
   a. Linearize dynamics and apply PD controller
2. What other linear controllers can we use?
   a. Pole placement and LQR
3. When should we use nonlinear controllers?
   a. When system behavior deviates from linearization
4. How to nonlinearly control a planar (2D) quadrotor?
   a. Nested architecture, nonlinear eq's and measurements
5. How to control the planar quadrotor in simulation?

1. How to linearly control a quadrotor?
    a. Linearize dynamics and apply PD controller
2. How to nonlinearly control a quadrotor?
    a. Nested architecture, nonlinear eq's and measurements
3. How to set up a commercially available drone?
    a. Training and knowledge, equipment choice and setup, planning, pilot mindset
4. What steps to fly a commercially available drone?

Q&A

Break