AER 1216 Ass II

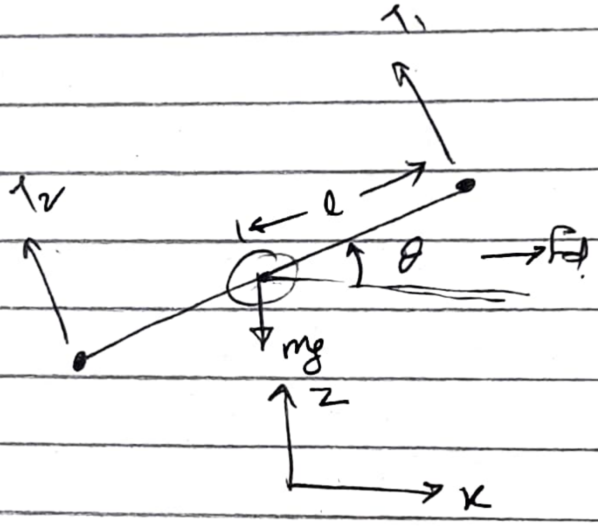
PAGE NO.:
DATE: / /

(a) inputs $\rightarrow$ $T_1(t)$ $T_2(t)$

outputs $\rightarrow$ $x(t)$, $z(t)$

state $\rightarrow$ $\{x(t), z(t), \theta(t), \dot{x}(t), \dot{z}(t), \dot{\theta}(t)\}$

$\therefore$ Governing Eq^n



$m\ddot{x} \Rightarrow F_d - (T_1+T_2)\sin\theta$

$m\ddot{z} = -mg + (T_1+T_2)\cos\theta$

$J\ddot{\theta} = (T_1 - T_2)\,l$

$$\begin{bmatrix}\ddot{x}\\ \ddot{z}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}K(V_0-\dot{x})^2/m\\ -g\\ 0\end{bmatrix} + \begin{bmatrix}-\frac{\sin\theta}{m} & -\frac{\sin\theta}{m}\\ \frac{\cos\theta}{m} & \frac{\cos\theta}{m}\\ -l/J & l/J\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix}$$

$$SS \rightarrow \dot{X} \Rightarrow \begin{bmatrix}\dot{x}\\ \dot{z}\\ \dot{\theta}\\ \ddot{x}\\ \ddot{z}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}\dot{x}\\ \dot{z}\\ \dot{\theta}\\ K(V_0-\dot{x})^2/m\\ -g\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ -\sin\theta/m & -\sin\theta/m\\ \cos\theta/m & \cos\theta/m\\ -l/J & l/J\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix}$$

$$y = \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ \theta \\ \dot{x} \\ \dot{z} \\ \dot{\theta} \end{bmatrix}$$

PAGE NO.:
DATE: / /

(b) $1 \longrightarrow F_d = (T_1 + T_2)\sin\theta$

$mg = (T_1 + T_2)\cos\theta$

$T_1 - T_2 = 0 \Rightarrow T_1 = T_2$

$(2T_1 \sin\theta = F_d)^2$
$+$
$(2T_1 \cos\theta = mg)^2$

$\rightarrow 4T_1^2 \Rightarrow F_d^2 + mg^2$

$T_{2e} = T_{1e} \Rightarrow \frac{1}{2}\sqrt{F_d^2 + mg^2}$

$\tan(\theta) \Rightarrow \left(\frac{F_d}{mg}\right) \Rightarrow \theta_e \Rightarrow \tan^{-1}\left(\frac{F_d}{mg}\right)$

An (∞) number of Equilib^r points Exist! as the copter can hover anywhere

(c) $\Rightarrow T_1 + T_2 = \frac{mg}{\cos(\theta)}$

For first Eq^n in x- dynamics

state $\rightarrow \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$

$m\ddot{x} = F_d - (T_1 + T_2)\sin\theta$

$\dot{x} \rightarrow \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix}$

$m\ddot{x} \Rightarrow F_d - mg\tan\theta$

Input $\rightarrow \{\theta\}$

$\therefore$ SS $\Rightarrow \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} \Rightarrow \begin{Bmatrix} \dot{x} \\ \left(\frac{F_d}{m}\right) \end{Bmatrix} - \begin{bmatrix} 0 \\ g\tan\theta \end{bmatrix}$

$\therefore$ Linearizing $\rightarrow f_1 \rightarrow \dot{x}$

$$\left.\frac{\partial f_1}{\partial x}\right|_{x_e}(x - x_e) + \cancel{\frac{\partial f}{\partial y}(y - y_e)}$$

$\rightarrow \ddot{x}\big|_{x_e} \Rightarrow 0!$

PAGE NO.:
DATE: / /

$$f_2 \rightarrow \left(\frac{F_d}{m}\right) - g\tan\theta$$

Linearizing the eqn

$$\frac{\partial f_2}{\partial X}$$

$\hookrightarrow$ state $\rightarrow x \rightarrow \frac{\delta f_2}{\delta x} = 0$

$\rightarrow \dot{x} \rightarrow \frac{\delta f_2}{\delta \dot{x}} = \frac{2K(V_{\infty} - \dot{x})(-1)}{m}\Big|_{x_e} (\dot{x} - \dot{x}_0)$

$$= \frac{-2KV_{\infty}}{m}$$

$$\frac{\delta f_2}{\delta u} \Rightarrow -g\sec^2\theta\Big|_{\theta_e} (\theta - \theta_0)$$

$$\Rightarrow -g\sec^2\theta_e$$

$$\Rightarrow -g\left(1 + \left(\frac{KV_{\infty}}{m}\right)^2\right)\delta\theta$$

$$\therefore \begin{bmatrix} \delta\dot{x} \\ \delta\ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{2KV_{\infty}}{m} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -g\left(1 + \left(\frac{KV_{\infty}}{mg}\right)\right)^2 \end{bmatrix} \delta\theta$$

$$\delta y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \end{bmatrix}$$