AER1216: Fundamentals of UAVs: Propulsion

P.R. Grant

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Introduction I

What we will cover:
Overview of Propulsion for Aero Vehicles (Video Part 1)

- Fixed-wing
- Helicopters
- Multi-rotors

Propulsion for small UAVs

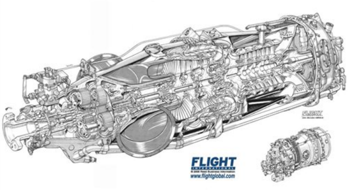
- Propellers (Video Part 2)
    - Introduction
    - Charts for axial flow
    - Actuator Disc (Momentum) Theory (axial flow)
    - Momentum-blade element theory
    - Forward Flight (Non-linear) Momentum Theory
    - brief introduction to general blade element models
- Powerplants for small UAVs (Video Part 3)
    - Electric
        - brushless DC (BLDC) motors
        - LiPo Batteries

Introduction II

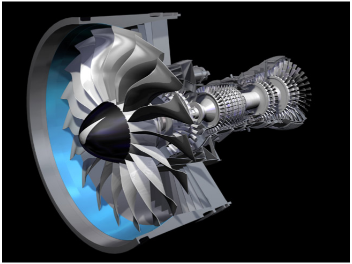
  - ESC's
- Piston Engine IC engines

Fixed-Wing Propulsion

1. **Electric motor+propeller**
2. Piston engine+propeller
3. Turboprop
4. Turbofan
5. Turbojet



(a) P&W PT6 Turboprop



(b) P&W Geared Turbofan

Electric Motor+Propeller

- Most small (<5kg) fixed wing UAVs are of this type
- Electric motors cheap, powerful, reliable
- Usually fixed pitch propeller
- Limited speed, altitude, range, endurance and payload
- Most popular motors BLDC
- Most popular batteries LiPo
- Requires Electronic Speed Control (ESC)
  - provide commutation instead of brushes

Piston engine+propeller

- Small to large fixed wing UAVs
- Endurance good, range (speed), payload limited
- Speed/altitude moderate
- Fixed pitch or variable pitch propellers
  - variable pitch higher performance

Turboprop

- Higher performance (cost) fixed wing UAVs
- Altitude, speed, range, endurance, payload improved compared to Piston Engine
- Almost exclusively variable pitch propellers
- Mainly military
- Commercial aviation commuter

(a) AT200

(b) Reaper

Turbofan

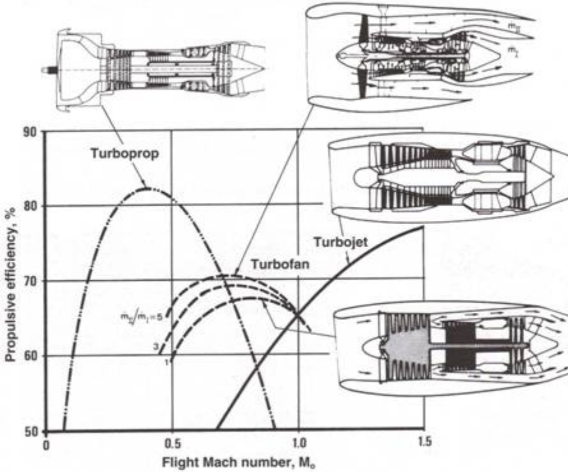
- Further improvements in performance
- Altitude, speed, range, payload
- High cost, military only
- Mainstay of commercial aviation

(a) Global Hawk

(b) Sentinel

Comparison of Types

Different engines are useful in different flight regimes...



Select the engine that fits your needs...

Helicopter Propulsion Overview

- Rotor(s) used for both lift and thrust through tilting of rotor tip-path (plane)
- Mainly 3 types of propulsion systems used for helicopter UAVs:
  - Gas Turbine+rotor(s)
  - Piston Engine+rotor
  - Electric Motor+rotor

Electric

- Mainly used for smaller UAV helicopters
- Range up to 10km
- Endurance up to 60min
- Payload up to 20kg
- Speed up to 30 mph

(a) Alpha

(b) AeroVironment Vapor

Piston Engine

- Can have larger range, endurance, speed compared to electric
- Reliability lower than Turbine

(a) Robinson R22

(b) Yamaha Rmax

Turbine Engine

- Most civil and military helicopters use turbines
- Highest performance and better reliability than piston

(a) Fire Scout

(b) UH60 Rascal

Multi-rotor Propulsion Overview

- The vast majority of multi-rotors are electric with fixed pitch propeller
- A few piston engine multi-rotors exist (with fixed pitch propellers)
  - Longer endurance, range
- Hybrid starting to become popular
  - Gas engine recharges batteries

(c) Electric DJI Phantom

(d) Gas-Powered Yeair

Electric/Piston Engine+Propeller

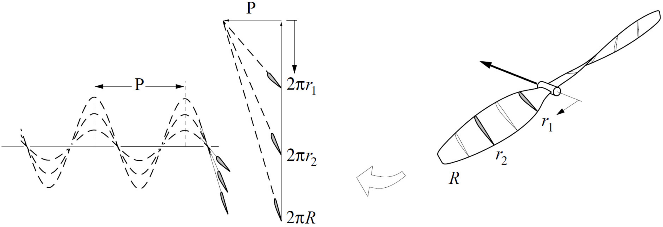
- Remainder of Lecture will focus on small Electric UAV propulsion
  - Vast majority of small fixed-wing/multi-rotor/hybrid UAVs are electric
  - For larger fixed-wing aircraft large body of knowledge available for reference
- Will cover:
  - Propellers
  - Electric propulsion
    - BLDC electric motors
    - LiPo batteries
    - Electronic Speed Controllers (ESC)
  - Piston Engines (extremely brief)

Propeller Characterization

- Propeller theory was developed for airplanes where flow is axial (i.e. aligned with axis of rotation)
- Often assume axial flow in following slides

Propeller Characterization

- A propeller mainly characterized by its diameter, D, and its pitch, p
- Chord distribution, airfoil shape etc... important but usually unavailable for small propellers
- 9 $\times$ 4 propeller diameter of 9 *in* and an average pitch of 4 *in*.

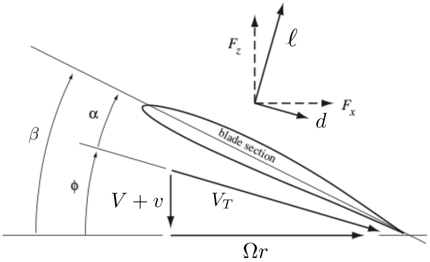


- Pitch of propeller defined same as wood screw: distance propeller would travel in one revolution if advanced through solid material

$$p = 2\pi r \tan \beta$$

Propeller Characterization

- Where β is the angle of blade relative to a plane perpendicular to the axis of rotation



- Therefore for a propeller with a constant pitch, p,

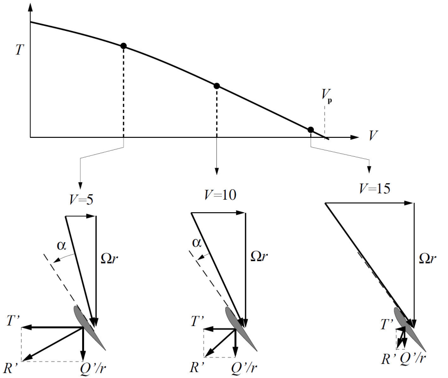
$$\beta = \tan^{-1} \frac{p/D}{\pi x}$$

- Where $x = r/R$

Propeller Characterization

For Axial Flow (fixed wing aircraft, vertical climb of multi-rotor):

- Thrust, $T = f(V, RPM)$ for given propeller
- For fixed RPM as airspeed V approaches pitch speed (V_p) angle attack of blades approaches zero, therefore $T \to 0$.



Note α shown here is not α_{zll} and we've ignored the induced velocity to make the argument more straightforward, induced velocity goes to zero at α_{zll}=0

- Blades usually cambered, so define α with respect to zll for T=0 at α=0.
- Increasing either RPM, or pitch of propeller will increase V_p

For axial flow:

- Propeller behaviour succinctly captured in non-dimensional charts of C_T, C_P, C_Q and η versus J[†] where:

$$J = \frac{V}{nD} \quad C_T = \frac{T}{\rho n^2 D^4} \quad C_P = \frac{P}{\rho n^3 D^5} \quad \eta = \frac{TV}{Q\Omega} = \frac{C_T J}{C_P} \quad C_Q = \frac{Q}{\rho n^2 D^5}$$

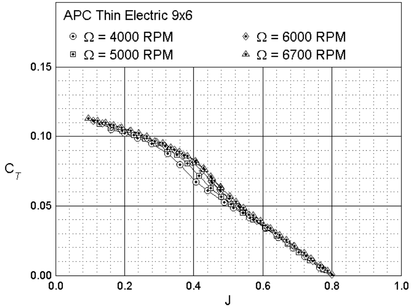
- note for these definitions $C_P = 2\pi C_Q$
- where n =rev/sec=$\frac{\Omega}{2\pi}$, P=Input Power=Q Ω, and Q=torque
- Ideally charts generated from measurements but can also be approximated using numerical techniques
- Collection of measured charts for small UAV propellers at UIUC website: *http://m-selig.ae.illinois.edu/props/propDB.html*

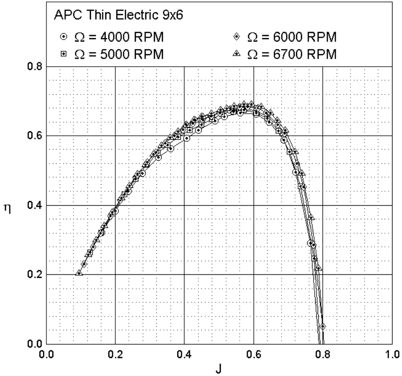
,

[†]Often there may still be changes in C_T and C_P with rpm/velocity (Re)

Propeller Characterization

Here are charts for APC 9x6E propeller†



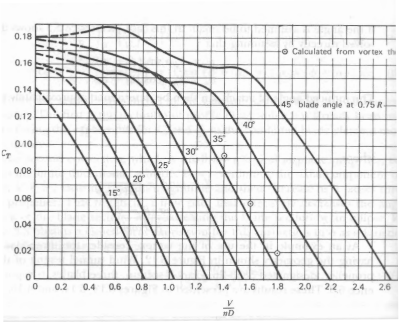


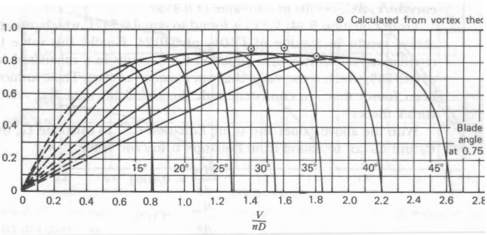
$V_p = 0.8\text{J} \rightarrow 24$ m/s for 8000RPM ($V_p \approx p\Omega/(2\pi)$=20.4 m/s)

†These charts are for a propeller with nothing but the motor behind it–the fuselage of the aircraft will further lower the thrust and efficiency

Propeller Characterization

- So far examined fixed pitch propellers
  - Most small UAVs use fixed pitch propellers
- Variable pitch propellers entire blade rotates around hub
  - change α over entire radius
  - can make V_p large without sacrificing low-speed thrust efficiency
  - take-off at low-pitch, fly fast high-pitch
  - quicker way to change thrust than change RPM?

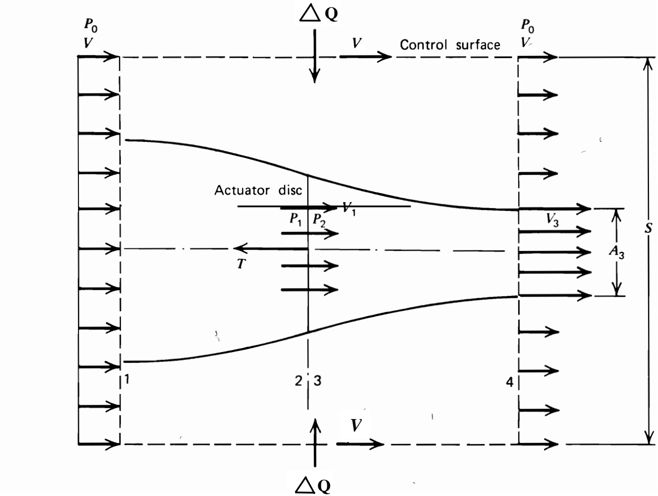




Actuator Disc (Momentum) Theory

- Propeller or fan losses can be through of in a similar way to wing: induced and profile drag
- Momentum theory gives **global** estimate of induced losses
- Very simple model/theory but provides insight and upper bound for efficiency
- Assumptions:
  - Infinitely thin disc with jump in static pressure across disc
  - velocity is constant over the disc
  - pressure is uniform over the disc
  - flow is irrotational
  - well-defined stream tube exists
  - incompressible flow
  - axial flow (aircraft forward flight or multi-rotor vertical flight)

Actuator Disc (Momentum) Theory: Axial Flight I



Actuator Disc Control Volume

Actuator Disc (Momentum) Theory: Axial Flight II

- Apply continuity to cylindrical CV. Flux out of surface at 4 minus that at 1 is,

$$\Delta Q = A_3 V_3 + (S - A_3)V - SV = A_3(V_3 - V) \tag{1}$$

- This flux must enter on sides of CV (at associated velocity V)
- Now apply momentum equation to cylindrical CV (noting pressures cancel out),

$$T = \rho \left[A_3 V_3^2 + (S - A_3)V^2 \right] - \rho S V^2 - \rho \Delta Q V \tag{2}$$

- substituting Equation 1 into this and simplifying yields,

$$T = \rho A_3 V_3 \left(V_3 - V \right) \tag{3}$$

- but, thrust also equal to (where A is disc area),

$$T = A(p_2 - p_1) \tag{4}$$

Actuator Disc (Momentum) Theory: Axial Flight III

- can relate p_1 and p_2 using Bernoulli's equation upstream of disc and downstream of disc (can't use it across disc since energy added),

$$\begin{aligned} p_0 + \frac{1}{2}\rho V^2 &= p_1 + \frac{1}{2}\rho V_1^2 && (5) \\ p_0 + \frac{1}{2}\rho V_3^2 &= p_2 + \frac{1}{2}\rho V_1^2 && (6) \end{aligned}$$

- subtracting these two yields,

$$p_2 - p_1 = \frac{1}{2}\rho(V_3^2 - V^2) \quad (7)$$

- substituting into 4 and simplifying yields,

$$T = \frac{1}{2}\rho A\left(V_3^2 - V^2\right) \quad (8)$$

Actuator Disc (Momentum) Theory: Axial Flight IV

- but from 3 and 8 and noting $A_3 V_3 = A V_1$ (A-area of disc),

$$V_1 = \frac{V_3 + V}{2} \tag{9}$$

- Velocity through disc is the average of velocity far ahead and behind it. If we call v the propeller induced velocity, $V_1 = V + v$, then

$$V_3 = V + 2v \tag{10}$$

- and from 8 and previous equation thrust becomes,

$$T = 2\rho A \left(V + v\right) v \tag{11}$$

- so total (or shaft) power in (to move air is),

$$P_{total} = T(V + v) \tag{12}$$

Actuator Disc (Momentum) Theory: Axial Flight V

- also can find required torque, Q as $Q\Omega = T(V+v)$ or,

$$Q = 2\rho A \left(V+v\right)^2 v/\Omega \tag{13}$$

- and useful power (for moving vehicle) is,

$$P_{useful} = TV \tag{14}$$

- so efficiency is given by,

$$\eta = \frac{TV}{T(V+v)} = \frac{1}{1+\frac{v}{V}} \tag{15}$$

- For highest efficiency need to make v as small as possible
- Thus from Eq 11 for given thrust need large A
  - So low disc loading T/A leads to higher efficiency
  - helicopters more efficient than multi-rotors
  - Turbo-fans getting larger

Actuator Disc (Momentum) Theory: Axial Flight VI

- This is appeal of propellers relative to turbo-fans and turbo-jets
- Defining induced power (loss) as difference between total and useful power yields $P_{ind} = Tv$
- Rearranging equation for T as a quadratic in v/V and solving gives,

$$\frac{v}{V} = \sqrt{\frac{1}{4} + \frac{T}{2\rho A V^2}} - \frac{1}{2} \tag{16}$$

- and for lightly loaded propeller $(T/(2\rho A V^2) \ll 1)$ can use binomial theorem to simplify to:

$$\frac{v}{V} \approx \frac{T}{2\rho A V^2} \tag{17}$$

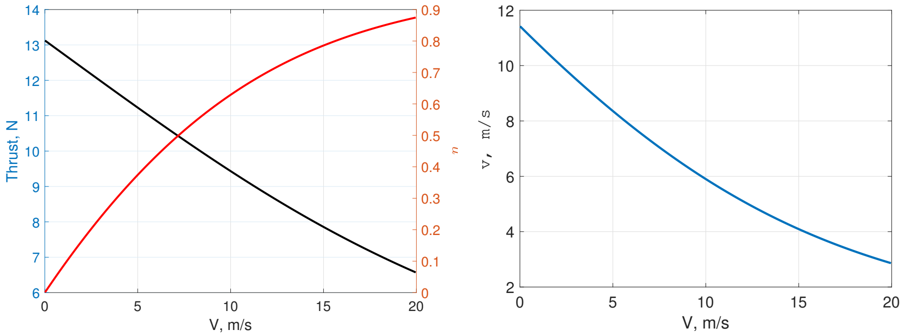
- thus

$$\eta_P \approx \frac{1}{1 + \frac{T}{2\rho A V^2}} \tag{18}$$

- thus can see low disc loading (T/A) leads to higher efficiency

Actuator Disc (Momentum) Theory: Axial Flight VIII

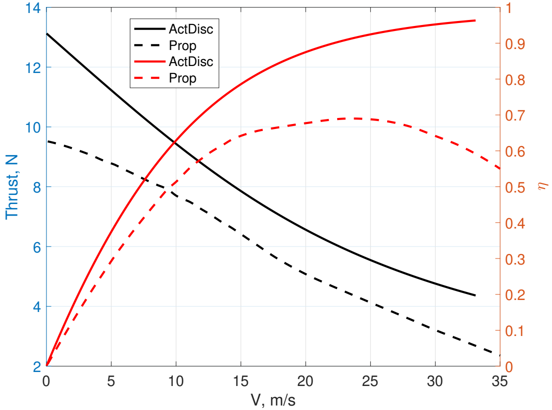
- Thrust, Efficiency, and Induced velocity (v) function of axial speed (V) for given Power, Area, Density
- Plot below shows Thrust and Efficiency for: D=9 in, P_{total}= 150W, ρ =1.225 kg/m^3



- Profile drag and pitch of blades will substantially effect behaviour so don't read too much into this slide yet!

Actuator Disc (Momentum) Theory: Axial Flight IX

- Compare thrust and efficiency of APC9x6E propeller to 9in actuator disc with fixed input power of 150W:



Actuator Disc (Momentum) Theory: Axial Flight X

- This theory can be used when the forward/climb velocity goes to zero, $V = 0$
  - aircraft static thrust
  - multi-rotor hover
  - In this case thrust is given as,

$$T = 2\rho A v_h^2$$

  - Rotor k of multi-rotor in hover, $T_k = W/N$, N-number of rotors, and take A_k as area of rotor k
  - Induced velocity is then,

$$v_h = \sqrt{\frac{W}{2\rho N A_k}} = \sqrt{\frac{W}{2\rho A}} \quad \text{since for N identical rotors } A = NA_k$$

  - The total power (or induced power since there is no 'useful' power in hover) is,

$$P_{total} = P_{ind} = NT_k v_h = N(W/N)v_h = W\sqrt{\frac{W}{2\rho A}} = \sqrt{\frac{W^3}{2\rho A}}$$

  - Efficiency as defined for disc theory not useful for hover, a more useful measure is $\bar{M} = T/P_{meas}$ where P_{meas} is total measured power which includes all losses

Actuator Disc (Momentum) Theory: Axial Flight XI

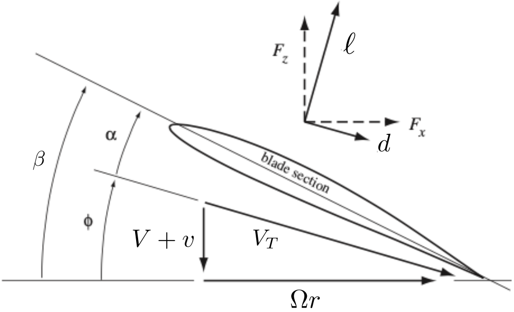
- Momentum theory is optimistic because:
  - does not include profile drag (skin friction and pressure)
  - does not include rotational (swirl) velocity imparted to air
  - assumes uniform inflow velocity, which can be shown to lead to minimum induced drag
    - hard to achieve in practice at multiple conditions
- For small negative axial velocities $(V + v \gg 1)$ theory works
- When velocity V negative and $V + v \approx 0$ then the propeller is going through its own wake and theory no longer holds–called **vortex ring state**
- For even larger negative velocities direction of flow through actuator disc reverses and when it becomes large enough a well defined slipstream forms again–called **windmill brake state**–theory needs to be adjusted
- State between **vortex ring state** and **windmill brake state** is called **turbulent wake state**
- More about these in multi-rotor performance lecture

Momentum-Blade Element Model I

- Next level of propeller model is the momentum-blade element model
- This model has some assumptions that make it only valid for axial flow
- The following derivation follows that of Johnston (Rotorcraft Aeromechanics)
- Consider the propeller shown below:

- If we consider a small element of a blade, dr

Momentum-Blade Element Model II



- The lift and the drag per unit span are given as,

$$\ell = 1/2\rho c_\ell V_T^2 c = 1/2\rho a V_T^2 c\,(\beta - \phi) \quad (19)$$

$$d = 1/2\rho c_d V_T^2 c \quad (20)$$

- where $V_T = \sqrt{\Omega^2 r^2 + (V+v)^2}$ and $\phi = \tan^{-1}\left(\frac{V+v}{\Omega r}\right)$

Momentum-Blade Element Model III

- The forces are then,

$$F_z = \ell \cos\phi - d \sin\phi \quad (21)$$

$$F_x = \ell \sin\phi + d \cos\phi \quad (22)$$

- and the elemental thrust and torque and power are,

$$dT = N F_z dr \quad (23)$$

$$dQ = N F_x r dr \quad (24)$$

$$dP = \Omega dQ = N F_x \Omega r dr \quad (25)$$

- For propeller in hover (static thrust for aircraft) or climb (forward flight for aircraft) we can assume $\Omega r \gg (V + v)$ except near root where dynamic pressure is low so loads are small thus,

$$\phi, \alpha \ll 1 \text{ so } \phi \approx \frac{V+v}{\Omega r}, \ \sin\phi \approx \phi, \ \cos\phi \approx 1 \text{ and } V_T \approx \Omega r$$

Momentum-Blade Element Model IV

- Note this assumption is better for multi-rotors than for a fast flying fixed wing aircraft
  - can adjust theory for fixed wing aircraft so only small angles need to be assumed for change in angle for induced velocity (see McCormick, Aerodynamics, Aeronautics and Flight Mechanics for example)
- So now blade section forces reduce to,

$$\begin{aligned} \ell &\approx 1/2\rho a \left(\Omega r\right)^2 c \underbrace{\left(\beta - \frac{V+v}{\Omega r}\right)}_{\alpha_{zll}} \\ d &\approx 1/2\rho c_d \left(\Omega r\right)^2 c \end{aligned}$$

- where a is the linear lift curve slope and β is the angle to the zero-lift line, and then,

$$\begin{aligned} dT &\approx N\ell dr \\ dQ &\approx N(\ell\phi + d) r dr \end{aligned}$$

Momentum-Blade Element Model V

- next non-dimensionalize all quantities using helicopter nomenclature to give,

$$
\begin{aligned}
dC_T &= \frac{\sigma a}{2}\left(\beta x^2 - \lambda x\right) dx \\
dC_Q &= \left[\frac{\sigma a}{2}\left(\beta x \lambda - \lambda^2\right) + \frac{\sigma c_d}{2} x^2\right] x dx
\end{aligned}
$$

- where $C_T = \frac{T}{\rho A \Omega^2 R^2}$, $C_Q = \frac{Q}{\rho A \Omega^2 R^3}$, $\lambda = \frac{V+v}{\Omega R}$ is the inflow ratio, and $\sigma = \frac{Nc}{\pi R}$ is the solidity (function of r generally), and $x = r/R$
- Note C_T and C_Q are a bit different in helicopter than for propellers
- Usually the inflow ratio is broken into $\lambda = \lambda_i + \lambda_c$ where $\lambda_i = v/\Omega R$ and $\lambda_c = V/\Omega R$,
- we can now assume actuator disc theory holds for a differential annulus and find the differential thrust for this annulus as,

$$dT = 2\rho(V + v)v dA$$

Momentum-Blade Element Model VI

- with $dA = 2\pi r dr$ and then making everything non-dimensional,

$$dC_T = 4\lambda\lambda_i x dx$$

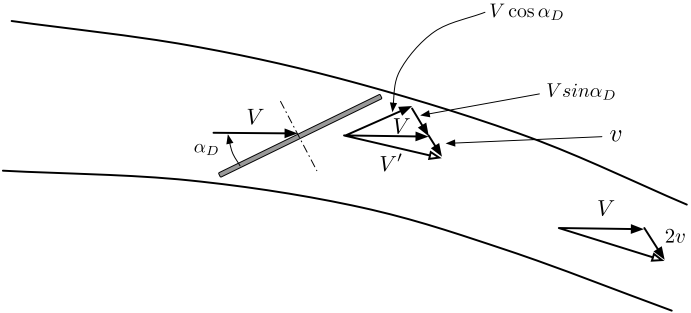
- equating the two expressions for dC_T we can get,

$$\lambda^2 + \left(\frac{\sigma a}{8} - \lambda_c\right)\lambda - \frac{\sigma a}{8}\beta x = 0$$

- If blade properties are know as function of r this can be solved at each r
- Results inserted into dC_T and numerically integrated over the blade(s) to produce total thrust coefficient
- Similarly C_Q (or C_P) found using numerical integration
- Note that linear lift was assumed, so blade angle of attack must be small over entire blade or stall will occur and drag will have large error

Momentum Theory: Forward Flight I

For level horizontal flight Glauert worked out an expression for the mean induced velocity by treating a rotor disc as an elliptically loaded circular wing as shown below:



$$v = \frac{T}{2\rho A \sqrt{V^2 cos^2 \alpha_D + (v + V sin\alpha_D)^2}} \tag{26}$$

Momentum Theory: Forward Flight II

- From momentum theory when pressure has reached ambient and the induced velocity is $2v$,

$$T = \dot{m} 2v$$

- mass flow is conserved so at the disc $\dot{m} = \rho A V^{'}$ and the total velocity at the disc is,

$$V^{'} = \sqrt{V^2 \cos^2 \alpha_D + (v + V \sin \alpha_D)^2}$$

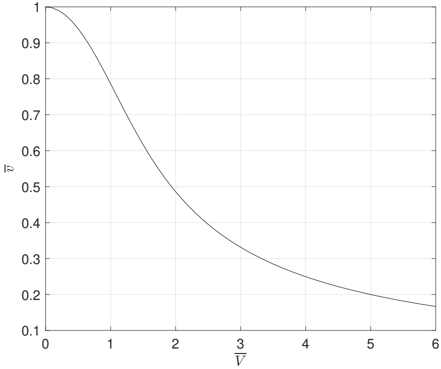
- Thus the Thrust is,

$$T = 2\rho A v \sqrt{V^2 \cos^2 \alpha_D + (v + V \sin \alpha_D)^2}$$

Momentum Theory: Forward Flight III

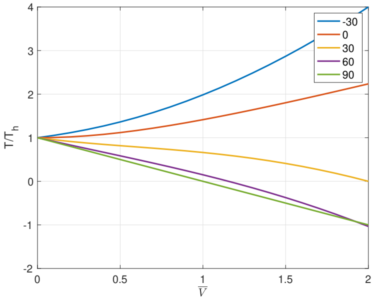
- Equation 26 simplifies to actuator disc theory if $\alpha_D = 90$
- No formal proof has ever been given, really empirical relation
- Define $\bar{v} = v/v_h$, $\bar{V} = V/v_h$ and assume $\alpha_D = 0$ (edgewise flow)
- Equation 26 simplifies to,

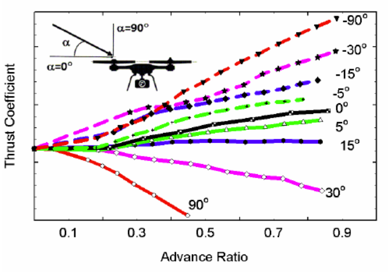
$$\bar{v}^4 + \bar{V}^2 \bar{v}^2 - 1 = 0$$



Momentum Theory: Forward Flight IV

- If we hold P/P_h constant then we can get the following plot



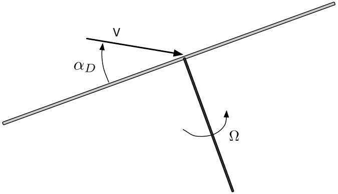


- v_h is large for multi-rotors so $\bar{V}$ will be relatively small
- Resembles the measured results from wind-tunnel tests you saw last week

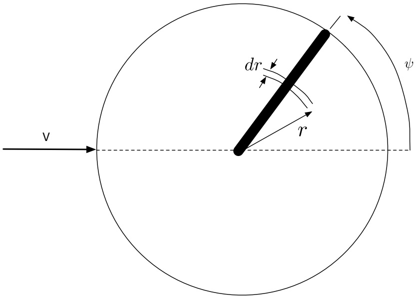
Momentum Theory: Forward Flight V

- While giving the correct trend the momentum method for forward flight is less accurate than actuator disc theory is for axial flight
  - has all the same limitations as actuator disc theory
  - wing generates upwash at leading edge, downwash at trailing edge – induced velocity constant over disc is much poorer assumption!
  - theory cannot predict roll moments that also occur
  - need to go to blade element theory or other numerical technique to improve estimates

General Blade Element Model I

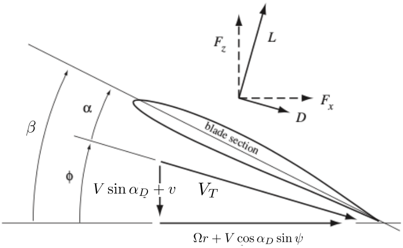


- for multi-rotors that fly with angles of attack between the rotor plane and air velocity $\neq 90^\circ$



General Blade Element Model II

- Now take a section of a blade at radius, r,



- When blade is at ψ=90° horizontal velocity is larger than Ωr and when ψ=270° horizontal velocity smaller than Ωr
- In addition, the disc acts like circular wing, thus there is upflow at front of disc and downflow at rear
- Unlike axial flow, cannot use momentum theory for a differential annulus to get inflow velocity
- The Pitt-Peters inflow model can be applied in this situation, it will be introduced during multi-rotor performance lecture

Brushless DC Motors I

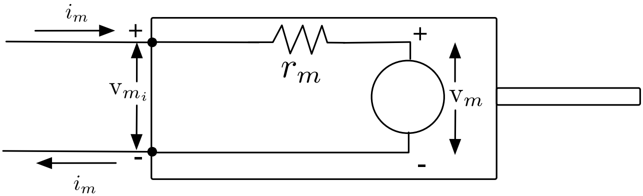
- Brushless DC motors are very common for small fixed-wing and multi-rotor UAVs
- Compared to IC engines they are quieter, easier to start, more reliable, cleaner
- For long range/duration however power density of batteries still limiting factor compared to IC
- Relative to other electric motors, DC brushless are very efficient, light and low maintenance (no brushes)
- Require an ESC for commutation and to change the voltage (using PWM)
- There are outrunners and inrunners:
    - Outrunners have permanent magnets that spin with case, stator winding fixed
    - Inrunners have permanent magnets on outside that are fixed, stator winding rotate
    - Outrunners have larger diameter cases
    - Outrunners develop more torque
    - Inrunners have higher RPM per volt
    - Inrunners dissipate heat better

Brushless DC Motors II

- Multi-rotors typically use outrunners
- Fixed-wing aircraft mostly use outrunners, but inrunners used in ducted fans and when low-drag needed

Simple Model of Brushless DC Motor I

- A very simple model for a BLDC motor is given as (see Drela MIT):



- This model can be approximated with following set of equations:

$$\Omega = K_{\mathrm{v}}(\mathrm{v}_{m_i} - i_m r_m) \quad (27)$$

$$Q = K_t(i_m - i_0) \quad (28)$$

$$Q\Omega = K_t K_{\mathrm{v}}(\mathrm{v}_{m_i} - i_m r_m)(i - i_0) \quad (29)$$

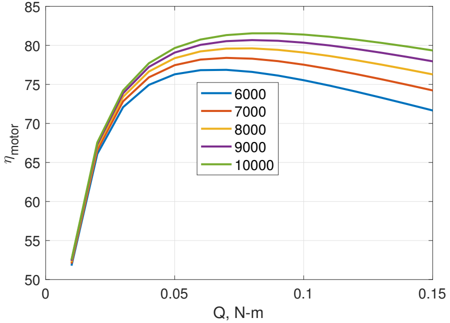
- K_{v} characterizes how back EMF of motor, $\mathrm{v}_{m_i} -_m ir$, varies with RPM
- Back EMF is voltage generated across motor's terminals that opposes drive voltage as windings move through magnetic field

Simple Model of Brushless DC Motor II

- K_t characterizes the torque generated by the current (minus no load current)
- r_m is the internal resistance of armature, i_0 is no-load current at specified motor voltage
- Equation 29 equates shaft power to the electric power minus electric losses
- Requires (for consistent units) $K_\mathrm{v} K_t = 1$ (Note: can be less than 1 to account for simple model)
- Convert K_v to engineering units $\frac{rad/s}{V}$ to use equations, K_t is then $\frac{Nm}{A}$
- Total input power $i_m \mathrm{v}_{m_i}$ so efficiency is $\frac{Q\Omega}{i_m \mathrm{v}_{m_i}}$

Simple Model of Brushless DC Motor III

- η_{motor} plotted against torque and RPM for varying voltage and current for Turnigy SK3 Aerodrive 2830-1020 below:



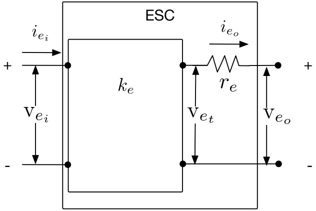
- Turnigy SK3 Aerodrive 2830-1020 outrunner brushless motor parameters:

$$K_{\mathrm{v}} = 1020\text{RPM/V}, \quad i_0 = 0.92\text{A at } 10\text{V}, \quad r = 0.110\text{ohm}$$

- Want to have motor operating near peak efficiency during majority of flight

ESC for BLDC Motors I

- The ESC performs two tasks, it commutes the voltage
  - turns DC voltage into rotating magnetic field by switching power to different windings
  - Allows different voltages/powers using PWM or equivalent
  - Modern ESCs are very efficient (>95%), simplest model is simply a resistor in series



  - The throttle adjustment is reflected in k_e (0-1)
  - $v_{e_o} = k_e v_{e_i}$ and $i_{e_o} = i_{e_i}/k_e$
  - Losses captured with resistance, r_e

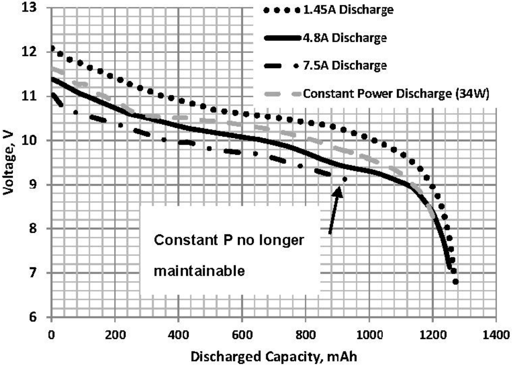
LiPo Batteries I

- LiPo batteries ubiquitous for powering BLDC motors for RC aircraft/multi-rotors
- Batteries characterized by 3 main numbers:
  - Number of cells, each cell varies in voltage depending on charge from 4.2V to 3.4V, 3.7V considered nominal
  - mAh – number of milli-amp hours that can be extracted from battery before voltage drops drops off a cliff
  - C value – indicated maximum discharge current that can safely be discharged: 30C mean 30$\times$ Ah rating
- **0th order model:** voltage constant at nominal value until energy (E_b) dissipated
- Example 3 cell, 1300mAh, 30 C battery

$$\mathrm{v}_{b_{nom}} = 3\text{cells} \times 3.7V = 11.1 \text{ Volts}$$
$$E_b = 11.1 \text{ Volts} \times 1.3\text{Ah} \times 60 \times 60 = 51948 \text{ Joules}$$
$$i_{b_{max}} = 30\text{C} \times 1.3\text{Ah} = 39\text{Amps}$$

LiPo Batteries II

- Note if battery current draw known, more accurate battery model integrates current and output drops to zero when reaches mAhr rating
- **1st order model:** LiPo batteries characterized by discharge curves†
- Discharge curve looks like:



- 1300mAH
- 3 cell
- 30 C

LiPo Batteries III

- Collapsed to single curve by plotting $\mathrm{v}_b i_b^{0.05}$ vs Discharge
- Discharge capacity found by integrating current
- Fit to collapsed curve given as:

$$\mathrm{v}_b i_b^{0.05}(D) = \frac{aD^2 + bD + c}{1 + dD + eD^2 + fD^3}$$

- Where D is the battery Discharge (in mAh)

$$D = D_0 + \int_0^T \frac{i_b}{1000} dt$$

- For numerical simulation

$$D_i = D_{i-1} + \frac{i_{b_{i-1}}}{1000} \Delta t$$

- Can just use a given representative discharge D_0 if interested in average performance

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† See *Calculation of Constant Power Lithium Battery Discharge Curves*, by L. W. Traub

Battery Models Summary

- **0th order model:** (useful if propulsion system details unknown)

$$\begin{aligned} \mathrm{v}_b &= \mathrm{v}_{b_{nom}} \text{ if } E < E_b \\ \mathrm{v}_b &= 0 \text{ otherwise} \end{aligned}$$

  where:

$$E = \int_0^T \mathrm{v}_b i_b dt$$

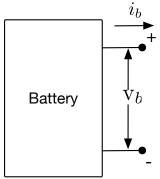
- **1st order model:**

$$\mathrm{v}_b i_b^{0.05}(D) = \frac{aD^2 + bD + c}{1 + dD + eD^2 + fD^3}$$

  Where D is the battery Discharge (in mAh)

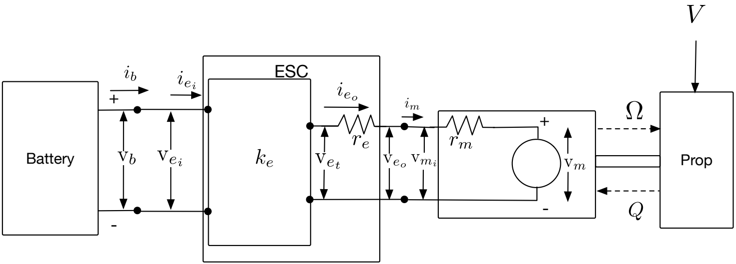
$$D = D_0 + \int_0^T \frac{i_b}{1000} dt$$

- v_b and i_b need to be solved for simultaneously



Electric Propulsion

- Putting it all together:

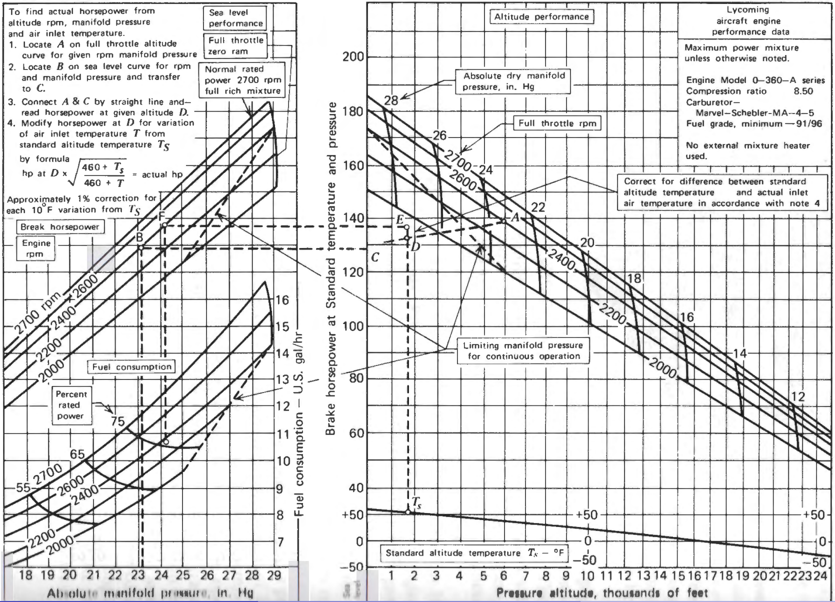


- so $i_m = i_{e_o} = i_{e_i}/k_e = i_b/k_e$
- and $\mathrm{v}_{m_i} = \mathrm{v}_{e_t} - i_{e_o} r_e = \mathrm{v}_{e_i} k_e - i_{e_o} r_e = \mathrm{v}_b k_e - i_m r_e$
- If use propeller charts can use linear interpolation on lookup tables that relate C_T, C_Q, C_P, and η to J
- Need to find a solution that is consistent with all equations in system
  - Q from motor model and prop model agree
  - If using 1st order battery model $\mathrm{v}_b i_b^{0.05} = f(D)$

Piston Engines

- Internal combustion, piston engines are also commonly used in UAVs
- As we are concentrating on smaller UAVs will only touch on Piston Engines
- Behaviour very complicated and depends on a large number of factors
- Larger four-stroke engines steady state behaviour captured in charts that relate power output/fuel consumption to rpm, manifold pressure, altitude and temperature
- Manifold pressure partly controlled by throttle position

Piston Engines



Piston Engines

- For simplified analysis can consider engine as producing constant power at a given altitude (given density)
  - propeller needs to be well matched with motor and flight condition
- Fuel consumption proportional to power