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Assg 1 AER121B

(1.)
- Symmetrical Aerofoil
- $\alpha = 3^\circ = 0.05233$ Rad
- $V_\infty = 40$ m/sec
- $c = 1$m

$\rho_\infty = 1.225$ kg/m³
$\mu = 1.789 \times 10^{-5}$ $\frac{kg\,m}{s}$

To find $\rightarrow$ Lift, $C_m\big|_{x=0}$, Re

According to Thin Airfoil theory,

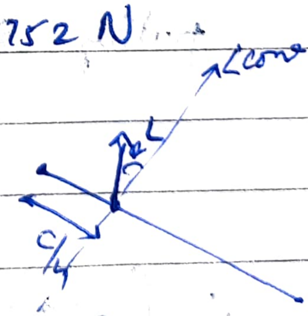
For Uncambered Airfoils,

$$L' = \rho_\infty V_\infty \Gamma = \rho_\infty V_\infty^2 \pi \alpha c$$

$$= 1.225 \times (40)^2 \times 3.14 \times 0.05233 \times 1$$

$$\Rightarrow 322.059752 \text{ N}$$

$$M\big|_{x=0} = (L\cos\theta) \times \frac{c}{4}$$



$$\Rightarrow L \times \frac{1}{4}$$

$$M_{LE} = \frac{-L}{4}$$

$$\Rightarrow 80.515 \text{ Nm}$$

Also Can be derived from

$$M_{LE} = -\rho_\infty V_\infty \int_0^c x\gamma(x)\,dx$$

(2.) • Camber line:-

$$\left(x-\frac{c}{2}\right)^2+\left(z+\frac{c}{8K}-\frac{Kc}{2}\right)^2=\left(\frac{c}{8K}+\frac{Kc}{2}\right)^2$$

• V_∞, α known

• $K<<1$

• $\gamma = f(V_\infty, \alpha, \theta, K)$

→ Basic eqn → $\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\,d\xi}{x-\xi} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$

putting $\xi \Rightarrow \frac{c}{2}(1-\cos\theta)$

$x \Rightarrow \frac{c}{2}(1-\cos\theta_0)$

$$\frac{1}{2\pi}\int \frac{\gamma(\theta)\sin\theta\,d\theta}{\cos\theta-\cos\theta_0} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$$

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Assuming $\gamma(\theta) = 2V_\infty \left[A_0 \left(\frac{1+\sin\theta}{\cos\theta} \right) + \sum_{n=1}^{\infty} A_n \sin(n\theta) \right]$

Subst in $\mathcal{E}q^n$

$$\frac{A_0}{\pi} \int_0^{\pi} \frac{1+\cos\theta}{\cos\theta - \cos\theta_0} \, d\theta + \sum_{n=1}^{\infty} \frac{A_n}{\pi} \int_0^{\pi} \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_0} \, d\theta$$

$$= \alpha - \frac{dz}{dx}$$

Solving

$$A_0 - \sum_{n=1}^{\infty} A_n \cos n\theta_0 = \alpha - \frac{dz}{dx}$$

$$\underbrace{(\alpha - A_0)}_{B_0} + \underbrace{\sum_{n=1}^{\infty} A_n \cos n\theta_0}_{B_1 \to B_n} = \underbrace{\frac{dz}{dx}}_{\text{fourier } \mathcal{E}q^n}$$

$$A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} \, d\theta_0 \; ; \; A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n\theta_0 \, d\theta$$

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Circular Arc Eq^n

$$\underbrace{\left(x-\frac{c}{2}\right)}_{x'}^2+\left(\underbrace{z+\frac{c}{8k}-\frac{kc}{2}}_{A}\right)^2=\left(\underbrace{\frac{c}{8k}+\frac{kc}{2}}_{B}\right)^2$$

$$(x')^2+(z+A)^2=B^2$$

$$z=-A+\sqrt{B^2-(x')^2} \qquad \left[-ve \text{ ignored}\right]$$

$$\frac{dz}{dx}=\frac{1}{2\sqrt{B^2-(x')^2}}\times(-2x')\times\frac{dx'}{dx}$$

$$\Rightarrow \frac{-\cancel{2}x'}{\cancel{2}\sqrt{B^2-(x')^2}}$$

$$x'=x-\frac{c}{2}$$

$$x'=\frac{c}{2}(1-\cos\theta_0)-\frac{c}{2}$$

$$\Rightarrow \frac{c}{2}(\cancel{1}-\cos\theta_0-\cancel{1})$$

$$x'\rightarrow -\frac{c}{2}\cos\theta_0$$

$$\frac{dz}{dx} \Rightarrow \frac{\frac{c}{2}\cos\theta_0}{\sqrt{B^2 - \frac{c^2}{4}\cos^2\theta_0}}$$

$$\Rightarrow \frac{\frac{c}{2}\cos\theta_0}{\sqrt{\left(\frac{c}{8K}\right)^2 + \left(\left(\frac{c}{2}\right)^2 - \cancel{\frac{c^2}{4}\cos^2\theta_0} + \frac{c^2}{8}\right)}} \rightarrow 0$$

$$\Rightarrow \frac{\cancel{c}\cos\theta_0}{2\left(\frac{\cancel{c}}{8K}\right)}$$

$$\frac{dz}{dx} \Rightarrow 4K\cos\theta_0$$

$$\therefore A_0 \Rightarrow \alpha - \frac{1}{\pi}\int_0^{\pi} 4K\cos\theta_0\, d\theta_0 \Rightarrow \alpha$$

$$A_1 \Rightarrow \frac{2}{\pi}\int_0^{\pi} 4K\cos\theta_0 \times \cos\theta_0\, d\theta_0 \Rightarrow 4K$$

$A_2, A_3 \ldots A_n \longrightarrow 0$; $K << 1 \longrightarrow$ Camber small.

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$$\gamma(\theta) \Rightarrow \left[2V_\infty \left(\alpha \left(\frac{1+\sin\theta}{\cos\theta} \right) + 4k\sin\theta \right] \right.$$

$$C_m\Big|_{c/4} \Rightarrow -\frac{\pi}{4} A_1 \Rightarrow -\pi k \sin\theta$$

$$\alpha_{L=0} \Rightarrow -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1)\, d\theta_0$$

$$\Rightarrow -\frac{1}{\cancel{\pi}} \times \left(\cancel{\pi}/2\right) \times \left(\frac{2}{\cancel{\pi}}k\right)$$

$$\Rightarrow \underline{\underline{-2k}}$$

(3.) • Weight → 60,000 N

• Span → 12m

• Speed → 70 m/s

∴ ~~Since~~ Since Elliptical loading,

$\alpha_i \longrightarrow$ Const for y, $\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{y}{b}\right)^2}$

$$\therefore \; D_i \Rightarrow \frac{L^2}{q_\infty \pi b^2}$$

$$\Rightarrow \frac{(60,000)^2}{(1.225 \times 70) \times \pi \times (12 \times 12)}$$

$$\Rightarrow \underline{92,848.786 \text{ N}}$$

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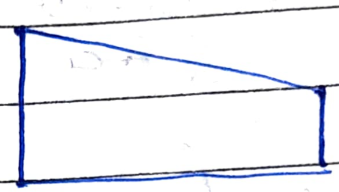
(4.)

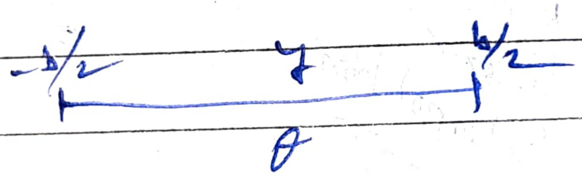
$AR = 8$

$C_{tip} = 0.4\, C_r$



$\alpha_{L=0} = -1.2$

$$\alpha_{eff} = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0)$$



$$\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{N} A_n \sin(n\theta)$$

$$\alpha(\theta_0) = \underbrace{\frac{2b}{\pi c(\theta_0)} \sum_{1}^{N} A_n \sin(n\theta_0) + \overset{-1.2}{\alpha_{L=0}(\theta_0)}}_{\alpha_{eff}} + \underbrace{\sum_{1}^{N} n A_n \frac{\sin(n\theta_0)}{\sin\theta_0}}_{\alpha_i}$$

$\cancel{\alpha = \frac{2b}{\pi c(\theta_0)}}$ (Taking for $N = 8$)

$$
(1)\quad \begin{cases} A_1\left(\dfrac{2b\sin\theta}{\pi c}+1\right)+A_3\left(\dfrac{2b\sin 3\theta}{\pi c}+\dfrac{3\sin 3\theta}{\sin\theta}\right) \\ +A_5\left(\dfrac{2b\sin 5\theta}{\pi c}+\dfrac{5\sin 5\theta}{\sin\theta}\right)+A_7\left(\dfrac{2b\sin 7\theta}{\pi c}+\dfrac{7\sin 7\theta}{\sin\theta}\right) \end{cases}
$$

Variable $c(\theta_0) \Rightarrow c_r\left(1-\frac{3}{4}\times\frac{(\pi/2-\theta)}{\pi/2}\right)$

Direct Linear Interpolation

$$
\left.\begin{aligned} &= c_r\left(1-\frac{3}{4}\times\frac{\pi-2\theta}{\pi}\right)\end{aligned}\right\}\ (2)
$$

After Inputting this formulae of c (2) into (1) & substituting, $\pi/8$, $\pi/4$, $3\pi/8$, $\pi/2$, we get the matrix (Done in Matlab)

$$
\begin{bmatrix} 3.118 & 14.771 & 19.599 & 10.118 \\ 4.033 & 7.033 & -9.033 & -11.033 \\ 4.053 & -2.921 & -3.750 & 11.053 \\ 3.565 & -6.565 & 8.565 & -10.565 \end{bmatrix} \begin{bmatrix} A_1 \\ A_3 \\ A_5 \\ A_7 \end{bmatrix} = \begin{bmatrix} \alpha+1.2 \\ \alpha+1.2 \\ \alpha+1.2 \\ \alpha+1.2 \end{bmatrix}
$$

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$$[A]\begin{bmatrix} A_1/\alpha+1.2^\circ \\ A_2/\alpha+1.2^\circ \\ A_3/\alpha+1.2^\circ \\ A_n/\alpha+1.2^\circ \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{B}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = (\alpha+1.2^\circ)\begin{bmatrix} [A^{-1}]\,[B] \end{bmatrix}$$

$$\Rightarrow (\alpha+1.2^\circ)\begin{bmatrix} 2.599\times10^{-1} \\ 2.2275\times10^{-2} \\ 8.6681\times10^{-3} \\ -1.3048\times10^{-3} \end{bmatrix}$$

$$\left(\alpha + 1.2\times\frac{\pi}{180}\right)$$

(At all places)

$$L = \int_{-b/2}^{b/2} L(y)\,dy = \rho_\infty U_\infty \int_{-b/2}^{b/2} \Gamma(y)\,dy$$

$$\Rightarrow \rho_\infty U_\infty \int_0^\pi \Gamma(\theta)\,d\theta$$

$$\Rightarrow \frac{2b}{\pi e}\times \rho_\infty U_\infty \int_0^\pi \left(A_1\sin\theta + A_3\sin3\theta + A_5\sin5\theta + A_7\sin7\theta\right)\times(\alpha+1.2^\circ)\,d\theta$$

$$C_L = A_1 \pi AR$$

$$\Rightarrow (0.2599)\times(3.14)\times(8)\times(\alpha + 1.2)\times\frac{\pi}{180}$$

$$\Rightarrow \underline{0.4783} \qquad (\text{for } \alpha = 3^\circ)$$

$$C_{Di} \Rightarrow \frac{C_L^2}{\pi AR e} \; ; \; e = \frac{1}{1+\delta} \; ; \; \delta = \sum_{2}^{N}\left(\frac{A_1}{A_n}\right)^2$$

$$\sum_{2}^{8}\left(\frac{A_1}{A_n}\right)^2 \Rightarrow 12.097\times10^{-4} \lll 1$$

$$\therefore e \simeq 1$$

$$C_{Di} \Rightarrow \frac{(0.4783)^2}{(3.14)\times 8\times 1} \Rightarrow \underline{0.0092}$$