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Final.

(1) $f_B = m(\dot{v}_B + \omega_B \times v_B)$

$$\left.\begin{aligned} X &= m(\dot{u} + qw - rv) \\ Y &= m(\dot{v} + ru - pw) \\ Z &= m(\dot{w} + pv - qu) \end{aligned}\right\} \rightarrow \begin{aligned} \dot{u} &= X/m - qw + rv \\ \dot{v} &= Y/m - ru + pw \\ \dot{w} &= Z/m - pv + qu \end{aligned}$$

$L \rightarrow$
$M \rightarrow$ $\cancel{\tau_B}$
$N \rightarrow$

$$\tau_B = \omega_B \times I_B \omega_B + I_B \dot{\omega}_B$$

$$\tau_B - \omega_B \times (I_B \omega_B) = I_B \dot{\omega}_B$$

$$\dot{\omega}_B \Rightarrow I_B^{-1} \left(\tau_B - \omega_B \times (I_B \omega_B) \right)$$

$I_{xy} = I_{yz} = 0$

$$\dot{\omega}_B \Rightarrow \begin{bmatrix} I_{xx} & \cancel{I_{xy}}^{0} & I_{xz} \\ \cancel{I_{yx}}^{0} & I_{yy} & \cancel{I_{yz}}^{0} \\ I_{zx} & \cancel{I_{zy}}^{0} & I_{zz} \end{bmatrix}^{-1} \left(\tau_B - \omega_B \times (I_B \omega_B) \right)$$

$$\underbrace{\dot{\omega}_B \Rightarrow \left(\begin{bmatrix} I_{yy} I_{zz} & 0 & -I_{yy} I_{zx} \\ 0 & I_{xx} I_{zz} - I_{zx} I_{xz} & 0 \\ -I_{yy} I_{xz} & 0 & I_{xx} I_{yy} \end{bmatrix} \right) \begin{bmatrix} L - pq I_{xz} + (I_{yy} - I_{zz}) qr \\ M - \cancel{I_{yx}(qr)}^{0} + I_{zx}(r^2 - p^2) + (I_{zz} - I_{xx}) pr \\ N + I_{zx} qr - \cancel{I_{zy} pr}^{0} + (I_{xx} - I_{yy}) pq \end{bmatrix}}$$

$\dot{\omega}_B$

All terms as mentioned in ques

(2.) Important Quantities :- (θ, ϕ, ψ), $\underset{\omega_B}{(p, q, r)}$, $\underset{V_B}{(u, v, w)}$.

$$C_{BE} \Rightarrow \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\theta\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

ϕ, θ, ψ calculates iteratively in next page

$$V_E \Rightarrow C_{EB} V_B \rightarrow \text{known}$$

$$\therefore \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \int_0^t [C_{EB}]_{3\times3} \begin{bmatrix} u \\ v \\ w \end{bmatrix}_{3\times1} dt$$

θ_0, ϕ_0, ψ_0 assumed & Runge-Kutta or any other iterative process used for guessing. This values of θ, ϕ, ψ then sent to C_{EB} & finally X, Y, Z Calculated

$$S_B \Rightarrow \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

Iterative Process

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \int_0^t [S_B]^{-1}_{3\times3} \begin{bmatrix} p \\ q \\ r \end{bmatrix} dt$$

All the Computation Can be Perform in matlab and hence have not been Elaborated here.

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3) $X - mg\sin\theta = m(\dot{u} + QW - RV)$

$(X_e + \Delta X) - mg(\sin\theta_e + \Delta\theta\cos\theta_e) = m\Delta\dot{u}$

$X_e - mg\sin\theta_e = 0$

$\Delta X - mg\cos\theta_e\Delta\theta = m\Delta\dot{u}$

$$\Delta X = \begin{bmatrix} X_u & X_w & X_q & X_{\dot{u}} - X_{\dot{w}} & X_{\dot{q}} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} + \delta_T$$

~~5) $X_e + \Delta X$~~

6) Linearized System of Eqn:-

$X_e + \Delta X - mg(\sin\theta_e + \Delta\theta\cos\theta_e) = m\Delta\dot{u}$

Similarly others derived

$Y_e + \Delta Y + mg(\Delta\phi\cos\theta_e) = m(\Delta\dot{v} + U_e r)$

$Z_e + \Delta Z + mg(\cos\theta_e - \Delta\theta\sin\theta_e) = m(\Delta\dot{w} - U_e q)$

$L_e + \Delta L = I_{xx}\dot{p} - I_{zx}\dot{r}$

$M_e + \Delta M = I_{yy}\dot{q}$

$N_e + \Delta N = -I_{zx}\dot{p} + I_{zz}\dot{r}$

$\Delta\dot{\theta} = q$

$\Delta\dot{\phi} = p + r\tan\theta_e$ $\quad\quad \Delta\dot{\psi} = r\sec\theta_e$

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(4) Longitudinal

→ State Variable → u, w, q, θ

→ Control Variable → δ_e, δ_p

Lateral

→ State Variable → v, p, r, ψ, ϕ

→ Control Variables → δ_a, δ_r

from Q3:-

$\dot{u} \Rightarrow \frac{1}{m}(\Delta X) - g\cos\theta_e \Delta\theta$

$\Delta X \rightarrow X_u u + X_w w + X_q q$;

∴ Similarly Expanding for other Eqⁿs, we get the matrix given in the notes;

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} X_u & X_w & X_q & -g\cos\theta_e \\ Z_u & Z_w & (Z_q + MU_e) & -g\sin\theta_e \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix}$$

$$+ \begin{pmatrix} X_{\delta e} & X_{\delta p} \\ Z_{\delta e} & 0 \\ M_{\delta e} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_e \\ \delta_p \end{pmatrix}$$

Variation with δ_e & δ_p Considered similar to Aerodynamic Derivatives in the Respective Directions

(mass included in the coefficients for better writing)

for Lateral:

$m\dot{v} \Rightarrow -\frac{U_e r}{m} + \left(\frac{\Delta Y}{m}\right) + g(\cos\theta)\phi$

$\Delta Y \Rightarrow Y_v v + Y_p p + Y_r r$

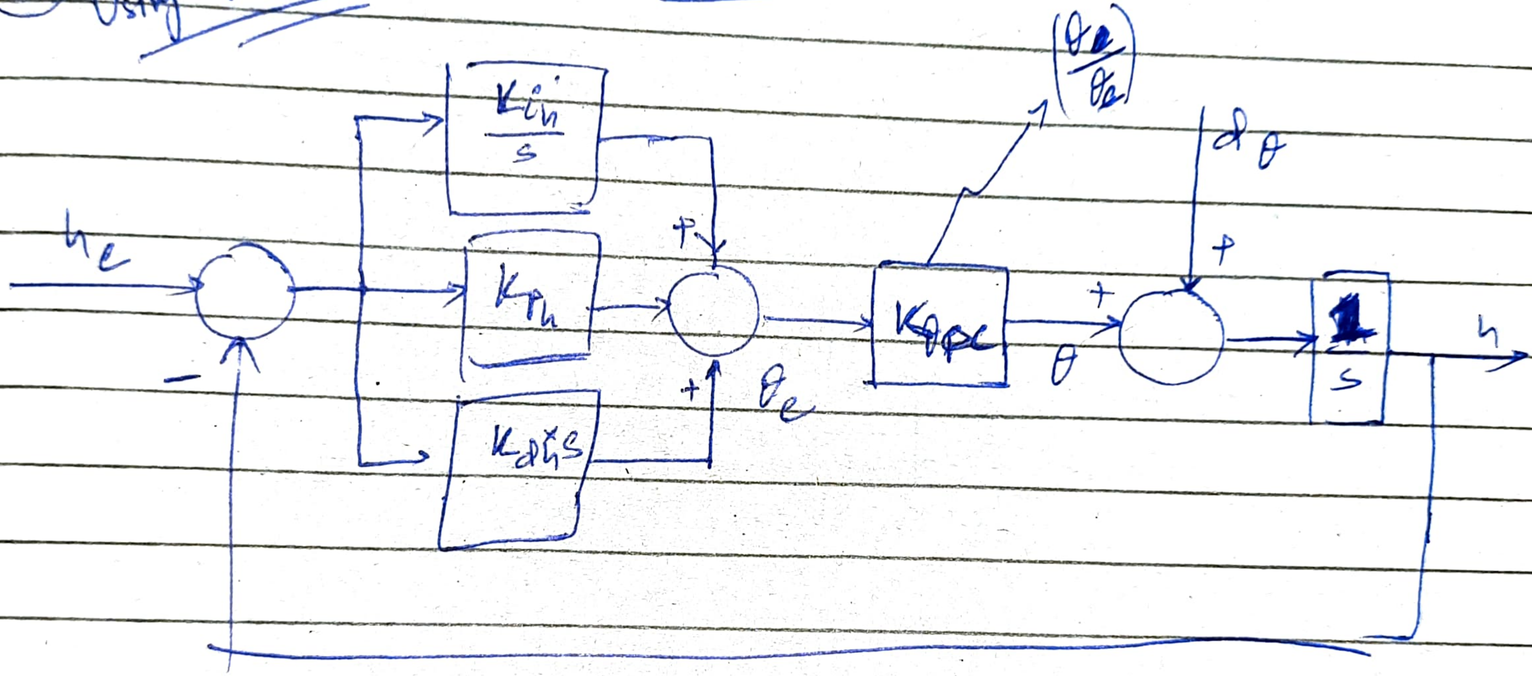
Similarly expanding other eq's

$$\begin{pmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} Y_v & Y_p & Y_r & g\cos\theta_e & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 1 & \tan\theta_e & 0 & 0 \\ 0 & 0 & \sec\theta_e & 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ p \\ r \\ \phi \\ \psi \end{pmatrix}$$

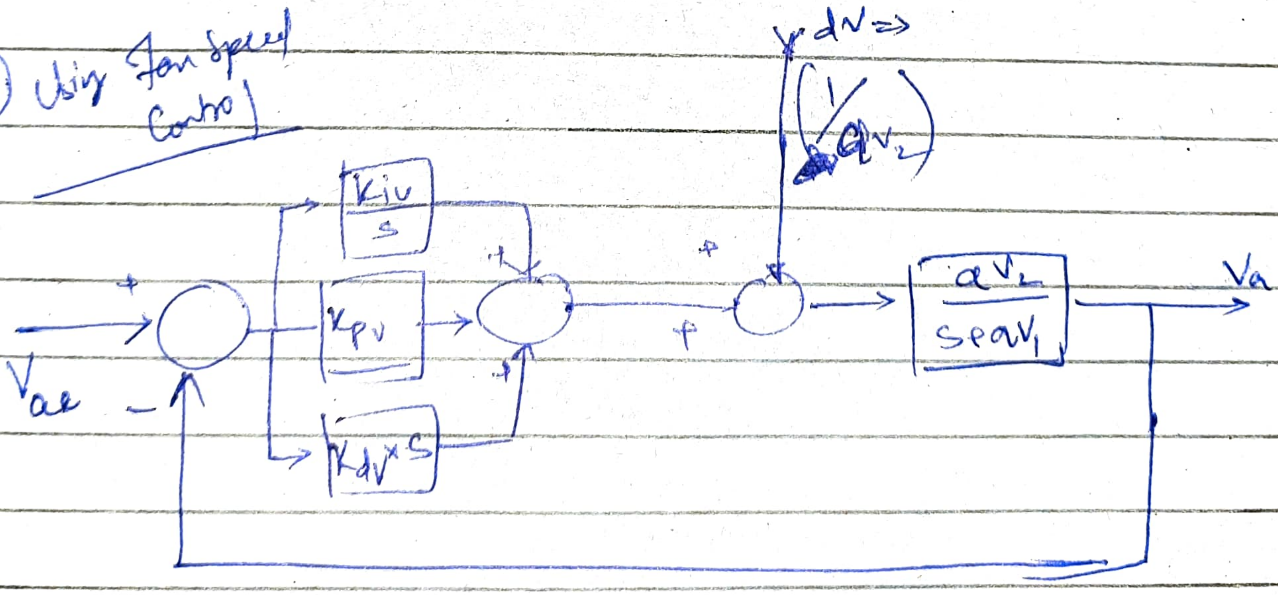
$$+ \begin{pmatrix} Y_{\delta a} & Y_{\delta r} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{\delta}a \\ \delta r \end{pmatrix}$$

Altitude Control → 2 Ways.

5. Using Pitch Control

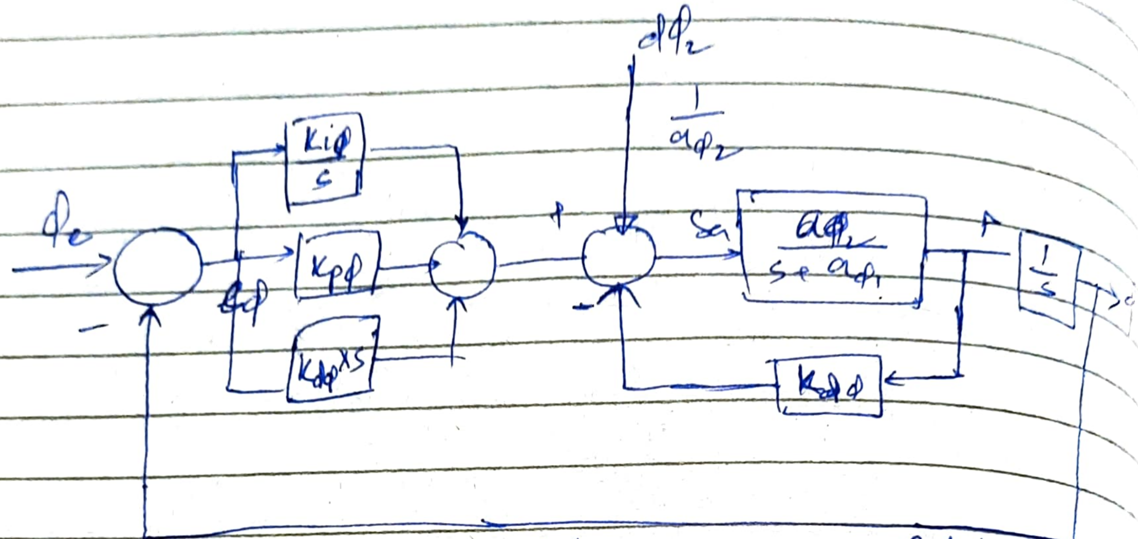


6. Using Fan Speed Control

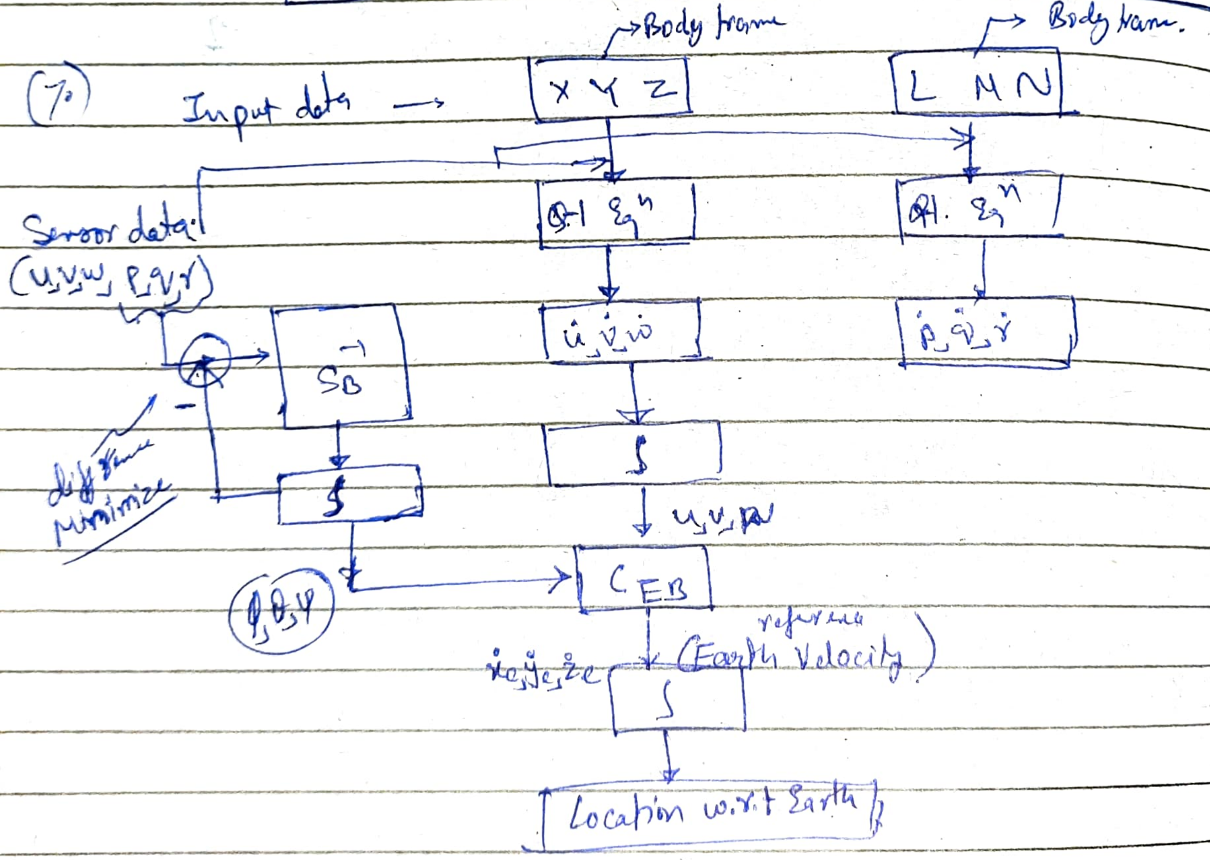


(6) Level Coordinated Turn

- Same level (height Control)
- Roll Required (Roll Control)



(7)



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(8) Non-linear Model

→ Accurate Modelling without any assumptions
→ Good for simulating simple scenarios.

Linearized Model

→ Can be Scaled & Easily used for Complex Modelling
→ Can be Easily used for Practical Scenarios with Complex force functions.

(9) Currently apart from linearization, few other assumptions are also made, like Symmetric airplane, etc. which need to be taken into considerations. Furthermore, the plane can be operated in a wide variety of operations & analysis needs to consider the mission requirements before using the equations & models.