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Assg 1 AER 1216

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(1.)
- Symmetrical Aerofoil
- $\alpha = 3^\circ = 0.05233$ Rad
- $V_\infty = 40$ m/sec
- $c = 1$ m

$\rho_\infty = 1.225$ kg/m³

$\mu = 1.789 \times 10^{-5}$ $\frac{kg\,m}{s}$

To find → Lift, $C_m\big|_{x=0}$, Re

According to Thin Airfoil theory,

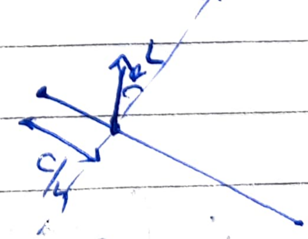
For Uncambered Airfoils,

$$L' = \rho_\infty V_\infty \Gamma = \rho_\infty V_\infty^2 \pi \alpha c$$

$$= 1.225 \times (40)^2 \times 3.14 \times 0.05233 \times 1$$

$$\Rightarrow 322.059752 \text{ N}$$

$$M\big|_{x=0} = (L\cos\theta) \times \frac{c}{4}$$



$$M_{LE} = \frac{-L}{4}$$

$$\Rightarrow L \times \frac{1}{4}$$

$$\Rightarrow 80.515 \text{ Nm}$$

Also can be derived from

$$M_{LE} = -\rho_\infty V_\infty \int_0^c x\gamma(x)\,dx$$

(2.) • Camber line :-

$$\left(x-\frac{c}{2}\right)^2+\left(z+\frac{c}{8k}-\frac{kc}{2}\right)^2 = \left(\frac{c}{8k}+\frac{kc}{2}\right)^2$$

• V_∞, α known

• $k << 1$

• $\gamma = f(V_\infty, \alpha, \theta, k)$

→ Basic eqn →
$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)\,d\xi}{x-\xi} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$$

putting $\xi \Rightarrow \frac{c}{2}(1-\cos\theta)$

$x \Rightarrow \frac{c}{2}(1-\cos\theta_0)$

$$\frac{1}{2\pi}\int \frac{\gamma(\theta)\sin\theta\,d\theta}{\cos\theta-\cos\theta_0} = V_\infty\left(\alpha-\frac{dz}{dx}\right)$$

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Assuming $\gamma(\theta) = 2V_\infty \left[A_0 \left(\frac{1+\sin\theta}{\cos\theta} \right) + \sum_{n=1}^{\infty} A_n \sin(n\theta) \right]$

Subst in ξ_1^n

$$\frac{A_0}{\pi} \int_0^{\pi} \frac{1+\cos\theta}{\cos\theta - \cos\theta_0} d\theta + \sum_{n=1}^{\infty} \frac{A_n}{\pi} \int_0^{\pi} \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_0} d\theta$$

$$= \alpha - \frac{dz}{dx}$$

Solving

$$A_0 - \sum_{n=1}^{\infty} A_n \cos n\theta_0 = \alpha - \frac{dz}{dx}$$

$$\underbrace{(\alpha - A_0)}_{B_0} + \underbrace{\sum_{n=1}^{\infty} A_n \cos n\theta_0}_{B_1 \to B_n} = \underbrace{\frac{dz}{dx}}_{\text{fourier } \xi_{\text{op}}^n}$$

$$A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} d\theta_0 \; ; \; A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n\theta_0 \, d\theta$$

Circular Arc Eq^n

$$\underbrace{\left(x-\frac{c}{2}\right)}_{x'}^2 + \left(z+\underbrace{\frac{c}{8K}-\frac{Kc}{2}}_{A}\right)^2 = \left(\underbrace{\frac{c}{8K}+\frac{Kc}{2}}_{B}\right)^2$$

$$(x')^2 + (z+A)^2 = B^2$$

$$z = -A + \sqrt{B^2-(x')^2} \qquad \left[-ve \text{ ignored}\right]$$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{B^2-(x')^2}} \times (-2x') \times \frac{dx'}{dx}$$

$$\Rightarrow \frac{-\cancel{2}x'}{\cancel{2}\sqrt{B^2-(x')^2}}$$

$$x' = x - \frac{c}{2}$$

$$x' = \frac{c}{2}(1-\cos\theta_0) - \frac{c}{2}$$

$$\Rightarrow \frac{c}{2}(\cancel{1} - \cos\theta_0 - \cancel{1})$$

$$x' \Rightarrow -\frac{c}{2}\cos\theta_0$$

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$$\frac{dz}{dx} \Rightarrow \frac{\frac{c}{2}\cos\theta_0}{\sqrt{B^2 - \frac{c^2}{4}\cos^2\theta_0}}$$

$$\Rightarrow \frac{\frac{c}{2}\cos\theta_0}{\sqrt{\left(\frac{c}{8K}\right)^2 + \left(\left(\frac{c}{2}\right)^2 - \cancel{\frac{c^2}{4}\cos^2\theta_0} + \frac{c^2}{8}\right)}} \quad \frac{\pi}{4} \quad \to 0$$

$$\Rightarrow \frac{c\cos\theta_0}{2x\left(\frac{c}{8K}\right)}$$

$$\frac{dz}{dx} \Rightarrow 4K\cos\theta_0$$

$$\therefore A_0 \Rightarrow \alpha - \frac{1}{\pi}\int_0^{\pi} 4K\cos\theta_0 \, d\theta_0 \Rightarrow \alpha$$

$$A_1 \Rightarrow \frac{2}{\pi}\int_0^{\pi} 4K\cos\theta_0 \times \cos\theta_0 \, d\theta_0 \Rightarrow 4K$$

$A_2, A_3 \ldots A_n \longrightarrow 0$; $K \ll 1 \longrightarrow$ Camber Small.

$$\gamma(\theta) \Rightarrow \left[2V_\infty \left(\alpha \left(\frac{1 + \sin\theta}{\cos\theta} \right) + 4k\sin\theta \right] \right.$$

$$C_m\Big|_{c/4} \Rightarrow -\frac{\pi}{4} A_1 \Rightarrow -\pi k \sin\theta$$

$$\alpha_{L=0} \Rightarrow -\frac{1}{\pi} \int_0^\pi \frac{dz}{dx} (\cos\theta - 1)\, d\theta.$$

$$\Rightarrow -\frac{1}{\cancel{\pi}} \times \left(\frac{\cancel{\pi}}{\cancel{2}} \right) \times (\cancel{2}k)$$

$$\Rightarrow \underline{\underline{-2k}}$$

(3.) • Weight → 60,000 N

• Span → 12m

• Speed → 70 m/s

∴ Since Elliptical loading,

$\alpha_i \rightarrow$ Const for y, $\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{y}{6}\right)^2}$

$$\therefore \quad D_i \Rightarrow \frac{L^2}{q_\infty \pi b^2}$$

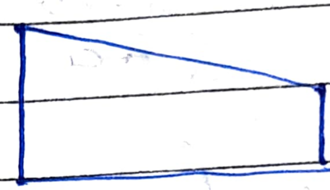
$$\Rightarrow \frac{(60,000)^2}{(1.225 \times 70) \times \pi \times (12 \times 12)}$$

$$\Rightarrow \underline{92,848.786 \text{ N}}$$

(4.)

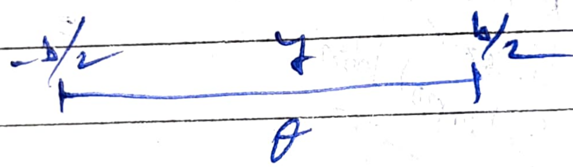
$AR = 8$

$C_{tip} = 0.4\, C_r$



$\alpha_{L=0} = -1.2$

$$\alpha_{eff} = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}(y_0)$$



$$\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{N} A_n \sin(n\theta)$$

$$\alpha(\theta_0) = \underbrace{\frac{2b}{\pi c(\theta_0)} \sum_{1}^{N} A_n \sin(n\theta_0) + \overset{-1.2}{\cancel{\alpha_{L=0}}}(\theta_0)}_{\alpha_{eff}} + \underbrace{\sum_{1}^{N} n A_n \frac{\sin(n\theta_0)}{\sin\theta_0}}_{\alpha_i}$$

$\cancel{\alpha = \frac{2b}{\pi c(\theta_0)}}$

(Taking for $N = 8$)

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$$(1)\left\{\begin{aligned} & A_1\left(\frac{2b\sin\theta}{\pi c}+1\right)+A_3\left(\frac{2b\sin3\theta}{\pi c}+\frac{3\sin3\theta}{\sin\theta}\right) \\ & +A_5\left(\frac{2b\sin5\theta}{\pi c}+\frac{5\sin5\theta}{\sin\theta}\right)+A_7\left(\frac{2b\sin7\theta}{\pi c}+\frac{7\sin7\theta}{\sin\theta}\right)\end{aligned}\right.$$

Variable $c(\theta_0) \Rightarrow C_r\left(1-\frac{3}{4}\times\frac{(\pi/2-\theta)}{\pi/2}\right)$

Direct Linear Interpolation

$$= C_r\left(1-\frac{3}{4}\times\frac{\pi-2\theta}{\pi}\right) \quad (2)$$

After Inputting this formulae of c (2) into (1) & substituting, $\pi/8$, $\pi/4$, $3\pi/8$, $\pi/2$, we get the matrix (Done in Matlab)

$$\begin{bmatrix} 3.188 & 14.771 & 19.599 & 10.118 \\ 4.033 & 7.033 & -9.033 & -11.033 \\ 4.053 & -2.921 & -3.750 & 11.053 \\ 3.585 & -6.585 & 8.585 & -10.585 \end{bmatrix}\begin{bmatrix} A_1 \\ A_3 \\ A_5 \\ A_7 \end{bmatrix} = \begin{bmatrix} \alpha+1.2 \\ \alpha+1.2 \\ \alpha+1.2 \\ \alpha+1.2 \end{bmatrix}$$

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$$[A]\begin{bmatrix} A_1/\alpha+1.2^\circ \\ A_2/\alpha+1.2^\circ \\ A_3/\alpha+1.2^\circ \\ A_n/\alpha+1.2^\circ \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{B}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = (\alpha+1.2^\circ)\begin{bmatrix} [A^{-1}][B] \end{bmatrix}$$

$$\Rightarrow (\alpha+1.2^\circ)\begin{bmatrix} 2.599\times10^{-1} \\ 2.2275\times10^{-2} \\ 8.6681\times10^{-3} \\ -1.3048\times10^{-3} \end{bmatrix}$$

$$\left(\alpha+1.2\times\frac{\pi}{180}\right)$$

(At all places)

$$L = \int_{-b/2}^{b/2} L(y)\,dy = \rho_\infty U_\infty \int_{-b/2}^{b/2} \Gamma(y)\,dy$$

$$\Rightarrow \rho_\infty U_\infty \int_0^\pi \Gamma(\theta)\,d\theta$$

$$\Rightarrow \frac{2b}{\pi e}\times \rho_\infty U_\infty \int_0^\pi \left(A_1\sin\theta + A_3\sin3\theta + A_5\sin5\theta + A_7\sin7\theta\right)d\theta \times(\alpha+1.2^\circ)$$

$$C_L = A_1 \pi AR$$

$$\Rightarrow (0.2599)\times(3.14)\times(8)\times(\alpha+1.2)\times\frac{\pi}{180}$$

$$\Rightarrow \underline{0.4783} \quad (\text{for } \alpha = 3^\circ)$$

$$C_{Di} \Rightarrow \frac{C_L^2}{\pi AR e} \; ; \; e = \frac{1}{1+\delta} \; ; \; \delta = \sum_{2}^{N}\left(\frac{A_n}{A_1}\right)^2$$

$$\sum_{2}^{8}\left(\frac{A_1}{A_n}\right)^2 \Rightarrow 12.097\times10^{-4} \lll 1$$

$$\therefore e \simeq 1$$

$$C_{Di} \Rightarrow \frac{(0.4783)^2}{(3.14)\times 8\times 1} \Rightarrow \underline{0.0092}$$

5.) Numerical Analyses such as Finite Element Analysis are very useful for performing stress analysis of the actual design and are capable of handling complex geometries. But the main problem with such analyses is that they are resource-intensive and are used generally for refining designs only. While Simple analyses like Prandtl's lifting line theory are perfect for doing a preliminary design, where we try to get a rough estimate of the wing shape and size. We use these methods for estimating key parameters like wingspan, chord width, and Aspect ratio. This helps us narrow down the domain for the variables. This domain is then used for further design purposes, where the parameters are refined to produce the final design of the wings.