Chapter 1

Atmospheric Flight

Contents

Lecture 1: Introduction to Atmospheric Flight

1.1 A Paper Plane Example

What are Guinness World Records about a paper airplane?

Using a standard A4 or letter size paper (a certain type of paper qualities may need to be defined beforehand), a produced paper plane may compete for how far it may travel, or how long it would stay in the air. In atmospheric flight business, the term of *range* (travel distance) and *endurance* (flying time) are identified to give the paper plane's flying quality some technical descriptions. That is essentially what aircraft performance is about (and we will discuss in detail in Chapter 4). Further, we can give some quantitive descriptions of such performance.

Let's say a paper plane is flying in the air with a steady speed and direction (straight line flight), with its head against wind in an angle (gliding angle), to generate aerodynamic forces (lift and drag), which will balance the weight of the plane.

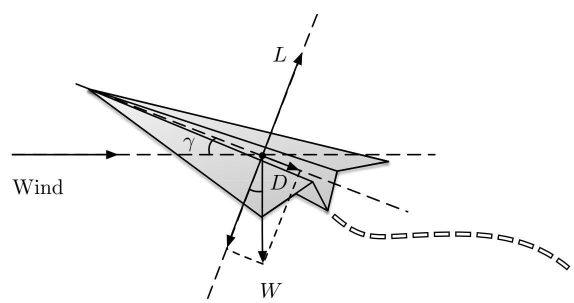

Figure 1.1: An illustration of paper plane

$$\begin{cases} L &= W\cos\gamma \\ D &= W\sin\gamma \end{cases} \tag{1.1}$$

The gliding angle, γ, then

$$\gamma = \tan^{-1}(\frac{1}{L/D}) = \tan^{-1}(\frac{1}{C_L/C_D}) \tag{1.2}$$

where C_L/C_D is an aerodynamic term, often called lift-drag ratio, and C_L, C_D represent non-dimensional life and drag coefficients (to be addressed later) respectively. The gliding angle is obviously affiliated with the range and endurance performance. One wishes that a minimum glide angle will lead to long flight range and long endurance. Such maximum performance criteria can be achieved by making the life-drag ratio to have its maximum value, $\gamma_{\min} \propto (C_L/C_D)_{\max}$.

Alternatively, the performance can be analyzed in a more systematic approach. A simple example of paper plane dynamic model is given. It is mainly taken from Stengel [Ste04] (Example 1.3-1) with modification.[1]

[Example 1.1] Paper Plane Longitudinal Dynamics
The point-mass longitudinal motions of the paper airplane is described by

$$m\dot{V} = mg\sin\gamma - D = mg\sin\gamma - C_D\left(\frac{1}{2}\rho V^2 S\right) \tag{1.3}$$

$$mV\dot{\gamma} = mg\cos\gamma - L = mg\cos\gamma - \left(\frac{1}{2}\rho V^2 S\right)C_L \tag{1.4}$$

where V, γ, r, h are the airspeed, gliding angle, range, and height respectively. The gliding angle takes positive sign when velocity direction is below the horizontal line, which is different from the so-called flight path angle (will address details in Chapter 4). The lift coefficient C_L

$$C_L = C_{L_\alpha}\alpha$$

and the lift-slope derivative is estimated as

$$C_{L_\alpha} = \frac{\pi AR}{1 + \sqrt{1 + (AR/2)^2}}$$

The drag coefficient

$$C_D = C_{D_o} + \varepsilon C_L^2$$

and the induced-drag factor $\varepsilon = 1/\pi eAR$, e is knwon as th Oswald efficiency factor. The relevant data are given. The configuration data of paper plane is given as below:

```
S = 0.017; % Reference Area, m^2
AR = 0.86; % Wing Aspect Ratio
e = 0.9; % Oswald Efficiency Factor;
m = 0.003; % Mass, kg
g = 9.8; % Gravitational acceleration, m/s^2
rho = 1.225; % Air density at Sea Level, kg/m^3
CLa = 3.141592 * AR/(1 + sqrt(1 + (AR / 2)^2));
                             % Lift-Coefficient Slope, per rad
CDo = 0.02; % Zero-Lift Drag Coefficient
epsilon = 1 / (3.141592 * e * AR);% Induced Drag Factor
```

Solving for the set of nonlinear differential equations, one may describe the dynamic behaviour quantitively. By plotting the horizontal and vertical displacement, we can even get the taste how exactly this paper airplane flies in terms of trajectory. Obviously, different trajectories are expected when pilots fly the same vehicle at different speed, angle, as illustrated in the following figure.

[1] The equations of motion are slightly different, mainly in the sign.

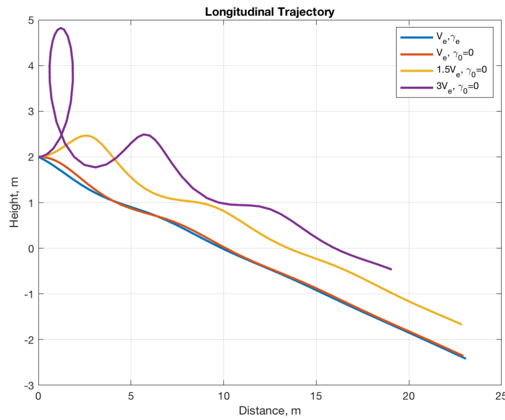

Figure 1.2: Longitudinal trajectory of the paper plane example

1.2 The Subject of Atmospheric Flight

One of the most popular textbooks in the study of flight dynamics, is " Dynamics of Flight – Stability and Control", by the well known Professors B. Etkin and L.D. Reid[2] [ER96]. In its introduction, the authors stated that " this book is about the motion of vehicles that fly in the atmosphere", where flying, or flight, is defined as a scientific term of "motion through a fluid medium or empty space" of flyable vehicles.

In this context, the subject of our study is the

- atmospheric flying vehicle (aircraft) - flight mechanics
- its dynamic behaviour,
  - performance,
  - stability and control, subject to
- governing physics laws

[2]Professor Reid and the late Professor Etkin became my mentor when I started my academic career at the UTIAS in 2000. When I inherited the teaching of AER1202: Advanced Flight Dynamics, and later got involved in AER302/402: Aircraft Flight, their textbook, additional teaching notes, plus many hours of lunch conversation, influenced me profoundly.

Lecture 2: Atmospheric Model

1.3 Earth's Atmosphere

The performance of an aircraft is strongly influenced by the local properties of the atmosphere through which it flies. The Earth's atmosphere is a dynamically changing system that is impractical to take into account all varying factors. Therefore, a standard atmosphere is defined to relate general aircraft design and performance to a common reference. The standard atmosphere is represented by mean values of air pressure p, air temperature T, and air density ρ in variation with respect to different altitudes. These values are obtained from experimental data.

1.4 Fundamental Properties of Air

1.4.1 Perfect Gas

The ideal gas law is a general relation between the pressure, temperature, and density of an ideal gas such as air. It is often referred to as an equation of state. Let p denote the air pressure, T denote the absolute temperature, and ρ denote the density of an element of air at a particular location and time. Pressure, temperature, and density of that element of air are related by the ideal gas law

$$p = \rho RT \tag{1.5}$$

where R is a constant appropriate to air, $R = 287\ \mathrm{m^2/s^2K}$.

1.4.2 Hydrostatic Equation

We will now begin to piece together a model which allow us to calculate variations of pressure, density, and temperature as functions of altitude. The foundation of this model is the *hydrostatic equation*. Consider the small stationary fluid element of air shown in the following figure:

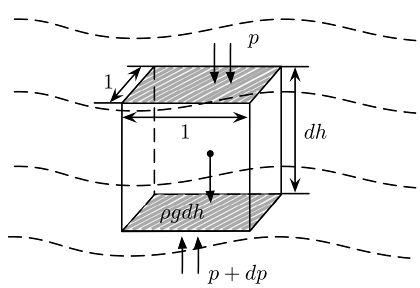

Figure 1.3: Force diagram for the hydrostatic equation

On the bottom face, the pressure p is felt, which gives rise to upward force of $p \times 1$ (unit area) exerted on the fluid element. The top face is flightly higher in altitude by the distance of an infinitesimally small height dh, and because pressure varies with altitude, the pressure on the top face will be slightly different from that on the bottom face, differing by the intinittesimally small value dp. Hence, on the top face, the pressure $p + dp$ is felt. It gives rise to a downward of $(p + dp) \times 1$ (unit area) on the fluid element. Moreover, the volume of the fluid element is $dh \times 1$ (unit area), and since ρ is the mass per unit volume, then the mass of the fluid element is simply $\rho(dh \times 1$ (unit area)). The three forces balance because the fluid element is not moving. Then the hydrostatic Equation:

$$\begin{aligned} p &= p + dp + \rho g dh \\ dp &= -\rho g dh \end{aligned} \tag{1.6}$$

1.4.3 Altitude

The standard atmospheric model that will be presented shortly is about the atmospheric properties associated with altitudes. In this section, we shall present several different concepts of altitudes. First of all, the vertical distance w.r.t. the earth centre is called the *absolute altitude*, represented by h_a; while the distance w.r.t. the earth surface is called the *geometric altitude*, represented by h_G.

According to Newton's Law of Gravitation, gravity at h_G

$$g = g_0 \left(\frac{r}{h_G + r} \right)^2 \tag{1.7}$$

where g_0 is the gravity at the earth surface. Strictly speaking, the hydrostatic equation (1.6) should write as $dp = -\rho g dh_G$ where g varies with altitude h_G, leading to

$$dp = -\rho g_0 \left(\frac{r}{h_G + r} \right)^2 dh_G.$$

In comparison with (1.6), $dp = -\rho g dh = -\rho g_0 dh$ when the gravity is assumed to be constant $g = g_0$, one gets the relationship of altitude h in (1.6) with the geometric altitude h_G:

$$dh = \left(\frac{r}{h_G + r} \right)^2 dh_G$$

that is,

$$h = \frac{r}{r + h_G} h_G \tag{1.8}$$

We shall call the altitude above h as the *geopotential altitude*. It is a different concept of altitude, getting from the hydrostatic equation assuming the constant gravity at the earth surface. Assume g is constant[3] throughout the atmosphere, equal to its value at sea level g_0, then the default altitude, without special expression otherwise, is assumed to be the geopotential altitude. It is good to know that, relationship between geopotential and geometric altitudes, up to 65km, the difference is less than 1%.

[3] In fact, the gravity change within 15km is less than 1%, quite negligible.

1.5 Standard Atmosphere Model

"The International Standard Atmosphere (ISA) is a static atmospheric model of how the pressure, temperature, density, and viscosity of the Earth's atmosphere change over a wide range of altitudes or elevations. It has been established to provide a common reference for temperature and pressure and consists of tables of values at various altitudes, plus some formulas by which those values were derived. The International Organization for Standardization (ISO) publishes the ISA as an international standard, ISO 2533:1975." – wikipedia. The defined variation of temperature T with altitude

1. Troposphere: from sea level to 11 km. Clouds form in this region and turbulent winds are present. Temperature drops linearly with altitude with a lapse rate approximately $-0.0065K/m$
2. Stratosphere: from the top of the troposphere to an altitude of approximately 50 km. Steady high velocity winds may be found (e.g. the jet stream) The temperature in the lower stratosphere remains fairly constant with altitude ($-56.5\mathring{C}$ or $-216.66K$)

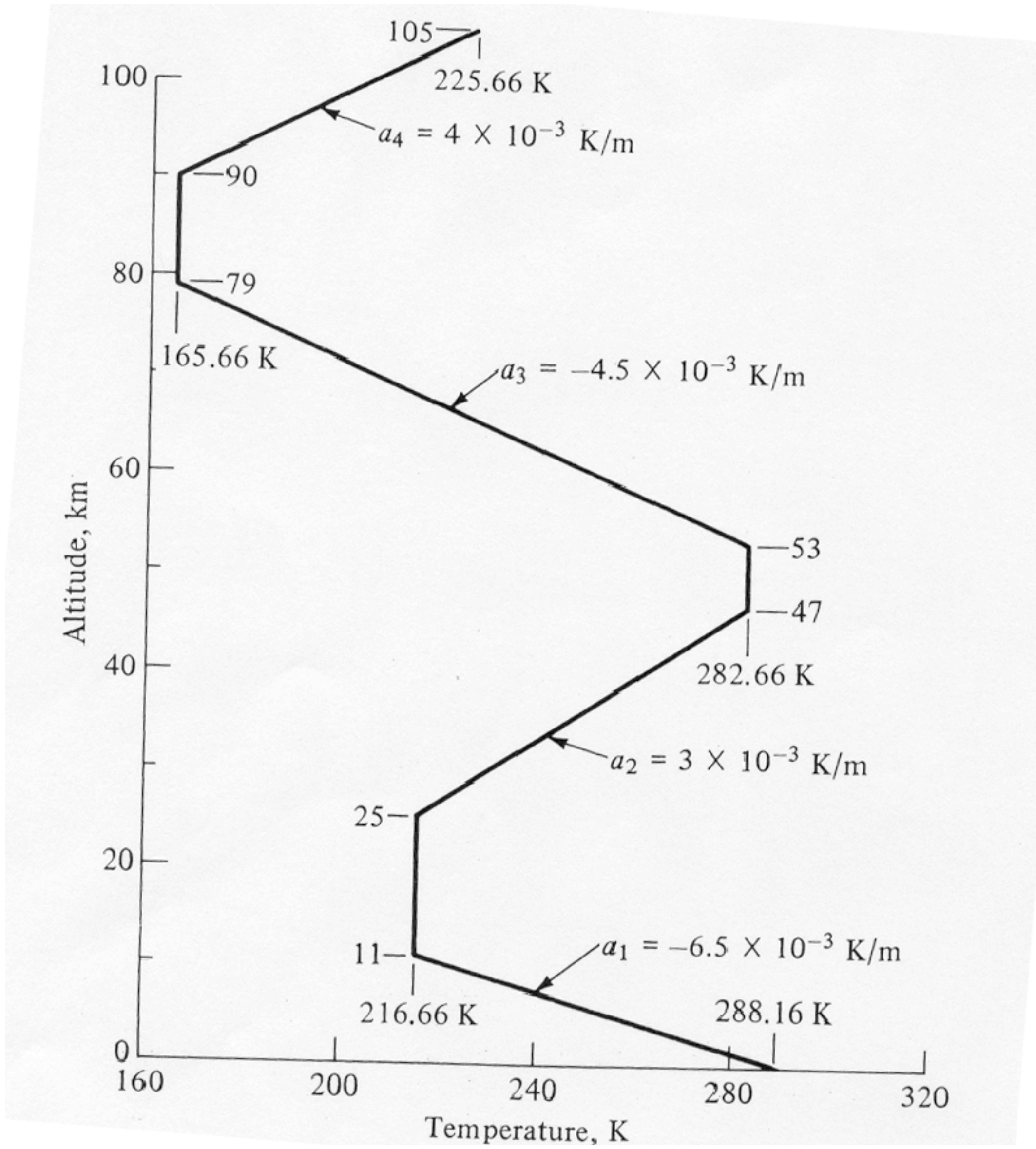

Figure 1.4: Temperature Distribution in the Standard Atmosphere

Temperature Functions of Altitude

$$\text{Isothermal Regions}: \ T = \text{constant} \tag{1.9}$$

$$\text{Gradient Regions}: \ T = T_0 + (\frac{dT}{dh})h = T_0 + ah \tag{1.10}$$

Pressure Functions of Altitude

The functions are derived from hydrostatic equation and the equation of state of a perfect gas:

$$\text{Isothermal Regions :} \qquad \delta = \frac{p}{p_1} = e^{-\frac{g_0}{RT}(h-h_1)} \tag{1.11}$$

where p_1 and $T_1 = T$ are the values at the base of the isothermal layer.

$$\text{Gradient Regions :} \qquad \delta = \frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}} \tag{1.12}$$

where p_1 and T_1 are the values of one altitude within the gradient region.

Density Functions of Altitude

The functions are derived from hydrostatic equation and the equation of state of a perfect gas:

$$\text{Isothermal Regions :} \qquad \sigma = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)} \tag{1.13}$$

where ρ_1, p_1 and $T_1 = T$ are the values at the base of the isothermal layer.

$$\text{Gradient Regions :} \qquad \sigma = \frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[\frac{g_0}{aR}+1]} \tag{1.14}$$

where ρ_1, p_1 and T_1 are the values of one altitude within the gradient region.

At sea level ($h = 0$), the standard sea level values are:

$$p_s = 1.01325 \times 10^5 \text{ N/m}^2 = 2116.2 \text{ lb/ft}^2 \tag{1.15}$$
$$\rho_s = 1.2250 \text{ kg/m}^3 = 0.002377 \text{ slug/ft}^3 \tag{1.16}$$
$$T_s = 288.16K = 518.69\overset{\circ}{R} \tag{1.17}$$

[**Example 1.2**] Calculate the standard atmosphere at a geopotential altitude 14km.

(1) Values at the base of the isothermal region where $h_1 = 11$ km:

$$\begin{aligned}
T_1 &= T_s + a_1 h_1 \\
&= 288.16 - 6.5 \times 10^{-3} \times 11,000 = 216.66 \ \ (K) \\
p_1 &= p_s \left(\frac{T_1}{T_s}\right)^{-g_0/(aR)} \\
&= 1.01 \times 10^5 \times \left(\frac{216.66}{288.16}\right)^{-9.8/(-0.0065\times 287)} = 2.2578 \times 10^4 \ \ (N/m^2) \\
\rho_1 &= \rho_s \left(\frac{T_1}{T_s}\right)^{-[g_0/(aR)+1]} \\
&= 1.2250 \times \left(\frac{216.66}{288.16}\right)^{-[9.8/(-0.0065\times 287)+1]} = 0.3642 \ \ (kg/m^3)
\end{aligned}$$

(2) Calculate temperature T at $h = 14\ km$: $T = T_1 = 216.66\ (K)$.
(3) Calculate pressure p at $h = 14\ km$:

$$\begin{aligned} p &= p_1 e^{-\frac{g_0}{RT}(h-h_1)} \\ &= 2.2578 \times 10^4 \times e^{-\frac{9.8}{287\times 216.66}(14000-11000)} = 1.4071 \times 10^4\ (N/m^2) \end{aligned}$$

(4) Calculate density ρ at $h = 14\ km$:

$$\begin{aligned} \rho &= \rho_1 e^{-\frac{g_0}{RT}(h-h_1)} \\ &= 0.3642 \times e^{-\frac{9.8}{287\times 216.66}(14000-11000)} = 0.2270\ (kg/m^3) \end{aligned}$$

■

1.6 Pressure Altitude

Another popular terminology in the aviation community is the so-called pressure altitude. Imagine the captain in the cockpit during flight, s/he needs to monitor the states of flight operation and, if necessary, communicates with traffic control tower to report the status. One can read the pressure, temperature, density values using SI units. Now that we have the ISA (international standard atmosphere) reference model, why don't we use the model data, i.e. the readings correspond to the altitude in the model. The result is the pressure altitude.

[**Example 1.3**] An airplane is flying at a certain altitude where the **actual** reading of pressure and temperature are 4.72×10^4 N/m^2 and 255.7 K respectively.

- In the standard atmosphere, the altitude value corresponding to the actual pressure reading is called the *pressure altitude*. In other words, the pressure altitude represents the pressure in the unit of altitude;
- In the standard atmosphere, the altitude value corresponding to the actual temperature reading is called the *temperature altitude*;
- In the standard atmosphere, the altitude value corresponding to the actual density reading is called the *density altitude*.

(1) The value of actual pressure reading is $p = 4.72 \times 10^4$ N/m^2. In the ISA model,

$$p = p_s \left(\frac{\hat{T}}{T_s}\right)^{-g_0/(aR)}$$

leading to $\hat{T} = 249.22K$. Obviously, this is NOT the actual temperature reading of this flight, rather, it is the temperature value in the ISA model associated with the actual p data.
(2) Now we will use $\hat{T}$ in the ISA model to find out its corresponding altitude, again in the ISA model,

$$\hat{T} = T_s + a_1 \hat{h}$$

leading to $\hat{h} = 5,990\ m$. In other words, the (geopotential) altitude of $5,990m$ in the ISA model correspond to $p = 4.72 \times 10^4\ \mathrm{N/m^2}$, the actual pressure reading in flight. Therefore, $5,990m$ is the pressure altitude.

■

Alternatively, one may treat the concept of pressure altitude simply as the pressure reading in the unit of altitude of ISA model.

Lecture 3: Airspeed

1.7 Speed of Sound

Sound waves travel through the air at a definite speed - the speed of sound or called *acoustic velocity*. First let us derive a formula to calculate the speed of sound. Consider a sound wave is created by a source, and moves into a stagnant gas,

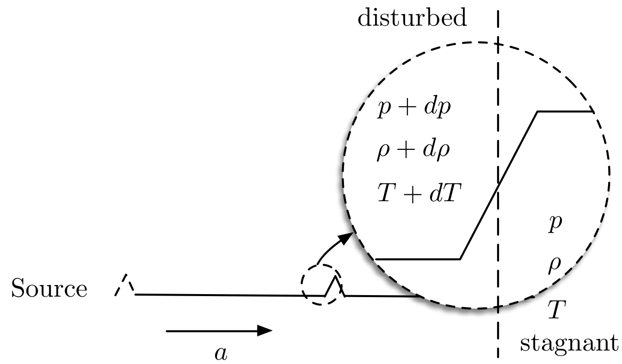

Figure 1.5: Sound wave model

The sound wave itself is a thin region of disturbance in the air, across which the pressure, temperature, and density change slightly. One would expect the speed will change slightly as well. Apply the *continuity equation* and the *momentum equation*:

$$\begin{aligned} \rho a &= (\rho + d\rho)(a + da) \\ dp &= -\rho a da \end{aligned}$$

the area of the stream tube running through the wave is constant .
Therefore

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)}$$

On a physical basis, the flow through a sound wave involves no heat addition, and effect of friction is negligible. Hence the flow through a sound wave is isentropic. For isentropic flow,

$$\frac{p_2}{\rho_2^\gamma} = \frac{p_1}{\rho_1^\gamma} = constant$$

We have

$$a = \sqrt{\gamma \frac{p}{\rho}}$$

For a perfect gas, we further have

$$a = \sqrt{\gamma RT} \tag{1.18}$$

It demonstrates a fundamental result: *The speed of sound in a perfect gas depends only on the temperature of the gas.*

1.7.1 Mach Number

The speed of sound leads to another concept, namely *Mach number*. Consider a point in a flow field. The flow velocity at that point is V, and the speed of sound is a. By definition, the Mach number at that point is the flow velocity divided by the speed of sound

$$M = \frac{V}{a} \tag{1.19}$$

We normally degine three different regimes of aerodynamic flows:

- If $M < 1$, the flow is subsonic
- If $M = 1$, the flow is sonic
- If $M > 1$, the flow is supersonic

Each of these regimes is characterized by its own special phenomena. In addition, two other specialized aerodynamic regimes are commonly defined, namely, *transonic* flow ($0.8 \leq M \leq 1.2$), and *hypersonic* flow where $M > 5$.

1.8 Airspeed of Subsonic Incompressible Flow

According to the principle of *Newton's Second Law*, we can have the following **Euler's equation**, or **momentum equation** on *a point of the streamline*, when we treat the flow as frictionless (*inviscid flow*) and gravity impact is negligible,

$$dp = -\rho V dV \tag{1.20}$$

Further, integrating the above equation along the stream line, we shall be able to derive to the **Bernoulli's Equation** for incompressible flow (air density is kept constant):

$$p_1 + \rho\frac{V_1^2}{2} = p_2 + \rho\frac{V_2^2}{2} \tag{1.21}$$

where point 1 and 2 are both on the same streamline; or

$$p + \frac{1}{2}\rho V^2 = \text{constant} \tag{1.22}$$

Airspeed measurement is critical for both flight test and normal aircraft operations. Pressure measurements are employed. The velocity of airflow is obtained by measuring the different pressure between total and static pressure.

1.8.1 Static and Total Pressure

Recall the Bernoulli's equation (for low speed, incompressible flow)

$$p + \frac{1}{2}\rho V^2 = \text{constant} = p_t$$

- *Static pressure* p is the local atmospheric pressure (random motion of molecules)
- *Dynamic pressure* $q = \frac{1}{2}\rho V^2$ represents the pressure associated with the flow motion (kinetic energy)
- *Total pressure* p_t is also measured by a pitot probe in which the moving air mass is brought to rest thereby correcting the air's kinetic energy into an increased air pressure.

1.8.2 Pitot-Static Probe

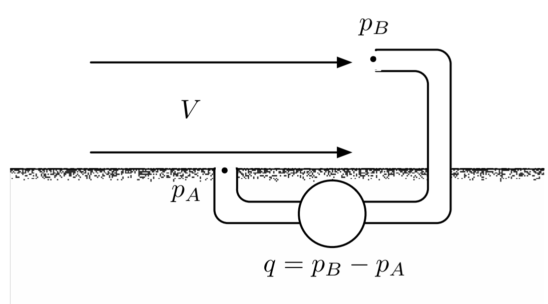

Figure 1.6: Pitot-static probe

Point B is a *stagnation point*, $V = 0$, which measures the total pressure. At point A, there is a uniform flow over the surface and this small hole will feel only the random motion of the gas molecules where static pressure is measured.
Consider low-speed flight, for velocities less than 91 m/s (300 ft/s), or Mach number less than 0.3, the airflow can be treated as incompressible flow:

$$\underset{\text{static pressure}}{p} + \underset{\text{dynamic pressure}}{\frac{1}{2}\rho V^2} = \underset{\text{total pressure}}{p_t} \tag{1.23}$$

and the airspeed is calculated by

$$V = \sqrt{\frac{2(p_t - p)}{\rho}} \tag{1.24}$$

$p_t - p$ comes directly from the pitot-static probe.

Figure 1.7: Pitot-static Probe: A Photo from Aircraft

1.8.3 Subsonic Wind Tunnel

The *streamline* is the path where the fluid element traces in a steady flow. The *mass flow* through area A is the mass crossing A per unit time.

$$\dot{m} = \rho A V$$

According to the principle of *conservation of mass*, the mass flow *bounded by the streamline* has the following **continuity equation**:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{1.25}$$

For *incompressible flow*, where the density of fluid element remains as constant, a special format of continuity equation is given:

$$A_1 V_1 = A_2 V_2 \quad \text{for incompressible flow} \tag{1.26}$$

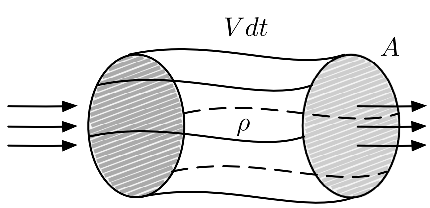

Figure 1.8: The continuity illustration

It serves as the scientific principle to build a subsonic wind tunnel.

1.9 Airspeed of Subsonic Compressible Flow

For higher-speed flows, but the Mach number is still less than 1, compressibility must be taken into account. As airspeed increases it is found that the air density in the flow in and around

the pitot-static tube begins to change due to compressibility effects. The relationship between pressure and airspeed is then expressed by the energy equation.
Define *enthalpy* as

$$h = e + pv = e + RT \text{ for perfect gas}$$

Then

$$dh = de + pdv + vdp$$

We have another form of first law of thermodynamics:

$$\delta q = dh - vdp, \quad \text{since } \delta q + \delta w = de, \delta w = -pdv$$

The specific heat at constant volume is defined by:

$$c_v = \left(\frac{\delta q}{dT}\right)_{\text{constant volume}}$$

$$de = \delta q - pdv = \delta q = c_v dT$$

leading to

$$e = c_v T \tag{1.27}$$

The specific heat at constant pressure is defined by:

$$c_p = \left(\frac{\delta q}{dT}\right)_{\text{constant pressure}}$$

$$\delta q = dh - vdp = dh = c_p dT$$

leading to

$$h = c_p T \tag{1.28}$$

Further, for perfect gas,

$$h = e + pv \Longrightarrow c_p T = c_v T + RT$$

Therefore,

$$c_p = \frac{\gamma}{\gamma - 1} R \tag{1.29}$$

where $\gamma = c_p/c_v = 1.4$ (for air). Now, we combine the first law equation and Euler's equation

$$\begin{aligned} \delta q &= dh - vdp \\ &= dh + v\rho V dV \\ &= dh + V dV \end{aligned}$$

Considering isentropic process, $\delta q = 0$, we have the **energy equation**:

$$h + \frac{V^2}{2} = c_p T + \frac{V^2}{2} = constant \tag{1.30}$$

$$\frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = \frac{\gamma}{\gamma - 1}\frac{p_t}{\rho_t} \tag{1.31}$$

where $\gamma = \frac{c_p}{c_v} \approx 1.4$ for air. c_p, c_v are the specific heats for air at constant pressure and volume. p and ρ are local values of static pressure and density respectively. p_t, ρ_t apply when $V = 0$, such as inside the pitot-static probe.
It is assumed that the air obeys isentropic gas law that

$$\frac{p}{\rho^\gamma} = \text{constant for an element of gas} \tag{1.32}$$

Combining the above equations obtain that

$$V = \sqrt{\frac{2}{(\gamma-1)}\frac{\gamma p}{\rho}\left[\left(\frac{p_t - p}{p} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{1.33}$$

Also, one can get the relationship with the local Mach number as follows:

$$\frac{T_t}{T_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right) \tag{1.34}$$

$$\frac{p_t}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \tag{1.35}$$

$$\frac{\rho_t}{\rho_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{1/(\gamma-1)} \tag{1.36}$$

As a matter of fact, we can use the above equation of ρ_t/ρ_1 to show why we can treat the flow is incompressible when $M < 0.3$.

1.10 Airspeed Calibration

- The true airspeed (TAS) in Eq. (1.24) and (1.33) requires the air density (and static pressure) that varies with altitude and temperature.
- It is calibrated by using sea-level values, as it is called *calibrated airspeed* (CAS)
- The *equivalent airspeed* (EAS) is the speed calibrated at sea level (with standard sea level density)

$$V_e \triangleq \sqrt{\frac{\rho}{\rho_s}}V = \sqrt{\sigma}V \tag{1.37}$$

1.10.1 Incompressible Flow

Note that at low airspeed the fluid can be considered as incompressible ($\rho = constant = \rho_s$) and the above equation can be simplified as: (use incompressible Bernoulli's equation)

$$V = \sqrt{\frac{2(p_t - p)}{\rho}} \tag{1.38}$$

$$V_c = \sqrt{\frac{2\left(p_t - p\right)}{\rho_s}} \tag{1.39}$$

$$V_e = V_c \tag{1.40}$$

1.10.2 Subsonic Compressible Flow

$$V = \sqrt{\frac{2}{(\gamma-1)}\frac{\gamma p}{\rho}\left[\left(\frac{p_t - p}{p}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{1.41}$$

$$V_c = \sqrt{\frac{2}{(\gamma-1)}\frac{\gamma p_s}{\rho_s}\left[\left(\frac{p_t - p}{p_s}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{1.42}$$

$$V_e = \sqrt{\frac{2}{(\gamma-1)}\frac{\gamma p}{\rho_s}\left[\left(\frac{p_t - p}{p}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{1.43}$$

When an airspeed instrument is designed, usually it is only fed $p_t - p$ from the pitot-static probe, and the $\frac{p}{\rho}$ factor and p are replaced by sea level values $\frac{p_s}{\rho_s}$ and p_s which are constants. For this reason the instrument only indicates *true airspeed* when the aircraft is at sea level.

[**Example 1.4**] Consider the isentropic flow over the airfoil. The free-stream pressure, velocity, and density are $1.013 \times 10^5\ N/m^2$, $223.5\ m/s$, and $1.225\ kg/m^3$ respectively. At a given point A on the airfoil, the pressure is $7.1676 \times 10^4\ N/m^2$. What is the Mach number at point A?

1.10.3 * Air Data System

A multi function pitot probe1 like the one shown here has a pitot tube, static port, and angle of attack vane in a single unit.[4]

A malfunctioning air data system can have severe consequences on the safety of flight. One of the most often encountered fault mode on an air data system is pitot tube blockage which is often caused by ice accumulation. Moisture and foreign objects like insect nests could also cause blockage in the pitot/static system. Currently, the approach used to achieve the requisite level of reliability is through extensive physical redundancy on sensors, actuators, and flight computers. Airbus A330, for instance, has an air data system that is triply redundant.

[4] Materials are taken from Lie and Gebre-Egziabher [LGE15].

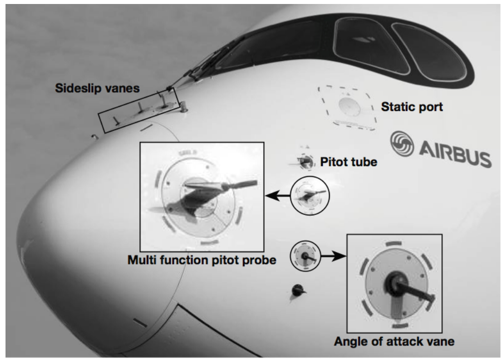

Figure 1.9: Air data sensor arrangement on an Airbus A350XWB

Date	Type	Flight	Probable Cause	Result
Dec 1, 1974	B727	Northwest Orient 6231	Pitot icing	LOC, total fatalities
Jul 28, 1984	LJ25	N1JR	Pitot cover not removed	RTO, hull loss
Feb 6, 1996	B752	Birgen Air 301	Pitot blocked by wasp nest	LOC, total fatalities
Oct 2, 1996	B752	Aero Peru 603	Static port blockage	LOC, total fatalities
Oct 7, 2008	A330	Qantas 72	Temporary Data spikes	Multiple injuries
Nov 27, 2008	A320	XL 888T	Frozen α vane	LOC, total fatalities
Jun 1, 2009	A330	Air France 447	Pitot icing	LOC, total fatalities

LOC: Loss of Control, RTO: Rejected Take Off

Figure 1.10: Examples of aircraft accidents that have been caused by air data system failure

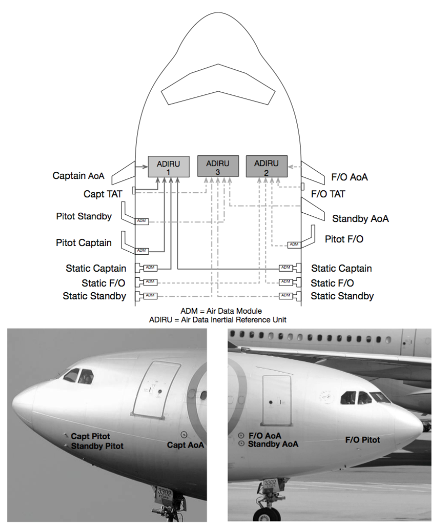

Figure 1.11: A schematic of Airbus A330 air data system [to-do: permit]

Lecture 4: Aircraft Anatomy

1.11 Aircraft Anatomy

1.11.1 Airfoil Nomenclature

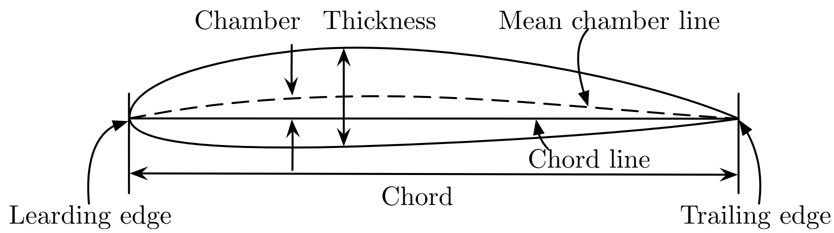

Figure 1.12: Airfoil momenclature

The cross-sectional shape obtained by the intersection of the wing with the perpendicular plane is called an *airfoil*. The *mean camber line* is the locus of points halfway between the upper and lower surfaces. The most forward and rearward points of the mean camber line are called the *leading* and *trailing edges*, respectively. The precise distance from the leading the trailing edge is called the *chord*, denoted by c. The *camber* is the maximum distance between the mean camber line and the chord line. There is an aerodynamic force created by the pressure and shear stress distributions over the wing surface.

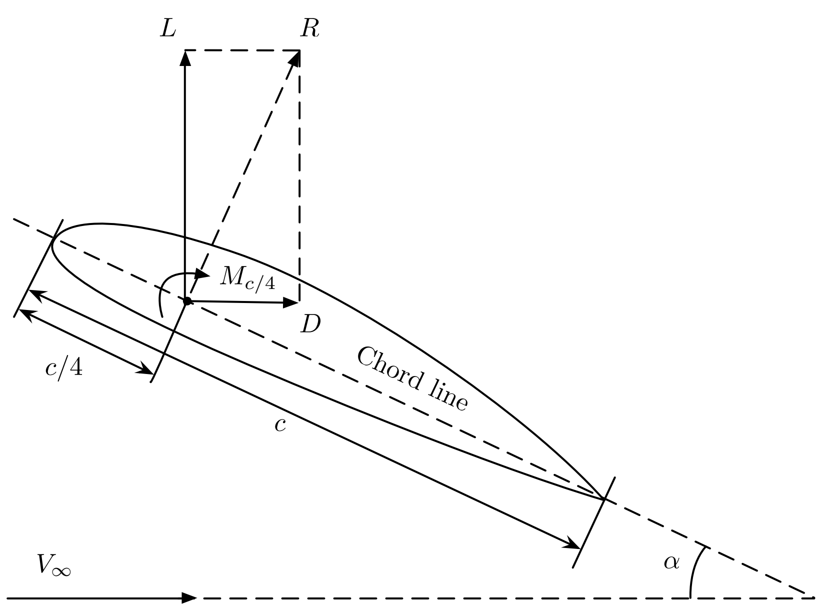

Figure 1.13: Aerodynamic forces

The direction of free-stream velocity V_∞ is defined as the *relative wind*. The angle between relative wind and chord line is called the *angle of attack*. The *drag* is always defined as the component of the aerodynamic force parallel to the relative wind. The *lift* is always defined as the component of the aerodynamic force perpendicular to the relative wind. We also take moments about a point. M_{LE} is the moment about the leading edge. It is common in the case of subsonic airfoils to take moments about a point on the quarter-chord $M_{c/4}$. Note that M_{LE} and $M_{c/4}$ are both functions of α. However, there exists a certain point on the airfoil about which moments do NOT vary with α. This point is defined as *aerodynamic center*, the moment about the aerodynamic center is denoted by M_{ac}. Accordingly, the moment about any point can be considered as the moment about the aerodynamic center, superposed by the moment generated from the aerodynamic force acting on the aerodynamic center to that point.

1.11.2 Wing Geometry

The wing is the large horizontal surface on an airplace, which provides most of the lift to support its weight.

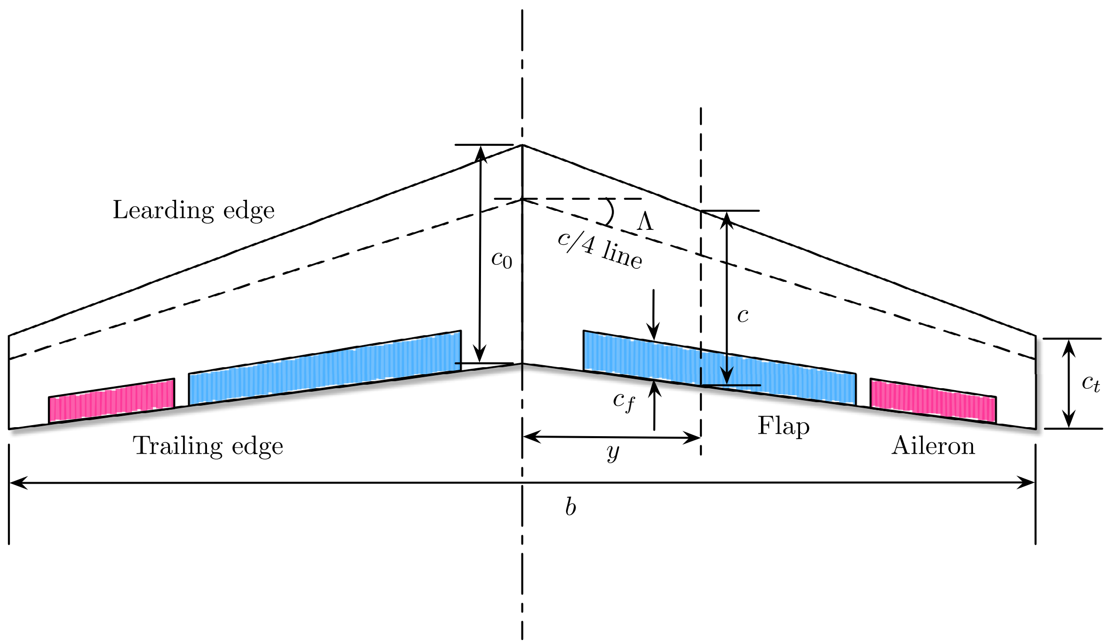

Figure 1.14: Wing geometry

The *planform* of a wing is the view of the wing, which one sees when looking directly up or down on the wing. The *wingspan* b is the distance from one wing tip to the other. The cross-section of a wing is an airfoil. *Aspect ratio* AR is a measure of the space length relative to the chord:

$$AR = \frac{b}{c} \tag{1.44}$$

when chord c is constant along the span, and

$$AR = \frac{b^2}{S} \tag{1.45}$$

when chord varies along the span which is generally true. Note that S is the planform area of the wing.

Almost all modern high-speed aircraft have *swept-back wings*. *Sweep or sweepback* is usually used to alleviate compressibility effects. The sweep angle is measured as the angle between the leading edge and the spanwise direction and denoted as Λ. Sweep is also frequently taken with reference to the quarter-chord line of the wing. The *quarter-chord line* of a wing is the locus of points one-quarter of the chord back from the leading edge. For a linearly tapered wing it is a straight line.

The *taper ratio* of a wing having straight leading and trailing edges is defined simply as the ratio of the tip chord to the midspace chord $\lambda = \frac{c_t}{c_0}$. For a linearly tapered wing, the chord, aspect ratio, and taper ratio have the following relationship:

$$c = c_0\left[1 - (1-\lambda)\frac{2y}{b}\right] \tag{1.46}$$

$$AR = \frac{2b}{c_0(1+\lambda)} \tag{1.47}$$

1.11.3 Control Surfaces

A typical transport aircraft has a number of movable aerodynamic control surfaces that must be set by the pilot to achieve the desired flight path.
From Plan View Controls include columns, wheel, trim wheels, flap selector, etc. Control surfaces include: elevator, aileron, flaps.

Elevator The column sets the elevator angle to change the pitching moment (nose up or down). It is normally placed at the tail. This surface contribution can be used to put aircraft into climb or dive (throttle must often be adjusted as well). Also it is used to set a new value for angle of attack α to allow level flight at different airspeed (again, throttle change will also be required).

Aileron On the outboard (outer one-third or so of the span), the trailing edge on one side of the wing deflects opposite to that on the other. These oppositely moving surfaces are called ailerons. The wheel sets the aileron angle (one up the other down) to cause the aircraft to roll about its longitudinal x-axis. Once the desired bank angle is achieved the aileron are centered leaving the lift vector tilted away from the earth vertical and this causes the aircraft to follow a curved flight path. This the sequence used to turn an aircraft.

From Side View Controls include rudder pedals (foot operated), turn wheel, throttle. Control surfaces include rudder.

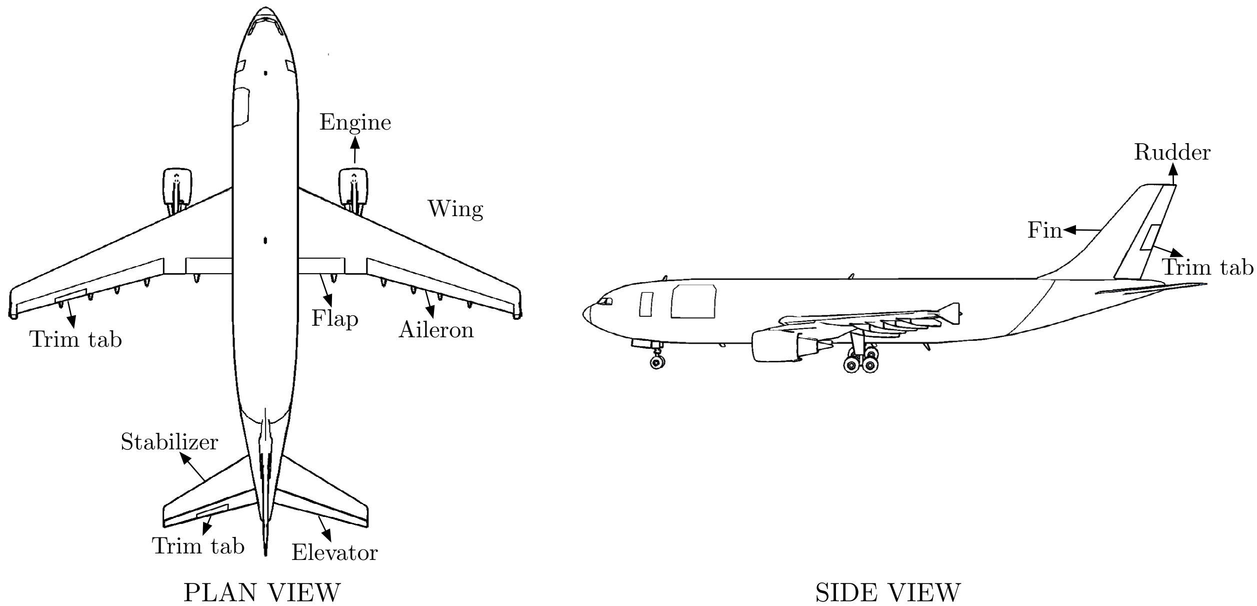

Figure 1.15: Control surfaces

Rudder The rudder is used to correct for yaw caused by an engine failure or to slip the aircraft using crossed controls (e.g. x-wind landing). Although it is not the primary control for turing, it may be used to correct nose pointing error in a turn.

Flaps for High Lift An aircraft normally encounters its lowest flight velocities at takeoff or landing. The slowest speed at which an aircraft can fly in straight and level flight is defined as the *stalling speed* V_{stall}. Hence, the aerodynamic methods of making V_{stall} as small as possible is of vital importance. [5]

[5] Details will be discussed in performance study.

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