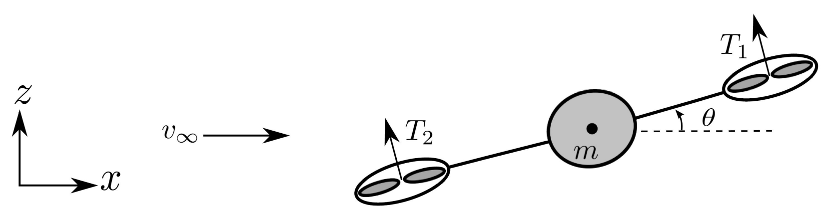
AER1216: Fundamentals of UAS

Assignment # 5
Due: Dec. 2^{nd}

1. We consider a quadrotor vehicle with a spherical body moving in the vertical $xz-$plane. The motion in the $y-$direction is assumed to be stabilized separately; that is, $y(t)$ is constant. The vehicle mass is m. The two propellers produce forces orthogonal to the vehicle arms. The forces are denoted by $T_1(t)$ and $T_2(t)$, respectively, with $T_1, T_2 > 0$. The angle $\theta(t)$ describes the vehicle's rotation around its center of mass. The position of the vehicle is $(x(t), z(t))$. A constant wind blows from the left with velocity v_∞. This produces a drag force on the quadrotor which can be modeled as:

$$F_d = \frac{1}{2}C_d\rho A(v_{rel})^2 = k(v_{rel})^2 \text{ with } k = \frac{1}{2}C_d\rho A$$

This force depends on the drag coefficient C_d, the air density ρ, the vehicle's cross sectional area A, and the relative flow velocity v_{rel}. We assume that the coefficient k is constant. The relative flow velocity is defined as the velocity of the air relative to the object. We neglect drag effects in the $z-$direction and on the rotational motion.



   (a) Derive the differential equations that describe the translational and rotational motion of the vehicle. For now, use $T_1(t)$ and $T_2(t)$ as inputs, and $x(t)$ and $z(t)$ as the outputs of the system. Formulate the equations in standard form; that is, as a system of nonlinear first-order differential equations of the form:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \qquad \mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))$$

where $\mathbf{x}(t)$ represents the system state, $\mathbf{u}(t)$ the system input, and $\mathbf{y}(t)$ the system output. The rotational inertia of the vehicle about its center of mass is J, and the arm length from the center of mass to the center of the propeller is l.

(b) Find the equilibrium points $(\mathbf{x}_o, \mathbf{u}_o, \mathbf{y}_o)$ where the quadrotor is stationary (i.e. $\dot{\mathbf{x}}(t) = 0$). How many equilibrium points exist?

(c) A control engineer helped us out and designed a controller that keeps the vehicle at a constant height. As a result, we can assume that $(T_1(t) + T_2(t)) = mg/\cos\theta(t)$, where g is the gravitational constant. Moreover, he designed an attitude controller for the rotational dynamics so that we can choose $\theta(t)$ as our input. Linearize the $x-$dynamics about the equilibrium point you found in part b) with the new input $u(t) = \theta(t)$ and the output $y(t) = x(t)$. Express your answer in the standard linear form:

$$\delta\dot{\mathbf{x}}(t) = \mathbf{A}\delta\mathbf{x}(t) + \mathbf{b}\delta\mathbf{u}(t)) \qquad \delta\mathbf{y}(t) = \mathbf{c}^T\delta\mathbf{x}(t)$$

where $\delta\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}_o$, $\delta\mathbf{u}(t) = \mathbf{u}(t) - \mathbf{u}_o$, and $\delta\mathbf{y}(t) = \mathbf{y}(t) - \mathbf{y}_o$ with equilibrium point $(\mathbf{x}_o, \mathbf{u}_o, \mathbf{y}_o)$. Note that the state, input and output have changed compared to part a) and b).

(d) How would you design a controller for the system in c)? Describe your approach to controller design in words only, no equations required.