

$$(3.) \quad I[f] = \int_0^2 \left[\left(\frac{df}{dx} \right)^2 + f \right] dx.$$

$$= \int_0^2 \left((f')^2 + f \right) dx$$

$$a=0$$

$$b=2$$

$$F \Rightarrow f + (f')^2$$

$$\therefore \frac{\partial F}{\partial f} - \frac{d}{dx} \left(\frac{\partial F}{\partial f'} \right) = 0$$

$$\Rightarrow \frac{\partial F}{\partial f} \Rightarrow 1 \quad ; \quad \frac{\partial F}{\partial f'} \Rightarrow 2(f')$$

$$\therefore 1 - \frac{d}{dx} (2f') = 0$$

$$1 - 2 \frac{d^2 f}{dx^2} = 0$$

$$\frac{d^2 f}{dx^2} = \frac{1}{2} \Rightarrow f \Rightarrow \frac{x^2}{4} + C_1 x + C_2$$

$$\begin{aligned} f(0) = 0 &\Rightarrow C_2 = 0, \\ f(2) = 2 &\Rightarrow C_1 = \frac{1}{2} \end{aligned} \quad \left. \begin{array}{l} \text{4. } f(2) = 2 \\ \text{5. } f(0) = 0 \end{array} \right\} \Rightarrow f \Rightarrow \underline{\underline{\left(\frac{x^2}{4} + \frac{x}{2} \right)}}$$

A random $f^n \rightarrow \frac{x^2 + 3x}{3}$

Another random f^n that
satisfies the b/c

$$f(2) \Rightarrow \frac{x^2}{8} + px \Big|_2 = 2$$

\Rightarrow

$$\frac{1 + 2p}{2} = 2$$

$$\Rightarrow p = \underline{\underline{3/4}}$$

$$\therefore f_{\text{new}}(x) \Rightarrow \frac{x^2}{8} + \frac{3}{4}x \quad \left| \quad \frac{df}{dx} = \frac{x}{4} + \frac{3}{4} \right.$$

$$\therefore J \Rightarrow \int_0^2 \left[\left(\frac{x+3}{4} \right)^2 + \left(\frac{x^2 + 3x}{8} \right) \right] dx$$

$$\Rightarrow \int_0^2 \left[\frac{x^2 + 9 + 6x}{4} + \frac{x^2}{8} + \frac{3x}{4} \right] dx$$

$$\Rightarrow \int_0^2 \left(\frac{3x^2}{8} + \frac{9x}{4} + \frac{9}{4} \right) dx$$

$$= \underline{\underline{10}} > 0$$