

(2)

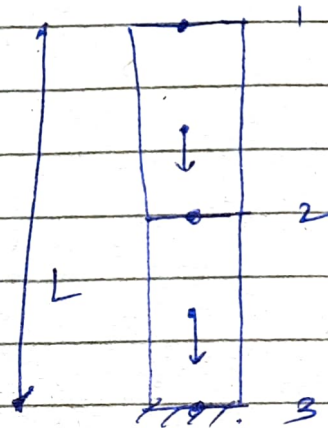
Global Stiffness Matrix



Element Stiffness Matrix



$$K_2 = K_1 = \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



$$d_3 = 0$$

$$\therefore K_G \Rightarrow \frac{AE}{(L/2)} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Force Vectors:-

$$F_2 \Rightarrow F_{21} \Rightarrow \frac{Mg}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \Rightarrow \frac{Mg}{4} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$F_G \Rightarrow \frac{Mg}{4} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Applying B/c  $d_3 = 0$  : gives us with the final setup of  $2^{\text{nd}}$  to be solved

$$K_u u = F$$

$$\frac{2AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{mg}{4} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$u_1 - u_2 \Rightarrow \frac{MgL}{8AE} \Rightarrow u_1 \Rightarrow u_2 + \frac{MgL}{8AE}$$

$$2u_2 - u_1 \Rightarrow \frac{MgL}{4AE}$$

⇓

$$2u_2 - \left( u_2 + \frac{MgL}{8AE} \right) = \frac{MgL}{4AE}$$

Downwards has been considered  
~~the~~ here all the +ve values mean downward force/displacement, etc.

$$u_2 \Rightarrow \frac{3}{8} \frac{MgL}{AE}$$

$$\rightarrow M = 8AL$$

$$\frac{3}{8} \frac{9gL^2}{E}$$

$$u_1 \Rightarrow \frac{MgL}{2AE}$$

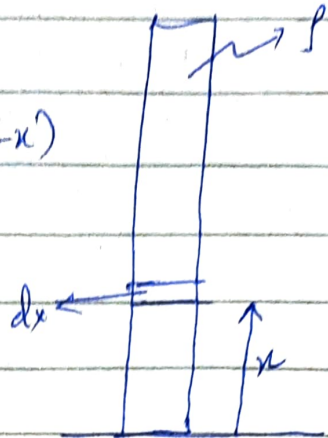
$$\frac{1}{2} \frac{9gL^2}{E}$$

$$u_3 = 0$$

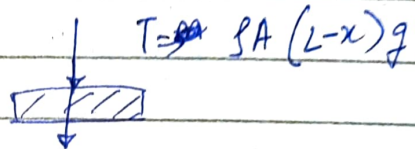
## Continuous System Calculation

Compression in Rod  $T = \rho A g (L - x)$

$$U \Rightarrow \int_0^L \frac{T dx}{AE}$$



$$\Rightarrow \int_0^L \frac{\rho A g (L - x) dx}{AE}$$



$$\Rightarrow \frac{\rho g}{E} \int_0^L (L - x) dx$$



$$dx \Rightarrow \frac{T dx}{AY}$$

$$\Rightarrow \frac{\rho g}{E} \left( Lx - \frac{x^2}{2} \right) \Big|_0^L$$



$$\Rightarrow \frac{\rho g}{E} \left( \frac{L^2}{2} \right) \Rightarrow \frac{\rho g L^2}{2E}$$

$$U \Big|_{L/2}^{L/2} \Rightarrow \int_0^{L/2} \frac{\rho A g (L - x) dx}{AE} \Rightarrow \frac{\rho g}{E} \left( Lx - \frac{x^2}{2} \right) \Big|_0^{L/2}$$

$$\Rightarrow \frac{\rho g}{E} \left( \frac{3L^2}{8} \right) \Rightarrow \frac{3\rho g L^2}{8E}$$