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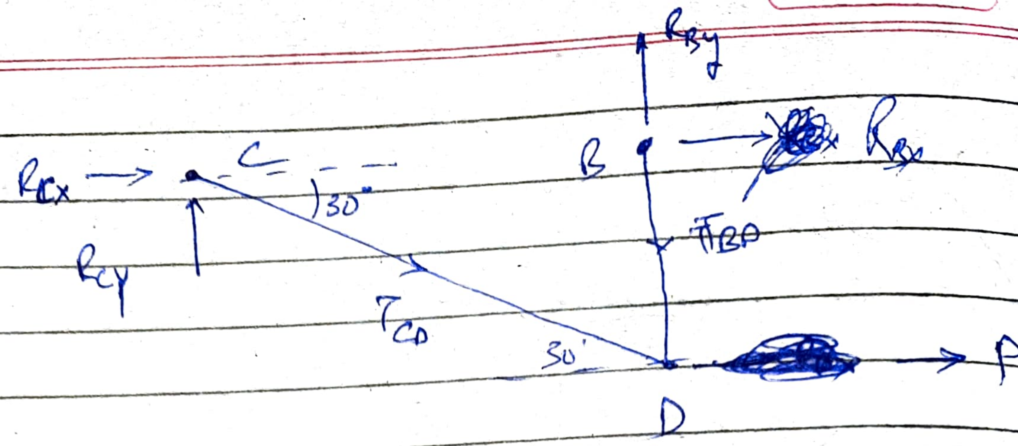
Actual System



$\Sigma F_x = 0$

$\Rightarrow R_{Cx} + R_{Bx} + P = 0$

$R_{Cy} + R_{By} = 0$

$\boxed{R_{Bx} = -7P/4}$

$\Rightarrow R_{By} = -R_{Cy} \Rightarrow \frac{\sqrt{3}}{4} P$

For Joint B, $R_{By} = T_{BD} \Rightarrow R_{By} = \frac{P\sqrt{3}}{4}$

For Joint C,

$R_{Cx} + T_{CD}\cos 30° = 0 \Rightarrow R_{Cx} \Rightarrow \frac{3}{4} P$

$R_{Cy} - T_{CD}\sin 30° = 0 \Rightarrow R_{Cy} \Rightarrow -\frac{\sqrt{3}}{4} P$

For Joint D,

$T_{CD}\cos 30° + P = 0 \Rightarrow T_{CD} = \left(-\frac{P\sqrt{3}}{2}\right)$ ✓

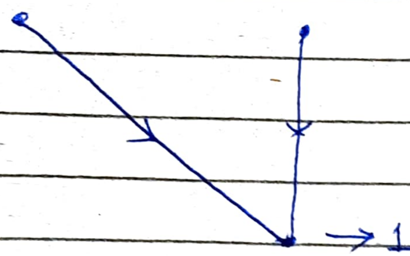
$T_{CD}\sin 30° + T_{BD} = 0 \Rightarrow T_{BD} = \left(+\frac{P\sqrt{3}}{2}\right) \times \frac{1}{2}$

$\Rightarrow \quad = \frac{P\sqrt{3}}{4}$ ✓

Virtual System

For X-disp →

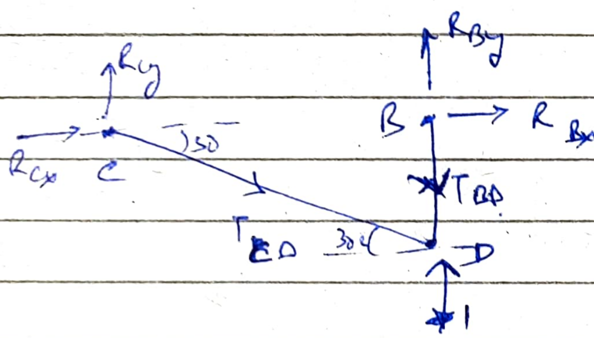
The Method of Joints applied



$T_{BD} \Rightarrow \frac{\sqrt{3}}{4}$

$T_{CD} = -\frac{\sqrt{3}}{2}$

For Y-disp →



For D →

$-T_{BD} + 1 = 0$

$-T_{CD}\sin 30°$

$T_{CD}\cos 30° = 0$ $\Rightarrow \underline{\underline{T_{CD} = 0}}$; $\underline{\underline{T_{BD} \Rightarrow 1}}$

Now Applying the principle of Virtual Work.

Member	N	n_V	n_H	Nn_V	Nn_H
BD	$\left(P\sqrt{3}/4\right)$	1	$\sqrt{3}/4$	$P\sqrt{3}/4$	$3P/16$
CD	$\left(-P\sqrt{3}/2\right)$	0	$-\sqrt{3}/2$	0	$3P/4$

$$\delta_D = \sum\left(\frac{nNL}{AE}\right)$$

$$(\delta_D)_x \Rightarrow \left(\left(3P/16\right) + \left(3P/4\right) \right) \times \frac{L}{AE}$$

$$\Rightarrow \left(\frac{15P}{16} \times \frac{L}{AE} \right)$$ [+ve x side]

$$(\delta_D)_y \Rightarrow \left(\frac{P\sqrt{3}}{4} \right) \times \frac{L}{AE}$$

$$\Rightarrow \left(\frac{\sqrt{3}P}{4} \times \frac{L}{AE} \right)$$ [-ve Y side]

$$\therefore (\delta_D) \Rightarrow \sqrt{(\delta D_x)^2 + (\delta D_y)^2}$$

$$\Rightarrow \frac{PL}{AE} \sqrt{\left(\frac{15}{16}\right)^2 + \left(\frac{3}{16}\right)}$$

$$\delta_D \Rightarrow \boxed{\frac{PL}{AE} \times 1.032}$$

(2.) Potential Energy in Elastic Beam

$\hookrightarrow \quad U \Rightarrow \frac{EI}{2}\int_0^a \left(\frac{d^2y}{dx^2}\right)^2 dx$

In our case, $\Rightarrow \frac{EI}{2}\int_0^L (y'')^2 dx$

$0 - L$

Potential Energy due to gravity $\rightarrow E \Rightarrow -mg \times y(L)$

Considering $F = \frac{EI}{2}(y'')^2$ & applying Variational Method,

$$\frac{EI}{2}\frac{d^2}{dx^2}\left(2y''\right) = 0$$

$$EI\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right) = 0$$

$$\therefore EI\frac{d^4y}{dx^4} = 0$$

App~~ying B/c~~

$$EI\frac{d^3y}{dx^3} \Rightarrow C_1 + m_1x$$

$$EI\frac{d^2y}{dx^2} \Rightarrow C_1x + C_2$$

$$EI\frac{dy}{dx} = \frac{C_1x^2}{2} + C_2x + C_3$$

$$y \Rightarrow \frac{1}{EI}\left(\frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4\right)$$

$\therefore y(0) = 0$

$$\Rightarrow \quad y \Rightarrow \frac{1}{EI}\left(\frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x\right)$$

☆ B/c for Mg on the End!

$$\theta = \frac{dy}{dx} \Rightarrow \theta(0) = 0 \Rightarrow \boxed{C_3 = 0}$$

$$y = \frac{1}{EI}\left(\frac{C_1x^3}{6} + \frac{C_2x^2}{2}\right)$$

Moment at free-End $=0$

$$\therefore \left.\frac{d\theta}{dx}\right|_{x=L} = 0 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=L} = 0.$$

$$\Rightarrow C_1 L + C_2 = 0$$

$$C_2 = -C_1 L \Rightarrow y \Rightarrow \frac{C_1}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)$$

$$V \Rightarrow \frac{-d}{dx} EI \frac{d^2u}{dx^2} = mg \quad \Big| \; \forall x \in (0, L)$$

$$\therefore -EI\left(\frac{1}{EI} \times C_1\right) = mg$$

$$\therefore C_1 = -mg.$$

$$\therefore \boxed{y = \frac{-mg}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)}$$

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(3.) $J[f] = \int_0^2 \left[\left(\frac{df}{dx} \right)^2 + f \right] dx.$

$$= \int_0^2 \left((f')^2 + f \right) dx \qquad a = 0, \quad b = 2$$

$\therefore F \Rightarrow f + (f')^2$

$$\therefore \frac{\partial F}{\partial f} - \frac{d}{dx}\left(\frac{\partial F}{\partial f'} \right) = 0$$

$$\Rightarrow \frac{\partial F}{\partial f} \Rightarrow 1 \quad ; \quad \frac{\partial F}{\partial f'} \Rightarrow 2(f')$$

$$\therefore 1 - \frac{d}{dx}(2f') = 0$$

$$1 - 2\frac{d^2 f}{dx^2} = 0$$

$$\frac{d^2 f}{dx^2} = \frac{1}{2} \Rightarrow f \Rightarrow \frac{x^2}{4} + c_1 x + c_2$$

$\frac{4}{4} + c_1(2) = 2$

$f(0) = 0 \Rightarrow c_2 = 0,$

$f(2) = 2 \Rightarrow c_1 = 1/2$

$$\Rightarrow f \Rightarrow \left(\frac{x^2}{4} + \frac{x}{2} \right)$$

A Random $f^n \Rightarrow \cancel{\frac{x^2}{3}} \cancel{+ \frac{3x}{3}}$

Another random f^n that satisfies the B/c

$$f(2) \Rightarrow \left.\frac{x^2}{8} + px\right|_2 = 2$$

$$\Rightarrow \frac{1}{2} + 2p = 2$$

$$\Rightarrow p \Rightarrow 3/4$$

$$\therefore f_{new}(x) \Rightarrow \frac{x^2}{8} + \frac{3}{4}x \qquad \frac{df}{dx} \Rightarrow \frac{x}{4} + \frac{3}{4}$$

$$\therefore J \Rightarrow \int_0^2 \left[\left(\frac{x+3}{4}\right)^2 + \left(\frac{x^2}{8} + \frac{3x}{4}\right)\right] dx$$

$$\Rightarrow \int_0^2 \left[\frac{x^2 + 9 + 6x}{4} + \frac{x^2}{8} + \frac{3x}{4}\right] dx$$

$$\Rightarrow \int_0^2 \left(\frac{3x^2}{8} + \frac{9x}{4} + \frac{9}{4}\right) dx$$

$$= 10 > 0$$