(2.) Potential Energy in Elastic Beam

$\hookrightarrow$ $U \Rightarrow \frac{EI}{2} \int_0^a \left(\frac{d^2y}{dx^2}\right)^2 dx$

In our case, $\Rightarrow \frac{EI}{2} \int_0^L (y'')^2 dx$

0 – L

Potential Energy due to gravity $\rightarrow$ $E \Rightarrow -mg \times y(L)$

Considering $F = \frac{EI}{2}(y'')^2$ & applying Variational Method,

$$\frac{EI}{2}\frac{d^2}{dx^2}\left(2y''\right) = 0$$

$$EI\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right) = 0$$

$$\therefore EI\frac{d^4y}{dx^4} = 0$$

Apply ~~B/c~~

$$EI\frac{d^3y}{dx^3} \Rightarrow c_1 + m_1 x$$

$$EI\frac{d^2y}{dx^2} \Rightarrow c_1 x + c_2$$

$$EI\frac{dy}{dx} = \frac{c_1 x^2}{2} + c_2 x + c_3$$

$$y \Rightarrow \frac{1}{EI}\left(\frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x + c_4\right)$$

$\therefore y(0) = 0$

$$\Rightarrow y \Rightarrow \frac{1}{EI}\left(\frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x\right)$$

☆ B/c for Mg on the End!

$$\theta = \frac{dy}{dx} \Rightarrow \theta(0) = 0 \Rightarrow \boxed{c_3 = 0}$$

$$y = \frac{1}{EI}\left(\frac{c_1 x^3}{6} + \frac{c_2 x^2}{2}\right)$$

Moment at free-End $=0$

$$\therefore \left.\frac{d\theta}{dx}\right|_{x=L} = 0 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=L} = 0.$$

$$\Rightarrow C_1 L + C_2 = 0$$

$$C_2 = -C_1 L \Rightarrow y \Rightarrow \frac{C_1}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)$$

$$V \Rightarrow -\frac{d}{dx} EI \frac{d^2u}{dx^2} = mg \quad \Big| \ \forall x \in (0,L)$$

$$\therefore -EI\left(\frac{1}{EI} \times C_1\right) = mg$$

$$\therefore C_1 = -mg.$$

$$\therefore \boxed{y = \frac{-mg}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)}$$