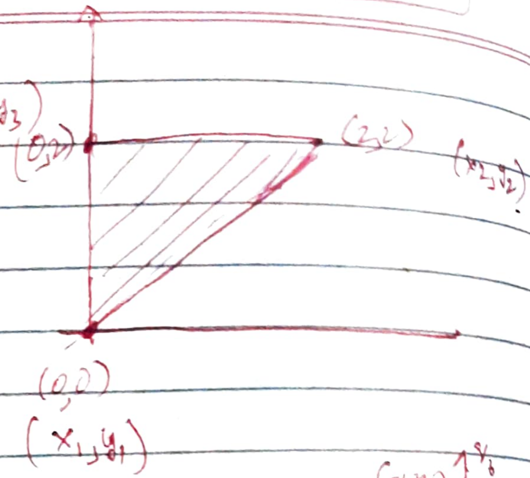
(Q1.)

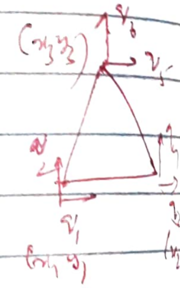
$E = 110\ kPa$

$\nu = 0.28$



~~Derivation~~

For Any triangle



$$N_1 \rightarrow \xi, \quad N_2 \rightarrow \eta, \quad N_3 = 1 - \xi - \eta$$

$$\begin{cases} u = N_1 q_1 + N_2 q_3 + N_3 q_5 \\ v = N_1 q_2 + N_2 q_4 + N_3 q_6 \end{cases}$$

$$\begin{cases} x = N_1 x_1 + N_2 x_2 + N_3 x_3 \\ y = N_1 y_1 + N_2 y_2 + N_3 y_3 \end{cases}$$

$$x = \underbrace{(x_1 - x_3)}_{x_{13}} \xi + \underbrace{(x_2 - x_3)}_{x_{23}} \eta + x_3$$

$$y = \underbrace{(y_1 - y_3)}_{y_{13}} \xi + \underbrace{(y_2 - y_3)}_{y_{23}} \eta + y_3$$

$$\frac{\partial u}{\partial \xi} \Rightarrow \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial \xi} \Rightarrow \quad J \Rightarrow \begin{bmatrix} \frac{\partial x}{\partial \xi} & \partial y/\partial \xi \\ \partial x/\partial \eta & \partial y/\partial \eta \end{bmatrix}$$

$$\frac{\partial u}{\partial \eta} \Rightarrow \frac{\partial u}{\partial x} \frac{\partial x}{\partial \eta} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial \eta}$$

PAGE NO.:
DATE: / /

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} \Rightarrow J^{-1} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix}$$

$$J^{-1} = \frac{1}{\det|J|} \begin{bmatrix} y_{23} & -y_{13} \\ -x_{23} & x_{13} \end{bmatrix}$$

$$\underline{\det|J| = 2A}$$

$$\begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{Bmatrix}$$

$$G \Rightarrow \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix}$$

$$G \Rightarrow \frac{1}{\det|J|} \begin{Bmatrix} y_{23}(q_1 - q_3) - y_{13}(q_3 - q_5) \\ -x_{23}(q_2 - q_6) + x_{13}(q_4 - q_6) \\ -x_{23} \quad - \; - \; - \; - \end{Bmatrix} \Longleftrightarrow \vec{B}\,\vec{q}$$

$$\therefore \vec{B} \Rightarrow \frac{1}{\det|J|} \begin{bmatrix} y_{23} & 0 & -y_{13} & 0 & \overbrace{y_{13} - y_{23}}^{y_{12}} & 0 \\ & & & & & \\ & & & & & \end{bmatrix}$$

$$\therefore \vec{B} \Rightarrow \frac{1}{\det|J|} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$\therefore$

My Derivation	$\Leftrightarrow$	Sir's Derivation
B	$\Leftrightarrow$	H
$\xi, \eta, 1-\xi-\eta$	$\Leftrightarrow$	$\alpha_0, \alpha_1, \alpha_2$
q	$\Leftrightarrow$	d

I have my own version Memorised & So proceeded with the original Notations

$$K^e \Rightarrow \int_{A^e} \vec{B}^T D \vec{B} \quad ; \quad D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

(i) $$B \Rightarrow \frac{1}{\det|J|}\begin{bmatrix} y_2-y_3 & 0 & y_3-y_1 & 0 & y_1-y_2 & 0 \\ 0 & x_3-x_2 & 0 & x_1-x_3 & 0 & x_2-x_1 \end{bmatrix}$$

Derived from Earlier

(ii) $$B = \frac{1}{4}\begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & 0 & 0 & 2 & 2 & -2 \end{bmatrix}$$

$x_1, y_1 \rightarrow 0, 0$

$x_2, y_2 \rightarrow 2, 2$

$x_3, y_3 \rightarrow 0, 2$

$\det|J| \Rightarrow 2A$

$\Rightarrow 4$

Acc^r of Sir $\Rightarrow A_e \times H^T D H$

or

$A_e \times B^T D B$

Matrix $\Rightarrow$ $9 \times$

$$\begin{bmatrix} 10.7422 & & & & & \\ 0 & 89.8384 & & & & \\ 0 & -8.3330 & 29.8394 & & & \\ -10.7422 & 0 & 0 & 10.7422 & & \\ -10.7422 & 8.3330 & -29.8394 & 10.7422 & 40.5816 & \\ 10.7422 & -29.8394 & 8.3330 & -10.7422 & -19.0972 & 40.5816 \end{bmatrix}$$

Diagonally Symmetric

$$\begin{bmatrix} 81.4844 & & & & & \\ 0 & 59.6788 & & & & \\ 0 & -16.7101 & 59.6788 & & & \\ -21.4844 & 0 & 0 & 81.4844 & & \\ -21.4844 & 16.7101 & -59.6788 & 81.4844 & 81.1632 & \\ 81.4844 & -59.6788 & 16.7101 & -21.4844 & -58.1746 & 81.1632 \end{bmatrix}$$

Diagonally Symmetric