Assignment 1

1.)

$f = n^2 - 8n + r^2 - 4r + 15$

$g_1 = r - 5 \leq 0$

$g_2 = r - 10/n \leq 0$

Case-1 : Both Constraints are active.

$L(n, r, \lambda_1, \lambda_2) = n^2 - 8n + r^2 - 4r + 15$
$+ \lambda_1 (r - 5)$
$+ \lambda_2 (r - 10/n)$

$L_n = 2n - 8 + \lambda_2 \left(\frac{10}{n^2}\right) = 0 \qquad \lambda_2 = 1.6$

$L_r = 2r - 4 + \lambda_1 + \lambda_2 = 0 \qquad \lambda_1 = -7.6.$

$L_{\lambda_1} = r - 5 = 0 \qquad \Rightarrow r = 5$

$L_{\lambda_2} = r - 10/n = 0 \qquad \Rightarrow n = 2$

$$\boxed{\begin{aligned} \lambda_1 + \lambda_2 &= -6 \\ 4 - 8 + \lambda_2 \left(\frac{10}{4}\right) &= 0 \\ \frac{16}{10} = \lambda_2 &\Rightarrow \lambda_2 = 1.6 \end{aligned}}$$

$\therefore$ here $\lambda_1 = -7.6$ $\rightarrow$ Solution is not optimal!

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Case 2 :-

None of Constraints are active.

$$L(x, r) = x^2 - 8x + r^2 - 4r + 15$$

$$L_x = 2x - 8 = 0 \Rightarrow x = 4$$

$$L_r = 2r - 4 = 0 \Rightarrow r = 2$$

$$g_1 = r - 5 \Rightarrow -3 \leq 0 \quad ✓$$

$$g_2 = r - 10/x \Rightarrow -0.5 \leq 0 \quad ✓$$

$$L(x, r) \Rightarrow 16 - 32 + 4 - 8 + 15 \Rightarrow \underline{\underline{-5}}$$

This is the optimal <u>solution</u>

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Case 3- g_1 is active

$$(n^2 - 8n + r^2 - 4r + 15) + \lambda_1 (r - 5) = 0$$

L_n $2n - 8 = 0$ $n = 4$

L_r $2r - 4 + \lambda_1 = 0$ $\lambda_1 \Rightarrow -6$ Not optimal.

L_{λ_1} $r = 5$

Case 4- g_2 is active

$$(n^2 - 8n + r^2 - 4r + 15) + \lambda_2 \left(r - \frac{10}{n}\right) = 0$$

L_n $2n - 8 + \lambda_2 (10/n^2)$

L_r $2r - 4 + \lambda_2 = 0$

L_{λ_2} $r - 10/n = 0$

$r = 10/n$

$$\frac{20}{n} - 4 + \lambda_2 = 0$$

$$\lambda_2 = \left(4 - \frac{20}{n}\right)$$

$$(2n - 8) + \left(4 - \frac{20}{n}\right)\left(\frac{10}{n^2}\right)$$

$$2n - 8 + \frac{40}{n^2} - \frac{200}{n^3} = 0$$

$$2n^4 - 8n^3 + 40n - 200 = 0$$

$$n^4 - 4n^3 + 20n - 100 = 0$$

$n = -2.8407$ $n = 4.2112$

$r = -3.521$ $r = 2.375$

$\lambda_2 = 11.042$ $\lambda_2 = -0.75$ X

$d = 72.2766$ → Not optimal

When the Problem Constraints change

$f = n^2 - 8n + r^2 - 4r + 15$

$g_1 = r - 5 \leq 0$

$g_2 = r - 6/n \leq 0$

since $(r - 6/n)$ for $r = 2$, $n = 4$ $\Rightarrow$ $0.5 \nleq 0$

$\therefore$ We take Case 1:- Constraint 2 (g_2) is active

$L(n, r, \lambda_2) = n^2 - 8n + r^2 - 4r + 15 + \lambda_2 (r - 6/n)$

$L_n = 2n - 8 + \lambda_2 (6/n^2) = 0$

$L_r = 2r - 4 + \lambda_2 = 0$

$L_{\lambda_2} = r - 6/n = 0$

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$x = \frac{6}{n}$; $2x + \lambda_2 = 4$; $2n + \frac{6\lambda_2}{n^2} = 8$

$2\left(\frac{6}{n}\right) + \lambda_2 = 4$

$2n + \frac{6}{n^2}\left(4 - \frac{12}{n}\right) = 8$

$\lambda_2 = 4 - \frac{12}{n}$

$2n + \frac{24}{n^2} - \frac{72}{n^3} = 8$

$2n^4 + 24n - 72 = 8n^3$

$2n^4 - 8n^3 + 24n - 72 = 0$

Real roots :- -2.1581 / 3.8232

$\lambda_2 = 9.570 \rightarrow +ve$

$n = -2.1580$

$x = -2.780$

$g_1 \leq 0$, $g_2 = 0$. } Constraints Satisfied.

$f \Rightarrow 55.7694$

$\lambda_2 = 0.862 \rightarrow +ve$

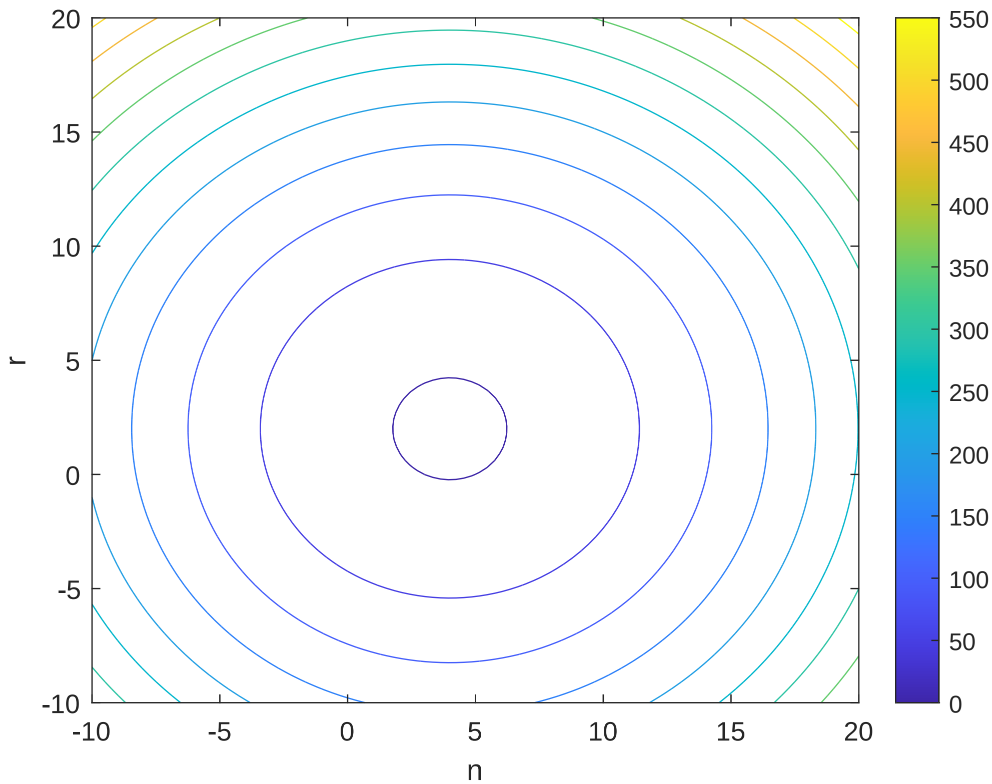
$n = 3.823$

$x = 1.5790$

$g_1 \leq 0$, $g_2 = 0$ {Constraints Satisfied

$f \Rightarrow -4.7829$

This is the optimal Solution



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