## Proportional Navigation

Robin Mills

December 22, 2015

Proportional navigation Assume this version of proportional navigation:

$$\frac{d\gamma}{dt} = N\frac{d\theta}{dt} \tag{1}$$

Here,  $\theta$  is the change in line-of-sight. The line-of-sight, in the angle of the vector connecting the predator and the prey. Thus the change in line-of-sight in the change in this relative angle (fixing the predator as the base of the coordinate system and measuring how fast the prey rotates around it. The  $\gamma$  is the angle of the velocity of the predator, and N is some proportionality constant that can be tweaked to optimize interception. Thus the change in the angle of the heading of the predator is proportional to the change in line-of-sight.

How can we implement this strategy in code?

This angular velocity of the line of sight is calculated with

$$\omega = \theta = \frac{|v_r|sin\psi}{|r|} = \frac{r \times v_r}{|r|^2} \tag{2}$$

Where r is the difference in position between prey and predator, and  $v_r$  is the difference in velocity,  $\times$  denotes the cross product,  $\psi$  is the angle between r and  $v_r$ . The reasoning behind the equation is as follows (see https://en.wikipedia.org/wiki/Angular\_velocity). We want to get the element of the difference in velocity which is perpendicular to the difference in position. We know that:

$$|\perp v| = \sin \psi * |v| = \frac{r \times v_r}{|r| * |v|} * |v| = \frac{r \times v_r}{|r|}$$
 (3)

We divide this by the length of r again to get  $\omega$ . This relation between the cross product and the sine can be found here: https://en.wikipedia.org/wiki/Cross\_product under geometric meaning.