07

SABARAGAMUWA UNIVERSITY OF SRI LANKA
FACULTY OF MANAGEMENT STUDIES

BSc DEGREE PROGRAMME IN BANKING & INSURANCE
YEAR III SEMESTER I EXAMINATION – SEPTEMBER/OCTOBER 2017

BASIC ACTUARIAL METHODS IN INSURANCE - BI 3153 ®

Instruction to Candidate;

- Answer All Questions.
- Time Allowed : Three (03) hours
- Total Number of Questions : 05
- Use of actuarial notations is mandatory when required
- Calculators are permitted

1 i Express the following terms in Words

a) P_{35}
b) $10|5q_{20}$
c) $10q_{40}$
d) $15q_{50}$

(2 x 4 = 08 Marks)

ii Express $4|5q_{50}$ in p and q terms. (04 Marks)

iii Express the following terms in l terms

a) P_{40}
b) $5|2q_{40}$
c) $5q_{25}$
d) $20q_{40}$

(2 x 4 = 08 Marks)
(Total 20 Marks)

2 i Use the Life table provided to you **(Male Table) and calculate the following probabilities.**

a) The probability of a life aged 20 to survive to age 40.
b) The probability of a life (65) will die between ages 80 and 90.
c) The probability that a person now aged 20 will reach retirement age of 65.
d) Probability that a life aged 30 dies between the ages 35 and 36.

(3 x 4 = 12 Marks)

ii Consider the following survival function.

$$s(x) = 1 - \frac{x^2}{36} \qquad (for\ 0 \leq x \leq 6)$$

and

$$s'(x) = \frac{-2x}{36}$$

Force of mortality : $\mu_x = \frac{s'(x)}{s(x)}$

Find an expression for μ_x

By using that calculate μ_3

(4 x 2 = 08 Marks)

(Total 20 Marks)

3 i Complete the following table for a particular population.

x	q_x	l_x	d_x	L_x	T_x	e_x
1	0.3	1000				
2	0.1	700				
3	0.2	630				
4	0.4	504				
5	0.7	30				
6	1.0	90				
		0				

(1/2 x 24 = 12 Marks)

Note :

$$d_x = l_x - l_{x+1}$$

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$T_x = l_x + T_{x+1}$$

$$e_x = \frac{T_x}{l_x}$$

2

ii If m_x, age specific death rate or the central death rate ate age x is defined by the following formula, calculate m_x for the above population for each and every x.

$$m_x = \frac{d_x}{L_x}$$

(1 x 6 = 06 Marks)

iii Calculate ${}_2P_2$ for the above population. (02 Marks)

(Total 20 Marks)

18

4 i Using the standard notations show that $iv = d$ (04 Marks)

ii A certain annuity (Assume Rs. X) paid in advance for 5 years under 6% effective annual rate of interest. Stating the actuarial notations correctly, find the present value and accumulated value of that annuity. (04 Marks)

iii Find the present value and accumulated value of annuity amount to Rs. 25,000 payable at the end of the each year for 10 years under the annual effective rate of interest of 7.5%. (06 Marks)

iv **Neraga** has borrowed **X** from **Harshani** for **10 years** at an **annual effective rate of 6.5%**. If Sandun pays the principal and accumulated interest in one lump sum (at once) at the end of 10 years, he would pay **Rs. 486.091** more in interest than if he had repaid the loan with 10 level payments (Annuities) at the end of the each year. **Calculate X.** (06 Marks)

5 Samanmali borrowed Rs. 20,000,000 for 10 years at a rate of interest of 9.5% per annum. She agreed to repay the loan by equal 10 installments which will be payable at the end of the each year.

i. Calculate the annual installment. (04 Marks)

ii Prepare the loan repayment schedule for four years starting from year 1 (1/2 x 12 = 06 Marks)

iii Calculate the interest portion of the 9^{th} payment by using the formula. (05 Marks)

iv Calculate the loan balance after payment of 08^{th} Payment. (05 Marks)

(Total 20 Marks)

Annexure - List of formulae

Annuity Immediate

$$a_{\overline{n}|i} = v + v^2 + \cdots + v^n = \frac{1-v^n}{i} \qquad s_{\overline{n}|i} = (1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1 = \frac{(1+i)^n - 1}{i}$$

$$a_{\overline{n}|i} = v^n \cdot s_{\overline{n}|i} \qquad s_{\overline{n}|i} = (1+i)^n \cdot a_{\overline{n}|i}$$

Annuity Due

$$\ddot{a}_{\overline{n}|i} = 1 + v + \cdots + v^{n-1} = \frac{1-v^n}{d} \qquad \ddot{s}_{\overline{n}|i} = (1+i)^n + (1+i)^{n-1} + \cdots + (1+i) = \frac{(1+i)^n - 1}{d}$$

$$\ddot{a}_{\overline{n}|i} = v^n \cdot \ddot{s}_{\overline{n}|i} \qquad \ddot{s}_{\overline{n}|i} = (1+i)^n \cdot \ddot{a}_{\overline{n}|i}$$

Perpetuity

$$a_{\overline{\infty}|i} = \lim_{n\to\infty} a_{\overline{n}|i} = v + v^2 + v^3 + \cdots = \frac{1}{i} \qquad \ddot{a}_{\overline{\infty}|i} = \lim_{n\to\infty} \ddot{a}_{\overline{n}|i} = \frac{1}{d}$$

Continuous Annuities

$$\bar{a}_{\overline{n}|i} = \frac{1-v^n}{\delta} = \frac{i}{\delta} a_{\overline{n}|i} \qquad \bar{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{\delta} = \frac{i}{\delta} s_{\overline{n}|i} \qquad \bar{a}_{\overline{n}|i} = \int_0^n v^t dt$$

$$PV = \int_0^n e^{-\int_0^t \delta(u)du} p(t)dt \qquad FV = \int_0^n e^{\int_t^n \delta(u)du} p(t)\,dt \quad \text{where } p(t) = \text{payment function}$$

Increasing Annuities— Payments are $1, 2, \ldots, n$

$$(Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i} \qquad (I\bar{a})_{\overline{n}|i} = \frac{i}{d}(Ia)_{\overline{n}|i} = (1+i)(Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

$$(Is)_{\overline{n}|i} = (1+i)^n (Ia)_{\overline{n}|i} = \frac{\ddot{s}_{\overline{n}|} - n}{i} \qquad (I\bar{s})_{\overline{n}|i} = \frac{i}{d}(Is)_{\overline{n}|i} = (1+i)^n (I\bar{a})_{\overline{n}|i} = \frac{\ddot{s}_{\overline{n}|} - n}{d}$$

$$(Ia)_{\overline{\infty}|i} = \lim_{n\to\infty}(Ia)_{\overline{n}|i} = \frac{1}{di} = \frac{1}{i} + \frac{1}{i^2} \qquad (I\bar{a})_{\overline{\infty}|i} = \lim_{n\to\infty}(I\bar{a})_{\overline{n}|i} = \frac{1}{d^2}$$

Decreasing Annuities— Payments are $n, n\text{-}1, \ldots, 2, 1$

$$(Da)_{\overline{n}|i} = \frac{n - a_{\overline{n}|i}}{i} \qquad (D\bar{a})_{\overline{n}|i} = \frac{i}{d}(Da)_{\overline{n}|i} = (1+i)(Da)_{\overline{n}|i} = \frac{n - a_{\overline{n}|i}}{d}$$

$$(Ds)_{\overline{n}|i} = (1+i)^n (Da)_{\overline{n}|i} = \frac{n(1+i)^n - s_{\overline{n}|i}}{i} \qquad (D\bar{s})_{\overline{n}|i} = (1+i)^n (D\bar{a})_{\overline{n}|i}$$

English Life Table No 15.1

Age	Males			Females		
x	l_x	q_x	e_x	l_x	q_x	e_x
0	100000	.00814	73.413	100000	.00532	78.956
1	99186	.00062	73.019	99368	.00055	78.462
2	99124	.00038	72.064	99313	.00030	77.506
3	99086	.00030	71.091	99283	.00022	76.528
4	99056	.00024	70.113	99261	.00018	75.545
5	99032	.00022	69.130	99243	.00016	74.559
6	99010	.00020	68.145	99228	.00015	73.570
7	98990	.00019	67.158	99213	.00014	72.581
8	98972	.00018	66.171	99199	.00014	71.591
9	98953	.00018	65.183	99185	.00013	70.601
10	98935	.00018	64.195	99172	.00013	69.610
11	98917	.00018	63.206	99159	.00014	68.620
12	98899	.00019	62.218	99145	.00014	67.629
13	98880	.00023	61.230	99131	.00015	66.638
14	98857	.00029	60.244	99116	.00018	65.649
15	98828	.00040	59.261	99098	.00022	64.660
16	98789	.00052	58.285	99077	.00026	63.674
17	98737	.00075	57.315	99051	.00031	62.691
18	98663	.00087	56.358	99020	.00031	61.710
19	98577	.00083	55.406	98989	.00032	60.729
20	98496	.00084	54.452	98957	.00031	59.748
21	98413	.00086	53.497	98926	.00032	58.767
22	98328	.00089	52.543	98894	.00033	57.786
23	98241	.00089	51.589	98862	.00033	56.805
24	98154	.00088	50.635	98829	.00033	55.823
25	98067	.00088	49.679	98797	.00034	54.842
26	97980	.00089	48.721	98763	.00035	53.860
27	97890	.00095	47.762	98729	.00036	52.878
28	97817	.00097	46.802	98694	.00038	51.897
29	97732	.00098	45.842	98655	.00040	50.917
30	97645	.00091	44.883	98617	.00043	49.937
31	97556	.00094	43.923	98574	.00047	48.958
32	97465	.00097	42.964	98528	.00052	47.981
33	97370	.00099	42.005	98477	.00057	47.006
34	97273	.00106	41.045	98420	.00063	46.032
35	97170	.00116	40.090	98359	.00069	45.061
36	97057	.00127	39.136	98291	.00075	44.092
37	96933	.00138	38.185	98217	.00082	43.124
38	96800	.00149	37.237	98136	.00090	42.160
39	96655	.00160	36.292	98048	.00098	41.197
40	96500	.00172	35.349	97952	.00107	40.237
41	96334	.00186	34.409	97847	.00117	39.279
42	96155	.00201	33.473	97733	.00129	38.325
43	95961	.00219	32.539	97607	.00142	37.374
44	95751	.00240	31.609	97469	.00158	36.426
45	95521	.00265	30.684	97315	.00177	35.483
46	95266	.00297	29.765	97142	.00198	34.546
47	94983	.00332	28.852	96950	.00219	33.612
48	94668	.00371	27.947	96738	.00241	32.685
49	94316	.00415	27.049	96504	.00266	31.763
50	93925	.00464	26.159	96247	.00294	30.846
51	93489	.00519	25.279	95964	.00326	29.936
52	93004	.00577	24.408	95652	.00357	29.032
53	92457	.00642	23.547	95310	.00390	28.134
54	91873	.00714	22.696	94938	.00428	27.242
55	91217	.00797	21.856	94532	.00475	26.357
56	90490	.00890	21.027	94082	.00531	25.481
57	89684	.00995	20.211	93583	.00592	24.614
58	88792	.01112	19.409	93029	.00660	23.757
59	87805	.01243	18.622	92415	.00739	22.912
60	86714	.01392	17.850	91732	.00830	22.079
61	85507	.01560	17.095	90971	.00922	21.259
62	84173	.01749	16.357	90132	.01015	20.452
63	82701	.01965	15.640	89217	.01129	19.657
64	81076	.02199	14.943	88210	.01256	18.875
65	79293	.02447	14.267	87093	.01399	18.111
66	77353	.02711	13.612	85875	.01523	17.361
67	75256	.02997	12.978	84567	.01676	16.621
68	73001	.03292	12.363	83160	.01844	15.896
69	70595	.03602	11.767	81617	.02017	15.185
70	68055	.03930	11.187	79970	.02190	14.487
71	65391	.04311	10.624	78219	.02399	13.800
72	62562	.04745	10.089	76343	.02693	13.127
73	59593	.05217	9.557	74287	.03014	12.476
74	56484	.05697	9.056	72049	.03264	11.848
75	53266	.06197	8.572	69682	.03569	11.234
76	49965	.06777	8.106	67195	.03919	10.631
77	46579	.07418	7.658	64551	.04358	10.044
78	43124	.08101	7.232	61749	.04833	9.478
79	39630	.08835	6.825	58766	.05373	8.930
80	36128	.09616	6.438	55607	.05991	8.413
81	32654	.10411	6.070	52283	.06668	7.914
82	29254	.11279	5.718	48858	.07216	7.435
83	25954	.12235	5.382	45332	.07933	6.974
84	22779	.13278	5.063	41736	.08757	6.532
85	19756	.14372	4.762	38081	.09731	6.111
86	16917	.15585	4.478	34375	.10933	5.715
87	14280	.16848	4.213	30651	.11859	5.349
88	11874	.18061	3.968	27017	.12850	5.002
89	9730	.19249	3.734	23542	.14146	4.667
90	7857	.20465	3.508	20212	.15580	4.334
91	6249	.21911	3.285	17069	.17096	4.065
92	4880	.23656	3.071	14186	.18573	3.797
93	3726	.25575	2.872	11535	.20126	3.551
94	2773	.27483	2.693	9214	.21780	3.322
95	2011	.29311	2.531	7206	.23619	3.112
96	1421	.31104	2.383	5504	.25344	2.925
97	979	.32918	2.244	4109	.26820	2.754
98	657	.34783	2.114	3007	.28352	2.588
99	428	.36712	1.991	2154	.30331	2.422
100	271	.38705	1.874	1501	.32489	2.269
101	165	.40760	1.764	1013	.34562	2.133
102	98	.42870	1.660	663	.36196	2.011
103	56	.45030	1.562	423	.37992	1.887
104	31	.47428	1.468	262	.40045	1.768
105	16	.49634	1.384	157	.43618	1.621
106	8	.51841	1.306	89	.45984	1.518
107	4	.54041	1.234	48	.48389	1.425
108	2	.56225	1.166	25	.50791	1.338
109	1	.58385	1.104	12	.53190	1.257
110				6	.55574	1.183
111				3	.57932	1.114
112				1	.60255	1.050
113						
114						

English Life Table No. 15.2

Age	Males			Females		
x	d_x	T_x	μ_x	d_x	T_x	μ_x
0	814	7341337		632	7895613	
1	62	7242424	.00080	55	7796600	.00073
2	38	7143272	.00043	30	7697262	.00035
3	30	7044168	.00033	22	7597965	.00025
4	24	6945097	.00027	18	7498693	.00020
5	22	6846053	.00023	15	7399441	.00017
6	20	6747032	.00021	13	7300206	.00015
7	18	6648032	.00019	14	7200986	.00014
8	19	6549051	.00018	14	7101780	.00014
9	18	6450088	.00018	13	7002588	.00014
10	18	6351144	.00018	13	6903409	.00013
11	18	6252218	.00018	14	6804244	.00014
12	19	6153309	.00019	14	6705092	.00014
13	23	6054419	.00021	15	6605953	.00014
14	29	5955550	.00026	18	6506829	.00017
15	59	5856707	.00034	21	6407722	.00020
16	52	5757897	.00045	26	6308634	.00024
17	74	5659132	.00064	31	6209570	.00029
18	86	5560431	.00083	31	6110535	.00031
19	81	5461810	.00085	32	6011530	.00032
20	83	5363274	.00083	31	5912557	.00032
21	85	5264819	.00085	32	5813615	.00032
22	87	5166448	.00088	32	5714704	.00032
23	87	5068163	.00089	33	5615826	.00033
24	87	4969966	.00089	32	5516980	.00033
25	84	4871856	.00087	34	5418167	.00033
26	83	4773833	.00085	34	5319387	.00034
27	83	4675889	.00084	35	5220641	.00035
28	85	4578030	.00086	38	5121929	.00037
29	87	4480255	.00088	39	5023254	.00039
30	89	4382566	.00090	43	4924617	.00042
31	91	4284966	.00092	46	4826021	.00045
32	95	4187455	.00096	51	4727470	.00050
33	97	4090037	.00098	57	4628967	.00054
34	102	3992715	.00102	61	4530518	.00060
35	113	3895493	.00111	68	4432128	.00066
36	124	3798379	.00122	74	4333803	.00072
37	133	3701383	.00133	81	4235549	.00079
38	145	3604515	.00144	89	4137372	.00086
39	153	3507787	.00155	96	4039279	.00094
40	166	3411208	.00166	105	3941278	.00102
41	179	3314791	.00179	114	3843377	.00112
42	194	3218545	.00193	126	3745586	.00123
43	210	3122486	.00210	138	3647916	.00135
44	230	3026629	.00229	154	3550377	.00149
45	255	2930992	.00253	173	3452984	.00167
46	283	2835596	.00281	192	3355754	.00187
47	315	2740469	.00314	212	3258706	.00208
48	352	2645640	.00352	234	3161861	.00230
49	391	2551145	.00393	257	3065238	.00253
50	436	2457021	.00440	283	2968860	.00280
51	485	2363311	.00492	312	2872752	.00310
52	537	2270060	.00549	342	2776942	.00342
53	594	2177320	.00610	372	2681459	.00374
54	656	2085145	.00679	406	2586332	.00408
55	727	1993595	.00757	450	2491594	.00451
56	806	1902735	.00845	499	2397283	.00503
57	892	1812641	.00945	554	2303446	.00562
58	987	1723395	.01057	614	2210135	.00626
59	1091	1635068	.01182	685	2117408	.00700

Age	Males			Females		
x	d_x	T_x	μ_x	d_x	T_x	μ_x
60	1207	1547820	.01323	761	2025328	.00786
61	1334	1461700	.01483	839	1933970	.00880
62	1472	1376849	.01664	915	1843412	.00972
63	1625	1293400	.01870	1007	1753731	.01074
64	1783	1211498	.02101	1117	1665008	.01203
65	1940	1131301	.02348	1218	1577348	.01342
66	2097	1052963	.02610	1308	1490856	.01470
67	2255	976647	.02893	1417	1405626	.01609
68	2403	902506	.03192	1533	1321758	.01774
69	2543	830695	.03505	1647	1239366	.01949
70	2674	761357	.03833	1751	1158563	.02123
71	2819	694628	.04198	1876	1079459	.02311
72	2969	630644	.04626	2056	1002166	.02559
73	3109	569554	.05105	2239	926835	.02897
74	3218	511505	.05609	2366	853654	.03203
75	3301	456622	.06123	2487	782780	.03480
76	3386	404999	.06694	2634	714351	.03803
77	3455	356720	.07352	2812	648439	.04214
78	3494	311864	.08068	2984	585270	.04694
79	3502	270485	.08840	3158	524998	.05228
80	3474	232697	.09675	3314	467798	.05827
81	3400	198221	.10544	3435	413837	.06464
82	3300	167274	.11464	3526	363253	.07131
83	3175	139679	.12491	3596	316152	.07861
84	3023	115324	.13627	3655	272612	.08691
85	2839	94071	.14857	3706	232699	.09574
86	2637	75751	.16208	3724	196467	.10841
87	2406	60170	.17689	3634	163956	.12052
88	2144	47113	.19190	3475	135134	.13174
89	1873	36334	.20647	3330	109867	.14462
90	1608	27563	.22114	3143	88003	.16053
91	1369	20530	.23754	2903	69381	.17751
92	1154	14985	.25793	2631	53784	.19573
93	953	10699	.28226	2321	40958	.21498
94	762	7466	.30837	2008	30610	.23490
95	590	5090	.33424	1702	22426	.25732
96	442	3387	.35974	1395	16097	.28114
97	322	2198	.38579	1102	11316	.30267
98	229	1389	.41313	853	7781	.32241
99	157	853	.44216	633	5219	.34628
100	105	508	.47312	488	3406	.37671
101	68	293	.50609	350	2161	.40887
102	42	163	.54117	240	1334	.43769
103	25	88	.57832	161	798	.46273
104	15	45	.61901	105	461	.49303
105	8	22	.66418	68	255	.53729
106	4	11	.70630	41	135	.59908
107	2	5	.75113	23	68	.63785
108	1	2	.79741	13	33	.68388
109	1	1	.84499	6	15	.73191
110				3	7	.78181
111				2	3	.83337
112				1	1	.88529
113						
114						