# Supplementary File for "Handling Constrained Many-objective Optimization Problems via Problem Transformation"

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## S-I. Parameter settings

The characteristics of the constrained PF and the maximum number of function evaluations for CF suite [1], MW suite [2], DAS-CMOP suite [3], C-DTLZ suite [4], and DC-DTLZ suite [5] are given in Table S-I and Table S-II, respectively, where $m$ and $n$ are the number of objectives and decision variables, respectively. $FEs$ represents the number of function evaluations. In our experiment, some constraint function parameters of these test problems were set as follows:

- For C1-DTLZ3: $r = \{9, 12.5, 12.5, 15, 15\}$ for $m = \{3, 5, 8, 10, 15\}$;
- For C2-DTLZ2: $r = \{0.4, 0.5, 0.5, 0.5, 0.5\}$ for $m = \{3, 5, 8, 10, 15\}$;
- For DC1-DTLZ1 and DC1-DTLZ3 problems: $a$=5, $b = 0.95$;
- For DC2-DTLZ1 and DC2-DTLZ3 problems: $a$=3, $b = 0.9$;
- For DC3-DTLZ1 and DC3-DTLZ3 problems: $a$=5, $b = 0.5$.

TABLE S-I
Characteristics of constrained PF of different test problems.

| Problem | Geometry | Connectivity |
|---|---|---|
| CF8 | Concave | Disconnected |
| CF9 | Concave | Disconnected |
| CF10 | Concave | Disconnected |
| MW4 | Linear | Connected |
| MW8 | Concave | Disconnected |
| MW14 | Scaled | Disconnected |
| DAS-CMOP7 | Linear | Disconnected |
| DAS-CMOP8 | Concave | Disconnected |
| DAS-CMOP9 | Concave | Disconnected |
| C1-DTLZ1 | Linear | Connected |
| C1-DTLZ3 | Concave | Connected |
| C2-DTLZ2 | Concave | Disconnected |
| C3-DTLZ1 | Convex | Connected |
| C3-DTLZ4 | Mixed | Connected |
| DC1-DTLZ1 | Linear | Disconnected |
| DC1-DTLZ3 | Concave | Disconnected |
| DC2-DTLZ1 | Linear | Connected |
| DC2-DTLZ3 | Concave | Connected |
| DC3-DTLZ1 | Linear | Disconnected |
| DC3-DTLZ3 | Concave | Disconnected |

## S-II. Detailed results in terms of IGD and HV indicators

Table S-III and Table S-IV list the average and standard deviation of IGD and HV values over 30 runs for seven CMaOEAs: A-NSGA-III [4], C-MOEA/D [4], I-DBEA [6], PPS [7], C-TAEA [5], C-AnD [8], and the proposed DCNSGA-III, respectively. The best results for each instance are highlighted with a gray background.

## S-III. Effectiveness of the mating selection

In the high-dimensional objective space, the choice of parents from the entire population is not a good idea since it may degrade the performance of a MaOEA [9], [10]. To improve the effects of the genetic operation, in the mating selection operator of DCNSGA-III, an $\varepsilon$-feasible solution was selected over an $\varepsilon$-infeasible solution or a solution with a smaller constraint violation between two $\varepsilon$-infeasible solutions is chosen. In order to verify this mating selection operator is necessary for DCNSGA-III, an additional experiment was executed. We compare two other mating selection operators. The first one is the same as C-NSGA-III and A-NSGA-III [4], where a feasible solution will be chosen over an infeasible one, or a solution with a smaller constraint violation will be selected between two infeasible solutions, or pick one at random from two feasible solutions. We denote this operator as $Variant2$. The other mating selection is randomly choosing one from two solutions in binary tournament selection regardless of their constraint violation, and this operator is referred as $Variant3$. The mating selection operator in the proposed DCNSGA-III is called $Variant1$. We compare the feasibility ratio of these three operators. The feasibility ratio equals the

TABLE S-III
COMPARATIVE RESULTS OF IGD VALUES (MEAN AND STANDARD DEVIATION) OBTAINED BY A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AND, AND DCNSGA-III, RESPECTIVELY.

| Problem | $m$ | A-NSGA-III | C-MOEA/D | I-DBEA | PPS | C-TAEA | C-AnD | DCNSGA-III |
|---|---|---|---|---|---|---|---|---|
| CF8 | 3 | Infeasible | Infeasible | Infeasible | 1.1713e-1 (1.06e-2)− | 3.8116e-1 (1.33e-1)+ | Infeasible | 2.6301e-1 (1.02e-1) |
| CF9 | 3 | 1.7495e-1 (1.18e-1)≈ | 9.0134e-2 (6.69e-3)− | 5.6681e-1 (2.03e-1)+ | 5.1617e-2 (6.90e-3)− | 1.1855e-1 (1.95e-2)≈ | 1.1281e-1 (2.64e-2)≈ | 1.6353e-1 (8.86e-2) |
| CF10 | 3 | Infeasible | Infeasible | Infeasible | 1.8890e-1 (7.75e-2)− | 5.2363e-1 (1.10e-1)+ | Infeasible | 3.0136e-1 (9.59e-2) |
| MW4 | 3 | 4.3700e-2 (1.44e-3)+ | 6.0256e-2 (5.41e-3)+ | 5.9749e-1 (1.78e-1)+ | Infeasible | 4.2191e-2 (1.49e-4)≈ | 4.9214e-2 (1.13e-2)+ | 4.2514e-2 (8.49e-4) |
| MW8 | 3 | 8.1382e-2 (1.09e-1)≈ | 5.1357e-2 (2.61e-3)+ | 7.9668e-1 (1.58e-1)+ | 1.3604e-1 (6.24e-2)+ | 4.9345e-2 (2.58e-3)≈ | 1.8619e-1 (1.38e-1)+ | 4.9645e-2 (2.15e-3) |
| MW14 | 3 | 1.2103e-1 (2.09e-2)− | 2.1889e-1 (3.00e-2)+ | 2.7402e+0 (6.39e-1)+ | 2.4732e-1 (8.36e-2)+ | 1.0784e-1 (7.81e-3)− | 4.3795e-1 (1.66e-1)+ | 1.3346e-1 (3.64e-3) |
| DASCMOP7 | 3 | 5.1396e-2 (8.71e-3)≈ | 1.2545e-1 (1.39e-1)+ | 1.1370e+0 (2.94e-1)+ | 9.2612e-2 (9.25e-2)+ | 4.3713e-2 (2.02e-2)− | 4.5122e-2 (2.18e-2)− | 4.8872e-2 (9.72e-3) |
| DASCMOP8 | 3 | 1.2892e-1 (1.59e-1)+ | 1.4310e-1 (1.06e-1)+ | 1.1315e+0 (2.31e-1)+ | 1.1646e-1 (5.06e-2)+ | 7.3195e-2 (2.88e-2)+ | 5.1216e-2 (3.36e-3)≈ | 5.1206e-2 (3.50e-3) |
| DASCMOP9 | 3 | 3.3171e-1 (3.60e-2)≈ | 9.9844e-2 (9.13e-3)− | 6.4437e-1 (7.38e-2)+ | 4.6456e-2 (5.87e-3)− | 1.6879e-1 (3.21e-2)− | 3.0212e-1 (9.26e-2)− | 3.9821e-1 (1.59e-1) |
| C1-DTLZ1 | 3 | 2.2535e-2 (1.68e-3)+ | 2.0813e-2 (3.40e-4)+ | 4.1450e-1 (7.19e-2)+ | 3.0376e-2 (6.72e-3)+ | 2.3287e-2 (6.84e-4)+ | 2.2823e-2 (3.74e-4)+ | 2.0375e-2 (3.21e-4) |
| | 5 | 5.2074e-2 (4.44e-4)≈ | 5.2288e-2 (1.58e-4)+ | 4.5171e-1 (8.68e-2)+ | Infeasible | 5.9592e-2 (4.45e-4)+ | 5.3650e-2 (3.70e-4)+ | 5.2082e-2 (3.98e-4) |
| | 8 | 1.0243e-1 (1.36e-2)+ | 9.3252e-2 (3.29e-4)− | 4.8486e-1 (7.87e-2)+ | 1.1148e-1 (1.59e-2)+ | 1.2115e-1 (9.64e-4)+ | 1.0898e-1 (1.74e-2)+ | 9.7726e-2 (9.32e-3) |
| | 10 | 1.1634e-1 (1.74e-2)+ | 1.0036e-1 (3.68e-4)− | 4.8149e-1 (9.52e-2)+ | 1.2379e-1 (4.08e-3)+ | 1.3979e-1 (1.70e-3)+ | 1.1292e-1 (9.06e-4)+ | 1.0917e-1 (5.09e-3) |
| | 15 | 1.8818e-1 (1.32e-2)+ | 1.2703e-1 (7.25e-4)− | 5.4833e-1 (4.11e-2)+ | 1.5581e-1 (1.66e-2)− | 2.1419e-1 (5.36e-3)+ | 1.6736e-1 (2.79e-3)− | 1.7643e-1 (1.13e-2) |
| C1-DTLZ3 | 3 | 1.0786e+0 (1.39e+0)− | 1.5652e+0 (4.38e+0)+ | 2.3378e+0 (1.24e+0)+ | 6.6099e-1 (1.66e+0)− | 1.4430e+0 (1.49e+0)+ | 2.5902e+0 (1.07e+0)+ | 1.2695e+0 (1.45e+0) |
| | 5 | 1.7576e-1 (1.30e-2)+ | 5.0539e-1 (1.02e+0)+ | 1.6531e-1 (3.58e-4)≈ | 6.6550e-1 (9.13e-1)+ | 3.5807e-1 (5.63e-2)+ | 1.7861e-1 (4.84e-3)+ | 1.6520e-1 (5.71e-5) |
| | 8 | 5.3340e-1 (1.27e-1)+ | 5.7731e-1 (5.74e-1)+ | 1.2085e+0 (2.97e-2)+ | 5.6555e-1 (3.48e-2)+ | 5.1777e-1 (2.08e-2)+ | 6.4301e-1 (3.56e-1)+ | 3.7026e-1 (1.14e-1) |
| | 10 | 6.2231e-1 (6.86e-2)+ | 4.7150e-1 (4.52e-3)+ | 1.2417e+0 (2.18e-6)+ | 6.6143e-1 (2.01e-2)+ | 5.4068e-1 (1.43e-2)+ | 5.5324e-1 (5.70e-2)+ | 4.7022e-1 (8.33e-2) |
| | 15 | 1.7201e+0 (1.28e+0)+ | 6.5922e-1 (6.27e-3)− | 1.2896e+0 (1.23e-5)+ | 8.6588e-1 (5.85e-2)+ | 7.9537e-1 (2.76e-1)≈ | 1.5927e+0 (7.63e-1)+ | 7.3584e-1 (3.35e-2) |
| C2-DTLZ2 | 3 | 4.4212e-2 (5.23e-4)− | 6.4049e-2 (3.14e-3)+ | 8.3458e-1 (1.64e-1)+ | 3.3002e-2 (5.84e-4)− | 5.6375e-2 (1.98e-3)+ | 6.3248e-2 (8.73e-2)+ | 4.8415e-2 (4.50e-4) |
| | 5 | 1.4334e-1 (4.64e-2)+ | 1.6285e-1 (2.80e-3)+ | 1.0622e+0 (1.88e-1)+ | 2.8406e-1 (4.29e-2)+ | 1.4676e-1 (1.08e-3)+ | 1.3854e-1 (1.05e-3)− | 1.3933e-1 (1.25e-3) |
| | 8 | 3.6654e-1 (2.19e-1)+ | 4.4142e-1 (2.55e-1)+ | 1.2439e+0 (1.58e-1)+ | 4.5042e-1 (7.50e-2)+ | 2.3792e-1 (1.48e-3)≈ | 2.8169e-1 (1.09e-1)+ | 2.4941e-1 (4.87e-2) |
| | 10 | 4.1815e-1 (1.94e-1)≈ | 3.6840e-1 (2.06e-1)+ | 1.2783e+0 (1.01e-1)+ | 4.6369e-1 (4.24e-2)+ | 3.5324e-1 (2.13e-2)+ | 3.1320e-1 (1.14e-1)− | 3.2193e-1 (1.14e-1) |
| | 15 | 6.9167e-1 (1.11e-1)+ | 6.0973e-1 (2.64e-1)≈ | 1.4795e+0 (1.24e-1)+ | 9.2356e-1 (1.62e-1)+ | 2.7072e-1 (4.98e-3)− | 3.5180e-1 (1.62e-1)− | 4.5615e-1 (5.81e-2) |
| C3-DTLZ1 | 3 | 6.4693e-2 (1.20e-2)+ | 5.4988e-2 (1.46e-2)≈ | 7.0098e-2 (6.30e-3)+ | 6.3010e-2 (9.09e-3)+ | 6.3393e-2 (2.65e-2)+ | 6.3114e-2 (4.08e-3)+ | 4.5845e-2 (1.70e-3) |
| | 5 | 1.1056e-1 (6.53e-3)+ | 1.1784e-1 (2.76e-3)+ | 2.6861e-1 (2.61e-1)≈ | 1.9334e-1 (3.41e-2)+ | 1.2471e-1 (8.90e-4)+ | 1.2161e-1 (3.38e-3)+ | 1.0797e-1 (1.24e-3) |
| | 8 | 2.3654e-1 (7.48e-2)+ | 2.0254e-1 (1.53e-3)− | 9.6476e-1 (3.88e-1)+ | 3.2248e-1 (2.65e-2)+ | 2.4484e-1 (5.66e-3)+ | 2.2855e-1 (5.02e-3)+ | 2.2330e-1 (7.13e-2) |
| | 10 | 3.2904e-1 (1.02e-1)+ | 2.1208e-1 (3.34e-3)− | 7.2867e-1 (1.94e-1)+ | 3.2983e-1 (2.48e-2)+ | 2.7945e-1 (3.23e-2)≈ | 2.3413e-1 (3.53e-3)≈ | 2.7966e-1 (7.98e-2) |
| | 15 | 4.1706e-1 (4.46e-2)≈ | 2.7696e-1 (5.98e-3)− | 1.2996e+0 (7.86e-1)+ | 4.7672e-1 (3.60e-1)+ | 4.9344e-1 (4.03e-2)+ | 3.3203e-1 (1.21e-2)− | 4.1768e-1 (2.70e-2) |
| C3-DTLZ4 | 3 | 1.6127e-1 (2.31e-1)− | 1.1056e-1 (5.65e-2)≈ | 4.0726e-1 (3.22e-1)+ | 1.1149e-1 (7.43e-2)− | 1.1255e-1 (2.82e-3)≈ | 1.1101e-1 (2.70e-3)≈ | 3.2507e-1 (3.45e-1) |
| | 5 | 2.6257e-1 (6.29e-2)≈ | 2.6852e-1 (2.05e-2)+ | 4.6611e-1 (9.07e-2)+ | 4.3049e-1 (3.28e-2)+ | 2.9092e-1 (2.56e-3)+ | 2.6579e-1 (2.55e-3)+ | 2.6156e-1 (6.47e-2) |
| | 8 | 7.9595e-1 (8.63e-2)≈ | 5.1933e-1 (7.29e-3)− | 8.4917e-1 (1.52e-1)≈ | 6.5860e-1 (2.45e-2)− | 5.4648e-1 (4.03e-3)− | 5.1464e-1 (2.66e-3)− | 7.9011e-1 (1.13e-1) |
| | 10 | 8.5162e-1 (7.27e-2)≈ | 6.0624e-1 (2.67e-3)− | 7.6730e-1 (6.30e-2)− | 7.3312e-1 (2.27e-2)− | 6.2175e-1 (5.73e-3)− | 5.6029e-1 (1.96e-3)− | 8.1663e-1 (8.95e-2) |
| | 15 | 1.3047e+0 (1.37e-1)+ | 8.0836e-1 (7.22e-4)− | 1.2760e+0 (2.96e-1)+ | 8.9505e-1 (1.32e-2)− | 8.0489e-1 (5.63e-4)− | 7.7564e-1 (6.06e-3)− | 1.1067e+0 (1.23e-1) |
| DC1-DTLZ1 | 3 | 1.7701e-2(2.22e-2)≈ | 1.1530e-1(7.62e-2)+ | 6.1464e-1(1.01e+0)+ | 2.5552e-2(2.78e-2)+ | 1.0115e-2(2.32e-4)≈ | 1.5944e-2(1.20e-3)+ | 1.0579e-2(2.48e-4) |
| | 5 | 3.5007e-2(2.65e-4)− | 9.5875e-2(7.72e-2)+ | 3.7816e-1(3.25e-1)+ | 1.1139e-1(1.03e-1)+ | 3.5557e-2(3.28e-4)≈ | 4.3671e-2(1.97e-3)+ | 3.5492e-2(2.70e-4) |
| | 8 | 1.1672e-1(8.09e-2)≈ | 8.7019e-2(6.11e-2)≈ | 8.6818e-1(2.28e+0) + | 1.4054e-1(6.20e-2)+ | 1.3913e-1(7.72e-2)+ | 1.4428e-1(7.11e-2)+ | 6.7155e-2(1.86e-3) |
| | 10 | 9.7234e-2(5.14e-2)+ | 1.2225e-1(5.84e-2)+ | 3.2460e-1(2.69e-1)+ | 1.4275e-1(5.61e-2)+ | 6.1893e-2(2.13e-2)≈ | 1.3037e-1(1.98e-2)+ | 6.9388e-2(3.29e-2) |
| | 15 | 2.5685e-1 (7.50e-2)+ | 1.7743e-1 (4.03e-2)≈ | 9.5391e-1 (1.66e+0)+ | 1.9669e-1 (4.03e-2)+ | 1.4850e-1 (1.05e-2)− | 1.8399e-1 (2.13e-2)≈ | 1.8928e-1 (3.27e-2) |
| DC1-DTLZ3 | 3 | 7.7459e-2(8.79e-2)≈ | 2.5210e+0(6.76e+0)+ | 2.6072e+1(2.08e+1)+ | 2.7010e-1(5.83e-1)≈ | 3.8855e-2(2.68e-2)+ | 2.7972e-2(6.35e-3)− | 3.8561e-2(3.96e-2) |
| | 5 | 1.0926e-1(4.34e-3)− | 5.4292e-1(1.99e-1)+ | 1.8586e+1(1.47e+1)+ | 2.1450e+0(5.74e+0)+ | 2.1411e-1(7.77e-2)+ | 1.3430e-1(8.42e-3)+ | 1.1255e-1(3.65e-3) |
| | 8 | 6.8658e-1(2.98e-1)≈ | 6.8627e-1(1.75e-1)− | 6.4296e+0(1.92e+1)+ | 9.8144e-1(2.17e+0)+ | 7.4596e-1(1.59e-1)≈ | 4.3032e-1(6.24e-2)− | 7.5589e-1(2.22e-1) |
| | 10 | 5.7757e-1(7.12e-2)+ | 4.4867e-1(1.46e-1)≈ | 5.3989e+0(1.21e+1)+ | 6.2885e-1(4.33e-2)+ | 5.3151e-1(1.98e-2)+ | 4.1334e-1(7.19e-3)≈ | 4.6197e-1(9.32e-2) |
| | 15 | 3.5472e+0 (3.35e+0)+ | 6.1183e-1 (1.98e-1)− | 2.0765e+1 (4.72e+1)≈ | 7.9473e+0 (1.81e+1)+ | 8.7800e-1 (2.29e-1)− | 9.6455e-1 (1.12e-1)≈ | 9.2587e-1 (1.27e-1) |
| DC2-DTLZ1 | 3 | Infeasible | Infeasible | Infeasible | Infeasible | 2.3392e-2(2.94e-4)+ | Infeasible | 2.0562e-2(4.76e-5) |
| | 5 | Infeasible | Infeasible | Infeasible | Infeasible | 6.1595e-2(2.15e-4)+ | Infeasible | 5.2710e-2(8.86e-5) |
| | 8 | Infeasible | Infeasible | Infeasible | Infeasible | 1.1132e-1(1.33e-3)+ | Infeasible | 9.9903e-2(3.51e-3) |
| | 10 | Infeasible | Infeasible | Infeasible | Infeasible | 1.3340e-1(1.99e-3)+ | Infeasible | 1.2312e-1(2.21e-3) |
| | 15 | Infeasible | Infeasible | Infeasible | Infeasible | 2.1108e-1(5.81e-3)+ | Infeasible | 2.0012e-1(2.81e-3) |
| DC2-DTLZ3 | 3 | Infeasible | Infeasible | Infeasible | Infeasible | 5.4695e-2(2.48e-4)≈ | Infeasible | 5.4566e-2(1.29e-4) |
| | 5 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 1.6515e-1(1.95e-5) |
| | 8 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 3.8180e-1(8.92e-2) |
| | 10 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 4.6032e-1(7.18e-2) |
| | 15 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| DC3-DTLZ1 | 3 | 9.4914e-2(9.76e-2)+ | 6.5415e-1(1.53e+0)+ | 7.4301e+0(8.10e+0)+ | 6.1489e-2(1.69e-1)+ | 1.0768e-2(4.03e-4)− | 1.0796e-1(1.43e-1)+ | 1.1489e-2(5.51e-4) |
| | 5 | 5.3751e-2(3.31e-2)+ | 1.5392e-1(8.28e-2)+ | 9.1535e+0(1.01e+1)+ | 4.1103e-1(3.72e-1)+ | 3.4773e-2(3.90e-3)+ | 3.1988e-2(1.23e-2)+ | 2.9012e-2(1.30e-2) |
| | 8 | 9.2965e-2(3.11e-2)+ | 1.8205e-1(9.13e-2)+ | 5.1377e+0(8.87e+0)+ | 1.0667e+0(1.22e+0)+ | 1.7178e+0(8.17e+0)+ | 6.9990e-2(2.06e-2)+ | 4.8288e-2(5.22e-3) |
| | 10 | 7.7301e-2(2.39e-2)+ | 1.3402e-1(8.53e-2)+ | 4.3745e+0(5.39e+0)+ | 2.8098e-1(8.20e-2)+ | Infeasible | 5.6680e-2(4.06e-3)+ | 4.6598e-2(4.17e-3) |
| | 15 | 7.7899e-1(8.81e-1)+ | 3.0366e+0(2.10e+0)+ | 1.4460e+1(9.22e+0)+ | 3.6798e+0(3.62e+0)+ | Infeasible | 6.8743e-1(6.40e-1)≈ | 3.3576e-1(6.97e-2) |
| DC3-DTLZ3 | 3 | 2.6568e+0(1.56e+0)+ | 3.2402e+1(3.39e+1)+ | 6.1044e+1(3.98e+1)+ | 8.1662e+0(1.59e+1)+ | 5.1671e-2(1.75e-2)≈ | 4.5776e+0(2.75e+0)+ | 2.7685e-1(5.79e-1) |
| | 5 | 2.6568e+0(1.56e+0)+ | 3.2402e+1(3.39e+1)+ | 6.1044e+1(3.98e+1)+ | 8.1662e+0(1.59e+1)+ | 5.1671e-2(1.75e-2)≈ | 4.5776e+0(2.75e+0)+ | 2.7685e-1(5.79e-1) |
| | 8 | 2.6568e+0(1.56e+0)+ | 3.2402e+1(3.39e+1)+ | 6.1044e+1(3.98e+1)+ | 8.1662e+0(1.59e+1)+ | 5.1671e-2(1.75e-2)≈ | 4.5776e+0(2.75e+0)+ | 2.7685e-1(5.79e-1) |
| | 10 | 7.4588e+0(3.06e+0)+ | 1.1923e+0(3.71e-1)≈ | 1.8891e+0(2.81e+0)≈ | 5.4020e+1(4.52e+1)+ | 3.7178e-1(7.65e-2)− | 5.2769e-1(1.57e-1)− | 1.4573e+0(1.54e+0) |
| | 15 | 1.3689e+1(1.07e+1)+ | 2.0934e+1(9.74e+0)+ | 1.5852e+1(1.89e+1)+ | 8.6460e+1(5.73e+1)+ | 2.1973e+0(2.01e+0)− | 4.3974e+0(4.66e+0)≈ | 2.3841e+0(1.43e+0) |

“Infeasible” stands for the algorithm cannot find feasible solutions in all 30 runs.

TABLE S-IV
COMPARATIVE RESULTS OF HV VALUES (MEAN AND STANDARD DEVIATION) OBTAINED BY A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AND, AND DCNSGA-III, RESPECTIVELY.

| Problem | $m$ | A-NSGA-III | C-MOEA/D | I-DBEA | PPS | C-TAEA | C-AnD | DCNSGA-III |
|---|---|---|---|---|---|---|---|---|
| CF8 | 3 | Infeasible | Infeasible | Infeasible | 4.2406e-1(1.09e-2)− | 1.9378e-1(6.51e-2)+ | Infeasible | 2.6138e-1(6.48e-2) |
| CF9 | 3 | 3.5940e-1(8.84e-2)≈ | 4.3778e-1(1.75e-2)− | 8.8064e-2(4.98e-2)+ | 5.0806e-1(1.63e-2)− | 3.9182e-1(1.56e-2)≈ | 3.8477e-1(2.62e-2)≈ | 3.5405e-1(8.00e-2) |
| CF10 | 3 | Infeasible | Infeasible | Infeasible | 2.7805e-1(7.12e-2)− | 1.0265e-1(3.36e-2)+ | Infeasible | 1.9366e-1(6.74e-2) |
| MW4 | 3 | 8.3825e-1(3.98e-3)+ | 8.1085e-1(8.36e-3)+ | 1.9614e-1(9.33e-2)+ | Infeasible | 8.4074e-1(2.52e-4)− | 8.3214e-1(1.15e-2)+ | 8.4033e-1(7.85e-4) |
| MW8 | 3 | 5.0368e-1(6.21e-2)≈ | 5.2983e-1(1.03e-2)≈ | 5.1517e-2(3.09e-2)+ | 3.5554e-1(9.95e-2)+ | 5.3083e-1(1.25e-2)≈ | 4.1824e-1(1.00e-1)+ | 5.3015e-1(9.53e-3) |
| MW14 | 3 | 4.6162e-1(1.13e-2)≈ | 4.1485e-1(6.76e-3)+ | 1.2004e-2(7.25e-3)+ | 4.1615e-1(2.15e-2)+ | 4.6793e-1(5.20e-3)− | 3.0300e-1(8.36e-2)+ | 4.6420e-1(2.34e-3) |
| DASCMOP7 | 3 | 2.8498e-1(1.22e-3)− | 2.4667e-1(6.44e-2)≈ | 1.3813e-2(3.95e-2)+ | 2.5942e-1(3.68e-2)+ | 2.8504e-1(5.34e-3)− | 2.8547e-1(8.26e-3)− | 2.8088e-1(2.26e-3) |
| DASCMOP8 | 3 | 1.9050e-1(3.09e-2)+ | 1.6520e-1(5.85e-2)+ | 6.4248e-3(1.50e-2)+ | 1.8102e-1(1.68e-2)+ | 2.0234e-1(3.75e-3)≈ | 2.0610e-1(1.74e-3)− | 2.0404e-1(7.56e-4) |
| DASCMOP9 | 3 | 1.1957e-1(7.78e-3)≈ | 1.9951e-1(8.83e-4)− | 4.2268e-2(5.27e-3)+ | 2.0636e-1(1.70e-3)− | 1.5702e-1(1.51e-2)− | 1.0660e-1(2.75e-2)≈ | 1.0852e-1(2.30e-2) |
| C1-DTLZ1 | 3 | 8.2037e-1(1.44e-2)+ | 8.3111e-1(2.77e-3)+ | 9.5747e-2(7.95e-2)+ | 7.9241e-1(3.07e-2)+ | 8.3039e-1(1.21e-2)≈ | 8.2947e-1(5.30e-3)+ | 8.3322e-1(7.53e-3) |
| | 5 | 9.6765e-1(1.25e-2)+ | 9.6698e-1(2.02e-3)+ | 1.1844e-1(1.16e-1)+ | Infeasible | 9.7679e-1(1.20e-3)≈ | 9.6519e-1(1.53e-2)+ | 9.7508e-1(4.66e-3) |
| | 8 | 9.6876e-1(2.37e-2)+ | 9.7730e-1(2.85e-3)+ | 1.2084e-1(1.02e-1)+ | 9.7384e-1(3.23e-2)≈ | 9.9566e-1(1.41e-3)− | 9.6808e-1(5.36e-2)≈ | 9.8179e-1(1.15e-2) |
| | 10 | 9.7799e-1(1.92e-2)+ | 9.8479e-1(1.92e-3)+ | 1.3506e-1(1.37e-1)+ | 9.9513e-1(1.97e-3)≈ | 9.9791e-1(3.65e-3)− | 9.8287e-1(1.83e-2)≈ | 9.9194e-1(6.94e-3) |
| | 15 | 9.6990e-1(3.04e-2)+ | 9.8176e-1(5.00e-3)+ | 6.1801e-2(4.25e-2)+ | 9.6221e-1(5.27e-2)+ | 9.9576e-1(7.66e-3)≈ | 9.8652e-1(1.05e-2)+ | 9.9684e-1(5.02e-3) |
| C1-DTLZ3 | 3 | 3.3338e-1(2.59e-1)≈ | 3.0303e-1(2.16e-1)≈ | 1.1294e-1(2.21e-1)+ | 3.9410e-1(2.47e-1)≈ | 2.8686e-1(2.73e-1)≈ | 7.2928e-2(1.89e-1)+ | 3.1273e-1(2.78e-1) |
| | 5 | 8.0076e-1(9.38e-3)+ | 6.0344e-1(1.67e-1)+ | 8.1139e-1(1.09e-3)≈ | 4.1268e-1(2.16e-1)+ | 6.6086e-1(4.26e-2)+ | 8.0234e-1(4.09e-3)+ | 8.1194e-1(7.80e-4) |
| | 8 | 7.5165e-1(1.21e-1)+ | 6.1442e-1(1.76e-1)+ | 9.7072e-2(3.39e-2)+ | 5.0676e-1(5.78e-2)+ | 7.9914e-1(2.60e-2)+ | 7.0275e-1(2.02e-1)+ | 8.8632e-1(7.96e-2) |
| | 10 | 8.2564e-1(5.83e-2)+ | 8.4843e-1(1.62e-2)+ | 9.0907e-2(3.18e-6)+ | 4.4896e-1(6.28e-2)+ | 9.0948e-1(1.27e-2)+ | 8.7843e-1(5.44e-2)+ | 9.4101e-1(5.12e-2) |
| | 15 | 1.0945e-1(1.87e-1)+ | 9.4066e-1(1.36e-2)− | 9.0897e-2(1.73e-5)+ | 1.7318e-1(5.91e-2)+ | 7.8556e-1(2.55e-1)≈ | 1.7984e-1(1.45e-1)+ | 8.3910e-1(7.22e-2) |
| C2-DTLZ2 | 3 | 5.0763e-1(3.32e-3)− | 4.7208e-1(7.16e-3)+ | 3.3824e-2(3.43e-2)+ | 5.0117e-1(2.78e-3)≈ | 5.0410e-1(2.82e-3)− | 5.0389e-1(5.74e-2)− | 5.0171e-1(2.11e-3) |
| | 5 | 7.4258e-1(3.22e-2)+ | 6.7281e-1(7.27e-3)+ | 3.1005e-2(3.72e-2)+ | 5.5906e-1(6.01e-2)+ | 7.4057e-1(2.06e-3)+ | 7.6111e-1(1.25e-3)− | 7.4693e-1(2.69e-3) |
| | 8 | 7.4129e-1(1.91e-1)≈ | 5.1812e-1(1.95e-1)+ | 4.8087e-2(5.35e-2)+ | 5.3041e-1(6.68e-2)+ | 7.9526e-1(8.04e-3)+ | 8.1807e-1(1.17e-1)≈ | 8.2979e-1(2.76e-2) |
| | 10 | 7.9787e-1(1.73e-1)+ | 7.3422e-1(2.12e-1)+ | 5.9943e-2(3.85e-2)+ | 6.1815e-1(4.39e-2)+ | 8.7485e-1(3.83e-3)− | 8.6816e-1(1.26e-1)≈ | 8.7289e-1(4.64e-2) |
| | 15 | 7.0308e-1(1.41e-1)+ | 6.3875e-1(3.13e-1)+ | 1.3028e-2(2.64e-2)+ | 2.7349e-1(1.51e-1)+ | 9.3182e-1(4.06e-3)− | 8.7940e-1(1.84e-1)+ | 8.8257e-1(3.08e-2) |
| C3-DTLZ1 | 3 | 8.4744e-1(1.17e-2)+ | 8.5779e-1(1.35e-2)+ | 8.4416e-1(6.35e-3)+ | 8.3818e-1(1.35e-2)+ | 8.5034e-1(2.97e-2)+ | 8.4763e-1(5.06e-3)+ | 8.6746e-1(2.17e-3) |
| | 5 | 9.7883e-1(1.35e-3)≈ | 9.7555e-1(1.02e-3)+ | 8.0668e-1(2.88e-1)≈ | 9.2596e-1(5.08e-2)+ | 9.7702e-1(2.40e-4)+ | 9.7332e-1(1.35e-3)+ | 9.7936e-1(4.43e-4) |
| | 8 | 9.8330e-1(4.69e-2)+ | 9.9577e-1(3.28e-4)− | 2.8764e-1(2.70e-1)+ | 9.2786e-1(2.97e-2)+ | 9.9628e-1(8.94e-4)− | 9.9450e-1(8.32e-4)− | 9.8670e-1(3.31e-2) |
| | 10 | 9.7089e-1(3.65e-2)+ | 9.9938e-1(7.73e-5)≈ | 5.5569e-1(2.16e-1)+ | 9.3357e-1(2.86e-2)+ | 9.9868e-1(3.03e-3)≈ | 9.9903e-1(1.77e-4)≈ | 9.8872e-1(1.63e-2) |
| | 15 | 9.8386e-1(1.59e-2)≈ | 9.9963e-1(1.45e-4)− | 2.2155e-1(2.40e-1)+ | 7.4041e-1(2.78e-1)+ | 9.6084e-1(4.27e-2)+ | 9.9746e-1(1.34e-3)− | 9.9208e-1(4.22e-3) |
| C3-DTLZ4 | 3 | 7.6642e-1(7.89e-2)− | 7.8329e-1(1.71e-2)≈ | 6.1919e-1(1.32e-1)+ | 7.9165e-1(2.26e-2)− | 7.8470e-1(1.50e-3)≈ | 7.8355e-1(2.34e-3)≈ | 7.0967e-1(1.19e-1) |
| | 5 | 9.6016e-1(1.03e-2)≈ | 9.6002e-1(1.21e-3)− | 8.8700e-1(2.56e-2)+ | 9.3693e-1(6.04e-3)+ | 9.5712e-1(6.44e-4)+ | 9.5892e-1(8.97e-4)+ | 9.5995e-1(1.10e-2) |
| | 8 | 9.6240e-1(1.82e-2)≈ | 9.9493e-1(2.46e-4)− | 9.2386e-1(5.04e-2)+ | 9.8054e-1(2.93e-3)− | 9.9439e-1(1.50e-4)− | 9.9465e-1(2.93e-4)− | 9.6102e-1(2.18e-2) |
| | 10 | 9.8517e-1(9.13e-3)≈ | 9.9934e-1(3.72e-5)− | 9.8846e-1(4.46e-3)≈ | 9.9156e-1(1.59e-3)≈ | 9.9903e-1(4.61e-5)− | 9.9921e-1(4.24e-5)− | 9.8755e-1(1.02e-2) |
| | 15 | 9.3349e-1(3.99e-2)+ | 9.9996e-1(7.28e-6)− | 8.1914e-1(2.56e-1)+ | 9.9722e-1(7.21e-4)− | 9.9997e-1(5.20e-6)− | 9.9991e-1(1.45e-5)− | 9.8393e-1(1.78e-2) |
| DC1-DTLZ1 | 3 | 6.3755e-1(3.15e-2)+ | 4.8228e-1(9.98e-2)+ | 1.6766e-1(1.83e-1)+ | 6.0783e-1(5.28e-2)+ | 6.4899e-1(1.11e-3)≈ | 6.2964e-1(5.37e-3)+ | 6.4935e-1(1.40e-3) |
| | 5 | 8.1004e-1(2.22e-4)+ | 7.0701e-1(1.73e-1)+ | 2.7291e-1(2.52e-1)+ | 6.6295e-1(2.05e-1)+ | 8.1108e-1(2.42e-4)− | 7.9934e-1(2.05e-3)+ | 8.1029e-1(2.06e-4) |
| | 8 | 6.2109e-1(5.76e-2)≈ | 5.1949e-1(5.99e-2)+ | 1.5709e-1(1.19e-1)+ | 3.4641e-1(6.86e-2)+ | 5.7134e-1(6.76e-2)+ | 5.3984e-1(5.70e-2)+ | 6.2786e-1(7.89e-3) |
| | 10 | 8.3343e-1(1.46e-2)+ | 7.4883e-1(1.40e-1)+ | 5.9205e-1(2.85e-1)+ | 8.1798e-1(6.55e-2)+ | 8.3601e-1(5.90e-5)− | 8.3432e-1(4.15e-4)− | 8.3411e-1(7.29e-3) |
| | 15 | 1.7451e-1(2.52e-1)≈ | 1.6984e-1(2.14e-1)≈ | 7.3000e-2(9.37e-2)+ | 5.2633e-1(8.62e-2)− | 3.0352e-1(1.17e-1)− | 4.3158e-2(4.04e-2)+ | 1.6245e-1(1.39e-1) |
| DC1-DTLZ3 | 3 | 4.4664e-1(4.25e-2)≈ | 2.4284e-1(1.23e-1)+ | 3.3604e-3(1.84e-2)+ | 3.6841e-1(1.55e-1)≈ | 4.6891e-1(1.60e-2)− | 4.8035e-1(4.16e-3)− | 4.6281e-1(1.59e-2) |
| | 5 | 7.8341e-1(1.39e-3)− | 5.9135e-1(1.72e-1)+ | 5.7259e-3(1.92e-2)+ | 2.1250e-1(2.56e-1)+ | 6.8475e-1(4.28e-2)+ | 7.7943e-1(2.53e-3)+ | 7.8236e-1(1.88e-3) |
| | 8 | 7.6723e-1(1.23e-1)≈ | 7.7455e-1(1.73e-1)− | 4.8599e-2(4.42e-2)+ | 4.3862e-1(1.23e-1)+ | 7.5935e-1(3.02e-2)≈ | 8.9110e-1(1.23e-2)− | 7.4255e-1(9.23e-2) |
| | 10 | 8.2849e-1(6.15e-2)+ | 8.6170e-1(1.85e-1)≈ | 5.6151e-2(3.85e-2)+ | 4.6553e-1(8.65e-2)+ | 8.8374e-1(1.55e-2)≈ | 9.4726e-1(5.59e-3)≈ | 9.0438e-1(6.62e-2) |
| | 15 | 2.5670e-2(5.96e-2)+ | 5.5659e-1(9.29e-2)− | 4.7228e-2(3.52e-2)+ | 1.9069e-2(3.34e-2)+ | 2.4744e-1(1.07e-1)+ | 1.6951e-1(1.01e-1)+ | 3.1949e-1(1.14e-1) |
| DC2-DTLZ1 | 3 | Infeasible | Infeasible | Infeasible | Infeasible | 8.3899e-1(4.56e-4)+ | Infeasible | 8.4165e-1(7.34e-4) |
| | 5 | Infeasible | Infeasible | Infeasible | Infeasible | 9.7691e-1(1.66e-4)+ | Infeasible | 9.7990e-1(1.99e-4) |
| | 8 | Infeasible | Infeasible | Infeasible | Infeasible | 9.9699e-1(4.36e-4)+ | Infeasible | 9.9716e-1(9.01e-4) |
| | 10 | Infeasible | Infeasible | Infeasible | Infeasible | 9.9941e-1(7.59e-5)+ | Infeasible | 9.9969e-1(9.67e-5) |
| | 15 | Infeasible | Infeasible | Infeasible | Infeasible | 9.9979e-1(3.09e-5)− | Infeasible | 9.9906e-1(3.44e-4) |
| DC2-DTLZ3 | 3 | Infeasible | Infeasible | Infeasible | Infeasible | 5.3491e-1(6.74e-2)≈ | Infeasible | 5.3576e-1(2.31e-2) |
| | 5 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 7.9899e-1(1.54e-3) |
| | 8 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 8.1380e-1(5.90e-2) |
| | 10 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | 9.0094e-1(5.31e-2) |
| | 15 | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| DC3-DTLZ1 | 3 | 4.3827e-1(2.17e-1)+ | 1.6143e-1(1.65e-1)+ | 0.0000e+0(0.00e+0)+ | 5.9171e-1(1.63e-1)+ | 6.5840e-1(9.94e-4)+ | 4.4886e-1(2.34e-1)+ | 6.6126e-1(4.08e-3) |
| | 5 | 6.8732e-1(8.92e-2)≈ | 4.8293e-1(2.01e-1)+ | 5.8107e-3(3.18e-2)+ | 2.5609e-1(2.13e-1)+ | 7.3385e-1(7.13e-3)+ | 7.4800e-1(3.21e-2)≈ | 7.5480e-1(5.70e-3) |
| | 8 | 5.9264e-1(1.13e-1)+ | 3.7597e-1(1.48e-1)+ | 5.2404e-3(2.02e-2)+ | 1.5454e-1(1.76e-1)+ | 3.5665e-1(1.61e-1)+ | 7.1013e-1(7.14e-2)− | 6.9623e-1(2.07e-2) |
| | 10 | 6.1449e-1(1.12e-1)+ | 3.7781e-1(1.55e-1)+ | 6.3148e-3(2.48e-2)+ | 3.2666e-1(1.74e-1)+ | Infeasible | 7.4425e-1(5.13e-3)− | 7.0015e-1(1.85e-2) |
| | 15 | 1.9004e-1(1.30e-1)+ | 4.0722e-2(8.53e-2)+ | 0.0000e+0(0.00e+0)+ | 5.7453e-2(9.89e-2)+ | Infeasible | 2.0784e-1(1.55e-1)≈ | 2.8513e-1(7.64e-2) |
| DC3-DTLZ3 | 3 | 3.0870e-3(1.29e-2)+ | 1.8764e-2(8.36e-2)+ | 0.0000e+0(0.00e+0)+ | 2.3058e-1(1.96e-1)+ | 4.1582e-1(2.46e-2) ≈ | 0.0000e+0(0.00e+0)+ | 3.5079e-1(1.73e-1) |
| | 5 | 2.7257e-1(2.34e-1)+ | 4.0867e-2(1.42e-1)+ | 0.0000e+0(0.00e+0)+ | 2.6815e-2(8.59e-2)+ | 6.7709e-1(8.16e-3)+ | 4.1649e-1(2.49e-1)+ | 7.2092e-1(3.29e-3) |
| | 8 | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0) |
| | 10 | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0) |
| | 15 | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 0.0000e+0(0.00e+0)≈ | 1.5777e-6(7.12e-6)− | 0.0000e+0(0.00e+0) |

"Infeasible" stands for the algorithm cannot find feasible solutions in all 30 runs.

TABLE S-II
NUMBER OF MAXIMUM FUNCTION EVALUATIONS FOR DIFFERENT TEST PROBLEMS.

| Problem | $m$ | $n$ | $FEs$ |
|---|---|---|---|
| CF8-10 | 3 | 10 | 150,000 |
| MW4, MW8, MW14 | 3 | 15 | 60,000 |
| DAS-CMOP7-9 | 3 | 30 | 300,000 |
| C1-DTLZ1 | 3 | 7 | 46,000 |
| | 5 | 9 | 127,200 |
| | 8 | 12 | 124,800 |
| | 10 | 14 | 276,000 |
| | 15 | 19 | 204,000 |
| C1-DTLZ3 | 3 | 12 | 92,000 |
| | 5 | 14 | 318,000 |
| | 8 | 17 | 390,000 |
| | 10 | 19 | 966,000 |
| | 15 | 24 | 680,000 |
| C2-DTLZ2 | 3 | 12 | 23,000 |
| | 5 | 14 | 74,200 |
| | 8 | 17 | 78,000 |
| | 10 | 19 | 207,000 |
| | 15 | 24 | 136,000 |
| C3-DTLZ1, DC1-DTLZ1, DC3-DTLZ1 | 3 | 7 | 69,000 |
| | 5 | 9 | 265,000 |
| | 8 | 12 | 312,000 |
| | 10 | 14 | 828,000 |
| | 15 | 19 | 544,000 |
| C3-DTLZ4, DC1-DTLZ3, DC3-DTLZ3 | 3 | 12 | 69,000 |
| | 5 | 14 | 265,000 |
| | 8 | 17 | 312,000 |
| | 10 | 19 | 828,000 |
| | 15 | 24 | 544,000 |
| DC2-DTLZ1 | 3 | 7 | 138,000 |
| | 5 | 9 | 530,000 |
| | 8 | 12 | 624,000 |
| | 10 | 14 | 1,656,000 |
| | 15 | 19 | 1,088,000 |
| DC2-DTLZ3 | 3 | 12 | 138,000 |
| | 5 | 14 | 530,000 |
| | 8 | 17 | 624,000 |
| | 10 | 19 | 1,656,000 |
| | 15 | 24 | 1,088,000 |

percentage of runs where at least one feasible solution was found. We employ CF10 and DC2-DTLZ3 as test problems.

Fig. S-1 shows the feasibility ratio of three mating selection operators on CF10 and DC2-DTLZ3, respectively. From Fig. S-1, we can clearly see that $Variant3$ has the worst performance among these three operators. The reason is that it neglects the constraint information, which cannot guarantee to generate enough solutions with small constraint violations. Particularly at the latter

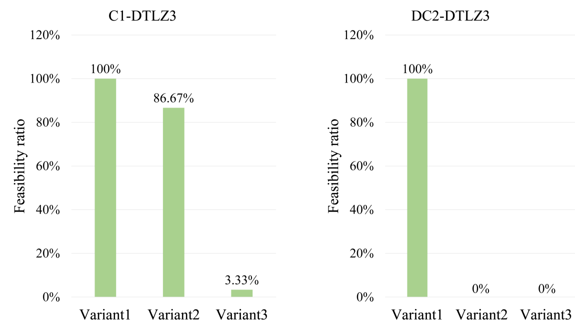


Fig. S-1. Comparison of feasibility ratio of three different mating selection operators on CF10 problem and DC2-DTLZ3 problem, respectively.

stage of evolution, the poor tournament selection cannot create enough offsprings with a low degree of constraint violation to push the population towards the feasible area. $Variant2$ performs better than $Variant3$, but it still cannot find feasible solutions consistently on CF10 and DC2-DTLZ3 problems. This is because excessively emphasize feasibility and ignore the information of objectives during the offspring generation process can lead to the population stagnate in infeasible regions easily. By contrast, the proposed mating selection operator ($Variant1$) can find feasible solutions in all runs. A key to the proposed algorithm is to generate more $\varepsilon$-feasible solutions. On the one hand, the majority of $\varepsilon$-feasible solutions assure that the population can focus on searching for well-balanced solutions between convergence and diversity, which is beneficial to the population to cross the large and separate infeasible local optima. On the other hand, the dynamically reduced $\varepsilon$ constraint boundary ensures these $\varepsilon$-feasible solutions towards the feasible region.

## S-IV. PARAMETER ANALYSIS

The shrink of the dynamic $\boldsymbol{\varepsilon}$ constraint boundary adopts the exponential function of the simulated annealing algorithm. It has a crux parameter $cp$, which controls the decreasing trend of the dynamic $\boldsymbol{\varepsilon}$ constraint boundary. In this section, the sensitivity analysis of $cp$ is conducted as follows.

Fig. S-2 plots the changes of the dynamic $\varepsilon$ constraint boundary with different $cp$ values. It can be observed that the dynamic $\varepsilon$ constraint boundary shrinks very fast with $cp$=1. When the value of $cp$ increases, the shrinkage speed of the dynamic $\varepsilon$ constraint boundary also slows down.

Figs. S-3-S-5 plot the performance, in terms of IGD, of DCNSGA-III with different $cp$ values (1-10) on C1-DTLZ1, C1-DTLZ3, and DC3-DTLZ1 over 30 independent runs, respectively. It can be observed that the optimal value of $cp$ is problem-dependent. To elaborate,

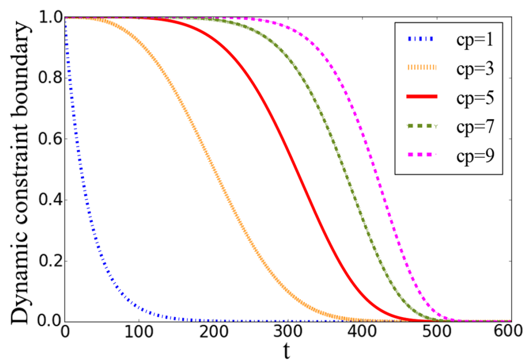


Fig. S-2. Changes of the dynamic $\boldsymbol{\varepsilon}$ constraint boundary with different $cp$ values ($\boldsymbol{\varepsilon}^{(0)} = 1$ and $T$=600).

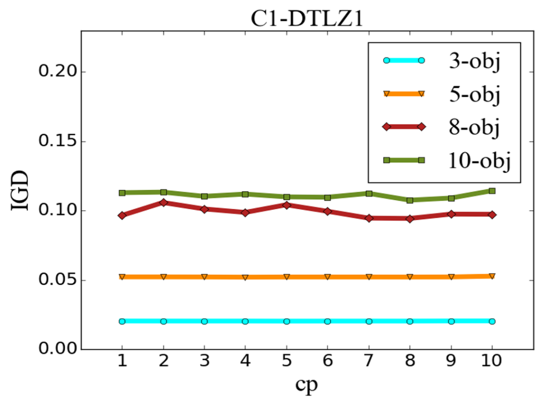


Fig. S-3. Mean IGD values obtained by DCNSGA-III with different values of $cp$ on C1-DTLZ1 test problem over 30 runs.

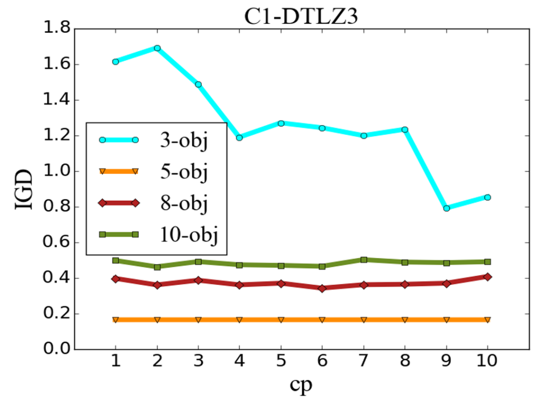


Fig. S-4. Mean IGD values obtained by DCNSGA-III with different values of $cp$ on C1-DTLZ3 test problem over 30 runs.

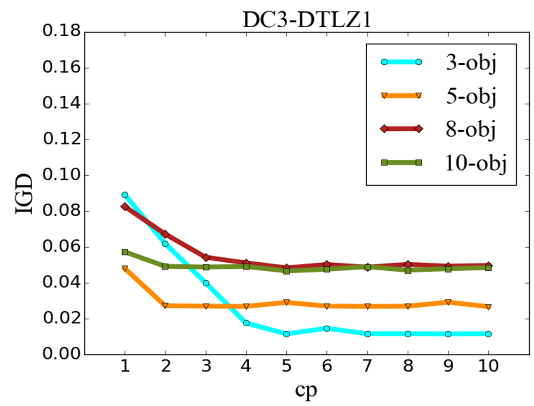


Fig. S-5. Mean IGD values obtained by DCNSGA-III with different values of $cp$ on DC3-DTLZ1 test problem over 30 runs.

the performance on the C1-DTLZ1 test problem in Fig. S-3 is not sensitive to the setting of $cp$. Except for the C1-DTLZ3 problem with 3 objectives, the performance of the C1-DTLZ3 problem with 5, 8, and 10 objectives does not fluctuate greatly for different $cp$ settings. For the C1-DTLZ3 problem with 3 objectives and DC3-DTLZ1 test problem, the results in Fig. S-4 and Fig. S-5 suggest that DCNSGA-III has the poor performance with $cp = 1, 2, 3$. The reason can be attributed that the dynamic $\varepsilon$ constraint boundary shrinks very fast when $cp$ has a small value, which results in the poor exploration ability in the early stage of evolution.

However, in general, DCNSGA-III is robust with regard to $cp = 4, 5, 6, 7, 8, 9, 10$. Therefore, $cp$ is in the range of 4 to 10 could be the better choice for most problems.

## S-V. Performance on Constrained Bi-objective Optimization Problems

Although the proposed algorithm is designed for constrained many-objective optimization problems, in this section, we compare the proposed DCNSGA-III with its competitors on two set of bi-objective optimization problems: C-DTLZ test suite [4] and DC-DTLZ test suite [5]. The constraint function parameters of these two-objective test problems were set as follows:

- For C1-DTLZ3: $r$=6;
- For C2-DTLZ2: $r$=0.1;
- For DC1-DTLZ1 and DC1-DTLZ3 problems: $a$=3, $b = 0.5$;
- For DC2-DTLZ1 and DC2-DTLZ3 problems: $a$=3, $b = 0.9$;
- For DC3-DTLZ1 and DC3-DTLZ3 problems: $a$=5, $b = 0.5$.

DCNSGA-III was compared with two MOEA/D variants (C-MOEA/D [4], MOEA/D-DAE [11]) and a NSGA-III variant (C-NSGA-III [4]). The population size and the maximal number of function evaluations are set to 100 and 60,000, respectively.

Table S-V presents both the average and standard deviation of the IGD values over 30 independent runs for the four compared algorithms, where the best average and standard deviation among four algorithms are highlighted with a gray background.

TABLE S-V
COMPARATIVE RESULTS OF IGD VALUES (MEAN AND STANDARD DEVIATION) ON CONSTRAINED BI-OBJECTIVE OPTIMIZATION PROBLEMS OBTAINED BY C-NSGA-III, C-MOEA/D, MOEA/D-DAE, AND DCNSGA-III, RESPECTIVELY.

| Problem | $m$ | C-NSGA-III | C-MOEA/D | MOEA/D-DAE | DCNSGA-III |
|---|---|---|---|---|---|
| C1-DTLZ1 | 2 | 1.8353e-3(7.24e-5) | 1.7824e-3(1.06e-6) | 1.8232e-3(1.20e-5) | 1.8251e-3(5.57e-5) |
| C1-DTLZ3 | 2 | 3.0009e+0(2.01e-3) | 2.5551e+0(3.42e+0) | 4.0296e-2(1.81e-1) | 3.0007e+0(9.91e-4) |
| C2-DTLZ2 | 2 | 1.3869e-1(9.86e-2) | 3.3352e-1(1.54e-1) | 1.1837e-3(2.90e-5) | 1.7711e-3(8.45e-5) |
| C3-DTLZ1 | 2 | 4.5938e-3(9.37e-4) | 8.2651e-3(2.37e-2) | 4.5747e-3(3.02e-4) | 7.6047e-3(2.97e-3) |
| C3-DTLZ4 | 2 | 9.2471e-3(5.48e-4) | 9.6789e-3(4.59e-4) | 8.7021e-3(2.73e-4) | 1.2063e-2(1.04e-3) |
| DC1-DTLZ1 | 2 | 4.9581e-4(8.97e-5) | 1.6595e-1(7.90e-2) | 6.3965e-2(1.00e-1) | 4.8899e-4(1.34e-4) |
| DC1-DTLZ3 | 2 | 2.4300e-2(6.97e-2) | 4.4291e-1(3.18e-1) | 2.6283e-1(2.96e-1) | 3.9511e-2(8.69e-2) |
| DC2-DTLZ1 | 2 | Infeasible | Infeasible | 1.18268e-3(2.96e-5) | 1.8224e-3(5.73e-5) |
| DC2-DTLZ3 | 2 | Infeasible | Infeasible | Infeasible | Infeasible |
| DC3-DTLZ1 | 2 | 7.4914e-2(6.14e-2) | 3.1199e-1(5.01e-1) | 4.6977e-1(1.11e-1) | 1.0533e-3(1.10e-4) |
| DC3-DTLZ4 | 2 | 6.3767e-1(1.78e-1) | 3.5385e+0(2.70e+0) | 1.5661e+1(1.81e+1) | 4.9956e-1(2.08e-1) |

"Infeasible" stands for the algorithm cannot find feasible solutions in all 30 runs.

Overall, MOEA/D-DAE performs best on the two-objective C-DTLZ test suite. For MOEA/D-DAE, if a feasible subarea is found, it will continue to search other areas which could dominate the current found feasible region, so that it can cross the insurmountable infeasible area to converge to the PF on C1-DTLZ3 problem. Furthermore, if the population gets stuck in local optima of the constraint violation, the diversity enhancement scheme will be trigged to jump out of this local region, which is suitable for solving problems with multi-modal of the constraint violation.

The proposed DCNSGA-III has the best performance on the bi-objective DC-DTLZ test suite. The constrains of DC1-DTLZ and DC3-DTLZ test instances split the feasible region into a couple of narrow tapered strips, the population has the risk of being trapped in a local feasible area and all feasible areas cannot easily be found. For DCNSGA-III, at first, the population gets to its approximated unconstrained PF. In the middle and late stages of the evolution, some feasible solutions keep unchanged due to some parts of the unconstrained PF are segments of the constrained PF. Other infeasible solutions move back to the segments of the constrained PF. That is why DCNSGA-III can perform best on such type of problem.

## S-VI. PLOTS OF THE OBTAINED APPROXIMATED SOLUTIONS

This section presents the plots of the final obtained approximated solutions obtained by DCNSGA-III and the six peer competitors on different test instances for 3-objective problems. Here the plots of the final obtained solutions are the one having the median IGD value. Note that the final obtained solutions may include feasible and infeasible solutions, due to an algorithm on some test problems may not obtain feasible solutions.

## S-VII. THE SETTING OF FIVE REPRESENTATIVE CONSTRAINT-HANDLING TECHNIQUES

In Section VI-C of the original paper, five well-known constraint-handling techniques: CDP [12], SR [13], self-adaptive penalty (SP) [14], $\varepsilon$ method [15], and adaptive trade-off model (ATM) [16] were embedded into the same framework of NSGA-III, namely CDP-NSGA-III, SR-NSGA-III, SP-NSGA-III, $\varepsilon$-NSGA-III, and ATM-NSGA-III, respectively. Some of them have parameters need to be set. The simulated binary crossover (SBX) [12] and polynomial mutation (PM) [12] were employed as reproduction operators for these five algorithms and the proposed DCNSGA-III:

- SBX: crossover probability $p_c$=0.9 and distribution index $\eta_c$=30;
- PM: mutation probability $p_m$=1/$n$ and distribution index $\eta_m$=20.

For $\varepsilon$-NSGA-III, as suggested in [17], $\theta = 0.1N, T_c = 0.6 \times Max_{gen}$, where $Max_{gen}$ is the maximum number of generations, and $cp = (-5 - \log \varepsilon_0)/\log(0.05)$. For SR-NSGA-III, the probability $P_f$=0.45.

The source code of DCNSGA-III is available from https://ruwangjiao.github.io/.

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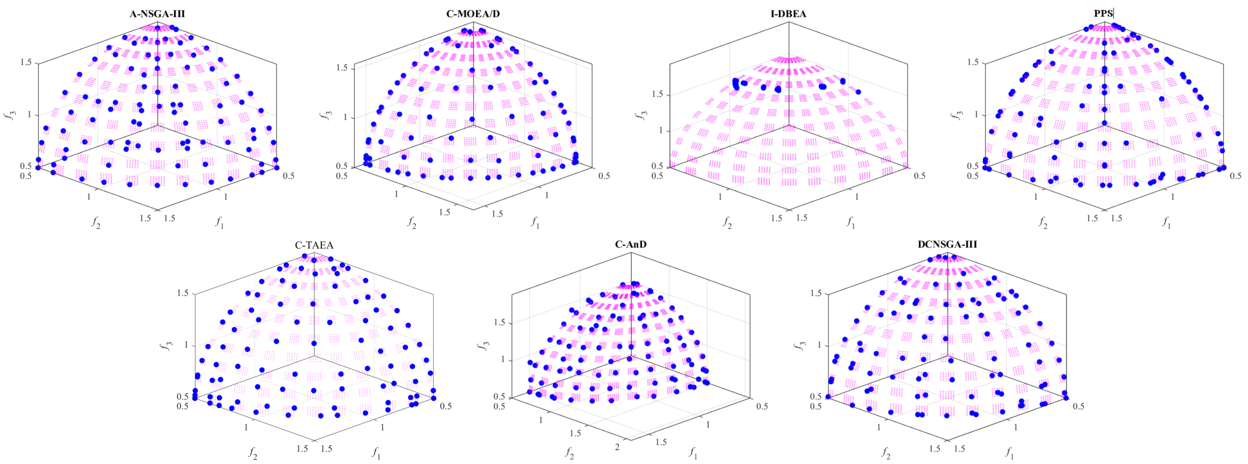


Fig. S-6. Obtained solutions for DAS-CMOP8 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

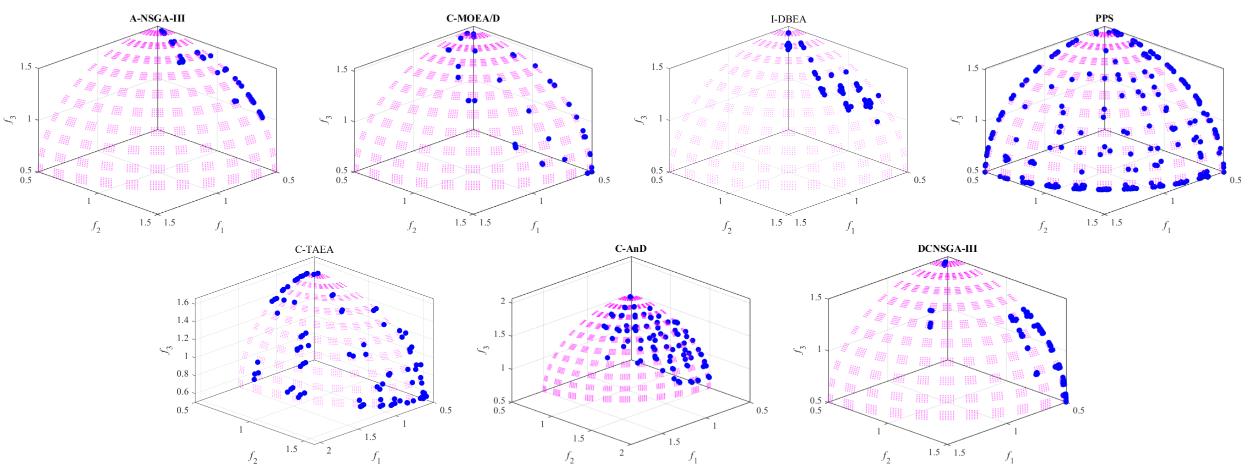


Fig. S-7. Obtained solutions for DAS-CMOP9 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

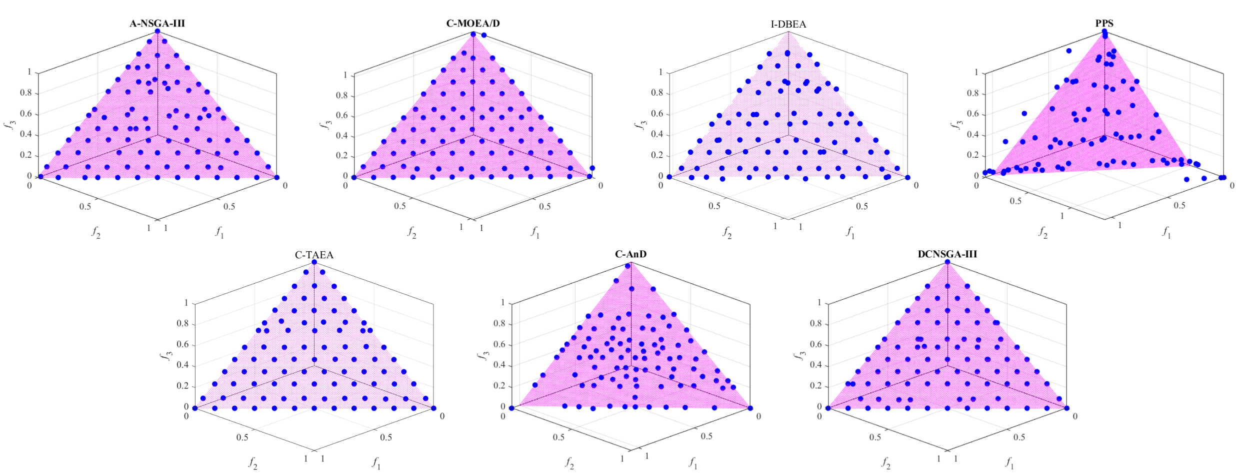


Fig. S-8. Obtained solutions for MW4 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

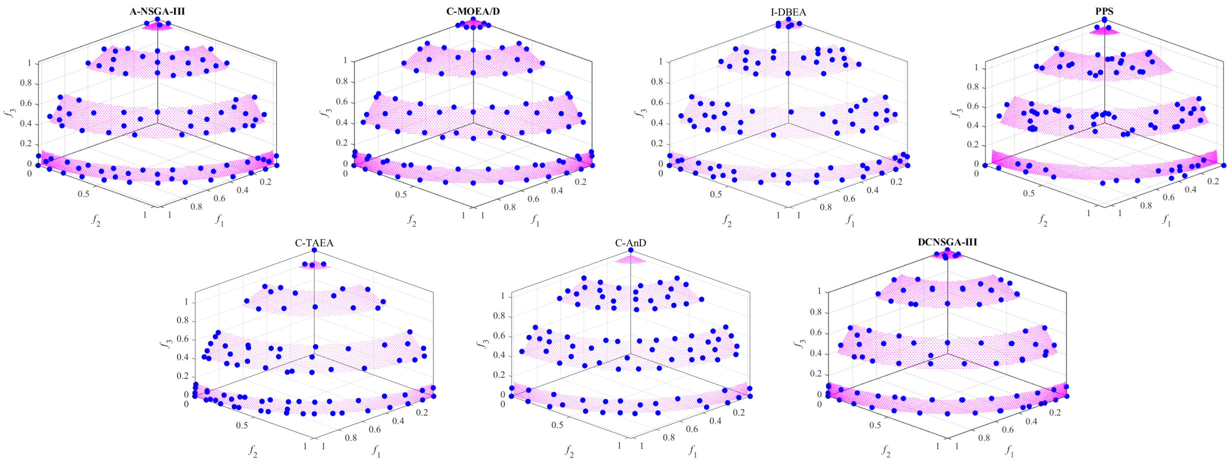


Fig. S-9. Obtained solutions for MW8 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

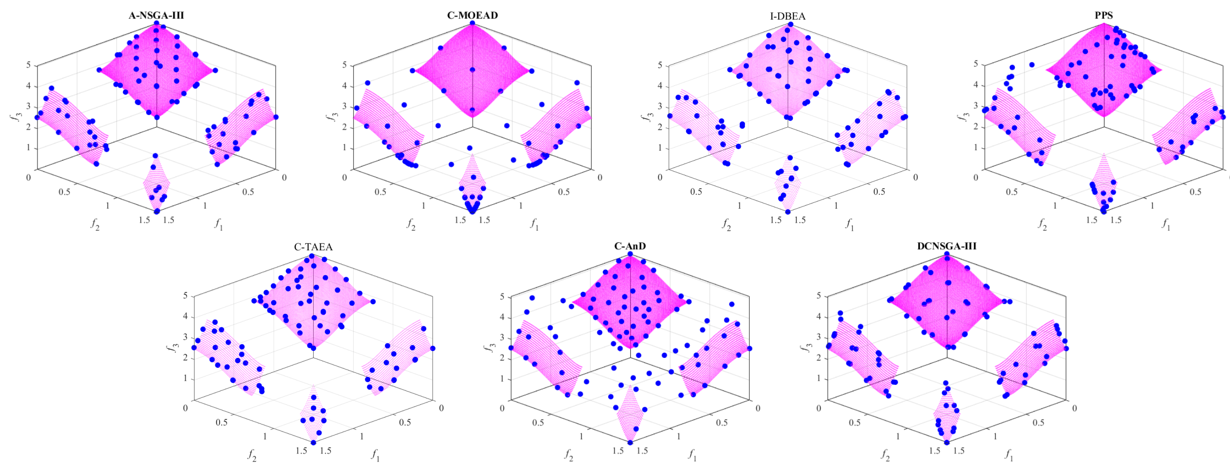


Fig. S-10. Obtained solutions for MW14 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

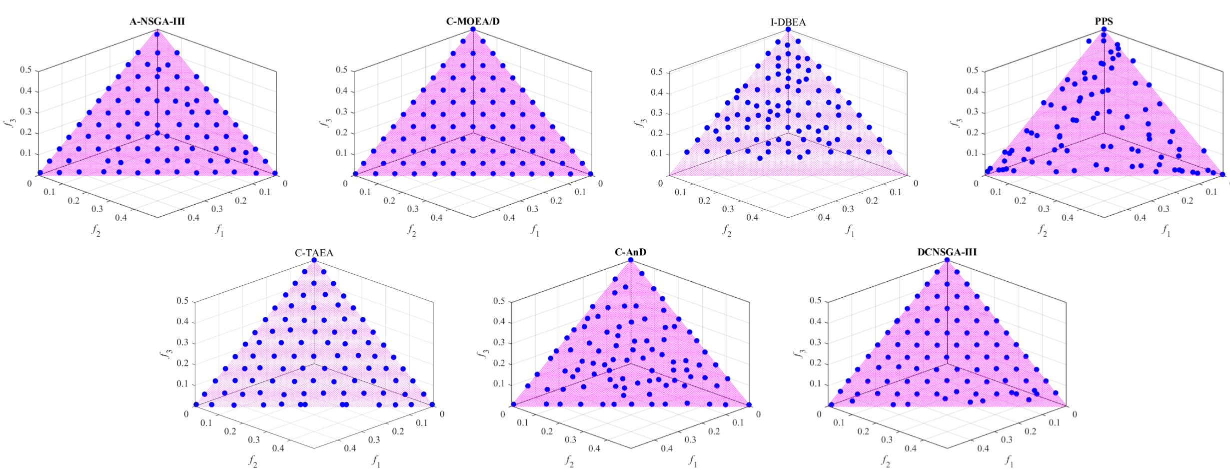


Fig. S-11. Obtained solutions for C1-DTLZ1 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

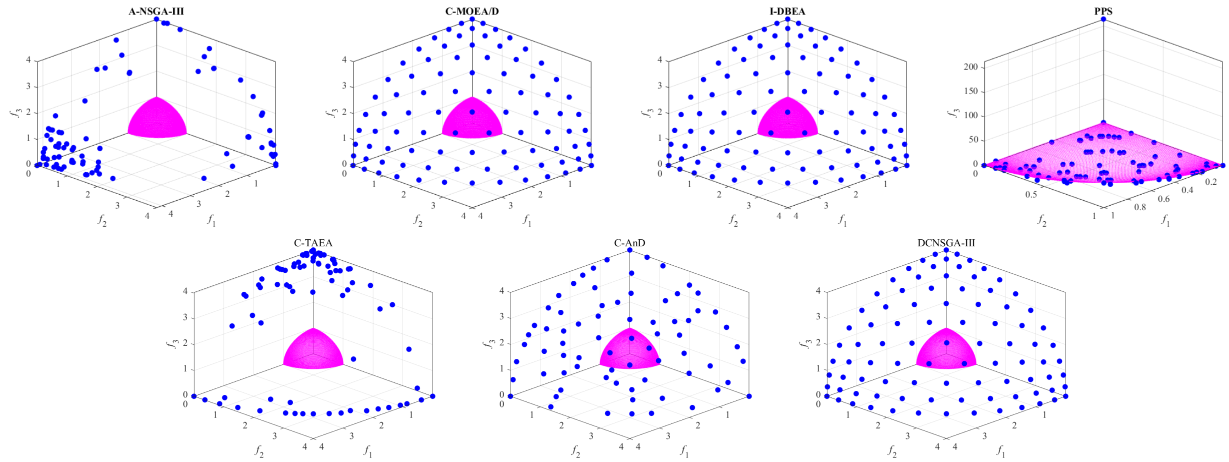


Fig. S-12. Obtained solutions for C1-DTLZ3 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

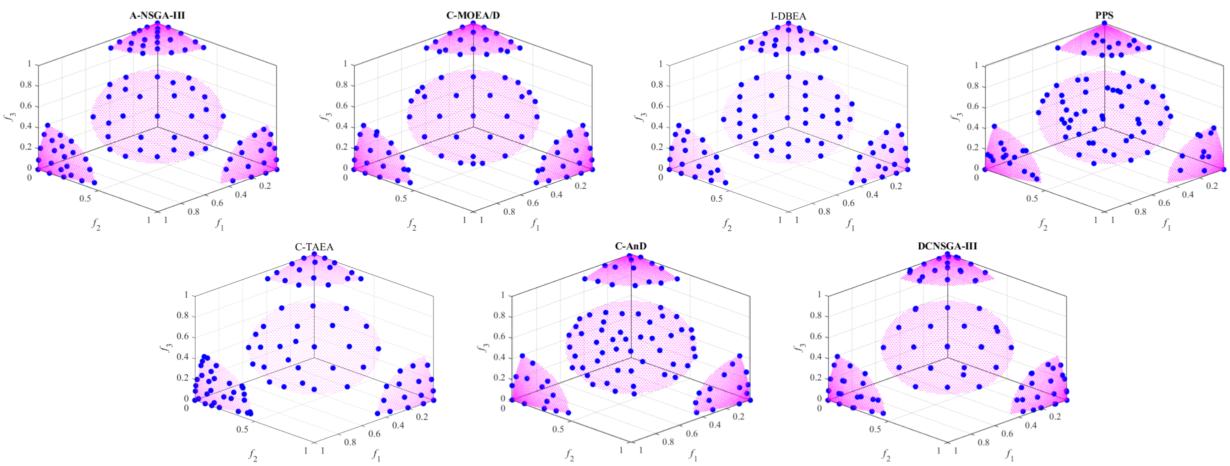


Fig. S-13. Obtained solutions for C2-DTLZ2 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

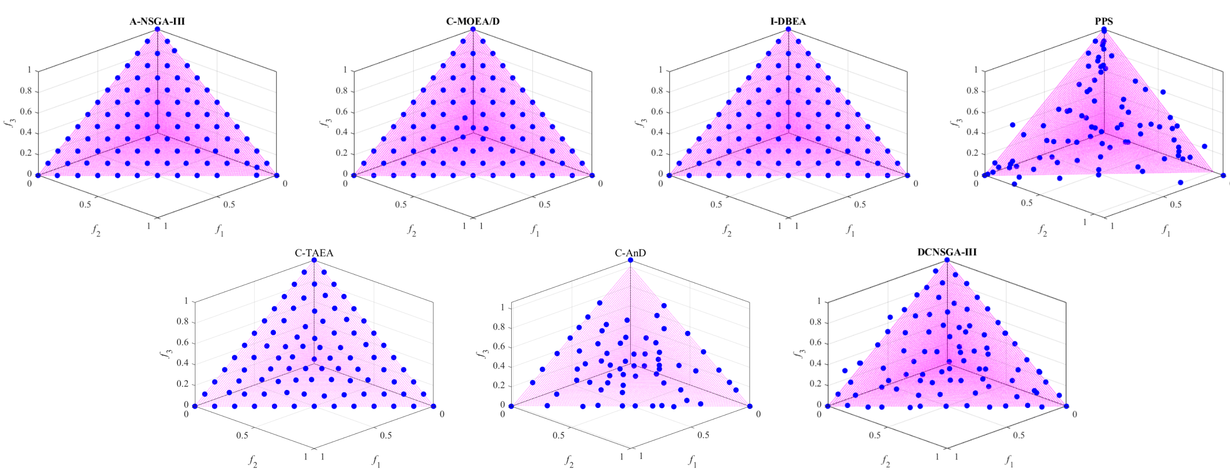


Fig. S-14. Obtained solutions for C3-DTLZ1 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

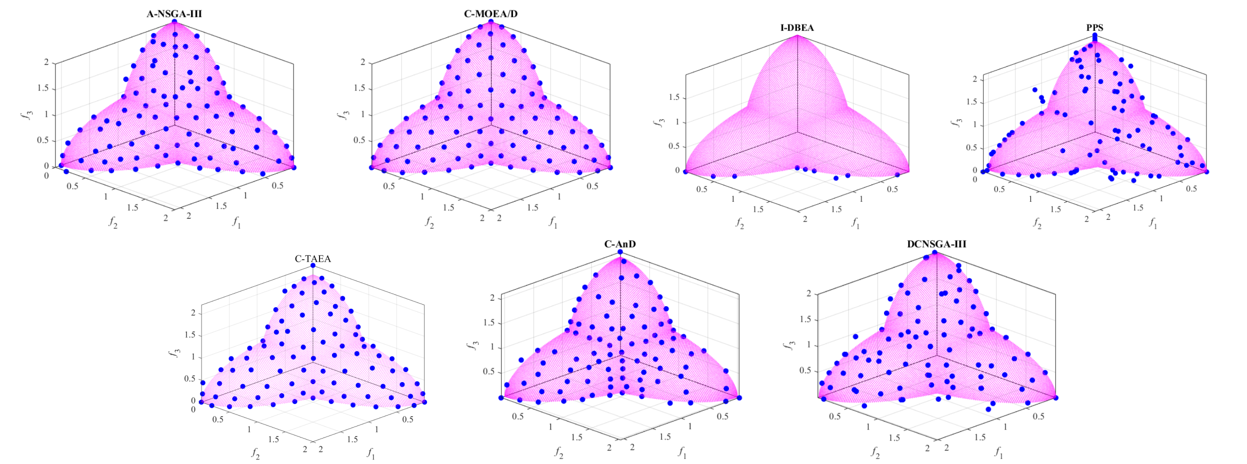


Fig. S-15. Obtained solutions for C3-DTLZ4 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

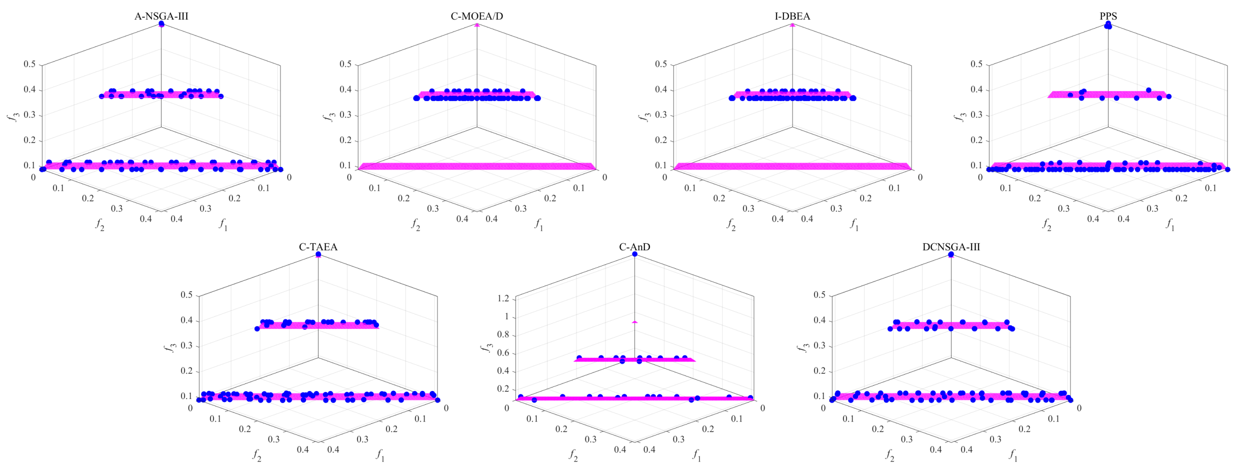


Fig. S-16. Obtained solutions for DC1-DTLZ1 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

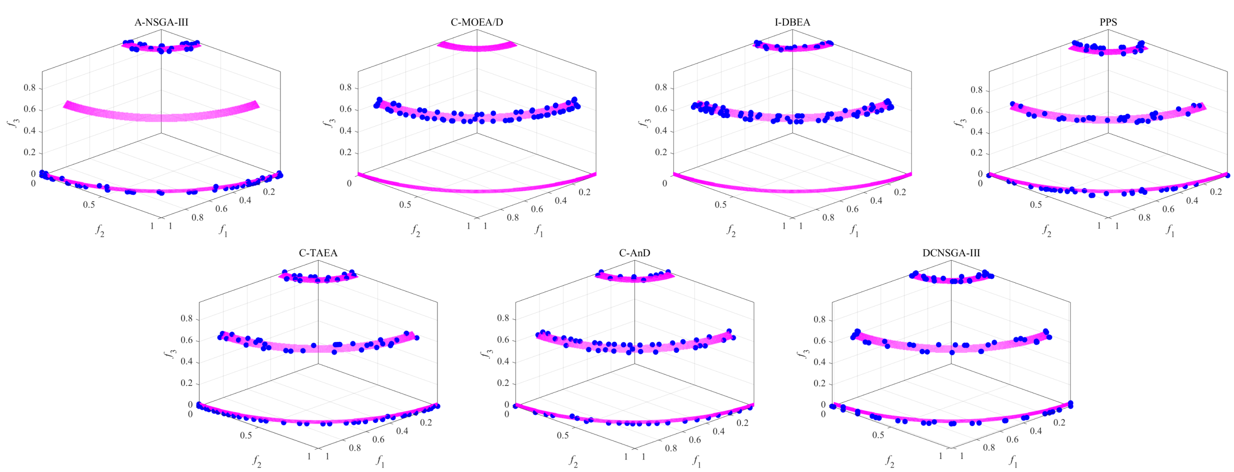


Fig. S-17. Obtained solutions for DC1-DTLZ3 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

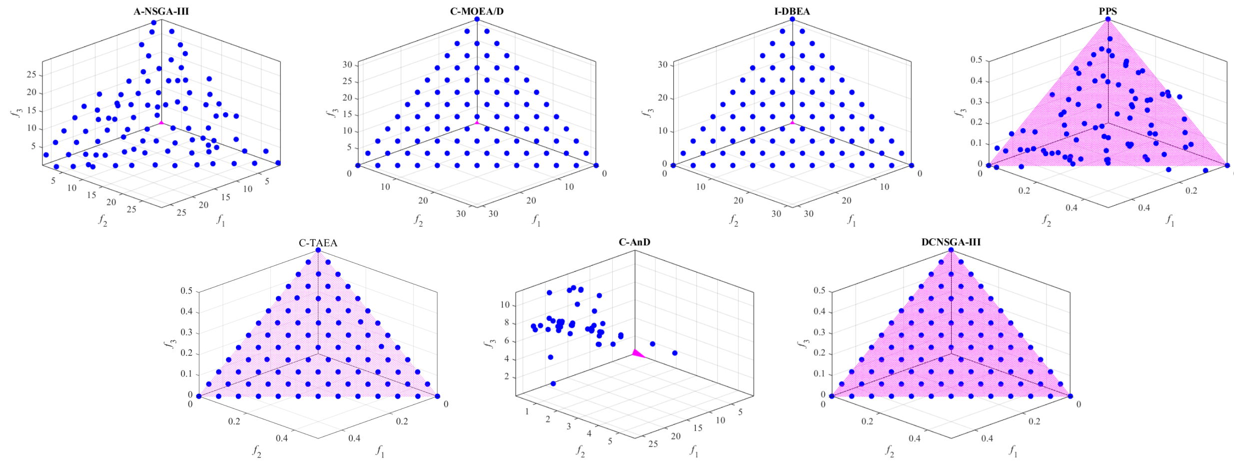


Fig. S-18. Obtained solutions for DC2-DTLZ1 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

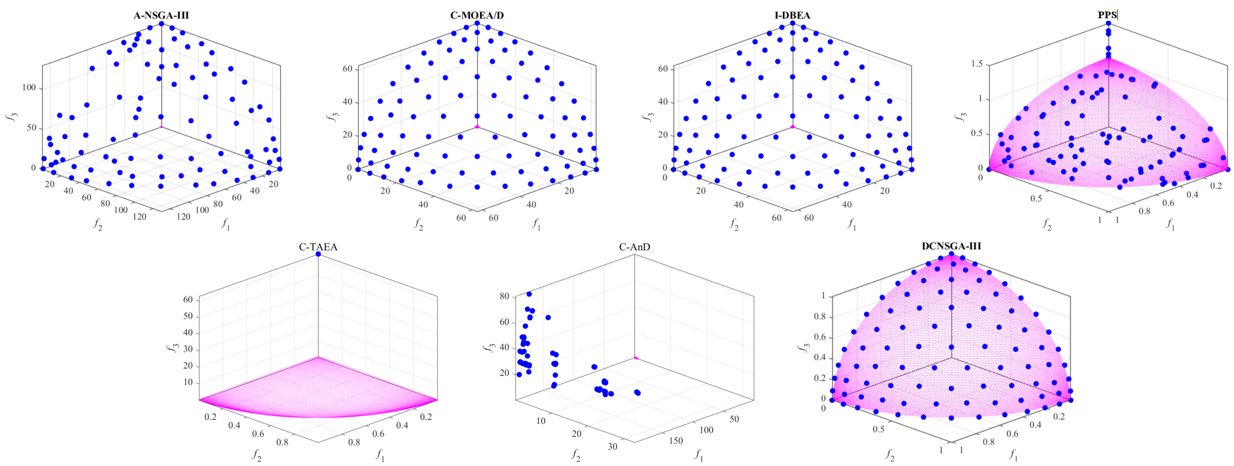


Fig. S-19. Obtained solutions for DC2-DTLZ3 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

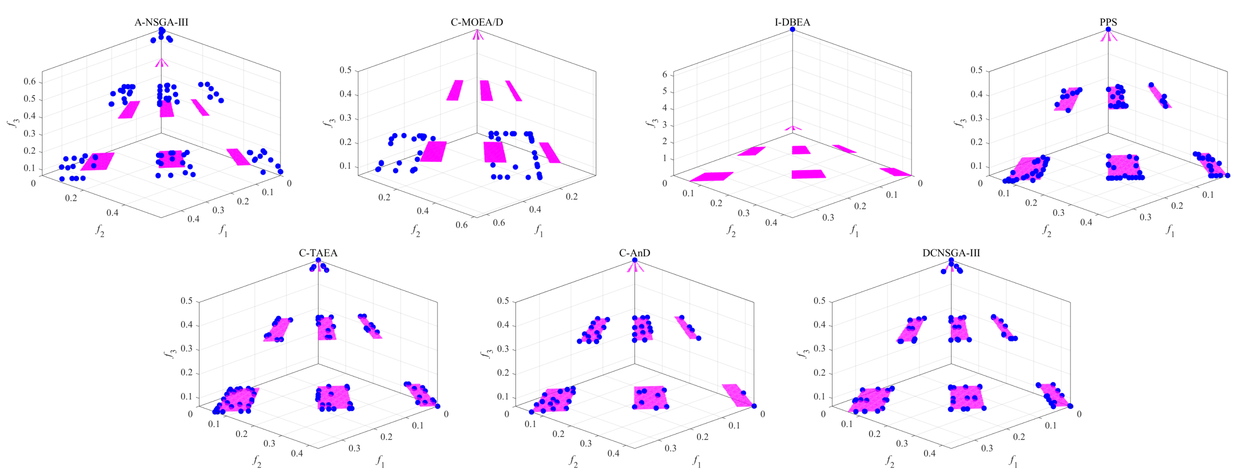


Fig. S-20. Obtained solutions for DC3-DTLZ1 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).

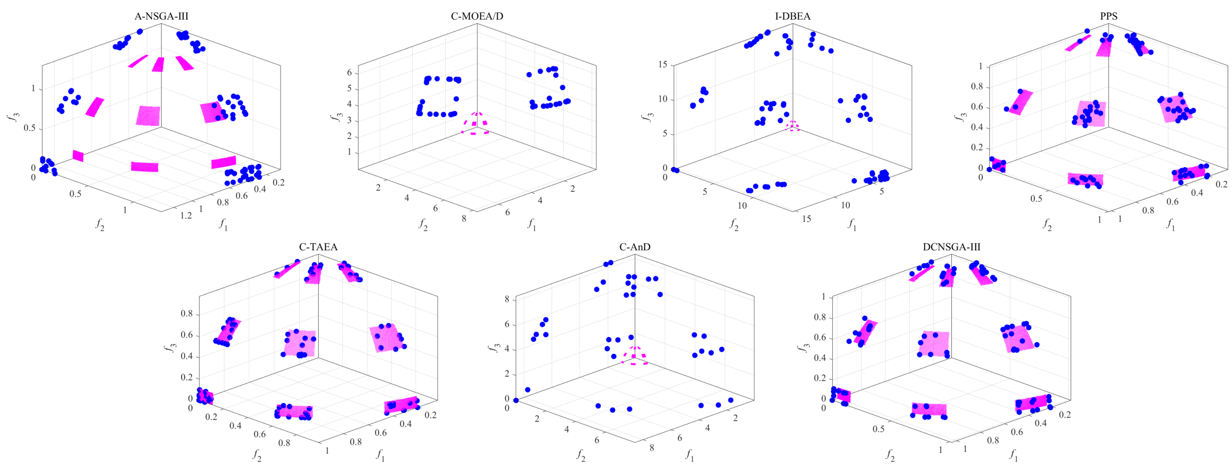


Fig. S-21. Obtained solutions for DC3-DTLZ3 problem using A-NSGA-III, C-MOEA/D, I-DBEA, PPS, C-TAEA, C-AnD, and DCNSGA-III from left to right (in median IGD value).