

# Quantizing neural networks

## Efficient Deep Learning - Session 3



**IMT Atlantique**  
Bretagne-Pays de la Loire  
École Mines-Télécom

## Sessions

- 1 Intro Deep Learning,
- 2 Data Augmentation and Self Supervised Learning,
- 3 Quantization,
- 4 Pruning,
- 5 Factorization,
- 6 Distillation,
- 7 Embedded Software and Hardware for DL,
- 8 Presentations for challenge.

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- 2 Data Augmentation and Self Supervised Learning,
- 3 **Quantization,**
- 4 Pruning,
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- 6 Distillation,
- 7 Embedded Software and Hardware for DL,
- 8 Presentations for challenge.

# Today's Summary

## 1 Objectives

## 2 Quantization : Basics

- Floating Point
- Integers, Fixed Point
- Quantization

## 3 Quantization : Neural Networks

- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

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# Motivation

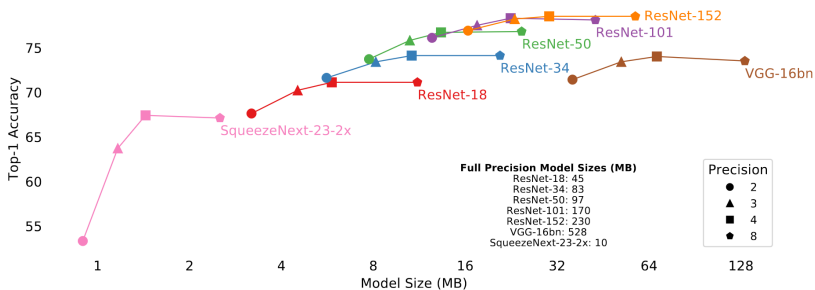
## 1 Reduce model size

- Fewer bits → Reduced memory footprint

## 2 Decrease memory access

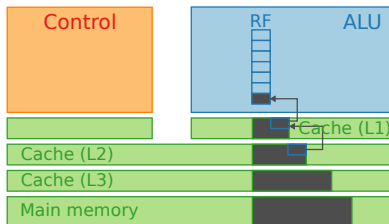
- GPU & CPU : reduce Cache usage

## 3 Computational complexity



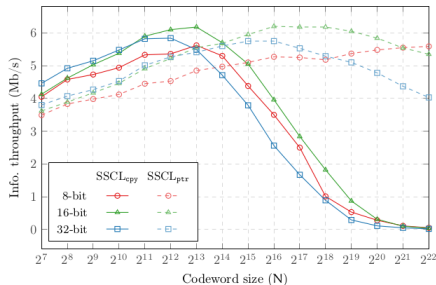
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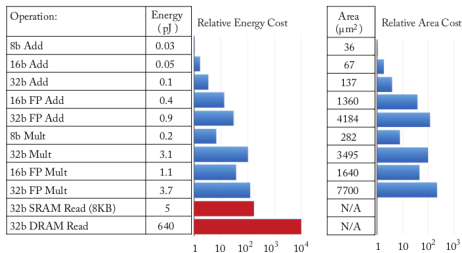
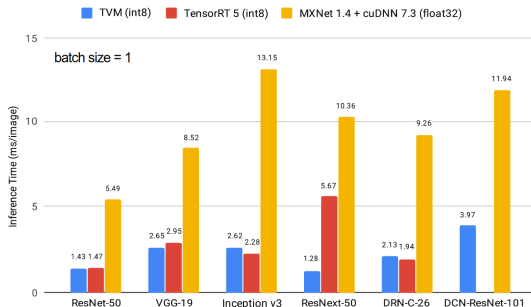


Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

# Motivation

- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- 2 Decrease memory access
  - GPU & CPU : reduce Cache usage
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From : <https://github.com/vinx13/tvm-cuda-int8-benchmark/>

## 1 Objectives

## 2 Quantization : Basics

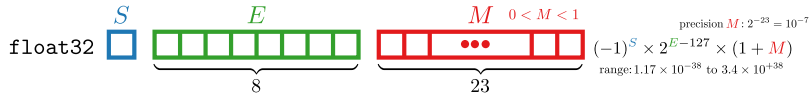
- Floating Point
- Integers, Fixed Point
- Quantization

## 3 Quantization : Neural Networks

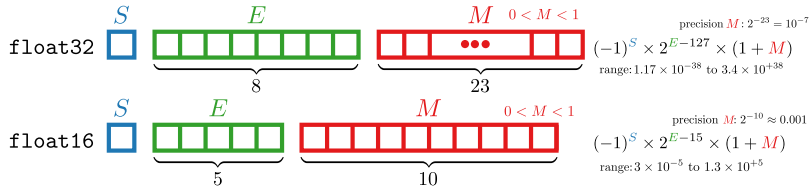
- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

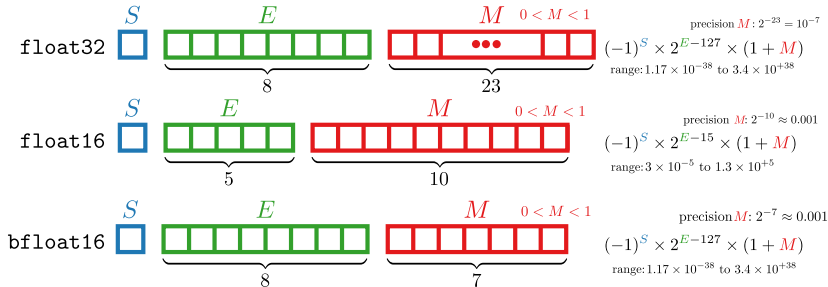
# Floating Point



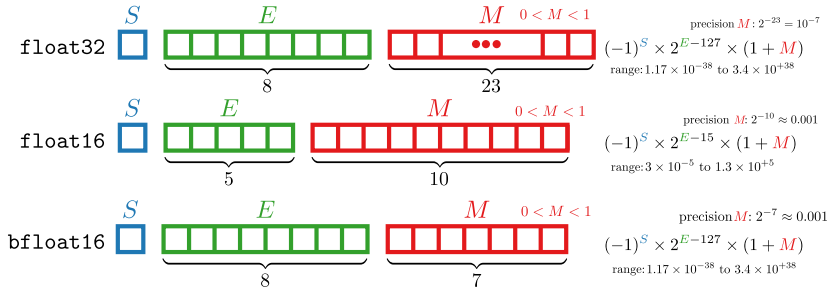
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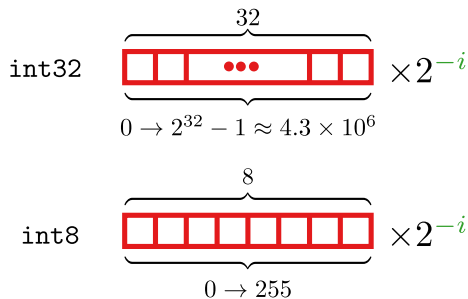
## ■ To add two FP numbers:

- Shift  $M$  according to  $E$  (int shift  $n_E$  bits)
- Add  $M$  (int add  $n_M$  bits)
- Normalize ( $0 < M < 1$ )

## ■ To multiply two FP numbers:

- Multiply  $M$  (int mult  $n_M$  bits)
- Add  $E$  (int mult  $n_E$  bits)
- Normalize ( $0 < M < 1$ )

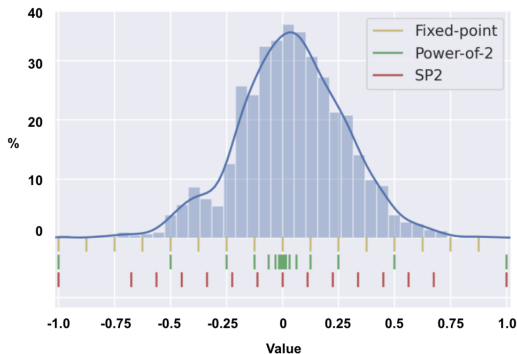
# Integers, fixed point



- Fixed point ( $-i$ )
- Short range
- Simple computation

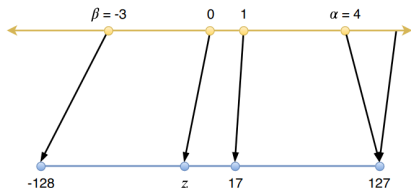


# Uniform and Non-Uniform Quantization

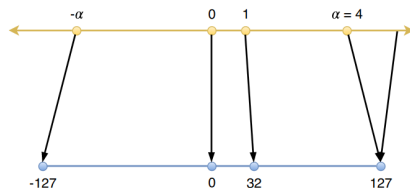


- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup → not straightforward for standard hardware (CPU, GPU)

# Affine and Scale Quantization



(a) Affine quantization

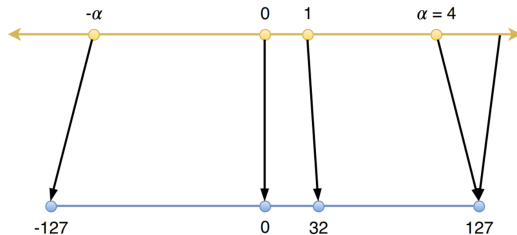


(b) Scale quantization

- 2 kinds of uniform quantization
- Assymetric vs Symmetric

Wu, Hao, et al. "Integer quantization for deep learning inference: Principles and empirical evaluation." arXiv preprint arXiv:2004.09602 (2020).

# Scale Quantization



$$\text{clip}(x, l, u) = \begin{cases} l, & x < l \\ x, & l \leq x \leq u \\ u, & x > u \end{cases}$$

$$s = \frac{2^{b-1} - 1}{\alpha}$$

$$x_q = \text{quantize}(x, b, s) = \text{clip}(\text{round}(s \cdot x), -2^{b-1} + 1, 2^{b-1} - 1)$$

$$\hat{x} = \text{dequantize}(x_q, s) = \frac{1}{s} x_q$$

$$y_{ij} = \sum_{k=1}^p x_{ik} \cdot w_{kj} \approx$$

$$\sum_{k=1}^p \text{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \text{dequantize}(w_{q,kj}, s_{w,kj}) =$$

$$\sum_{k=1}^p \frac{1}{s_{x,ik}} x_{q,ik} \cdot \frac{1}{s_{w,kj}} w_{q,kj}$$

And, in order to use integer multiplication, the scaling factor  $s$  must be independent of  $k$  :

$$\frac{1}{s_{x,i} \cdot s_{w,j}} \sum_{k=1}^p x_{q,ik} \cdot w_{q,kj}$$

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# Quantization Post Training : Weights

## Quantize a set of parameters

- Select the number of quantization bits,
- Select the set of parameters to quantize,
- Determine range of values,
- Determine the scaling factor (and zero if affine).

## Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation → better accuracy.

# Quantization Post Training : Activation

## Quantize a set of parameters

- Select the number of quantization bits,
- Select the set of activations to quantize,
- Determine range of values **according to the training set or a subset**,
- Determine the scaling factor (and zero if affine).

## Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

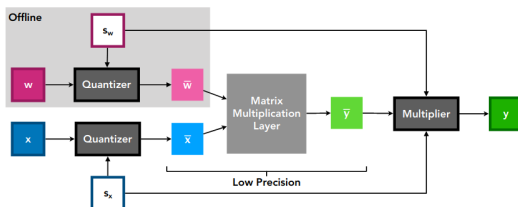
Finer sets segmentation → better accuracy.

# Quantization Aware Training

- Train the network while quantizing the weights and / or the activations,
- Quantization Aware Techniques yield way better accuracy,
- Especially for extremely low-bit precision (2-3-4 bit precision).



# Learned Step Size Quantization



- $s$  quantizer step size
- $Q_P$  and  $Q_N$ , the number of positive and negative quantization levels

$$\bar{v} = \lfloor \text{clip}(v/s, -Q_N, Q_P) \rfloor, \quad (1)$$

$$\hat{v} = \bar{v} \times s. \quad (2)$$

- $s$  is learned with :

$$\frac{\partial \hat{v}}{\partial s} = \begin{cases} -v/s + \lfloor v/s \rfloor & \text{if } -Q_N < v/s < Q_P \\ -Q_N & \text{if } v/s \leq -Q_N \\ Q_P & \text{if } v/s \geq Q_P \end{cases} \quad (3)$$

# Quantization while Learning - Binary Connect

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**Algorithm 1** SGD training with BinaryConnect.  $C$  is the cost function for minibatch and the functions  $\text{binarize}(w)$  and  $\text{clip}(w)$  specify how to binarize and clip weights.  $L$  is the number of layers.

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**Require:** a minibatch of (inputs, targets), previous parameters  $w_{t-1}$  (weights) and  $b_{t-1}$  (biases), and learning rate  $\eta$ .

**Ensure:** updated parameters  $w_t$  and  $b_t$ .

**1. Forward propagation:**

$$w_b \leftarrow \text{binarize}(w_{t-1})$$

For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$

**2. Backward propagation:**

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$

For  $k = L$  to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$

**3. Parameter update:**

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$

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Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.

"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015.

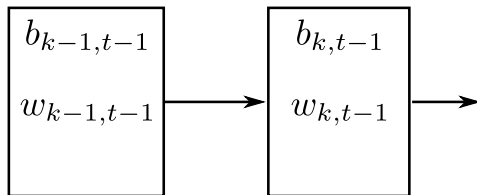
<https://arxiv.org/pdf/1511.00363.pdf>

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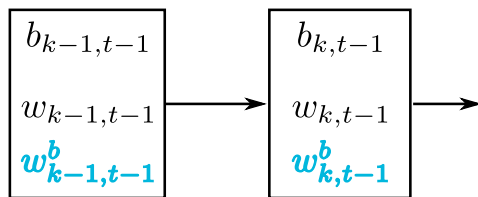
For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$



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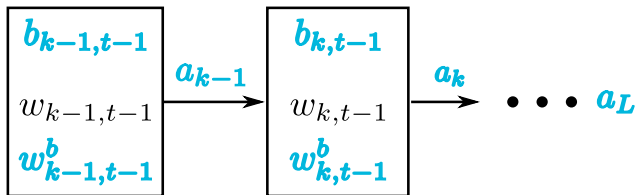
$$w^b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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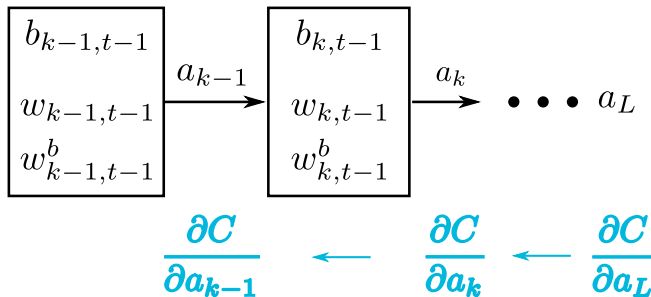


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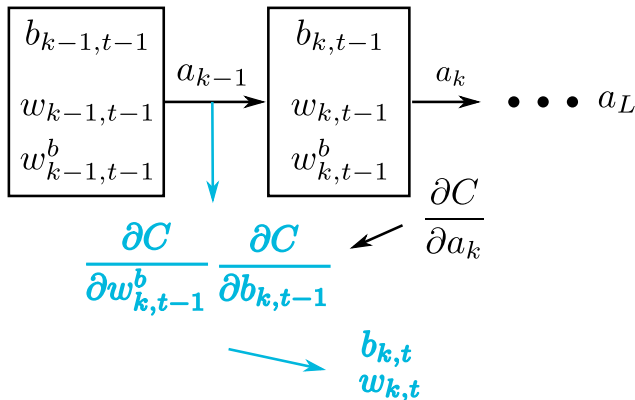
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## 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$



# Binarization : Stochastic vs Deterministic

## ■ Deterministic

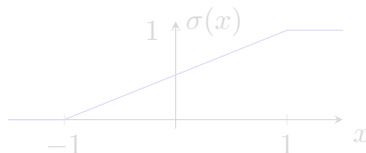
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## ■ Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1 - p \end{cases}$$

avec

$$\sigma(x) = \text{clip}\left(\frac{x+1}{2}, 0, 1\right) = \max\left(0, \min\left(1, \frac{x+1}{2}\right)\right)$$





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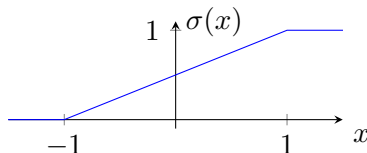
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# Quantization in Pytorch

- 1 Dynamic Quantization
- 2 Static Quantization
- 3 Quantization Aware Training

`https://pytorch.org/docs/stable/quantization.html`

And for our need :

`https://pytorch.org/tutorials/prototype/pt2e\_quant\_ptq.html`