Section 1.1

no

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1 Problem 1

Distributive Law 1:
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$ (Converting to logical statement)
$(A \wedge B) \vee (A \wedge C) = (A \wedge B) \vee (A \wedge C)$ (Using DeM Law applying to logic)
$(A \cap B) \cup (A \cap C) = (A \cap B) \cup (A \cap C)$ (Converting back to set theory)
Proved ✓

Distributive Law 2:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$ (Converting to logical statement)
$(A \vee B) \wedge (A \vee C) = (A \vee B) \wedge (A \vee C)$ (Using DeM Law applying to logic)
$(A \cup B) \cap (A \cup C) = (A \cup B) \cap (A \cup C)$ (Converting back to set theory)
Proved ✓

DeMorgan's Law 1:
$A - (B \cup C) = (A - B) \cap (A - C)$
$x \in A \wedge x \notin B \wedge x \notin C = (A - B) \cap (A - C)$
$(x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C) = (A - B) \cap (A - C)$
$(x \in (A - B)) \wedge (x \in (A - C)) = (A - B) \cap (A - C)$
$(A - B) \wedge (A - C) = (A - B) \cap (A - C)$ (Remove the instance x)
$(A - B) \cap (A - C) = (A - B) \cap (A - C)$ (Convert back to set theory)
Proved ✓

DeMorgan's Law 2:
$A - (B \cap C) = (A - B) \cup (A - C)$
$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C) = (A - B) \cup (A - C)$
$(x \in (A - B) \vee x \in (A - C) = (A - B) \cup (A - C)$
$(A - B) \vee (A - C) = (A - B) \cup (A - C)$ (Remove the instance x)
$(A - B) \cup (A - C) = (A - B) \cup (A - C)$ (Convert back to set theory)
Proved ✓

2 Problem 2

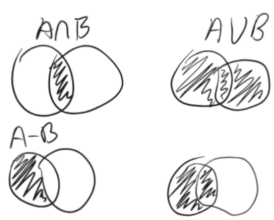
2.1 a)

$A \subset B$ and $A \subset C \leftrightarrow A \subset (B \cup C)$
$x \in A \cap x \in B \cap x \in C$
$x \in A \rightarrow x \in (B \cup C)$
$x \in A \rightarrow x \in B$ or $x \in C$
$A \subset B$ and $A \subset C \rightarrow A \subset (B \cup C)$ The arrow only works going to the right, because A could be a subset of just B and not C.
Therefore, this statement is false.

2.2 e)

A - (A - B) = B
A - B gives you all the elements in A but not intersecting with or in B.
Then, when you subtract that from the set A, you get ONLY the elements intersecting with B (if any).
This proves that the set containing only elements in the B set is not the only possibility if you take an instance x in a set.
Therefore, this statement is false.
If you were to change the equals to a $\subset$ sign, then the statement would technically true because the intersection between A and B is a subset of B.

2.3 i)



If we combine the two Venn-diagrams on the left side of the above image (which is the procedure for unions in a Venn-diagram), we get the set of elements contained in the entire A set, regardless of it's presence in B or not.
Therefore, this statement is true.

2.4 o)

$A \times (B - C) = (A \times B) - (A \times C)$
$a \in A$ and $(x \in B$ and $x \notin C) = (A \times B) - (A \times C)$
$(a \in A$ and $x \in B)$ and $(a \notin A$ and $x \notin C) = (A \times B) - (A \times C)$
$(A \times B)$ and $\neg(A \times C) = (A \times B) - (A \times C)$
$(A \times B) - (A \times C) = (A \times B) - (A \times C)$

Therefore, this statement is true because after using Cartesian identities, the statements are the same.

3 Problem 3

3.1 a)

x <0 $\rightarrow$ x^2 - x >0 (Statement, Counter: between 0 and -1, false)
x^2 - x $\leq$ 0 $\rightarrow$ x $\geq$ 0 (Contrapos., Counter: None, true)
x^2 - x >0 $\rightarrow$ x <0 (Converse, Counter: 3, false)

3.2 b)

If x >0 $\rightarrow$ $x^2 - x$ >0 (Statement, Counter: between 0 and 1, false)
x^2 - x $\leq$ 0 $\rightarrow$ x $\leq$ 0 (Contrapos., Counter: between 0 and 1, false)
Converse $x^2 - x$ >0 $\rightarrow$ x <0 (Converse, Counter: 3, false)

4 Problem 4

4.1 a)

a$\in$A$\rightarrow$ a^2 $\in$B
$\neg$a$\in$A$\vee$$a^2$ $\in$B
$\forall$a(a$\in$A$\rightarrow$ a^2 $\in$B)
$\exists$a(a$\in$A$\cap$$a^2$ $\notin$B)
There exists an a where it's an element of A but a^2 is not an element of B.

4.2 b)

$\exists$a(a$\in$A $\implies$ a^2 $\in$B)
$\neg\exists$a(a$\in$A $\implies$ a^2 $\in$B)
$\forall$a($\neg$(a$\in$A $\implies$ a^2 $\in$B))
$\forall$a(a$\in$A $\wedge\neg$(a^2 $\in$B))
$\forall$a(a$\in$A $\cap$$a^2$ $\notin$B))
For every a in A, a^2 is in B.

4.3 c)

$\forall$a(a$\in$A $\implies$ a^2 $\notin$B)
$\exists$a($\neg$(a$\in$A $\implies$ a^2 $\notin$B)
$\exists$a(a$\in$A $\wedge\neg$(a^2 $\notin$B))
$\exists$a(a$\in$A $\cap$$a^2$ $\in$B)
There exists an a in A, such that a^2 is in B.

4.4 d)

$\exists$a(a$\notin$A$\rightarrow$ $a^2 \in$ B)
$\forall$a(a$\notin$A$\cap a^2$ $\notin$B)
For every a not in A, not true for a^2 not in B.

5 Problem 5

5.1 a)

This means that x is somewhere in the combination of all of the sets of $\mathcal{A}$, and because you can not have an x that is an element of a set of sets, such as $\mathcal{A}$, this means that x has to be an element of one of the sets that is a subset of $\mathcal{A}$.
STATEMENT: TRUE
The converse is also true because if the entire set A is included in $\mathcal{A}$ then x has to be in $\mathcal{A}$ as well.
CONVERSE: TRUE

5.2 b)

This means that if x is an element of the union, then no MATTER what, it has to be in the set A, irregardless of if there are additional sets in $\mathcal{A}$, which is a false conception.
STATEMENT: FALSE
This statement means that for all x that are in A, the x also have to be in $\mathcal{A}$, which is true because all the elements of A have to be in $\mathcal{A}$, so no matter where x lies in A, conversely, it has to be in $\mathcal{A}$.
CONVERSE: TRUE

5.3 c)

This statement means that if x is an element of the intersection of A and all the other sets in $\mathcal{A}$, then it must resultingly be in the set A, which is true.
This is because if the intersection means that any arbitrary x is in every single set, then the set A falls under that category.
STATEMENT: TRUE
The converse means that if x is in A, then there is a place in the set you can place that x in A such that it is in intersection with all other subsets of $\mathcal{A}$.
This conclusion is false because just because x is an element of a does not necessarily always imply that x is also inside of the intersection.
CONVERSE: FALSE

5.4 d)

This statement says that in all cases where x is in the intersection of all subsets of $\mathcal{A}$, x will also always be in the subset A, which is true, because intersection

requires the existence in all sets to be satisfied.
STATEMENT: TRUE
This converse states that if x is in A, then it ALWAYS has to be in the intersection between all sets of $\mathcal{A}$, which is a misconception because correlation does not necessarily mean causation, which means that part of the subset A could not be present in the intersection, so there is an x that is in A but not in $\mathcal{A}$.
CONVERSE: FALSE

6 Problem 6

6.1 a)

$\forall$x(x$\notin$A) $\rightarrow$ x$\notin \bigcup_{A\in\mathcal{A}}$A

6.2 b)

$\exists$x(x$\notin$A) $\rightarrow$ x$\notin \bigcup_{A\in\mathcal{A}}$A

6.3 c)

$\forall$x(x$\notin$A) $\rightarrow$ x$\notin \bigcap_{A\in\mathcal{A}}$A

6.4 d)

$\exists$x(x$\notin$A) $\rightarrow$ x$\notin \bigcap_{A\in\mathcal{A}}$A

7 Problem 7

7.1 D)

(A$\cap$B)$\cup$(A$\cap$C)

7.2 E)

(A$\cup$C)$\cap$(B$\cup$C)

7.3 F)

(A - C)$\cup$(A$\cap$B)

8 Problem 8

A = a,b
$\mathcal{P}$(A) = {$\varnothing$, {a}, {b}, {a,b}}
1 element $\rightarrow$ 2

0 elements $\rightarrow$ 1
2 elements $\rightarrow$ 4
3 elements $\rightarrow$ 8
A has n elements: $\mathcal{P}$(A) has 2^n elements
n....4 2 1
$\mathcal{P}$(n+1)
$\mathcal{P}$(n)$\cup${b} $= 2^n + 2^n = 2^{n+1}$
$\mathcal{A}$
Using this proof, we can conclude that $\mathcal{P}$(A) is called the power set of A because it has 2^n elements, which is a power function.

9 Problem 9

9.1 a)

A - $\bigcup_{B\in\mathcal{B}}$B
x$\in$A $\wedge\neg$(x$\in$B)
x$\in$A $\wedge$ x$\notin$B
x$\in$(A-B)
$\bigcap_{B\in\mathcal{B}}$(A - B)

9.2 b)

A - $\bigcap_{B\in\mathcal{B}}$B
x$\in$A $\wedge\neg$(x$\in$B)
x$\in$A $\wedge$ x$\notin$B
x$\in$(A-B)
$\bigcup_{B\in\mathcal{B}}$(A - B)

10 Problem 10

10.1 a)

$\mathbb{R}\times$ (0,1]
True

10.2 b)

True

10.3 c)

(0, 1) and (2, 3) breaks the equations because the cross products cause y to not always be greater than x.
False

10.4 d)

True

10.5 e)

If you take the points (0.9, 0) and (0, 0.9), then when you take the cross product point (0.9, 0.9) it is not in the point, so the equation becomes not a set of $\mathbb{R} \times \mathbb{R}$. False