Project Euler 243

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January 2024

1 Problem 243

For this problem, I will start by evaluating the times at which the resilience values for a denominator d is at a minimum. After printing the first 50 resilience values for denominators 1-50, I found that the resilience is at it's lowest at 2, 6, and 30.

My initial thought was that we might be multiplying fibonacci numbers, or maybe multiplying 1*2*3..., but when I verified by checking greater numbers, the pattern didn't continue.

After some observation, Mr. Bailey brought to my attention that 30 = 2*3*5, and that those were all prime numbers. I knew that since the resilience is determined by the amount of factors that are coprime with the numerator, it must have something to do with a multiple of primes.

So, I found that when you multiply the first n prime numbers (i.e. 2*3*5*7*11), you get increasingly low resilience values, but not necessarily the FIRST resilience value to pass a given number (as seen that 12 is the first denominator positive integer that has a $R(d)$ value lower than 4/10, but 12 is not a multiple of primes).
I initially wrote code to manually check for every single coprime (and non-coprime) factor with a given denominator to compute its resilience value by dividing the result by $(d-1)$, but this was super slow and even when looking at the prime multiples only it still was not worth waiting through checking each value even after optimization of my functions.

When I was researching easier ways to find coprime values, I found a function related to products of sequential prime factors, which was similar to what I was doing so I made the assumption that it would save time. As it turns out, Euler did something exactly for the resilience value's numerator, a.k.a. an Euler formula to find the coprime factor count that I was looking for. This was called the Euler totient function.
To save time, I searched for a python implementation for the totient function and ran my code as $\frac{\phi(d)}{d-1}$, where $\phi(d)$ represents the totient function.
Now, I have the resilience values of the first 8 or so prime number multiples. With this, I can find the two prime number multiples in which $R(d) < \frac{15499}{94744}$ lies between.

The last step is where the process gets kind of confusing, because Mr. Bailey and I could not find out why this was the step to take (we had to search it up).
For the next step, you need to multiply the closest prime multiple that is greater than this resilience fraction n times until you get a resilience value less than that given resilience fraction.
Q.E.D the first integer is 892371480.