Section 1.3

RAJHOB

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1 Problem 1

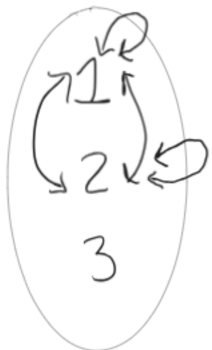
aCa
$aCb \leftrightarrow bCa$
$aCb \cap bCc \leftrightarrow aCc$
$y_0 - x_0^2 = y_1 - x_1^2$
$y_0 - x_0^2 = y_0 - x_0^2$
$y_0 - x_0^2 = y_1 - x_1^2$
$y_1 - x_1^2 = y_0 - x_0^2$
$-(y_0 - x_0^2) = (y_1 - x_1^2)$
(x_0, y_0)
(x_1, y_1)
(x_2, y_2)
$y_0 - x_0^2 = y_1 - x_1^2$
$y_1 - x_1^2 = y_2 - x_2^2$
$y_0 - x_0^2 = y_2 - x_2^2$
We can generalize this to $y = x^2 + a$ where $a \in \mathbb{R}$ such that all parabolas of $x^2 + a$ are partitions.

2 Problem 2

$C_0 = A_0 \times A_0$
Reflexivity, Symmetry, and Transitivity still all remain true because if the original function that C is a relation on ONLY consists of equivalence relations for any two points (or three for transitivity) you take from the function to check if it is an equivalence relation, we know these properties will still be true.
When we take the restriction (which is a subset of the original function) of the equivalence relation C, the subset will still only contain points that satisfy those properties of an equivalence relation.
More generally, if I have a set of red beads, and I take a handful of those red beads, every single bead in my new handful will still be red (in this analogy, red represents an equivalence relation).
(a_1, a_2, a_3)
$\therefore (a_1 \leftrightarrow a_2)$ and $a_1 \sim a_3$

3 Problem 3

$A = \{1, 2, 3\}$
$C = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$



$\therefore\ a\,C\,b$

3 is not related to anything OR itself

3 is in this set, but not related to anything, so the relation fails reflexivity, but still passes transitivity and symmetry.

4 Problem 4

4.1 a)

Reflexivity:
$f(a) = f(a) \to a \sim a$
Symmetry:
$a, b; f(a) = f(b) \leftrightarrow a \sim b$
$f(b) = f(a) \leftrightarrow b \sim a$
Transitivity: $a, b, c; (a \sim b \wedge b \sim c) \leftrightarrow f(a) = f(b) \wedge f(b) = f(c)$
$\therefore f(a) = f(c) \leftrightarrow a \sim c$

4.2 b)

Equivalence Class:
$E = \{y | y \sim x\} = \{y | f(y) = f(x)\}$ (Injectivity)
Because these two things are related, if they map to the same output then they are the same object in different formats, because we are mapping the entire equivalence class to the same output. So it is injective because there is only a single object being mapped to the equivalence class' value.
$b \in B \to$ F is surjective, so $\forall b \in B$, b has a corresponding equivalence class. (Surjectivity)

By the definition of Bijectivity, if the relation is both injective and surjective, then it is also bijective. Therefore, this equivalence relation is also bijective.