I_1, I_2, I_3

V_{ab}, V_{bd}, V_{cb}

l

I_1

I_2

I_3

$$I_a$$

I_b

$$\sum V_{abda}$$

=

$$-32 + 2I_a + 8(I_a - I_b) = 0$$

$$\sum V_{bcdb}$$

$$8(I_b - I_a) + 4I_b + 20 = 0$$

$$\begin{cases} 10I_a & -8I_b & = & 32 \\ -8I_a & +12I_b & = & -20 \end{cases} \quad \Longrightarrow \quad \begin{cases} I_a = 4 \\ I_b = 1 \end{cases}$$

$$I_1 = I_a = 4$$

$$I_2 = -I_b = -1$$

$$I_3 = I_a - I_b = 3$$

$$V_a = 32V$$

$$V_b = 32 - 2 \times 4 = 24V$$

$$V_c = 20V$$

$$V_{ab} = V_a - V_b = 8\,V$$

$$V_{bd} = V_b - V_d = V_b = 24\,V$$

$$V_{cb} = V_c - V_b = -4$$

I_c

$$-32 + 2(I_a + I_c) + 8(I_a - I_b) = 0$$

$$8(I_b - I_a) + 4(I_b + I_c) + 20 = 0$$

$$\sum V_{abcda}$$

$$-32 + 2(I_a + I_c) + 4(I_b + I_c) + 20 = 0$$

$$-32 + 2(I_a + I_c) + 8I_a = 0$$

$$-32 + 2(I_a + I_c) + 4I_c + 20 = 0$$

$$\left\{\begin{array}{rrcl} 10I_a & +2I_c & = & 32 \\ 2I_a & +6I_c & = & 12 \end{array}\right. \qquad \Longrightarrow \qquad \left\{\begin{array}{l} I_a = 3 \\ I_c = 1 \end{array}\right.$$

$$I_1 = I_a + I_c = 4$$

$$I_2 = -I_c = -1$$

n

$n - 1$

b

d

a

c

$$V_b = V_{bd}$$

v

$$\sum_i I_i = 0 = I_1 + I_2 + I_3$$

$$I_1 = \frac{V_a - V_b}{R_{ab}} = \frac{32 - V_b}{2}, \quad I_2 = \frac{V_c - V_b}{R_{cb}} = \frac{20 - V_b}{4}, \quad I_3 = \frac{V_d - V_b}{R_{bd}} = \frac{0 - V_b}{8}$$

$$I_1 + I_2 + I_3 = \frac{32 - V_b}{2} + \frac{20 - V_b}{4} + \frac{0 - V_b}{8} = 0$$

$$V_b = 24$$

$$I_1 = (V_a - V_b)/2 = (32 - 24)/2 = 4A$$

$$I_2 = (V_c - V_b)/4 = (20 - 24)/4 = -1A$$

$$I_3 = I_1 + I_2 = 3A$$

$$\sum_i I_i = 0 = -I_1 - I_2 - I_3$$

V

V_i

R_i

I_j

R_k

$$\sum_i \frac{V - V_i}{R_i} + \sum_k \frac{V}{R_k} = \sum_j I_j$$

$$V = \frac{\sum_i V_i/R_i + \sum_j I_j}{\sum_i 1/R_i + \sum_k 1/R_k} = \frac{\sum I}{\sum G}$$

$$G = 1/R$$

$$\sum_j (I - I_j)R_j + \sum_k IR_k = \sum_i V_i$$

I

$$I = \frac{\sum_i V_i + \sum_j R_j I_j}{\sum_i R_j + \sum_k R_k} = \frac{\sum V}{\sum R}$$

$$\begin{aligned}
&\text{loop bacb:} \quad -V_{s1} + R_1 I_a + R_3(I_a - I_b) + R_4(I_a - I_c) + V_{s4} = 0 \\
&\text{loop adca:} \quad R_2 I_b + V_{s2} + R_5(I_b - I_c) + R_3(I_b - I_a) = 0 \\
&\text{loop bcdb:} \quad -V_{s4} + R_4(I_c - I_a) + R_5(I_c - I_b) + R_6 I_c - V_{s6} = 0
\end{aligned}$$

$$
\begin{array}{ll}
\text{node a:} & (V_a - (V_b + V_{s1}))/R_1 + (V_a - V_{s2})/R_2 + (V_a - V_c)/R_3 = 0 \\
\text{node b:} & (V_b - (V_a - V_{s1}))/R_1 + (V_b + V_{s4} - V_c)/R_4 + (V_b - V_{s6})/R_6 = 0 \\
\text{node c:} & (V_c - (V_b + V_{s4}))/R_4 + (V_c - V_a)/R_3 + V_c/R_5 = 0
\end{array}
$$

$$I = 0.5A$$

$$V = 6V$$

$$R_1 = 3\Omega$$

$$R_2 = 8\Omega$$

$$R_3 = 6\Omega$$

$$R_4 = 4\Omega$$

$$I_1 = I = 0.5A$$

$$\begin{cases} R_2(I_2 - I_1) + R_3(I_2 - I_3) - V = 0 \\ R_1(I_3 - I_1) + R_4 I_3 + R_3(I_3 - I_2) = 0 \end{cases}$$

$$\begin{cases} 14I_2 - 6I_3 = 10 \\ -6I_2 + 13I_3 = 1.5 \end{cases} \quad \Longrightarrow \quad \begin{cases} I_2 = 0.952 \\ I_3 = 0.555 \end{cases}$$

$$V_3 = -6V$$

$$V_2 = R_2(I_1 - I_2) = 8(0.5 - 0.952) = -3.616V$$

$$V_1 = V_3 + R_4 I_3 = -3.78V$$

V_1

V_2

V_3

$$V_3 = -V = -6V$$

$$\begin{cases} (V_1 - V_3)/R_4 + (V_1 - V_2)/R_1 = I \\ (V_2 - V_1)/R_1 + (V_2 - V_3)/R_3 + V_2/R_2 = 0 \end{cases}$$

$$\begin{cases} 7V_1 - 4V_2 = -12 \\ -8V_1 + 15V_2 = -24 \end{cases} \quad \Longrightarrow \quad \begin{cases} V_1 = -3.78 \\ V_2 = -3.616 \end{cases}$$

$$V_1 = 12V$$

$$V_2 = 6V$$

$$R_1$$

R_2

R_3

$$R_1 = 100\Omega$$

$$R_2 = 5\Omega$$

$$R_3 = 200\Omega$$

$$R_4 = 50\Omega$$

$$V = 50V$$

$$I = 0.2A$$

x

y

$$y = F(x)$$

$$F(ax) = aF(x)$$

$$F(x_1 + x_2) = F(x_1) + F(x_2)$$

$$F(ax_1 + bx_2) = aF(x_1) + bF(x_2)$$

F

x_1

x_2

$$F(x_1)$$

$F(x_2)$

$$V^2/R$$

$$I^2R$$

$$I_{12} = \frac{aV_1 + bV_2}{R} = a\frac{V_1}{R} + b\frac{V_2}{R} = aI_1 + bI_2$$

$$I_1 = V_1/R$$

$$I_2 = V_2/R$$

$$V = R(aI_1 + bI_2) = aI_1R + bI_2R = aV_1 + bV_2$$

$$V_1 = I_1 R$$

$$V_2 = I_2R$$

$$P_{12} = \frac{(V_1 + V_2)^2}{R} \neq \frac{V_1^2}{R} + \frac{V_2^2}{R} = P_1 + P_2, \qquad P_{12} = R(I_1 + I_2)^2 \neq RI_1^2 + RI_2^2 = P_1 + P_2$$

$$I_1' = \frac{32}{2+8||4} = \frac{48}{7}, \quad I_2' = -I_1'\frac{8}{8+4} = -\frac{32}{7}, \quad I_3' = I_1'\frac{4}{8+4} = \frac{16}{7}$$

$$I_2'' = \frac{20}{4 + 8||2} = \frac{25}{7}, \quad I_1'' = -I_2''\frac{8}{2+8} = -\frac{20}{7}, \quad I_3'' = I_2''\frac{2}{8+2} = \frac{5}{7}$$

$$I_1 = I_1' + I_1'' = \frac{48}{7} - \frac{20}{7} = 4, \quad I_2 = I_2'' + I_2' = \frac{25}{7} - \frac{32}{7} = -1, \quad I_3 = I_3' + I_3'' = \frac{16}{7} + \frac{5}{7} = 3$$

R

R_L

(V_T, R_T)

(I_N, R_N)

V_T

$$R_T$$

$$V_T = V_{oc}$$

$$I = V/R_L$$

$$V' = V_{oc}$$

$$V'' = -R_0 I$$

R_0

$$V = V' + V'' = V_{oc} - R_0 I$$

$$V = V_T - R_T I$$

$$R_T = R_0$$

I_N

R_N

$$I_N = I_{sc}$$

$$R_N = R_T$$

$$V = IR_L$$

$$I' = I_{sc}$$

$$I'' = -V/R_0$$

$$I = I' + I'' = I_{sc} - V/R_0$$

$$I = I_N - V/R_N$$

$$R_N = R_0$$

$$R_T = R_N$$

$$V_{oc}/R_T = V_T/R_T = I_{sc} = I_N$$

$$R_T = R_N = \frac{V_{oc}}{I_{sc}}$$

N

$i = 1, \cdots, n$

$$V = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

R_j

$$V_j = 0$$

$$V_j = I_j R_j$$

Δ

Y

$$R'_{ab}$$

$$\begin{cases} R'_{ab} = R_a + R_b = R_{ab}||(R_{ac} + R_{bc}) \\ R'_{ac} = R_a + R_c = R_{ac}||(R_{ab} + R_{bc}) \\ R'_{bc} = R_b + R_c = R_{bc}||(R_{ab} + R_{ac}) \end{cases}$$

R_{ab}

R_{ac}

R_{bc}

R_a

R_b

R_c

$$\begin{cases} R_a = R_{ab}R_{ac}/(R_{ab} + R_{ac} + R_{bc}) \\ R_b = R_{ab}R_{bc}/(R_{ab} + R_{ac} + R_{bc}) \\ R_c = R_{ac}R_{bc}/(R_{ab} + R_{ac} + R_{bc}) \end{cases}$$

$$\begin{cases} R_{ab} = R_a + R_b + R_a R_b / R_c \\ R_{ac} = R_a + R_c + R_a R_c / R_b \\ R_{bc} = R_b + R_c + R_b R_c / R_a \end{cases}$$

Z_1

Z_3

Z_4

$$\begin{cases} Y_1 = Z_1 Z_3/(Z_1 + Z_2 + Z_3) \\ Y_2 = Z_2 Z_3/(Z_1 + Z_2 + Z_3) \\ Y_3 = Z_1 Z_2/(Z_1 + Z_2 + Z_3) + Z_4 \end{cases}$$

Z_2

$$
\left\{
\begin{array}{l}
X_1 = Z_1 + Z_4 + Z_1 Z_4 / Z_2 \\
X_2 = Z_2 + Z_4 + Z_2 Z_4 / Z_1 \\
X_3 = (Z_1 + Z_2 + Z_1 Z_2 / Z_4) || Z_3
\end{array}
\right.
$$

$$R_T = R_N = R_1$$

$$V_T = V_{oc} = V_{ab} = V_0 + I_0 R_1$$

$$I_N = I_{sc} = I_{ab} = V_0/R_1 + I_0$$

$$I_N = V_T/R_T = V_0/R_1 + I_0, \qquad V_T = I_N R_N = V_0 + I_0 R_1$$

V_0

I_0

$$IR_2 + (I_0 + I)R_1 - V_0 = 0$$

$$I = \frac{V_0 - I_0 R_1}{R_1 + R_2}$$

$$I_2 = I$$

$$I_0 + I_1 + I_2 = I_0 + \frac{V - V_0}{R_1} + \frac{V}{R_2} = 0,$$

$$I_0R_1R_2 + R_2(V - V_0) + R_1V = 0$$

$$V = \frac{R_2 V_0 - I_0 R_1 R_2}{R_1 + R_2} = V_0 \frac{R_2}{R_1 + R_2} - I_0 \frac{R_1 R_2}{R_1 + R_2}$$

$$I = I_2 = \frac{V}{R_2} = \frac{V_0}{R_1 + R_2} - I_0 \frac{R_1}{R_1 + R_2}$$

V'

I'

$$I' = \frac{V_0}{R_1 + R_2}, \quad V' = I' R_2 = V_0 \frac{R_2}{R_1 + R_2}$$

$$V''$$

I''

$$I'' = -I_0 \frac{R_1}{R_1 + R_2}, \quad V'' = I'' R_2 = -I_0 \frac{R_1 R_2}{R_1 + R_2}$$

$$I = I' + I'' = \frac{V_0}{R_1 + R_2} - I_0 \ \frac{R_1}{R_1 + R_2}$$

$$V = V' + V'' = V_0 \ \frac{R_2}{R_1 + R_2} - I_0 \ \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = V_0/R_1$$

$$I = \left(\frac{V_0}{R_1} - I_0 \right) \frac{R_1}{R_1 + R_2} = \frac{V_0}{R_1 + R_2} - I_0 \frac{R_1}{R_1 + R_2}$$

$$V = R_2 I = V_0 \frac{R_2}{R_1 + R_2} - I_0 \frac{R_1 R_2}{R_1 + R_2}$$

$$R_T = R_1, \quad V_T = V_0 - I_0 R_1$$

$$I = \frac{V_T}{R_T + R_2} = \frac{V_0 - I_0 R_1}{R_1 + R_2} = \frac{V_0}{R_1 + R_2} - I_0 \frac{R_1}{R_1 + R_2}$$

$$R_N = R_1, \quad I_N = V_0/R_1 - I_0$$

$$I = \left(\frac{V_T}{R_1} - I_0 \right) \frac{R_T}{R_T + R_2} = \frac{V_0}{R_1 + R_2} - I_0 \frac{R_1}{R_1 + R_2}$$

$$V_0 = 18V$$

$$R_1 = R_2 = 3\Omega$$

$$R_4 = 1.5\Omega$$

$$R_5 = 2\Omega$$

R_5

1Ω

3Ω

R_4

$$\begin{cases} R_2(I_0 - I_1) + R_4(I_0 - I_2) = 3(I_0 - I_1) + 1.5(I_0 - I_2) = 18 \\ R_1 I_1 + R_5(I_1 - I_3) + R_2(I_1 - I_0) = 3I_1 + 2(I_1 - I_3) + 3(I_1 - I_0) = 0 \\ R_3 I_2 + R_4(I_2 - I_0) + R_5(I_2 - I_1) = 6I_2 + 1.5(I_2 - I_0) + 2(I_2 - I_1) = 0 \end{cases}$$

$$\left\{\begin{array}{l} 3I_0 - 2I_1 - I_2 = 12 \\ -3I_0 + 8I_1 - 2I_2 = 0 \\ 3I_0 + 4I_1 - 19I_2 = 0 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} I_0 = 32/5 \\ I_1 = 14/5 \\ I_2 = 8/5 \end{array}\right.$$

$$V_a = R_3 I_2 = 6 \times \frac{8}{5} = \frac{48}{5}, \qquad V_b = R_4(I_0 - I_2) = 1.5 \times \frac{24}{5} = \frac{36}{5}, \qquad V_{ab} = V_a - V_b = \frac{12}{5}$$

$$I_1 - I_2 = \frac{14}{5} - \frac{8}{5} = \frac{6}{5} = 1.2\ A,$$

$$I = \frac{V_a - V_b}{R_5} = \frac{12/5\ V}{2\ \Omega} = 1.2\ A$$

$$V_0 = 18\,V$$

$$\frac{V_a - V_0}{R_1} + \frac{V_a - V_b}{R_5} + \frac{V_a}{R_3} = \frac{V_a - 18}{3} + \frac{V_a - V_b}{2} + \frac{V_a}{6} = 0$$

$$\frac{V_b - V_0}{R_2} + \frac{V_b - V_a}{R_5} + \frac{V_b}{R_4} = \frac{V_b - 18}{3} + \frac{V_b - V_a}{2} + \frac{V_b}{1.5} = 0$$

$$\begin{cases} -V_a + 3V_b = 12 \\ 2V_a - V_b = 12 \end{cases} \implies \begin{cases} V_a = 48/5 \\ V_b = 36/5 \end{cases}$$

$$I = \frac{V_a - V_b}{R_5} = \frac{(48/5 - 36/5)\,V}{2\,\Omega} = \frac{12/5\ V}{2\ \Omega} = \frac{6}{5} = 1.2\ A$$

$$I = 0.5$$

$$I = (V_a - V_b)/2$$

$$\frac{V_a - 18}{3} + 0.5 + \frac{V_a}{6} = 0, \qquad \frac{V_b - 18}{3} - 0.5 + \frac{V_b}{1.5} = 0$$

$$V_a = 11\,V$$

$$V_b = 6.5\,V$$

$$R_5 = \frac{V_a - V_b}{I} = \frac{11 - 6.5}{0.5} = 9\ \Omega$$

$$\Delta - Y$$

$$R_c = \frac{R_1 R_2}{R_1 + R_2 + R_5} = \frac{9}{8}, \quad R_a = \frac{R_1 R_5}{R_1 + R_2 + R_5} = \frac{3}{4}, \quad R_b = \frac{R_2 R_5}{R_1 + R_2 + R_5} = \frac{3}{4}$$

$$R_0 = R_c + (R_a + R_3) \mid\mid (R_b + R_4) = \frac{45}{16}$$

$$I_0 = \frac{V_0}{R_0} = \frac{18}{45/16} = \frac{32}{5}$$

$$I_a = I_0 \; \frac{R_b + R_4}{(R_a + R_3) + (R_b + R_4)} = \frac{8}{5}$$

$$I_b = I_0 \ \frac{R_a + R_3}{(R_a + R_3) + (R_b + R_4)} = \frac{24}{5}$$

$$V_a = I_a \times R_3 = \frac{8}{5} \times 6 = \frac{48}{5}, \qquad V_b = I_b \times R_4 = \frac{24}{5} \times 1.5 = \frac{36}{5}$$

$$I = \frac{V_a - V_b}{R_5} = \frac{12}{5} \times \frac{1}{2} = 1.2A$$

$$R_5 = 1\Omega$$

$$R_5 = 3\Omega$$

$$V_T = V_{oc} = V_0 \frac{R_3}{R_1 + R_3} - V_0 \frac{R_4}{R_2 + R_4} = 18 \left(\frac{6}{9} - \frac{1.5}{4.5} \right) = 6V$$

$$R_T = R_1 \,||\, R_3 + R_2 \,||\, R_4 = \frac{3 \times 6}{3 + 6} + \frac{3 \times 1.5}{3 + 1.5} = 3\Omega$$

$$I = \frac{V_T}{R_T + R_5}$$

$$R_5 = 1$$

$$I = 6/(3+1) = 1.5A$$

$$R_5 = 2$$

$$I = 6/(3+2) = 1.2A$$

$$R_5 = 3$$

$$I = 6/(3+3) = 1.0A$$

$$R_5 = V_T/I - R_T = 6/0.5 - 3 = 9$$

$$V_1 = 72V, \quad V_2 = 80V, \quad R_1 = R_3 = 1.5\Omega, \quad R_2 = 3\Omega, \quad R_4 = 2.5\Omega$$

I_L

$$R_L = 1.5\Omega$$

I_i

V_a

V_b

I'_L

$$V_1 = 72$$

$$I''_L$$

$$V_2 = 80$$

$$I_L = I'_L + I''_L$$

$$I'_L = 8$$

$$R_3 = 1.5$$

$$R_4 = 2.5$$

$$I_3 = 5$$

$$I_4 = 3$$

$$V_b = I_3 R_3 = I_4 R_4 = 5 \times 1.5 = 3 \times 2.5 = 7.5$$

$$V_a = R_L I'_L + V_b = 1.5 \times 8 + 7.5 = 19.5$$

$R_2 = 3$

$$I_2 = V_a / R_2 = 19.5/3 = 6.5$$

$$I_1 = I_2 + I_L' = 6.5 + 8 = 14.5$$

$$V_1' = R_1 \times I_1 + V_a = 1.5 \times 14.5 + 19.5 = 41.25$$

$$V_1/V_1' = 72/41.25 = 96/55$$

$$I'_L = 8 \times 96/55$$

$$I''_L = 3$$

$$R_1 = 1.5$$

$$I_1 = 2$$

$$I_2 = 1$$

$$V_a = R_1 I_1 = R_2 I_2 = 1.5 \times 2 + 3 \times 1 = 3$$

$$V_b = R_L I_L'' + V_a = 1.5 \times 3 + 3 = 7.5$$

$$I_3 = V_b/R_3 = 7.5/1.5 = 5$$

$$I_4 = I_3 + I_L'' = 5 + 3 = 8$$

$$V_2' = R_4 \times I_4 + V_b = 2.5 \times 8 + 7.5 = 27.5$$

$$V_2/V_2' = 80/27.5 = 32/11$$

$$I''_L = 3 \times 32/11 = 96/11$$

$$I_L = I'_L - I''_L = \frac{96 \times 8}{55} - \frac{96}{11} = \frac{288}{55}$$

$$V_{ab} = V_a - V_b$$

$$I_L = V_{ab}/(R + R_L)$$

$$V_a = V_1 \times R_2/(R_1 + R_2) = 72 \times 3/(1.5 + 3) = 48$$

$$V_b = V_2 \times R_3/(R_3 + R_4) = 80 \times 1.5/(1.5 + 2.5) = 30$$

$$V_T = V_{ab} = V_a - V_b = 48 - 30 = 18$$

$$R_T = R_1||R_2 + R_3||R_4 = 3||1.5 + 2.5||1.5 = 7.75/4$$

$$I_L = \frac{V_T}{R_T + R_L} = \frac{18}{7.75/4 + 1.5} = \frac{288}{55}$$

$$R_1 = 6\Omega$$

$$R_2 = 2\Omega$$

$$R_3 = 3\Omega$$

$$R_4 = 5\Omega$$

$$V = 5V$$

$$\frac{V_a - 5}{2} + \frac{V_a}{2} + \frac{V_a - V_b}{3} = 0, \qquad \frac{V_b - V_a}{3} + \frac{V_b - 5}{6} + 0.5 = 0$$

$$V_a = 2.45$$

$$V_b = 2.3V$$

$$R = V_b/I = 2.3/0.5 = 4.6\Omega$$

$$R_2 = R_3 = 15\Omega$$

$$R_4 = 10\Omega$$

$$V_1 = 6V$$

$$V_2 = 9V$$

$$I_1 = 2A$$

$$I_2 = 1A$$

$$\left\{ \begin{array}{l} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{array} \right. \qquad \left[\begin{array}{c} V_1 \\ V_2 \end{array} \right] = \left[\begin{array}{cc} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{array} \right] \left[\begin{array}{c} I_1 \\ I_2 \end{array} \right] = \mathbf{Z} \left[\begin{array}{c} I_1 \\ I_2 \end{array} \right]$$

Z_{11}

$$Z_{12}$$

$$Z_{21}$$

Z_{22}

$$Z_{12} = V_1/I_2$$

Z

$$\left\{\begin{array}{l} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{array}\right. \qquad \left[\begin{array}{c} I_1 \\ I_2 \end{array}\right] = \left[\begin{array}{cc} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \end{array}\right] = \mathbf{Y} \left[\begin{array}{c} V_1 \\ V_2 \end{array}\right]$$

Y_{11}

Y_{12}

$$Y_{21}$$

$$Y_{22}$$

Y

$$\begin{cases} V_1 = A_{11}V_2 + A_{12}(-I_2) \\ I_1 = A_{21}V_2 + A_{22}(-I_2) \end{cases} \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

A_{11}

$$A_{22}$$

A_{12}

$$A_{21}$$

$$\left\{\begin{array}{l} V_1 = H_{11}I_1 + H_{12}V_2 \\ I_2 = H_{21}I_1 + H_{22}V_2 \end{array}\right. \qquad \left[\begin{array}{c} V_1 \\ I_2 \end{array}\right] = \left[\begin{array}{cc} H_{11} & H_{12} \\ H_{21} & H_{22} \end{array}\right] \left[\begin{array}{c} I_1 \\ V_2 \end{array}\right] = \mathbf{H} \left[\begin{array}{c} I_1 \\ V_2 \end{array}\right]$$

H_{12}

H_{21}

H_{11}

H_{22}

$$z = f(x, y)$$

$\triangle x$

$\triangle y$

$$\triangle z = \triangle f(x, y) = \frac{\partial f}{\partial x}\triangle x + \frac{\partial f}{\partial y}\triangle y$$

$$\frac{\partial f}{\partial x} = \lim_{\triangle x \to 0} \frac{\triangle f}{\triangle x}\bigg|_{\triangle y = 0} \qquad \frac{\partial f}{\partial y} = \lim_{\triangle y \to 0} \frac{\triangle f}{\triangle y}\bigg|_{\triangle x = 0}$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}, \quad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}, \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}, \quad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}, \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{Y_1=0},$$

$$A_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad A_{12} = \left.\frac{V_1}{I_2}\right|_{V_2=0}, \quad A_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \quad A_{22} = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$H_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, \quad H_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}, \quad H_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \quad H_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$\mathbf{V} = [V_1, V_2]^T, \qquad \mathbf{I} = [I_1, I_2]^T$$

$$\mathbf{V} = \mathbf{Z}\mathbf{I}, \qquad \mathbf{I} = \mathbf{Y}\mathbf{V}, \quad \mathbf{Y} = \mathbf{Z}^{-1}$$

$$\left\{ \begin{array}{l} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{array} \right.$$

$$I_2 = 0$$

$$Z_{11} = V_1/I_1 = j\omega L + 1/j\omega C, \qquad Z_{21} = V_2/I_1 = 1/j\omega C$$

$$I_1 = 0$$

$$Z_{22} = V_2/I_2 = 1/j\omega C, \qquad Z_{12} = V_1/I_2 = 1/j\omega C$$

Z

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \begin{bmatrix} j\omega L + 1/j\omega C & 1/j\omega C \\ 1/j\omega C & 1/j\omega \end{bmatrix}^{-1} = \begin{bmatrix} 1/j\omega L & -1/j\omega L \\ -1/j\omega L & j\omega C + 1/j\omega L \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2$$

$$\mathbf{A} = \mathbf{A}_1 \cdot \mathbf{A}_2$$

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 4\Omega & j3\Omega \\ j3\Omega & 2\Omega \end{bmatrix}$$

$$V_0 = 3\angle 0^\circ$$

$$Z_0 = 5\Omega$$

$$R_L = 4\Omega$$

$$\begin{cases} V_1 = Z_{11}I_1 + Z_{12}I_2 = 4I_1 + j3I_2 \\ V_2 = Z_{21}I_1 + Z_{22}I_2 = j3I_1 + 2I_2 \end{cases}$$

$$\begin{cases} V_1 = V_0 - Z_0 I_1 = 3 - 5I_1 \\ V_2 = -Z_L I_2 = -4I_2 \end{cases}$$

$$\begin{cases} 9I_1 + j3I_2 = 3 \\ j3I_1 + 6I_2 = 0 \end{cases}$$

$$I_1 = \frac{2}{7}, \qquad I_2 = -\frac{j}{7}$$

$$V_1 = \frac{11}{7}, \qquad V_2 = \frac{j4}{7}$$

$$Z_T = V_2/I_2$$

$$V_0 = 0$$

$$V_1 = V_0 - I_1 Z_0 = 0 - 5I_1$$

$$4I_1 + j3I_2 = -5I_1,$$

$$I_1 = -\frac{j}{3} I_2$$

$$V_2 = j3I_1 + 2I_2 = I_2 + 2I_2 = 3I_2$$

$$Z_T = \frac{V_2}{I_2} = 3$$

$$\begin{cases} V_1 = Z_{11} I_1 = 4 I_1 \\ V_2 = Z_{21} I_1 = j3 I_1 \end{cases}$$

$$I_1 = \frac{V_0 - V_1}{Z_0} = \frac{3 - 4I_1}{5}$$

$$I_1 = 1/3$$

$$V_T = V_2$$

$$V_2 = Z_{21} I_1 = j3\frac{1}{3} = j$$

$$V_2 = \frac{V_T}{Z_T + Z_L}\ Z_L = \frac{j}{3+4}\ 4 = \frac{j4}{7}$$

$$I_2 = -\frac{V_2}{Z_L} = -\frac{j4}{7}\ \frac{1}{4} = -\frac{j}{7}$$

$$\dot{I}_2$$

$$\dot{I}_2 = \frac{V}{Z_1 + Z_2 Z_3/(Z_2 + Z_3)} \; \frac{Z_3}{Z_2 + Z_3} = V \frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$\dot{I}_1 = \frac{V}{Z_2 + Z_1 Z_3/(Z_1 + Z_3)} \frac{Z_3}{Z_1 + Z_3} = V \frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$\dot{I}_1$$

Π

$$\left\{ \begin{array}{l} V_1 = (Z_1 + Z_3)I_1 + Z_3 I_2 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 = Z_{21} I_1 + Z_{22} I_2 \end{array} \right.$$

$$Z_{11} = Z_1 + Z_3, \qquad Z_{22} = Z_2 + Z_3, \qquad Z_{12} = Z_{21} = Z_3$$

$$Z_1 = Z_{11} - Z_{12}, \quad Z_2 = Z_{22} - Z_{21}, \quad Z_3 = Z_{12} = Z_{21}$$

$$\begin{cases} I_1 = Y_1 V_1 + Y_3(V_1 - V_2) = (Y_1 + Y_3)V_1 - Y_3 V_2 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_3(V_2 - V_1) + Y_2 V_2 = -Y_3 V_1 + (Y_2 + Y_3)V_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases}$$

$$Y_{11} = Y_1 + Y_3, \qquad Y_{22} = Y_2 + Y_3, \qquad Y_{12} = Y_{21} = -Y_3$$

Y_1

$$Y_2$$

$$Y_3$$

$$Y_1 = Y_{11} + Y_{12}, \quad Y_2 = Y_{22} + Y_{21}, \quad Y_3 = -Y_{12} = -Y_{21}$$

$$Z_1 = j5$$

$$Z_2 = -j5$$

$$Z_3 = 1$$

$$Z_{11} = Z_1 + Z_3 = 1 + j5, \quad Z_{22} = Z_2 + Z_3 = 1 - j5, \quad Z_{12} = Z_{21} = Z_3 = 1$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1+j5 & 1 \\ 1 & 1-j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1+j5 & 1 \\ 1 & 1-j5 \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} 1-j5 & -1 \\ -1 & 1+j5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_1 = Y_{11} + Y_{12} = -j/5, \quad Z_2 = Y_{22} + Y_{21} = j/5, \quad Y_3 = -Y_{21} = -Y_{12} = 1/25$$

$$Z_1 = 1/Y_1 = 5j, \quad Z_2 = 1/Y_2 = -j5, \quad Z_3 = 1/Y_3 = 25$$

$$R_0 = 10\Omega$$

$$R_L = 5\Omega$$

$$n = n_1/n_2 = 2$$

$$\begin{cases} V_1 = 10\ I_1 + 2\ V_2 \\ I_2 - V_2/5 = -2\ I_1 \end{cases}$$

$$V_2 = 10\ I_1 + 5\ I_2$$

$$V_1 = 30\ I_1 + 10\ I_2$$

$$\left[\begin{array}{c} V_1 \\ V_2 \end{array}\right] = \left[\begin{array}{cc} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{array}\right] \left[\begin{array}{c} I_1 \\ I_2 \end{array}\right] = \left[\begin{array}{cc} 30 & 10 \\ 10 & 5 \end{array}\right] \left[\begin{array}{c} I_1 \\ I_2 \end{array}\right]$$

$$Z_{12} = Z_{21} = 10$$

$$\begin{cases} I_1 = (V_1 - 2\ V_2)/10 \\ I_2 = -2\ I_1 + V_2/5 \end{cases}$$

$$I_1 = V_1/10 - V_2/5$$

$$I_2 = -V_1/5 + 2V_2/5 + V_2/5 = -V_1/5 + 3V_2/5$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1/10 & -1/5 \\ -1/5 & 3/5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\mathbf{Z}^{-1} = \mathbf{Y}$$

$$\mathbf{Z}^{-1} = \begin{bmatrix} 30 & 10 \\ 10 & 5 \end{bmatrix}^{-1} = \frac{1}{50} \begin{bmatrix} 5 & -10 \\ -10 & 30 \end{bmatrix} = \begin{bmatrix} 1/10 & -1/5 \\ -1/5 & 3/5 \end{bmatrix} = \mathbf{Y}$$

r_{in}

$$r_{in} = \infty$$

r_{out}

G_v

A

v_{out}

v_{in}

$$v_{out} = G_v v_{in}$$

G_a

β

i_{out}

i_{in}

$$i_{out} = G_a i_{in}$$

$$i_s = v_s / R_S$$

$$v_s = i_s R_S$$

R_S

$$v_{in} = i_s(R_S \,||\, r_{in}) = i_s \frac{R_S \; r_{in}}{R_S + r_{in}} = v_s \frac{r_{in}}{R_S + r_{in}}$$

$$v_{out} = G_v v_{in} \frac{R_L}{R_L + r_{out}} = G_v i_s \frac{R_S \, r_{in}}{R_S + r_{in}} \frac{R_L}{R_L + r_{out}} = G_v v_s \frac{r_{in}}{R_S + r_{in}} \frac{R_L}{R_L + r_{out}}$$

$$R_{in}$$

$$R_{in} = \frac{\text{voltage } v_{in} \text{ across the input port}}{\text{current } i_{in} \text{ through the input port}}$$

R_{out}

v_T

$$R_{out} = \frac{\text{open-circuit voltage}}{\text{short-circuit current}} = \frac{v_{oc} = v_T \quad (R_L = \infty)}{i_{sc} = v_T / R_T \quad (R_L = 0)} = R_T$$

v_s

$$A_{oc}$$

v_{OC}

$$R_L = \infty$$

$$A_{oc} = \frac{\text{open-circuit voltage}}{\text{ideal voltage source}} = \frac{v_{oc}}{v_{in}}$$

$$R_{in} = R_1 + r_{in}$$

$$v_{oc} = v_T R_2/(r_{out} + R_2)$$

$$i_{sc} = v_T / r_{out}$$

$$R_{out} = \frac{v_{oc}}{i_{sc}} = \frac{r_{out} R_2}{r_{out} + R_2} = r_{out} \mid\mid R_2$$

$$R_{out} = r_{out} \,||\, R_2$$

V_{CD}

V_{AB}

v_{in}

$$V_{AB} \frac{r_{in}}{R_1 + r_{in}}$$

$$V_{CD}$$

$$Av_{in}\frac{R_2}{r_{out}+R_2} = AV_{AB}\frac{r_{in}}{R_1+r_{in}}\frac{R_2}{r_{out}+R_2}$$

$$A_{oc}$$

$$\frac{V_{CD}}{V_{AB}} = \frac{AR_2 r_{in}}{(R_1 + r_{in})(r_{out} + R_2)} \overset{r_{in}\to\infty \;\; r_{out}=0}{\Longrightarrow} A$$

v_1

$$v_{in} = v_s \frac{r_{in}}{R_1 + r_{in}}, \qquad v_1 = v_s \frac{R_1}{R_1 + r_{in}}$$

$$v_{oc} = v_1 + Av_{in} = v_s \frac{R_1}{R_1 + r_{in}} + Av_s \frac{r_{in}}{R_1 + r_{in}} = v_s \frac{R_1 + Ar_{in}}{R_1 + r_{in}}$$

$$A_{oc} = \frac{v_{oc}}{v_s} = \frac{R_1 + Ar_{in}}{R_1 + r_{in}} < A$$

$$R_1 = 0$$

$$A_{oc} = A$$

i_{sc}

$$v_s = (R_S + R_1 + r_{in})i_{in} - R_1 i_{sc}$$

$$Av_{in} = Ar_{in}i_{in} = (r_{out} + R_1)i_{sc} - R_1 i_{in}$$

$$i_{sc} = v_s \frac{A r_{in} + R_1}{(1 - A) R_1 r_{in} + r_{out}(R_S + r_{in} + R_1) + R_S R_1}$$

$$v_{oc} = Ar_{in}i_{in} + R_1 i_{in} = (Ar_{in} + R_1)\frac{v_s}{R_S + r_{in} + R_1}$$

$$R_{out} = \frac{v_{oc}}{i_{sc}} = \frac{(1-A)r_{in}R_1 + r_{out}(R_S + r_{in} + R_1) + R_S R_1}{R_S + r_{in} + R_1} = r_{out} + \frac{(1-A)r_{in}R_1 + R_S R_1}{R_1 + r_{in} + R_S}$$

$$R_S = 0$$

$$R_{out} \overset{R_S=0}{\Longrightarrow} r_{out} - (A-1)r_{in} \,||\, R_1 < r_{out}$$

$$R_{out} = r_{out}$$

$$R_{in} = v_s / i_{in}$$

$$v_s = (r_{in} + R_1)i_{in} - R_1 i_{out}$$

$$Av_{in} = Ar_{in}i_{in} = (r_{out} + R_L + R_1)i_{out} - R_1 i_{in}$$

$$i_{in} = v_s \frac{R_1 + R_L + r_{out}}{(R_L + r_{out})(R_1 + r_{in}) + (1 - A)R_1 r_{in}} \xRightarrow{R_L \to \infty} \frac{v_s}{r_{in} + R_1}$$

$$R_{in} = \frac{v_s}{i_{in}} = \frac{(R_L + r_{out})(R_1 + r_{in}) + (1 - A)R_1 r_{in}}{R_1 + R_L + r_{out}} \overset{R_L \to \infty}{\Longrightarrow} R_1 + r_{in} > r_{in}$$

$$R_{in} = r_{in}$$

$$v_s \uparrow \rightarrow i_{in}, v_{out} \uparrow \rightarrow v_1 \uparrow \rightarrow i_{in}, v_{out} \downarrow$$

$$r_{in} >> r_{out}$$

$$R_{in} = V_s / i_{in}$$

V_s

$$R_{out} = v_{oc}/i_{sc}$$

$$A_{oc} = V_{oc}/V_{sc}$$

A_1

r_{i1}

r_{o1}

A_2

r_{i2}

r_{O2}

G_p

$$v_s = 1.5V$$

$$R_S = 5k\Omega$$

$$v_{in} = 1.25V$$

$$R_1 = 150\Omega$$

$$R_2 = 200\Omega$$

$$v_1 = 18.75V$$

$$v_2 = 20$$