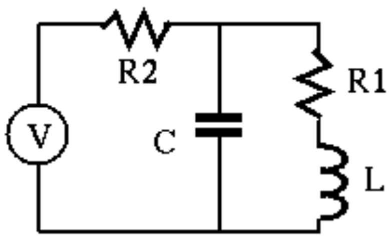
E84 Home Work 9

1. The load of a voltage soruce of $v(t)=110\sqrt{2}\,\cos(2\pi 60\,t)$ is shown in the figure, where $R_1=100\Omega$, $R_2=50\Omega$, $C=6.63\mu F$, $L=0.53H$. Is the load capacitive ($\phi=\tan^{-1}(X/R)<0$) or inductive ($\phi>0$)? Find the power factor, the apparent power, the real power and the reactive power.



Solution:

$$Z_C=-\frac{j}{\omega C}=-\frac{j}{2\pi 60\times 6.63\times 10^{-6}}=-j400\Omega$$

$$Z_L=j\omega L=j2\pi 60\times 0.53=j200\Omega,\quad Z_{RL}=100+j200$$

$$Z_{CRL}=\frac{Z_C Z_{RL}}{Z_C+Z_{RL}}=\frac{(100+j200)(-j400)}{100+j200-j400}=\frac{800-j400}{1-j2}$$

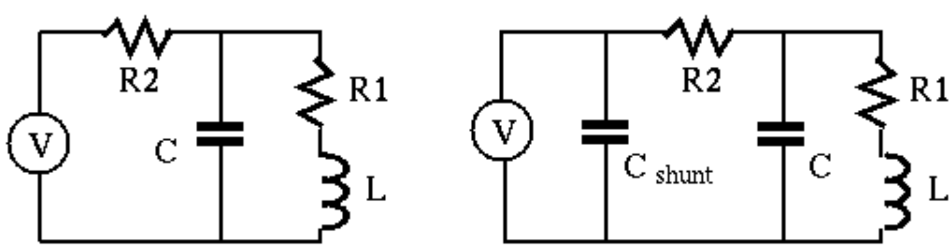
$$Z_{total} = Z_{CRL} + R_2 = \frac{800 - j400}{1 - j2} + 50 = 370 + j240 = 441\angle 33^\circ$$

The load is inductive as $\phi > 0$.

$$\dot{I} = \frac{\dot{V}}{Z_{total}} = \frac{110}{441\angle 33^\circ} = 0.25\angle - 33^\circ$$

power factor is $\lambda = cos(-33^\circ) = 0.839$, the apparent power is $S = 110 \times 0.25 = 27.5W$, the real power is $P = S\cos 33^\circ = 27.5 \times 0.84 = 23W$ the reactive power is $Q = S\sin 33^\circ = 27.5 \times 0.54 = 15W$

2. To improve the power factor of the circuit above to 0.9, a shunt capacitor is added. What should the capacitance C be? What should C be if the power factor is required to be 1?



Solution:

Adding a shunt capacitor with impedance $1/j\omega C = -jX$ ($X = 1/\omega C$), the overall load impedance is

$$Z_{all} = -jX || Z_{total} = \frac{-jX(370 + j240)}{-jX + (370 + j240)} = \frac{240X - j370X}{370 - j(X - 240)} = |Z|\angle Z = |Z|\angle\phi$$

For the power factor to be 0.9, this impedance need to have a phase angle $\phi = \cos^{-1} 0.9 = 25.84°$, and we need to have:

$$\tan^{-1}[\frac{-370X}{240X}] - \tan^{-1}[\frac{-(X-240)}{370}] = -57° + \tan^{-1}[\frac{X-240}{370}] = 25.84°$$

$$\tan^{-1}[\frac{X-240}{370}] = 82.84°, \quad \frac{X-240}{370} = \tan\ 82.84° = 7.96, \quad X = 3185.4$$

$$\frac{1}{\omega C} = X = 3185.4, \quad C = \frac{1}{2\pi 60 \times 3185.4} = 0.83\mu F$$

For the power factor to be 1, we need to have

$$\tan^{-1}[\frac{-370X}{240X}] - \tan^{-1}[\frac{-(X-240)}{370}] = -57° + \tan^{-1}[\frac{X-240}{370}] = 0°$$

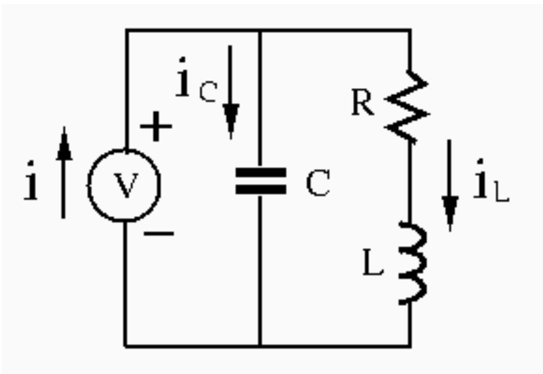
i.e.,

$$\tan^{-1}[\frac{X-240}{370}] = 57°, \quad \frac{X-240}{370} = \tan\ 57° = 1.54, \quad X = 810$$

$$\frac{1}{\omega C} = X = 810, \quad C = \frac{1}{2\pi 60 \times 810} = 3.27\mu F$$

3. In the circuit shown below, the voltage source $v(t) = 100\sqrt{2}\, sin\, 314t$ volts, and the effective values of the three currents i, i_L and i_C are

the same. The total real energy comsumed by the circuit is 866 W. Find the values of R, L and C. (Hint: represent all currents $i(t)$, $i_C(t)$, $i_L(t)$ and voltage $v(t)$ as phasors $\dot{I}$, $\dot{I}_C$, $\dot{I}_L$, $\dot{V}$, and draw them as vectors to figure out how they are related.)



Solution: Let $\dot{V}=100\angle 0$ be the phasor representation of $v(t)$ so that

$$v(t)=Im[\sqrt{2}\dot{V}e^{j\omega t}]=Im[\sqrt{2}\dot{V}e^{j314t}]$$

where $\omega=2\pi f=314$, i.e., $f=50$ Hz. We have

$$\dot{I}_C=j\omega C\dot{V}=100\omega C\angle 90^\circ,\quad \dot{I}_L=\frac{\dot{V}}{R+j\omega L}=\frac{100}{\sqrt{R^2+\omega^2L^2}}\angle-\phi$$

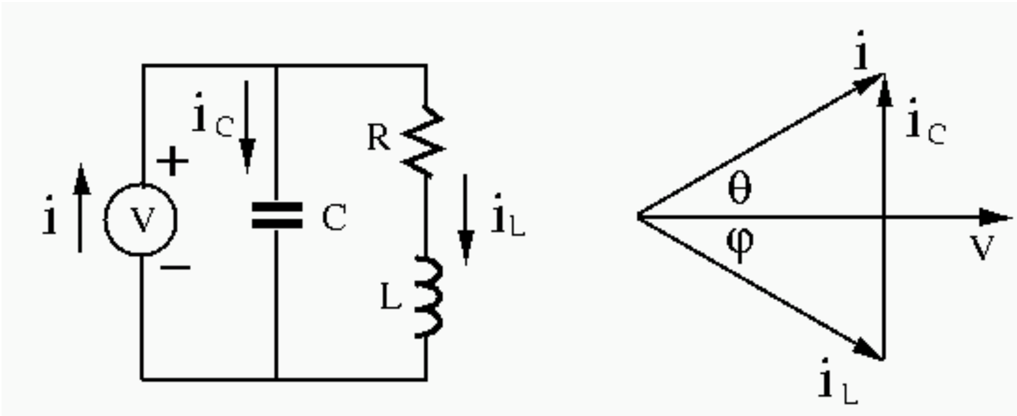
where $\phi=tan^{-1}(\omega L/R)$. Due to KCL, we hav

$$\dot{I}=\dot{I}_C+\dot{I}_L$$

but also as given

$$|\dot{I}|=|\dot{I}_C|=|\dot{I}_L|=I$$

we conclude that these three currents are equal in magnitude and 60° apart in phase angle, as shown below:



where $\dot{I}_C$ is 90° ahead of $\dot{V}$, $\dot{I}$ is $\theta = 30^\circ$ ahead of $\dot{V}$, which in turn is $\phi = 30^\circ$ ahead of $\dot{I}_L$. Therefore,

$$\dot{I} = I\angle\theta = I\angle 30^\circ, \quad \dot{I}_L = I\angle\phi = I\angle -30^\circ, \quad \dot{I}_C = I\angle 90^\circ$$

As the real power is

$$P = 866 = VI\cos\phi = 100I\cos(-30^\circ) = 86.6I$$

we get

$$I = 10A$$

therefore we also get

$$P = 866 = RI^2 = R100, \quad R = 8.66\Omega$$

and

$$\dot{I} = 10\angle 30^\circ, \quad \dot{I}_L = 10\angle -30^\circ, \quad \dot{I}_C = 10\angle 90^\circ$$

But from above

$$\dot{I}_C = 100\omega C\angle 90^\circ = 10\angle 90^\circ$$

we get:

$$\omega C = 0.1, \quad C = 0.1/314 = 3.18 \times 10^{-4}\ F$$

Since we also have:

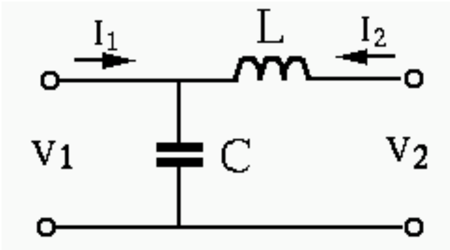
$$\dot{I}_L = \frac{100}{\sqrt{R^2+\omega^2 L^2}}\angle -\phi = 10\angle -30^\circ$$

solving this we get

$$\omega L = 5, \qquad L = 5/\omega = 5/314 = 0.0159\ H$$

4. Read and understand the notes about two-port networks on this page (before the title ``Principle of reciprocity'', which will not be covered), especially the two examples, and then you should be able to do the following three problems.

   Find the Z-model and Y-model of the circuit shown in the figures, by assuming one of the two known variables (currents or voltages) is zero at a time. Then verify your results by checking whether $\mathbf{Z}^{-1} = \mathbf{Y}$.



Solution:

For the Z-model, item Assume $I_2 = 0$ (open-circuit), then $Z_{11} = V_1/I_1 = 1/j\omega C$, $Z_{21} = V_2/I_1 = 1/j\omega C$

- Assume $I_1 = 0$ (open-circuit), then $Z_{12} = V_1/I_2 = 1/j\omega C$, $Z_{22} = V_2/I_2 = j\omega L + 1/j\omega C$

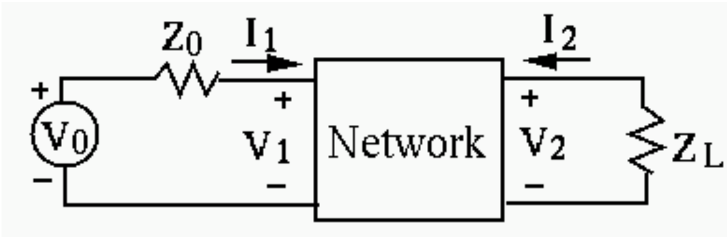
For the Y-model,

- Assume $V_2 = 0$ (short-circuit), then $Y_{11} = I_1/V_1 = j\omega C + 1/j\omega L$, $Y_{21} = I_2/V_1 = 1/j\omega L$
- Assume $V_1 = 0$ (short-circuit), then $Y_{12} = I_1/V_2 = 1/j\omega L$, $Y_{22} = I_2/V_2 = 1/j\omega L$

verify:

$$\mathbf{Z}^{-1} = \begin{bmatrix} 1/j\omega C & 1/j\omega C \\ 1/j\omega C & j\omega L + 1/j\omega C \end{bmatrix}^{-1} = \frac{C}{L} \begin{bmatrix} j\omega L + 1/j\omega C & -1/j\omega C \\ -1/j\omega C & 1/j\omega C \end{bmatrix} = \begin{bmatrix} j\omega C + 1/j\omega L & -1/j\omega L \\ -1/j\omega L & 1/j\omega L \end{bmatrix}$$

5. The parameters of the Y-model of the two-port network are $Y_{11} = -4$, $Y_{12} = 3$, $Y_{21} = 3$, and $Y_{22} = -2$. The voltage source is $V_0 = 5V$, $Z_0 = 1\Omega$, $Z_L = j1\Omega$. Find variables I_1, I_2, V_1, V_2. (Hint: refer to Method 1 in the example shown in the web notes)



Solution:

- Convert Y-model to Z-model:

$$\mathbf{Z} = \mathbf{Y}^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

$$\begin{cases} V_1 = 2I_1 + 3I_2 \\ V_2 = 3I_1 + 4I_2 \end{cases}$$

- Setup additional equations:

$$\begin{cases} V_1 = V_0 - R_0 I_1 \\ V_2 = -Z_L I_2 \end{cases}$$

- Find I_1 and I_2:

$$\begin{cases} 2I_1 + 3I_2 = V_0 - R_0 I_1 \\ 3I_1 + 4I_2 = -Z_L I_2 \end{cases} \qquad \text{i.e.} \qquad \begin{cases} (2 + R_0)I_1 + 3I_2 = V_0 \\ 3I_1 + (4 + Z_L)I_2 = 0 \end{cases}$$

These equations can be solved to get

$$I_1 = \frac{5(4+j)}{3(1+j)}, \qquad I_2 = -\frac{5}{1+j}$$

- Find V_1, V_2:

$$V_2 = -I_2 Z + L = \frac{j5}{1+j}, \qquad V_1 = -\frac{5(1-j2)}{3(1+j)}$$

6. Repeat the previous problem but this time use Thevenin's theorem to find V_2 across load R_L. (Hint: refer to Method 2 in the example shown in the web notes)

Solution:

$$\begin{cases} V_1 = 2I_1 + 3I_2 \\ V_2 = 3I_1 + 4I_2 \end{cases}$$

$$\begin{cases} V_1 = V_0 - R_0 I_1 \\ V_2 = -Z_L I_2 \end{cases}$$

First, find Z_{Th} when the voltage souce is short circuit:

- Equating $V_1 = V_0 - R_0 I_1 = -I_1$ to $V_1 = 2I_1 + 3I_2$, we get $2I_1 + 3I_2 = -I_1$, i.e., $I_1 = -I_2$.
- Substituting $I_1 = -I_2$ into $V_2 = 3I_1 + 4I_2$, we get $V_2 = I_2$, i.e., $Z_{Th} = V_2/I_2 = 1$.

Second, find V_{Th} when the load is open circuit, i.e., $I_2 = 0$:

- Since $I_2 = 0$, the Z-model equations become $V_1 = 2I_1$, $V_2 = 3I_1$.
- We also get $I_1 = (V_0 - V_1)/R_0 = 5 - V_1 = 5 - 2I_1$, which can be solved to get $I_1 = 5/3$.

- Then $V_{Th}=V_2=3I_1=5$.

Finally, we can find voltage across R_L as

$$V_2=V_{Th}\,\frac{Z_L}{Z_{Th}+Z_L}=\frac{j5}{1+j}$$

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