$$H(j\omega) = \frac{V_{out}}{V_{in}} = \begin{cases} -R_2/R_1 & \omega = 0 \\ 0 & \omega \to \infty \end{cases}$$

$$Z_C = 1/j\omega C \to 0$$

$$\omega \to \infty$$

$$H(j\omega)$$

=

$$-\frac{Z_2}{Z_1} = -\frac{R_2 \parallel (1/j\omega C)}{R_1} = -\frac{1}{R_1}\frac{R_2/j\omega C}{R_2 + 1/j\omega C} = -\frac{R_2}{R_1}\frac{1}{j\omega R_2 C + 1}$$

$$-H(0)\frac{1}{1+j\omega\tau}$$

$$H(0) = R_2/R_1$$

$$\omega = 0$$

$$\omega_c = 1/\tau = 1/R_2C$$

$$|H(j\omega_c)| = H(0)/\sqrt{2}$$

$$Z_2(j\omega)$$

$$\tau = 10^{-3}$$

$$\omega_c = 1/\tau = 10^3$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \begin{cases} 0 & \omega \to 0 \\ -R_2/R_1 & \omega \to \infty \end{cases}$$

$$Z_C = 1/j\omega C \to \infty$$

$$-\frac{Z_2(j\omega)}{Z_1(j\omega)} = -\frac{R_2}{R_1 + 1/j\omega C} = -\frac{R_2}{R_1}\ \frac{1}{1 + 1/j\omega R_1 C}$$

$$-H(\infty)\ \frac{1}{1+1/j\omega\tau}$$

$$H(\infty) = R_2/R_1$$

$$\omega_c = 1/\tau = 1/R_1 C$$

$$|H(j\omega_c)| = H(\infty)/\sqrt{2}$$

$$Z_1(j\omega)$$

$$\tau = 10^{-6}$$

$$\omega_c = 1/\tau = 10^6$$

$$R_1 = R_2$$

$$H(0) = 1$$

180°

$$s = j\omega$$

$$H_{lp}(j\omega) = \frac{\omega_c}{j\omega + \omega_c} = \frac{1}{j\omega\tau + 1}, \qquad H_{hp}(j\omega) = \frac{j\omega}{j\omega + \omega_c} = \frac{j\omega\tau}{j\omega\tau + 1}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \begin{cases} 0 & \omega = 0 \\ 0 & \omega \to \infty \end{cases}$$

$$Z_1(\omega) = R_1 + (1/j\omega C_1), \qquad Z_2(\omega) = R_2||(1/j\omega C_2)$$

$$H_{BP}(j\omega)$$

$$-\frac{Z_2(\omega)}{Z_1(\omega)} = -\frac{R_2||(1/j\omega C_2)}{R_1 + 1/j\omega C_1} = -\frac{R_2/j\omega C_2}{(R_1 + 1/j\omega C_1)(R_2 + 1/j\omega C_2)}$$

$$-\frac{j\omega R_2 C_1}{(j\omega R_1 C_1 + 1)(j\omega C_2 R_2 + 1)} = -\frac{j\omega \tau_3}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)} = -\,(j\omega \tau_3)\,\left(\frac{1}{1 + j\omega \tau_1}\right)\,\left(\frac{1}{1 + j\omega \tau_2}\right)$$

$$= -j\omega\tau_3 \ \left(\frac{1/\tau_1\tau_2}{(j\omega)^2 + j\omega(\tau_1 + \tau_2)/\tau_1\tau_2 + 1/\tau_1\tau_2} \right) = -j\omega\tau_3 \ \frac{\omega_n^2}{\omega^2 + j\omega\Delta\omega + \omega_n^2}$$

$$\omega_{c_1} = 1/\tau_1 = 1/R_1C_1$$

$$\omega_{c_2} = 1/\tau_2 = 1/R_2C_2$$

$$\omega_{c_3} = 1/\tau_3 = 1/R_2C_1$$

$$\Delta\omega = (\tau_1 + \tau_2)/\tau_1\tau_2$$

$$\omega_n = 1/\sqrt{\tau_1 \tau_2}$$

$$\omega_n = 1/\tau_n$$

τ_n

τ_1

τ_2

$$\tau_n = \sqrt{\tau_1 \tau_2} = (\tau_1 \tau_2)^{1/2}, \qquad \log \tau_n = \frac{1}{2}\left(\log \tau_1 + \log \tau_2\right)$$

$$\log \omega_n = \frac{1}{2} \left(\log \omega_1 + \log \omega_2 \right)$$

ω_n

ω_1

ω_2

$$\tau_1 = 10^{-6}$$

$$\tau_2 = 10^{-8}$$

$$\tau_3 = 10^{-3}$$

Lm

$$-Lm_{BP}(j\omega) = -20\log|H_{BP}(j\omega)| = -20\log\left|\frac{Z_2(\omega)}{Z_1(\omega)}\right| = 20\log\left|\frac{Z_1(\omega)}{Z_2(\omega)}\right| = Lm_{BS}(j\omega)$$

$$Z_1(\omega)$$

$$Z_2(\omega)$$

$H_{BS}(j\omega)$

$$-\frac{Z_1(\omega)}{Z_2(\omega)} = -\frac{R_2 + 1/j\omega C_2}{R_1||1/j\omega C_1} = -\frac{R_2 + 1/j\omega C_2}{R_1/j\omega C_1/(R_1 + 1/j\omega C_1)}$$

$$\frac{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)}{j\omega R_1 C_2} = \frac{(1 + j\omega \tau_1)(1 + j\omega \tau_1)}{j\omega \tau_3}$$

$$V_b \approx V_{out}$$

$$\frac{V_a - V_{in}}{Z_1} + \frac{V_a - V_{out}}{Z_3} + \frac{V_a - V_{out}}{Z_2} = \frac{V_a - V_{in}}{Z_1} + (V_a - V_{out})\left(\frac{1}{Z_3} + \frac{1}{Z_2}\right) = 0$$

$$\frac{V_{out} - V_a}{Z_2} + \frac{V_{out}}{Z_4} = 0,$$

V_a

$$V_a = V_{out}\frac{Z_2 + Z_4}{Z_4}$$

$$V_{out}\ \left(\frac{Z_2+Z_4}{Z_1Z_4}+\frac{Z_2}{Z_3Z_4}+\frac{Z_2}{Z_2Z_4}\right)=\frac{V_{in}}{Z_1}$$

$$V_{out}$$

V_{in}

H

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4}$$

$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = 1/j\omega C_1$$

$$Z_4 = 1/j\omega C_2$$

$$H(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + \Delta\omega\ j\omega + \omega_n^2}$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \qquad \Delta\omega = \frac{1}{(R_1||R_2)C_1} = \frac{1}{R_p C_1}, \quad R_p = R_1||R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$Q = \frac{\omega_n}{\Delta\omega} = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2)C_2}, \qquad \zeta = \frac{1}{2Q} = \frac{(R_1 + R_2)C_2}{2\sqrt{R_1 R_2 C_1 C_2}}$$

ζ

$$Q = 1/2\zeta$$

R_1

R_2

C_1

C_2

$$R_1 = R_2 = 1$$

$$\omega_n = \frac{1}{\sqrt{C_1 C_2}}, \qquad \Delta\omega = \frac{1}{C_1}$$

$$Z_1 = 1/j\omega C_1$$

$$Z_2 = 1/j\omega C_2$$

$$Z_3 = R_1$$

$$Z_4 = R_2$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + \Delta\omega\ j\omega + \omega_n^2}$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \qquad \Delta\omega = \frac{1}{C_s R_2}, \qquad C_s = \frac{C_1 C_2}{C_1 + C_2}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(C_1 + C_2) R_1}, \qquad \zeta = \frac{1}{2Q} = \frac{(C_1 + C_2) R_1}{2\sqrt{R_1 R_2 C_1 C_2}}$$

$$V_2 = \frac{R_a}{R_a + R_b} V_{out} = k V_{out}, \qquad \left(k = \frac{R_a}{R_a + R_b} \right)$$

V_2

$$\frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_4}, \qquad \text{i.e.} \qquad V_1 = V_2 \left(1 + \frac{Z_2}{Z_4}\right)$$

V_1

$$\frac{V_{in} - V_1}{Z_1} + \frac{V_{out} - V_1}{R_f} + \frac{V_2 - V_1}{Z_2} = \frac{V_1}{Z_3}$$

$$V_2(1 + Z_2/Z_4)$$

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{R_f} + \frac{V_2}{Z_2} = V_1 \left(\frac{1}{Z_1} + \frac{1}{R_f} + \frac{1}{Z_2} + \frac{1}{Z_3} \right)$$

$$V_2 \left(1 + \frac{Z_2}{Z_4}\right) \left(\frac{1}{Z_1} + \frac{1}{R_f} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$kV_{out}$$

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{R_f} = kV_{out}\left[\left(1 + \frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1} + \frac{1}{R_f} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) - \frac{1}{Z_2}\right]$$

$$\frac{V_{in}}{Z_1}$$

$$kV_{out}\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{V_{out}}{R_f}$$

$$V_{out}\left\{k\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{1}{R_f}\right\}$$

$$H = \frac{V_{out}}{V_{in}}$$

$$\frac{1}{Z_1\left\{k\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{1}{R_f}\right\}}$$

$$\frac{1/k}{Z_1\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{R_f}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right]-\frac{Z_1}{kR_f}}$$

$$\frac{1/k}{1+\frac{Z_1}{R_f}+\frac{Z_1}{Z_3}+\frac{Z_2}{Z_4}+\frac{Z_1 Z_2}{Z_4 R_f}+\frac{Z_1}{Z_4}+\frac{Z_1 Z_2}{Z_3 Z_4}-\frac{Z_1}{k R_f}}$$

$$Z_1 = R_1, \quad Z_4 = R_2, \quad Z_2 = \frac{1}{j\omega C_2}, \quad Z_3 = \frac{1}{j\omega C_1}$$

$$\frac{\left(1+\frac{R_b}{R_a}\right)\frac{1}{R_1C_1}j\omega}{(j\omega)^2+\left(\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{1}{R_1C_1}-\frac{R_b}{R_aR_fC_1}\right)j\omega+\frac{R_1+R_f}{R_fR_1R_2C_1C_2}}$$

$$\frac{Aj\omega}{(j\omega)^2 + \Delta\omega j\omega + \omega_n^2}, \qquad A = (1 + R_b/R_a)/R_1C_1$$

$$\omega_n = \sqrt{\frac{R_1 + R_f}{R_f R_1 R_2 C_1 C_2}}$$

$$\Delta\omega = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_1} - \frac{R_b}{R_a R_f C_1}$$

$$1 + R_b/R_a$$

$$R_1 = R_2 = R$$

$$C_3 = 2C$$

$$C_1 = C_2 = C$$

$$R_3 = R/2$$

$$Z_L = \infty$$

$$\omega \approx 0$$

$$\frac{v_{out}}{v_{in}} = \frac{(j\omega)^2 + \omega_n^2}{(j\omega)^2 + 4\omega_n j\omega + \omega_n^2}$$

$$\frac{(j\omega)^2 + \omega_n^2}{(j\omega)^2 + j\omega\omega_n/Q + \omega_n^2} = \frac{\omega^2 - \omega_n^2}{\omega^2 - j\omega\Delta\omega - \omega_n^2}$$

$$\omega_n = \frac{1}{RC} = \frac{1}{\tau}$$

$$Q = 1/4 = 0.25$$

$$\Delta\omega = \omega_n / Q = 4\omega_n$$

$$\omega_n = 1/\tau$$

$$|H(j\omega)| = \begin{cases} H(0) = \omega_n^2/\omega_n^2 = 1 & \omega = 0 \\ H(j\omega_n) = 0 & \omega = \omega_n = 1/\tau \\ H(\infty) = \lim_{\omega\to\infty} H(j\omega) = \omega^2/\omega^2 = 1 & \omega \to \infty \end{cases}$$

$$Q = 1/4$$

$$A = 1$$

R_4

R_5

$$V_1 = \frac{R_5}{R_4 + R_5}\ V_{out} = \alpha v_{out}$$

$$\alpha = R_5/(R_4 + R_5)$$

$$1 - \alpha = R_4/(R_4 + R_5)$$

$$V_{in} - V_1$$

$$V_{out} - V_1$$

$$H(j\omega)$$

$$V_{out} - V_1 = H(j\omega)(V_{in} - V_1)$$

$$V_1 = V_{out}\ R_5/(R_4 + R_5)$$

$$H(j\omega)V_{in}$$

$$V_{out} + (H(j\omega) - 1)V_1 = V_{out} + (H(j\omega) - 1)\frac{R_5}{R_4 + R_5}V_{out}$$

$$\left(1 + (H(j\omega) - 1)\ \frac{R_5}{R_4 + R_5}\right)\ V_{out} = \frac{R_4 + H(j\omega)R_5}{R_4 + R_5}\ V_{out}$$

$$H_{active}(j\omega) = \frac{V_{out}}{V_{in}} = \frac{H(j\omega)(R_4 + R_5)}{R_4 + H(j\omega)R_5}$$

$$H(j\omega) = ((j\omega)^2 + \omega_n^2)/((j\omega)^2 + 4\omega_n j\omega + \omega_n^2)$$

$$H_{active}(j\omega)$$

$$\frac{(\omega_n^2 - \omega^2)(R_4 + R_5)}{R_4(\omega_n^2 - \omega^2 + 4\omega_n j\omega) + (\omega_n^2 - \omega^2)R_5}$$

$$\frac{(\omega_n^2 - \omega^2)(R_4 + R_5)}{4\omega_n R_4 j\omega + (\omega_n^2 - \omega^2)(R_4 + R_5)}$$

$$\frac{\omega_n^2 - \omega^2}{j\omega 4\omega_n R_4/(R_4 + R_5) + \omega_n^2 - \omega^2}$$

$$\frac{\omega_n^2 - \omega^2}{\omega_n^2 - \omega^2 + \omega_n/Q_{active} j\omega} = \frac{\omega^2 - \omega_n^2}{\omega^2 - \Delta\omega_{active} j\omega - \omega_n^2}$$

$$Q_{active} = \frac{R_4 + R_5}{4R_4}, \qquad \Delta\omega_{active} = \frac{\omega_n}{Q_{\ active}}$$

$$\frac{\omega_n^2 - \omega^2}{\omega_n^2 + 4j\omega\omega_n R_4/(R_4 + R_5) - \omega^2}$$

$$\frac{\omega_n^2 - \omega^2}{\omega_n^2 - \omega^2 + \omega_n/Q_{active} j\omega} = \frac{\omega_n^2 - \omega^2}{\omega_n^2 - \omega^2 + \Delta\omega_{active} j\omega}$$

$$\Delta\omega_{active}$$

$$R_5 = 0$$

$$V_1 = 0$$

$$Q_{active} = 1/4 = Q$$

$$R_4 = 0$$

$$V_1 = V_{out}$$

$$Q_{active} = \infty$$

$$\Delta\omega_{active} = \omega_n / Q_{active} = 0$$

$$C = 0$$

$$Z_3' = 2R$$

$$Z_3'' = 2(1 + j\omega RC)/R(j\omega C)^2$$

$$Z_3 = Z_3' || Z_3'' = \frac{Z_3' Z_3''}{Z_3' + Z_3''} = \frac{2R(1 + j\omega RC)}{1 + j\omega RC + (j\omega RC)^2}$$

$$\frac{Z_2}{Z_2 + Z_3} = \frac{R + 1/j\omega C}{R + 1/j\omega C + 2R(1 + j\omega RC)/(1 + j\omega RC + (j\omega RC)^2)}$$

$$\frac{1/C}{1/j\omega C + 2R/(1 + j\omega RC + (j\omega RC)^2)} = \frac{1}{1 + 2j\omega RC/(1 + j\omega RC + (j\omega RC)^2)}$$

$$\frac{1 + j\omega RC + (j\omega RC)^2}{1 + 3j\omega RC + (j\omega RC)^2} = \frac{(j\omega)^2 + j\omega/RC + 1/(RC)^2}{(j\omega)^2 + 3j\omega/RC + 1/(RC)^2}$$

$$\omega_n = 1/RC$$

$$H(j\omega) = \frac{(j\omega)^2 + \omega_n j\omega + \omega_n^2}{(j\omega)^2 + 3\omega_n j\omega + \omega_n^2} = \frac{(j\omega)^2 + \Delta\omega_n j\omega + \omega_n^2}{(j\omega)^2 + \Delta\omega_d j\omega + \omega_n^2} = \frac{\omega_n^2 - \omega^2 + \Delta\omega_n j\omega}{\omega_n^2 - \omega^2 + \Delta\omega_d j\omega}$$

$$\Delta\omega_n = \omega_n, \qquad \Delta\omega_d = 3\omega_n$$

$$H(j\omega) = H(0) = 1$$

$$H(j\omega) = 1$$

$$\omega = \omega_n = 1/RC$$

$$H(j\omega_n) = 1/3$$

$$\frac{R_3}{R_4} = \frac{R_2 + 1/j\omega C_2}{R_1||1/j\omega C_1} = \frac{(j\omega R_1 C_1 + 1)(j\omega R_2 C_2 + 1)}{j\omega R_1 C_2} = \frac{1 - \omega^2 R_1 R_2 C_1 C_2 + j\omega(R_1 C_1 + R_2 C_2)}{j\omega R_1 C_2}$$

$$1 - \omega^2 R_1 R_2 C_1 C_2 = 0, \qquad \text{i.e.,} \qquad \omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\frac{R_3}{R_4} = \frac{R_1 C_1 + R_2 C_2}{R_1 C_2} = \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{\sqrt{R^2 C^2}} = \frac{1}{RC}$$

$$\frac{R_3}{R_4} = \frac{C_1}{C_2} + \frac{R_2}{R_1} = 1 + 1 = 2, \qquad \text{i.e.} \qquad R_4 = 2R_3$$

$$\frac{V_{in}}{R_4} + \frac{V_{out}}{R_3} + \frac{V_1}{R_2} = 0 \qquad (1)$$

$$V_2 = \frac{R + 1/j\omega C}{R + 1/j\omega C + R/j\omega C/(R + 1/j\omega C)} V_1 = \frac{(j\omega\tau + 1)^2}{(j\omega\tau + 1)^2 + j\omega\tau} V_1,$$

$$\tau = RC$$

$$V_1 = \left(1 + \frac{j\omega\tau}{(j\omega\tau + 1)^2}\right) V_2, \qquad (2)$$

$$\frac{V_1 - V_2}{R_1} + \frac{V_{out} - V_2}{2R_1} = 0, \qquad \text{i.e.} \qquad V_{out} = 3V_2 - 2V_1, \qquad (3)$$

$$V_{out} = 3V_2 - 2V_1 = 3V_2 - 2\left(1 + \frac{j\omega\tau}{(j\omega\tau + 1)^2}\right)V_2 = V_2 - \frac{j\omega 2\tau}{(j\omega\tau + 1)^2}\ V_2 = \frac{(j\omega\tau)^2 + 1}{(j\omega\tau + 1)^2}\ V_2$$

$$V_2 = \frac{(j\omega\tau + 1)^2}{(j\omega\tau)^2 + 1} V_{out}$$

$$V_1 = \left(1 + \frac{j\omega\tau}{(j\omega\tau+1)^2}\right) V_2 = \left(1 + \frac{j\omega\tau}{(j\omega\tau+1)^2}\right) \frac{(j\omega\tau+1)^2}{(j\omega\tau)^2+1} V_{out} = \frac{(j\omega\tau)^2 + 3j\omega\tau + 1}{(j\omega\tau)^2+1}\ V_{out}$$

$$\frac{V_{in}}{R_4} + \frac{V_{out}}{R_3} + \frac{(j\omega\tau)^2 + 3j\omega\tau + 1}{(j\omega\tau)^2 + 1} \; \frac{V_{out}}{R_2} = 0$$

$$\frac{V_{in}}{R_4} = -\left(\frac{1}{R_3} + \frac{(j\omega\tau)^2 + 3j\omega\tau + 1}{(j\omega\tau)^2 + 1}\ \frac{1}{R_2}\right) V_{out}$$

$$\frac{R_2}{R_4} = -\left(\frac{R_2}{R_3} + \frac{(j\omega\tau)^2 + 3j\omega\tau + 1}{(j\omega\tau)^2 + 1}\right)\frac{V_{out}}{V_{in}}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{R_2 R_3}{R_4(R_2 + R_3)} \; \frac{(j\omega\tau)^2 + 1}{(j\omega\tau)^2 + 3j\omega\tau R_3/(R_2 + R_3) + 1}$$

$$H(j\omega) = A \; \frac{(j\omega)^2 + \omega_n^2}{(j\omega)^2 + \Delta\omega/Q \; j\omega + \omega_n^2} = \left\{ \begin{array}{ll} A & \omega = 0 \\ 0 & \omega = \omega_n = 1/\tau \\ A & \omega \to \infty \end{array} \right.$$

$$A = -R_2 R_3 / R_4 (R_2 + R_3) = (R_2 // R_3) / R_4$$

$$Q = (R_2 + R_3)/3R_3$$

$$\omega_c = 1/\tau$$

n

$$H(j\omega) = \frac{1}{j\omega\tau + 1} = \frac{1}{j\omega/\omega_c + 1} = \frac{\omega_c}{j\omega + \omega_c}$$

$$H(j\omega) = \left(\frac{\omega_c}{\omega_c + j\omega} \right)^n$$

$$\omega_{cn}$$

$$\left|\frac{\omega_c}{\omega_c + j\omega}\right|^n = \left|\frac{\omega_c}{\sqrt{\omega_c^2 + \omega^2}}\right|^n = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$\omega = \omega_{cn} = \omega_c \sqrt{2^{1/n} - 1}$$

$$\omega_{c4} = 2\pi 1000 = 6.2832 \times 10^3$$

$$\omega_c = \frac{\omega_{c4}}{\sqrt{2^{1/4} - 1}} = \frac{6.2832 \times 10^3}{0.435} = 1.445 \times 10^4$$

$$\tau = RC = 1/\omega_c = 6.92 \times 10^{-5}$$

$$C = 0.1\ \mu F = 10^{-7}\ F$$

$$R = \frac{\omega_c}{C} = \frac{6.92 \times 10^{-5}}{10^{-7}} = 6.92 \times 10^2 = 692\ \Omega$$

ω_c

$$|H_{lp}(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^{2n}}} = \begin{cases} 1 & \omega = 0 \\ 1/\sqrt{2} & \omega = \omega_c \\ 0 & \omega = \infty \end{cases}$$

$$|H_{lp}(j\omega_c)| = 1/\sqrt{2}$$

$n = 0$

$$n = 1$$

$$n = \infty$$

$$|H_{lp}(j\omega)| = 1/\sqrt{2}$$

$$|H_{lp}(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^2}} = \frac{\omega_c}{\sqrt{\omega^2+\omega_c^2}} = \left|\frac{\omega_c}{j\omega+\omega_c}\right|$$

$$|H_{lp}(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^{\infty}}} = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega > \omega_c \end{cases}$$

$$|H_{hp}(j\omega)| = \frac{1}{\sqrt{1+(\omega_c/\omega)^{2n}}} = \frac{(\omega/\omega_c)^n}{\sqrt{1+(\omega/\omega_c)^{2n}}} = \begin{cases} 0 & \omega = 1 \\ 1/\sqrt{2} & \omega = \omega_c \\ 1 & \omega = \infty \end{cases}$$

$$\omega = \omega/\omega_c$$

$$\omega_c = 1$$

$$|H(j\omega)|^2 = H(j\omega)H(-j\omega) = \frac{1}{1+(\omega/\omega_c)^{2n}} = \frac{1}{1+\omega^{2n}} = \frac{1}{1+(\omega^2)^n}$$

$$|H(j\omega)|$$

$$H(s)$$

$$s^2 = (j\omega)^2 = -\omega^2$$

$$|H(j\omega)|^2 = H(j\omega)\,H(-j\omega) = \frac{1}{1+(\omega^2)^n} = \frac{1}{1+(-s^2)^n} = \frac{1}{1+(-1)^n s^{2n}} = |H(s)|^2 = H(s)H(-s)$$

$$H(-s)$$

$$1 + (-1)^n s^{2n} = 0, \qquad \text{i.e.} \qquad \begin{cases} 1 + s^{2n} = 0 & n \text{ is even} \\ 1 - s^{2n} = 0 & n \text{ is odd} \end{cases}$$

$2n$

$$\begin{cases} s = (-1)^{1/2n} = (e^{j(2k+1)\pi})^{1/2n} = e^{j(2k+1)\pi/2n} & n \text{ is even} \\ s = 1^{1/2n} = (e^{j2k\pi})^{1/2n} = e^{jk\pi/n} & n \text{ is odd} \end{cases} \qquad (k = 0, \cdots, 2n-1)$$

$$s_k = e^{j(2k+1)\pi/2n}, \qquad (k = 0, \cdots, 2n-1)$$

$$s_k = e^{j(2k+1)\pi/2n}$$

$k = 0, \cdots, n-1$

s_{2n-1-k}

$$s_{2n-1-k} = e^{j(2(2n-1-k)+1)\pi/2n} = e^{j\pi(4n-2k-1)/2n} = e^{j2\pi}\ e^{-j(2k+1)\pi/2n} = e^{-j(2k+1)\pi/2n} = s_k^*$$

$$\frac{(2k+1)\pi}{2n} > \frac{\pi}{2}, \qquad \text{i.e.} \qquad k > \frac{n-1}{2}$$

$H(s)$

$$\frac{1}{\prod_{k=\lceil (n-1)/2 \rceil}^{n-1} (s - s_k)(s - s_{2n-1-k})} = \frac{1}{\prod_{k=\lceil (n-1)/2 \rceil}^{n-1} (s - s_k)(s - s_k^*)}$$

$$\frac{1}{\prod_{k=\lceil (n-1)/2 \rceil}^{n-1} (s^2 - (s_k + s_k^*)s + s_k s_k^*)} = \frac{1}{\prod_{k=\lceil (n-1)/2 \rceil}^{n-1} (s^2 - 2\cos((2k+1)\pi/2n) + 1)}$$

$$\lceil x \rceil$$

x

$$s_k + s_k^* = e^{j(2k+1)\pi/2n} + e^{-j(2k+1)\pi/2n} = 2\cos((2k+1)\pi/2n), \qquad s_k s_k^* = 1$$

$$s_k = e^{j2k\pi/2n} = e^{jk\pi/n}, \qquad (k = 0, \cdots, 2n-1)$$

$$s_0 = e^{j0} = 1$$

$$s_n = e^{j\pi} = -1$$

$n - 1$

$$s_k = e^{jk\pi/n}$$

$k = 1, \cdots, n-1$

s_{2n-k}

$$s_{2n-k} = e^{j(2n-k)\pi/n} = e^{-jk\pi/n} = s_k^*$$

s_k

$$\frac{k\pi}{n} > \frac{\pi}{2}, \qquad \text{i.e.} \qquad k > \frac{n}{2}$$

$$\frac{1}{(s-s_n)\prod_{i=\lceil n/2\rceil}^{n-1}(s-s_k)(s-s_{2n-k})} = \frac{1}{(s+1)\prod_{i=\lceil n/2\rceil}^{n-1}(s-s_k)(s-s_k^*)}$$

$$\frac{1}{(s+1)\prod_{i=\lceil n/2\rceil}^{n-1}(s^2-(s_k+s_k^*)s+s_ks_k^*)}=\frac{1}{(s+1)\prod_{i=\lceil n/2\rceil}^{n-1}(s^2-2\cos(k\pi/n)+1)}$$

$n = 2, \cdots, 6$

$$n = 2$$

$$1 + s^4 = 0$$

$$s^4 = -1 = e^{j(2k+1)\pi}$$

$$s_k = e^{j(2k+1)\pi)/4}$$

$k = 0, \cdots, 3$

$$s_0 = e^{j\pi/4} = \frac{1+j}{\sqrt{2}}, \quad s_1 = e^{j3\pi/4} = \frac{-1+j}{\sqrt{2}}, \quad s_2 = e^{j5\pi/4} = \frac{-1-j}{\sqrt{2}}, \quad s_3 = e^{j7\pi/4} = \frac{1-j}{\sqrt{2}}$$

s_1

$$s_2 = s_1^*$$

$$\frac{1}{(s-s_1)(s-s_2)} = \frac{1}{(s-(-1+j)/\sqrt{2})(s-(-1-j)/\sqrt{2})}$$

$$\frac{1}{s^2 + \sqrt{2}s + 1}$$

$$-2\cos(3\pi/2) = \sqrt{2}$$

$$n = 3$$

$$1 - s^6 = 0$$

$$s^6 = 1 = e^{j2k\pi}$$

$$s_k = e^{jk2\pi/6} = e^{jk\pi/3}$$

$k = 0, \cdots, 5$

$$s_0 = e^{j0} = 1, \quad s_1 = e^{j\pi/3} = \frac{1 + j\sqrt{3}}{2}, \quad s_2 = e^{j2\pi/3} = \frac{-1 + j\sqrt{3}}{2}$$

$$s_3 = e^{j3\pi/3} = e^{j\pi} = -1, \quad s_4 = e^{j4\pi/3} = \frac{-1 - j\sqrt{3}}{2}, \quad s_5 = e^{j5\pi/3} = \frac{1 - j\sqrt{3}}{2}$$

$$s_2$$

$$s_3 = -1$$

$$s_4 = s_2^*$$

$$\frac{1}{(s-s_2)(s-s_3)(s-s_4)} = \frac{1}{(s+1)(s-(-1+j\sqrt{3})/2)(s-(-1-j\sqrt{3})/2)}$$

$$\frac{1}{(s+1)(s^2+s+1)}$$

$$-2\cos(2\pi/3) = 1$$

$n = 4$

$$1 + s^8 = 0$$

$$s^8 = -1 = e^{j(2k+1)\pi}$$

$$s_k = e^{j(2k+1)pi/8}$$

$k = 0, \cdots, 7$

$$-2\cos((2k+1)\pi/2n)$$

$k = 2$

$$k = 3$$

$$-2\cos(5\pi/8) = 0.7654$$

$$-2\cos(7\pi/8) = 1.8478$$

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

$n = 5$

$$1 - s^{10} = 0$$

$$s^{10} = 1 = e^{j2k\pi}$$

$$s_k = e^{j(2k\pi)/10} = e^{j(k\pi)/5}, \qquad (k = 0, \cdots, 9)$$

$$-2\cos(k\pi/n)$$

$$-2\cos(2\pi/5) = 0.618$$

$$-2\cos(3\pi/5) = 1.618$$

$$H(s) = \frac{1}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

$$n = 6$$

$$1 + s^{12} = 0$$

$$s^{12} = -1 = e^{j(2k+1)\pi}$$

$$s_k = e^{j(2k+1)\pi)/12}, \qquad (k = 0, \cdots, 11)$$

$$k = 4$$

$$k = 5$$

$$-2\cos(7\pi/12) = 0.5176$$

$$-2\cos(9\pi/12) = 1.4142$$

$$-2\cos(9\pi/12) = 1.319$$

$$H(s) = \frac{1}{(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9319s + 1)}$$

$$1/(s^2 + a\,s + 1)$$

$$1/(s+1)$$

$$H(s) = \frac{1/\tau}{s + 1/\tau} = \frac{\omega_c}{s + \omega_c} = \frac{1}{s + 1}$$

$$\omega_c = 1/\tau = 1/RC$$

$$R = 1$$

$$C = 1/\omega_c = 1$$

$$H(s) = \frac{1/R_1C_1R_2C_2}{s^2 + s(R_1 + R_2)/R_1R_2C_1 + 1/R_1C_1R_2C_2} = \frac{1}{s^2 + \Delta\omega s + \omega_n^2} = \frac{1}{s^2 + as + 1}$$

$$\omega_n^2 = 1/R_1R_2C_1C_2$$

$$2/C_1 = a, \qquad 1/C_1 C_2 = \omega_n^2 = 1$$

$$C_1 = 2/a, \qquad C_2 = 1/C_1 = a/2$$

$$H(s) = \frac{s}{s+\omega_c} = \frac{s}{s+1}, \qquad H(s) = \frac{s^2}{s^2+\Delta\omega s+\omega_c^2} = \frac{s^2}{s^2+as+1}$$

$$H(s) = \begin{cases} s^n/(1+s^{2n}) & n \text{ is even} \\ s^n/(1-s^{2n}) & n \text{ is odd} \end{cases}$$

$$\omega_n = 1$$

$$\omega_n \neq 1$$

C

$$C' = C/\omega_c$$

$$1/RC' = \omega_c/C' = \omega_c$$

$$C_1' = C_1/\omega_n$$

$$C_2' = C_2/\omega_n$$

$$1/C_1'C_2' = \omega_n^2/C_1C_2 = \omega_n^2$$

$$\left\{\begin{array}{l} Y_3(s) = Y_2(s)/s \Longrightarrow Y_2(s) = Y_3(s)s \\ Y_2(s) = Y_1(s)/s \Longrightarrow Y_1(s) = Y_2(s)s = Y_3(s)s^2 \\ Y_1(s) = Y_0(s)/s \Longrightarrow Y_0(s) = Y_1(s)s = Y_3(s)s^3 \end{array}\right.$$

$$Y_0(s) = X(s) - (k_1 Y_1(s) + k_2 Y_2(s) + k_3 Y_3(s))$$

$$X(s) = Y_0(s) + k_1 Y_1(s) + k_2 Y_2(s) + k_3 Y_3(s) = (s^3 + k_1 s^2 + k_2 s + k_3) Y_3(s)$$

$$H(s) = \frac{Y_3(s)}{X(s)} = \frac{1}{s^3 + k_1 s^2 + k_2 s + k_3}$$

$$\begin{cases} Y_2(s) = -c_2 Y_1(s)/s \Longrightarrow Y_1(s) = -sY_2(s)/c_2 \\ Y_1(s) = -c_1 Y_0(s)/s \Longrightarrow Y_0(s) = -sY_1(s)/c_1 = s^2 Y_2(s)/c_1 c_2 \\ Y_0(s) = k_0 X(s) + k_1 Y_1(s) + k_2 Y_2(s) \end{cases}$$

$$\frac{s^2}{c_1 c_2} Y_2(s) = k_0 X(s) + k_1 (-\frac{s}{c_2}) Y_2(s) + k_2 Y_2(s)$$

$$H(s) = \frac{Y_2(s)}{X(s)} = \frac{k_o}{\frac{s^2}{c_1 c_2} + \frac{s}{c_2}s - k_2} = \frac{k_o c_1 c_2}{s^2 + k_1 c_1 s - c_1 c_2 k_2}$$

$$c_1 = c_2 = c$$

$$H(s) = k_0 \frac{c^2}{s^2 + ck_1 s - k_2 c^2}$$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$c = \omega_n$$

$$k_1 = 2\zeta$$

$k_2 < 0$

k_0

$$w = u + jv = |w|\angle w = |w|e^{j\angle w}$$

$$z = x + jy = |z|\angle z = |z|e^{j\angle z}$$

$$\begin{cases} |w| = \sqrt{u^2 + v^2}, & \angle w = \tan^{-1}(v/v) \\ |z| = \sqrt{x^2 + y^2}, & \angle z = \tan^{-1}(y/x) \end{cases}$$

$$wz = (u + jv)(x + jy) = |w|e^{j\angle w}\ |z|e^{j\angle z}$$

$$|wz| = |w||z|, \qquad e^{j\angle w} e^{j\angle z} = e^{j(\angle w + \angle z)}, \quad \text{or} \quad \angle (wz) = \angle w + \angle z$$

$$\frac{w}{z} = \frac{u + jv}{x + jy} = \frac{|w|e^{j\angle w}}{|z|e^{j\angle z}}$$

$$\left|\frac{w}{z}\right| = \frac{|w|}{|z|}, \qquad \frac{e^{j\angle w}}{e^{j\angle z}} = e^{j(\angle w - \angle z)}, \quad \text{or} \quad \angle\left(\frac{w}{z}\right) = \angle w - \angle z$$

$$\log\,(ab) = \log a + \log b$$

$$\log\,(a/b) = \log a - \log b$$

$$\log\left(a^{n}\right)=n\ \log a$$

$$\log\left(a^{-n}\right) = -n\ \log a$$

R

v

i

$$R = v/i$$

Z

$$Z = \frac{V}{I} = \frac{v_m e^{j\phi}}{i_m e^{j\psi}} = \frac{v_m}{i_m} e^{j(\phi - \psi)}$$

V

I

L

$$Ie^{j\omega t}$$

$$Ve^{j\omega t} = L\frac{d}{dt}[Ie^{j\omega t}] = j\omega LIe^{j\omega t} \quad \text{i.e.} \qquad Z_L = \frac{V}{I} = \frac{j\omega LI}{I} = j\omega L$$

$$Ve^{j\omega t}$$

$$Ie^{j\omega t} = C\frac{d}{dt}[Ve^{j\omega t}] = j\omega CVe^{j\omega t} \qquad \text{i.e.} \qquad Z_C = \frac{V}{I} = \frac{V}{j\omega CV} = \frac{1}{j\omega C}$$

$$Z_R = \frac{V}{I} = \frac{v_m e^{j\phi}}{i_m e^{j\phi}} = \frac{v_m}{i_m} = R$$

$$P_{in} = 100\ mW$$

$$P_{out} = 10\ W$$

$$P_{out}/P_{in} = 100$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} \frac{10}{0.1} = \log_{10} 100 = 2 \ bel(B)$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} 100 = 2\ B = 20\ dB, \quad \text{or} \quad L_{dB} = 10 \log_{10} \frac{P_{out}}{P_{in}} = 20\ dB$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} 1000 = 3\ B = 30\ dB, \quad \text{or} \quad L_{dB} = 10 \log_{10} \frac{P_{out}}{P_{in}} = 30\ dB$$

P_{in}

$$L_{dB} = 30\ dB$$

$$\frac{P_{out}}{P_{in}} = 10^{L_{dB}/10} = 10^{30/10} = 10^3, \quad \text{i.e.} \quad P_{out} = 10^3\ P_{in} = 1,000\ P_{in}$$

W/m^2

$$10^{-12}\ W/m^2$$

10^{14}

$10^2\ W/m^2$

$$P = V^2/R = I^2R$$

$$E = mv^2/2$$

$$E = kx^2/2$$

$$L_{dB} = 10 \log_{10} \frac{V_{out}^2}{V_{in}^2} = 20 \log_{10} \frac{V_{out}}{V_{in}} \ dB$$

$$20\log_{10}\frac{V_{out}}{V_{in}} = 20\log_{10}\frac{1,000}{10} = 40\ dB$$

$$20 \log_{10} \frac{V_{out}}{V_{in}} = 20 \log_{10} \frac{10,000}{10} = 60\ dB$$

$$L_{dB} = 60\ dB$$

$$\frac{V_{out}}{V_{in}} = 10^{L_{dB}/20} = 10^{60/20} = 10^3, \quad \text{i.e.,} \quad V_{out} = 10^3 V_{in} = 1,000 V_{in}$$

$$\zeta < 0.2$$

$$\omega = \omega_p \approx \omega_n$$

$$\omega_1 < \omega_n < \omega_2$$

$$|H(j\omega_1)|^2 = |H(j\omega_2)|^2 = \frac{1}{2}|H(j\omega_p)|^2 \quad \text{i.e.,} \quad |H(j\omega_{1,2})| = 0.707\ |H(j\omega_p)|$$

$$20\log_{10}\left(\frac{|H(j\omega_{1,2})|}{|H(j\omega_p)|}\right) = 20\log_{10} 0.707 = -3.01\ dB \approx -3\ dB$$

$$\omega = 2\pi f$$

$$H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)} = |H(j\omega)|\angle H(j\omega)$$

$$\angle H(j\omega)$$

$-\infty$

$$10^{-\infty} = 0$$

$$Lm\ H(j\omega) = 20\ log_{10}|H(j\omega)|\ dB$$

ω

10ω

$$\begin{cases} Lm(H_1H_2) = Lm\ H_1 + Lm\ H_2, & \angle(H_1H_2) = \angle H_1 + \angle H_2 \\ Lm(H_1/H_2) = Lm\ H_1 - Lm\ H_2, & \angle(H_1/H_2) = \angle H_1 - \angle H_2 \\ Lm\ H^n = n\ Lm\ H, & \angle H^n = n\ \angle H \\ Lm(1/H) = -Lm\ H, & \angle(1/H) = -\angle H \end{cases}$$

k

$$(j\omega)$$

$$e^{\pm j\omega\tau}$$

$$(1 + j\omega\tau)$$

$$(j\omega)^2 + 2\zeta\omega_n\omega j + \omega_n^2 = (\omega_n^2 - \omega^2) + j\,2\zeta\omega_n$$

$$H(j\omega) = \frac{N(j\omega)}{1 + j\omega\tau}$$

$$H(j\omega) = \frac{N(j\omega)}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{N(j\omega)}{(\omega_n^2 - \omega^2) + j\ (2\zeta\omega_n\omega)}$$

$$\begin{cases} \text{If } k > 0, & k = |k|e^{j0}, \;\; Lm\ k = 20\ \log_{10}|k|, \;\; \angle k = 0 \\ \text{If } k < 0, & k = -|k| = |k|e^{j\pi}, \;\; Lm\ k = 20\ \log_{10}|k|, \;\; \angle k = \pi \end{cases}$$

$$Lm\ e^{j\omega\tau} = 20\ \log_{10}|e^{j\omega\tau}| = 20\ \log_{10} 1 = 0, \quad \angle e^{j\omega\tau} = \pm\omega\tau$$

$$j\omega = \omega\ e^{j\pi/2}$$

$$Lm\ (j\omega) = 20\ log_{10}\omega\ dB, \qquad \angle(j\omega) = \frac{\pi}{2}$$

$$\omega = 1$$

$$Lm\ (1) = 20\ \log_{10} 1 = 0\ dB$$

$$Lm\ (j10\omega) = 20\ \log_{10} 10\omega = 20\ \log_{10} 10 + 20\ \log_{10} \omega = 20 + Lm(j\omega)$$

$$j\omega$$

$$(j\omega)^{\pm m} = \omega^{\pm m} e^{\pm jm\pi/2}$$

$$Lm(j\omega)^{\pm m} = \pm m \ Lm(j\omega), \qquad \angle(j\omega)^{\pm m} = \pm m\pi/2$$

$$\pm 20m/dec$$

$m = 2$

$$Lm\ (j\omega)^2 = 40 \log_{10} \omega, \qquad \angle\ (j\omega)^2 = \pi$$

$j\omega\tau$

$$\omega\tau = 1$$

$$\omega = 1/\tau$$

$$1/j\omega = (j\omega)^{-1}$$

$$Lm\ (j\omega)^{-1} = -Lm\ (j\omega) = -20\ log_{10}\omega\ dB, \quad \angle\ (j\omega)^{-1} = -\angle(j\omega) = -\frac{\pi}{2}$$

$1/j\omega$

$$1 + j\omega\tau$$

$$1 + j\omega\tau = \sqrt{1 + (\omega\tau)^2}\, e^{j\tan^{-1}(\omega\tau)} = \sqrt{1 + (\omega\tau)^2}\, \angle \tan^{-1}(\omega\tau)$$

$$Lm(1+j\omega\tau) = 20\ \log_{10}\sqrt{1+(\omega\tau)^2} = 20\ \log_{10}(1+(\omega\tau)^2)^{1/2} = 10\ \log_{10}(1+(\omega\tau)^2)$$

$$\angle(1 + j\omega\tau) = \tan^{-1}(\omega\tau)$$

$$Lm(1+j) = 20\ \log_{10}\sqrt{1^2+1^2} = 20\ \log_{10} 0.707 \approx 3.01\ dB, \qquad \angle(1+j) = \frac{\pi}{4}$$

$$\omega\tau \ll 1$$

$\omega\tau \leq 10$

$$Lm(1 + j\omega\tau) \approx 10\ \log_{10}(1) = 0, \qquad \angle(1 + j\omega\tau) \approx \angle(1) = 0$$

$$\omega\tau \gg 1$$

$\omega\tau \geq 10$

$$Lm(1 + j\omega\tau) \approx 20\ \log_{10}(\omega\tau), \quad \angle(1 + j\omega\tau) \approx \angle(j\omega\tau) = \frac{\pi}{2}$$

$$Lm(1 + j\omega\tau)$$

$\omega\tau < 1$

$\omega\tau > 1$

$$\angle(1 + j\omega\tau)$$

$\omega\tau < 0.1$

$\pi/2$

$\omega\tau > 10$

$45^\circ/dec$

$$1/(1 + j\omega\tau) = (1 + j\omega\tau)^{-1}$$

$$Lm\ (1 + j\omega\tau)^{-1} = -Lm(1 + j\omega\tau) = -10\ \log_{10}(1 + (\omega\tau)^2)$$

$$\angle\,(1 + j\omega\tau)^{-1} = -\angle(1 + j\omega\tau) = -\tan^{-1}(\omega\tau)$$

$$1/(1 + j\omega\tau)$$

$$\omega_1 = 100$$

$$\omega_2 = 1000$$

$$H(j\omega) = \frac{1}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{1}{(\omega_n^2 - \omega^2) + 2\zeta\omega_n j\omega} = \frac{\frac{1}{\omega_n^2}}{1 - (\frac{\omega}{\omega_n})^2 + j\,2\zeta\,\frac{\omega}{\omega_n}}$$

$$\Delta = b^2 - 4ac = (2\zeta\omega_n)^2 - 4\omega_n^2 = 4\omega_n^2(\zeta^2 - 1) \geq 0$$

$\zeta \geq 1$

$$p_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1})\omega_n < 0$$

$$H(j\omega) = \frac{1}{(j\omega - p_1)(j\omega - p_2)} = \frac{1/p_1 p_2}{(j\omega/p_1 - 1)(j\omega/p_2 - 1)} = \frac{\tau_1}{1 + j\omega\tau_1} \ \frac{\tau_2}{1 + j\omega\tau_2} = H_1(j\omega)H_2(j\omega)$$

$$\tau_1 = -1/p_1 > 0$$

$$\tau_2 = -1/p_2 > 0$$

$$Lm\ (H_1 H_2) = Lm\ H_1 + Lm\ H_2, \quad \angle(H_1 H_2) = \angle H_1 + \angle H_2$$

$$\omega_{c1} = 1/\tau_1 = p_1$$

$$\omega_{c1} = 1/\tau_2 = p_2$$

$$0 < \zeta < 1$$

$$20\log_{10}\omega_n^{-}2 = -40\log_{10}\omega_n$$

$$|H(j\omega)| = [(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta \frac{\omega}{\omega_n})^2]^{-1/2}$$

$$Lm\ H(j\omega) = 20\log_{10}|H(j\omega)| = -10\ \ \log_{10}[\ (1-(\frac{\omega}{\omega_n})^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2\ ]$$

$$\angle H(j\omega) = -\tan^{-1} \frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}$$

$$\omega/\omega_n = 1$$

$$H(j\omega) = 1/j2\zeta = -j/2\zeta$$

$$Lm\ H(j\omega) = -20\ \log_{10} 2\zeta, \qquad \angle H(j\omega) = -\frac{\pi}{2}$$

$$\omega/\omega_n \ll 1$$

$$\omega \ll \omega_n$$

$$Lm\ H(j\omega) \approx -10\ \log_{10}(1) = 0, \qquad \angle H(j\omega) = 0^\circ$$

$$\omega/\omega_n \gg 1$$

$$Lm\ H(j\omega) \approx -10\ \log_{10}[\ (\frac{\omega}{\omega_n})^4] = -40\ \log_{10} \frac{\omega}{\omega_n}$$

$$\angle H(j\omega) \approx -\tan^{-1}(-2\zeta\omega_n/\omega) \approx -\tan^{-1}(-0) = -\pi = -180^\circ$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} = \frac{1}{\sqrt{(1-u)^2 + 4\zeta^2 u}}$$

$$u = (\omega/\omega_n)^2$$

$$u = 1$$

$$\omega = \omega_n$$

$$|H(j\omega_n)| = \frac{1}{2\zeta} = Q$$

ω_p

u

$$\frac{d}{du}[u^2 + (4\zeta^2 - 2)u + 1] = 2u + 4\zeta^2 - 2 = 0$$

$$u = \frac{\omega^2}{\omega_n^2} = 1 - 2\zeta^2, \quad \text{i.e.,} \quad \omega = \omega_n\sqrt{1 - 2\zeta^2} < \omega_n$$

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}$$

$$|H(j\omega_p)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}} > \frac{1}{2\zeta} = |H(j\omega_n)|$$

$$\zeta^2 > 1/2$$

$$\zeta > 0.707$$

V_C

$$H_C(j\omega) = \frac{V_C}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega\tau + 1}$$

$$V_R$$

$$H_R(j\omega) = \frac{V_R}{V_{in}} = \frac{Z_R}{Z_R + Z_C} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1} = \frac{j\omega\tau}{j\omega\tau + 1}$$

$H_R(j\omega)$

$$H_R(j\omega) = \frac{1}{j\omega\tau + 1} j\omega\tau$$

$$H_C(j\omega)$$

$$Lm\ H_R(j\omega) = 20\log_{10}\left|\frac{1}{j\omega\tau+1}\right| + 20\log_{10}|j\omega\tau| = Lm\ H_C(j\omega) + 20\log_{10}(\omega\tau)$$

$$\omega = \omega_c = 1/\tau$$

$$\angle H_R(j\omega) = \angle\left(\frac{1}{j\omega\tau + 1}\right) + \angle j\omega\tau = \angle H_C(j\omega) + \frac{\pi}{2}$$

$\tau = 0.01$

$$\omega_C = 100$$

$$\omega = \omega_c$$

$$|H_R(j\omega)| = |H_C(j\omega)| = 1/\sqrt{2}$$

$$H_C(j\omega)$$

$$\frac{V_C}{V_{in}} = \frac{Z_C}{Z_L + Z_R + Z_C} = \frac{1/j\omega C}{j\omega L + R + 1/j\omega C} = \frac{1}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{1/LC}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{1}{(1 - \frac{\omega^2}{\omega_n^2}) + j2\zeta\frac{\omega}{\omega_n}}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$|H_C(j\omega)| = \frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} = \frac{1}{\sqrt{(1-u)^2+4\zeta^2 u}}$$

$$u = \omega/\omega_n = 1$$

$$H_R(j\omega)$$

$$\frac{V_R}{V_{in}} = \frac{Z_R}{Z_L + Z_R + Z_C} = \frac{R}{j\omega L + R + 1/j\omega C} = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{j\omega R/L}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{2\zeta\omega_n j\omega}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = H_C(j\omega)\ 2\zeta\omega_n\ j\omega$$

$$Lm\ H_R(j\omega) = Lm\ H_C(j\omega) + Lm\ (2\zeta\omega_n\ j\omega), \quad \angle H_R(j\omega) = \angle H_C(j\omega) + \angle(2\zeta\omega_n\ j\omega)$$

$$20 \log_{10} 2\zeta\omega_n^2$$

90°

$$R + j(\omega L - 1/\omega C)$$

$$j\omega L = 1/j\omega C$$

$$\omega = \omega_n = 1/\sqrt{LC}$$

$$|H_R(j\omega)|$$

V_L

$$H_L(j\omega)$$

$$\frac{V_L}{V_{in}} = \frac{Z_L}{Z_L + Z_R + Z_C} = \frac{j\omega L}{j\omega L + R + 1/j\omega C} = \frac{(j\omega)^2 LC}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{(j\omega)^2}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{(j\omega)^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = H_C(j\omega)\ (j\omega_n)^2$$

$$Lm\ H_L(j\omega) = Lm\ H_C(j\omega) + Lm\ (j\omega)^2, \quad \angle H_L(j\omega) = \angle H_C(j\omega) + \angle(j\omega)^2$$

$$20 \log_{10} \omega_n^2$$

$$\omega_n = 100$$

$$\zeta = 0.05$$

$$20\log_{10}(2\zeta\omega_n^2) = 20\log_{10} 1000 = 60$$

$$20 \log_{10}(\omega_n^2) = 20 \log_{10} 10,000 = 80$$

$$H(j\omega) = -\frac{Z_2(j\omega)}{Z_1(j\omega)} = -\frac{R_2||1/j\omega C_2}{R_1 + 1/j\omega C_1} = -\frac{R_2/(1 + j\omega R_2 C_2)}{(1 + j\omega R_1 C_1)/j\omega C_1} = -\frac{j\omega\tau_3}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

$$\tau_1 = R_1 C_1$$

$$\tau_2 = R_2 C_2$$

$$\tau_3 = R_2 C_1$$