Chapter 20: Special-Purpose Electric Machines – Instructor Notes

The content of Chapter 20 is somewhat unusual for a textbook of this nature. The intent of this chapter is to provide a reasonably quantitative overview of the operation of small electric machines (mostly motors). In many practical industrial applications, ranging from servos for robots, to drug delivery systems, to actuation devices for control systems, to manufacturing equipment, to fluid power systems, small motors find widespread application.

Section 20.1 discusses the brushless DC motor, including the basics of the electronic circuits that make its operation possible. The second section introduces the stepper motor and its drive. In section 20.3, the switched reluctance machine is introduced; this is a new section in the third edition, motivated by the increasing interest in this family of machines for industrial applications. Next, single phase AC motors are discussed in Section 20.4, starting with the universal motor, and continuing with a classification of single phase induction motors, which includes split-phase, capacitor-type and shaded-pole motors. The presentation detail is sufficient to permit quantitative analysis of these motors using circuit models. The final section, 20.5, on motor selection and application, introduces some of the basic ideas behind motor selection and performance calculations. This section, which describes calculations related to reflected load inertias in the presence of mechanical gear reductions, and calculations of acceleration, torque, efficiency, and thermal loading, could be covered at any point in Chapter 17 or 20, even if the material in Sections 20.1-20.4 is not covered.

The examples given in the chapter are supplemented forty homework problems, some of which are extensions of the examples presented in the text. Problems 20.5 and 20.9 require some background in digital logic circuits (Chapter 13); problems 20.7, 20.36 and 20.3 and 20.38 7 require some background in thermal system dynamics; all remaining problems can be solved strictly on the basis of the material covered in the chapter.
The 5th Edition of this book includes 3 new problems, increasing the end-of-chapter problem count from 37 to 40.

Learning Objectives

1. Understand the basic principles of operation of brushless DC motors, and the trade-offs between these and brush-type DC motors. *Section 20.1.*
2. Understand the operation and basic configurations of step motors, and understand step sequences for the different classes of step motors. *Section 20.2.*
3. Understand the operating principles of switched-reluctance machines. *Section 20.3*.
4. Classify and analyze single-phase AC motors, including the universal motor and various types of single-phase induction motors, using simple circuit models. *Section 20.4.*
5. Outline the selection process for an electric machine given an application; perform calculations related to load inertia, acceleration, efficiency, and thermal characteristics. *Section 20.5.*

Section 20.1: Brushless DC Motors

Problem 20.1

Solution:

Known quantities:

A permanent magnet six-pole two-phase synchronous machine. $\lambda_m = 0.1 V \cdot s$.

Find:

The amplitude of the open-circuit phase voltage measured when the rotor is turned at $\lambda = 60\, rev/\sec$.

Assumptions:

None.

Analysis:

We know that

$$\lambda_m = 0.1 V \cdot s$$
$$p = 6$$
$$m = 2$$
$$\omega_m = 60\, rev/s = 2\pi \times 60\, rad/s$$

Let flux linkage $\lambda = \lambda_m \sin \omega t$, where

$$\omega = \frac{p}{2}\omega_m = 3 \times 60 = 180\, rev/s$$
$$= 2\pi \times 180\, rad/s = 360\pi\, rad/s$$

Then, the generated voltage:

$$V = e = \left|\frac{d\lambda}{dt}\right| = \frac{d(\lambda_m \sin \omega t)}{dt} = \omega\lambda_m \cos \omega t$$
$$= V_m \cos \omega t$$
$$V_m = \omega\lambda_m = 360\pi \times 0.1 = 113.1 V$$

Problem 20.2

Solution:

Known quantities:
A four-pole two-phase brushless dc motor. $n = 3600\, rev/\min$. The open-circuit voltage across one of the phases is $50V$.

Find:
λ.
The no-load rotor speed ω in rad/s when the mechanical source is removed and $V_a = \sqrt{2}25\cos\theta, V_b = \sqrt{2}25\sin\theta, where\, \theta = \omega_e t$.

Assumptions:
None.

Analysis:
We know that

$$p = 4$$
$$m = 2$$
$$\omega_m = 3600\, rev/\min$$
$$V_n = 50\,V$$

a)
let $e = V = \sqrt{2}V\sin\theta = \sqrt{2}V\sin\omega t$

$$\omega = \frac{p}{2}\omega_m \cdot 2\pi\, rad/\min = 2\times 60\times 2\pi\, rad/s$$

$$\lambda = \int_0^t e dt = \int_0^t \sqrt{2}V\sin\omega t dt = \frac{\sqrt{2}}{\omega}V\cos\omega t$$
$$= \frac{\sqrt{2}}{240\pi}50\cos 240\pi t = 0.094\cos\omega t$$

b)
Symmetric voltages in symmetric windings produce a rotational field in voltage with frequency f_s.
Let $f_s = 3600\, rev/\min$. Then, the rotor speed is

$$\omega_m = \frac{\omega}{p/2} = \frac{3600}{2} = 1800\, rev/\min = \frac{2\pi}{60}\times 1800\, rad/s = 60\pi\, rad/s$$

Problem 20.3

Solution:

Known quantities:

$T_1 = T_3 = 1$ s (see Figure 18.7); maximum motor rpm, $n_{max} = 1{,}800$ rev/min.

Find:

T_2.

Assumptions:

The motor covers 0.5 m in 100 revolutions.

Analysis:

The maximum rotational velocity of the motor is: $v = n_{max}/60 = 1{,}800/60 = 30$ rev/s. Using the expression derived in Example 18.2, we know that the maximum motor rotational velocity is:

$$v = \frac{d}{\left(\frac{1}{2}T_1 + T_2 + \frac{1}{2}T_3\right)}$$

and we can calculate T_2 as follows: $T_2 = \frac{d}{v} - \frac{1}{2}T_1 - \frac{1}{2}T_3 = \frac{100\text{ rev}}{30\text{ rev/s}} - 1\text{ s} = 2.33\text{ s}$

Thus, the total trapezoidal profile has been shortened by 2/3 s.

Problem 20.4

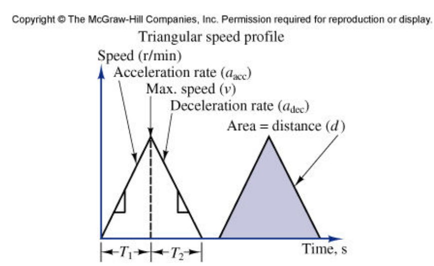
Solution:

Known quantities:

Desired load motion profile (Figure P20.4). The motor covers 0.5 m (100 revolutions) in 3 s.

Find:

Maximum motor speed, acceleration and deceleration times.



Assumptions:

Assume a triangular speed profile..

Analysis:

To simplify the analysis, choose a symmetrical speed profile; thus, the motor will accelerate for 1.5 s and decelerate for 1.5 s, or $T_1 = T_2 = 1.5$ s.
Using the results of Example 18.2, if we set the flat portion of the speed profile (T_2 in Example 18.2) to zero, we can write an expression for the total motor travel.

$$d = v\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right)$$

and calculate the maximum motor speed to be:

$$v = \frac{d}{\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right)} = \frac{100\text{ rev}}{\left(\frac{1}{2}1.5 + \frac{1}{2}1.5\right)\text{ s}} = 66.67\text{ rev/s}$$

which corresponds to $n_{max} = 66.67 \times 60 = 4{,}000$ rev/min.

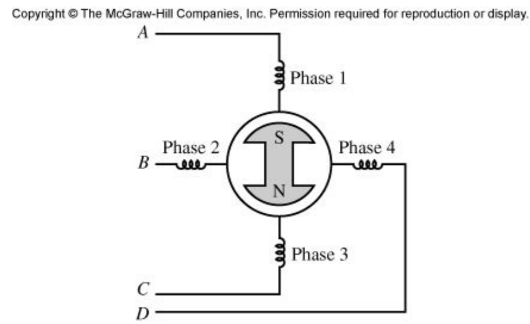
Section 20.2: Stepping Motors

Problem 20.5

Solution:

Known quantities:

Variable-reluctance step motor of Example 20.4 (Figure 20.11)



Find:

Design a logic circuit to achieve the step sequence given in Table 20.4 (see below)

Assumptions:

Hint: Use a counter and logic gates

Analysis:

Table 20.4: Current Excitation Sequence for VR Step Motor

S_A	S_B	S_C	S_D	Rotor Position
1	0	0	0	0°
1	1	0	0	45°
0	1	0	0	90°
0	1	1	0	135°
0	0	1	0	180°
0	0	1	1	225°
0	0	0	1	270°
1	0	0	1	315°
1	0	0	0	360°

There are eight possible configurations for the motor. Hence, a 3-bit binary counter was chosen that pulses every 45°. The table below lists the corresponding logic for the binary counter.

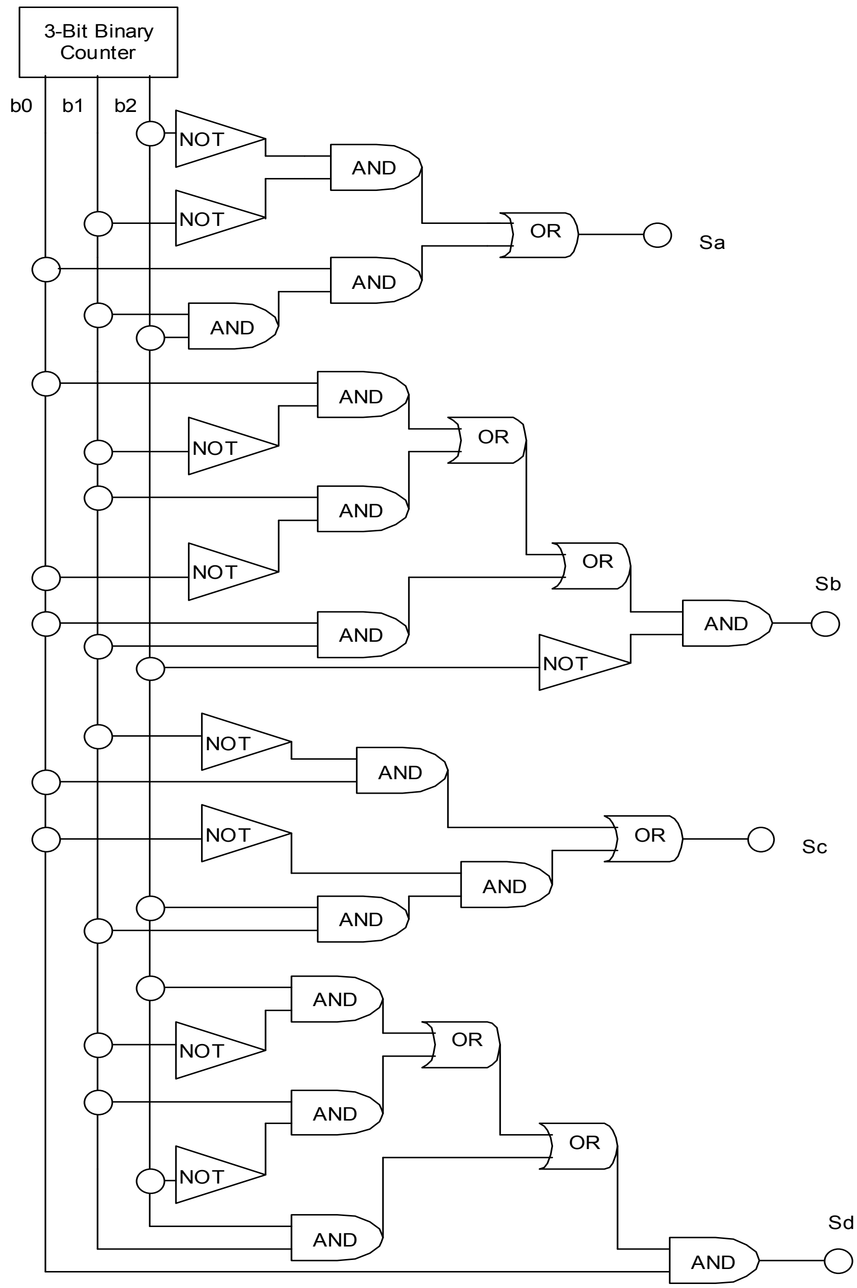
Binary Counter			Step Motor Response				
b_0	b_1	b_2	S_A	S_B	S_C	S_D	Rotor Position
0	0	0	1	0	0	0	0°
1	0	0	1	1	0	0	45°
0	1	0	0	1	0	0	90°
1	1	0	0	1	1	0	135°
0	0	1	0	0	1	0	180°
1	0	1	0	0	1	1	225°
0	1	1	0	0	0	1	270°
1	1	1	1	0	0	1	315°
0	0	0	1	0	0	0	360°

Next, convert the truth table to a logical expression for each of the outputs:

$$S_A = \overline{b}_0 \cdot \overline{b}_1 \cdot \overline{b}_2 + b_0 \cdot \overline{b}_1 \cdot \overline{b}_2 + b_0 \cdot b_1 \cdot b_2 = \overline{b}_1 \cdot \overline{b}_2 \left(\overline{b}_0 + b_0\right) + b_0 \cdot b_1 \cdot b_2$$

$$S_A = \overline{b}_1 \cdot \overline{b}_2 + b_0 \cdot b_1 \cdot b_2$$
$$S_B = b_0 \cdot \overline{b}_1 \cdot \overline{b}_2 + \overline{b}_0 \cdot b_1 \cdot \overline{b}_2 + b_0 \cdot b_1 \cdot \overline{b}_2 = \overline{b}_2 \left(b_0 \cdot \overline{b}_1 + \overline{b}_0 \cdot b_1 + b_0 \cdot b_1\right)$$
$$S_C = b_0 \cdot b_1 \cdot \overline{b}_2 + \overline{b}_0 \cdot \overline{b}_1 \cdot b_2 + b_0 \cdot \overline{b}_1 \cdot b_2 = \overline{b}_1 \cdot b_2 \left(\overline{b}_0 + b_0\right) + b_0 \cdot b_1 \cdot \overline{b}_2$$
$$S_C = \overline{b}_1 \cdot b_2 + b_0 \cdot b_1 \cdot \overline{b}_2$$
$$S_D = b_0 \cdot \overline{b}_1 \cdot b_2 + \overline{b}_0 \cdot b_1 \cdot b_2 + b_0 \cdot b_1 \cdot b_2 = b_2 \left(b_0 \cdot \overline{b}_1 + \overline{b}_0 \cdot b_1 + b_0 \cdot b_1\right)$$

From these four expressions, a logic circuit diagram can be made for each individual output, and tied together to achieve the desired step response.



Problem 20.6

Solution:

Known quantities:
PM stepper motor with 6 poles, bipolar supply,

Find:
Smallest achievable step size.

Assumptions:
None

Analysis:
With reference to Example 20.3, we see that the half-step sequence for the 2-phase 4-pole motor leads to 45-degree steps. The addition of two poles will reduce the step size by 50%, resulting in 30-degree steps.

Problem 20.7

Solution:

Known quantities:
J_m, J_L, D, T_f.

Find:
The dynamic equation for a stepping motor coupled to a load.

Assumptions:
None.

Analysis:
The equation will have the following form:

$$V = Ri + L\frac{di}{dt} + K_E\omega$$

$$T = K_T i = (J_m + J_L)\frac{d\omega}{dt} + D\omega + T_F + T_L$$

Problem 20.8

Solution:

Known quantities:

A hybrid stepper motor capable of 18° steps.

Find:

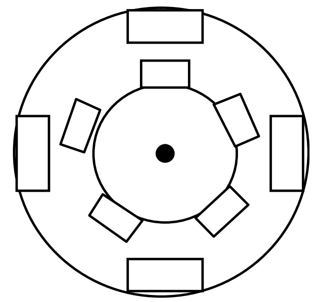
Sketch the rotor-stator configuration of the motor.

Assumptions:

None.

Analysis:

The rotor and stator configuration is shown below:



The motor has 5 rotor teeth and 4 stator teeth (two phases).

Problem 20.9

Solution:

Known quantities:

Shown in Check Your Understanding following Example 20.4.

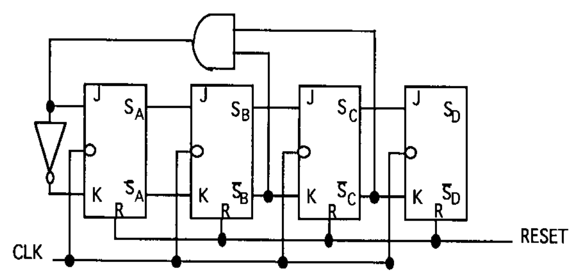
Find:

A binary counter and logic gates to implement the stepping motor binary sequence.

Assumptions:

None.

Analysis:



Problem 20.10

Solution:

Known quantities:

A two-phase permanent magnet stepper motor has 50 rotor teeth. When driven at $\omega = 100\, rad/s$, the measured open circuit phase peak-to-peak voltage is $25V$.

Find:

a) Calculate λ
b) Express the developed torque when $i_a = 1A$ and $i_b = 0$.

Assumptions:

The winding resistance is 0.1Ω.

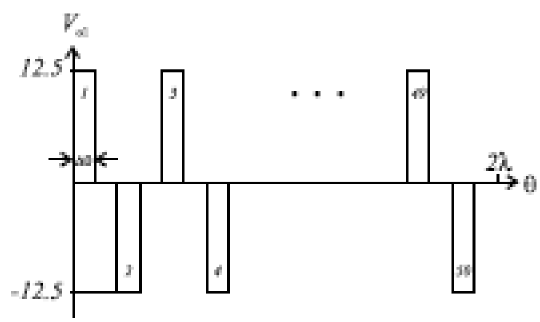
Analysis:

a)
We know that

$$p = 50$$
$$m = 2$$
$$\omega_m = 100\, rad/s$$

Strikes per rev.:

$$N = p \cdot m = 100$$



$$\Delta\theta = \frac{2\pi}{N} = \frac{2\pi}{100} = \frac{2\pi}{50}$$

$$\lambda = \int_0^t e dt = \int_0^{\frac{\pi}{50}} \frac{V_0}{\omega_m} d\theta$$

$$= \frac{V_0}{\omega_m} \cdot \frac{\pi}{50} = \frac{12.5}{100} \cdot \frac{\pi}{50} = 0.00785 V \cdot s$$

b)
Let the winding resistance be represented by R_w, then

$$V = k_a \omega_m + R_w I$$
$$T = k_T I$$

where $k_a = k_T = \frac{V - R_w I}{\omega_m}$

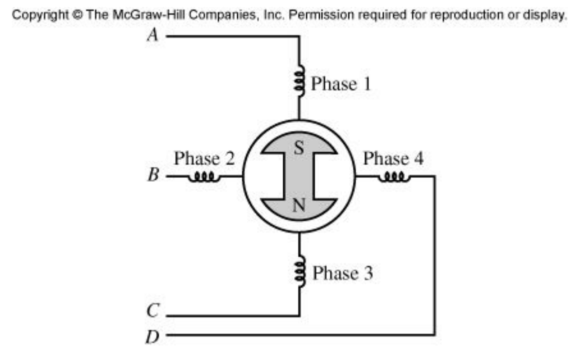
Then $T = k_a I = \frac{V - R_w I}{\omega_m} N \cdot m$

Problem 20.11

Solution:

Known quantities:

The schematic diagram of a four-phase, two-pole PM stepper motor is shown in Figure P20.11. The phase coils are excited in sequence by means of a logic circuit.



Find:

The no-load voltage of the generator and terminal voltage at half load.

Assumptions:

The logic schedule for full-stepping of this motor.
The displacement angle of the full step .

Analysis:

a)

For full step clockwise rotation is:

$$Phase1 \rightarrow phase4 \rightarrow phase3 \rightarrow phase2 \rightarrow phase1$$

b)

The displacement angle of the full step sequence is 90°.

Problem 20.12

Solution:

Known quantities:

A PM stepper motor provides a full-step angle of 15°.

Find:

The number of stator and rotor poles.

Assumptions:

None.

Analysis:

The motor will require 24 stator teeth and 2 rotor teeth.

Problem 20.13

Solution:

Known quantities:

A bridge driver scheme for a two-phase stepping motor is as shown in Figure P20.13.

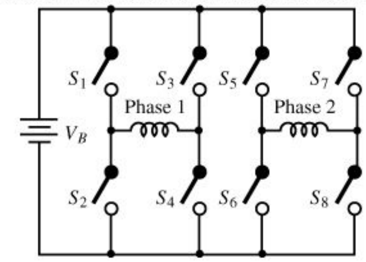
Find:

The excitation sequences of the bridge operation.

Assumptions:

None.

Analysis:



Clock state	Reset	1	2	3	4	5	6	7	8
S_1									
S_2									
S_3									
S_4									
S_5									
S_6									
S_7									
S_8									

CK	R	1	2	3	4	5	6	7	8
S1	1	1	0	0	0	0	0	1	1
S2	0	0	0	1	1	1	0	0	0
S3	0	0	0	1	1	1	0	0	0
S4	1	1	0	0	0	0	0	1	1
S5	0	1	1	1	0	0	0	0	0
S6	0	0	0	0	0	1	1	1	0
S7	0	0	0	0	0	1	1	1	0
S8	0	1	1	1	0	0	0	0	0

Where "1" means switch is closed.

Problem 20.14

Solution:

Known quantities:

A PM stepper motor provides a full-step angle of 15°. It is used to directly drive a $0.100\,in.$ lead screw.

Find:

a) The resolution of the stepper motor in steps/revolution.

b) The distance the screw travels in inches for each step 15° of motor.

c) The number of full 15° steps required to move the lead screw and the stepper motor shaft through 17.5 revolution.

d) The shaft speed (in rev/min) when the stepping frequency is $220\,pps$.

Assumptions:

None.

Analysis:

a)

$$steps/revolution = \frac{360}{15} = 24$$

b)

$$d = 0.1'' \times \frac{15^\circ}{360^\circ} = 0.0042\,''$$

c)

$$steps = 175\,rev \times 24\frac{steps}{rev} = 420\,steps$$

d)

$$\#\,pulses = \#\,steps$$

$$\Rightarrow n_{SH} = 220\frac{steps}{s} \times \frac{1\,rev}{24\,steps} \times \frac{60\,s}{\min} = 550\,rpm$$

Section 20.4: Single-Phase AC Motors

Problem 20.15

Solution:

Known quantities:

The motor data are the following:

$\frac{3}{4} hp, 900\, rev/\min$

$1\frac{1}{2} hp, 3600\, rev/\min$

$\frac{3}{4} hp, 1800\, rev/\min$

$1\frac{1}{2} hp, 6000\, rev/\min$

Find:

Whether the following motors are integral- or fractional-horse power motors.

Assumptions:

None.

Analysis:

a)

The power is $0.75\frac{1800}{900} = 1.5\, hp$.

Integral.

b)

The power is $1.5\frac{1800}{3600} = 0.75\, hp$.

Fractional.

c)

The power is $0.75\frac{1800}{1800} = 0.75\, hp$.

Fractional.

d)

The power is $1.5\frac{1800}{6000} = 0.45\, hp$.

Fractional.

Problem 20.16

Solution:

Known quantities:

$F_1 = F_{1(peak)} \cos\theta, \qquad F_{1(peak)} = F_{1(\max)} \cos\theta,$.

Find:

The expression for F_1 and verify that for a single-phase winding, both forward and backward components are present.

Assumptions:

None.

Analysis:

The stator mmf F_1 can be expressed as:

$$\begin{aligned}
F_1 &= F_{1\max} \cos(\omega t) \cos\theta \\
&= \frac{1}{2} F_{1\max} \cos\theta \cos(\omega t) - \frac{1}{2} F_{1\max} \cos\theta \cos(\omega t) \\
&\qquad + \frac{1}{2} F_{1\max} \cos\theta \cos(\omega t) + \frac{1}{2} F_{1\max} \cos\theta \cos(\omega t) \\
&= F_{CW} + F_{CCW}
\end{aligned}$$

where:

F_{CW} is a clockwise-rotating mmf.

F_{CCW} is a counter clockwise-rotating magnetic mmf.

Problem 20.17

Solution:

Known quantities:

A $200\,V, 60\,Hz, 10\,hp$ single-phase induction motor operates at an efficiency of 0.86 a power factor of 0.9.

Find:

The capacitor that should be placed in parallel with the motor so that the feeder supplying the motor will operate at unity power factor.

Assumptions:

None.

Analysis:

We have:

$$P_{out} = 746 \times 10 = 7460\,W$$

$$P_{in} = \frac{P_{out}}{eff.} = 8674.4\,W$$

From $P_{in} = V_S I_S \cos\theta_S = 8674.4\,W$, we have:

$$|I_S| = 48.2\,A, \qquad \theta = 25.84^\circ \ lagging$$

Therefore,

$$I_S = 48.2\angle -25.84^\circ = 43.38 - j21.01\,A$$

To get unit power factor, $I_C = j21.01$

From $j21.01 = \dfrac{-200}{-j\dfrac{1}{\omega C}}$, $\omega = 377\,rad/s$, we have:

$$C = 278.6\,\mu F$$

Problem 20.18

Solution:

Known quantities:

A $230\,V, 50\,Hz$ single-phase two-pole induction motor operates at 3 percent slip.

Find:

The slip in the opposite direction of rotation. Find the speed of the motor in the normal direction of rotation.

Assumptions:

None.

Analysis:

For a 2-pole machine, the synchronous speed is $3000\,rev/\min$ for an excitation frequency of $50\,Hz$. From $s = 0.03$, the slip in the opposite direction of rotation is 0.97, the motor speed is $2910\,rev/\min = 304.7\,rad/s$.

Problem 20.19

Solution:

Known quantities:

A stepper motor with a 15° step angle operates in one-phase excitation mode.

Find:

The amount of time to take for the motor to rotate through $28\,rev$ when the pulse rate is $180\,pps$.

Assumptions:

None.

Analysis:

$\omega = 180 \times 15 = 2700^\circ / s = 7.5 rev/s t = \dfrac{28 rev}{7.5\, rev/s} = 3.73 s$

Problem 20.20

Solution:

Known quantities:

A $\frac{1}{4} hp, 110 V, 60 Hz$, four-pole capacitor-start motor has the following parameters:

$$R_S = 2.02\Omega \quad X_S = 2.8\Omega R_R = 4.12\Omega \quad X_R = 2.12\Omega X_m = 66.8\Omega \quad s = 0.05$$

Find:

a) The stator current.
b) The mechanical power.
c) The rotor speed.

Assumptions:

None.

Analysis:

$$0.5 Z_b = 0.991 + j1.057 = 0.5(R_b + jX_b)$$

a) $$0.5 Z_f = 15.93 + j20.07 = 0.5(R_f + jX_f)$$

$$Z_{in} = 18.94 + j23.93 = 30.52\angle 51.64^\circ \therefore I_1 = \frac{V_1}{Z_{in}} = 3.60\angle -51.64^\circ A$$

b) $$P_f = I_1^2(0.5R_f) = 207.0W P_b = I_1^2(0.5R_b) = 12.84W P_{mech} = (1-s)(P_f - P_b) = 184.45W$$

c) For a 4-pole machine, $\omega_s = 188.5\, rad/s$, $\omega_m = 179.1\, rad/s$

Thus, the rotor speed is: $179.1\, rad/s = 1710\, rev/\text{min}$

Problem 20.21

Solution:

Known quantities:

A $\frac{1}{4} hp, 110 V, 60 Hz$, four-pole, single-phase induction motor has the following parameters:

$$R_S = 1.86\Omega \quad X_S = 2.56\Omega \quad R_R = 3.56\Omega \quad X_R = 2.56\Omega \quad X_m = 53.5\Omega \quad s = 0.05$$

Find:

The mechanical power output.

Analysis:

$$0.5 Z_b = 0.830 + j1.248 \quad 0.5 Z_f = 12.41 + j16.98$$

$$Z_{in} = 15.1 + j20.79 = 25.7\angle 54.0^\circ \quad I_1 = \frac{V_1}{Z_{in}} = 4.28\angle -54.0^\circ A$$

$$P_f = I_1^2(0.5R_f) = 227.5W \quad P_b = I_1^2(0.5R_b) = 15.2W \quad P_{mech} = (1-s)(P_f - P_b) = 201.68W$$

Problem 20.22

Solution:

Known quantities:

A $115V, 60\,Hz$, four-pole, one-phase induction motor has the following parameters:

$$R_S = 0.5\,\Omega \qquad X_S = 0.4\,\Omega$$
$$R_R = 0.25\,\Omega \qquad X_R = 0.4\,\Omega$$
$$X_m = 35\,\Omega$$

Find:

The input current and developed torque when the motor speed is $1{,}730\,rev/\min$.

Assumptions:

None.

Analysis:

The synchronous speed is $1{,}800\,rev/\min$ for $1{,}730\,rev/\min$.

$(1-s) = 0.961$, therefore the slip $s = 0.039$. We have:

$$0.5Z_b = 0.064 + j0.2$$
$$0.5Z_f = 3.034 + j0.747$$
$$Z_{in} = 3.842\angle 20.52^\circ$$
$$I_S = 29.9\angle -20.52^\circ\ A$$
$$P_f = 2717.5W$$
$$P_b = 57.22W$$
$$P_{mech} = 2660.3W$$
$$\omega_m = 181.2\,rad/s$$

The torque developed is:

$$T_{dev} = \frac{2551.6}{181.2} = 14.68\,N \cdot m$$

Problem 20.23

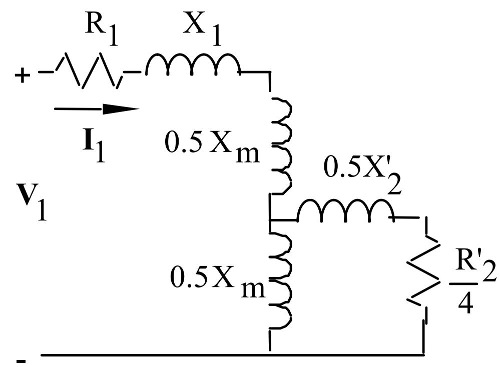
Solution:

Known quantities:
No-load test of a single-phase induction motor at rated voltage and rated frequency.

Find:
The equivalent circuit of a single-phase induction motor for the no-load test.

Assumptions:
None.

Analysis:
At no load, $s \approx 0$. The circuit model is shown below:



Problem 20.24

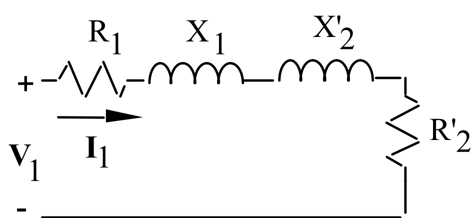
Solution:

Known quantities:
The locked-rotor test of the single-phase induction motor.

Find:
The equivalent circuit.

Assumptions:
Neglect the magnetizing current.

Analysis:
For locked rotor, $\omega_m = 0$, $s = 1$. The circuit is shown below:



Problem 20.25

Solution:

Known quantities:

A $\frac{1}{8} hp, 115 V$ two-pole universal motor has the effective resistances of the armature and series field as $4\,\Omega$ and $6\,\Omega$. The output torque is $0.17 N \cdot m$ when the motor is drawing rated current of $1.5 A$ at a power factor of 0.88 at rated speed.

Find:

a) The full-load efficiency.
b) The rated speed.
c) The full-load copper losses.
d) The combined windage, friction, and iron losses.
e) The motor speed when the rms current is $0.5 A$.

Assumptions:

Phase differences and saturation.

Analysis:

From $P_f = 0.88$, we have $\theta = -28.4^\circ$ and $I_S = 1.5\angle -28.4^\circ\ A$.

The rated speed is: $\frac{1/8 \times 746}{0.17} = 548.53\, rad/s$

a) $P_{in} = 151.8 W$ and the efficiency is: $eff = \frac{93.25}{151.8} = 61.43\%$

b) The speed is $5238.1\, rev/\min$.

c) The copper loss is: $1.5^2 \times 10 = 22.5 W$

d) Other loss will be: $151.8 - 22.5 - 93.25 = 36.05 W$

e) $$P_{in} = (115)(0.5 + (0.5^2 \times 10) + P_{out})(0.88)$$
$$= 12.05 W$$

Assume T is proportional to I^2.

$$T_{new} = (\frac{0.5}{1.5})^2 (0.17) = 0.019\, N \cdot m$$

$$\omega = \frac{12.05}{0.019} = 637.9\, rad/s$$

$$n = 6091.9\, rpm$$

Problem 20.26

Solution:

Known quantities:

A $240V, 60Hz$, two-pole universal motor operates at $12{,}000\,rev/\min$ on full load and draws a current of $6.5A$ at a power factor of 0.94 lagging. The series field-winding impedance is $4.55 + j3.2\Omega$ and the armature circuit impedance is $6.15 + j9.4\Omega$

Find:

a) The back emf of the motor.
b) The mechanical power developed by the motor.
c) The power output if the rotational loss is $65W$.
d) The efficiency of the motor.

Assumptions:

None.

Analysis:

From $P_f = 0.94(lagging)$, we have

$$\theta = -19.95^\circ \text{ and } I_S = 6.5\angle -19.95^\circ\ A.$$

a)

$$E_b = 146.68 - j53.25$$
$$= 156.05\angle -19.95^\circ\ V$$

b)

$$P_{dev} = I_S E_b \cos 0^\circ = 1014.3W$$

c)

$$P_{out} = 1014.3 - 65 = 949.3W$$

d)

$$P_{in} = 1466.4W$$
$$eff = 64.7\%$$

Problem 20.27

Solution:

Known quantities:
A single-phase motor is drawing $20\,A$ from a $400\,V, 50\,Hz$ supply. The power factor is 0.8 lagging.

Find:
The value of capacitor connected across the circuit to raise the power factor to unity.

Assumptions:
None.

Analysis:
$V_S = 400\angle 0^\circ, \theta = -36.9^\circ$, therefore,

$$I_S = 20\angle 36.9^\circ = 16 - j12$$

For a unity power factor, $I_C = j12$.

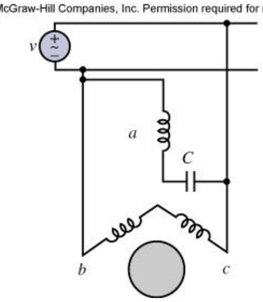
We have $12 = 400\omega C, \omega = 314.16\, rad/s$.

Therefore, $C = 95.5\,\mu F$

Problem 20.28

Solution:

Known quantities:
A way of operating a three-phase induction motor as a single-phase source is shows in Figure P20.28.



Find:
Whether the motor will work. Explain why or why not.

Assumptions:
None.

Analysis:
It will work. The b and c windings will produce a magnetic field similar to a single phase machine, that is, two components rotating in opposite directions and the a winding would act as a starting winding. The phase shift provided by the capacitor is needed to provide a nonzero starting torque.

Problem 20.29

Solution:

Known quantities:

A $\frac{1}{4}hp$ capacitor-start motor with its output adjusted to rated value.

$E = 115\,volts;$ $I = 3.8\,A;$ $P = 310\,W;$ $rev/\min = 1725.$

Find:

a) Efficiency.
b) Power factor.
c) Torque in pound-inches.

Assumptions:

None.

Analysis:

a)

$$P_{out} = \frac{1}{4} \times 746 = 186.5\,W$$

$$eff = \frac{P_{out}}{P_{in}} = 0.602 = 60.2\%$$

b)

$$pf = \frac{P}{VI} = \frac{310}{(115)(3.8)} = 0.709\,lagging$$

c)

$$T = 7.04\frac{P_{out}}{n_R} = 7.04\frac{186.5}{1725} = 0.761\,lb \cdot ft \times 12\frac{in}{ft}$$
$$= 9.13\ lb \cdot in$$

Section 20.5 Motor Selection and Applications

Problem 20.30

Solution:

Known quantities:

The tasks are the following:

a) Vacuum clearner
b) Refrigerator.
c) Air conditioner compressor.
d) Air conditioner fan.
e) Variable-speed sewing machine.
f) Clock.
g) Electric drill.
h) Tape drive.
i) X-Y plotter.

Find:

The type of motor that can perform the above tasks.

Assumptions:

None.

Analysis:

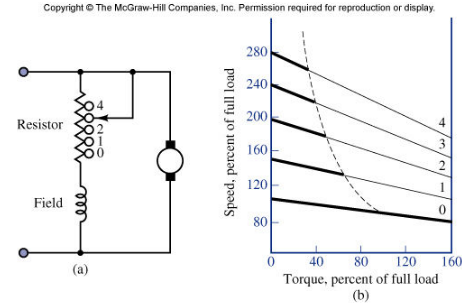
The universal motor speed is easily controlled and thus it would be used for variable speed, that is, (e) and (g). The vacuum cleaner motors are often universal motors. This motor could also be used for the fan motors.
A single-phase induction motor is used for (b) and (c).
The clock should use a single-phase synchronous motor.
The tape drive would be a single-phase synchronous motor also.
An X-Y plotter uses a stepper motor.

Problem 20.31

Solution:

Known quantities:

A $5\,hp, 1150\,rev/\min$ shunt motor. The speed control by means of a tapped field resistor is shown Figure P20.31.



Find:

The speed of the motor and the torque available at the maximum permissible load with the tap at position 3.

Assumptions:

None.

Analysis:

$$n = 230\% \times 1150 = 2645\,rpm \quad T = 40\% \times T_{rated} = (0.4)\frac{(33{,}000)(5)}{(2\pi)(1150)} = 9.13\,lb \cdot ft$$

Problem 20.32

Solution:

Known quantities:

The applications are the following:

a) Inexpensive analog electric clock.
b) Bathroom ventilator fan.
c) Escalator which must start under all load conditions.
d) Kitchen blender.
e) Table model circular saw operating at about $3{,}500\,rev/\min$.
f) Hand-held circular saw operating at $15{,}000\,rev/\min$.
g) Water pump.

Find:

The single-phase motor that can apply to the above cases.

Assumptions:

None.

Analysis:

a) reluctance
b) shaded-pole
c) capacitor-start
d) universal
e) permanent split capacitor
f) universal
g) permanent split capacitor

Problem 20.33

Solution:

Known quantities:

The power required to drive a fan varies as a cube of the speed. The motor driving a shaft-mounted fan is loaded to 100 percent of its horsepower rating on the top speed connection.

Find:

The horsepower output in percent of rating at the following speed reduction.
a) 20 percent.
b) 30 percent.
c) 50 percent.

Analysis:

a) $HP = (0.8)^3 = 0.512 = 51.2\%\ \textit{of rated}$

b) $HP = (0.7)^3 = 0.343 = 34.3\%\ \textit{of rated}$

c) $HP = (0.5)^3 = 0.125 = 12.5\%\ \textit{of rated}$

Problem 20.34

Solution:

Known quantities:

An industrial plant has a load of $800\,kW$ at a power factor of 0.8 lagging. A synchronous motor will be used to change them to $200\,kW$ and 0.92.

Find:

The KVA input rating and the power factor at which the motor operates.

Assumptions:

The motor has an efficiency of 91 percent.

Analysis:

$$P_m = \frac{200K}{0.91} = 219.8KW \quad P_{new} = 800K + 219.8K = 1019.8KW$$

$$Q_{new} = P_{new}\tan(\cos^{-1}0.92) = 434.4KVAR \quad Q_{old} = 800K\tan(\cos^{-1}0.8) = 600KVAR$$

$$Q_m = Q_{old} - Q_{new} = 165.6KVAR \quad S_m = \sqrt{P_m^2 + Q_m^2} = 274.8KVA$$

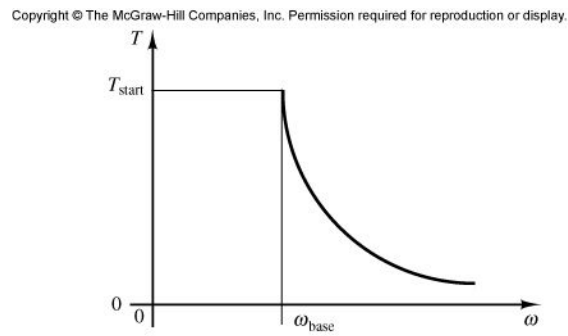
$$pf_m = \cos\left(\tan^{-1}\left(\frac{Q_m}{P_m}\right)\right) = 0.8 \text{ leading}$$

Problem 20.35

Solution:

Known quantities:

An electric machine is controlled so that its torque-speed characteristics exhibit a constant-torque region and a constant-power region as shown in Figure P20.35.
Average efficiency of the electric drive is 87%.
Machine torque is constant at 150 N-m from 0 to 2500 rpm.
The constant power region is from 2500 to 6000 rpm.
Machine drives a constant torque load requiring 75 N-m.



Find:

a) Operating speed of the machine
b) Electric power needed to operate the machine

Assumptions:

None

Analysis:

a) Since the load requires less than the starting torque, the motor should be operating in the constant power regime. The constant power is determined from the starting torque and the base speed:

$$P = \varpi_{base} T_{start} = (2500 r/\min)\frac{(2\pi rad/rev)}{(60 s/\min)}(150 N-m) = 39.3 kW$$

The operating speed is determined from the load torque:

$$\varpi = \frac{P}{T_L} = \frac{(39300 W)}{(75 N-m)} = 523.6 rad/s$$

$$n = (523.6 rad/s)\frac{(60 s/\min)}{(2\pi rad/rev)} = 5000 r/\min$$

b) The electric power needed to operate the machine:

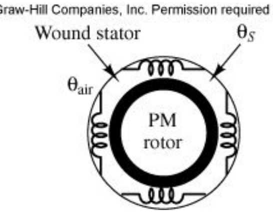
$$P_e = \frac{P}{\eta} = \frac{39.3 kW}{0.87} = 45.1 kW$$

Problem 20.36

Solution:

Known quantities:

The PM synchronous (brushless DC) motor in Figure P20.36
Electrical subsystem parameters: R_S, L_S, k. (motor constant), $V_S(t)$, $I_S(t)$.
Mechanical subsystem parameters: inertia and damping coefficient, J, b.
Thermal subsystem parameters: thermal resistance, specific heat, mass, R_t, c, m.



Find:

Write the differential equations describing the electrothermomechanical dynamics of the systems

Assumptions:

All heat is generated in the stator by the stator current (i.e. the heat generated in the rotor is negligible)
The rotor and stator are at the same temperature and specific heat c
The stator is highly thermally conductive
The dominant heat-transfer term is convection
Overall thermal resistance R_t from stator to air
The motor generates torque according to the equation $T_m = kI_S$
The back emf is equal to $E_b = kw$

Analysis:

Mechanical subsystem

$$J\frac{d\omega(t)}{dt} = T_m - b\omega(t) = kI_S(t) - b\omega(t)$$

Electrical subsystem

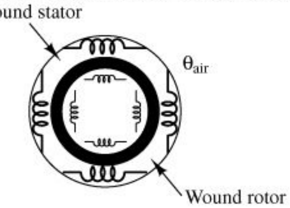
$$V_S(t) = L_S\frac{dI_S(t)}{dt} + R_S I_S(t) + E_b = L_S\frac{dI_S(t)}{dt} + R_S I_S(t) + k\omega(t)$$

Thermal subsystem

$$R_S I_S^2(t) - \frac{\theta_S(t) - \theta_{air}}{R_t} = mc\frac{d\theta_S(t)}{dt}$$

Problem 20.37

Solution:



Known quantities:

The wound separately excited motor in Figure P20.37
Electrical subsystem parameters: R_f, L_f, R_a, L_a, (motor field and armature electrical parameters), k_f. k_a, k_T (motor field and armature constants), $V_S(t)$, $V_f(t)$, $I_a(t)$, $I_f(t)$.
Mechanical subsystem parameters: load inertia, damping coefficient, load torque, J, b, T_L.
Thermal subsystem parameters: $C_{t\text{-rotor}}$, $h_{t\text{-rotor}}$, A_{rotor}(rotor thermal capacitance, film coefficient of heat transfer from rotor surface to air and from air to stator inner surface, rotor and inner stator surface area (assumed equal). $C_{t\text{-stator}}$, $h_{t\text{-stator}}$, A_{stator}(stator thermal capacitance, film coefficient of heat transfer from stator outer surface to air, stator outer surface area.

Find:

Write the differential equations describing the electrothermomechanical dynamics of the systems

Assumptions:

Heat is generated in the stator and the rotor by the respective currents
The stator and rotor are highly thermally conductive
The dominant heat-transfer term is convection through the air gap and to ambient
Heat storage in the air gap is negligible, and the air gap is infinitely thin
The motor generates torque according to the equation $T_m = k_T I_a$
The back emf is equal to $E_b = k_a w$
The stator and rotor each act as a lumped thermal mass

Analysis:

<u>Mechanical subsystem</u>

$$J\frac{d\omega(t)}{dt} = T_m - b\omega(t) - T_L = k_T I_a(t) - b\omega(t) - T_L$$

$$k_T = k_a\phi, \phi = k_f I_f(t), k_T = k_a k_f I_f(t)$$

$$J\frac{d\omega(t)}{dt} = k_a k_f I_f(t) I_a(t) - b\omega(t) - T_L$$

<u>Electrical subsystem</u>

Armature Circuit:

$$V_a(t) = L_a\frac{dI_a(t)}{dt} + R_a I_a(t) + E_b = L_a\frac{dI_a(t)}{dt} + R_a I_a(t) + k_a\omega(t)$$

Field Circuit:

$$V_f(t) = L_f\frac{dI_f(t)}{dt} + R_f I_f(t)$$

<u>Thermal subsystem</u>

Rotor:

$$R_a I_a^2(t) - h_{rotor}A_{rotor}\left(\theta_R(t) - \theta_S(t)\right) = mc_{rotor}\frac{d\theta_R(t)}{dt}$$

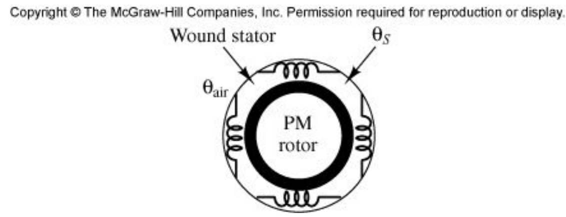
Stator:

$$R_f I_f^2(t) - h_{stator}A_{stator}\left(\theta_S(t) - \theta_{air}\right) + h_{rotor}A_{rotor}\left(\theta_R(t) - \theta_S(t)\right) = mc_{stator}\frac{d\theta_S(t)}{dt}$$

Problem 20.38

Solution:

Known quantities:



The PM synchronous (brushless DC) motor in Figure P20.36
Electrical subsystem parameters: R_S, L_S, k. (motor constant), $V_S(t)$, $I_S(t)$.
Thermal subsystem parameters: thermal resistance, specific heat, mass, R_t, c, m.

Find:

Write the differential equations describing the electrothermomechanical dynamics of the systems

Assumptions:

All heat is generated in the stator by the stator current (i.e. the heat generated in the rotor is negligible)
The rotor and stator are at the same temperature and specific heat c
The stator is highly thermally conductive
The dominant heat-transfer term is convection
Overall thermal resistance R_t from stator to air
The back emf is equal to $E_b = kw$
The brief acceleration transient to get the motor up to speed takes a short time

Analysis:

Electrical subsystem

$$V_S = R_S I_S + L_S \frac{dI_S}{dt} + k_b \omega_m$$

We first solve for I_S; since the motor is at constant speed, we can assume the current to be constant, thus:

$$V_S = R_S I_S + k_b \omega_m$$

$$I_S = \frac{V_S - k_b \omega_m}{R_S}$$

Thermal subsystem

$$q_{in} - q_{out} = q_{stored}$$

$$R_S I_S^2 - \frac{\theta_S - \theta_{air}}{R_t} = mc \frac{d\theta_S}{dt}$$

$$mc \frac{d\theta_S}{dt} + \frac{\theta_S}{R_t} = R_S I_S^2 + \frac{\theta_{air}}{R_t}$$

Hence the final differential equation is:

$$R_t mc \frac{d\theta_S}{dt} + \theta_S = R_t R_S I_S^2 + \theta_{air} = R_t R_S \left(\frac{V_S - k_b \omega_m}{R_S} \right)^2 + \theta_{air}$$

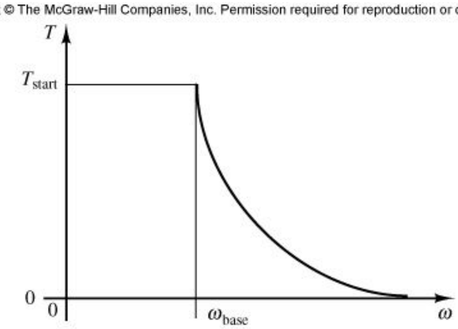
And the time constant $\tau = R_t mc$

Problem 20.39

Solution:

Known quantities:

Torque-speed characteristics as shown in figure 20.39.



Find:

a) Operating speed of electrical machine
b) Electrical power needed to operate the machine.

Assumptions:

Constant torque region and constant power region.

Analysis:

a) In constant torque region, maximum mechanical power EM can provide is

$$P_{Mech} = T \times \omega = 150 \times \frac{2500 \text{ rpm}}{9.55} = 39.3 \text{ KW}$$

In the constant power region, mechanical power EM can provide is a constant, so we can calculate the operating

speed $\omega_{O.P.} = \dfrac{P_{Mech}}{T_{O.P.}} = \dfrac{150 \times \dfrac{2500 \text{ rpm}}{9.55}}{75} = \dfrac{5000 \text{ rpm}}{9.55} = 523.4 \text{ rad/s} = 5000 \text{ rpm}$

b) $P_{Elec} = \dfrac{P_{Mech}}{\eta} = \dfrac{39.3 \text{ KW}}{0.87} = 45.1 \text{ KW}$

Problem 20.40

Solution:

Known quantities:

Load torque as a function of speed, electrical subsystem parameters and value of voltage source.

Find:

The speed of motor and fan.

Assumptions:

Inductance is negligible.

Analysis:

<u>Mechanical subsystem</u>

$$J\frac{d\omega(t)}{dt} = T_m - T_L = k_T I_a(t) - T_L = 2.42 I_a(t) - \left(5 + 0.05 \times \omega(t) + 0.001 \times \omega(t)^2\right)$$

In steady state, $2.42 I_a(t) = \left(5 + 0.05 \times \omega(t) + 0.001 \times \omega(t)^2\right)$

<u>Electrical subsystem</u>

Armature Circuit: $V_a(t) = L_a \dfrac{dI_a(t)}{dt} + R_a I_a(t) + E_b = 0 + R_a I_a(t) + k_a \omega(t) = 0.2 I_a(t) + 2.42\omega(t)$

Solve the 2 equations above, we can have $\omega = 20.4$ rad/s $I = 3.16$ A