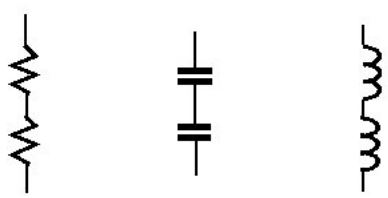
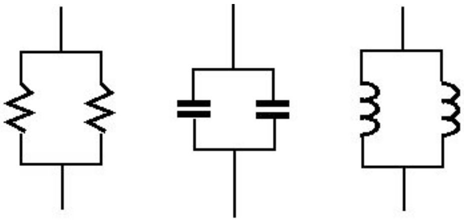
Handout 1: Review

- **Basic Circuit Variables**
  - **Voltage:**
    Voltage is an "across variable" as it measures the difference between two electrical potentials at two points. When a reference point is used (e.g., the "ground"), the voltage at a certain point is defined as the potential difference between this point and the reference point.
  - **Current:**
    Current is a "through variable" as it measures the amount of electrical charge that flow through a pass per unit time.
- **Basic Elements**
  - **Resistor:**
$$V = RI, \quad I = \frac{V}{R}, \quad R = \frac{V}{I}$$
  - **Capacitor:**
$$V = \frac{Q}{C}, \quad i(t) = C\frac{d}{dt}v(t), \quad v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$
  - **Inductor:**
$$V = \frac{W}{Q}, \qquad v(t) = \frac{d}{dt}\psi(t) = L\frac{d}{dt}i(t), \quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$$
- **Series and Parallel Connections**
  - **Series Connections:**



$$R_s = R_1 + \cdots + R_n = \sum_{i=1}^{n} R_i$$



$$L_s = L_1 + \cdots + L_n = \sum_{i=1}^{n} L_i$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \cdots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}, \quad or \quad C_s = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$

If $n = 2$,

$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

– **Parallel Connections:**

$$\frac{1}{R_p} = \frac{1}{R_1} + \cdots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}, \quad or \quad R_p = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

If $n = 2$,

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{L_p} = \frac{1}{L_1} + \cdots + \frac{1}{L_n} = \sum_{i=1}^{n} \frac{1}{L_i}, \quad or \quad L_p = \frac{1}{\sum_{i=1}^{n} \frac{1}{L_i}}$$

If $n = 2$,

$$L_p = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$

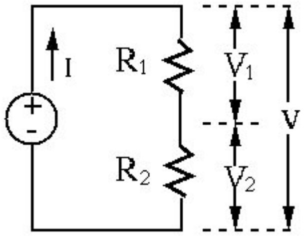
$$C_p = C_1 + C_2$$

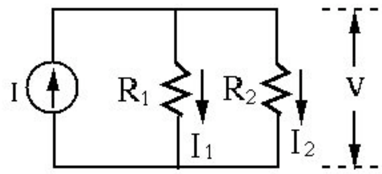
If $n = 2$, we have:

	Resistor	Inductor	Capacitor
v-i relation	$v = R\ i$	$v = L\ \frac{di}{dt}$	$v = \frac{1}{C} \int i\ dt$
Series	$R_s = R_1 + R_2$	$L_s = L_1 + L_2$	$C_s = C_1 C_2 / (C_1 + C_2)$
Parallel	$R_p = R_1 R_2 / (R_1 + R_2)$	$L_p = L_1 L_2 / (L_1 + L_2)$	$C_p = C_1 + C_2$

Q: Why are R and L similar to each other while C is different?

A: Observe how R, L and C are related to voltage v and current i differently.





- **Energy and Power**

$$V = \frac{W}{Q}, \quad W = VQ, \quad P = \frac{dW}{dt} = V\frac{dQ}{dt} = VI = \frac{V^2}{R} = RI^2$$

- **Voltage/Current Dividers**
  - **Voltage Divider**

$$I = \frac{V}{R_1 + R_2}, \quad V_1 = IR_1 = V\ \frac{R_1}{R_1 + R_2}, \quad V_2 = IR_2 = V\ \frac{R_2}{R_1 + R_2}$$

Voltage across R is proportional to its **own** resistance.
When there are more than two resistors in series, we simply have:

$$V_i = IR_i = V\ \frac{R_i}{\sum_j R_j}$$

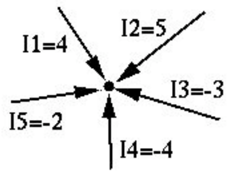
  - **Current Divider**

$$V = I\frac{R_1 R_2}{R_1 + R_2}, \quad I_1 = \frac{V}{R_1} = I\frac{R_2}{R_1 + R_2}, \quad I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$$

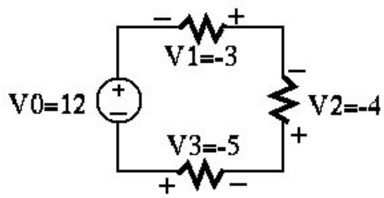
Current through R is proportional to **the other** resistance.
When there are more than two resistors in parallel, we have:

$$V = I\frac{1}{\sum_j 1/R_j}, \quad I_i = \frac{V}{R_i} = I\frac{1/R_i}{\sum_j 1/R_j} = I\frac{G_i}{\sum_j G_j}$$

where $G_i = 1/R_i$ is the *conductance* of the ith resistor.



I1+I2+I3+I4+I5=0



V0+V1+V2+I3=0

- **Kirchoff's Laws**
  - **KCL:** Due to conservation of electric charge, Kirchoff current Law (KCL) states: The algebraic sum of all currents into a node is zero $\sum_k I_k = 0$ (Currents leaving the node take negative values.)
  - **KVL:** Due to conservation of electric energy, Kirchoff voltage Law (KCL) states: The algebraic sum of voltage around a loop is zero $\sum_k V_k = 0$ (Voltages with opposite polarity take negative values.)
- **Reference Point or Ground**

  The voltage between any two nodes in a circuit is the difference between the potentials at the two nodes. However, a particular node in the circuit is usually chosen as the reference point, called the ground. The voltage at any node in the circuit is therefore defined as the voltage between this node and the common ground.