1. (5 points) FoEE 5.59 (just part c), 2) 5pts FoEE 5.61 (just part a)

5.59 Find the Laplace transform of (a) $(2e^{-8t} - e^{-2t})u(t)$, (b) $(6 + 2e^{-6t} - 12e^{-t})u(t)$, (c) $(2 + 3t)e^{-2t}u(t)$, and (d) $e^{-3t}(\cos 4t - \sin 4t)u(t)$.

5.61 Find the inverse Laplace transform of each of the following functions:

(a) $\dfrac{600}{s(s+10)(s+30)}$ (b) $\dfrac{60(s+4)}{s(s+2)(s+12)}$

5.59 c)

$$\mathcal{L}\{(2+3t)e^{-2t}u(t)\}$$

$$= \mathcal{L}\{2e^{-2t}u(t)\} + \mathcal{L}\{3te^{-2t}u(t)\}$$

$$= \frac{2}{s+2} + \frac{3}{(s+2)^2} = \boxed{\frac{2s+7}{s^2+4s+4}}$$

5.61 a) $$\frac{600}{s(s+10)(s+30)} = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+30}$$

$$K_1 = \left.\frac{600}{(s+10)(s+30)}\right|_{s=0} = \frac{600}{300} = 2$$

$$K_2 = \left.\frac{600}{s(s+10)}\right|_{s=-30} = \frac{600}{600} = 1$$

$$K_3 = \left.\frac{600}{s(s+30)}\right|_{s=-10} = \frac{600}{-200} = -3$$

So, $$\mathcal{L}^{-1}\left\{\frac{600}{s(s+10)(s+30)}\right\} = \mathcal{L}^{-1}\left\{\frac{2}{s} + \frac{1}{s+10} + \frac{-3}{s+30}\right\}$$

$$= (2 + e^{-30t} - 3e^{-10t})u(t)$$

3. (5 points) Using Laplace Transforms, find the solution for $\dfrac{d^2x(t)}{dt^2} + 6\dfrac{dx(t)}{dt} + 8x(t) = 2u(t)$;

where $v(0) = 1$ and $v'(0) = -2$

$$\frac{d^2x(t)}{dt} + 6\frac{dx(t)}{dt} + 8x(t) = 2u(t) \qquad x(0)=1 \quad x'(0)=-2$$

$\mathcal{L}$ of both sides:

$$[s^2X(s) - sx(0) - x'(0)] + 6[sX(s) - x(0)] + 8X(s) = \frac{2}{s}$$

$$s^2X(s) - s + 2 + 6sX(s) - 6 + 8X(s) = 2/s$$

$$(s^2+6s+8)X(s) = s + 4 + \frac{2}{s} = \frac{s^2+4s+2}{s}$$

$$X(s) = \frac{s^2+4s+2}{s(s^2+6s+8)} = \frac{s^2+4s+2}{s(s+4)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+2}$$

Partial Fraction Expansion:

$$K_1 = X(s)s\big|_{s=0} \longrightarrow K_1 = \frac{2}{8} = \frac{1}{4}$$

$$K_2 = X(s)(s+4)\big|_{s=-4} \longrightarrow K_2 = \frac{2}{8} = \frac{1}{4}$$

$$K_3 = X(s)(s+2)\big|_{s=-2} \longrightarrow K_3 = \frac{-2}{-4} = \frac{1}{2}$$

So, $$X(s) = \frac{\frac{1}{4}}{s} + \frac{\frac{1}{4}}{s+4} + \frac{\frac{1}{2}}{s+2}$$

$$\boxed{x(t) = \frac{1}{4}\left(1 + e^{-4t} + 2e^{-2t}\right)u(t)}$$

4. (10 points) FoEE 5.73

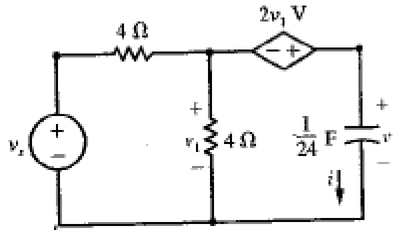
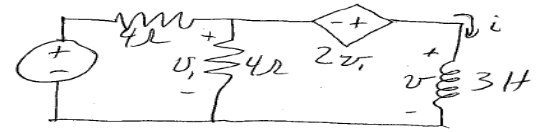


Fig. P5.72

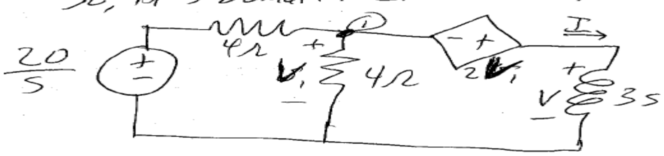
5.73 For the circuit shown in Fig. P5.72, replace the capacitor with a 3-H inductor, and find the step responses $v(t)$ and $i(t)$ when $v_s(t) = 20u(t)$ V.

FoEE 5.73



$v_s = 20u(t)$ V

So, in s-domain: circuit looks like this



KCL at node ①: $\dfrac{\frac{20}{s} - V_1}{4} = \dfrac{V_1}{4} + \dfrac{V}{3s}$

$3s(20/s - V_1) = V_1 3s + 4V$

$60 - 3sV_1 = 3sV_1 + 4V$

$60 = 6sV_1 + 4V$

Note $V_1 = \frac{V}{3}$ $\quad 60 = 6s\frac{V}{3} + 4V = (2s+4)V$

$30 = (s+2)V \Rightarrow V = \frac{30}{s+2}$

$$\boxed{\text{So, } v(t) = 30e^{-2t}u(t) \text{ V}}$$

$I = \frac{V}{3s} = \frac{10}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$

Part. Frac. Exp.: $K_1 = Is\big|_{s=0} \Rightarrow K_1 = 5$

$K_2 = I(s+2)\big|_{s=-2} \Rightarrow K_2 = -5$

So $I(s) = \frac{5}{s} + \frac{-5}{s+2}$

$$\boxed{\text{So, } i(t) = (5 - 5e^{-2t})u(t) \text{ A}}$$

5. (10 points) FoEE 5.84 (just find $v(t)$) (Hint: Notice the initial conditions, see example 5.23 for something similar)

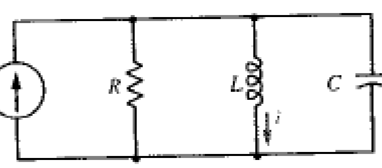
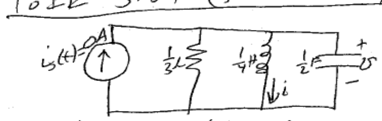


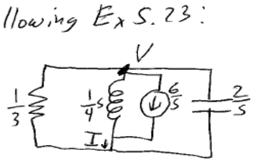
Fig. P5.83

5.84 For the parallel *RLC* circuit shown in Fig. P5.83, suppose that $R = \frac{1}{3}$ Ω, $L = \frac{1}{4}$ H, $C = \frac{1}{2}$ F, and $i_s(t) = 0$ A. Find $v(t)$ and $i(t)$ when $i(0) = 6$ A and $v(0) = 0$ V.

FoEE 5.84 (just find $v(t)$) Following Ex 5.23:



becomes



$i(0) = 6$A, $v(0) = 0$V

KCL: $\dfrac{V}{1/3} + \dfrac{V}{\frac{1}{4}s} + \dfrac{6}{s} + \dfrac{Vs}{2} = 0$

$3V + \frac{4V}{s} + \frac{6}{s} + \frac{Vs}{2} = 0$

$6sV + 8V + s^2V = -12$

$V = \dfrac{-12}{s^2+6s+8} = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+4}$

By Part. Frac. Exp.:

$K_1 = V(s)(s+2)\big|_{s=-2} \Rightarrow K_1 = -6$

$K_2 = V(s)(s+4)\big|_{s=-4}$ $\quad K_2 = 6$

$V(s) = \frac{-6}{s+2} + \frac{6}{s+4}$

$$\boxed{v(t) = -6e^{-2t}u(t) + 6e^{-4t}u(t) \text{ V}}$$

6. (5 points) FoEE 5.103 (Use the fact that in 5.49b from HW8, we already found the transfer function to be: $\frac{V_2}{V_1} = \frac{1}{3s^2+4s+1}$.)

5.103 For the op-amp circuit shown in Fig. P5.49, suppose that $C = 1$ F. Find the step response $v_2(t)$ when $v_1(t) = 3u(t)$ V.

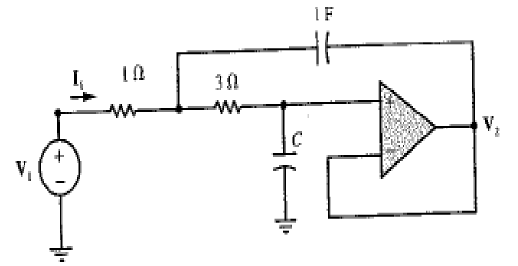


Fig. P5.49

$$V_1(s) = \frac{3}{s}$$

$$\text{So, } V_2(s) = \mathcal{L}^{-1}\left\{\frac{3}{s(3s^2+4s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3}{s(3s+1)(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{K_1}{s} + \frac{K_2}{3s+1} + \frac{K_3}{s+1}\right\}$$

$$K_1 = \left.\frac{3}{(3s+1)(s+1)}\right|_{s=0} = 3$$

$$K_2 = \left.\frac{3}{s(3s+1)}\right|_{s=-1} = \frac{3}{2}$$

$$K_3 = \left.\frac{3}{s(s+1)}\right|_{s=-\frac{1}{3}} = -\frac{27}{2}$$

$$\text{So, } V_2(s) = \frac{3}{s} + \frac{(3/2)}{s+1} - \frac{(27/2)}{3s+1}$$

$$v_2(t) = \mathcal{L}^{-1}\{V_2(s)\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3}{s} + \frac{3/2}{s+1} - \frac{9/2}{s+1/3}\right\}$$

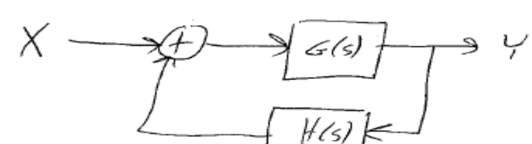
$$= \left(3 + \frac{3}{2}e^{-t} - \frac{9}{2}e^{-\frac{1}{3}t}\right)u(t)$$

7. (0 points) FoEE 5.57

5.57 For the feedback system given in Fig. 5.35 on p. 304, suppose that $\mathbf{H}(s) = (s+1)/(s+2)$. Determine $\mathbf{G}(s)$ such that the resulting transfer function is $\mathbf{Y}/\mathbf{X} = (s+2)^2/(s+1)(s+4)$.

Optional Problems

FoEE 5.57

For 

$$[X + Y\,H(s)]\,G(s) = Y$$

$$GX + YGH = Y$$

$$GX = Y(1-GH)$$

$$\frac{Y}{X} = \frac{G}{1-GH}$$

If $H(s) = \frac{s+1}{s+2}$ &

If $\frac{Y}{X} = \frac{(s+2)^2}{(s+1)(s+4)}$, what is $G(s)$

$$\frac{G}{1 - G\left[\frac{s+1}{s+2}\right]} = \frac{(s+2)^2}{(s+1)(s+4)}$$

$$G = \frac{(s+2)^2}{(s+1)(s+4)} - G\frac{(s+2)^2}{(s+1)(s+4)}\cdot\frac{s+1}{s+2}$$

$$G\left(1 + \frac{s+2}{s+4}\right) = \frac{(s+2)^2}{(s+1)(s+4)}$$

$$G = \frac{\frac{(s+2)^2}{(s+1)(s+4)}}{1 + \frac{s+2}{s+4}} = \frac{\frac{(s+2)^2}{(s+1)}}{s+4+s+2} = \frac{(s+2)^2}{(s+1)(2s+6)}$$

$$G(s) = \frac{(s+2)^2}{2(s+1)(s+3)}$$

8. (0 points) FoEE 5.69

5.69 For the series *RL* circuit shown in Fig. P5.69, suppose that $R = 5\ \Omega$ and $L = 5$ H. Find the step responses $i(t)$ and $v(t)$ when $v_s(t) = 20u(t)$ V.

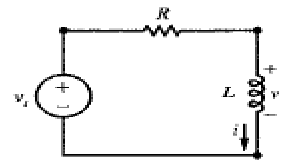
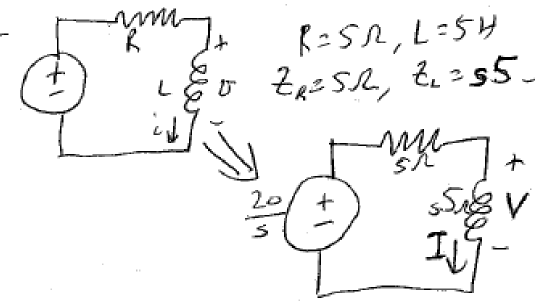


Fig. P5.69

FoEE 5.69



$R = 5\Omega,\ L = 5H$

$Z_R = 5\Omega,\ Z_L = s5\ \Omega \qquad v_s(t) = 20u(t)$

$$V = \left(\frac{s5}{s5+5}\right)\frac{20}{s} = \frac{20}{s+1}$$

$$\boxed{v(t) = 20e^{-t}u(t)\ V}$$

$$I = \frac{\frac{20}{s+1}}{5s} = \frac{4}{s(s+1)} = \frac{K_1}{s} + \frac{K_2}{s+1}$$

$$K_1 = I\,s\big|_{s=0} = 4$$

$$K_2 = I(s+1)\big|_{s=-1} = -4$$

$$\text{So } I(s) = \frac{4}{s} + \frac{-4}{s+1}$$

$$\boxed{i(t) = 4u(t) - 4e^{-t}u(t)\ A}$$

9. (0 points) FoEE 5.77

5.77 For the op-amp circuit shown in Fig. P5.8, suppose that $R = 2\ \Omega$ and $C = \frac{1}{8}$ F. Find the step response $v_2(t)$ when $v_1(t) = 3u(t)$ V.

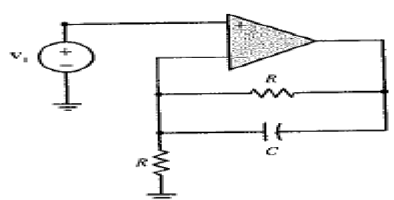
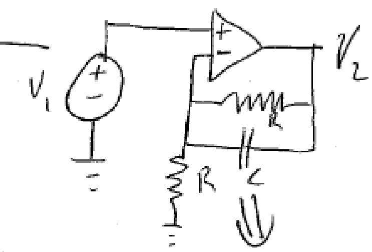


Fig. P5.8

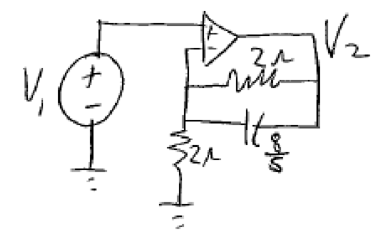
FoEE 5.77



$R = 2\Omega$

$C = \frac{1}{8}F$

$v_1(t) = 3e^{-4t}u(t)\ V$



$$\frac{V_1}{2} + \frac{V_1 - V_2}{2} + \frac{V_1 -}{8/s}$$

$$V_1 = \frac{3}{s}$$

So,

$$\frac{3}{2s} + \frac{3}{2s} - \frac{V_2}{2} + \frac{3s}{s8} - \frac{V_2 s}{8} = 0$$

$$\frac{3}{s} + \frac{3s}{8s} = V_2\left(\frac{1}{2} + \frac{s}{8}\right)$$

$$V_2 = \frac{\frac{24+3s}{8s}}{\frac{1}{2}+\frac{s}{8}} = \frac{\frac{24+3s}{8s}}{\frac{4+s}{8}} = \frac{3(s+8)}{(s+4)s}$$

So, partial fraction expansion: $\frac{K_1}{s} + \frac{K_2}{s+4}$

$$K_1 = V_2 s\big|_{s=0} = 6$$

$$K_2 = V_2(s+4)\big|_{s=-4} = -3$$

$$\text{So, } V_2 = \frac{6}{s} + \frac{-3}{s+4} \Rightarrow \boxed{v(t) = 6u(t) - 3e^{-4t}u(t)\ V}$$