Chapter 19: Introduction to Electrical Machines – Instructor Notes

The last part of the book presents an introduction to electro-magneto-mechanical systems. **Some of the foundations needed for this material (AC power) were discussed in Chapter 7; the polyphase AC power material in Chapter 7 may be introduced prior to covering Chapter 17, or together with the AC machine material of Chapter 17.**

The emphasis in this chapter (and the next two) is on preparing the student for the use of electro-magneto-mechanical systems as practical actuators for industrial applications. Thus, more emphasis is placed on describing the performance characteristics of linear motion actuators and of rotating machines than on a description of their construction details. The material in Chapters 18-20 has been used by several instructors over the last many years in a second (quarter-length) course in system dynamics (System Dynamics and Electromechanics) designed for mechanical engineering juniors.

Section 18.1 reviews basic laws of electricity and magnetism, which should already be familiar to the student from an earlier Physics course. The box *Focus on Measurements: Linear Variable Differential Transformer* (pp. 915-916) presents an example related to sensors with a discussion of the LVDT as a position transducer. Section 18.2 discusses approximate linear magnetic circuits and the idea of reluctance, and introduces magnetic structures with air gaps and simple electro-magnets. The box *Focus on Methodology: Magnetic Structures and Magnetic Equivalent Circuits* (p. 924) summarizes the analysis methods used in this section. A magnetic reluctance position sensor is presented in *Focus on Measurements: Magnetic Reluctance Position Sensor* (pp. 931-932) and *Focus on Measurements: Voltage Calculation in Magnetic Reluctance Position Sensor* (pp. 932-934). The non-ideal properties of magnetic materials are presented in Section 18.3, where hysteresis, saturation, and eddy currents are discussed qualitatively. Section 18.4 introduces simple models for transformers; more advanced topics are presented in the homework problems.

Section 18.5 is devoted to the analysis of forces and motion in electro-magneto-mechanical structures characterized by linear motion. The boxes *Focus on Methodology: Analysis of Moving-Iron Electromechanical Transducers* (pp. 943-944) and *Focus on Methodology: Analysis of Moving-Coil Electromechanical Transducers* (p. 956) summarize the analysis methods used in this section. The author has found that it is pedagogically advantageous to introduce the *Bli* and *Blu* laws for linear motion devices before covering these concepts for rotating machines: the student can often visualize these ideas more clearly in the context of a loudspeaker or of a vibration shaker. Example 18.9 (pp. 944-945) analyzes the forces in a simple electromagnet, and Examples 18.10 (pp. 945-947) and 18.12 (p. 951) extend this concept to a solenoid and a relay. Example 18.11 (p. 947-948) ties the material presented in this chapter to the transient analysis topics of Chapter 5. Example 18.13 (pp. 956-959) performs a dynamic analysis of a loudspeaker, showing how the frequency response of a loudspeaker can be computed from an electromechanical analysis of its dynamics. Finally, The box *Focus on Measurements: Seismic Transducer* (pp. 959-960) presents the dynamic analysis of an electromechanical seismic transducer.

The homework problems are divided into four sections. The first reviews basic concepts in electricity and magnetism; the second presents basic and more advanced problems related to the concept of magnetic reluctance; the third offers some problems related to transformers. Section 5 contains a variety of applied problems related to electromechanical transducers, and is divided into two separate sections on moving-iron and moving-coil transducers; some of the problems in this last section emphasize dynamic analysis (18.32, 18.41-18.44, and 18.48-53) and are aimed at a somewhat more advanced audience.

Learning Objectives

1. Review the basic principles of electricity and magnetism. *Section 18.1.*
2. Use the concepts of reluctance and magnetic circuit equivalents to compute magnetic flux and currents in simple magnetic structures. *Section 18.2.*
3. Understand the properties of magnetic materials and their effects on magnetic circuit models. *Section 18.3.*
4. Use magnetic circuit models to analyze transformers. *Section 18.4.*
5. Model and analyze force generation in electro-magneto-mechanical systems. Analyze moving iron transducers (electromagnets, solenoids, relays), and moving-coil transducers (electro-dynamic shakers, loudspeakers, seismic transducers. *Section 18.5.*

Section 19.1: Rotating Electric Machines

Problem 19.1

Solution:

Known quantities:

The relationship of the power rating and the ambient temperature is shown in the table. A motor with $P_e = 10\,kW$ is rated up to $85^\circ C$.

Find:

The actual power for the following conditions.

a) Ambient temperature is $50^\circ C$.

b) Ambient temperature is $25^\circ C$.

Assumptions:

None.

Analysis:

a)

The power at ambient temperature $50^\circ C$:

$$P_e' = 10 - 10 \times 0.125 = 8.75\,kW$$

b)

The power at ambient temperature $30^\circ C$:

$$P_e' = 10 + 10 \times 0.08 = 10.8\,kW$$

Problem 19.2

Solution:

Known quantities:

The speed-torque characteristic of an induction motor is shown in the table. The load requires a starting torque of $4\,N\cdot m$ and increase linearly with speed to $8\,N\cdot m$ at $1500\,rev/\min$.

Find:

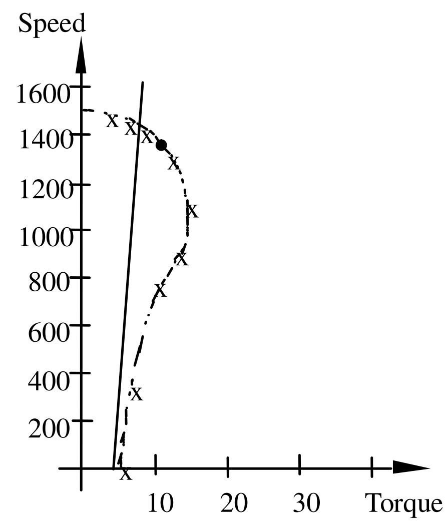
a) The steady state operating point of the motor.
b) The change in voltage if the load torque increases to $10\,N\cdot m$.

Assumptions:

None.

Analysis:

The characteristic is shown below:



a)
The operating point is:

$$n_m = 1425\,rev/\min, \qquad T = 7\,N\cdot m$$

b)
From the following equation:

$$\frac{T_{new}}{T_{old}} = \left|\frac{V_{s,new}}{V_{s,old}}\right|^2$$

$$\Rightarrow \frac{10}{7} = \left|\frac{KV_s}{V_S}\right|^2 = K^2$$

$$\Rightarrow K = 1.195$$

$$\therefore V_{s,new} = 1.195 V_{s,old}$$

Section 19.2: Direct Current Machines

Problem 19.3

Solution:

Known quantities:

Each conductor of the DC motor is $6\,in.$ long. The current is $90\,A$. The field density is $5.2\times10^{-4}\,Wb/in^2$.

Find:

The force exerted by each conductor on the armature.

Assumptions:

None.

Analysis:

$$F = BI\times l = 5.2\times10^{-4}\frac{Wb}{in^2}\times\frac{in^2}{(0.0254\,m)^2}\times 90\times 6\,in\times\frac{0.0254\,m}{in}$$
$$= 11.06\,Nt$$

Problem 19.4

Solution:

Known quantities:

The air-gap flux density of the DC machine is $4\,Wb/m^2$. The area of the pole face is $2\,cm\times 4\,cm$.

Find:

The flux per pole in the machine.

Assumptions:

None.

Analysis:

With $B = 4\,kG = 0.4\,T = 0.4\,Wb/m^2$, we can compute the flux to be:

$$\phi = BA = 0.4\times 0.02\times 0.04 = 0.32\,mWb$$

Section 19.3: Direct Current Generators

Problem 19.5

Solution:

Known quantities:

A $120V, 10A$ shunt motor. The armature resistance is 0.6Ω . The shunt field current is $2A$.

Find:

The LVDT equations.

Assumptions:

None.

Analysis:

V_L at full load is $120V$ and

$$E_b = 120 + (2+10) \times 0.6 = 127.2V$$

$$R_f = \frac{120}{2} = 60\Omega$$

Assuming E_b to be constant, we have:

$$i_a = i_f = \frac{127.2}{0.6+60} = 2.1A$$

Therefore:

$$V_L = 127.2 - 2.1 \times 0.6 = 125.9V$$

$$Voltage\,reg. = \frac{125.9-120}{120} = 0.049 = 4.9\%$$

Problem 19.6

Solution:

Known quantities:

A $20\,kW, 230\,V$ separately excited generator. The armature resistance is $0.2\,\Omega$. The load current is $100\,A$.

Find:

a) The generated voltage when the terminal voltage is $230\,V$.
b) The output power.

Assumptions:

None.

Analysis:

If we assume rated output voltage, that is $V_L = 230\,V$, we have
a) The generated voltage is $230\,V$.
b) The output power is $23\,kW$.
If we assume rated output power, that is $P_{out} = 20\,kW$, we have
a) The generated voltage is $200\,V$.
b) The output power is $20\,kW$.
If we assume $E_b = 230\,V$, and compute the output voltage to be: $V_L = 230 - 100 \times 0.2 = 210\,V$
We have:
a) The generated voltage is $210\,V$.
b) The output power is $21\,kW$.

Problem 19.7

Solution:

Known quantities:

A $10\,kW, 120\,VDC$ series generator. The armature resistance is $0.1\,\Omega$ and a series field resistance is $0.05\,\Omega$.

Find:

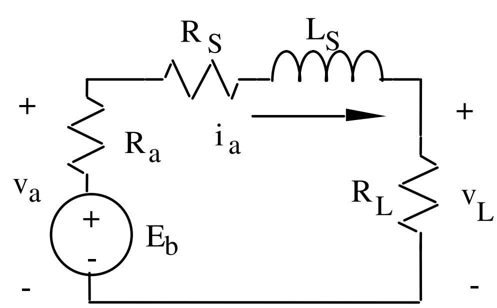
a) The armature current.
b) The generated voltage.

Assumptions:

The generator is delivering rated current at rated speed.

Analysis:

The circuit is shown below:



a) $$i_a = \frac{P}{V_L} = \frac{10 \times 10^3}{120} = 83.33\,A$$

b) $$V_a = 120 + i_a R_S = 124.17\,V$$

Problem 19.8

Solution:

Known quantities:

A $30\,kW, 440\,V$ shunt generator. The armature resistance is $0.1\,\Omega$ and a series field resistance is $200\,\Omega$.

Find:

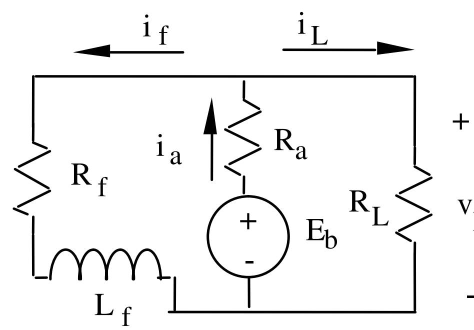
a) The power developed at rated load.
b) The load, field, and armature currents.
c) The electrical power loss.

Assumptions:

None.

Analysis:

The circuit is shown below:



$$i_L = \frac{30\times10^3}{440} = 68.2\,A$$

$$i_f = \frac{440}{200} = 2.2\,A$$

$$i_a = 70.4\,A$$

a)

$$E_b = V_L + i_a R_a = 440 + 70.4\times0.1 = 447.04\,V$$

$$P = E_b i_a = 31.471\,kW$$

b)

$$i_L = 62.8\,A$$

$$i_f = 2.2\,A$$

$$i_a = 70.4\,A$$

c)

$$P_{loss} = i_a^2 R_a + i_f^2 R_f = 1464\,W$$

Problem 19.9

Solution:

Known quantities:

A four-pole $450\,kW, 4.6\,kV$ shunt generator. The armature resistance is 2Ω and a series field resistance is 333Ω. The generator is operating at the rated speed of $3600\,rev/\min$.

Find:

The no-load voltage of the generator and terminal voltage at half load.

Assumptions:

None.

Analysis:

For

$$n = 3600\,rev/\min, \omega_m = 377\,rad/\sec : i_L = \frac{450\times 10^3}{4.6\times 10^3} = 97.8\,A \qquad i_f = \frac{4.6\times 10^3}{333} = 13.8\,A \qquad i_a = i_f + i_L = 111.6\,A$$

Using the relation: $E_b = V_L + i_a R_a = 4823.2V$

At no-load, $V_L = E_b - i_a R_a = 4820.4V$

At half load, $i_L = 48.9\,A \quad i_a = i_f + i_L = 62.7\,A \; V_L = E_b - i_a R_a = 4810.7V$

Problem 19.10

Solution:

Known quantities:

A $30\,kW, 240V$ generator is running at half load at $1800\,rev/\min$ with efficiency of 85 percent.

Find:

The total losses and input power.

Assumptions:

None.

Analysis:

$$P_{out} = \frac{1}{2} rated\ load = 15\,kW$$

At an efficiency of 0.85, the input power can be computed to be: $P_{in} = \dfrac{15\times 10^3}{0.85} = 17.647\,kW$

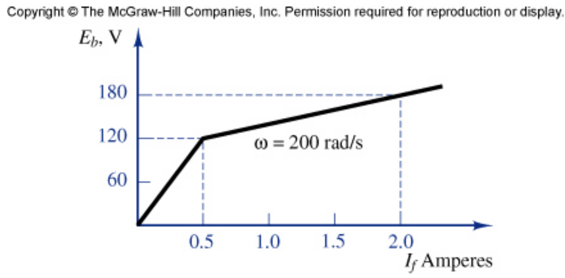
The total loss is: $P_{loss} = P_{in} - P_{out} = 2.647\,kW$

Problem 19.11

Solution:

Known quantities:

A self excited DC shunt generator. At $200\,rev/\min$, it delivers $20\,A$ to a $100V$ line. The armature resistance is 1.0Ω and a series field resistance is $100\,\Omega$. The magnetization characteristic is shown in Figure P19.11. When the generator is disconnected from the line, the drive motor speed up to $220\,rad/s$.



Find:

The terminal voltage.

Assumptions:

None.

Analysis:

From the figure, for $I_f > 0.5\,A, \omega = 200\,rad/\sec$

$$E_b = 40I_f + 100$$

For $\omega = 220\,rad/\sec$, we have:

$$\frac{E_b'}{220} = \frac{100 + 40I_f}{200}$$

Therefore,

$$E_b' = \frac{220}{200}(100 + 40I_f) = 110 + 44I_f$$

For no load, $I_a = I_f$. Therefore,

$$110 + 44I_f = 101I_f$$

$$\therefore I_f = 1.93\,A$$

The terminal voltage is:

$$V = I_f R_f = 193V$$

Problem 19.12

Solution:

Known quantities:

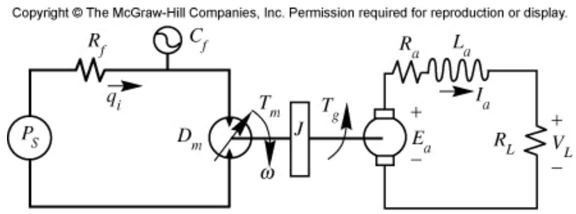
Hydralic system, mechanical system and electrical system shown in figure 19.12.

Find:

a) Write down differential equations for each system.
b) Derive transfer function from of the system from supply pressure, P_S to load voltage, V_L

Assumptions:

None.



Analysis:

Solution to Problem 19.12.
a) Derive the differential equations of the system.
Solution
We recognize that the key variables in the system are the hydraulic system pressure, p, the angular velocity of the pump, ω, and the armature current, i_a.
Hydraulic system:

$$q_i - q_o = q_{stored}$$

$$\left(\frac{P_S - p}{R_f}\right) - D_m\omega = C_f \frac{dp}{dt}$$

$$C_f \frac{dp}{dt} + \frac{p}{R_f} + D_m\omega = \frac{P_S}{R_f}$$

Mechanical subsystem:
Apply Newton's Second Law to obtain

$$J\frac{d\omega}{dt} = T_m - T_g = D_m p - k_{PM} i_a$$

Reorganizing the equation:

$$-D_m p + J\frac{d\omega}{dt} + k_{PM} i_a = 0$$

Electrical subsystem:
Apply KVL to write

$$E_a = k_{PM}\omega = L_a \frac{di_a}{dt} + R_a i_a + R_L i_a \text{ where } k_{PM} = k_a\phi$$

Reorganizing the equation:

$$-k_{PM}\omega + L_a \frac{di_a}{dt} + (R_a + R_L)i_a = 0$$

b) Compute the transfer function from input flow rate, P_S , to load voltage, V_L, using the method of determinants.
We first recognize that the load voltage V_L is given by the expression $V_L = i_a R_L$, and that we need to solve for the armature current. Next, we set up the matrix form of the equations obtained in part a:

$$\begin{bmatrix} \left(C_f s + \dfrac{1}{R_f}\right) & D_m & 0 \\ -D_m & Js & k_{PM} \\ 0 & -k_{PM} & \left(L_a s + (R_a + R_L)\right) \end{bmatrix} \begin{bmatrix} p(s) \\ \omega(s) \\ i_a(s) \end{bmatrix} = \begin{bmatrix} \dfrac{P_S(s)}{R_f} \\ 0 \\ 0 \end{bmatrix}$$

To solve for the armature current, we use Cramer's rule:

$$i_a(s) = \frac{\begin{vmatrix} \left(C_f s + \dfrac{1}{R_f}\right) & D_m & \dfrac{P_S(s)}{R_f} \\ -D_m & Js & 0 \\ 0 & -k_{PM} & 0 \end{vmatrix}}{\begin{vmatrix} \left(C_f s + \dfrac{1}{R_f}\right) & D_m & 0 \\ -D_m & Js & k_{PM} \\ 0 & -k_{PM} & \left(L_a s + (R_a + R_L)\right) \end{vmatrix}} = \frac{D_m k_{PM} \dfrac{P_S(s)}{R_f}}{\Delta(s)}$$

$$\Delta(s) = \left(C_f s + \frac{1}{R_f}\right)\left[Js\left(L_a s + R_{eq}\right) + k_{PM}^2\right] + D_m^2\left(L_a s + R_{eq}\right)$$

where $R_a + R_L = R_{eq}$.

Expanding Δ(s), we obtain:

$$\Delta(s) = C_f J L_a s^3 + \left(\frac{JL_a}{R_f} + JR_{eq}C_f\right)s^2 + \left(\frac{JR_{eq}}{R_f} + C_f k^2 + D_m^2 L_a\right)s + \left(\frac{k_{PM}^2}{R_f} + D_m^2 R_{eq}\right)$$

ANSWER:

$$\frac{V_L(s)}{P_S(s)} = \frac{R_L i_a(s)}{P_S(s)} = \frac{R_L D_m k_{PM}}{\Delta(s) R_f}$$

$$= \frac{D_m k_{PM} R_L}{C_f J L_a R_f s^3 + \left(JL_a + JR_{eq}C_f\right)s^2 + \left(JR_{eq} + C_f k^2 + D_m^2 L_a\right)s + \left(k_{PM}^2 + D_m^2 R_{eq}\right)}$$

Section 17.4: Direct Current Motors

Problem 19.13

Solution:

Known quantities:

A $220V$ shunt motor. The armature resistance is $0.32\,\Omega$. A field resistance is $110\,\Omega$. At no load the armature current is $6\,A$ and the speed is $1800\,rpm$.

Find:

a) The speed of the motor when the line current is $62\,A$.
b) The speed regulation of the motor.

Assumptions:

The flux does not vary with load. Assume a $8\,N \cdot m$ brush drop.

Analysis:

$$\text{a)}\,1800 = \frac{220-2-6(0.32)}{K_a\phi} \Rightarrow K_a\phi = 0.12 \qquad \therefore n = \frac{220-2-6(0.32)}{K_a\phi} = 1657\,rpm$$

$$\text{b)}\,\%reg = \frac{1800-1657}{1657}\times 100 = 8.65\%$$

Problem 19.14

Solution:

Known quantities:

A $50\,hp, 550\,volt$ shunt generator. The armature resistance including brushes is $0.36\,\Omega$. Operating at rated load and speed, the armature current is $75\,A$.

Find:

What resistance should be inserted in the armature circuit to get a 20 percent speed reduction when the motor is developing 70 percent of rated torque.

Assumptions:

There is no flux change.

Analysis:

$$T = K_a\phi I_a \Rightarrow I_a = 0.7(75) = 52.5\,A \qquad n_R = \frac{550-75(0.36)}{K_a\phi} \Rightarrow K_a\phi\, n_R = 523$$

$$0.8 n_R = \frac{550-52.5R_T}{K_a\phi} \Rightarrow 0.8\times 523 = 550-52.5R_T$$

$$\therefore R_T = 2.51\Omega \qquad R_{add} = 2.51-0.36 = 2.15\Omega$$

Problem 19.15

Solution:

Known quantities:

A $100\,kW, 440\,V$ shunt DC motor. The armature resistance is $0.2\,\Omega$ and a series field resistance is $400\,\Omega$. The generator is operating at the rated speed of $1200\,rev/\min$. The full-load efficiency is 90 percent.

Find:

a) The motor line current.
b) The field and armature currents.
c) The counter emf at rated speed.
d) The output torque.

Assumptions:

None.

Analysis:

At $n = 1200\,rev/\min, \omega_m = 125.7\,rad/\sec$, the output power is $100\,hp = 74.6\,kW$.
From full-load efficiency of 0.9, we have:

$$P_{in} = \frac{74.6}{0.9} = 82.9\,kW$$

a)
From $P_{in} = i_S V_S = 82.9\,kW$, we have:

$$i_S = \frac{82.9 \times 10^3}{440} = 188.4\,A$$

b)

$$i_f = \frac{440}{400} = 1.1\,A$$

$$i_a = 187.3\,A$$

c)

$$E_b = V_L - i_a R_a = 402.5\,A$$

d)

$$T_{out} = \frac{P_{out}}{\omega_m} = 593.5\,N \cdot m$$

Problem 19.16

Solution:

Known quantities:

A $240V$ series motor. The armature resistance is 0.42Ω and a series field resistance is 0.18Ω . The speed is $500\,rev/\min$ when the current is $36\,A$.

Find:

What is the motor speed when the load reduces the line current to $21\,A$.

Assumptions:

A $3\,volts$ brush drop and the flux is proportional to the current.

Analysis:

$$500 = \frac{240-3-36(0.6)}{K_a\phi} \Rightarrow K_a\phi = 0.431$$

$$n = \frac{240-3-21(0.6)}{(\frac{21}{36})(0.431)} = 893\,rpm$$

Problem 19.17

Solution:

Known quantities:

A $220VDC$ shunt motor. The armature resistance is 0.2Ω. The rated armature current is $50\,A$.

Find:

a) The voltage generated in the armature.
b) The power developed.

Assumptions:

None.

Analysis:

a)

$$E_b = V_L - i_a R_a = 220 - 50\times 0.2 = 210V$$

b)

$$P = E_b i_a = 210\times 50 = 10.5\,kW = 14.07\,hp$$

Problem 19.18

Solution:

Known quantities:

A $550V$ series motor. The armature resistance is $0.15\,\Omega$. The speed is $820\,rev/\min$ when the current is $112\,A$ and the load is $75\,hp$.

Find:

The horsepower output of the motor when the current drops to $84\,A$.

Assumptions:

The flux is reduced by 15 percent.

Analysis:

$$HP = \frac{2\pi \cdot n \cdot T}{33{,}000} \quad 75 = \frac{2\pi(820)T}{33{,}000} \Rightarrow T = 480.4\,lb \cdot ft$$

$$T = K_a\phi I_a \Rightarrow 480.4 = K_a\phi(112) \Rightarrow K_a\phi = 4.29$$

$$T_n = 4.29(0.85)(84) = 306.2\,lb \cdot ft$$

$$n_n = \frac{550 - 84(0.15)}{0.85(0.65)} = 973\,rpm \quad HP_n = \frac{2\pi(973)(306.2)}{33{,}000} = 56.7\,hp$$

Problem 19.19

Solution:

Known quantities:

A $220VDC$ shunt motor. The armature resistance is 0.1Ω and a series field resistance is $100\,\Omega$. The speed is $1100\,rev/\min$ when the current is $4\,A$ and there is no load.

Find:

E and the rotational losses at $1100\,rev/\min$.

Assumptions:

The stray-load losses can be neglected.

Analysis:

Since $n = 1100\,rev/\min$ corresponds to $\omega = 115.2\,rad/\sec$, we have:

$$i_S = 4\,A \quad i_f = \frac{200}{100} = 2\,A \quad i_a = i_S - i_f = 2\,A$$

Also, $E_b = 200 - 2 \times 0.1 = 199.8V$

The power developed by the motor is: $P = P_{in} - P_{copper\_loss} = 200 \times 4 - (2^2 \times 100 + 2^2 \times 0.1) = 399.6W$

Problem 19.20

Solution:

Known quantities:

A $230VDC$ shunt motor. The armature resistance is 0.5Ω and a series field resistance is 75Ω. At $1100\, rev/\min$, $P_{rot} = 500W$. When loaded, the current is $46A$.

Find:

a) The speed P_{dev} and T_{sh}.
b) $i_a(t)$ and $\omega_m(t)$ if $L_f = 25H, L_a = 0.008H$ and the terminal voltage has a $115V$ change.

Assumptions:

None.

Analysis:

$$i_f = \frac{230}{75} = 3.07A$$

$$i_a = i_L - i_f = 42.93A$$

$$\omega_m = 117.3\, rad/\sec$$

At no load, $117.3 = \dfrac{230}{K_a\phi}$, therefore,

$$K_a\phi = 1.96$$

At full load,

$$\omega_m = \frac{230 - 0.5\times 42.93}{K_a\phi}$$

The back emf is:

$$E_b = 230 - 0.5\times 42.93 = 208.5V$$

The power developed is:

$$P_{dev} = E_b I_a = 8.952\, kW$$

The power available at the shaft is:

$$P_o = P_{dev} - P_{rot} = 8952 - 500 = 8452W$$

The torque available at the shaft is:

$$T_{sh} = \frac{P_o}{\omega_m} = 72.1\, N\cdot m$$

Problem 19.21

Solution:

Known quantities:

A $200VDC$ shunt motor. The armature resistance is 0.1Ω and a series field resistance is 100Ω . At $955\,rev/\min$ with no load, $P_{rot} = 500W$, the line current is $5A$.

Find:

The motor speed, the motor efficiency, total losses, and the load torque when the motor draws $40A$ from the line.

Assumptions:

Rotational power losses are proportional to the square of shaft speed.

Analysis:

$$i_S = 5A$$

$$i_f = \frac{200}{100} = 2A$$

$$i_a = i_S - i_f = 3A$$

The copper loss is: $P_{copper} = i_f^2 R_f + i_a^2 R_a = 400.9W$

The input power is: $P_{in} = 5 \times 200 = 1kW$

Therefore, $P_{rot} + P_{SL} = 1000 - 409 = 599.1W$

at $\omega_m = 2\pi \frac{955}{60} = 100\,rad/\sec$.

Also, E_b at no load is:

$$E_b = 200 - 3 \times 0.1 = 199.7V$$

$$K_a\phi = 1.997$$

When $i_S = 40A$ with $i_f = 2A$ and $i_a = 38A$,

$$E_b = 200 - 38 \times 0.1 = 196.2V$$

$$\omega_m = \frac{E_b}{K_a\phi} = 98.25\,rad/\sec = 938.2\,rev/\min$$

The power developed is:

$$P = E_b I_a = 196.2 \times 38 = 7456W$$

The copper loss is: $P_{copper} = i_f^2 R_f + i_a^2 R_a = 544.4W$

The input power is: $P_{in} = 40 \times 200 = 8kW$

And

$$P_{SH} = 7456 - \frac{98.25}{100} \times 599.1 = 6867.4W$$

$$T_{SH} = \frac{6867.4}{98.25} = 69.9\,N \cdot m$$

Finally, the efficiency is: $eff = \frac{P_{SH}}{P_{in}} = 85.84\%$

Problem 19.22

Solution:

Known quantities:

A $50\,hp, 230\,V$ shunt motor operates at full load when the line current is $181\,A$ at $1350\,rev/\min$. The field resistance is $17.7\,\Omega$. To increase the speed to $1600\,rev/\min$, a resistance of $5.3\,\Omega$ is cut in via the field rheostat. The line current is increased to $190\,A$.

Find:

a) The power loss in the field and its percentage of the total power input for the $1350\,rev/\min$ speed.
b) The power losses in the field and the field rheostat for the $1600\,rev/\min$ speed.
c) The percent losses in the field and in the field rheostat at $1600\,rev/\min$ speed.

Assumptions:

None.

Analysis:

a)

$$I_f = \frac{230}{17.7} = 13.0\,A$$

$$P_f = (230)(13.0) = 2988.7\,W$$

$$\frac{P_f}{P_m} = \frac{2988.7}{(230)(181)} = 0.072 = 7.2\%$$

b)

$$I_f = \frac{230}{(17.7+5.3)} = 10\,A$$

$$P_f = 10^2(17.7) = 1770\,W$$

$$P_R = 10^2(5.3) = 530\,W$$

c)

$$P_{in} = (230)(190) = 43{,}700\,W$$

$$\%P_f = \frac{1770}{43700}\times 100 = 4.05\%$$

$$\%P_R = \frac{530}{43700}\times 100 = 1.21\%$$

Problem 19.23

Solution:

Known quantities:

A $10\,hp, 230\,V$ shunt-wound motor. The armature resistance is $0.26\,\Omega$ and a series field resistance is $225\,\Omega$. The rated speed is $1000\,rev/\min$. The full-load efficiency is 86 percent.

Find:

The effect on counter emf, armature current and torque when the motor is operating under rated load and the field flux is very quickly reduced to 50 percent of its normal value. The effect on the operation of the motor and its speed when stable operating conditions have been regained.

Assumptions:

None.

Analysis:

$E_C = K\phi n$; counter emf will decrease.

$I_a = \dfrac{V - E_C}{r_a}$; armature current will increase.

$T = K\phi I_a$; effect on torque is indeterminate.

Operation of a dc motor under weakened field conditions is frequently done when speed control is an important factor and where decreased efficiency and less than rated torque output are lesser considerations.

$$n = \frac{V - I_a r_a}{K\phi} \Rightarrow 1000 = \frac{V - I_a r_a}{K\phi}$$

$$n_{new} = \frac{V - I_a r_a}{0.5K\phi}$$

Assume small change in the steady-state value of I_a. Then:

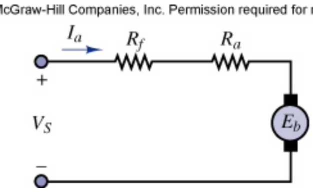
$$\frac{1000}{n_{new}} = \frac{0.5}{1} \Rightarrow n_{new} = 2000\,rpm$$

Problem 19.24

Solution:

Known quantities:

The machine is the same as that in Example 19.7. The circuit is shown in Figure P19.24. The armature resistance is $0.2\,\Omega$ and the field resistance is negligible.
$n = 120\,rev/\text{min}, I_a = 8A$. In the operating region, $\phi = kI_f, k = 200$.



Find:

a) The number of field winding turns necessary for full-load operation.
b) The torque output for the following speeds:

1. $n' = 2n$ 3. $n' = n/2$
2. $n' = 3n$ 4. $n' = n/4$

c) Plot the speed-torque characteristic for the conditions of part b.

Assumptions:

None.

Analysis:

in Example 17.7, $i_f = 0.6A$, the mmf F is:

$$F = 200 \times 0.6 = 120\,At$$

For a series field winding with:
a)
$i_{series} = i_a = 8\,A$, we have:

$$N_{series} = \frac{120}{8} = 15\,turns$$

b)

$$n_m = 120\,rev/\text{min}$$
$$\omega_m = 12.57\,rad/\text{sec}$$
$$E_b = V_S - i_a(R_a + R_S)$$

Neglecting R_S, we have

$$E_b = 7.2 - 8 \times 0.2 = 5.6V = k_a\phi\omega_m$$
$$k_a\phi = \frac{5.6}{12.57} = 0.446$$

From $T = k_T\phi i_a$ and $k_T = k_a$,

$$T = 0.446 \times 8 = 3.56\,N \cdot m$$

By using $\phi = ki_a$, we have:

$$E_b = k_a k i_a \omega_m = V_S - i_a R_a$$
$$T = k_T(ki_a)i_a = k_a k i_a^2$$

From $i_a = \dfrac{V_S}{R_a + K\omega_m}$, where $K = k_a k = \dfrac{5.6}{8 \times 12.57} = 0.056$

And $T \propto (\frac{1}{R_a + K\omega_m})^2$, we have:

$$\frac{T_X}{T} = (\frac{R_a + K\omega_m}{R_a + K\omega_X})^2 = (\frac{\frac{R_a}{K} + \omega_m}{\frac{R_a}{K} + \omega_X})^2$$

$$\because \frac{R_a}{K} = 3.59$$

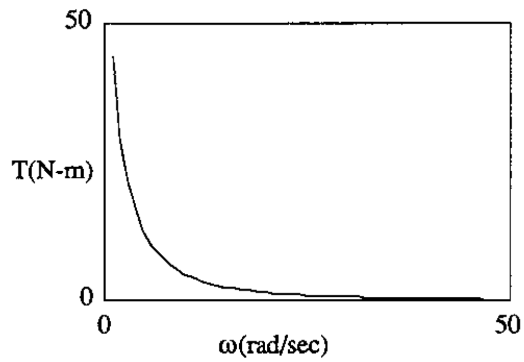
$$\therefore T_X = 3.56(\frac{3.59 + \omega_m}{3.59 + \omega_X})^2$$

1. at $\omega_X = 2\omega_m = 25.12\, rad/\sec$,
$T_X = 1.13\, N \cdot m$

2. at $\omega_X = 3\omega_m = 37.71\, rad/\sec$,
$T_X = 0.55\, N \cdot m$

3. at $\omega_X = 0.5\omega_m = 6.28\, rad/\sec$,
$T_X = 9.54\, N \cdot m$

4. at $\omega_X = 0.25\omega_m = 3.14\, rad/\sec$,
$T_X = 20.53\, N \cdot m$



c) The diagram is shown in the right

Problem 19.25

Solution:

Known quantities:

PM DC motor circuit model; mechanical load model. Example 19.9.

Find:

Voltage-step response of motor.

Assumptions:

None.

Analysis:

Applying KVL and equation 19.47 to the electrical circuit we obtain:

$$V_L(t) - R_a I_a(t) - L_a \frac{dI_a(t)}{dt} - E_b(t) = 0 \;\text{ or }\; L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + K_{aPM}\,\omega_m(t) = V_L(t)$$

Applying Newton's Second Law and equation 19.46 to the load inertia, we obtain:

$$J\frac{d\omega(t)}{dt} = T(t) - T_{Load}(t) - b\omega \;\text{ or }\; -K_{TPM} I_a(t) + J\frac{d\omega(t)}{dt} + b\omega(t) = 0$$

since the load torque is assumed to be zero. To derive the transfer function from voltage to speed, we use the result of Example 19.9 with $T_{load} = 0$:

$$\Omega_m(s) = \frac{K_{T\ PM}}{(sL_a + R_a)(sJ + b) + K_{a\ PM} K_{T\ PM}} V_L(s)$$

The step response of the system can be computed by assuming a unit step input in voltage:

$$\Omega_m(s) = \frac{K_{T\ PM}}{(sL_a + R_a)(sJ + b) + K_{a\ PM} K_{T\ PM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{(sL_a + R_a)(sJ + b) + K_{aPM} K_{TPM}} \frac{1}{s}$$

$$F(s) = \frac{K_{TPM}}{s[JL_a s^2 + (L_a b + R_a J)s + R_a b + K_a K_T]}$$

$$F(s) = \frac{K_{TPM}}{s\left[s^2 + \frac{(L_a b + R_a J)}{JL_a}s + \frac{(R_a b + K_a K_T)}{JL_a}\right]}$$

Set:

$$m = \frac{(L_a b + R_a J)}{JL_a} \qquad n = \frac{(R_a b + K_a K_T)}{JL_a}$$

$$F(s) = \frac{K_{TPM}}{s[s^2 + ms + n]} = \frac{K_{TPM}}{s\left[\left(s^2 + ms + \frac{m^2}{4}\right) + n - \frac{m^2}{4}\right]} = \frac{K_{TPM}}{s\left[\left(s + \frac{m}{2}\right)^2 + \left(n - \frac{m^2}{4}\right)\right]}$$

For

$$F(s) = \frac{1}{s[(s+a)^2 + b^2]} \qquad f(t) = \frac{1}{b_o^2} + \frac{1}{bb_o} e^{-at}\sin(bt - \phi)$$

$$\text{where}: \phi = \tan^{-1}\frac{b}{-a} \;\text{ and }\; b_o = \sqrt{b^2 + a^2}$$

Therefore:

$$a=\frac{m}{2}=\frac{(L_a b+R_a J)}{2JL_a} \qquad b=\sqrt{n-\frac{m^2}{4}}=\sqrt{\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}}$$

$$b_o^2=\left[\frac{(L_a b+R_a J)}{2JL_a}\right]^2+\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}=\frac{R_a b+K_a K_T}{JL_a}$$

$$\phi=\tan^{-1}\frac{\sqrt{\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}}}{-\frac{(L_a b+R_a J)}{2JL_a}}$$

Thus giving the step response:

$$\Omega_m(t)=\frac{JL_a}{R_a b+K_a K_T}+\frac{1}{\left(\sqrt{\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}}\right)\left(\sqrt{\frac{R_a b+K_a K_T}{JL_a}}\right)}e^{-\left[\frac{(L_a b+R_a J)}{2JL_a}\right]t} *$$

$$\sin\left[\left(\sqrt{\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}}\right)t-\tan^{-1}\frac{\sqrt{\frac{R_a b+K_a K_T}{JL_a}-\left(\frac{L_a b+R_a J}{JL_a}\right)^2\frac{1}{4}}}{-\frac{(L_a b+R_a J)}{2JL_a}}\right]$$

Expressions for the natural frequency and damping ratio of the second-order system may be derived by comparing the motor voltage-speed transfer function to a standard second-order system transfer function:

$H(s)=\dfrac{K_S}{s^2+2\zeta\omega_n s+\omega_n^2}$. The motor transfer function is:

$$\frac{\Omega_m(s)}{V_L(s)}=\frac{K_{T\ \ PM}}{(sL_a+R_a)(sJ+b)+K_{a\ \ PM}K_{T\ \ PM}}=\frac{K_{T\ \ PM}}{JL_a s^2+(JR_a+bL_a)s+R_a b+K_{a\ \ PM}K_{T\ \ PM}}$$

$$=\frac{\frac{K_{T\ \ PM}}{JL_a}}{s^2+\frac{(JR_a+bL_a)}{JL_a}s+\frac{R_a b+K_{a\ \ PM}K_{T\ \ PM}}{JL_a}}$$

Comparing terms, we determine that:

$$\omega_n^2=\frac{R_a b+K_{aPM}K_{TPM}}{JL_a} \qquad 2\zeta\omega_n=\frac{(JR_a+bL_a)}{JL_a}$$

or

$$\omega_n=\sqrt{\frac{R_a b+K_{aPM}K_{TPM}}{JL_a}} \qquad \zeta=\frac{1}{2}\frac{(JR_a+bL_a)}{JL_a}\sqrt{\frac{JL_a}{R_a b+K_{aPM}K_{TPM}}}$$

From these expressions, we can see that both natural frequency and damping ratio are affected by each of the parameters of the system, and that one cannot predict the nature of the damping without knowing numerical values of the parameters.

Problem 19.26

Solution:

Known quantities:

Torque-speed curves of motor and load: $T_m = a\omega + b$ (motor), $T_L = c\omega^2 + d$ (load),

Find:

Equilibrium speeds and their stability.

Assumptions:

All coefficients of torque-speed curve functions are positive constants

Analysis:

The first consideration is that the motor static torque, $T_{0,m} = b$ must exceed the load static torque, $T_{0,L} = d$; thus, the first condition is $b>d$.
The next step is to determine the steady state speed of the motor-load pair. If we set the motor torque equal to the load torque, the resulting angular velocities will be the desired solutions.

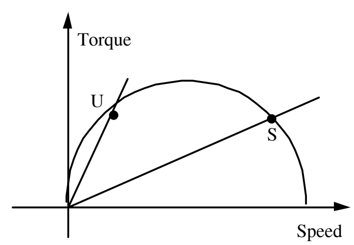
$$T_m = T_L \Rightarrow a\omega + b = c\omega^2 + d$$

resulting in the quadratic equation

$$c\omega^2 + a\omega + (d-b) = 0$$

with solution $\omega = \dfrac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$.

Both solutions are positive, and therefore physically acceptable. The question of stability can be addressed by considering the following sketch.



In the figure, we see that the intersection of a line with a quadratic function when both solutions are positive leads to two possible situations: the line intersecting the parabola when the rate of change of both curves is positive, and the line intersecting the parabola when the rate of change of torque w.r. to speed of the latter is negative. The first case leads to an unstable operating point; the second case to a stable operating point (you can argue each case qualitatively by assuming a small increase in load torque and evaluating the consequences). We can state this condition mathematically by requiring that the following steady-state stability condition hold:

$$\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega}$$

Evaluating this for our case, we see that

$$\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega} \Rightarrow 2c\omega > a .$$

From the expression we obtained earlier,

$$\omega = \frac{a + \sqrt{a^2 + 4c(b-d)}}{2c}$$

it is clear that $2c\omega>a$ always olds, since the term under the square root is a positive constant. Thus, this motor-load pair always leads to stable solutions. To verify this conclusion intuitively, you might wish to plot the motor and load torque-speed curves and confirm that the condition $\frac{dT_L}{d\omega} > \frac{dT_m}{d\omega}$ is always satisfied (note that the sketch above is **not** an accurate graphical representation of the two curves).

Problem 19.27

Solution:

Known quantities:

Expression for friction and windage torque, T_{FW}, functional form of motor torque, T, or load torque, T_L.

Find:

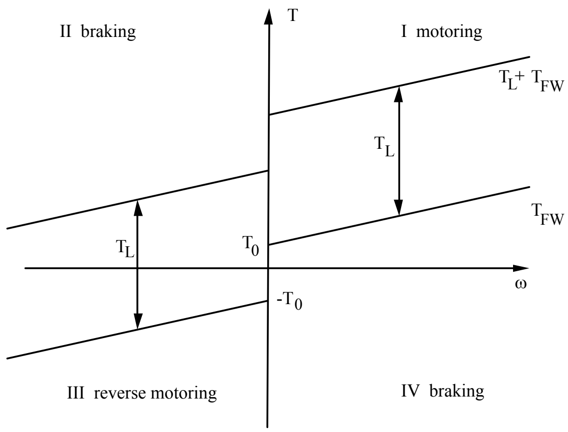
Sketch torque-speed curve

Assumptions:

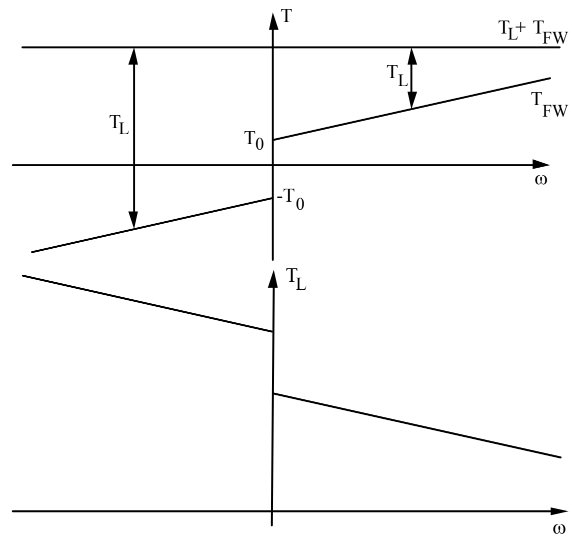
None.

Analysis:

The sketches are shown below.



Torque-speed curve for constant load torque



Torque-speed curves for variable load torque

Problem 19.28

Solution:

Known Quantities:

A PM DC motor and parameters when 1) in steady-state, no load conditions, and 2) connected to a pump

Find:

a) A damping coefficient, sketch of the motor, the dynamic equations, the transfer function, and 3 dB bandwidth.
b) A sketch of the motor, dynamic equations, transfer function, and 3 dB bandwidth.

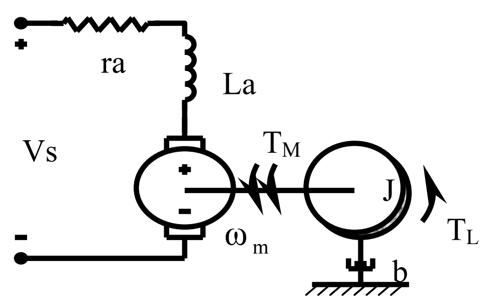
Assumptions:

$k_t = k_a = k_{PM}$

Analysis:

a)
The magnetic torque balanced the damping torque gives s:
$k_t * i_a = b * \omega_m$
or

$$b = \frac{k_{PM} * i_a}{\omega_m} = \frac{7*10^{-3}\ \frac{\text{N}-\text{m}}{\text{A}} * .15\ \text{A}}{\frac{3350\ \text{rev}}{\text{min}} * \frac{2\pi\ \text{rad}}{\text{rev}} * \frac{1\ \text{min}}{60\ \text{sec}}} = 2.993*10^{-6}\ \text{N-m-sec}$$



Sketch: PM DC Motor-Load System
Dynamic equations:

$$V_s = L_a * \frac{di_a}{dt} + r_a * i_a + E_b$$

$$E_b = k_{PM} * \omega_m$$

$$V_s = L_a * \frac{di_a}{dt} + r_a * i_a + k_{PM} * \omega_m$$

$$J * \frac{d\omega_m}{dt} = T_m - T_L - b * \omega_m$$

$$T_m = k_{PM} * i_a$$

$$-T_L = J * \frac{d\omega_m}{dt} + b * \omega_m - k_{PM} * i_a$$

To get the transfer function:

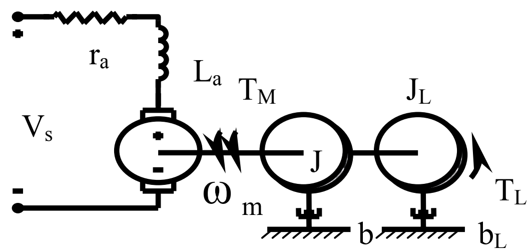
$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (Js + b) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & (Js + b) \end{bmatrix}}$$

$$\left. \frac{\omega_m}{V_s}(s) \right|_{T_L = 0} = \frac{k_{PM}}{(J * L_a)s^2 + (r_a * J + L_a * b)s + r_a * b + k_{PM}^2}$$

b)

Sketch:



Dynamic Equations:

$$V_s = L_a * \frac{di_a}{dt} + r_a * i_a + E_b$$
$$E_b = k_{PM} * \omega_m$$
$$V_s = L_a * \frac{di_a}{dt} + r_a * i_a + k_{PM} * \omega_m$$
$$(J + J_L) * \frac{d\omega_m}{dt} = T_m - T_L - (b + b_L) * \omega_m$$
$$T_m = k_{PM} * i_a$$
$$-T_L = (J + J_L) * \frac{d\omega_m}{dt} + (b + b_L) * \omega_m - k_{PM} * i_a$$

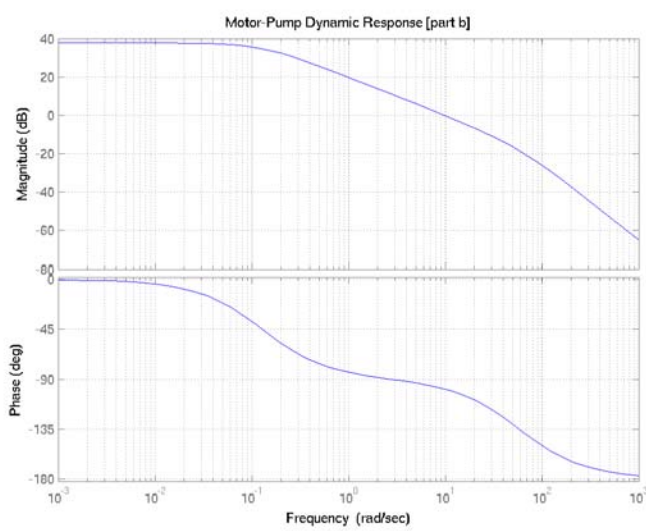
To get the transfer function:

$$\begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J + J_L)s + (b + b_L)) \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} V_s \\ -T_L \end{bmatrix}$$

$$\omega_m(s) = \frac{\det \begin{bmatrix} (L_a s + r_a) & V_s \\ -k_{PM} & -T_L \end{bmatrix}}{\det \begin{bmatrix} (L_a s + r_a) & k_{PM} \\ -k_{PM} & ((J + J_L)s + (b + b_L)) \end{bmatrix}}$$

$$\left. \frac{\omega_m}{V_s}(s) \right|_{T_L = 0} = \frac{k_{PM}}{((J + J_L) * L_a)s^2 + (r_a * (J + J_L) + L_a * (b + b_L))s + r_a * (b + b_L) + k_{PM}^2}$$

Frequency Response:



Problem 19.29

Solution:

Known Quantities:

A PM DC motor is used to power a pump

Find:

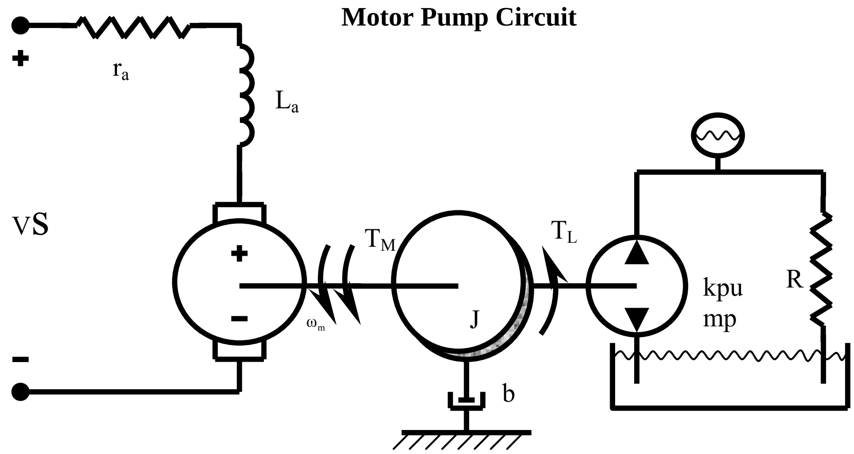
The dynamic equations of the system and the transfer function between the motor voltage and pressure.

Assumptions:

The inertia and damping of the motor and pump can be lumped together.

Analysis:

Sketch:



$$V_s = L_a \frac{di_a}{dt} + r_a i_a + E_b$$

$$V_s = L_a \frac{di_a}{dt} + r_a i_a + k_{PM}\,\omega_m$$

$$J\frac{d\omega_m}{dt} = T_m - T_L - b\omega_m$$

$$-T_L = J\frac{d\omega_m}{dt} + b\omega_m - k_{PM}\,\omega_m$$

$$k_p\omega_m - \frac{p-0}{R} = C_{acc}\frac{dp}{dt}$$

$$C_{acc}\frac{dp}{dt} + \frac{p}{R} - k_p\omega_m = 0$$

To get the transfer function:

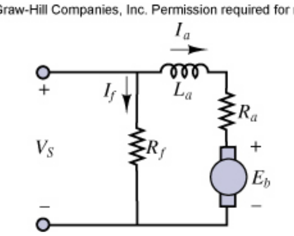
$$\begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ p \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

$$p(s) = \frac{\det \begin{bmatrix} L_a s + r_a & k_{PM} & V_s \\ -k_{PM} & Js + b & 0 \\ 0 & -Rk_p & 0 \end{bmatrix}}{\det \begin{bmatrix} L_a s + r_a & k_{PM} & 0 \\ -k_{PM} & Js + b & k_p \\ 0 & -Rk_p & RC_{acc}s + 1 \end{bmatrix}}$$

$$\frac{p}{V_s}(s) = \frac{k_{PM} R k_p}{(L_a s + r_a)(Js + b)(RC_{acc}s + 1) + k_{PM}^2 (RC_{acc}s + 1) + Rk_p^2 (L_a s + r_a)}$$

Problem 19.30

Solution:



Known quantities:

Motor circuit shown in Figure P19.30 and magnetization parameters, load parameters. Operating point.
Note correction to the operating point: $I_{a0} = 186.67$ A;
Note correction to the parameter: $k_f = 0.12$ V-s/A-rad

Find:

a) System differential equations in symbolic form
b) Linearized equations

Assumptions:

The dynamics of the field circuit are negligible.

Analysis:

a) Differential equations
Applying KVL and equation 19.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} + R_f I_f(t) = V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f \quad I_f(t)\omega_m(t) = V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 19.46 to the load inertia, we obtain:

$$J\frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m \quad \text{or} \quad -k_f \quad I_f(t)I_a(t) + J\frac{d\omega_m(t)}{dt} + b\omega_m(t) = T_L(t)$$

Since the dynamics of the field circuit are much faster than those of the armature circuit (time constant $\frac{L_f}{R_f} << \frac{L_a}{R_a}$), we can write $I_f = \frac{V_S}{R_f}$ and the system of equations is now:

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f \quad \frac{V_S(t)}{R_f}\omega_m(t) = V_S(t)$$

$$-k_f \quad \frac{V_S(t)}{R_f} I_a(t) + J\frac{d\omega_m(t)}{dt} + b\omega_m(t) = T_L(t)$$

b) Linearization

Define perturbation variables:

$$I_a(t) = \bar{I}_a + \delta I_a(t)$$
$$\omega_m(t) = \bar{\omega}_m + \delta\omega_m(t)$$
$$V_S(t) = \bar{V}_S + \delta V_S(t)$$

Next, we write the steady-state equations (all derivatives equal to zero):

$$R_a \bar{I}_a + k_f \quad \frac{\bar{V}_S}{R_f}\bar{\omega}_m = \bar{V}_S$$

$$-k_f \quad \frac{\bar{V}_S}{R_f}\bar{I}_a + b\bar{\omega}_m = \bar{T}_L$$

These equations must be satisfied at the operating point. We can verify this using numerical values:

$$0.75 \times \bar{I}_a + 0.12\frac{150}{60}200 = 200$$

resulting in $\bar{I}_a = \frac{1}{0.75}\left(200 - 0.12\frac{150}{60}200\right) = 186.67$

$$-0.12\frac{150}{60}186.67 + 0.6 \times 200 = \bar{T}_L$$

resulting in $\bar{T}_L = -56 + 120 = 64$ N - m

Now, given that the system is operating at the stated operating point, we can linearize the differential equation for the perturbation variables around the operating point. To linearize the equation we recognize the nonlinear terms:

$k_f \quad \frac{V_S(t)}{R_f}\delta\omega_m(t)$ and $k_f \quad \frac{V_S(t)}{R_f}I_a(t)$.

To linearize these terms, we use the first-order term in the Taylor series expansion:

$$k_f \quad \frac{V_S(t)}{R_f}\omega_m(t) \approx \frac{k_f}{R_f}\frac{\partial\left(V_S(t)\omega_m(t)\right)}{\partial V_S}\Bigg|_{\bar{V}_S,\bar{\omega}_m} \delta V_S(t) + \frac{k_f}{R_f}\frac{\partial\left(V_S(t)\omega_m(t)\right)}{\partial\omega_m}\Bigg|_{\bar{V}_S,\bar{\omega}_m} \delta\omega_m(t)$$

$$= \frac{k_f}{R_f}\left(\bar{\omega}_m\delta V_S(t) + \bar{V}_S\delta\omega_m(t)\right)$$

$$k_f \quad \frac{V_S(t)}{R_f}I_a(t) \approx \frac{k_f}{R_f}\frac{\partial\left(V_S(t)I_a(t)\right)}{\partial V_S}\Bigg|_{\bar{V}_S,\bar{I}_a} \delta V_S(t) + \frac{k_f}{R_f}\frac{\partial\left(V_S(t)I_a(t)\right)}{\partial I_a}\Bigg|_{\bar{V}_S,\bar{I}_a} \delta\bar{I}_a(t)$$

$$= \frac{k_f}{R_f}\left(\bar{I}_a\delta V_S(t) + \bar{V}_S\delta I_a(t)\right)$$

Now we can write the linearized differential equations in the perturbation variables:

$$L_a\frac{d\delta I_a(t)}{dt} + R_a\delta I_a(t) + \frac{k_f}{R_f}\bar{V}_S\delta\omega_m(t) = \delta V_S(t) - \frac{k_f}{R_f}\bar{\omega}_m\delta V_S(t)$$

$$-\frac{k_f}{R_f}\bar{V}_S\delta I_a(t) + J\frac{d\delta\omega_m(t)}{dt} + b\delta\omega_m(t) = \bar{I}_a\delta V_S(t) + \delta T_L(t)$$

This set of equations is now linear, and numerical values can be substituted to obtain a numerical answer, valid in the neighborhood of the operating point, for given voltage and load torque inputs to the system.

Problem 19.31

Solution:

Known Quantities:

$T_L = 5 + 0.05\omega + 0.001\omega^2$
$k_{TPM} = k_{APM} = 2.42$
$R_a = 0.2\Omega$
$V_S = 50V$

Find:

What will the speed of rotation be of the fan?

Assumptions:

The fan is operating at constant speed.

Analysis:

Applying KVL for a PM DC motor (note at constant current short inductor)

$V_S = i_a R_a + E_b$ (eqn.19.48)

$$i_a = \frac{T}{k_{TPM}}$$

$$E_b = k_{aPM} \cdot \omega_m$$

$$V_s = \frac{T}{k_{TPM}} R_a + k_{aPM} \cdot \omega_m$$

$$T = T_L = 5 + 0.05\omega_m + 0.001\omega_m^2$$

$$V_s = \frac{\left(5 + 0.05\omega_m + 0.001\omega_m^2\right)}{k_{TPM}} R_a + k_{aPM} \cdot \omega_m$$

Plug in the known variables and solving for ω_m.

$$-50V = \left(5 + 0.05\omega_m + 0.001\omega_m^2\right)\frac{0.02\Omega}{2.42\,\mathrm{N \cdot m}/\mathrm{Amp}} + \left(2.42\,V \cdot \sec/rad\right)\omega_m$$

$$8.26x10^{-5}\omega_m^2 + \left(2.42 + 4.13x10^{-3}\right)\omega_m + (-50 + .413) = 0$$

$$\omega_m = 20.5, \text{ or } -29318 \text{ rad/sec}$$

$$\omega_m = 20.5 \text{ rad/sec}$$

$$N_m = 20.5 \text{ rad/sec}\left(\frac{60 \sec}{\min}\right)\left(\frac{\text{rev}}{2\pi}\right) = 196RPM$$

Problem 19.32

Solution:

Known Quantities:

A separately excited DC motor

$R_a = 0.1\Omega, R_f = 100\Omega, L_a = 0.2H, L_f = 0.02H, K_a = 0.8, K_f = 0.9$

$J = 0.5kg - m^2, b = 2N - m - rad / s$

Find:

a) A sketch of the system and its three differential equations
b) Sketch a simulation block diagram
c) Put the diagram into Simulink
d) Run the simulation with *Armature Control* with a constant field voltage $V_f = 100V$

   Plot the current and angular speed responses

   Run the simulation with *Field Control* with a constant armature voltage $V_f = 100V$

   Plot the current and angular speed responses

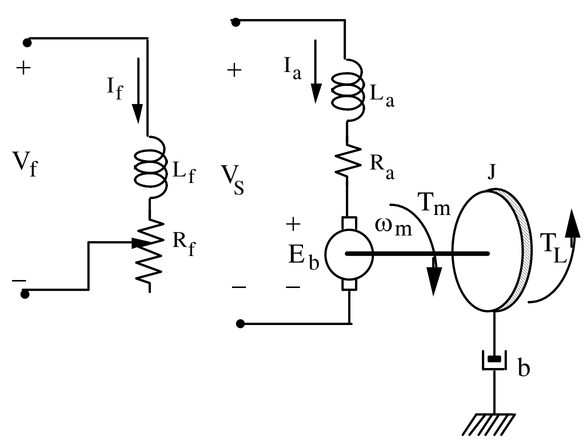
Assumptions:

No external load torque is applied

Analysis:

a) Sketch:

Separately Excited DC Motor



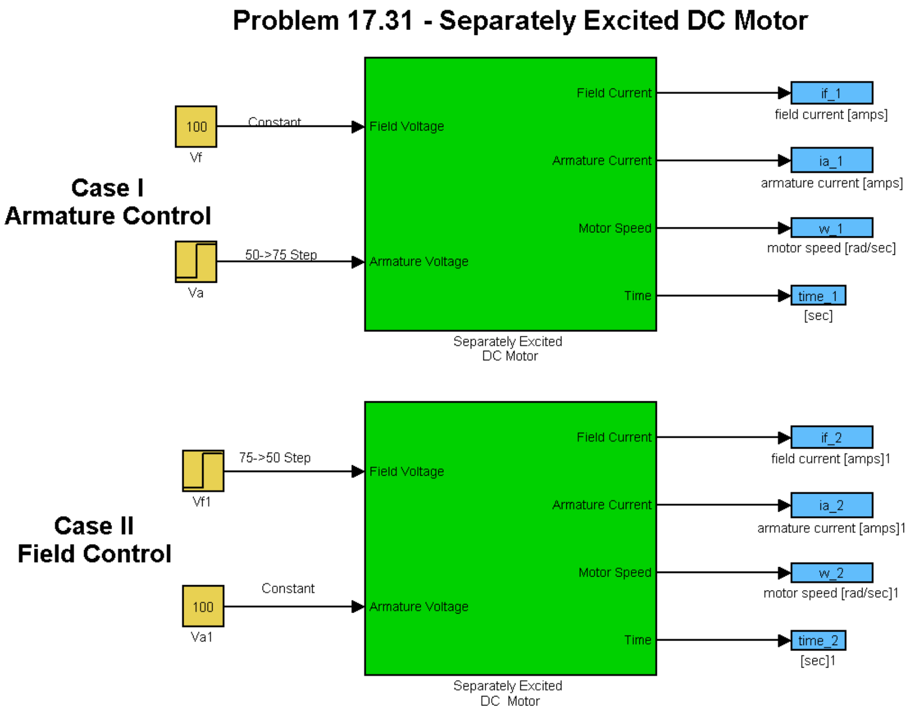
The three dynamic equations are:

$$V_f = L_f \frac{dI_f}{dt} + R_f I_f$$

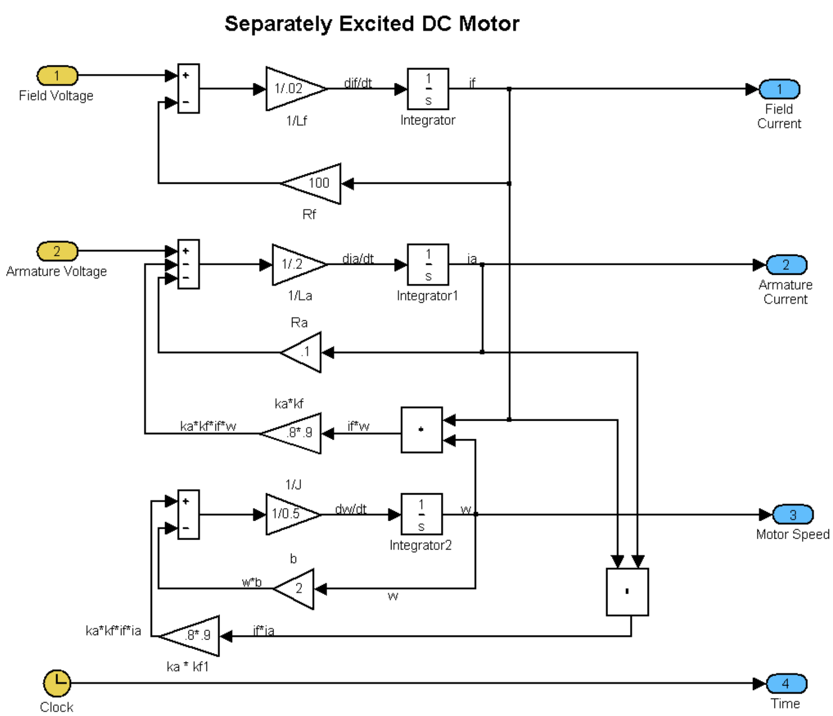
$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + E_b$$

$$J\frac{d\omega}{dt} + b\omega - T_m = J\frac{d\omega}{dt} + b\omega - k_a i_a \phi = J\frac{d\omega}{dt} + b\omega - k_a I_a k_f I_f$$

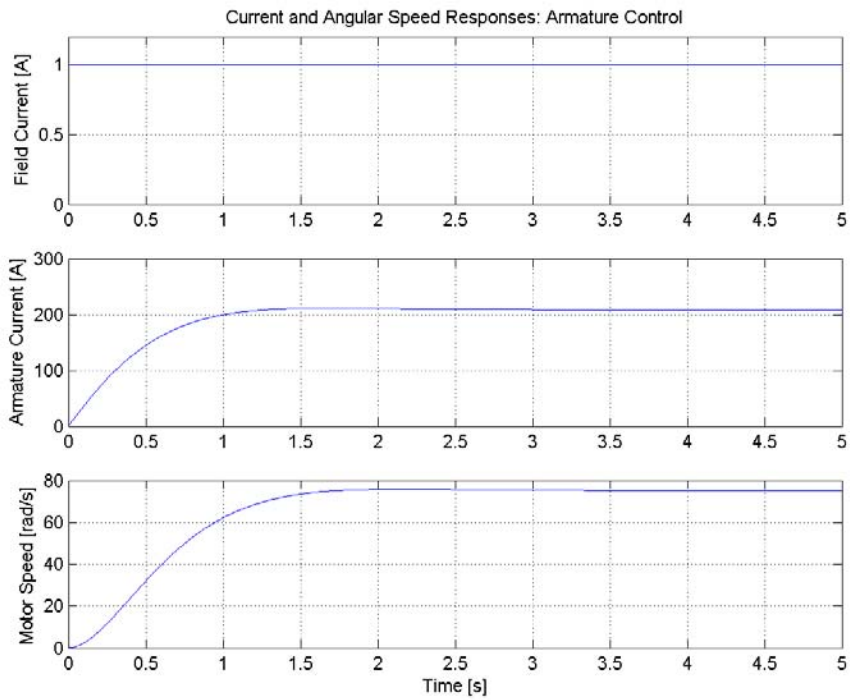
b) Simulink block diagram



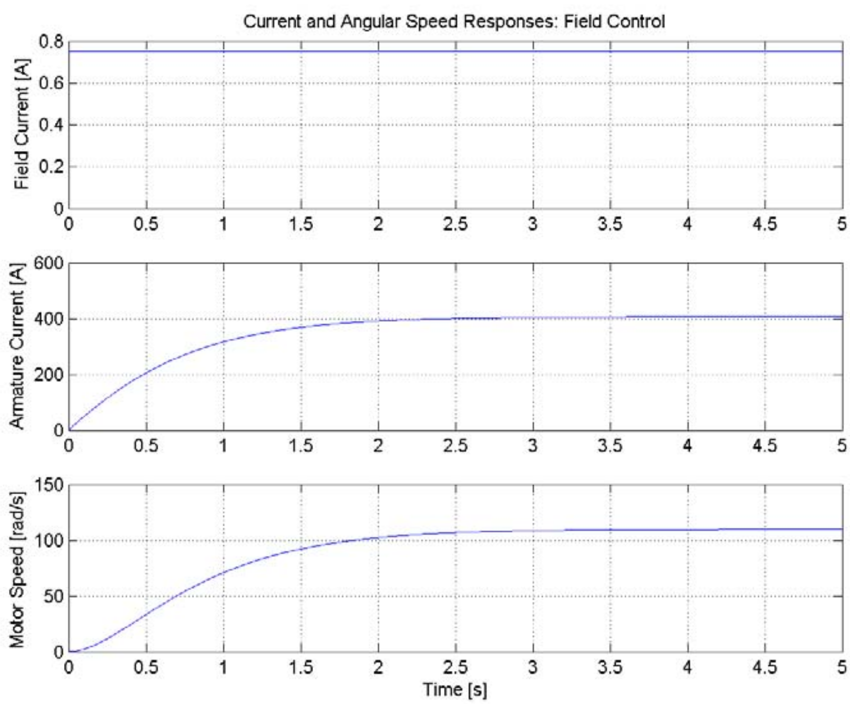
Simulink DC Motor Subsystem



d) Simulink Responses:
Armature Control:



Field Control:



Problem 19.33

Solution:

Known quantities:

R_a, L_a, $k_a = k_T$, J_m, b_m, J, b, T_L.

Find:

Transfer functions from armature voltage to angular velocity and from load torque to angular velocity.

Schematics, diagrams, circuits and given data.

See equations 19.16-18 and Figure 19.20.

Assumptions:

Analysis:

Applying KVL and equation 19.47 to the electrical circuit we obtain:

$$V_a(t) - R_a I_a(t) - L_a \frac{dI_a(t)}{dt} - E_b(t) = 0$$

$$\text{or } L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a \quad \omega_m(t) = V_a(t)$$

Applying Newton's Second Law and equation 19.46 to the load inertia, we obtain:

$$(J_m + J)\frac{d\omega(t)}{dt} = T_m(t) - T_L(t) - (b_m + b)\omega$$

$$\text{or } -k_T \quad I_a(t) + (J_m + J)\frac{d\omega(t)}{dt} + (b_m + b)\omega(t) = T_L(t)$$

To derive the transfer function, we Laplace transform the two equations to obtain:

$$(sL_a + R_a)I_a(s) + k_a \quad \Omega(s) = V_a(s)$$

$$-k_a I_a(s) + \left(s(J_m + J) + (b_m + b)\right)\Omega(s) = T_L(s)$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $\Omega_m(s)$ as a function of $V_a(s)$ and $T_L(s)$.

$$\begin{bmatrix} (sL_a + R_a) & k_a \\ -k_a & \left(s(J_m + J) + (b_m + b)\right) \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Omega_m(s) \end{bmatrix} = \begin{bmatrix} V_a(s) \\ T_L(s) \end{bmatrix}$$

$$\text{with solution } \Omega_m(s) = \frac{\det\begin{bmatrix} (sL_a + R_a) & V_a(s) \\ k_a & T_L(s) \end{bmatrix}}{\det\begin{bmatrix} (sL_a + R_a) & k_a \\ -k_a & \left(s(J_m + J) + (b_m + b)\right) \end{bmatrix}}$$

$$\text{or } \Omega_m(s) = \frac{(sL_a + R_a)}{(sL_a + R_a)\left(s(J_m + J) + (b_m + b)\right) + k_a^2} T_L(s) + \frac{k_a}{(sL_a + R_a)\left(s(J_m + J) + (b_m + b)\right) + k_a^2} V_a(s)$$

$$\text{and finally } \left.\frac{\Omega_m(s)}{T_L(s)}\right|_{V_a(s)=0} = \frac{(sL_a + R_a)}{(sL_a + R_a)\left(s(J_m + J) + (b_m + b)\right) + k_a^2}$$

$$\left.\frac{\Omega_m(s)}{V_a(s)}\right|_{T_L(s)=0} = \frac{k_a}{(sL_a + R_a)\left(s(J_m + J) + (b_m + b)\right) + k_a^2}$$

Problem 19.34

Solution:

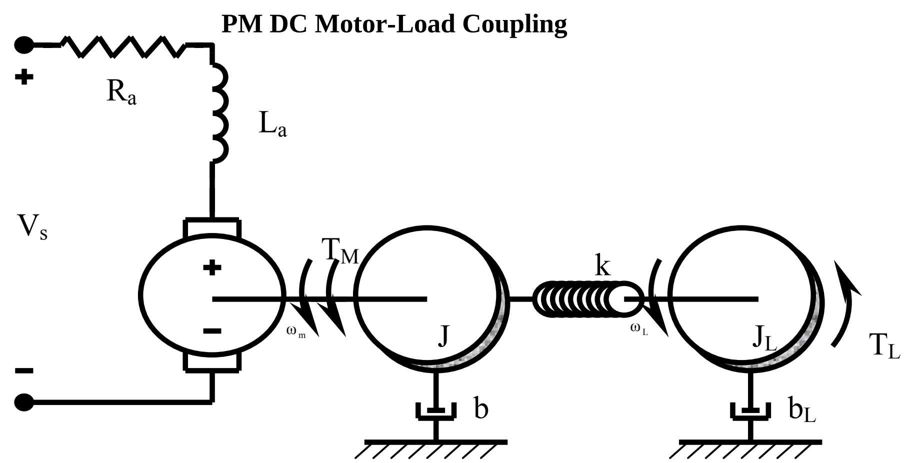
Known Quantities:
A PM DC motor that is coupled to a pump with a long shaft.

Find:
The dynamic equations of the system and the transfer function from input voltage to load inertia speed.

Assumptions:
The energy conversion is ideal.

Analysis:

Sketch:



Knowing:

$$\omega = \frac{d\theta}{dt} \qquad \theta = \int \omega\, dt = \frac{1}{s}\omega$$

Then the three dynamic equations for the system are:

$$V_a = L_a \frac{di_a}{dt} + R_a i_a + E_a = L_a \frac{di_a}{dt} + R_a i_a + k_a \omega_m$$

$$J_m \frac{d\omega_m}{dt} + b_m \omega_m + K\frac{\omega_m}{s} - k_a i_a - K\frac{\omega_L}{s} = 0$$

$$J_L \frac{d\omega_L}{dt} + b_L \omega_L + K\frac{\omega_L}{s} - K\frac{\omega_m}{s} = -T_L$$

Putting them in matrix form:

$$\begin{bmatrix} L_a s + R_a & k_a & 0 \\ -k_a & J_m s^2 + b_m s + K/s & -K/s \\ 0 & -K/s & J_L s^2 + b_L s + K/s \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \omega_L \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -T_L \end{bmatrix}$$

$$\left.\frac{\omega_L}{V_a}(s)\right|_{T_L=0} = \frac{k_a K/s}{(L_a s + R_a)(J_m s + b_m + K/s)(J_L s + b_L + K/s) - (-K/s)^2 (L_a s + R_a) + (J_L s + b_L + K/s)(-k_a)(k_a)}$$

Problem 19.35

Solution:

Known quantities:

Field and armature circuit parameters; magnetization and armature constants; motor and load inertia and damping coefficients.

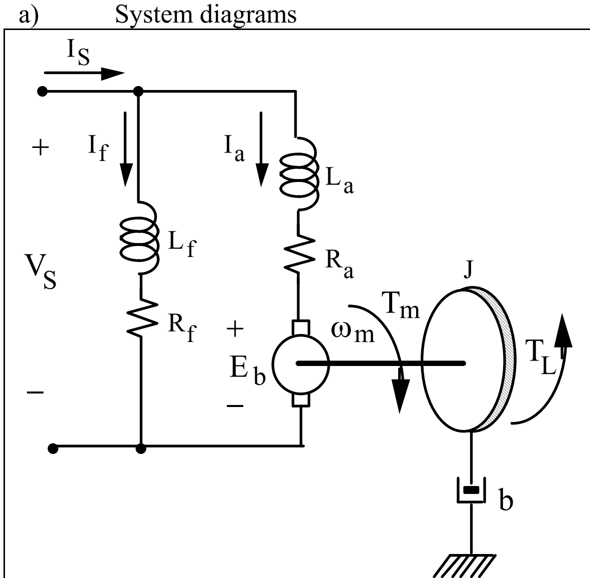
Find:

a) sketch system diagrams for shunt and series configuration
b) write expression for torque-speed curves for each configuration
c) write the differential equations for each configuration
d) determine whether equations are linear or nonlinear and how they could be linearized

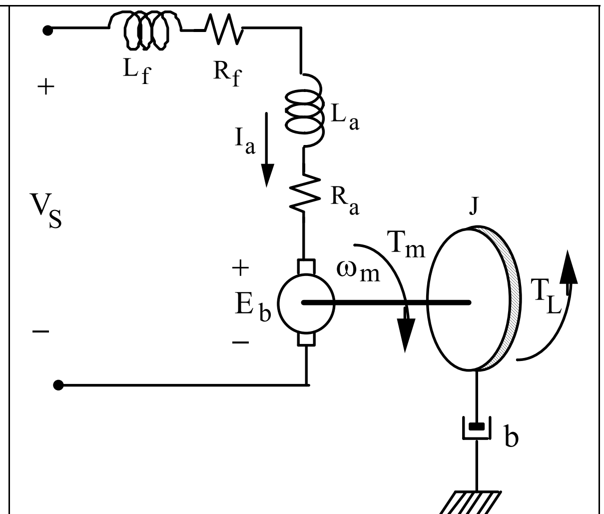
Assumptions:

Analysis:

a) System diagrams



Shunt motor-load system



Series motor-load system

b) Write expressions for the torque-speed curves

Shunt configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$V_S = R_f I_f$ field circuit

and

$V_S = R_a I_a + k_f I_f \omega_m$ armature circuit

or

$$V_S = R_a I_a + k_f \frac{V_S}{R_f}\omega_m$$

$$T_m = k_f I_f I_a = k_f \frac{V_S}{R_f} I_a = b\omega_m + T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$T_m = k_f I_f I_a$$

$$I_a = \frac{T_m}{k_f I_f} = \frac{T_m R_f}{k_f V_S}$$

and substitute the expression for I_a in the electrical circuit equation:

$$V_S = R_a I_a + k_f \frac{V_S}{R_f}\omega_m = \frac{R_a R_f}{k_f V_S} T_m + k_f \frac{V_S}{R_f}\omega_m$$

or

$$T_m = \frac{k_f V_S}{R_a R_f}\left(V_S - k_f \frac{V_S}{R_f}\omega_m\right) = \frac{k_f V_S^2}{R_a R_f} - \frac{k_f^2 V_S^2}{R_a R_f^2}\omega_m$$

Series configuration

Applying KVL and Newton's Second Law for the steady-state system we write:

$$V_S = (R_a + R_f) I_a + k_f I_a^2 \omega_m$$

$$T_m = k_f I_a^2 = b\omega_m + T_L$$

To obtain the torque-speed curve of the motor (there will also be a load torque-speed equation, but we do not have any information on the nature of the load), we write:

$$I_a = \sqrt{\frac{T_m}{k_f}}$$

$$V_S = (R_a + R_f) I_a + k_f I_a^2 \omega_m = (R_a + R_f)\sqrt{\frac{T_m}{k_f}} + T_m \omega_m$$

which leads to a quadratic equation in T_m and ω_m.

c) Write the differential equations

Shunt configuration

Applying KVL and equation 19.47 to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} + R_f I_f(t) = V_S(t) \quad \text{field circuit}$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f I_f(t)\omega_m(t) = V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 19.46 to the load inertia, we obtain:

$$J\frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m$$

or

$$-k_f I_f(t) I_a(t) + J\frac{d\omega_m(t)}{dt} + b\omega_m(t) = T_L(t)$$

Note that we have three differential equations that must be solved simultaneously. If the dynamics of the field circuit are much faster than those of the armature circuit (time constant $\frac{L_f}{R_f} << \frac{L_a}{R_a}$, as is often the case) one can assume that the field current varies instantaneously with the supply voltage, leading to $I_f = \frac{V_S}{R_f}$ and to the equations:

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_f \frac{V_S(t)}{R_f} \omega_m(t) = V_S(t)$$

$$-k_f \frac{V_S(t)}{R_f} I_a(t) + J \frac{d\omega_m(t)}{dt} + b\omega_m(t) = T_L(t)$$

Series configuration

Applying KVL and equation 19.47 to the electrical circuit we obtain:

$$\left(L_a + L_f\right)\frac{dI_a(t)}{dt} + \left(R_a + R_f\right)I_a(t) + k_f I_a(t)\omega_m(t) = V_S(t)$$

Applying Newton’s Second Law and equation 19.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m$$

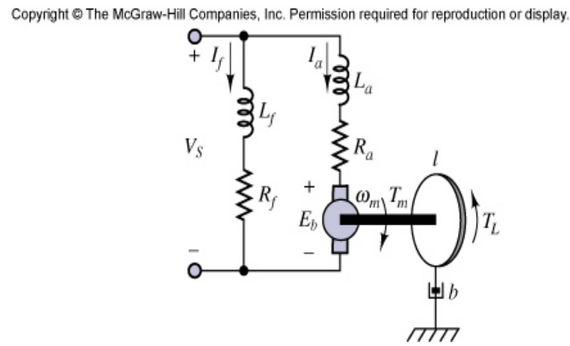
or

$$-k_f I_a^2(t) + J \frac{d\omega_m(t)}{dt} + b\omega_m(t) = T_L(t)$$

d) Determine whether the equations are nonlinear

Both systems of equations are nonlinear. In the shunt case, we have product terms in I_f and ω_μ, and in I_f and I_a (or in V_S and ω, and in V_S and I_a if we use the simplified system of two equations). In the series case, we have a quadratic term in I_a^2 and a product term in I_f and ω_m. In either case, no simple assumption leads to a linear set of equations; thus either linearization or nonlinear solution methods (e.g.: numerical simulation) must be employed.

Problem 19.36



Solution:

Known quantities:

A shunt-connected DC motor shown in Figure P19.36
Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant

Find:

Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:

None

Analysis:

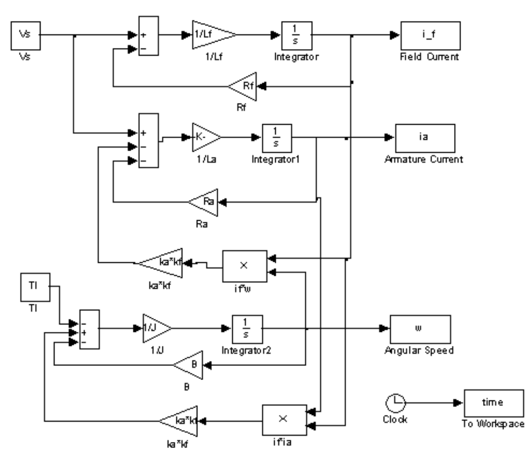
Electrical subsystem

$$V_S(t) = L_f \frac{dI_f(t)}{dt} + R_f I_f(t) \text{field}$$

$$V_S(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a \phi \omega_m(t) = L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_a k_f I_f(t) \omega_m(t) \text{armature}$$

Mechanical subsystem

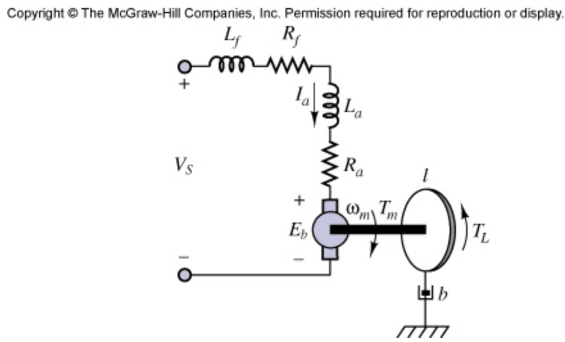
$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m(t) = k_a \phi I_a(t) - T_L(t) - b\omega_m(t) = k_a k_f I_f(t) I_a(t) - T_L(t) - b\omega_m(t)$$ Simulation block diagram:



Problem 19.37

Solution:

Known quantities:



A series-connected DC motor shown in Figure P19.37
Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant

Find:

Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:

None

Analysis:

Electrical subsystem

$$V_S(t) = (L_a + L_f)\frac{dI_a(t)}{dt} + (R_a + R_f)I_a(t) + k_a\phi\omega_m(t)$$

$$V_S(t) = L\frac{dI_a(t)}{dt} + RI_a(t) + k_a k_f I_a(t)\omega_m(t)$$

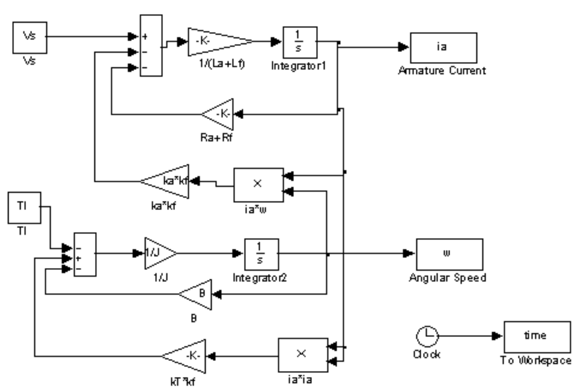
$$\frac{dI_a(t)}{dt} = \frac{1}{L}V_S(t) - \frac{R}{L}I_a(t) - \frac{k_a k_f}{L}I_a(t)\omega_m(t)$$

Mechanical subsystem

$$J\frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m(t) = k_T\phi I_a(t) - T_L(t) - b\omega_m(t) = k_T k_f I_a^2(t) - T_L(t) - b\omega_m(t)$$

$$\frac{d\omega_m(t)}{dt} = \frac{k_T k_f}{J}I_a^2(t) - \frac{1}{J}T_L(t) - \frac{b}{J}\omega_m(t)$$

Simulation block diagram:



Problem 19.38

Solution:

Known quantities:

A shunt DC motor with electrical parameters.

Find:

Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:

None

Analysis:

Differential equations
Applying KVL to the electrical circuit we obtain:

$$L_f \frac{dI_f(t)}{dt} + R_f I_f(t) = V_S(t) \quad \text{field circuit}$$

and

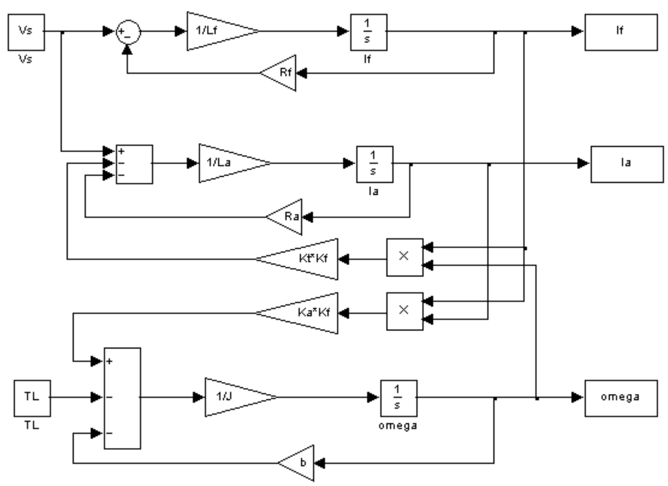
$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + k_T k_f I_f(t)\omega_m(t) = V_S(t) \quad \text{armature circuit}$$

Applying Newton's Second Law and equation 19.46 to the load inertia, we obtain:

$$J \frac{d\omega_m(t)}{dt} = T_m(t) - T_L(t) - b\omega_m$$

or

$$-k_a k_f I_f(t) I_a(t) + J \frac{d\omega_m(t)}{dt} + b\omega_m(t) = -T_L(t)$$

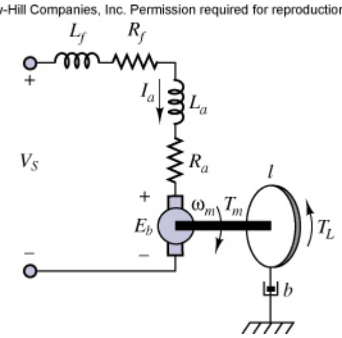


Problem 19.39

Solution:

Known quantities:

A series-connected DC motor shown in Figure P19.37
Motor parameters: k_a, k_T = armature and torque reluctance constant and k_f = field flux constant



Find:

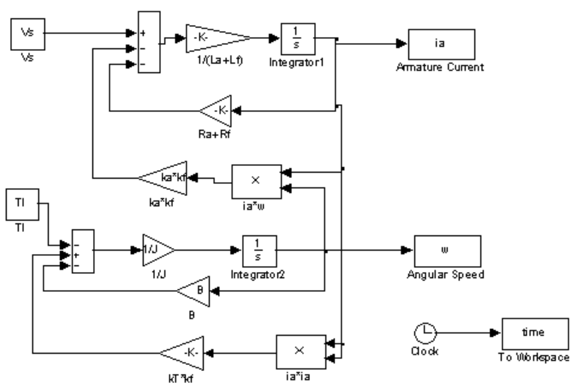
Derive the differential equations describing the electrical and mechanical dynamics of the motor
Draw a simulation block diagram of the system

Assumptions:

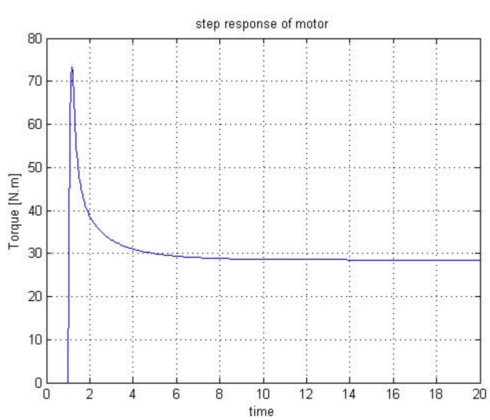
None

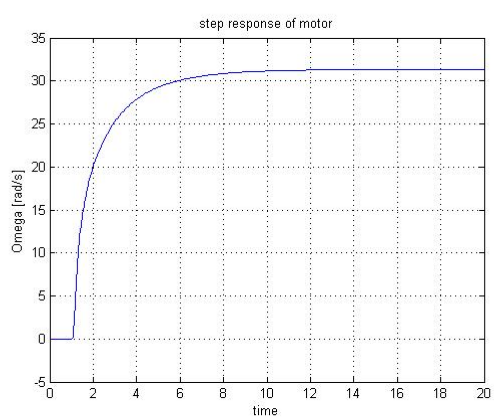
Analysis:

As shown in Problem 19.37, we can have the following Simulation block diagram:



Use a step voltage input and load torque input at 1 sec, we can have the following step response curves.
$V = 10$ V, $T_L = 5$ $N.m$





Section 19.6: The Alternator (Synchronous Generator)

Problem 19.40

Solution:

Known quantities:

A $550 V \cdot A, 20 V$ rated automotive alternator. At rated $V \cdot A$, the power factor is 0.85. The resistance per phase is 0.05Ω. The field takes $2 A$ at $12 V$. The friction and windage loss is $25W$ and core loss is $30W$.

Find:

The percent efficiency under rated conditions.

Assumptions:

None.

Analysis:

$$I_a = \frac{500}{20} = 25 A \quad P_a = I_a^2 R_a = 31.25W$$

$$P_{out} = 500(0.85) = 425W \quad P_f = 2(12) = 24W \quad P_{in} = P_{out} + P_a + 25 + 30 + 24 = 535.25W$$

$$\% = \frac{425}{535.25} \times 100 = 79.4\%$$

Problem 19.41

Solution:

Known quantities:

A three-phase $2300V, 500 kV \cdot A$ synchronous generator. $X_S = 8.0\Omega, r_a = 0.1\Omega$. The machine is operating at rated load and voltage at a power factor of 0.867 lagging.

Find:

The generated voltage per phase and the torque angle.

Assumptions:

None.

Analysis:

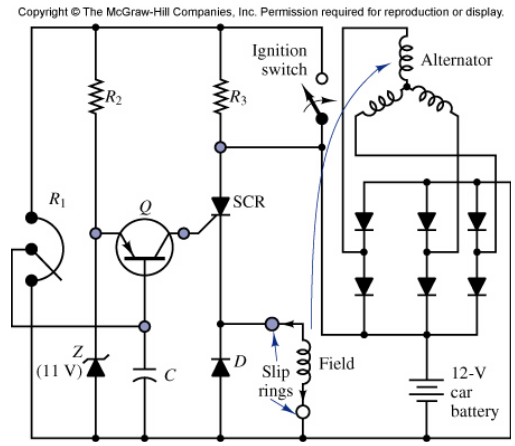
$$I = \frac{500k}{\sqrt{3}(2300)} = 125.5 A$$

$$E = \frac{2300}{\sqrt{3}} \angle 0^\circ + 125.5\angle -30^\circ (0.1 + j0.8) = 1327.9 + 101.2\angle 52.9^\circ = 1389 + j80.7 = 1391.3\angle 3.3^\circ V$$

$$\therefore E = 1391.3V \quad \delta = 3.3^\circ$$

Problem 19.42

Solution:



Known quantities:

As shown in Figure P19.42.

Find:

Explain the function of Q, D, Z, and SCR.

Assumptions:

None.

Analysis:

Q : The setting of R_1 determines the biasing of Q. When Q conducts, the SCR will fire, energizing the alternator's field.

D : This diode serves as a "free-wheeling" element, allowing the field current to circulate without interfering with the commutation of the SCR.

Z : The Zener diode provides a fixed reference voltage at the emitter of transistor Q; i.e., determination of when Q conducts is controlled solely by the setting of R_1.

SCR : The SCR acts as a half-wave rectifier, providing field excitation for the alternator. Without the field, of course, the alternator cannot generate.

Section 19.7: The Synchronous Motor

Problem 19.43

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, two-pole synchronous machine. The synchronous reactance is 7Ω and the resistance and rotational losses are negligible. One point on the open-circuit characteristic is given by $V_0 = 400V$ (phase voltage) for a field current of $3.32\,A$. The machine operates as a motor, with a terminal voltage of $400V$ (phase voltage). The armature current is $50\,A$, with power factor 0.85 leading.

Find:

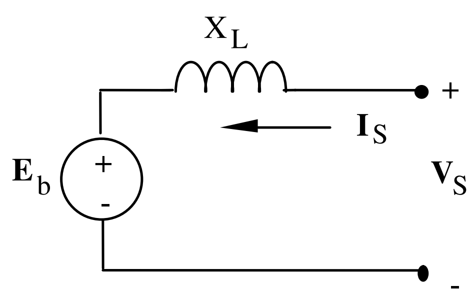
E_b, field current, torque developed, and power angle δ.

Assumptions:

None.

Analysis:

The per phase circuit is shown below:



Since the power factor is 0.85, we have:

$$\theta = 31.79^\circ$$

$$\omega_m = \frac{2\pi}{60}3600 = 377\,rad/\sec$$

From $V_{OC} = 400V$, we have

$$E_b = 400V\,(open\,circuit) = k\omega_m i_f$$

Therefore $k = \dfrac{400}{377\times 3.32} = 0.3196$

$$E_b = 400\angle 0^\circ - 50\angle 31.79^\circ \times 7\angle 90^\circ$$
$$= 400 + 184.38 - j297.49$$
$$= 655.74\angle -26.98^\circ\,V$$

$$i_f = \frac{E_b}{120.48} = 5.44\,A$$

$$\theta_T = 31.79^\circ + 26.98^\circ = 58.77^\circ$$

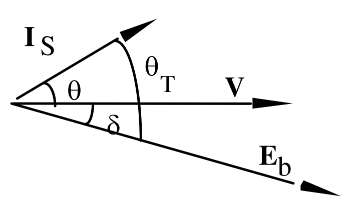
The torque developed is:

$$T = \frac{3}{377}|E_b||I_S|\cos\theta_T = 135.27\,N\cdot m$$

δ is the angle from V to E_b:

$$\delta = -26.98^\circ$$

The phase diagram is shown below:



Problem 19.44

Solution:

Known quantities:

A factory load of $900\,kW$ at 0.6 power factor lagging is increased by adding a $450\,kW$ synchronous motor.

Find:

The power factor this motor operates at and the KVA input if the overall power factor is 0.9 lagging.

Assumptions:

None.

Analysis:

$P_{old} = 900\,kW \qquad Q_{old} = 1200\,kVAR$

$P_m = 450\,kW \qquad P_T = 1350\,kW \qquad Q_T = 653.8\,kVAR$

$Q_m = 653.8 - 1200 = -546.2\,kVAR \qquad pf_m = \cos(\tan^{-1}\frac{Q_m}{P_m}) = 0.636\,leading$

$S_m = P_m / pf_m = 708\,kVA$

Problem 19.45

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, two-pole synchronous generator is connected to a $400V$ (line to line), $60\,Hz$, three-phase line. The stator impedance is $0.5 + j1.6$ (per phase). The generator is delivering rated current $36\,A$ at unity power factor to the line.

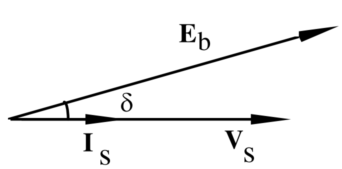
Find:

The power angle for this load and the value of E_b for this condition. Sketch the phasor diagram, showing E_b, I_S, and V_S.

Assumptions:

None.

Analysis:



$V_L = \frac{400}{\sqrt{3}}\angle 0^\circ = 230.9\angle 0^\circ\ V \qquad I_L = 36\angle 0^\circ\ A$

$Z_S = 0.5 + j1.6 = 1.676\angle 72.65^\circ\ \Omega$

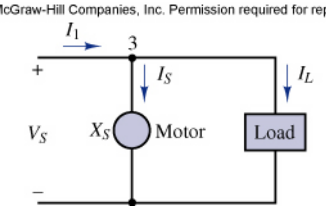
$E_b = V_L + I_L Z_S = 248.9 + j57.6 = 255.5\angle 13.03^\circ\ V$

The power angle is 13.03°.

Problem 19.46

Solution:

Known quantities:



A non-salient pole, three phase, two-pole synchronous generator is connected in parallel with a three-phase, Y-connected load. The equivalent circuit is shown in Figure P19.46. The parallel combination is connected to a $220V$ (line to line), , three-phase line. The load current is $25\,A$ at a power factor of 0.866 inductive. $X_S = 2\Omega$. The motor is operating with $I_f = 1A, T = 50\,N \cdot m$ at a power angle of -30°.

Find:

I_S, P_{in} (to the motor), the overall power factor and the total power drawn from the line.

Assumptions:

Neglect all losses for the motor.

Analysis:

The phasor per-phase voltage is:

$$V_S = 127\angle 0^\circ\,V$$

$$T_{dev} = 50\,N \cdot m = -\frac{3}{377}\frac{|E_b||V_S|}{X_S}\sin\delta$$

Therefore,

$$|E_b| = -\frac{50(377)2}{3(127)\sin(-30^\circ)} = 197.9V$$

$$E_b = 197.9\angle -30^\circ\,V$$

For $i_f = 1A$,

$$I_S = 49.47 + j22.2 = 54.23\angle 24.16^\circ\,A$$

The load current is:

$$I_L = 25\angle -\cos^{-1}0.866 = 21.65 - j12.5$$

and

$$I_1 = I_L + I_S = 71.12 + j9.7 = 71.78\angle 7.77^\circ\,A$$

$$P_{in\_motor} = 3 \times 54.23 \times 127 \times \cos 24.16^\circ = 18.85\,kW$$

$$P_{in\_total} = 3 \times 71.78 \times 127 \times \cos 7.77^\circ = 27.10\,kW$$

The power factor is:

$$pf = \cos 7.77^\circ = 0.991\,leading$$

Problem 19.47

Solution:

Known quantities:

A non-salient pole, Y-connected, three phase, four-pole synchronous machine. The synchronous reactance is 10Ω. It is connected to a $230\sqrt{3}V$ (line to line), $60\,Hz$, three-phase line. The load requires a torque of $T_{shaft} = 30\,N \cdot m$. The line current is $15\,A$ leading the phase voltage.

Find:

The power angle δ and E for this condition. The line current when the load is removed. Is it leading or lagging the voltage.

Assumptions:

All losses can be neglected.

Analysis:

At $\omega_m = 188.5\,rad/\sec$, we can calculate

$$P_{out} = 30 \times 188.5 = 5655W$$

Since $P_{in} = P_{out}$ and $P_{in}^{'}(per\ phase) = 1885W = 230 \times 15\cos\theta$, we calculate

$$\theta = \cos^{-1} 0.5464 = 56.88^\circ$$

Since $V_S = 230\angle 0^\circ\,V$, $I_S = 15\angle 56.88^\circ\,A$

$$E_b = 355.6 - j81.96 = 364.92\angle -12.98^\circ\,V$$

The power angle is:

$$P_{in} = P_{out}$$

If the load is removed, the power angle is 0° and from

$$364.92\angle 0^\circ = 230\angle 0^\circ - 10\angle 90^\circ$$

$$\Rightarrow I = 13.495\angle 90^\circ\,A$$

The current is leading the voltage.

Problem 19.48

Solution:

Known quantities:

A $10\,hp, 230V, 60\,Hz$ Y-connected, three phase synchronous motor delivers full load at a power factor of 0.8 leading. The synchronous reactance is 6Ω. The rotational loss is $230W$, and the field loss is $50W$.

Find:

a) The armature current.
b) The motor efficiency.
c) The power angle.

Assumptions:

Neglect the stator winding resistance.

Analysis:

$$P_{out} = 10\,hp = 7460W$$

$$P_{in} = P_{out} + P_r + P_{copper} = 7740$$

$$\therefore P_{in}(per\ phase) = 2580 = V_S I_S 0.8$$

$$V_S = \frac{230}{\sqrt{3}} = 132.8V$$

$$\therefore I_S = \frac{2580}{132.8 \times 0.8} = 24.3\,A$$

That is:

$$V_S = 132.8\angle 0^\circ V,\ I_S = 24.3\angle 36.87^\circ\,A$$

$$E_b = V_S - I_S(6\angle 90^\circ) = 249.2\angle -27.9^\circ V$$

a)

$$I_S = 24.3\angle 36.87^\circ\,A$$

b)

$$efficiency = \frac{7460}{7740} = 0.964 = 96.4\%$$

c)

$$power\,angle = -27.9^\circ$$

Problem 19.49

Solution:

Known quantities:

A three-phase $2300 V, 60 Hz, 30\ poles, 2000 hp$, unity power factor synchronous motor. $X_S = 1.95 \Omega$ per phase.

Find:

The maximum power and torque.

Assumptions:

Neglect all losses.

Analysis:

$$n_S = \frac{3600}{15} = 240\, rev/\min$$

$$\omega_S = 25.13\, rad/\sec$$

At full load,

$$P_{in} = 746 \times 2000 = 1.492\, MW$$

$$V_S = \frac{2300}{\sqrt{3}} = 1327.9\angle 0^\circ\, V$$

For unity power factor,

$$I_S = 374.5\angle 0^\circ\, A$$

$$E_b = V_S - I_S jX_S = 1327.9 - j730.3$$

$$= 1515.5\angle -28.2^\circ\, V$$

The maximum power and torque are:

$$P_{\max} = 3\frac{|E_b||V_S|}{X_S} = 3.096\, MW$$

$$T_{\max} = \frac{P_{\max}}{\omega_S} = 123.2\, kN \cdot m$$

Problem 19.50

Solution:

Known quantities:

A $1200V$ Y-connected, three phase synchronous motor takes $110\,kW$ when operated under a certain load at $1200\,rev/\min$. The back emf of the motor is $2000V$. The synchronous reactance is 10Ω per phase.

Find:

The line current and the torque developed by the motor.

Assumptions:

Winding resistance is negligible.

Analysis:

$$V_S = \frac{1200}{\sqrt{3}} = 692.8\angle 0^\circ$$

The input power per phase is:

$$L = \frac{N^2}{\Re_T} = \frac{100^2}{12.51\times 10^3} = 0.8\,H$$

The power developed is:

$$P = -3\frac{|E_b||V_S|}{X_S}\sin\delta$$

$$\therefore \sin\delta = -0.2646$$

$$\delta = -15.34^\circ$$

The torque developed is:

$$T = \frac{P}{\omega_S} = 875.1\,N\cdot m$$

Problem 19.51

Solution:

Known quantities:

A $600V$ Y-connected, three phase synchronous motor takes $24kW$ at a leading power factor of 0.707. The per-phase impedance is $5 + j50\Omega$.

Find:

The induced voltage and the power angle of the motor.

Assumptions:

None.

Analysis:

$$V_S = \frac{600}{\sqrt{3}} = 346.4\angle 0^\circ V$$

$$Z_S = 5 + j50 = 50.25\angle 84.29^\circ \Omega$$

From $pf = 0.707$, we have $\theta = 45^\circ$.

From $P_{in} = 3|V_S||I_S \cos\theta|$, we have

$$|I_S| = \frac{24\times 10^3}{3\times 346.4}\times 0.707 = 32.67\,A$$

$$I_S = 32.67\angle 45^\circ\,A$$

$$E_b = V_S - I_S Z_S = 1385 - j1270.6$$

$$= 1880.3\angle -42.51^\circ V$$

The power angle is:

$$\delta = -42.51^\circ$$

The power developed and the copper loss are:

$$P_{dev} = 3|E_b||I_S|\cos 87.51^\circ = 8.006\,kW$$

$$P_{loss} = 3|I_S|^2 R_S = 16.01\,kW$$

Section 19.8: The Induction Motor

Problem 19.52

Solution:

Known quantities:

A $74.6\,kW$ three-phase, $440V$ (line to line), four-pole, $60\,Hz$ induction motor. The equivalent circuit parameters are:

$R_S = 0.06\,\Omega \qquad R_R = 0.08\,\Omega$

$X_S = 0.3\,\Omega \qquad X_R = 0.3\,\Omega \qquad X_m = 5\,\Omega$

The no-load power input is $3240W$ at a current of $45\,A$.

Find:

The line current, the input power, the developed torque, the shaft torque, and the efficiency at $s = 0.02$.

Assumptions:

None.

Analysis:

$$V_S = \frac{400}{\sqrt{3}} = 254\angle 0^\circ\ V$$

$$Z_{in} = 0.06 + j0.3 + \frac{j5(4 + j0.3)}{4 + j5.3} = 2.328 + j2.294 = 3.268\angle 44.59^\circ\ \Omega$$

$$I_S = 77.7\angle -44.59\,A$$

$$P_{in} = 3 \times 254 \times 77.7\cos(-44.59^\circ) = 42.16\,kW$$

$$I_2 = \frac{j5}{4 + j5.3} I_S = 58.51\angle -7.55^\circ\ A$$

The total power transferred to the rotor is:

$$P_T = 3\frac{R_S}{S}\left|I_2\right|^2 = 41.1\,kW$$

$$P_m = P_T - P_{copper\_loss\_in\_rotor} \quad = 41.1 \times 10^3 (1 - s) = 40.25\,kW$$

$$\omega_m = (1 - s)\omega_S = 0.98 \times 188.5 = 184.7\,rad/\text{sec}$$

Therefore, the torque developed is: $T_{dev} = \frac{P_m}{184.7} = 218\,N \cdot m \quad = 1880.3\angle -42.51^\circ\ V$

The rotational power and torque losses are: $P_{rot} = 3240 - 3 \times 45^2 \times 0.06 = 2875.5W \qquad T_{rot} = 15.56\,N \cdot m$

The shaft torque is: $T_{sh} = 218 - 15.56 = 202.4\,N \cdot m$

Efficiency is: $T_{sh} = \frac{P_{out}}{P_{in}} = \frac{202.4 \times 184.7}{42.16 \times 10^3} = 0.887$

Problem 19.53

Solution:

Known quantities:

A $60\,Hz$, four-pole, Y-connected induction motor is connected to a three-phase, $400V$ (line to line), $60\,Hz$ line. The equivalent circuit parameters are:

$R_S = 0.2\,\Omega$ $R_R = 0.1\,\Omega$

$X_S = 0.5\,\Omega$ $X_R = 0.2\,\Omega$ $X_m = 20\,\Omega$

When the machine is running at $1755\,rev/\min$, the total rotational and stray-load losses are $800W$.

Find:

The slip, input current, total input power, mechanical power developed, shaft torque and efficiency.

Assumptions:

None.

Analysis:

From $n_S = 1800\,rev/\min$, we have

$$s = 0.025$$

$$\frac{R_R}{s} = 4$$

$$Z_{in} = 0.2 + j0.5 + \frac{j20(4 + j0.2)}{4 + j20.2}$$

$$= 3.972 + j1.444 = 4.226\angle 19.98^\circ\,\Omega$$

Therefore,

$$I_S = 54.6\angle -19.98^\circ\,A$$

$$P_{in} = 3(54.6)(\frac{400}{\sqrt{3}}\cos(-19.98^\circ)) = 35.6\,kW$$

$$P_t = P_{in} = 3\left|I_S\right|^2 R_S$$

$$= 35.6\times 10^3 - 3(54.6)^2 \times 0.2 = 33.81\,kW$$

$$P_m = (1-s)P_t = 32.97\,kW$$

$$P_{sh} = P_{out} = P_m - 800 = 32.17\,kW$$

$$\omega_m = 183.8\,rad/\sec$$

$$T_{sh} = 175\,N\cdot m$$

$$efficiency = \frac{32.17}{35.6} = 0.904$$

Problem 19.54

Solution:

Known quantities:

A three-phase, $60\,Hz$, eight-pole induction motor operates with a slip of 0.05 for a certain load.

Find:

a) The speed of the rotor with respect to the stator.
b) The speed of the rotor with respect to the stator magnetic field.
c) The speed of the rotor magnetic field with respect to the rotor.
d) The speed of the rotor magnetic field with respect to the stator magnetic field.

Assumptions:

None.

Analysis: $n_S = 900\,rev/\min$, $\omega_S = 94.25\,rad/\sec$

a) $n_m = (1-s)n_S = 855\,rev/\min$

b) The speed of the stator field is $900\,rev/\min$, the rotor speed relative to the stator field is $-45\,rev/\min$.

c) $45\,rev/\min$

d) $0\,rev/\min$

Problem 19.55

Solution:

Known quantities:

A three-phase, $60\,Hz$, $400V$ (per phase), two-pole induction motor develops $P_m = 37\,kW$ at a certain speed. The rotational loss at this speed is $800W$.

Find:

a) The slip and the output torque if the total power transferred to the rotor is $40\,kW$.
b) I_S and the power factor if $P_m = 45\,kW, R_S = 0.5\,\Omega$.

Assumptions:

Stray-load loss is negligible.

Analysis:

$$P_m = 3(1-s)P_t = 37\,kW$$

$$1-s = 0.925 \Rightarrow s = 0.075$$

a) $n_S = 3600\,rev/\min$, $\omega_S = 377\,rad/\sec$ $\omega_m = (1-s)\omega_S = 348.7\,rad/\sec$

$$P_{sh} = P_{out} = 37 - 0.8 = 36.2\,kW$$

$$T_{sh} = \frac{P_{sh}}{348.7} = 103.8\,N \cdot m$$

b) The power factor is: $\cos\theta = 0.65\,lagging$

Problem 19.56

Solution:

Known quantities:

The nameplate speed of a $25\,Hz$ induction motor is $720\,rev/\min$. The speed at no load is $745\,rev/\min$.

Find:

a) The slip.
b) The percent regulation.

Assumptions:

None.

Analysis:

a)

$$p \approx \frac{120(25)}{720} = 4.17 \Rightarrow p = 4$$

$$n_{sync} = \frac{120(25)}{4} = 750\,rpm$$

$$slip = \frac{750-720}{750} = 0.04 = 4\%$$

b)

$$reg = \frac{745-720}{720} = 0.035 = 3.5\%$$

Problem 19.57

Solution:

Known quantities:

The name plate of a squirrel-cage four-pole induction motor has $25\,hp, 220\,V, 60\,Hz, 830\,rev/\min, 64\,A$, three-phase line current. The motor draws $20{,}800\,W$ when operating at a full load.

Find:

a) slip.
b) Percent regulation if the no-load speed is $895\,rpm$.
c) Power factor.
d) Torque.
e) Efficiency.

Assumptions:

None.

Analysis:

$$n_{sync} = 900\,rpm$$

a) $$slip = \frac{900-830}{900} = 0.078 = 7.8\%$$

b) $$reg = \frac{895-830}{830} = 0.078 = 7.8\%$$

c) $$pf = \frac{20{,}800}{\sqrt{3}(220)(64)} = 0.853\,lagging$$

d) $$T = \frac{7.04(25\times 746)}{830} = 158.2\,lb\cdot ft$$

e) $$eff = \frac{25\times 746}{20{,}800} = 0.897 = 89.7\%$$

Problem 19.58

Solution:

Known quantities:

A $60\,Hz$, four-pole, Y-connected induction motor is connected to a $200V$ (line to line), three-phase, $60\,Hz$ line.
The equivalent circuit parameters are:

$R_S = 0.48\,\Omega$ Rotational loss torque $= 3.5\,N \cdot m$

$X_S = 0.8\,\Omega$ $R_R = 0.42\,\Omega$ (referred to the stator)

$X_m = 30\,\Omega$ $X_R = 0.8\,\Omega$ (referred to the stator)

The motor is operating at slip $s = 0.04$.

Find:

The input current, input power, mechanical power, and shaft torque.

Assumptions:

Stray-load losses are negligible.

Analysis:

$$V_S = 115.5V$$

$$\omega_m = (1-s)188.5 = 181\,rad/\sec$$

$$Z_{in} = 0.48 + j0.8 + \frac{j30(10.5 + j0.8)}{10.5 + j30.8} = 9.404 + j4.63 = 10.48\angle 26.2^\circ$$

$$\therefore I_S = 11.02\angle -26.2^\circ\,A$$

$$P_{in}\,(\text{per phase}) = 115.5 \times 11.02 \times \cos(-26.2^\circ)$$

$$P_{in}\,(\text{total}) = 3426\,W$$

$$P_t = P_{in}\,(\text{total}) - 3R_S\left|I_S\right|^2 = 3251\,W \quad \therefore P_m = (1-s)P_t = 3121\,W$$

$$T_{sh} = \frac{3121}{181} = 17.24\,N \cdot m$$

Problem 19.59

Solution:

Known quantities:

a) A three-phase, $220V$, $60\,Hz$ induction motor runs at $1140\,rev/\min$.

b) A three-phase squirrel-cage induction motor is started by reducing the line voltage to $V_S/2$ in order to reduce the starting current.

Find:

a) The number of poles (for minimum slip), the slip, and the frequency of the rotor currents.
b) The factor the starting torque and the starting current reduced.

Assumptions:

None.

Analysis:

a) For minimum slip, the synchronous speed, $\frac{3600}{p/2}$, should be as close as possible to $1140\,rev/\min$,

therefore, $n_S = 1200$, $p = 6\ poles$ $s = (1200 - 1140)/1200 = 0.05$ $f_{rotor} = 3\,Hz$

b) If the line voltage is reduced to half, the starting current is reduced by a factor of 2. The developed torque is proportional to $|I_S|^2$. Therefore, the starting torque is reduced by a factor of 4.

Problem 19.60

Solution:

Known quantities:

A six-pole induction machine has a $50\,kW$ rating and is 85 percent efficient. If the supply is $220V$ at $60\,Hz$.

6 poles
60 Hz 50 kW 85% efficient 220 Volt 4% slip

Find:

The motor speed and torque at a slip $s = 0.04$·

Assumptions:

None.

Analysis:

a) $n_s = \frac{120f}{p} = \frac{120 \times 60}{6} = 1200 rev/\min$ @ slip of 4% $n = n_s(1-s) = (1200\text{ rev/min})(1-0.04) = 1152\text{ rev/min}$

b) $P_{out} = P_{in} \times \text{efficiency} = (50kW)(0.85) = 42.5\text{ kW}$ $T_{out} = \frac{P_{out}}{\omega} = \frac{42500\text{ W}}{1152\text{ rev/min}} * \frac{1\text{ rev}}{2\pi\text{ rad}}\frac{60\text{ sec}}{\text{min}} = 352.3\text{ N - m}$

Problem 19.61

Solution:

Known quantities:

6 poles 60 Hz 240 Volt rms
10% slip
Torque = 60 N-m

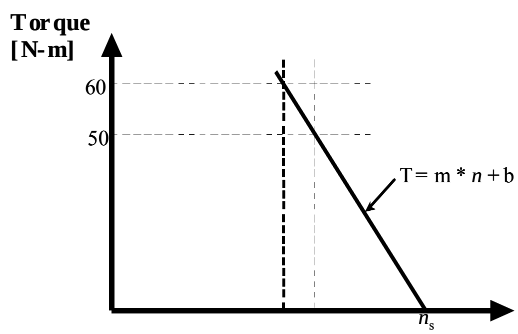
Find:

a) The speed and the slip of the induction machine if a load torque of 50 N-m opposes the motor.
b) The rms current when the induction machine is operating under the load conditions of part a.

Assumptions:

The speed torque curve is linear in the region of our interests.

Analysis:



a) $$n_s = \frac{120 f}{p} = \frac{120 \times 60}{6} = 1200 rev/\min$$

@ slip of 4%

$$n = n_s(1-s) = (1200 \text{ rev/ min})(1-0.04) = 1152 \text{ rev/ min}$$

torque

$$T = m \cdot n + b$$

$$m = \frac{60-0 \text{ N}-\text{m}}{1080-1200 \text{ rev/min}} = -0.5\frac{\text{N}-\text{m}}{\text{rev/min}}$$

$$b = T - m\cdot n = (60 \text{ N}-\text{m}) - \left(-0.5\frac{\text{N}-\text{m}}{\text{rev/min}}\cdot 1080 \text{ rev/min}\right) = 600 \text{ N - m}$$

motor speed (@ 50 N-m)

$$n = \frac{(T-b)}{m} = \frac{\text{rev/min}}{.5 \text{ N}-\text{m}}[(50 \text{ N - m}) - (600 \text{ N - m})] = 1000 \text{ rev/min}$$

n

slip (@ 50 N-m)

$$s = \frac{n_s - n}{n_s} = \frac{1200-1100}{1200} = .0833 \Rightarrow 8.33\%$$

b)

Output Power

$$P_{out} = T\omega = (50 \text{ N - m})\left(1100\frac{\text{rev}}{\text{min}}\frac{2\pi \text{ rad}}{\text{rev}}\frac{1\text{min}}{60\text{sec}}\right) = 5760\,W$$

Input Power

$$P_{in} = \frac{P_{out}}{\text{efficiency}} = I_{rms}\cdot V_{rms}$$

Current

$$I_{rms} = \frac{P_{out}}{\text{efficiency}\cdot V_{rms}} = \frac{5760 \text{ W}}{(0.92)(240 \text{ volts})} = 26.1 \text{amps}$$

Problem 19.62

Solution:

Known quantities:

A three-phase, $5\,hp$, $220V$, $60\,Hz$ induction motor. $V = 8V, I = 18\,A, P = 610W$.

Find:

a) The equivalent stator resistance per phase, R_S.
b) The equivalent rotor resistance per phase, R_R.
c) The equivalent blocked-rotor reactance per phase, X_R.

Assumptions:

None.

Analysis:

a)

$$R_S = \frac{1}{2}\frac{P_{BR}}{3I_{BR}^2} = 0.314\,\Omega$$

b)

$$R_R = 0.314\,\Omega$$

c)

$$Z_S = \frac{V_{BR}/\sqrt{3}}{I_{BR}} = \frac{48/\sqrt{3}}{18} = 1.54\,\Omega$$

$$X_R = \sqrt{Z_S^2 - R^2} = \sqrt{(1.54)^2 - (0.628)^2} = 1.4\,\Omega$$

Problem 19.63

Solution:

Known quantities:

The starting torque equation is:

$$T = \frac{m}{\omega_e} \cdot V_S^2 \cdot \frac{R_R}{(R_R + R_S)^2 + (X_R + X_S)^2}$$

Find:

a) The starting torque when it is started at $220V$.

b) The starting torque when it is started at $110V$.

Assumptions:

None.

Analysis:

a)

$$T = \frac{1}{\omega_S} \frac{q_1 V_1^2 (R_R/s)}{R^2 + X^2}; \qquad s = 1$$

$$\therefore T = \frac{1}{377} \frac{3(127)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 17.1\, N \cdot m$$

b)

$$T = \frac{1}{377} \frac{3(63.5)^2 (0.314)}{(0.628)^2 + (1.4)^2} = 4.28\, N \cdot m$$

Problem 19.64

Solution:

Known quantities:

A four-pole, three-phase induction motor drives a turbine load with torque-speed characteristic given by

$T_L = 20 + 0.006\varpi^2$

At a certain operating point, the machine has 4% slip and 87% efficiency.

Find:

Torque at the motor-turbine shaft
Total power delivered to the turbine
Total power consumed by the motor

Assumptions:

Motor run by 60-Hz power supply

Analysis:

Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120f}{P} = \frac{60s/\min \times 60r/s}{4/2} = 1800r/\min$$

$$\varpi_s = 1800rev/\min \times \frac{2\pi rad/rev}{60s/\min} = 188.5rad/s$$

Rotor mechanical speed at 4% slip:

$$\varpi_m = (1-s)\varpi_s = (1-0.04)(188.5rad/s) = 181.0rad/s$$

Load torque at the shaft:

$$T_L = 20 + 0.006(181.0rad/s)^2 = 216N-m$$

Total power delivered to the turbine:

$$P = T_L\varpi_m = (216N-m)(181.0rad/s) = 39.1kW$$

Total power consumed by the motor:

$$P_m = \frac{P}{\eta} = \frac{39.1kw}{0.87} = 44.9kW$$

Problem 19.65

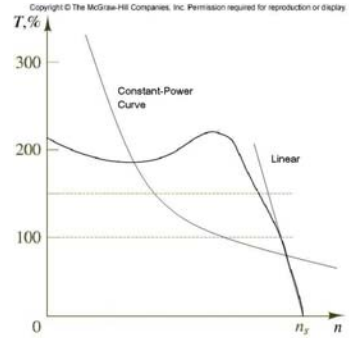
Solution:

Known quantities:

A four-pole, three-phase induction motor rotates at 1700 r/min when the load is 100 N-m. The motor is 88% efficient.

Find:

a) Slip
b) For a constant-power, 10-kW load, the operating speed of the machine
c) Total power consumed by the motor
d) Sketch the motor and load torque-speed curves on the same graph. Show numerical values.



Assumptions:

Motor run by 60-Hz power supply

Analysis:

a) Synchronous speed of four-pole induction motor at 60-Hz:

$$n_s = \frac{120f}{p} = \frac{60s/\min \times 60r/s}{4/2} = 1800r/\min$$

Slip: $s = \dfrac{n_s - n}{n_s} = \dfrac{1800r/\min - 1700r/\min}{1800r/\min} = 0.056 = 5.6\%$

b) Operating speed of machine for a constant-power load of 10-kW. Assume that it is linear around the original operating point and ideal synchronous speed point. We can use a line to approximate the torque-speed curve of the motor, and then use the intersection of such line and the constant power curve to decide the operating point.

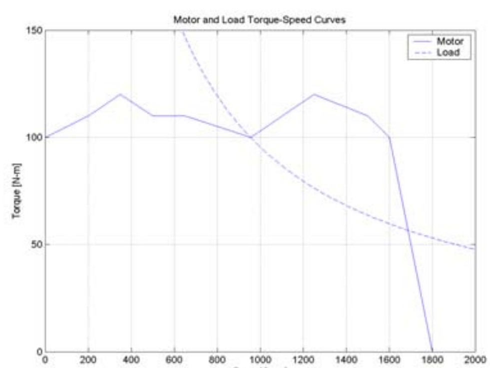
The line connects original operating point and ideal synchronous speed point satisfies:

$T_L = \dfrac{100-0}{1700-1800}(n-1800)\ N.m$ .And for the constant power curve, we have $T_L = \dfrac{10000}{\omega} = \dfrac{10000}{n\dfrac{2\pi rad/rev}{60s/\min}}$

Solve the two equation above, we can get the operating point as following $n = 1746\ r/\min \quad \omega = 183 rad/\sec$

c) Total power consumed by the motor $P_m = \dfrac{P}{\eta} = \dfrac{10kW}{0.88} = 11.4kW$

d) Sketch of motor and load torque-speed curves on the same graph, with the operating point at the first intersection:



Problem 19.66

Solution:

Known quantities:
A six-pole, three-phase motor.

Find:
The speed of the rotating field when the motor is connected to:
a) a $60\,Hz$ line.
b) a $50\,Hz$ line.

Assumptions:
None.

Analysis:
a)
For $60\,Hz$, $\omega_m = \dfrac{4\pi f}{P} = 125.7\,rad/\sec$, $\quad n_m = 1200\,rev/\min$

b)
For $50\,Hz$, $\omega_m = 104.72\,rad/\sec$, $\quad n_m = 1000\,rev/\min$

Problem 19.67

Solution:

Known quantities:
A six-pole, three-phase, $440V, 60\,Hz$ induction motor. The model impedances are: $R_S = 0.8\,\Omega \quad X_S = 0.7\,\Omega \quad R_R = 0.3\,\Omega \quad X_R = 0.7\,\Omega \quad X_m = 35\,\Omega$

Find:
The input current and power factor of the motor for a speed of $1200\,rev/\min$.

Assumptions:
None.

Analysis:
$$V_S = \frac{440}{\sqrt{3}} = 254\angle 0^\circ\,V$$

For $n_m = n_S = 1200\,rev/\min$, $s = 0\,(no\,load)$.

$Z_{in} = R_S + j(X_S + X_m) = 0.8 + j35.7 = 35.71\angle 88.7^\circ\,\Omega \qquad I_S = 7.11\angle -88.7^\circ\,A$

The power factor is: $\quad \cos 88.7^\circ = 0.0224\,lagging \quad P_{in} = 3|I_S||V_S|\cos\theta = 121.4\,W$

Problem 19.68

Solution:

Known quantities:

A eight-pole, three-phase, $220V, 60\,Hz$ induction motor. The model impedances are:

$$R_S = 0.78\,\Omega \qquad X_S = 0.56\,\Omega$$
$$R_R = 0.28\,\Omega \qquad X_R = 0.84\,\Omega$$
$$X_m = 32\,\Omega$$

Find:

The input current and power factor of the motor for $s = 0.02$.

Assumptions:

None.

Analysis:

For 8 poles,

$$n_S = \frac{3600}{4} = 900\,rev/\min$$
$$\omega_S = 94.25\,rad/\sec$$
$$\omega_m = (1-s)\omega_S = 92.4\,rad/\sec$$

By using the equivalent circuit, we have:

$$Z_{in} = 0.78 + j0.56 + \frac{j32(\frac{0.28}{0.02} + j0.84)}{14 + j32.84}$$
$$= 12.03 + j6.17 = 13.52\angle 27.15^\circ\ \Omega$$
$$V_S = 127\angle 0^\circ\ V$$
$$I_S = 9.39\angle -27.15^\circ\ A$$
$$pf = \cos(-27.15^\circ) = 0.8898 \text{ lagging}$$

Problem 19.69

Solution:

Known quantities:
The nameplate is as given in Example 19.2.

Find:
The rated torque, rated volt amperes, and maximum continuous output power for this motor.

Assumptions:
None.

Analysis:
The speed is:

$$n_m = 3565\, rev/\min$$

$$\omega_m = \frac{2\pi \times 3565}{60} = 373.3\, rad/\sec$$

The rated volt $\cdot$ amperes is:

$$\sqrt{3} \times (230V) \times (106A) = 42.23\, kVA$$

$$or\ \sqrt{3} \times (460V) \times (53A) = 42.23\, kVA$$

The maximum continuous output power is:

$$P_O = 40 \times 746 = 29840W$$

The rated output torque is:

$$T = \frac{P_O}{\omega_m} = 79.93\, N \cdot m$$

Problem 19.70

Solution:

Known quantities:

At rated voltage and frequency, the 3-phase induction machine has a starting torque of 140 percent and a maximum torque of 210 percent of full-load torque.

Find:

a) The slip at full load.
b) The slip at maximum torque.
c) The rotor current at starting as a percent of a full-load rotor current.

Assumptions:

Neglect stator resistance and rotational losses. Assume constant rotor resistance.

Analysis:

a)

$$T_R = \frac{KV^2 (R_2/s_R)}{(R_2/s_R)^2 + X^2}$$

$$T_{ST} = 1.4 T_R \qquad s_{ST} = 1.0$$

$$T_{MT} = 2.1 T_R \qquad s_{MT} = \frac{R_2}{X}$$

The above leads to 3 equations in 3 unknowns:

(1) $4.2 X R_2 = 1.4 R_2^2 + 1.4 X^2$

(2) $\frac{R_2^2}{s_R} + s_R X^2 = 1.4 R_2^2 + 1.4 X^2$

(3) $4.2 \frac{X R_2}{s_R} = (\frac{R_2}{s_R})^2 + X^2$

Solving the equations, we have:

$$\frac{R_2}{X} = 0.382$$

$$s_R = 0.097$$

b)

$$s_{MT} = \frac{R_T}{X} = 0.382$$

c)

$$I_R = \frac{KV}{4.06}, \qquad I_{ST} = \frac{KV}{1.07}$$

$$\frac{I_{ST}}{I_R} \times 100 = 379\%$$

Problem 19.71

Solution:

Known quantities:

Parameters of an induction motor.

Find:

a) Torque delivered to the load.
b) The total electrical (input) power consumed by the motor.

Assumptions:

Analysis:

Synchronous speed of four-pole induction motor at 60-Hz:

$$n_S = \frac{120f}{p} = 1800 \text{ RPM}$$

$$\omega_S = \frac{2\pi n_S}{60} = 188 \text{ rad/s}$$

Rotor mechanical speed at 4% slip:

$$\omega_r = (1 - 0.04)\omega_S = 181 \text{ rad/s}$$

Load torque at the shaft:

$$T_L = \frac{P_{Mech}}{\omega_r} = \frac{35 \times 10^3}{181} = 193.4 \text{ N - m}$$

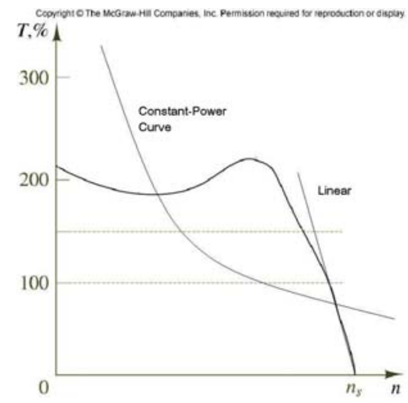
Total power consumed by the motor:

$$P_{Elec} = \frac{P_{Mech}}{\eta} = \frac{35 \times 10^3}{0.87} = 40.2 \text{ kW}$$

Problem 19.72

Note: 16,800 rev/min in this problem should read 1680 rev/min.

Solution:



Known quantities:

Parameters of an induction motor.

Find:

a) Determine the slip at this operating condition.
b) For a constant-power, 20-kW load, determine the operating speed of the machine.
c) Sketch the motor and load torque-speed curves for the load of part b. on the same graph. Show numerical values.

Analysis:

a) Synchronous speed of four-pole induction motor at 60-Hz:

$$n_S = \frac{120f}{p} = 1800\ \text{RPM}$$

Calculate the slip

$$s = \frac{n_s - n_r}{n_s} = \frac{1800 - 1680}{1800} = 0.067$$

b) Operating speed of machine for a constant-power load of 20-kW. Assume that it is linear around the original operating point and ideal synchronous speed point. We can use a line to approximate the torque-speed curve of the motor, and then use the intersection of such line and the constant power curve to decide the operating point.

The line connects original operating point and ideal synchronous speed point satisfies:

$$T_L = \frac{140 - 0}{1680 - 1800}(n - 1800)\ N.m$$

And for the constant power curve, we have

$$T_L = \frac{20000}{\omega} = \frac{20000}{n\dfrac{2\pi rad/rev}{60s/\min}}$$

Solve the two equation above, we can get the operating point as following

$$n = 1704\ r/\min$$

$$\omega = 178 rad/\sec$$

c) It is shown in the right

Problem 19.73

Solution:

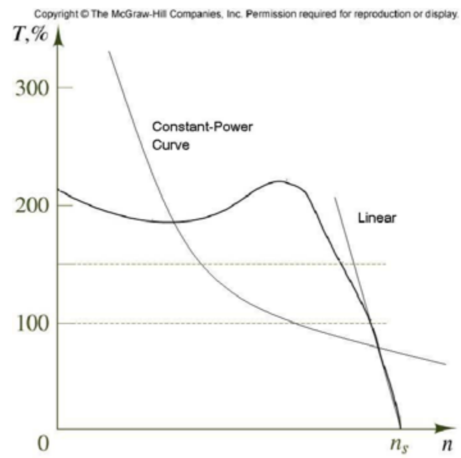
Known quantities:

Parameters of an induction motor.

Find:

a) Determine the speed and the slip at the operating condition.
b) If the machine has an efficiency of 89 percent, what minimum rms current is required for operation with the load of part a?

Analysis:



a) Synchronous speed of six-pole induction motor at 60-Hz:

$$n_S = \frac{120f}{p} = 1200\,\text{RPM}$$

Assume that it is linear around the original operating point and ideal synchronous speed point. We can use a line to approximate the torque-speed curve of the motor, and then use the intersection of such line and the constant power curve to decide the operating point.

The line connects original operating point and ideal synchronous speed point satisfies:

$$T_L = \frac{60-0}{1080-1200}(n-1200)\ N.m$$

And for the constant power curve, we have

$$T_L = \frac{800}{\omega} = \frac{800}{n\dfrac{2\pi rad/rev}{60s/\min}}$$

Solve the two equation above, we can get the operating point as follows

$$n = 1187\ r/\min$$

$$\omega = 124 rad/\sec$$

Calculate the slip

$$s = \frac{n_s - n_r}{n_s} = \frac{1200-1187}{1200} = 0.01$$

b) Calculate the electrical power required

$$P_{Mech} = T \times \varpi = 800\text{ W}$$

$$P_{Elec} = \frac{P_{Mech}}{\eta} = \frac{800}{0.89} = 899\text{ W}$$

$$I = \frac{P_{Elec}}{U} = \frac{899\text{ W}}{240\text{ V}} = 3.75\text{ A}$$