Chapter 15: Electronic Instrumentation and Measurements – Instructor Notes

Chapter 15 continues the discussion of integrated circuit electronics begun in Chapter 8 with op-amps. **The Chapter is extremely modular, and the degree of coverage can vary widely, depending on the requirements of each individual Instructor**. The first two sections cover measurement systems and transducers, providing a summary in Table 15.1 (p. 762), and noise problems. The third covers the instrumentation amplifier (which had been introduced in Chapter 8) in greater depth. **Section 15.3 might be of interest by itself as an extension of Chapter 8, and can be covered immediately following Section 8.2**. **Section 15.3 also discusses practical active filters, focusing on Butterworth and Chebyshev designs, and is a logical continuation of Section 8.3 on active filters;** the material in this section is fairly advanced, and will require the student to have had a rigorous introduction to the Laplace Transform and to systems concepts. Thus, this section will be appropriate for a second course on electronics and instrumentation. The active filter design material would be nicely complemented by laboratory exercises; the importance of filter design and analysis in instrumentation problems is not to be underestimated. This material could also be supplemented very effectively by a review of filter design and analysis procedures using computer aids (e.g., MATLAB™).

The material in Section 15.4, on the subject of signal interface (A/D, D/A conversion and sample-and-hold amplifiers) can be presented separately, for example, in conjunction with Chapter 8, or with Chapters 13 and 14. There are excellent possibilities for very useful laboratory experiments in connection with this material. The emphasis is on illustrating the important parameters and performance limitations in the application of commercial ADCs, DACs, and sample-and-hold amplifiers. A commercial data acquisition board is also described in *Focus on Measurements: Data Acquisition Card for Personal Computer* (pp. 797-800). It is this Author's opinion that this material is of great practical importance to non-majors, many of whom will at some point make use of a microcomputer-based digital data acquisition system, regardless of their specialty.

Section 15.5 can also be viewed as an extension of Chapter 8, and could be covered in a first course. Again, the section is independent of the other sections in the chapter. In this section, the op-amp comparator and the Schmitt trigger are introduced first. The section closes with a functional description of timing circuits including a one-shot IC (74123) and the NE 555 timer IC.

Section 15.6 provides a summary of other IC instrumentation circuits, to give the student a flavor of the capabilities of modern integrated electronics. The box *Focus on Measurements: Using the ADXL202 Accelerometer as a Multifunction Sensor in Car Alarms* (pp. 812-816) provides a detailed look at an application note (Courtesy: Analog Devices). The aim of this box is to present Application Notes as a valuable design resource for practicing engineers.

The homework problems present a variety of analysis and design problems on instrumentation amplifiers and active filters. Several design problems are also given to complement the section on timing circuits; a few of the problems require the student to explore the data sheets for the AD 625 instrumentation amplifier, the 555 timer, and the 74123 one-shot. Although these problems are fairly simple, they can be used to educate the student to search for design parameters in the data sheets. The data sheets are provided in the CD-ROM and on the website..

Also included is a series of problems on DAC and ADC analysis and design. Some emphasis is again placed on reading and understanding the device data sheets of commercial ADCs and DACs. Issues in sampling frequency selection and resolution are approached in a few applied problems, where practical measurement situations pertaining to the measurement of angular position (problem 15.60), torque (problem 15.62), and altitude (problem 15.63) are described. The chapter problems end with a few simple problems on data transmission and coding.

Learning Objectives

1. Review the major classes of sensors. *Section 15.1.*
2. Learn how to properly ground circuits, and methods for noise shielding and reduction. *Section 15.2.*
3. Design signal conditioning amplifiers and filters. *Section 15.3.*
4. Understand A/D and D/A conversion, and select the specifications of a the appropriate conversion system for a given application. *Section 15.4.*
5. Analyze and design simple comparator and timing circuits using integrated circuits. Review other common instrumentation integrated circuits. *Section 15.5, 15.6.*

Section 15.1: Measurement Systems and Transducers

Problem 15.1

Solution:

Find:

Explain the differences between tachometers and speedometers.

Analysis:

Frequency - engine speed is normally measured right at the crankshaft prior to any gearing in rpm - typically several thousands. The transducers used with speedometers measure speed at the axle in rpm - typically much lower than at the engine output.
Scale factors - the tachometer would require none, the speedometer requires a conversion factor from rpm of the axle (rotational) to mph (linear).

Problem 15.2

Solution:

Find:

Explain the differences between the engineering specifications you would write for a transducer to measure the frequency of an audible sound wave and a transducer to measure the frequency of a visible light wave.

Analysis:

Audio frequencies: $0 < f \le 15\ \mathbf{kHz}$

Visible frequencies: $3.9 \times 10^{14}\ \mathbf{Hz} \le f \le 7.9 \times 10^{14}\ \mathbf{Hz}$

Devices used to measure quantities at audio frequencies will be incapable of sensing or measuring accurately those same quantities at frequencies in the visual range.
Various types of photocells are available for use as light sensors. For audio frequencies, more conventional devices like bridges may be used for measuring signals.

Problem 15.3

Solution:

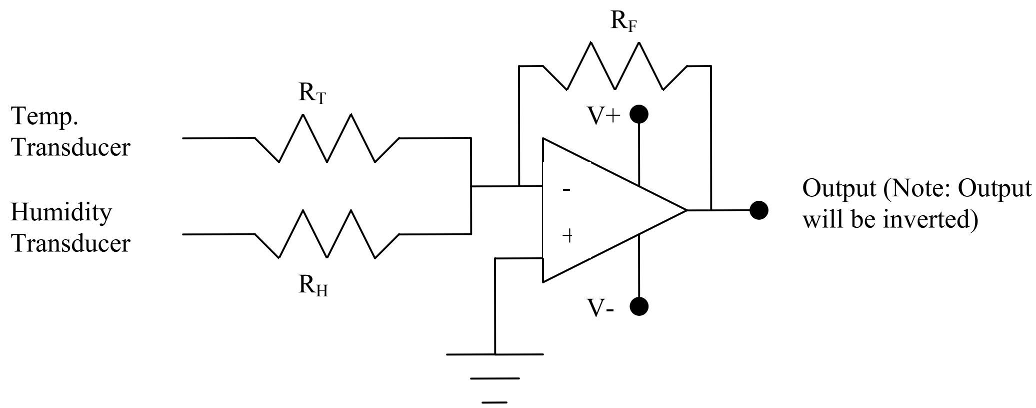
Find:

A simple schematic diagram of a circuit to compute the sum of a temperature and a percentage relative humidity signal.

Analysis:

Use a transducer that will convert temperature in degrees Fahrenheit to volts between the values of the 2-sided supply voltage.
Similarly measure the percentage relative humidity and convert the transducer output to the requisite voltage.

Summing Amplifier

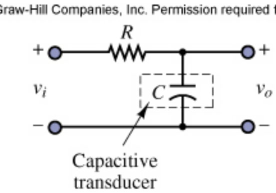


Problem 15.4

Solution:

Known quantities:

The capacity of a capacitive displacement transducer $C = \frac{0.255\ A}{d}$ **F**, where A is the cross-sectional area of the transducer plate (in^2), and d the air-gap length (in).



Find:

The change in voltage (Δv_0) when the air-gap changes from 0.01 in to 0.015 in.

Analysis:

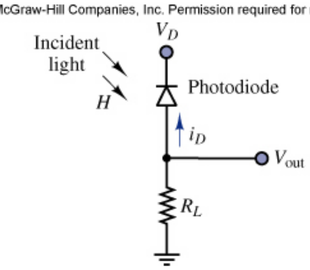
$i = C\frac{\Delta v_o}{\Delta t}$, $\Delta q = C\Delta v_o$, $\frac{\Delta q}{C} = \Delta v_o$

Assume no change in charge.

$C_{new} = \frac{2}{3}C_{old} \Rightarrow \Delta v_o = \frac{3}{2}$ **V**

Problem 15.5

Solution:



Known quantities:

The circuit of Figure P15.5 in which
$i_D/H = 0.5\ 10^{-6}\ \mu\mathbf{Am^2/W}$.

Find:

a) Show that the output voltage varies linearly with H.
b) If $H = 1{,}500\ \mathbf{W/m^2}$, $V_D = 7.5\ \mathbf{V}$, and an output voltage of 1 V is desired, determine an appropriate value for R_L.

Analysis:

a) $V_{out} = i_D R_L$
Given a large enough value of V_D,
$V_{out} = 0.5\ 10^{-6} H R_L$; hence, varies linearly with H.
b) $V_{out} = i_D/H\ R_L \;\Rightarrow\; 1 = 0.5\ 10^{-6}(1500)R_L$, $R_L = 1333\Omega$

Problem 15.6

Solution:

Known quantities:

The value of the constant G for quartz in compressive stress, $0.055\dfrac{\mathbf{V\cdot m}}{\mathbf{N}}$, and for polyvinylidene fluoride in axial stress, $0.22\dfrac{\mathbf{V\cdot m}}{\mathbf{N}}$.
The quartz element of a force sensor is 0.25 in thick and has a rectangular cross section of 0.09 in^2.
The fluoride film of a piezoelectric sensor is 30 μm thick, 1,5 cm wide, and 2.5 cm in the axial direction.

Find:

a) The output of the force sensor in V/N.
b) The output of the load sensor in V/N.

Analysis:

a) $V_{out} = 0.055\dfrac{\mathbf{V\cdot m}}{\mathbf{N}}\dfrac{1\ \mathbf{in}}{0.0254\,\mathbf{m}}\times\dfrac{1}{0.25\ \mathbf{in}} = 8.66\dfrac{\mathbf{V}}{\mathbf{N}}$

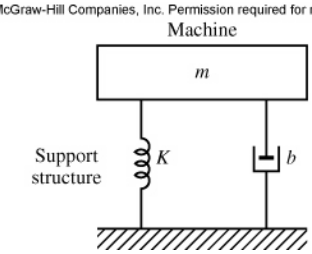
b) $0\ \dfrac{\mathbf{V}}{\mathbf{N}}$

Problem 15.7

Solution:

Known quantities:

The allowable levels of error in the measurement of *K*, *m* and ζ: ±5 %, ±2 %, ±10 %. The expression for *b*, $b = 2\zeta\sqrt{Km}$.



Find:

The percentage error limit for *b*.

Analysis:

$$error = \pm\left(10 + \frac{5+2}{2}\right) = \pm 13.5\ \%$$

Problem 15.8

Solution:

Known quantities:

The measurements taken by a sensor that measure the thickness of a wet pulp layer every 2 feet along the sheet: 8.2, 9.8, 9.92, 10.1, 9.98, 10.2, 10.2, 10.16, 10.0, 9.94, 9.9, 9.8, 10.1, 10.0, 10.2, 10.3, 9.94, 10.14, 10.22, 9.8. The roller speed is adjusted based on the last 20 measurements unless the probability that the mean thickness lies within ±2 % of the sample mean exceeds 0.99.

Find:

The adjustment of the speed roller.

Analysis:

$$mean = \frac{\sum measurements}{20} = 9.945 \pm 2\ \%, \quad \max. = 10.144, \quad \min. = 9.746$$

$$average\ deviation = \frac{\sum |deviations|}{20} = 0.225$$

$$standard\ deviation = \sqrt{\frac{\sum |deviations|^2}{20}} = 0.427$$

Measurement #1 exceeds the standard deviation σ probability<0.99 $\Rightarrow$ roller speed will be adjusted

---

Problem 15.9

Solution:

Find:

Discuss:
a) Measurement accuracy.
b) Instrument accuracy.
c) Measurement error.
d) Precision.

Analysis:

a) This term and instrument accuracy are used interchangeably if only one instrument is involved and if the measurement method is appropriate. Basically, the accuracy of the measurement is given by the instrument's specifications (ordinarily in terms of percent of indicated value or full scale value).
b) See answer to part a).
c) Measurement error can be synonymous with measurement accuracy, but can also refer to sloppy methods of data acquisition, use of multiple transducers and/or instruments whose individual errors combine, or simply a lack of reliable, multiple data points.
d) Precision and resolution are interchangeable terms and refer to the smallest increment of measured quantity that can be detected by the instrument.

Problem 15.10

Solution:

Known quantities:

Four sets of measurements shown in Figure P15.10 taken on the same response variable of a process using four different sensors.

Find:

Rank these data sets with respect to:
a) Precision.
b) Accuracy.

Analysis:

(b) and (c) are precise, (a) and (d) are not.
(a) and (c) are accurate, (b) and (d) are not.

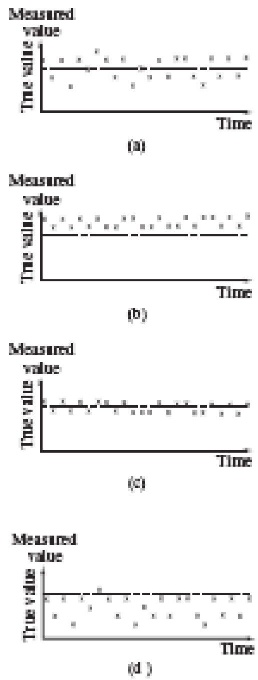
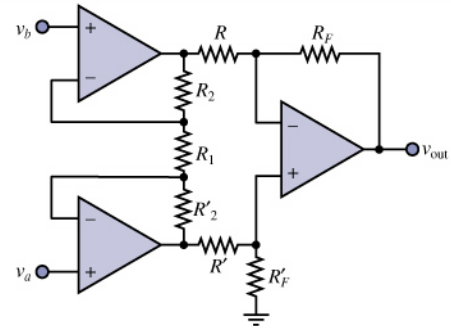


Figure P15.10

Section 15.3: Signal Conditioning

Problem 15.11

Solution:



Known quantities:

The resistances for the instrumentation amplifier of Figure P15.11, $R_1 = 1\ \mathbf{k\Omega}$, $R_2 = 5\ \mathbf{k\Omega}$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(5\ \mathbf{k\Omega})}{1\ \mathbf{k\Omega}} = 1 + 10 = 11$$

Problem 15.12

Solution:

Known quantities:

The resistance $R_1 = 1\ \mathbf{k\Omega}$ for the instrumentation amplifier of Figure P15.11, and the value of the gain for the input stage, 50.

Find:

The resistance R_2 to make that gain for the amplifier.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} \Rightarrow R_2 = \frac{1}{2}R_1(A-1) = \frac{1}{2}(1\ \mathbf{k\Omega})(50-1) = 24.5\ \mathbf{k\Omega}$$

Problem 15.13

Solution:

Known quantities:

The resistance $R_2 = 10\ \mathbf{k\Omega}$ for the instrumentation amplifier of Figure P15.11, and the value of the gain for the input stage, 16.

Find:

The resistance R_1 to make that gain for the amplifier.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} \Rightarrow R_1 = \frac{2R_2}{A-1} = \frac{2(10\ \mathbf{k\Omega})}{16-1} \approx 1333\ \Omega$$

Problem 15.14

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 1\ \mathbf{k\Omega}$, $R_2 = 10\ \mathbf{k\Omega}$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(10\ \mathbf{k\Omega})}{1\ \mathbf{k\Omega}} = 1 + 20 = 21$$

Problem 15.15

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 1.5\ \mathbf{k\Omega}$, $R_2 = 80\ \mathbf{k\Omega}$.

Find:

The gain of the input stage.

Analysis:

$$A = 1 + \frac{2R_2}{R_1} = 1 + \frac{2(80\ \mathbf{k\Omega})}{1.5\ \mathbf{k\Omega}} = 1 + 106.7 = 107.7$$

Problem 15.16

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = R^{'} = R = 1$ **k**Ω , $R_2 = 5$ **k**Ω , $R_f = 10$ **k**Ω .

Find:

The differential gain for the IA.

Analysis:

$$A_{dif} = A\frac{R_f}{R} = \left(1 + \frac{2R_2}{R_1}\right)\frac{R_f}{R} = \left(1 + \frac{2(5)}{1}\right)\frac{10}{1} = 110$$

Problem 15.17

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure P15.11, $R = 1$ **k**Ω , $R_f = 200$ **k**Ω , $\Delta R = 2$ **%** of R.

Find:

The CMMR of the IA.

Analysis:

$$CMRR_{dB} = 20\log_{10}\left|\frac{A_{dif}}{\frac{R_f}{R}\left(\frac{R + R_f}{R_f + R + \Delta R} - 1\right)A}\right|$$

$$A_{dif} = A\frac{R_f}{R}, \ \Delta R = 0.02 \cdot R = 0.02 \cdot 1 \ \mathbf{k\Omega} = 20 \ \Omega$$

$$CMRR_{dB} = 20\log_{10}\left|\frac{A\frac{R_f}{R}}{\frac{R_f}{R}\left(\frac{R + R_f}{R_f + R + \Delta R}\right)A}\right| = 20\log_{10}\left|\frac{1}{\frac{R + R_f}{R_f + R + \Delta R} - 1}\right| \approx 80dB$$

Problem 15.18

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure P15.11, $R = 1\ \mathbf{k\Omega}$, $R_f = 200\ \mathbf{k\Omega}$, $\Delta R = 2\ \mathbf{\%}$ of R.

Find:

The mismatch in gains for the differential components id dB.

Analysis:

Assume $A = 10$. Then $A_{dif} = \frac{R_f}{R} A = 2000$

$20\log_{10} A_{dif} \approx 66\ \mathbf{dB}$

$CMRR_{dB} = 80\ \mathbf{dB}$ (from Problem 15.17)

$20\log_{10} A_{dif} - CMRR_{dB} = -14\ \mathbf{dB}$

Problem 15.19

Solution:

Known quantities:

The resistances for the instrumentation amplifier of Figure 15.16, $R_1 = 2\ \mathbf{k\Omega}$, $R_f = 10\ \mathbf{k\Omega}$, and the differential gain, 900.

Find:

The resistances R and R_2 to achieve that gain.

Analysis:

$$A_{dif} = \frac{\mathrm{R_f}}{\mathrm{R}}\left(\mathbf{1} + \frac{\mathbf{2}\mathrm{R_2}}{\mathrm{R_1}}\right) = \frac{\mathbf{10}}{\mathrm{R}}\left(\mathbf{1} + \frac{\mathbf{2}\mathrm{R_2}}{\mathbf{2}}\right) = \frac{\mathbf{10}}{\mathrm{R}}\left(\mathbf{1} + \mathrm{R_2}\right) = 900$$

Thus, $\mathbf{1} + \mathrm{R_2} = 90R$

Choose: $R = 1\ \mathbf{k\Omega} \Rightarrow \mathrm{R_2} = 89\ \mathbf{k\Omega}$

Problem 15.20

Solution:

Known quantities:

The cutoff specification of Example 15.3, $\omega_c = 10$ **rad/s**.

Find:

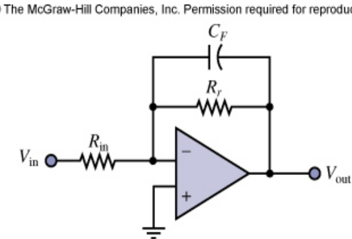
The order of the filter required to achieve 40 dB attenuation at $\omega_s = 24$ **rad/s**.

Analysis:

$20\log_{10}\sqrt{1+\omega_s^{2n}} \le 40 \;\; @ \;\; \omega_s = 25 \; \mathbf{rad/s}$

Solving the equation, we obtain $n = 1.43$. Thus $n = 2$ is desired.

Problem 15.21



Solution:

Known quantities:

The circuit for a low-pass filter shown in Figure P15.21 with gain.

Find:

a) The relationship between output amplitude and input amplitude.
b) The relationship between output phase angle and input phase angle.

Analysis:

This is an inverting amplifier circuit, with

$$V_{out} = -\frac{Z_F}{Z_{in}}V_{in}, \text{ where: } Z_F = R_F \left\| \frac{1}{j\omega C_F} \right. = \frac{R_F \cdot \dfrac{1}{j\omega C_F}}{R_F + \dfrac{1}{j\omega C_F}} = \frac{R_F}{1 + j\omega R_F C_F} \text{ and } Z_{in} = R_{in}.$$

Therefore, $V_{out} = -\dfrac{R_F}{R_{in}}\dfrac{1}{1 + j\omega R_F C_F}V_{in}$

a) $\left|\dfrac{V_{out}}{V_{in}}\right| = \dfrac{R_F}{R_{in}}\dfrac{1}{\sqrt{1+(\omega R_F C_F)^2}}$

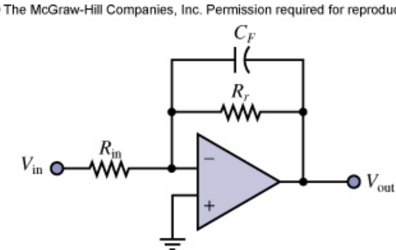
b) $\angle V_{out} - \angle V_{in} = \pi - \tan^{-1}(\omega R_F C_F)$ rad, or $\angle V_{out} - \angle V_{in} = 180° - \tan^{-1}(\omega R_F C_F)$ deg.

Problem 15.22

Solution:

Known quantities:

The circuit for a low-pass filter shown in Figure P15.21 with gain, with $R_{in} = 20$ **kΩ**, $R_F = 100$ **kΩ**, $C_F = 100$ **pF**, $v_{in} = 2\sin(2{,}000\pi t)$ **V** .



Find:

The expression of v_{out}.

Analysis:

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_F}{R_{in}}\frac{1}{\sqrt{1+(wR_F C_F)^2}} = 4.99$$

$$\angle V_{out} - \angle V_{in} = \pi - \tan^{-1}\left(2000\pi \cdot 100 \cdot 10^3 \cdot 100 \cdot 10^{-12}\right) = \pi - \tan^{-1}\left(62.8 \cdot 10^{-3}\right) = 3.079 \textbf{ rad}$$

$$v_{out}(t) = 9.98\sin(2000\pi t + 3.079) \textbf{ V}$$

Problem 15.23

Solution:

Known quantities:

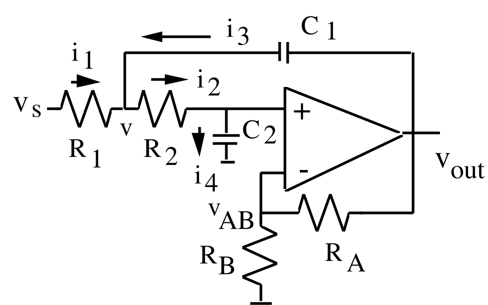
The circuit of the low-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

Analysis:

The circuit is shown below:



We have:

$$V_{AB} = \frac{R_B}{R_A + R_B} V_{out}$$

$$i_1 = \frac{V_s - V}{R_1},\ i_2 = \frac{V - V_{AB}}{R_2},\ i_3 = j\omega C_1\left(V_{out} - V\right),$$

$$i_4 = j\omega C_2 V_{AB}.$$

From $i_2 = i_4$, we have: $V = V_{AB}\left(1 + j\omega R_2 C_2\right)$

From $i_1 + i_3 = i_2$, we have: $V_{out}\left(-j\omega C_1 + \frac{R_B}{R_A + R_B}\left(\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + j\omega R_2 C_2\right)\right) - \frac{1}{R_2}\right) = \frac{V_s}{R_1}$

$$\frac{V_{out}}{V_s}(j\omega) = \frac{1}{R_1\left(-j\omega C_1 + \frac{R_B}{R_A+R_B}\left(\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2}\right)(1 + j\omega R_2 C_2)\right) - \frac{1}{R_2}\right)}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K(1/R_1R_2C_1C_2)}{(j\omega)^2 + \left[\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}(1-K)\right](j\omega) + \frac{1}{R_1R_2C_1C_2}}$$

where: $K = 1 + \frac{R_A}{R_B}$

Problem 15.24

Solution:

Known quantities:

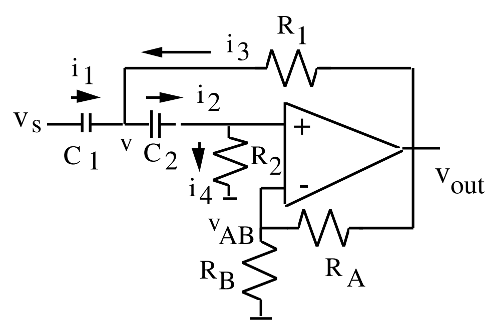
The circuit of the high-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

Analysis:

The circuit is shown below:



We have:

$$V_{AB} = \frac{R_B}{R_A+R_B}V_{out}$$

$$i_1 = j\omega C_1(V_s - V),\ i_2 = j\omega C_2(V - V_{AB}),\ i_3 = \frac{V_{out} - V}{R_1},$$

$$i_4 = \frac{V_{AB}}{R_2}.$$

From $i_2 = i_4$, we have: $V = V_{AB}\left(1 + \frac{1}{j\omega R_2 C_2}\right)$

From $i_1 + i_3 = i_2$, we have:

$$j\omega C_1 V_s = V_{out}\left\{\left(\frac{R_B}{R_A+R_B}\right)\left[\left(\frac{1}{R_1} + j\omega C_1 + j\omega C_2\right)\left(\frac{1}{j\omega C_2 R_2} + 1\right) - j\omega C_2\right] - \frac{1}{R_1}\right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K(j\omega)^2}{(j\omega)^2 + \left[\frac{1}{R_1C_1}(1-K) + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right](j\omega) + \frac{1}{R_1R_2C_1C_2}}$$

where: $K = 1 + \frac{R_A}{R_B}$

Problem 15.25

Solution:

Known quantities:

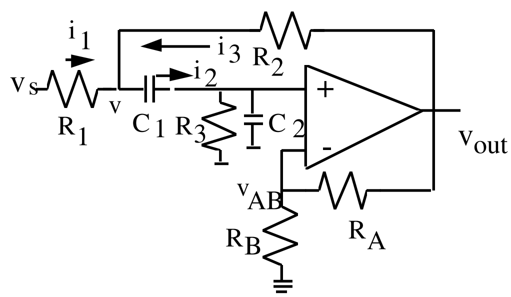
The circuit of the band-pass filter of Figure 15.22.

Find:

The frequency response of the filter.

Analysis:

The circuit is shown below:



We have:

$$V_{AB} = \frac{R_B}{R_A + R_B} V_{out}$$

$$i_1 = \frac{V_s - V}{R_1},\ i_2 = j\omega C_1 \left(V - V_{AB}\right),\ i_3 = \frac{V_{out} - V}{R_2}.$$

From $i_2 = V_{AB}\left(\frac{1}{R_3} + j\omega C_2\right)$, we have:

$$V = V_{AB}\left(1 + \frac{C_2}{C_1} + \frac{1}{j\omega R_3 C_1}\right)$$

From $i_1 + i_3 = i_2$, we have:

$$\frac{V_s}{R_1} = V_{out}\left\{\left(\frac{R_B}{R_A + R_B}\right)\left[\left(\frac{R_1 + R_2}{R_1} + j\omega C_1\right)\left(\frac{1}{j\omega C_1 R_3} + \frac{C_1 + C_2}{C_1}\right) - j\omega C_1\right] - \frac{1}{R_2}\right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{Kj\omega \frac{1}{R_1 C_2}}{(j\omega)^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}(1 - K)\right](j\omega) + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

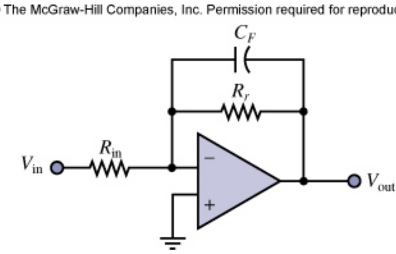
where: $K = 1 + \frac{R_A}{R_B}$

Problem 15.26

Solution:

Known quantities:

The circuit of Figure P15.21, where $C_F = 100$ **pF**, and the desired cutoff frequency and gain magnitude are respectively 20 kHz and 5.



Find:

The appropriate values of R_F and R_{in}.

Analysis:

$$f_C = 20 \textbf{ kHz} \quad \Rightarrow \quad \omega_C = 2\pi f_C = 40\pi \textbf{ k}\frac{\textbf{rad}}{\textbf{s}}$$

$$\omega_C = \frac{1}{R_F C_F} \quad \Rightarrow \quad R_F = \frac{1}{\omega_C C_F} = \frac{1}{40\pi \cdot 10^3 \cdot 100 \cdot 10^{-12}} = 79.6 \textbf{ k}\boldsymbol{\Omega}$$

$$\frac{R_F}{R_{in}} = 5 \quad \Rightarrow \quad R_{in} = \frac{R_F}{5} = 15.9 \textbf{ k}\Omega$$

Problem 15.27

Solution:

Known quantities:

The cutoff frequency, 10 kHz, and the DC gain, 10, for a second-order Butterworth high pass filter, with $Q = 5$ and $V_s = \pm 15$ **V**.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 10; \text{ choosing } R_A = 9 \textbf{ k}\boldsymbol{\Omega} \quad \Rightarrow \quad R_B = 1 \textbf{ k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 10 \textbf{ kHz}$$

$$\left[\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - 9\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] = 0.2$$

Choose C1 = C2 = 0.01 μF and solve for R_1 = 540 Ω and R_2 = 4.7 kΩ. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.28

Solution:

Known quantities:

The cutoff frequency, 25 kHz, and the DC gain, 15, for a second-order Butterworth high pass filter, with Q = 10 and $V_s = \pm 15\ \mathbf{V}$.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 15\text{; choosing } R_A = 14\ \mathbf{k\Omega} \;\Rightarrow\; R_B = 1\ \mathbf{k\Omega}$$

$$f = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}} = 25\ \mathbf{kHz}$$

$$\left[\sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} - 14\sqrt{\frac{R_1C_1}{R_2C_2}}\right] = 0.1$$

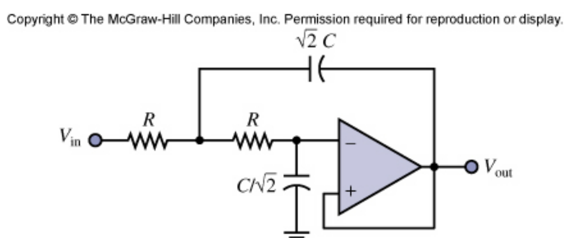
Choose $C_1 = C_2$ = 1 μF and solve for R_1 = 1.8 Ω and R_2 = 23 Ω. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.29

Solution:

Known quantities:

The circuit of Figure P15.29.



Find:

The characteristic of the filter.

Analysis:

Note that $V_b \approx V_{out}$. Therefore,

$$I = \frac{V_b}{\left(\dfrac{1}{j\omega\dfrac{C}{\sqrt{2}}}\right)} = j\omega\frac{C}{\sqrt{2}}V_{out}$$

and

$$V_a = RI + V_b = R\left(j\omega\frac{C}{\sqrt{2}}V_{out}\right) + V_{out} = \left(1 + j\frac{\omega RC}{\sqrt{2}}\right)V_{out}$$

Now, writing a KCL equation at node a,

$$\frac{V_a - V_{in}}{R} + I + \frac{V_a - V_{out}}{\left(\frac{1}{j\omega\sqrt{2}C}\right)} = 0$$

$$\frac{\left(1 + j\frac{\omega RC}{\sqrt{2}}\right)V_{out} - V_{in}}{R} + j\omega\frac{C}{\sqrt{2}}V_{out} + \frac{\left(1 + j\frac{\omega RC}{\sqrt{2}}\right)V_{out} - V_{out}}{\left(\frac{1}{j\omega\sqrt{2}C}\right)} = 0$$

$$\frac{1}{R}V_{in} = \left[\frac{\left(1 + j\frac{\omega RC}{\sqrt{2}}\right)}{R} + j\omega\frac{C}{\sqrt{2}} + \frac{\left(1 + j\frac{\omega RC}{\sqrt{2}} - 1\right)}{\left(\frac{1}{j\omega\sqrt{2}C}\right)}\right]V_{out} = \left[\frac{1}{R} + j\frac{\omega C}{\sqrt{2}} + j\frac{\omega C}{\sqrt{2}} - \omega^2 RC^2\right]V_{out}$$

$$V_{in} = \left(1 + j\sqrt{2}\omega RC - \omega^2 R^2 C^2\right)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 - (\omega RC)^2 + j\sqrt{2}\omega RC}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 - 2(\omega RC)^2 + (\omega RC)^4 + 2(\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^4}}$$

which is a second-order Butterworth low-pass function with cutoff frequency $\omega_C = \frac{1}{RC}$

Problem 15.30

Solution:

Known quantities:

The cutoff frequency, 15 kHz, and the DC gain, 15, for a second-order Butterworth low pass filter, with Q = 5 and $V_S = \pm 15\ \mathbf{V}$.

Find:

The design of the filter.

Analysis:

$$K = 1 + \frac{R_A}{R_B} = 15; \text{ choosing } R_A = 14\ \mathbf{k\Omega} \Rightarrow R_B = 1\ \mathbf{k\Omega}$$

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 15\ \mathbf{kHz}$$

$$\left[\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - 14\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right] = 0.2$$

Choose $C_1 = C_2 = 1\ \mu$F and solve for $R_1 = 2.6\ \Omega$ and $R_2 = 43.9\ \Omega$. Then, substitute the values thus obtained in the high-pass filter of Figure 15.22.

Problem 15.31

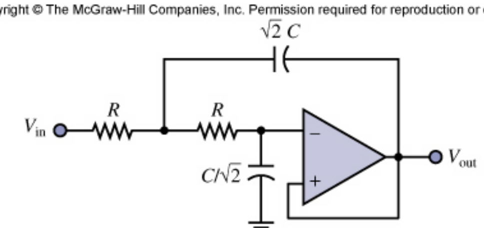
Solution:

Known quantities:
The low cutoff frequency, 200 Hz, the high cutoff frequency, 1 kHz, and the pass band gain, 4.

Find:
The value of Q for the filter, and the approximate frequency response of this filter.

Analysis:
$$Q = \frac{\sqrt{f_H f_L}}{f_H - f_L} = 0.6 \qquad K = 1 + \frac{R_A}{R_B} = 2 \text{; choosing } R_A = 1\ \mathbf{k\Omega} \Rightarrow R_B = 1\ \mathbf{k\Omega}$$
$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 1\ \mathbf{k}\text{Hz}$$
Choose $R_1 = R_2$ and $C_1 = C_2 = 1\ \mu\text{F}$
$R_1 = R_2 = 160\ \Omega$. Then, substitute the obtained values in the high-pass filter of Figure 15.22.
$$K = 1 + \frac{R_A}{R_B} = 2 \text{; choosing } R_A = 1\ \mathbf{k\Omega} \Rightarrow R_B = 1\ \mathbf{k\Omega}$$
$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} = 200\ \text{H}\mathbf{z}$$
Choose $R_1 = R_2$ and $C_1 = C_2 = 1\ \mu\text{F}$
$R_1 = R_2 = 800\ \Omega$. Then, substitute the obtained values in the high-pass filter of Figure 15.22.
By connecting the output of the high-pass filter to the input of the low-pass filter, we obtain the desired filter.

Problem 15.32



Solution:

Known quantities:
The circuit of Figure P15.29 and the cutoff frequency, 10 Hz, for a second-order Butterworth low pass filter.

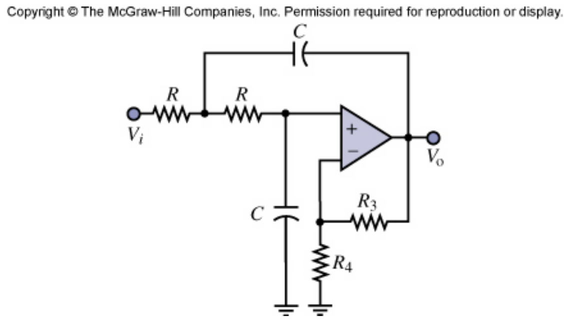
Find:
The design of the filter.

Analysis:
$$f_C = 10\ \mathbf{Hz} = \frac{1}{2\pi RC} \quad \Rightarrow \quad \omega_C = \frac{1}{RC} = 2\pi \cdot 10 = 20\pi\ \frac{\mathbf{rad}}{\mathbf{s}}$$
Choose $R = 20\ \mathbf{k\Omega}$. Then, $\frac{1}{(20\ \mathbf{k\Omega})C} = 20\pi \Rightarrow C = \frac{1}{20 \cdot 10^3 \cdot 20\pi} = 796\ \mathbf{nF}$

and the two capacitors have values given by $\sqrt{2}C = 1.125\ \mathbf{\mu F}$ and $\frac{C}{\sqrt{2}} = 563\ \mathbf{nF}$.

Problem 15.33

Solution:



Known quantities:

The low pass Sallen Key filter of Figure P15.33.

Find:

The voltage gain $\frac{V_{out}}{V_{in}}$ as a function of frequency and generate its Bode magnitude plot. Show that the cutoff frequency is $\frac{1}{2\pi RC}$ and that the low frequency gain is $\frac{R_4}{R_3}$.

Analysis:

We have:

$$V_{AB} = \frac{R_4}{R_3 + R_4} V_{out}$$

$i_1 = j\omega C\left(V_{out} - V\right)$, $i_2 = \frac{V_s - V}{R}$, $i_3 = \frac{V - V_{AB}}{R}$.

From $i_2 = j\omega C V_{AB}$, we have: $V = V_{AB}\left(1 + j\omega RC\right)$

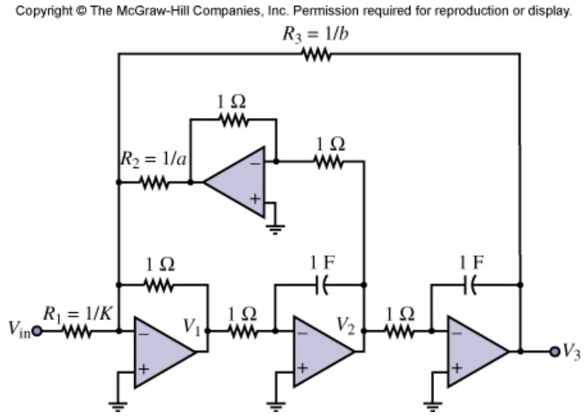
From $i_1 + i_3 = i_2$, we have:

$$V_s = V_{out}\left\{\left(\frac{R_4}{R_3 + R_4}\right)\left[\left(2 + j\omega RC\right)\left(1 + j\omega RC\right) - 1\right] - j\omega RC\right\}$$

Therefore, the frequency response is:

$$\frac{V_{out}}{V_s}(j\omega) = \frac{K\frac{1}{(RC)^2}}{(j\omega)^2 + \left[\frac{1}{RC}(3 - K)\right](j\omega) + \frac{1}{(RC)^2}} \quad \text{where: } K = 1 + \frac{R_3}{R_4}$$

Problem 15.34



Solution:

Known quantities:

The circuit shown in Figure P15.34

Find:

The transfer functions relating each of the three outputs to the input V_{in}.

Analysis:

Using Laplace transforms, note that

$$V_2 = -\frac{1}{s}V_1$$

$$V_3 = -\frac{1}{s}V_2 = \frac{1}{s^2}V_1$$

and

$$V_4 = -V_2 = \frac{1}{s}V_1$$

Writing a KCL equation at the inverting input to the leftmost op-amp,

$$-KV_{in} - V_1 - aV_4 - bV_3 = 0 \text{ or } -KV_{in} - V_1 - \frac{a}{s}V_1 - \frac{b}{s^2}V_1 = 0 \quad \Rightarrow \quad \frac{V_1}{V_{in}} = -\frac{Ks^2}{s^2 + as + b}$$

which is a second-order high-pass function.

Also,

$$\frac{V_2}{V_{in}} = -\frac{1}{s}\frac{V_1}{V_{in}} = \frac{Ks}{s^2 + as + b}$$

which is a bandpass function, and

$$\frac{V_3}{V_{in}} = \frac{1}{s^2}\frac{V_1}{V_{in}} = -\frac{K}{s^2 + as + b}$$

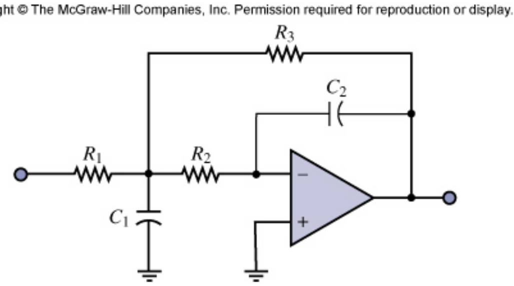
which is a low-pass function.

Problem 15.35

Solution:

Known quantities:

The filter shown in Figure P15.35.



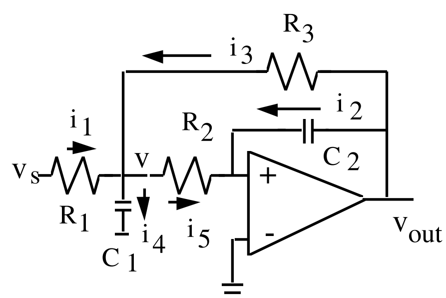
Find:

Verify that the filter's frequency response has the following expression:

$$H(j\omega) = \frac{-\left(1/R_3R_2C_1C_2\right)R_3/R_1}{(j\omega)^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_3C_1}\right)j\omega + \frac{1}{R_3R_2C_1C_2}}$$

Analysis:

The circuit is shown below:



We have:

$i_1 = \frac{V_s - V}{R_1}$, $i_2 = j\omega C_2 V_{out}$, $i_3 = \frac{V_{out} - V}{R_3}$, $i_4 = j\omega C_1 V$,

$i_5 = \frac{V}{R_2}$.

From $i_5 = -i_2$, we have: $V = -j\omega R_2 C_2 V_{out}$

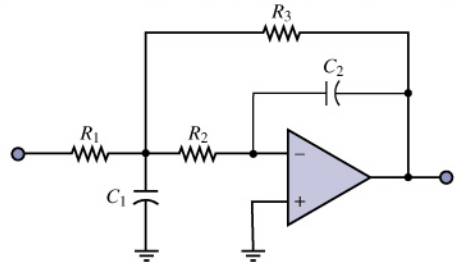
From $i_1 + i_3 = i_5 + i_4$, we have:

$$\frac{V_s}{R_1} + \frac{j\omega C_2 R_2}{R_1}V_{out} + \frac{V_{out}}{R_3} + \frac{j\omega C_2 R_2}{R_3}V_{out} = -(j\omega)^2 C_1C_2R_2V_{out} - j\omega C_2 V_{out}$$

Therefore, the frequency response is:

$$H(j\omega) = \frac{-\left(1/R_3R_2C_1C_2\right)R_3/R_1}{(j\omega)^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_3C_1}\right)j\omega + \frac{1}{R_3R_2C_1C_2}}$$

Problem 15.36



Solution:

Known quantities:

The filter shown in Figure P15.36.

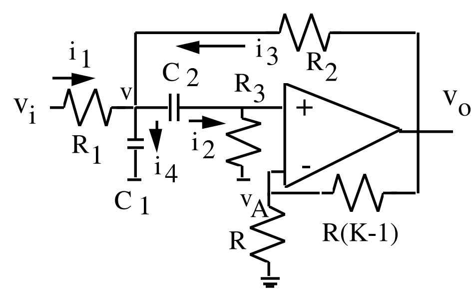
Find:

Verify that the filter's frequency response has the following expression:

$$H(j\omega)=\frac{j\omega K/R_1C_1}{(j\omega)^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{1-K}{R_2C_1}\right)j\omega+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

Analysis:

The circuit is shown below:



We have:

$$V_{AB}=\frac{R}{R+R(K-1)}V_{out}=\frac{V_{out}}{K}$$

$i_1=\frac{V_i-V}{R_1}$, $i_2=j\omega C_2(V-V_A)=\frac{V_A}{R_3}$, $i_3=\frac{V_{out}-V}{R_2}$,

$i_4=j\omega C_1 V$.

From $i_2=j\omega C_2(V-V_A)=\frac{V_A}{R_3}$, we have:

$$V=V_A\left(1+\frac{1}{j\omega R_3C_2}\right)=\frac{V_{out}}{K}\left(1+\frac{1}{j\omega R_3C_2}\right)$$

From $i_1+i_3=i_2+i_4$, we have:

$$\frac{V_i}{R_1}-\left(\frac{1}{j\omega C_2R_3}+1\right)\frac{V_{out}}{KR_1}+\frac{V_{out}}{R_2}-\left(\frac{1}{j\omega C_2R_3}+1\right)\frac{V_{out}}{KR_2}=\frac{V_{out}}{KR_3}+\left(\frac{j\omega C_1}{j\omega C_2R_3}+j\omega C_1\right)\frac{V_{out}}{K}$$

Therefore, the frequency response is:

$$H(j\omega)=\frac{j\omega K/R_1C_1}{(j\omega)^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{1-K}{R_2C_1}\right)j\omega+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

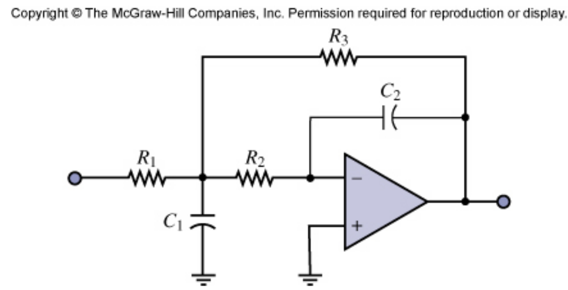
where: $K=1+\frac{R_A}{R_B}$

Problem 15.37

Solution:

Known quantities:

The filter shown in Figure P15.35.



Find:

Verify that for the filter:

$$\frac{1}{Q}=\sqrt{R_2R_3\frac{C_2}{C_1}}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

Analysis:

From the expression, we can see that

$$\omega_c=\sqrt{\frac{1}{C_1C_2R_2R_3}} \text{ and } \frac{\omega_c}{Q}=\frac{1}{C_1R_1}+\frac{1}{C_1R_2}+\frac{1}{C_1R_3}$$

or $$\frac{1}{Q}=\sqrt{C_1C_2R_2R_3}\left(\frac{1}{C_1R_1}+\frac{1}{C_1R_2}+\frac{1}{C_1R_3}\right)$$

or $$\frac{1}{Q}=\sqrt{R_2R_3\frac{C_2}{C_1}}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

Section 15.4: Analog-to-Digital and Digital-to-Analog Conversion

Problem 15.38

Solution:

Find:
List two advantages of digital signal processing over analog signal processing.

Analysis:
1. Digital signals are less subject to noise, since one only needs to discriminate between two voltages.
2. Digital signals are directly compatible with.digital computers, and can therefore be easily stored on a disk, or exchanged between computers. Thus, digital signals are intrinsically more portable than analog signals.

Problem 15.39

Solution:

Find:
Discuss the role of a multiplexer in a data acquisition system.

Analysis:
It sequentially switches a set of analog inputs to the system input.

Problem 15.40

Solution:

Find:
Discuss the purpose of using a sample-and-hold circuits in data acquisition systems.

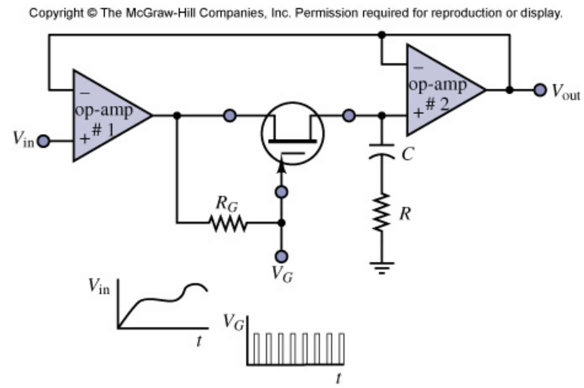
Analysis:
A sample-and-hold circuits "freezes" (holds) the value of a signal at the input to an ADC to allow the ADC to convert the analog signal without its changing. The sample-and-hold circuit is necessary because of the finite conversion time of the ADC.

Problem 15.41

Solution:

Known quantities:

The circuit shown in Figure P15.41.



Find:

Explain the operation of the circuit.

Analysis:

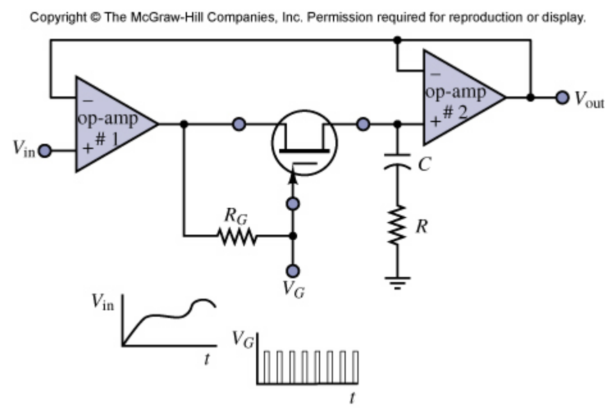
Op-amp #1 is an input buffer. The JFET behaves as a low-leakage diode which enables and disables the RC holding circuit, and op-amp #2 is a voltage-follower whose purpose is to isolate the circuit from the load.

Problem 15.42

Solution:

Known quantities:

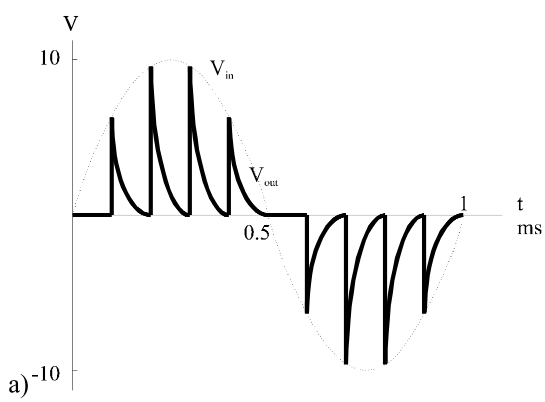
The circuit shown in Figure P15.41. The input is a 1 kHz sinusoidal signal with 0° phase angle, 0 V DC offset and 20 V peak to peak amplitude. V_G is a rectangular pulse train with 10 μs, and a period 100 μs, with a leading edge of the first pulse at *t=0*.

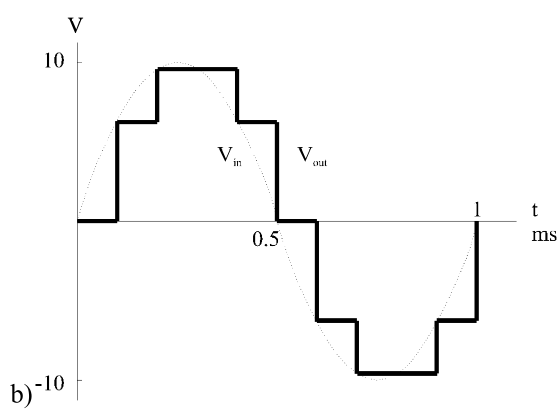


Find:

a) Sketch V_{out} if the *RC* circuit has a time constant equal to 20 μs.
b) Sketch V_{out} if the *RC* circuit has a time constant equal to 1 ms.

Analysis:





Problem 15.43

Solution:

Known quantities:

The input to a four-digit DAC, 12_{10}, given that $R_F = R_0/15$, logic 0 corresponds to 0 V, and logic 1 corresponds to 4.5 V.

Find:

a) The output of the DAC.
b) The maximum voltage that can be outputted from the DAC.
c) The resolution over the range 0 to 4.5 V.
d) The number of bits required if an improved resolution of 20 mV is desired.

Analysis:

$$V_a = -4.5\frac{R_F}{R_0}\left[2^3 b_3 + 2^2 b_2 + 2^1 b_1 + b_0\right]$$

a) $V_a = -4.5\frac{1}{15}[12] = -3.6$ **V**

b) $(V_a)_{\max} = -4.5\frac{1}{15}(15) = -4.5$ **V**

c) $\delta V_a = 4.5\frac{1}{15} = 0.3$ **V**

d) $$n \geq \frac{\log\left(\frac{\left|(V_a)_{\max} - (V_a)_{\max}\right|}{(\delta V_a)_{req}} + 1\right)}{\log 2} = 7.82$$

Therefore, we choose $n = 8$.

Problem 15.44

Solution:

Known quantities:

The input to a eight-digit DAC, 215_{10}, given that $R_F = R_0/255$, logic 0 corresponds to 0 V, and logic 1 corresponds to 10 V.

Find:

a) The output of the DAC.
b) The maximum voltage that can be outputted from the DAC.
c) The resolution over the range 0 to 10 V.
d) The number of bits required if an improved resolution of 3 mV is desired.

Analysis:

$215_{10} = 11010111_2$

$$V_a = -10\frac{R_F}{R_0}\left[2^7 b_7 + 2^6 b_6 + \ldots + 2^1 b_1 + b_0\right]$$

a) $V_a = -10\frac{1}{255}[215] = -8.341$ **V**

b) $(V_a)_{\max} = -10\frac{1}{255}[255] = -10$ **V**

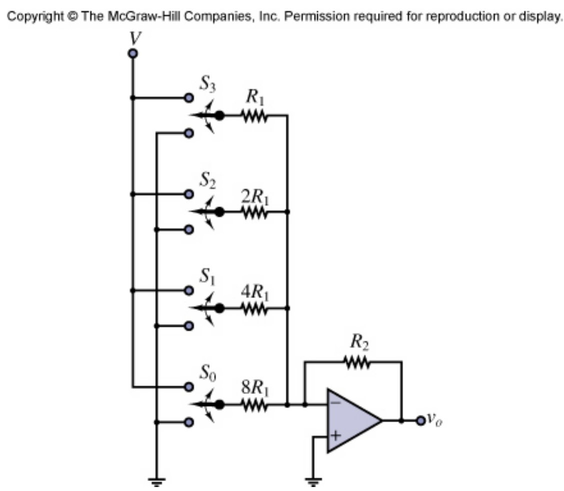
c) $\delta V_a = 10\frac{1}{255} = 39.2$ **mV**

d) $$n \geq \frac{\log\left(\frac{\left|(V_a)_{\max} - (V_a)_{\max}\right|}{(\delta V_a)_{req}} + 1\right)}{\log 2} = 11.703$$

Therefore, we choose n = 12.

Problem 15.45

Solution:



Known quantities:

The circuit of Figure P15.45, a simple 4-bit DAC. In the circuit if the bit is 1, the corresponding switch is up, if the bit is 0 the switch is down.

Find:

If the digital number is represented by $b_3b_2b_1b_0$, determine an expression relating v_0 to the binary input bits..

Analysis:

This circuit is just a summing amplifier, with

$$v_O = -\frac{R_2}{R_1}b_3V - \frac{R_2}{2R_1}b_2V - \frac{R_2}{4R_1}b_1V - \frac{R_2}{8R_1}b_0V = -\frac{R_2V}{R_1}\left(b_3 + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8}\right)$$

$$= -\frac{R_2V}{8R_1}\left(8b_3 + 4b_2 + 2b_1 + b_0\right)$$

Problem 15.46

Solution:

Known quantities:

The input to a eight-digit DAC, 98_{10}, given that $R_F = R_0/255$, logic 0 corresponds to 0 V, and logic 1 corresponds to 4.5 V.

Find:

a) The output of the DAC.
b) The maximum voltage that can be outputted from the DAC.
c) The resolution over the range 0 to 4.5 V.
d) The number of bits required if an improved resolution of 0.5 mV is desired.

Analysis:

a) $V_a = 4.5\frac{1}{255}[98] = 1.729\ \mathbf{V}$

b) $(V_a)_{\max} = 4.5\frac{1}{255}[255] = 4.5\ \mathbf{V}$

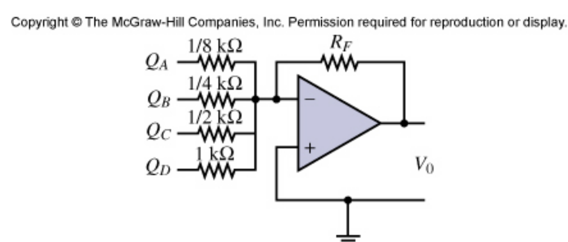
c) $\delta V_a = 4.5\frac{1}{255} = 17.6\ \mathbf{mV}$

d) $n \ge \frac{\log\left(\frac{\left|(V_a)_{\max} - (V_a)_{\max}\right|}{(\delta V_a)_{req}} + 1\right)}{\log 2} = 13.136$

Therefore, we choose n = 14.

Problem 15.47

Solution:



Known quantities:

The four-digit DAC of Figure P15.47, with an output range $-10 \le V_O \le 0\ \mathbf{V}$.

Find:

The value of R_F that will give that output.

Assumptions:

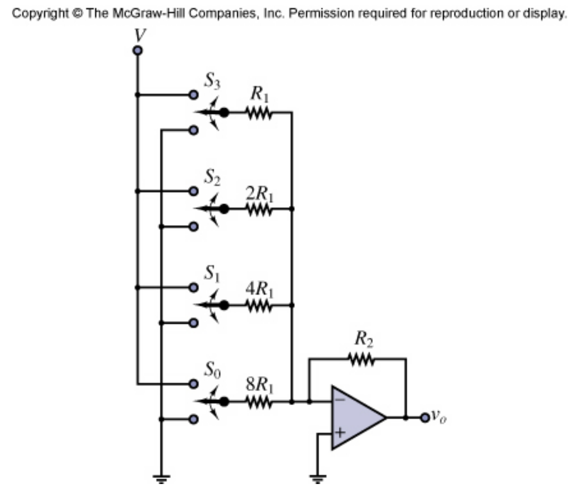
The logic 0 = 0 V and the logic 1 = 5 V.

Analysis:

For the circuit of Figure P15.47, $R_O = 1\ \mathbf{k\Omega}$ and n = 4.

Therefore, $(V_a)_{\max} = -10\ \mathbf{V}$, and $-10 = -5\frac{R_f}{1000}15$, or $R_f = 133.3\ \Omega$.

Problem 15.48



Solution:

Known quantities:

The circuit of Figure P15.45.

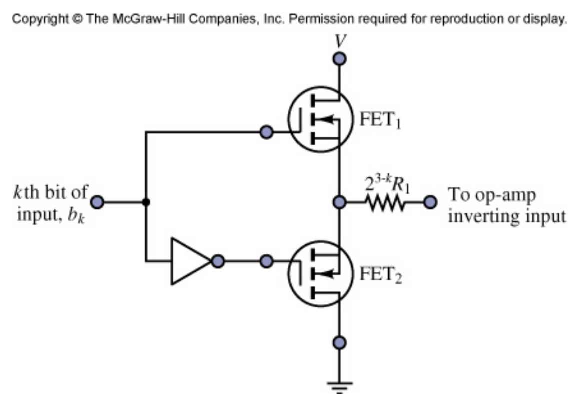
Find:

Explain how to redesign that circuit so that it becomes a "noninverting" device.

Analysis:

The simplest way would be to cascade the circuit of Figure P15.45 with a single-input inverting amplifier like the one shown in Figure 8.5 in the text. The two resistors in that circuit could be chosen equal in value so that all the second amplifier does is change the sign of the output.

Problem 15.49



Solution:

Known quantities:

The circuit of Figure P15.49.

Find:

Explain how the circuit works as a means of implementing the switches needed for the digital to analog converter of figure P15.45.

Analysis:

If $b_k = 0$, FET_1 is cutoff and FET_2 is on. Under these conditions, the left-hand end of the resistor is grounded.

If $b_k = 1$, FET_2 is cutoff and FET_1 is on. Under these conditions, the left-hand end of the resistor is connected to V.

Problem 15.50

Solution:

Known quantities:

The input to a twelve-digit DAC, 345_{10}, given that $R_F = R_0/4{,}095$, logic 0 corresponds to 0 V, and logic 1 corresponds to 10 V.

Find:

a) The output of the DAC.
b) The maximum voltage that can be outputted from the DAC.
c) The resolution over the range 0 to 10 V.
d) The number of bits required if an improved resolution of 0.5 mV is desired.

Analysis:

a) $V_a = -10\frac{1}{4095}[345] = -0.8425\ \mathbf{V}$

b) $(V_a)_{\max} = -10\frac{1}{4095}[4095] = -10\ \mathbf{V}$

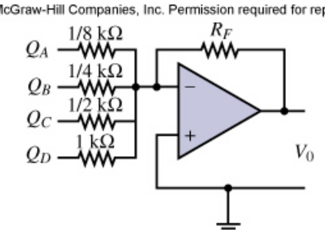
c) $\delta V_a = 10\frac{1}{4095} = 2.44\ \mathbf{mV}$

d) $n \ge \dfrac{\log\left(\dfrac{\left|(V_a)_{\max} - (V_a)_{\max}\right|}{(\delta V_a)_{req}} + 1\right)}{\log 2} = 14.288$

Therefore, we choose a 15-bit ADC.

Problem 15.51

Solution:



Known quantities:

The four-digit DAC of Figure P15.47, with an output range $-15 \le V_O \le 0\ \mathbf{V}$.

Find:

The value of R_F that will give that output.

Assumptions:

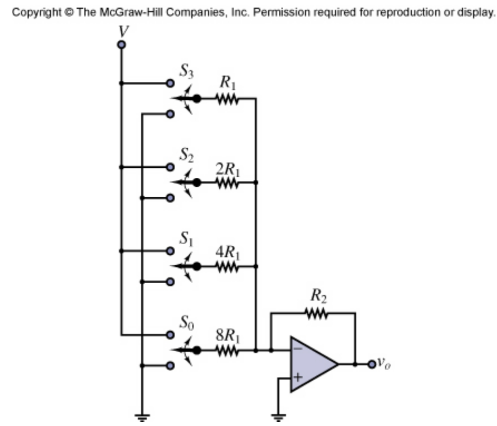
The logic 0 = 0 V and the logic 1 = 5 V.

Analysis:

Here, $-15 \le V_O \le 0\ \mathbf{V}$. For the circuit of Figure P15.46, $R_O = 1\ \mathbf{k\Omega}$ and n = 4.

Therefore, $(V_a)_{\max} = -15\ \mathbf{V}$, and $-15 = -5\frac{R_f}{1000}15$, or $R_f = 200\ \Omega$.

Problem 15.52



Solution:

Known quantities:

The circuit of Figure P15.45, and the desired output of the 4-bit DAC, $V_0 = -\frac{1}{10}(8b_3 + 4b_2 + 2b_1 + b_0)$ **V** .

Find:

The design of the DAC.

Analysis:

From the results of Problem 15.45, we see that we must choose V_0, R_2 and R_1 such that: $\frac{R_2 \cdot V_0}{8 \cdot R_1} = \frac{1}{10}$

One possible choice is to let $V = 15\,\mathbf{V}$, $R_1 = 30\ \mathbf{k\Omega}$ and $R_2 = 1.6\ \mathbf{k\Omega}$.

Problem 15.53

Solution:

Known quantities:

The range, □15 V, and the resolution, 0.01 V, of a data acquisition system.

Fin

The number of bits of the DAC.

Analysis:

$n \ge \log_2\left(\frac{15-(-15)}{0.01}+1\right) = 11.55$. Choose n = 12.

Problem 15.54

Solution:

Known quantities:

The range, ±10 V, and the resolution, 0.04 V, of a data acquisition system.

Find:

The number of bits of the DAC.

Analysis:

$n \ge \log_2\left(\frac{10-(-10)}{0.04}+1\right) = 8.96$. Choose n = 9.

Problem 15.55

Solution:

Known quantities:

The range, –10 – 15 V, and the resolution, 0.004 V, of a data acquisition system.

Find:

The number of bits of the DAC.

Analysis:

$n \ge \log_2\left(\frac{15-(-10)}{0.004}+1\right) = 12.6$. Choose $n = 13$.

Problem 15.56

Solution:

Known quantities:

The range, 0 – 2,500 rev/min, and the resolution, 1 rev/min, of a DAC used to deliver velocity commands to a motor.

Find:

The number of bits of the DAC.

Analysis:

$n \ge \log_2\left(\frac{2500-(0)}{1}+1\right) = 11.29$. Choose $n = 12$.

With this choice we compute the following resolution: $res = \frac{2500}{2^{12}} = 0.61$ **rev/min**

Problem 15.57

Solution:

Known quantities:

The range, 0 – 10 V, for an ADC.

Find:

a) The resolution if this is a 3-bit device.
b) The resolution if this is a 8-bit device.
c) A general comment about the relationship between the number of bits and the resolution.

Analysis:

a) $res = 2^{-3} \cdot 10 \text{ } \mathbf{V} = 1.25 \text{ } \mathbf{V}$.

b) $res = 2^{-8} \cdot 10 \text{ } \mathbf{V} = 39.0625 \text{ } \mathbf{mV}$.

c) more bits give better resolution.

Problem 15.58

Solution:

Known quantities:

The range, -5 – 15 V, and the resolution required, 0.05 %, for a DAC.

Find:

The number of bits required.

Analysis:

The range is 15 - (-5) = 20 V

Thus, $n \ge \log_2\left(\dfrac{20}{20 \cdot 0.0005} + 1\right) = 10.97$. Choose $n = 11$.

Problem 15.59

Solution:

Known quantities:

The number of channels of an ADC, eight, the time required for ADC conversion, 100 μs, the time required for computation and output time for four of the channels, 500 μs, and for the other four, 250 μs.

Find:

The number of bits required.

Analysis:

We assume a data acquisition system of the type shown in Figure 15.32. Therefore, each channel will be sampled at $\frac{1}{8}$ of the external clock rate and the slowest channels will determine the rate.

Thus, *sampling rate* = 8(100 μs + 500 μs) = 4.8 ms.

Thus, $f_s = \dfrac{1}{4.8 \cdot 10^{-3}} = 208.3$ **Hz** and $f_{\max} = \dfrac{f_s}{2} = 104.15$ **Hz**

Problem 15.60

Solution:

Known quantities:

The range of the potentiometer, 270° and 10 V, and the maximum displacement to be measured, 180°.

Find:

a) The voltage to be resolved by an ADC to resolve an angular displacement of 0.5°, and the number of bits to do that.
b) If the ADC requires a 10 V input voltage find the optimum amplifier gain to take advantage of the full range of the ADC.

Analysis:

For a dynamic range of 10 V for 270° of rotation, we compute the following resolution:

$$res = \frac{10}{270} 0.5 = 18.52 \ \mathbf{mV}$$

a) Finding the range for 180° rotation it is possible to determine the bits requirement for the ADC.

$$n = \log_2 \left(\frac{10\left(\frac{180}{270}\right)}{18.52 \cdot 10^{-3}} + 1 \right) = 8.50$$, and we choose n = 9.

b) The voltage gain of the amplifier is

$$gain = \frac{270}{180} = 1.5$$

Problem 15.61

Solution:

Known quantities:

The maximum frequency of the signal to be digitized, 250 kHz, and the number of bits, 10, of the successive approximation ADC used.

Find:

The maximum permissible conversion time.

Analysis:

The conversion time should be no more than 10% of the signal period. For this case, the signal period is

$$T = \frac{1}{250 \cdot 10^3} = 4\ \mathbf{\mu s}$$

Therefore, the conversion time should be no longer than 400 ns.

Problem 15.62

Solution:

Known quantities:

The maximum frequency of the signal to be digitized, a torque signal from a torque sensor mouted on a farm tractor engine, is twice the shaft rotation frequency; the rotational speed of the crankshaft is 800 rpm.

Find:

The minimum sampling period according to Nyquist criterion.

Analysis:

Shaft rotation frequency is 800 rpm or 13.33 rev/sec; therefore, the fluctuation frequency is 26.67 Hz:

$$T_S = \frac{1}{26.67 \cdot 2} = 18.75\ \text{ms}$$

Problem 15.63

Solution:

Known quantities:

The range of an aircraft altimeter, from 0 V at 0 m to 10 V at 10000m, and the allowable error in sensing, 10 m.

Find:

The minimum number of bits for the ADC.

Analysis:

The allowable error is ±10 m. Therefore, an equivalent 20 m step size is allowable and

$$n = \log_2\left(\frac{10000}{20} + 1\right) = 8.97$$

Thus, a 9-bit ADC is required.

Problem 15.64

Solution:

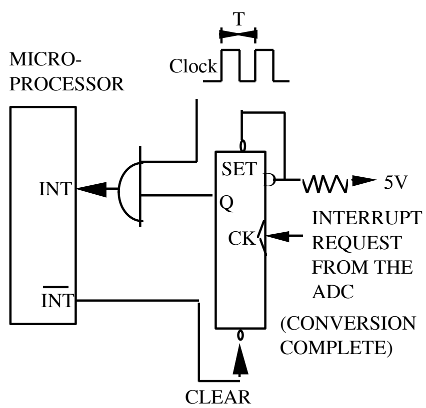
Known quantities:

The characteristic of the circuit needed, a circuit that generates interrupts at fixed time intervals.

Find:

The design of the circuit using a square wave that has a period equal to the desired time interval between interrupts.

Analysis:



Problem 15.65

Solution:

Find:

The minimum number of bits required to digitize an analog signal with a resolution of:
a) 5%. b) 2%. c) 1%.

Analysis:

a) $5\% \quad \Rightarrow \quad 2^{-n} \le 0.05 \quad \Rightarrow \quad n = 5$

b) $2\% \quad \Rightarrow \quad 2^{-n} \le 0.02 \quad \Rightarrow \quad n = 6$

c) $1\% \quad \Rightarrow \quad 2^{-n} \le 0.01 \quad \Rightarrow \quad n = 7$

Section 15.5: Comparator and Timing Circuits

Problem 15.66

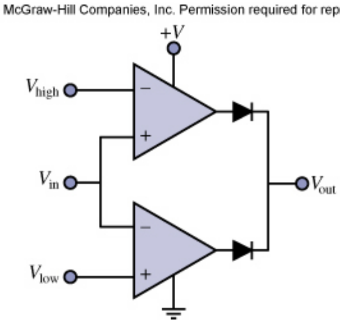
Solution:

Known quantities:

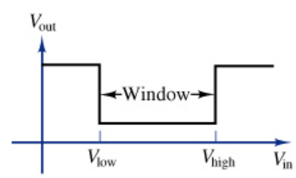
The window comparator circuit of Figure P15.66.

Find:

Show that $V_{out} = 0$ whenever $V_{low} \le V_{in} \le V_{high}$, and that $V_{out} = V_{in}$ otherwise.



(a)



(b)

Analysis:

If $V_{in} \le V_{low}$,

$$\left.\begin{array}{l} V_1 = 0 \\ V_2 = V^+ \end{array}\right\} \Rightarrow V_{out} = V^+$$

If $V_{low} \le V_{in} \le V_{high}$,

$$\left.\begin{array}{l} V_1 = 0 \\ V_2 = 0 \end{array}\right\} \Rightarrow V_{out} = 0$$

If $V_{in} \ge V_{high}$,

$$\left.\begin{array}{l} V_1 = V^+ \\ V_2 = 0 \end{array}\right\} \Rightarrow V_{out} = V^+$$

Problem 15.67

Solution:

Known quantities:

The noise peak amplitude, ±150 mV, the reference value around which the circuit is to switch, -1 V, and the characteristic of the op-amp, ±10 V supplies (V_{sat} = 8.5 V).

Find:

The design of the circuit.

Analysis:

This is very similar to Example 15.14, Therefore,

$$v^+ = \frac{R_2}{R_2 + R_1} v_{out} + \frac{R_2}{R_2 + R_3} v_s^+$$

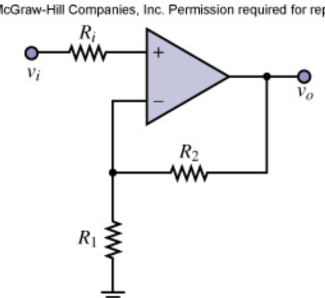
Since the required noise protection level is ±150 mV, R_1 and R_2 can be computed from:

$$\frac{\Delta v}{2} = \frac{R_2}{R_2 + R_1} v_{sat} = \frac{R_2}{R_2 + R_1} 8.5 = 0.15 \textbf{ V} \quad \text{where } \Delta v = 300 \textbf{ mV}$$

Assuming $R_1 = 100 \textbf{ k}\boldsymbol{\Omega}$, R_2 can be calculated to be approximately 1.8 kΩ. Since the required reference voltage is -1 V, we can find R_3 by solving the equation

$$\frac{R_2}{R_2 + R_3} v_s^- = \frac{R_2}{R_2 + R_3} 10 = 1 \textbf{ V} \quad \text{to obtain: } R_3 = 16.2 \textbf{ k}\Omega .$$

Problem 15.68



Solution:

Known quantities:

The circuit of Figure P15.68; $R_1 = 100 \ \boldsymbol{\Omega}$, $R_2 = 56 \textbf{ k}\Omega$,

$R_i = R_1 \| R_2$, and v_{in} is a 1 V peak to peak sine wave.

Find:

The threshold voltages and the output waveform.

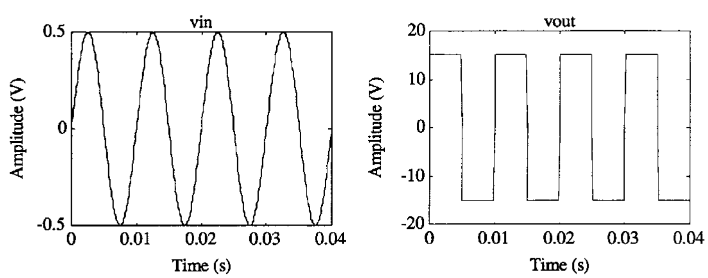
Assumptions:

The supply voltages are ±15 V.

Analysis:

Applying KCL: $\frac{V_o}{V_{in}} = 1 + \frac{R_2}{R_1}$. Therefore,

$$\frac{V_o}{V_{in}} = 561$$

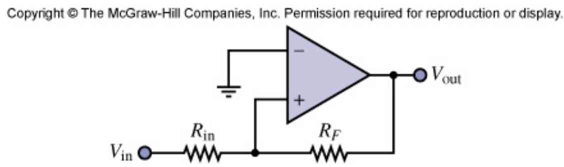


Problem 15.69

Solution:

Known quantities:

The circuit of Figure P15.69.



Find:

Explain the operation of the circuit.

Analysis:

Operation of this circuit depends on the magnitude of the input voltage, V_{in}, being much larger than zero. When $V_{in} >> 0$, the op-amp will be saturated, and V_{out} will be at its saturation limit, V_R (positive rail).

When the op-amp is saturated, the approximation $V_+ = V_-$ is no longer valid. Writing KCL at the non-inverting input, we have

$$\frac{V_+ - V_{in}}{R_{in}} + \frac{V_+ - V_{out}}{R_F} = 0 \quad \text{or} \quad V_+\left(\frac{1}{R_{in}} + \frac{1}{R_F}\right) = \frac{1}{R_{in}}V_{in} + \frac{1}{R_F}V_{out}$$

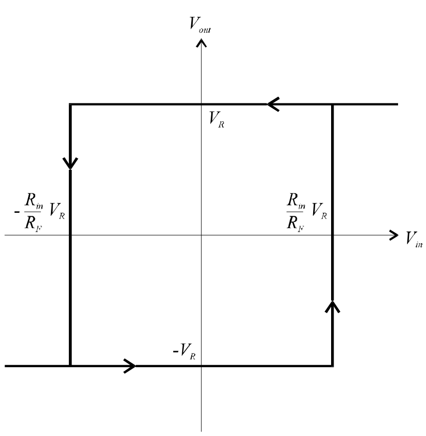
The output changes from positive rail $(+V_R)$ to negative rail $(-V_R)$ when $V_+ = V_- = 0$. When V_{in} is small, then $V_+ \approx V_- = 0$, and $V_{out} = -\left(\frac{R_F}{R_{in}}\right)V_{in}$

If $V_{out} = V_R$, then: $V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_R$

What this means is the following: As V_{in} drops from a large positive value, through zero, to a negative value, the output of the op-amp "switches" (from $+V_R$ to $-V_R$) at the point $V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_R$.

Similarly, as V_{in} increases in the opposite direction, the "switch" from $-V_R$ to $+V_R$ occurs at $V_{in} = +\left(\frac{R_{in}}{R_F}\right)V_R$.

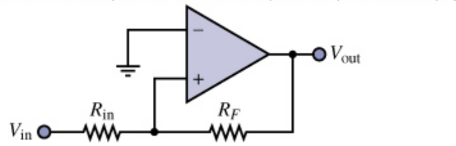
This is a form of voltage hysteresis, as shown in the figure below.



Problem 15.70

Solution:

Known quantities:



The circuit of Figure P15.69. The op-amp is a LM741 with ±15 V, $R_F = 104$ **kΩ**, and V_{in} is a 1 kHz sinusoidal signal with 1 V amplitude.

Find:

a) The appropriate value for Rin if the output is to be high whenever $|V_{in}| \geq 0.25$ **V**.

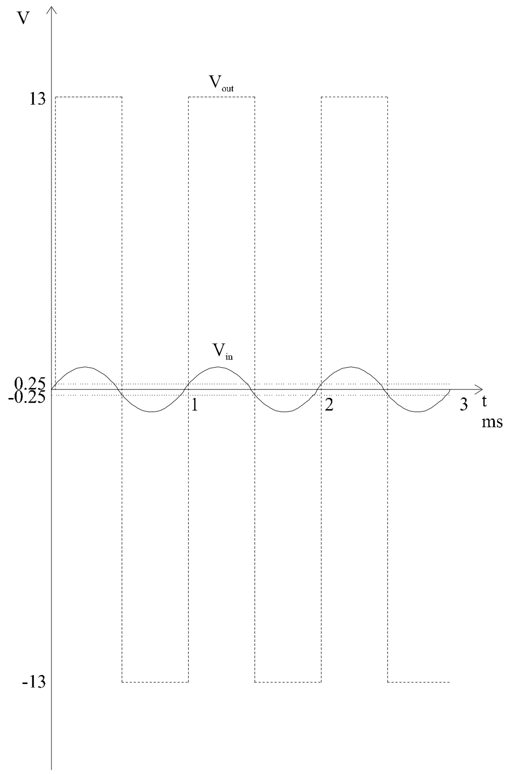
b) Sketch the input and the output waveforms.

Analysis:

a) We know that $V_R \approx 13V$ when the LM741 op-amp is used with $\pm 15V$ bias supplies. Then, from the discussion in the answer to Problem 15.69,

$$V_{in} = -\left(\frac{R_{in}}{R_F}\right)V_{out} \quad \Rightarrow \quad R_{in} = \left(\frac{V_{in}}{V_{out}}\right)R_F = \left(\frac{0.25}{13}\right)(104\ \mathbf{k\Omega}) = 2\ \mathbf{k\Omega}$$

b) The input and output waveforms are sketched below.

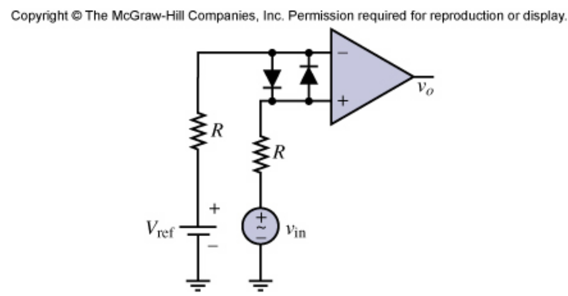


Problem 15.71

Solution:

Known quantities:

The circuit of Figure P15.71.

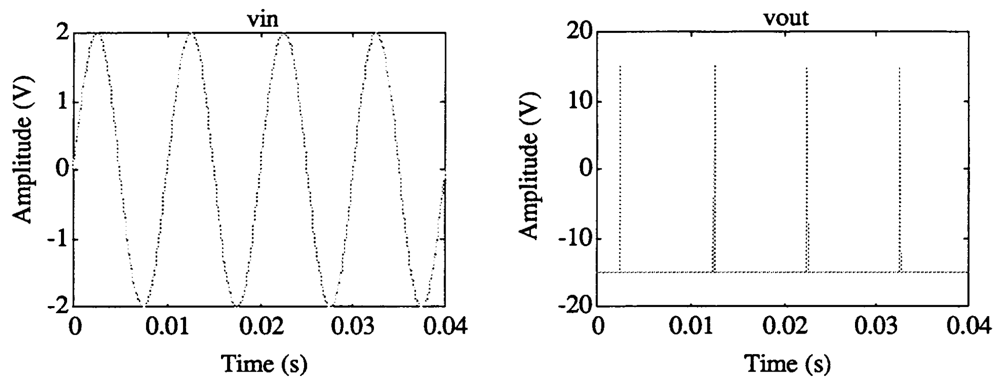


Find:

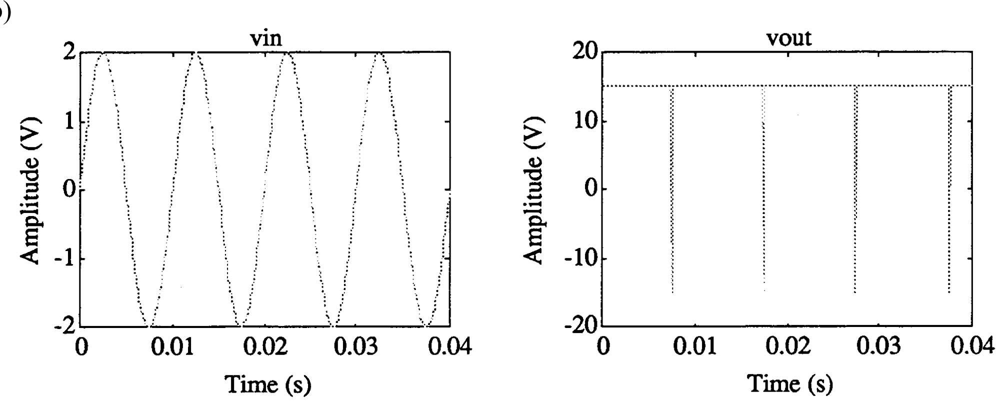
a) The output waveform for v_{in} a 4 V peak to peak sine wave at 100 Hz and $V_{ref} = 2$ V.
b) The output waveform for v_{in} a 4 V peak to peak sine wave at 100 Hz and $V_{ref} = -2$ V.

Analysis:

a)



b)

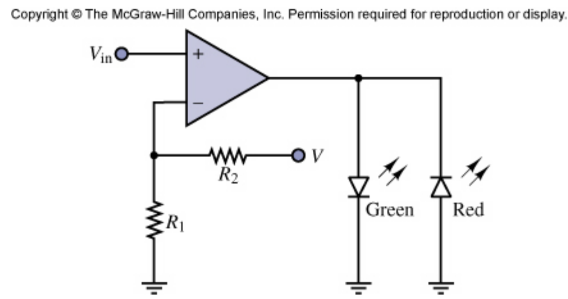


Problem 15.72

Solution:

Known quantities:

The go-no go detector application circuit of Figure P15.72.



Find:

a) Explain how the circuit works.
b) Design a circuit such that the green LED will turn on when Vin exceeds 5 V, and the LED will be on whenever Vin is less than 5 V.

Assumptions:

Only 15 V supplies are available.

Analysis:

a) Define $V_2 = \frac{R_1}{R_2 + R_1}V$ as the voltage at the inverting input of the op-amp. Then:

When $V_{in} > V_2$ the output of the op-amp will be positive and the green LED will turn on (*go*).

When $V_{in} < V_2$ the output of the op-amp will be negative and the red LED will turn on (*no go*)

b) For this design, $V_2 = 5\ \mathbf{V}$ and $V = 15\ \mathbf{V}$.

$$\frac{R_1}{R_2 + R_1}15\ \mathbf{V} = 5\ \mathbf{V} \quad \Rightarrow \quad \frac{R_1}{R_2 + R_1} = \frac{1}{3}$$

or $\frac{R_2}{R_1} + 1 = 3 \quad \Rightarrow \quad \frac{R_2}{R_1} = 2$

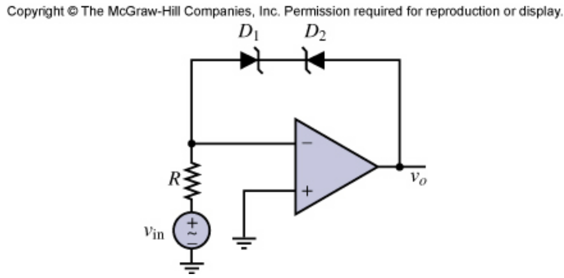
Choose $R_1 = 10\ \mathbf{k\Omega}$ and $R_2 = 20\ \mathbf{k\Omega}$ to complete the design.

Problem 15.73

Solution:

Known quantities:

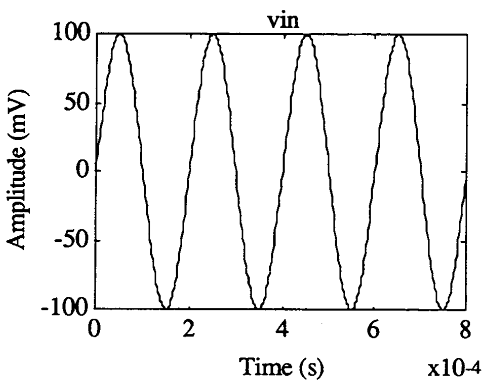
The circuit of Figure P15.73, where v_{in} is a 100 mV peak sine wave at 5 kHz, R = 10 kΩ, and *D1* and *D2* are 6.2 V Zener diodes.

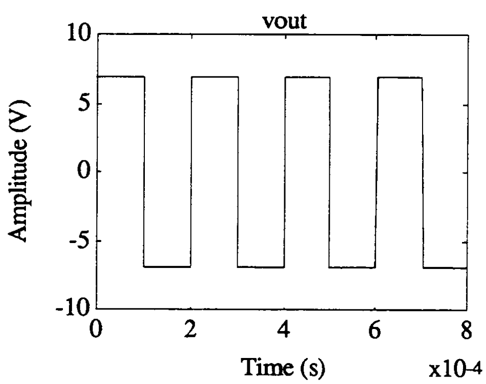


Find:

Draw the output voltage waveform.

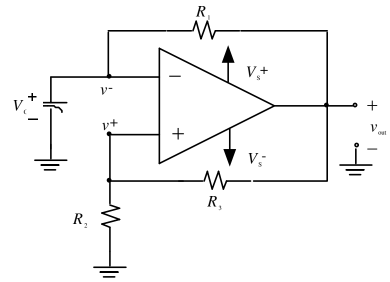
Analysis:





Problem 15.74

Note: this problem is missing a figure in the text.



Solution:

Known quantities:

An op-amp astable multivibrator, shown in the figure.

Find:

Show that the period of it is given by the expression $T = 2R_1 C \log_e\left(\frac{2R_2}{R_3}+1\right)$

Analysis:

The capacitor voltage is: $v_C(t) = \left(v_C(0) - v_C(\infty)\right)e^{-\frac{t}{R_1 C}} + v_C(\infty)$

Let the period be $T = t_1 + t_2$

where t_1 corresponds to the falling exponential and t_2 corresponds to the rising case. In the falling half period

$$v_C(0) = \frac{R_2}{R_2+R_3}v_{sat} \qquad v_C(\infty) = -v_{sat}$$

$$v_C(t) = \left(\frac{R_2}{R_2+R_3}v_{sat} + v_{sat}\right)e^{-\frac{t_1}{R_1 C}} - v_{sat}$$

when $t = t_1, v_C(t_1) = -\frac{R_2}{R_2+R_3}v_{sat}$

Therefore, we have $\frac{R_3}{2R_2+R_3} = e^{-\frac{t_1}{R_1 C}}$

The time t_1 is: $t_1 = -R_1 C \log_e\left(\frac{R_3}{2R_2+R_3}\right)$

In the rising half period $v_C(0) = -\frac{R_2}{R_2+R_3}v_{sat}$ $\qquad v_C(\infty) = v_{sat}$

when $t = t_2, v_C(t_2) = \frac{R_2}{R_2+R_3}v_{sat}$

We have $v_C(t) = \left(-\frac{R_2}{R_2+R_3}v_{sat} - v_{sat}\right)e^{-\frac{t_2}{R_1 C}} + v_{sat}$

From $\frac{R_3}{2R_2+R_3} = e^{-\frac{t_2}{R_1 C}}$, we have $t_2 = -R_1 C \log_e\left(\frac{R_3}{2R_2+R_3}\right)$

The period of the square-wave waveform therefore is:

$$T = t_1 + t_2 = -R_1 C \log_e\left(\frac{R_3}{2R_2+R_3}\right) - R_1 C \log_e\left(\frac{R_3}{2R_2+R_3}\right)$$

$$= -2R_1 C \log_e\left(\frac{R_3}{2R_2+R_3}\right) = 2R_1 C \log_e\left(\frac{2R_2+R_3}{R_3}\right) = 2R_1 C \log_e\left(\frac{2R_2}{R_3}+1\right)$$

Problem 15.75

Solution:

Known quantities:

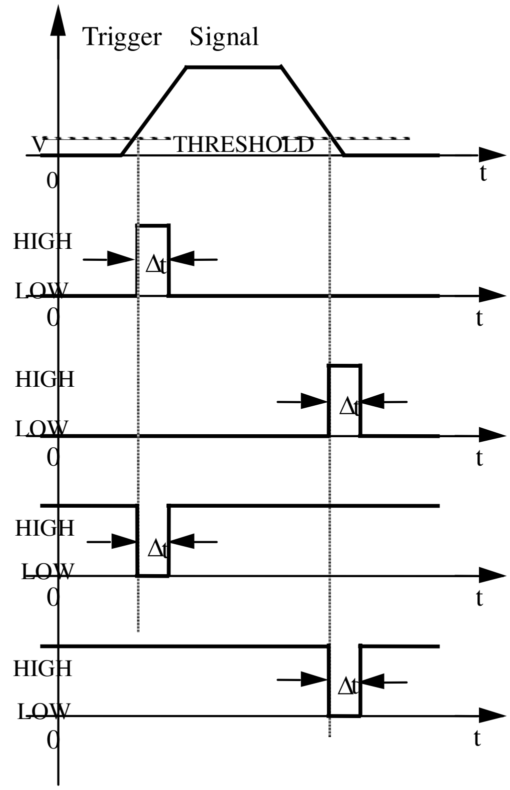
Data sheets for the 74123 monostable multivibrator and the circuit shown in Figure 15.54 in the text.

Find:

Draw a timing diagram indicating the approximate duration of each pulse, assuming that the trigger signal consists of a positive-going transition

Analysis:

Looking at the data sheet for the 74123 monostable multivibrator, we can see that for a positive going transition, A must be LOW and B must be HIGH.



Problem 15.76

Solution:

Known quantities:

NE 555 timer circuit of Figure 15.55 in the text, which represents a one-shot multivibrator circuit. $R_1 = 10\text{k}\Omega$ and $T = 10\text{ms}$.

Find:

Determine the value of *C*.

Analysis:

In one-shot multivibrator circuit, $T = 1.1R_1C$

In this case, $T = 10\text{ms}$, so

$$C = \frac{T}{1.1R_1} = \frac{10\text{ms}}{1.1\times 10\text{k}\Omega} = 0.91\mu\text{F}$$
