

5. Important circuit concepts such as the principle of superposition and Thévenin's theorem are also applicable in the frequency domain.

6. The instantaneous power absorbed by an element is equal to the product of the voltage across it and the current through it.

7. The average power absorbed by a resistance  $R$  having a sinusoidal current of amplitude  $I$  and voltage of amplitude  $V$  is

$$P_R = \frac{1}{2}VI = \frac{1}{2}RI^2 = \frac{1}{2}\frac{V^2}{R}$$

8. The average power absorbed by a capacitance or an inductance is zero.

9. A circuit whose Thévenin-equivalent (output) impedance is  $Z_o$  transfers maximum power to a load  $Z_L$  when  $Z_L$  is equal to the complex conjugate of  $Z_o$ .

10. For the case in which  $Z_L$  is restricted to be purely resistive, maximum power is transferred when  $Z_L$  equals the magnitude of  $Z_o$ .

11. The effective or rms value of a sinusoid of amplitude  $A$  is  $A/\sqrt{2}$ .

12. The average power absorbed by a resistance  $R$  having a current whose effective value is  $I_e$  and a voltage whose effective value is  $V_e$  is

$$P_R = V_e I_e = RI_e^2 = \frac{V_e^2}{R}$$

13. The power factor (pf) is the ratio of average power to apparent power.

14. If current lags voltage, the pf is lagging. If current leads voltage, the pf is leading.

15. Average or real power can be generalized with the notion of complex power.

16. The ordinary household uses a single-phase, three-wire electrical system.

17. The most common polyphase electrical system is the balanced three-phase system.

18. Three-phase sources are generally Y connected, and three-phase loads are generally  $\Delta$  connected.

19. The device commonly used to measure power is the wattmeter.

20. Three-phase load power measurements can be taken with the two-wattmeter method.

## Problems

4.1 Find the exponential form of the following complex numbers given in rectangular form: (a)  $4 + j7$ , (b)  $3 - j5$ , (c)  $-2 + j3$ , (d)  $-1 - j6$ , (e)  $4$ , (f)  $-5$ , (g)  $j7$ , (h)  $-j2$ .

4.2 Find the rectangular form of the following complex numbers given in exponential form: (a)  $3e^{j70^\circ}$ , (b)  $2e^{j120^\circ}$ , (c)  $5e^{-j60^\circ}$ , (d)  $4e^{-j150^\circ}$ , (e)  $6e^{j90^\circ}$ , (f)  $e^{-j90^\circ}$ , (g)  $2e^{j180^\circ}$ , (h)  $2e^{-j180^\circ}$ .

4.3 Find the rectangular form of the product  $A_1 A_2$  given that: (a)  $A_1 = 3e^{j30^\circ}$ ,  $A_2 = 4e^{j60^\circ}$ ; (b)  $A_1 = 3e^{j30^\circ}$ ,  $A_2 = 4e^{-j30^\circ}$ ; (c)  $A_1 = 5e^{-j60^\circ}$ ,  $A_2 = 2e^{j120^\circ}$ ; (d)  $A_1 = 4e^{j45^\circ}$ ,  $A_2 = 2e^{-j90^\circ}$ .

4.4 Find the rectangular form of the quotient  $A_1/A_2$  for  $A_1$  and  $A_2$  given in Problem 4.3.

4.5 Find the rectangular form of the sum  $A_1 + A_2$  for  $A_1$  and  $A_2$  given in Problem 4.3.

4.6 For the ac circuit shown in Fig. P4.6, suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find  $v_o(t)$  by using voltage division. Draw a phasor diagram. Is this circuit a lag network or a lead network?

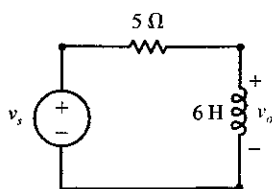


Fig. P4.6

**4.7** Connect a  $5\text{-}\Omega$  resistor in parallel with the inductor in the circuit shown in Fig. P4.6. Suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find the voltage  $v_o(t)$  across the inductor by using voltage division. Draw a phasor diagram. Is this circuit a lag network or a lead network?

**4.8** Connect a  $5\text{-}\Omega$  resistor in parallel with the inductor in the circuit shown in Fig. P4.6. Suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find the voltage  $v_o(t)$  across the inductor by using nodal analysis. Draw a phasor diagram. Is this circuit a lag network or a lead network?

**4.9** For the circuit given in Fig. P4.9, suppose that  $i_s(t) = 5 \cos 3t$  A. Find  $v_o(t)$  and  $v_s(t)$  by using current division.

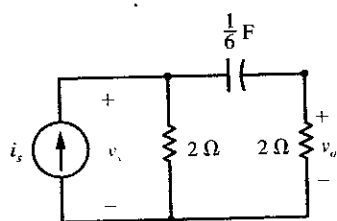


Fig. P4.9

**4.10** For the circuit given in Fig. P4.9, suppose that  $i_s(t) = 5 \cos 3t$  A. Find  $v_o(t)$  and  $v_s(t)$  by using nodal analysis.

**4.11** A voltage of  $v_s(t) = 10 \cos \omega t$  V is applied to a series  $RLC$  circuit. If  $R = 5\text{ }\Omega$ ,  $L = \frac{1}{5}$  H, and  $C = \frac{1}{5}$  F, by how many degrees does  $v_C(t)$  lead or lag  $v_s(t)$  when (a)  $\omega = 1$  rad/s, (b)  $\omega = 5$  rad/s, and (c)  $\omega = 10$  rad/s?

**4.12** A voltage of  $v_s(t) = 10 \cos \omega t$  V is applied to a series  $RLC$  circuit. If  $R = 5\text{ }\Omega$ ,  $L = \frac{1}{5}$  H, and  $C = \frac{1}{5}$  F, by how many degrees does  $v_R(t)$  lead or lag  $v_s(t)$  when (a)  $\omega = 1$  rad/s, (b)  $\omega = 5$  rad/s, and (c)  $\omega = 10$  rad/s?

**4.13** For the  $RLC$  connection given in Fig. P4.13, find the impedance  $Z$  when  $\omega$  is (a) 2, (b) 4, and (c) 8 rad/s.

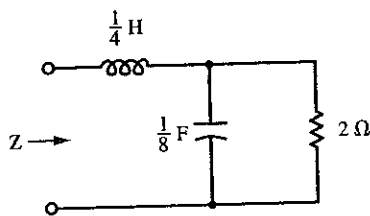


Fig. P4.13

**4.14** For the  $RLC$  connection shown in Fig. P4.14, find the admittance  $Y$  when  $\omega$  is: (a) 1, (b) 3, and (c) 7 rad/s.

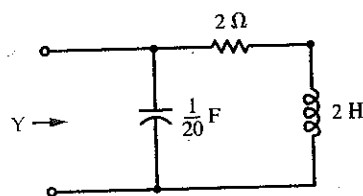


Fig. P4.14

**4.15** Show that a general expression for the impedance  $Z$  depicted in Fig. P4.13 is

$$Z = \frac{32}{\omega^2 + 16} + j \frac{\omega(\omega^2 - 16)}{4(\omega^2 + 16)}$$

**4.16** Show that a general expression for the admittance  $Y$  depicted in Fig. P4.14 is

$$Y = \frac{1}{2(\omega^2 + 1)} + j \frac{\omega(\omega^2 - 9)}{20(\omega^2 + 1)}$$

**4.17** For the circuit shown in Fig. P4.17, find the Thévenin equivalent of the circuit in the shaded box when  $v_s(t) = 4 \cos(4t - 60^\circ)$  V. Use this to determine  $v_o(t)$ .

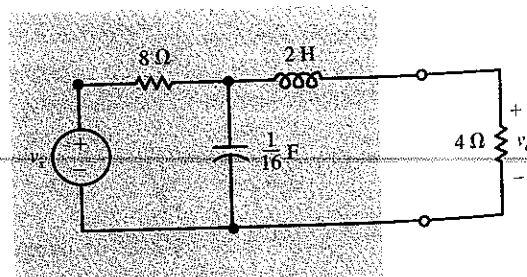


Fig. P4.17

**4.18** For the circuit shown in Fig. P4.17, find the Thévenin equivalent of the circuit in the shaded box when  $v_s(t) = 4 \cos(2t - 60^\circ)$  V. Use this to determine  $v_o(t)$ .

**4.19** Find the frequency-domain Thévenin equivalent (to the left of terminals  $a$  and  $b$ ) of the circuit shown in Fig. 4.20 on p. 211. (Hint: Use the fact that  $Z_o = V_{oc}/I_{sc}$ .)

**4.20** The frequency-domain Thévenin equivalent of a circuit having  $\omega = 5$  rad/s has  $V_{oc} = 3.71 \angle -15.9^\circ$  V and  $Z_o = 2.38 - j0.667 \Omega$ . Determine a corresponding time-domain Thévenin-equivalent circuit.

**4.21** For the op-amp circuit shown in Fig. P4.21, find  $v_o(t)$  when  $v_s(t) = 6 \sin 2t$  V.

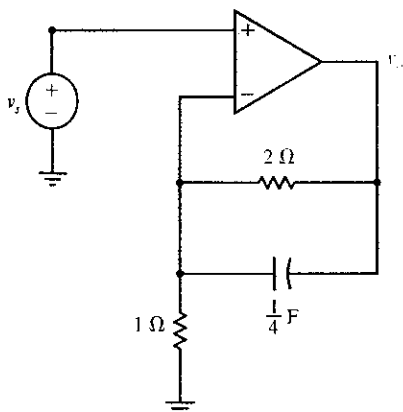


Fig. P4.21

**4.22** For the op-amp circuit given in Fig. P4.22, find  $v_o(t)$  when  $v_s(t) = 3 \cos 2t$  V.

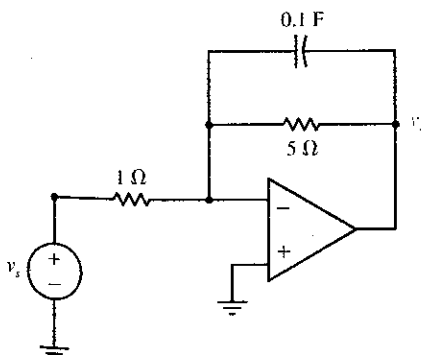


Fig. P4.22

**4.23** For the op-amp circuit shown in Fig. P4.23, find  $v_o(t)$  when  $v_s(t) = 4 \cos(2t - 30^\circ)$  V. (See p. 258.)

**4.24** For the circuit shown in Fig. P4.24, find the currents  $I_1$  and  $I_2$  when  $V_{s1} = 250\sqrt{2} \angle -30^\circ$  V,  $V_{s2} = 250\sqrt{2} \angle -90^\circ$  V, and  $Z = 78 - j45 \Omega$ .

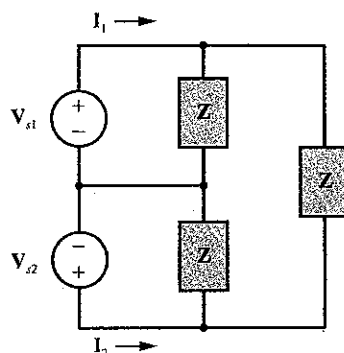


Fig. P4.24

**4.25** Use mesh analysis to find  $I_1$  and  $I_2$  for the circuit given in Fig. P4.25 when  $V_{s1} = 250\sqrt{2} \angle -30^\circ$  V,  $V_{s2} = 250\sqrt{2} \angle -90^\circ$  V, and  $Z = 26 - j15 \Omega$ .

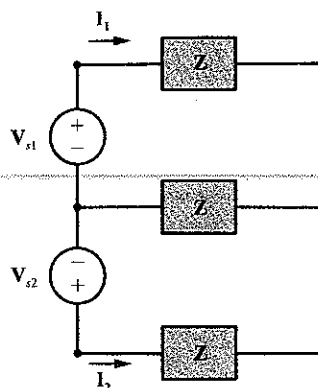


Fig. P4.25

**4.26** For the circuit shown in Fig. P4.9, when  $i_s(t) = 5 \cos 3t$  A then  $v_o(t) = 4.47 \cos(3t + 26.6^\circ)$  V. Find the average power absorbed by each element in the circuit.

**4.27** For the circuit shown in Fig. P4.17, when  $v_s(t) = 10 \cos 4t$  V, then the Thévenin equivalent of the portion of the circuit in the shaded box is  $V_{oc} =$

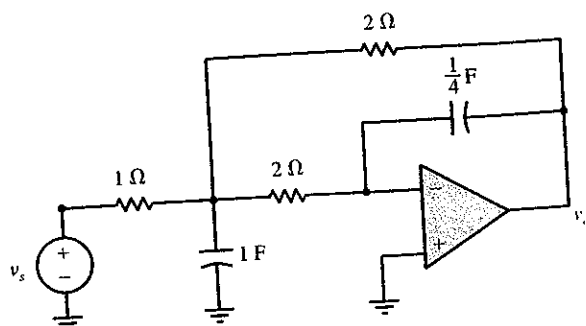


Fig. P4.23

$4.47 \angle -63.4^\circ$  V and  $Z_o = 1.6 + j4.8 \Omega$ . (a) Replace the  $4\text{-}\Omega$  load resistor by an impedance  $Z_L$  that absorbs the maximum average power, and determine this maximum power. (b) Replace the  $4\text{-}\Omega$  load resistor with a resistance  $R_L$  that absorbs the maximum power for resistive loads, and determine this power.

**4.28** For the *RLC* circuit shown in Fig. P4.28, suppose that  $v_s(t) = 10 \cos 3t$  V. Find the average power absorbed by the  $4\text{-}\Omega$  resistor for the case that (a)  $C = \frac{1}{6}$  F; (b)  $C = \frac{1}{18}$  F; (c)  $C = \frac{1}{30}$  F.

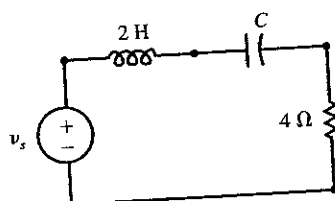


Fig. P4.28

**4.29** For the circuit shown in Fig. P4.29, suppose that  $v_s(t) = 8 \cos 2t$  V. Find the average power absorbed by each element in the circuit for the case that  $Z_L = 1 \Omega$ .

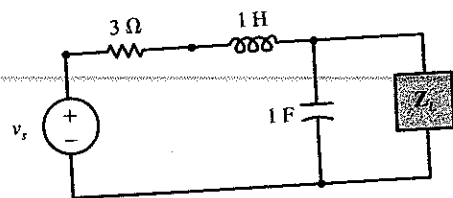


Fig. P4.29

**4.30** For the circuit shown in Fig. P4.29, change the value of the resistor to  $2 \Omega$  and the value of the capacitor to  $\frac{1}{4}$  F. Suppose that  $v_s(t) = 8 \cos 2t$  V. (a) Find the load impedance  $Z_L$  that absorbs the maximum average power, and determine this power. (b) Find the load resistance  $R_L$  that absorbs the maximum power for resistive loads, and determine this power.

**4.31** For the op-amp circuit given in Fig. P4.21, when  $v_s(t) = 6 \sin 2t$  V, then the output voltage  $v_o(t) = 13.4 \cos(2t - 117^\circ)$  V. Find the average power absorbed by each element.

**4.32** For the op-amp circuit given in Fig. P4.22, when  $v_s(t) = 3 \cos 2t$  V, then the output voltage  $v_o(t) = 10.6 \cos(2t + 135^\circ)$  V. Find the average power absorbed by each element.

**4.33** For the op-amp circuit given in Fig. P4.23, when  $v_s(t) = 4 \cos(2t - 30^\circ)$  V, then  $v_i(t) = 1.6 \cos(2t - 66.9^\circ)$  V and  $v_o(t) = 1.6 \cos(2t + 23.1^\circ)$  V. Find the average power absorbed by each element.

**4.34** For the circuit given in Fig. P4.24, when  $V_{s1} = 250\sqrt{2} \angle -30^\circ$  V,  $V_{s2} = 250\sqrt{2} \angle -90^\circ$  V, and  $Z = 78 - j45 \Omega$ , then  $I_1 = 6.8 \angle 30^\circ$  A and  $I_2 = 6.8 \angle -90^\circ$  A. (a) Find the average power absorbed by each impedance. (b) Find the average power supplied by each source.

**4.35** For the circuit given in Fig. P4.25, when  $V_{s1} = 250\sqrt{2} \angle -30^\circ$  V,  $V_{s2} = 250\sqrt{2} \angle -90^\circ$  V, and  $Z = 26 - j15 \Omega$ , then  $I_1 = 6.8 \angle 30^\circ$  A and  $I_2 = 6.8 \angle -90^\circ$  A. (a) Find the average power absorbed by each impedance. (b) Find the average power supplied by each source.

**4.36** For the op-amp circuit shown in Fig. P4.36, find the average power absorbed by each element for the case that  $v_s(t) = \cos \omega t$  V.

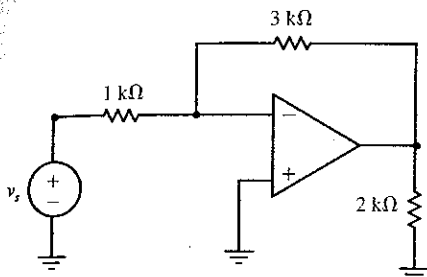


Fig. P4.36

**4.37** For the op-amp circuit shown in Fig. P4.37, find the average power absorbed by each element for the case that  $v_s(t) = \cos \omega t$  V.

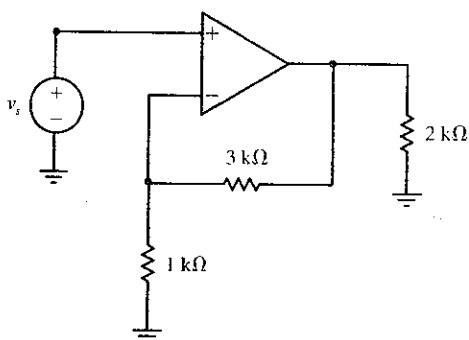


Fig. P4.37

**4.38** Find the rms value of each function given in Fig. P4.38. (See p. 260.)

**4.39** Find the rms value of the “half-wave rectified” sine wave that is shown in Fig. P4.39. [Hint:  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ .]

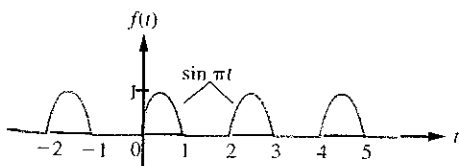


Fig. P4.39

**4.40** Find the rms value of the “full-wave rectified” sine wave that is shown in Fig. P4.40. [Hint:  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ .]

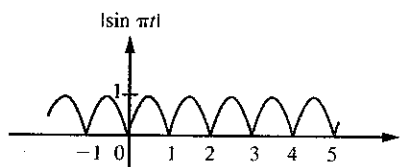


Fig. P4.40

**4.41** The load shown in Fig. P4.41 operates at 60 Hz. (a) What are the pf and the pf angle of this load? (b) Is the pf leading or lagging? (c) To what value should the capacitor be changed to get a unity pf (pf = 1)?

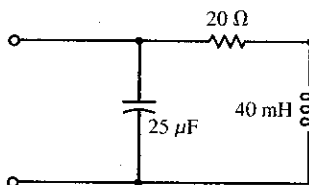


Fig. P4.41

**4.42** A 115-V rms, 60-Hz electric hair dryer absorbs 500 W at a lagging pf of 0.95. What is the rms value of the current drawn by this dryer?

**4.43** An electric motor which operates at 220 V rms, 20 A rms, 60 Hz, absorbs 2200 W. (a) What is the pf of the motor. (b) For the case that the pf is lagging, what value capacitor should be connected in parallel with the motor such that the resulting combination has a unity pf (pf = 1)?

**4.44** An electric motor operating at 220 V rms, 60 Hz, draws a current of 20 A rms at a pf of 0.75 lagging. (a) What is the average power absorbed by the motor? (b) What value capacitor should be connected in parallel with the motor such that the resulting combination has a unity pf (pf = 1)?

**4.45** Two loads, which are connected in parallel, operate at 230 V rms. One load absorbs 500 W at a pf of 0.8 lagging, and the other absorbs 1000 W at