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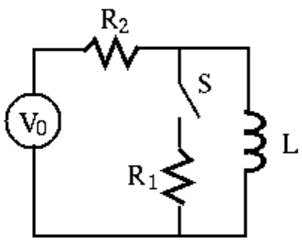
E84 Home Work 7

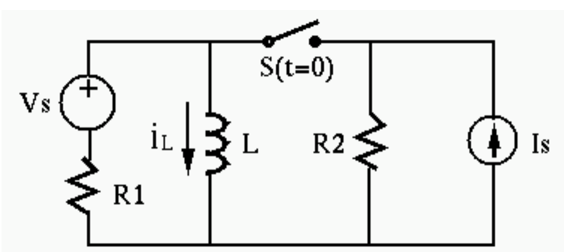
1. Find the current $i_1(t)$ through resistor R_1 after the switch is closed at $t=0$, assuming the input is a DC voltage and the circuit is already in steady state before $t=0$. (Hint: current through an inductor cannot change instantaneously.)



Solution:

First, as current through L does not change when the switch is closed at $t=0$, the current through R_1 is zero $i_1(0_+)=0$. Second, when the circuit reaches steady state after the switch is closed at $t=0$, the inductor is short-circuit, i.e., no current goes through R_1. Therefore, $v_1(t)=0$.

2. In the circuit below, $V_s=6V$, $R_1=6\Omega$, $R_2=3\Omega$, $L=0.5H$, $I_s=2A$. Assume before the switch is closed at $t=0$, the system is already stablized. Find current $i_L(t)$ through L and voltage v_{R_1} across R_1.



Solution:

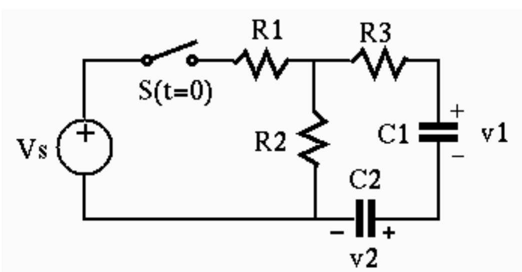
$i(0)=V_s/R_1=6/6=1A$, $i(\infty)=V_s/R_1+I_s=1+2=3A$ Find equivalent resistance (when both energy sources are turned off

$R = R_1 || R_2 = 3 \times 6/(3+6) = 2$, $\tau = L/R = 0.5/2 = 0.25\ sec.$ Final solution: $i_L(t) = 3 + (1-3)e^{-t/0.25} = 3 - 2e^{-4t}\ (A)$ Find v_{R_1}: apply KVL to the loop of V_s, R_1 and L, get

$$L\frac{di_L}{dt} + v_{R_1} = V_s$$

$$v_{R_1} = V_s - L di_L/dt = 6 - 0.5\frac{d(3-2e^{-4t})}{dt} = 6 - 4e^{-4t}$$

3. In the circuit below, $V_s = 12V$, $R_1 = 5\Omega$, $R_2 = 20\Omega$, $R_3 = 6\Omega$, $C_1 = 10\mu F$, $C_2 = 30\mu F$. Assume before the switch is closed at $t=0$, the system is already stablized. Find voltages $v_1(t)$ and $v_2(t)$ across capacitors C_1 and C_2, respectively. (Hint, C_1 and C_2 are two capacitors in series with an equivalent capacitance is $C = C_1C_2/(C_1+C_2)$. C_1 and C_2 have share the same time constant $\tau = RC$.)



Solution:

$$v_1(0) = v_2(0) = 0$$

$$v_1(\infty) + v_2(\infty) = V_s\frac{R_2}{R_1+R_2} = 12\frac{20}{5+20} = 9.6V$$

As voltage across capacitor is inversely proportional to C, we have

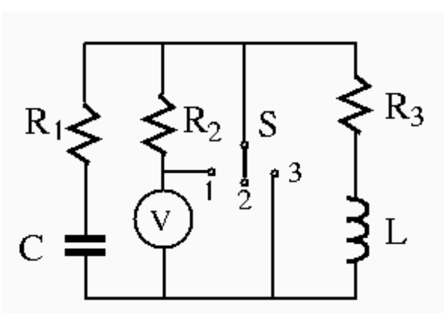
$$\frac{v_1(\infty)}{v_2(\infty)} = \frac{C_2}{C_1} = 3$$

i.e., $v_1(\infty) = 3v_2(\infty)$, and we get $v_1(\infty) = 7.2V$, $v_2(\infty) = 2.4V$. Find equivalent resistance:

$$R = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 6 + \frac{5 \times 20}{5 + 20} = 10\Omega$$

Find equivalent capacitance: $C_1 C_2/(C_1 + C_2) = 30 \times 10/(30 + 10) = 7.5$. Find time constant: $\tau = RC = 10 \times 7.5 \times 10^{-6} = 7.5 \times 10^{-5}$. Find $v_1(t)$ and $v_2(t)$: $v_1(t) = 7.2(1 - e^{-t/(7.5\times10^{-5})})$ $v_2(t) = 2.4(1 - e^{-t/(7.5\times10^{-5})})$

4. In the circuit below, $R_1 = 100\Omega$, $R_1 = 150\Omega$, $R_3 = 100\Omega$, $H = 0.1H$, $C = 20\mu F$, $V = 60V$. The circuit is in steady state initially when the switch is at position 2 (not connected). Find $v_C(t)$ and $i_L(t)$ for the following two independent cases:
   - after the switch is changed to position 1 at $t = 0$;
   - after the switch is changed to position 3 at $t = 0$;



Solution

Find initial values:

$$v_C(0) = \frac{100}{150 + 100} \times 60 = 24V, \qquad i_L(0) = \frac{6}{150 + 100} = 0.24A$$

- after the switch is put on position 1 at $t = 0$,

$$v_C(\infty) = 60V, \qquad i_L(\infty) = 60/100 = 0.6A$$

Find the time constants:

$$\tau_1 = R_1 C = 2 \times 10^{-3} s, \qquad \tau_2 = L/R_2 = 10^{-3} s$$

The complete responses:

$$v_C(t) = v_C(\infty)+[v_C(0)-v_C(\infty)]e^{-t/\tau_1} = 60-36e^{-500t}, \quad i_L(t) = i_L(\infty)+[i_L(0)-i_L(\infty)]e^{-t/\tau_2} = 0.6-0.36e^{-1000t}$$

- after the switch is put on position 3 at $t=0$,

$$v_C(\infty) = 0, \qquad i_L(\infty) = 0$$

The time constants are the same as above. The complete responses:

$$v_C(t) = v_C(\infty)+[v_C(0)-v_C(\infty)]e^{-t/\tau_1} = 24e^{-500t}, \quad i_L(t) = i_L(\infty)+[i_L(0)-i_L(\infty)]e^{-t/\tau_2} = 0.24e^{-1000t}$$

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