1. (10 points) FoEE 4.41

4.41 The load shown in Fig. P4.41 operates at 60 Hz. (a) What are the pf and the pf angle of this load? (b) Is the pf leading or lagging? (c) To what value should the capacitor be changed to get a unity pf (pf = 1)?

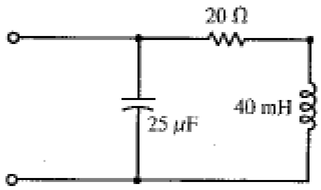
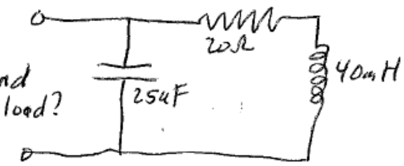


Fig. P4.41

FoEE 4.41

a) What is the pf and pf angle of this load?



Given $f = 60\text{Hz}$, $\omega = 120\pi \ \frac{rad}{s}$

First, we need to find Z_T and the angle of the load.

$Z_R = 20\Omega$, $Z_L = j\,4.800\pi\Omega$, $Z_C = \frac{-j}{120\pi(.000025)} = -j106.1\Omega$

$Z_T = -j106.1 \,//\, 20 + j15.08$

$= \frac{(-106.1j)(20+15.08j)}{-106.1j + 20 + j15.08} = \frac{-2122j + 1600}{-91j + 20}$

$= \frac{2657.6\angle -53^\circ}{93.2\angle -77.6^\circ}$

$Z_T = 28.5\angle 24.6^\circ$

$\boxed{\text{So, pf angle is } 24.6^\circ\text{, pf is } \cos(24.6) = 0.909}$

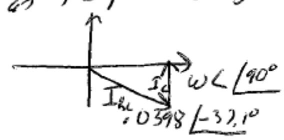
b) Load angle is positive, so the pf is lagging!

c) A shortcut way to view this is to assume a 1 Volt 0° source.

So, $I_C = \frac{1\angle 0^\circ}{\frac{-j}{\omega C}} = \omega C\angle 90^\circ$

$I_{RL} = \frac{1\angle 0^\circ}{Z_R + Z_L} = \frac{1\angle 0^\circ}{20 + j15.1} = \frac{1\angle 0^\circ}{25.1\angle 37.1^\circ} = .0398\angle -37.1^\circ$

Looking at the phasor diagram: $I = I_{RL} + I_C$ such that $I\angle 0^\circ$.

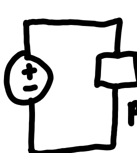


So, $\frac{\omega C}{39.8\times 10^{-3}} = \sin 37.1^\circ$ $\boxed{C = 63.7\mu F}$

2. (10 points) FoEE 4.48

4.48 A load, which operates at 220 V rms, draws 5 A rms at a lagging pf of 0.95. (a) Find the complex power absorbed by the load. (b) Find the average power absorbed by the load. (c) Find the reactive power absorbed by the load. (d) Find the apparent power absorbed by the load. (e) Find the impedance of the load.

$\vec{V}_{rms} = 220\angle 0^\circ$



$\vec{I}_L = 5\angle\phi_2$

pf $= \cos\theta = 0.95 \Rightarrow \theta = 18.2^\circ$

Since it is lagging: $\phi_2 = -18.2^\circ$

a) $\vec{S} = \vec{V}_{rms}\vec{I}_{rms}^{\,*} = 220\angle 0^\circ \; 5\angle 18.2^\circ$

"+" due to conjugate

$\vec{S} = 1100\angle 18.2^\circ \text{ VA} = 1045 + j346 \text{ VA}$

b) $P = Re\{\vec{S}\} = 1045 \text{ W}$

c) $Q = Im\{\vec{S}\} = 346 \text{ VAR}$

d) $|\vec{S}| = 1100 \text{ VA}$

e) $Z = \frac{V_s}{I_L} = \frac{220\angle 0^\circ}{5\angle -18.2^\circ} = 44\angle 18.2^\circ = 41.8 + j13.7\,\Omega$

3. (10 points) FoEE 4.59

4.59 For the single-phase, three-wire circuit shown in Fig. P4.58, suppose that $\mathbf{V}_s = 115\angle 0°$ V rms. Find the average power supplied by each source if $\mathbf{Z}_1 = 60\ \Omega$, $\mathbf{Z}_2 = 80\ \Omega$, $\mathbf{Z}_3 = 40\ \Omega$, $R_g = 1\ \Omega$, and $R_n = 2\ \Omega$.

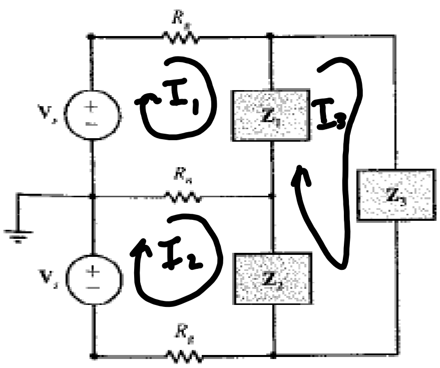


Fig. P4.58

Using Mesh analysis:

Mesh 1: $1 \cdot I_1 + 60(I_1 - I_3) + 2(I_1 - I_2) = 115\angle 0°$

$63 I_1 - 2I_2 - 60 I_3 = 115 \quad (1)$

Mesh 2: $2(I_2 - I_1) + 80(I_2 - I_3) + 1 I_2 = 115\angle 0°$

$-2I_1 + 83 I_2 - 80 I_3 = 115 \quad (2)$

Mesh 3: $40 I_3 + 80(I_3 - I_2) + 60(I_3 - I_1) = 0$

$-3I_1 - 4I_2 + 9I_3 = 0$

Solving $\begin{bmatrix} 63 & -2 & -60 \\ -2 & 83 & -80 \\ -3 & -4 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 115 \\ 0 \end{bmatrix}$

$I_1 = 7.18$ Arms, $I_2 = 6.76$ Arms, $I_3 = 5.40$ Arms

$P_{s1} = |V_s||I_1| \cos(\angle V_s - \angle I_1) = 115 \cdot 7.18 \cos 0°$

$P_{s1} = 826\,W$

$P_{s2} = |V_s||I_1| \cos(\angle V_s - \angle I_2) = 115 \cdot 6.76 \cos 0°$

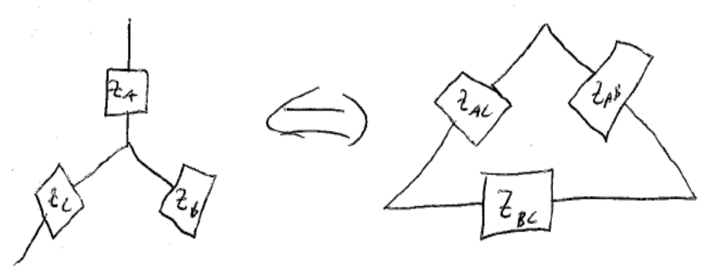
$P_{s2} = 777\,W$

4. (5 points) FoEE 4.70

4.70 A Y-connected load has impedances $\mathbf{Z}_A = 1\ \Omega$, $\mathbf{Z}_B = 0.6\ \Omega$, and $\mathbf{Z}_C = 1.5\ \Omega$. Find the equivalent Δ-connected load.

FoEE 4.70

A Y-connected load has: $Z_A = 1\,\Omega$
$Z_B = 0.6\,\Omega$
$Z_C = 1.5\,\Omega$



So, $Z_{AB} = \dfrac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C} = \dfrac{0.6 + (0.6)(1.5) + (1.5)}{(1.5)}$

$Z_{AB} = 2\,\Omega$

$Z_{BC} = \dfrac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_A} = \dfrac{0.6 + (0.6)(1.5) + 1.5}{1}$

$Z_{BC} = 3\,\Omega$

$Z_{AC} = \dfrac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_B} = \dfrac{0.6 + (0.6)(1.5) + 1.5}{0.6}$

$Z_{AC} = 5\,\Omega$

5. (10 points) FoEE 4.84

4.84 Find the wattmeter readings for the circuit shown in Fig. P4.84.

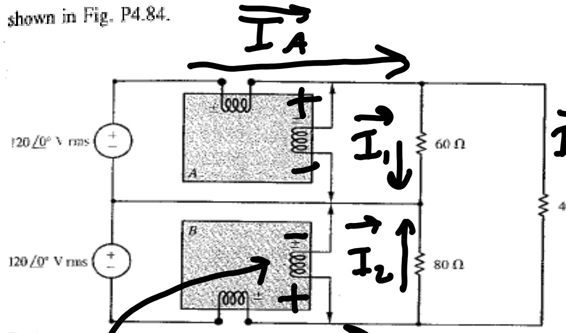


Fig. P4.84

$\vec{I}_1 = \frac{\vec{V_S}}{60} = \frac{120\angle 0°}{60} = 2A$; $\vec{I}_2 = \frac{-\vec{V_S}}{80} = -\frac{3}{2} = 1.5\angle 180° A$

$\vec{I}_3 = \frac{\vec{V_S}+\vec{V_S}}{40} = \frac{\vec{V_S}}{20} = 6A$ KCL

So, $\vec{I}_A = \vec{I}_1 + \vec{I}_3 = 2+6 = 8A$

Thus, $P_A = |\vec{V_S}||\vec{I_A}|\cos(ang(\vec{V_S}) - ang(\vec{I_A}))$

$P_A = 120\cdot 8 \cos\angle 0° = 960W$

$\vec{I}_B = \vec{I}_2 - \vec{I}_3 = -\frac{3}{2} - 6 = -\frac{15}{2} = 7.5\angle 180°$

$P_B = |-\vec{V_S}||\vec{I_B}|\cos(ang(-\vec{V_S}) - ang(\vec{I_B}))$

note the negative due to this polarity.

$P_B = 120\cdot 7.5\cdot\cos(180-180) = 900W$

6. (10 points) FoEE 5.3

5.3 Sketch the amplitude response of V_2/V_1 for the op-amp circuit shown in Fig. P5.3. Determine the half-power frequency. What type of filter is this circuit?

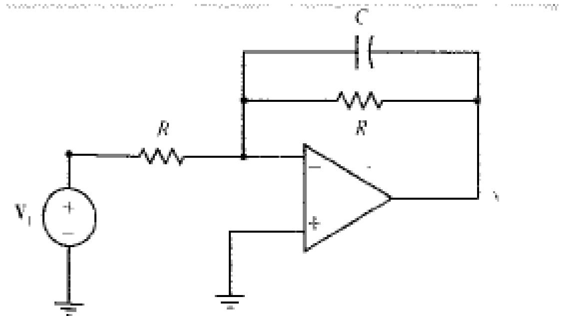
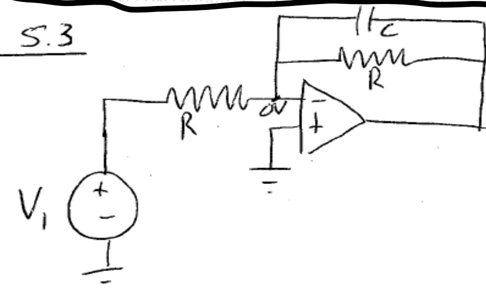


Fig. P5.3

FoEE 5.3



- Find / Sketch amplitude response V_2/V_1
- Find half power frequency.
- Determine type of filter:

KCL: $\frac{V_1}{R} = \frac{-V_2}{R} - \frac{V_2}{1/j\omega C} = \left(\frac{-1}{R} - j\omega C\right)V_2$

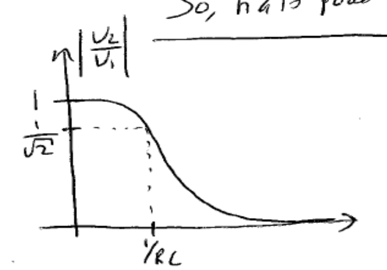
So, $\frac{V_2}{V_1} = \frac{-1/R}{1/R + j\omega C} = \frac{-1}{1+j\omega RC}$

So, $\left|\frac{V_2}{V_1}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

For $\omega = 0$, $|V_2/V_1| = 1$
$\omega = \infty$, $|V_2/V_1| = 0$

$\frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{1+(\omega RC)^2} = \frac{1}{2} \Rightarrow (\omega RC)^2 = 1$

So, half power frequency is $\frac{1}{RC}$.



Low-Pass Filter

7. (10 points) FoEE 5.4

5.4 Show that for the circuit given in Fig. P5.4 the voltage transfer function is

$$H(j\omega) = \frac{V_2}{V_1} = \frac{R_2(1 + j\omega R_1 C_1)}{(R_1 + R_2) + j\omega R_1 R_2 (C_1 + C_2)}$$

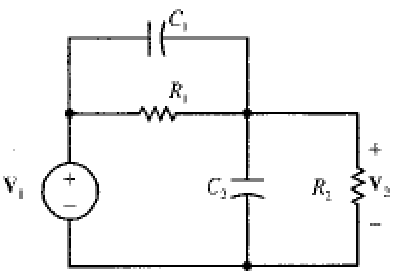


Fig. P5.4

$$Z_1 = \frac{R_1\left(\frac{1}{j\omega C}\right)}{R_1 + \frac{1}{j\omega C}} = \frac{R_1}{1+j\omega R_1 C_1}$$

$$Z_2 = \frac{R_2}{1+j\omega R_2 C_2}$$

$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{R_1}{1+j\omega R_1 C_1} + \frac{R_2}{1+j\omega R_2 C_2}}$$

$$= \frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{R_1(1+j\omega R_2 C_2) + R_2(1+j\omega R_1 C_1)}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}}$$

$$= \frac{R_2(1+j\omega R_1 C_1)}{R_1(1+j\omega R_2 C_2) + R_2(1+j\omega R_1 C_1)}$$

$$\frac{\vec{V_2}}{\vec{V_1}} = \frac{R_2(1+j\omega R_1 C_1)}{R_1+R_2+j\omega R_1 R_2 (C_1+C_2)}$$

Optional Problems

8. (0 points) FoEE 4.44

4.44 An electric motor operating at 220 V rms, 60 Hz, draws a current of 20 A rms at a pf of 0.75 lagging. (a) What is the average power absorbed by the motor? (b) What value capacitor should be connected in parallel with the motor such that the resulting combination has a unity pf (pf = 1)?

③ FoEE 4.44 $V_m = 220\angle 0^\circ$

Can assume initial phase.

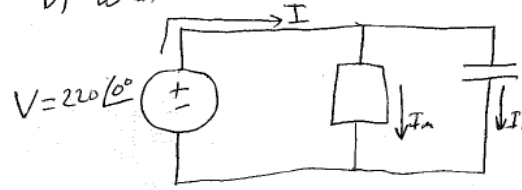
$I_m = 20\angle\theta^\circ$ θ is determined by pf. We are told lagging so, θ is negative.

$\theta = \cos^{-1}(pf) = \cos^{-1}(0.75) = 41.41^\circ$

a) Avg. Power absorbed by motor: $P_M = V_e I_e \cos\theta$ & $V_e = V_{rms}$, $I_e = I_{rms}$

$\boxed{P_M = (220)(20)(.75) = 3300\,W}$

b) What value capacitor should be connected in parallel to create a pf = 1.



$C \Rightarrow \frac{-j}{\omega C}\,\Omega = \frac{-j}{120\pi C}$

$I = I_m + I_c = 20\angle 41.41 + \frac{220}{-j/120\pi C}$

We want pf=1, so we want real current:

$I = 15 - 13.23j + 26{,}400\pi C j$

So, $26{,}400\pi C = 13.23$ is

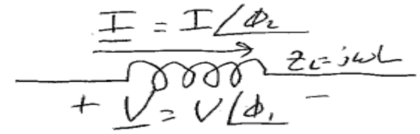
$\boxed{C = 159\,\mu F}$

9. (0 points) FoEE 4.56

4.56 An L-henry inductor has the voltage $v(t) = V\cos(\omega t + \phi_1)$ across it and it has the current $i(t) = I\cos(\omega t + \phi_2)$ through it. Show that the complex power absorbed by the inductor is given by

$$\mathbf{S}_L = \frac{j\omega L I^2}{2} = \frac{jV^2}{2\omega L}$$

(4) FoEE 4.56



$\underline{V} = j\omega L\,\underline{I}$

$V\angle\phi_1 = \omega L\angle 90^\circ\; I\angle\phi_2 = \omega L I\angle\phi_2 + 90^\circ$

So $V = \omega L I$ $\quad \phi_1 = \phi_2 + 90^\circ$

$I = \frac{V}{\omega L}$ $\quad \phi_2 = \phi_1 - 90^\circ$

$S_L = \frac{1}{2}\underline{V}\,\underline{I}^* = \frac{1}{2} V\angle\phi_1\; I\angle -\phi_2$

$= \frac{1}{2}\omega L I\angle 90 + \phi_2\; I\angle -\phi_2$

$= \frac{1}{2}\omega L I^2\angle 90^\circ$

$\boxed{S_L = \frac{j\omega L I^2}{2}}$

$S_L = \frac{1}{2} V\angle\phi_1\; I\angle -\phi_2$

$= \frac{1}{2} V\angle\phi_1\; \frac{V}{\omega L}\angle -\phi_1 + 90^\circ$

$= \frac{1}{2}\frac{V^2}{\omega L}\angle 90^\circ$

$S_L = \frac{jV^2}{2\omega L}$

10. (0 points) FoEE 4.82

4.82 The unbalanced Δ-connected load shown in Fig. 4.45 on p. 250 has rms line voltages $\mathbf{V}_{ab} = 220\angle 30^\circ$ V, $\mathbf{V}_{bc} = 220\angle -90^\circ$ V, $\mathbf{V}_{ca} = 220\angle -210^\circ$ V, rms phase currents $\mathbf{I}_{AB} = 11\angle 30^\circ$ A, $\mathbf{I}_{BC} = 5.5\angle -135^\circ$ A, $\mathbf{I}_{CA} = 22\angle 60^\circ$ A. Find the wattmeter readings for the two-wattmeter method.

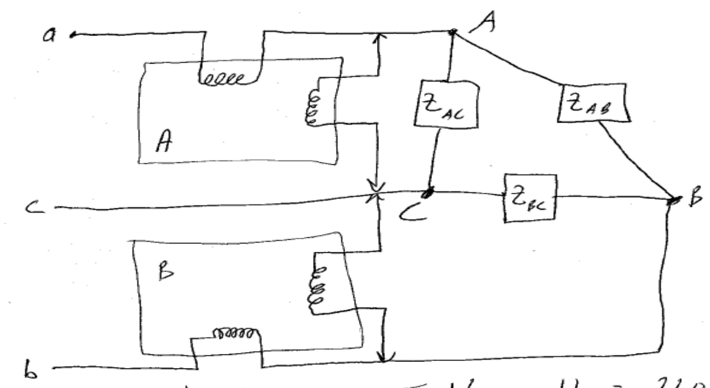
(5) FoEE 4.82

rms line voltages: $V_{ab} = 220\angle 30^\circ$ V, $V_{bc} = 220\angle -90^\circ$ V, $V_{ca} = 220\angle -210^\circ$ V

rms phase currents:

$I_{AB} = 11\angle 30^\circ$ A, $I_{BC} = 5.5\angle -135^\circ$, $I_{CA} = 22\angle 60^\circ$ A

Find wattmeter readings for 2 wattmeter method:



$P_A = |V_{ac}||I_{aA}|\cos\theta_A \Rightarrow$

$P_A = |220||13.7|\cos(66.2^\circ)$

$\boxed{P_A = 1216\ \text{W}}$

$P_B = |V_{bc}||I_{bB}|\cos\theta_B$

$= |220||16.4|\cos(55^\circ)$

$\boxed{P_B = 2069\ \text{W}}$

$V_{ac} = -V_{ca} = -220\angle -210^\circ = 220\angle -30^\circ$ V

$I_{aA} = I_{AB} - I_{CA} = 11\angle 30^\circ - 22\angle 60^\circ$

$= (9.526 + 5.5j) - (11 + 19.05j)$

$= -1.47 - 13.55j = 13.7\angle -96.2^\circ$

$V_{bc} = 220\angle -90^\circ$

$I_{bB} = I_{BC} - I_{AB} = 5.5\angle -135^\circ - 11\angle 30^\circ$

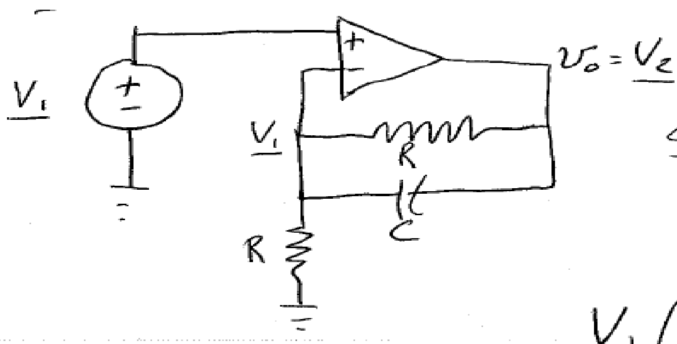
$= (-3.89 - 3.89j) - (9.53 + 5.5j)$

$= 13.42 - 9.39j$

$= 16.4\angle -145^\circ$ A

11. (0 points) FoEE 5.8

5.8 For the op-amp circuit shown in Fig. P5.8, sketch the amplitude response of $\mathbf{V}_2/\mathbf{V}_1$, indicating the half-power frequency. What type of filter is this circuit?



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So, by KCL:

$$\frac{V_1}{R} + \frac{V_1 - V_2}{R} + \frac{V_1 - V_2}{\frac{1}{j\omega C}} = 0$$

$$V_1\left(\frac{1}{R} + \frac{1}{R} + j\omega C\right) = V_2\left(\frac{1}{R} + j\omega C\right)$$

$$\text{So, } \frac{V_2}{V_1} = \frac{\left(\frac{2}{R} + j\omega C\right)}{\left(\frac{1}{R} + j\omega C\right)} = \frac{2 + j\omega RC}{1 + j\omega RC}$$

$$\left|\frac{V_2}{V_1}\right| = \frac{\sqrt{4 + (\omega RC)^2}}{\sqrt{1 + (\omega RC)^2}}$$

So, some points of interest:

$\omega = 0, \quad \left|\frac{V_2}{V_1}\right| = 2$

$\omega = \infty, \quad \left|\frac{V_2}{V_1}\right| = 1$

$\omega = \frac{1}{RC}, \quad \left|\frac{V_2}{V_1}\right| = \frac{\sqrt{5}}{\sqrt{2}} = 1.58$

Half power frequency : $X = \omega RC$

max amplitude

$$\frac{\sqrt{4 + x^2}}{\sqrt{1 + x^2}} = \frac{2}{\sqrt{2}}$$

$$4 + x^2 = 2(1 + x^2)$$

$$2 = x^2$$

$$x = \sqrt{2} \Rightarrow \omega RC = \sqrt{2} \Rightarrow$$

$$\boxed{\text{half power frequency} \rightarrow \omega = \frac{\sqrt{2}}{RC}}$$

Amplitude Response Sketch