E84 Home Work 8

1. An RCL series circuit composed of $R=10\Omega$, $L=10mH$ and $C=1\mu F$ is connected to an input AC voltage $v_{in}(t)=cos\omega t$.
   - Find the quality factor Q and resonant frequency ω_0.
   - Assume the voltage $v_R(t)$ across R is taken as the output voltage. Find the bandwidth of the circuit. Also, use any software (e.g., Matlab) to plot the ratio of the magnitudes between the output and input voltages v_R/v_{in} as a function of frequency ω.
   - Assume the voltage $v_L(t)$ across L is taken as the output voltage. plot the ratio v_L/v_{in} as a function of frequency ω.
   - Assume the voltage $v_C(t)$ across C is taken as the output voltage. plot the ratio v_C/v_{in} as a function of frequency ω.

   Solution:

   - $$Q=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{10}\sqrt{\frac{10^{-2}}{10^{-6}}}=10$$

     $$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{10^{-2}10^{-6}}}=\frac{1}{\sqrt{10^{-8}}}=10^4$$

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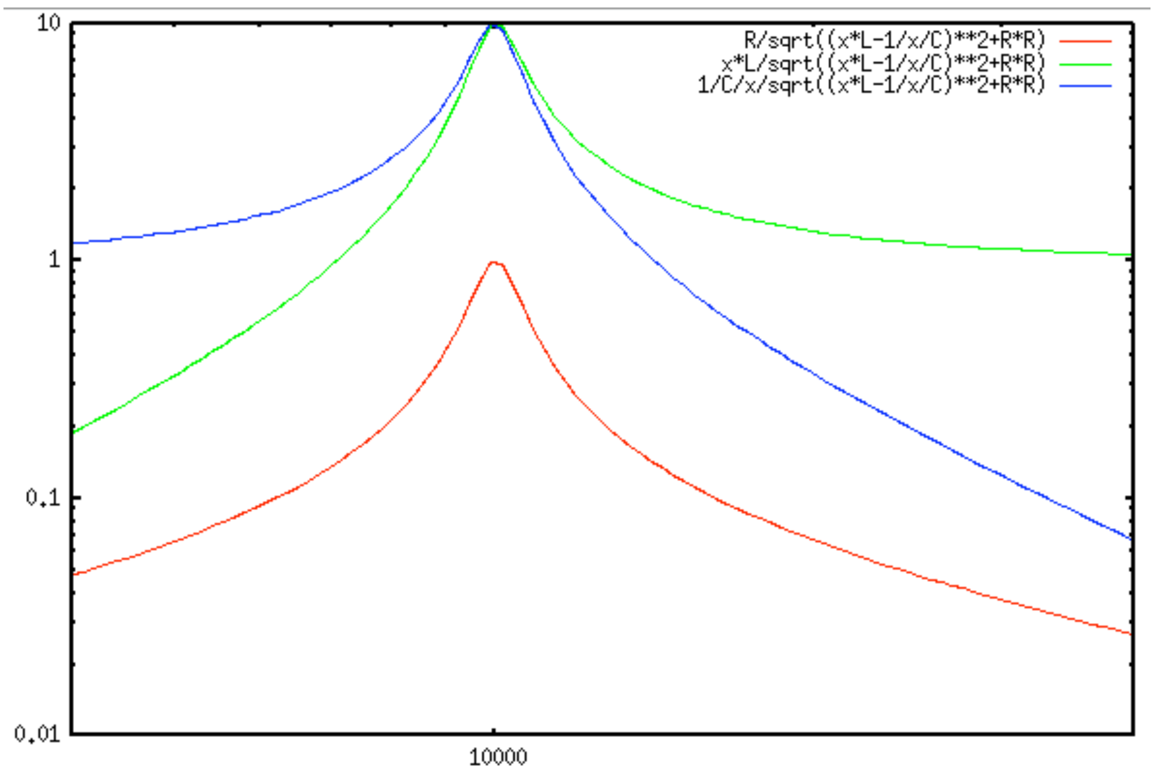
$$\triangle\omega = \frac{\omega_0}{Q} = \frac{10^4}{10} = 1,000$$



2. A series circuit composed of a capacitor and an inductor is to be resonant at 800 kHz with voltage input. Specify the value of C for the capacitor required for the given inductor with $L = 40\mu H$ and an internal resistance $R_L = 4.02\Omega$, and predict the bandwidth. Assume the capacitor is ideal, i.e., it introduces no resistance.

   Solution: As $\underline{\omega_0 = 1/\sqrt{LC} = 2\pi 8 \times 10^5}$, and $\underline{L = 4 \times 10^{-5} H}$, we can find C to be

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi 8 \times 10^5)^2 \times 4 \times 10^{-5}} = 0.99\ nF$$

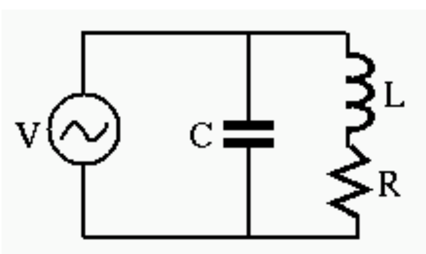
Next find the quality factor:

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi 8 \times 10^5 \times 4 \times 10^{-5}}{4.02} = 50$$

then the bandwidth is

$$f_2 - f_2 = \frac{f_0}{Q} = \frac{800 \times 10^3}{50} = 16\ kHz$$

3. In reality, all inductors have a non-zero resistance, therefore a parallel LC resonance circuit should be modeled as shown in the figure:



Assuming $\omega_0 L/R > 20$, give the expression of the resonant frequency at which the admittance of the circuit will be minimized.

Hint: Unlike pure series and parallel RCL circuits, for this mixed RCL circuit, both the real and imaginary parts of its admittance are a function of ω. However, if the quality factor of the inductor is large enough, i.e., $Q = \omega_0 L/R > 20$, we can still assume at the resonant frequency, the imaginary part of the admittance is zero $Im[Y(\omega_0)] = 0$.

Next, assume $C = 1\,nF = 10^{-9}F$, $L = 25\mu H = 2.5 \times 10^{-5}H$, and $R = 5\Omega$, confirm the assumption $\omega_0 L/R > 20$ is valid, and find the resonant frequency.

Solution:

The admittance is:

$$Y(\omega) = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L + j\omega C(R^2 + \omega^2 L^2)}{R^2 + \omega^2 L^2}$$

Assuming $\omega_0 L/R > 20$, the resonant frequency can be found by letting the imaginary part of Y be zoro $Im[Y] = 0$:

$$\omega_0 L = \omega_0 C(R^2 + \omega_0^2 L^2)$$

Solving this for ω_0, we get:

$$\omega_0 = \sqrt{\frac{1}{LC} - (\frac{R}{L})^2} = \sqrt{\frac{1}{2.5 \times 10^{-14}} - (\frac{5}{2.5 \times 10^{-5}})^2} = \sqrt{4 \times 10^{13} - 4 \times 10^{10}} \approx 6.3{\times}10^6$$

i.e., $f_0 = \omega_0/2\pi = 6.3 \times 10^6/6.283\ Hz = 10^6 = 1000KHz$. Check to see the validity of the approximation:

$$\frac{\omega_0 L}{R} = \frac{6.3 \times 10^6 \times 2.5 \times 10^{-5}}{5} = 31.5 > 20$$

As the admittance reaches minimum at $\omega = \omega_0$, the current drawn from the voltage source $\dot{I} = Y\dot{V}$ reaches minimum, i.e., it behaves as a band stop filter.

Also note that for ω_0 to be real, we must have

$$\frac{1}{LC} > (\frac{R}{L})^2, \quad \text{i,e,} \quad R < \sqrt{\frac{L}{C}}$$

When $R \ll \sqrt{L/C}$, the resonant frequency is

$$\omega_0 = \sqrt{\frac{1}{LC} - (\frac{R}{L})^2} \approx \frac{1}{\sqrt{LC}}$$

4. Design a parallel circuit to be resonant at 800 kHz with a bandwidth of 32 kHz. The inductor has $L = 40\mu H$ and $R_L = 4.02\Omega$. Find the capacitance C needed for the desired resonant frequency. In order to satisfy the desired bandwidth, you may also need to include a resistor in the circuit.

   Solution: Based on the desired resonant frequency and bandwidth, the quality factor needs to be

$$Q = \frac{f_0}{f} = \frac{800 \times 10^3}{32 \times 10^3} = 25$$

   Since $Q > 20$, the resonant frequency is approximately

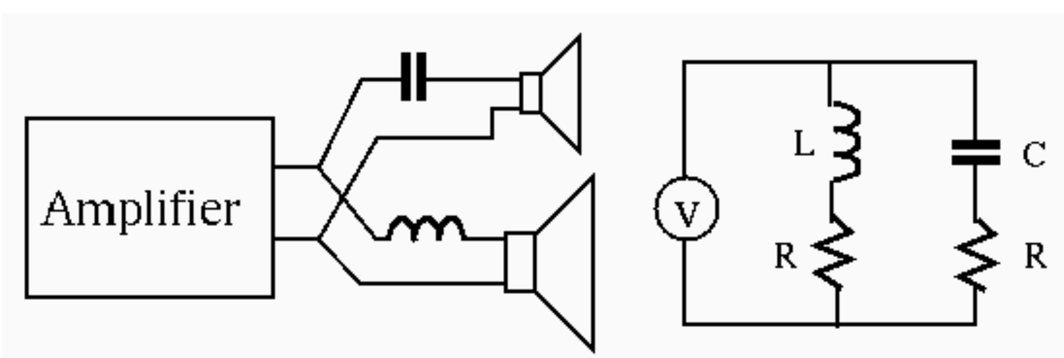
$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \text{i.e.,} \quad C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi 8 \times 10^5)^2 \times 4 \times 10^{-5}} = 0.99nF$$

However, the quality factor of the parallel circuit is

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi 8 \times 10^5 \times 4 \times 10^{-5}}{4.02} = 50$$

twice the desired $Q = 25$, we have to double the resistance $R = 4.02$ to $R = 8.04$ to reduce Q by half.

5. The function of a loudspeaker crossover network is to channel frequencies higher than a given crossover frequency f_c into the high-frequency speaker (``tweeter'') and frequencies below $f_c$ into the low-frequency speaker (``woofer''). One such circuit is shown below. Assume the resistances of the tweeter is $R_1 = 8\Omega$ and that of the woofer is $R_2 = 8\Omega$, the voltage amplifier can be modeled as an ideal voltage source, and the crossover frequency is $f_c = 2000\ Hz$. Design the network in terms of L and C so that f_c is the corner freqnency or half-power point of each of the two speaker circuits. Give the expression of the power $P_1(f)$ and $P_2(f)$ of the speakers as a function of frequency f and crossover frequency f_c, and sketch them. Assume the RMS of the input voltage is 1V.



Solution

The RMS voltage across the tweeter is

$$V_1 = \left|\frac{R}{1/j2\pi fC + R}\right| = \frac{2\pi fCR}{\sqrt{1+(2\pi fCR)^2}}$$

If $f = f_c = 2000$ is at half-power point ($V_1 = V_{in}/\sqrt{2}$), the real and imaginary parts of the denominator should be equal and we get

$$\frac{1}{2\pi fC} = R; \quad \text{i.e.} \quad C = \frac{1}{2\pi f_c R} = 9.95\ \mu F$$

The RMS voltage across the woofer is

$$V_2 = \left|\frac{R}{j2\pi fL + R}\right| = \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}$$

If $f = f_c = 2000$ is at half-power point ($V_2 = V_{in}/\sqrt{2}$), the real and imaginary parts of the denominator should be equal and we get

$$2\pi fL = R; \quad \text{i.e.} \quad L = \frac{R}{2\pi f_c} = 0.637\ mH$$

The power plots:

$$P_1(f) = \frac{V_1^2(f)}{R} = \frac{1}{8}\left(\frac{1}{1+(f_c/f)^2}\right)$$

$$P_2(f) = \frac{V_1^2(f)}{R} = \frac{1}{8}(\frac{1}{1+(f/f_c)^2})$$

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