Chapter 2: Fundamentals of Electrical Circuits – Instructor Notes

Chapter 2 develops the foundations for the first part of the book. Coverage of the entire chapter would be typical in an introductory course. The first four sections provide the basic definitions (Section 2.1) and cover Kirchoff's Laws and the passive sign convention (Sections 2.2, 2.3 And 2.4); A special feature, *Focus on Methodology: The Passive Sign Convention* (p. 39) and two examples illustrate this very important topic. A second feature, that will recur throughout the first six chapters, is presented in the form of sidebars. *Make The Connection: Mechanical Analog of Voltage Sources* (p. 24) and *Make The Connection: Hydraulic Analog of Current Sources* (p. 26) present the concept of analogies between electrical and other physical domains.

Sections 2.5and 2.6 introduce the *i-v* characteristic and the resistance element. Tables 2.1 and 2.2 on p. 45 summarize the resistivity of common materials and standard resistor values; Table 2.3 on p. 48 provides the resistance of copper wire for various gauges. The sidebar *Make The Connection: Electric Circuit Analog of Hydraulic Systems – Fluid Resistance* (p. 44) continues the electric-hydraulic system analogy.

Finally, Sections 2.7 and 2.8 introduce some basic but important concepts related to ideal and non-ideal current sources, and measuring instruments. In the context of measuring instruments, Chapter 2 introduces a third feature of this book, the *Focus on Measurements* boxes, referenced in the next paragraph.

The Instructor will find that although the material in Chapter 2 is quite basic, it is possible to give an applied flavor to the subject matter by emphasizing a few selected topics in the examples presented in class. In particular, a lecture could be devoted to *resistance devices*, including the resistive displacement transducer of *Focus on Measurements: Resistive throttle position sensor* (pp. 56-59), the resistance strain gauges of *Focus on Measurements: Resistance strain gauges* (pp. 58-59), and *Focus on Measurements: The Wheatstone bridge and force measurements* (pp. 59-60). The instructor wishing to gain a more in-depth understanding of resistance strain gauges will find a detailed analysis in the references[1].

Early motivation for the application of circuit analysis to problems of practical interest to the non-electrical engineer can be found in the *Focus on Measurements: The Wheatstone bridge and force measurements*. The Wheatstone bridge material can also serve as an introduction to a laboratory experiment on strain gauges and the measurement of force (see, for example[2]). Finally, the material on practical measuring instruments in Section2.8b can also motivate a number of useful examples.

The homework problems include a variety of practical examples, with emphasis on instrumentation. Problem 2.51 illustrates analysis related to fuses; problems 2.65 relates to wire gauges; problem 2.70 discusses the thermistor; problems 2.71 discusses moving coil meters; problems 2.79 and 2.80 illustrate calculations related to strain gauge bridges; a variety of problems related to practical measuring devices are also included in the last section. The 5th Edition of this book includes 26 new problems; some of the 4th Edition problems were removed, increasing the end-of-chapter problem count from 66 to 80.

It has been the author's experience that providing the students with an early introduction to practical applications of electrical engineering to their own disciplines can increase the interest level in the course significantly.

Learning Objectives for Chapter 2

6. Identify the principal *elements of electrical circuits*: nodes, loops, meshes, branches, and voltage and current sources.
7. Apply *Kirchhoff's Laws* to simple electrical circuits and derive the basic circuit equations.
8. Apply the *passive sign convention* and *compute power* dissipated by circuit elements.
9. Apply the *voltage and current divider laws* to calculate unknown variables in simple series, parallel and series-parallel circuits.
10. Understand the rules for connecting *electrical measuring instruments* to electrical circuits for the measurement of voltage, current, and power.

[1] E. O. Doebelin, Measurement Systems – Application and Design, 4th Edition, McGraw-Hill, New York, 1990.
[2] G. Rizzoni, A Practical Introduction to Electronic Instrumentation, 3rd Edition, Kendall-Hunt, 1998.

Section 2.1: Definitions

Problem 2.1

Solution:

Known quantities:

Initial Coulombic potential energy, $V_i = 17kJ/C$; initial velocity, $U_i = 93M\,\frac{m}{s}$; final Coulombic potential energy, $V_f = 6kJ/C$.

Find:

The change in velocity of the electron.

Assumptions:

$$\Delta PE_g << \Delta PE_c$$

Analysis:

Using the first law of thermodynamics, we obtain the final velocity of the electron:

$$Q_{heat} - W = \Delta KE + \Delta PE_c + \Delta PE_g + ...$$

Heat is not applicable to a single particle. W=0 since no external forces are applied.

$$\Delta KE = -\Delta PE_c$$

$$\frac{1}{2} m_e (U_f^2 - U_i^2) = -Q_e (V_f - V_i)$$

$$U_f^2 = U_i^2 - \frac{2Q_e}{m_e}(V_f - V_i)$$

$$= \left(93\,M\,\frac{m}{s}\right)^2 - \frac{2\left(-1.6\times10^{-19}\,C\right)}{9.11\times10^{-37}\,g}\left(6kV - 17kV\right)$$

$$= 8.649\times10^{15}\,\frac{m^2}{s^2} - 3.864\times10^{15}\,\frac{m^2}{s^2}$$

$$U_f = 6.917\times10^{7}\,\frac{m}{s}$$

$$\left|U_f - U_i\right| = 93\,M\,\frac{m}{s} - 69.17\,M\,\frac{m}{s} = 23.83\,M\,\frac{m}{s}.$$

Problem 2.2

Solution:

Known quantities:
MKSQ units.

Find:
Equivalent units of volt, ampere and ohm.

Analysis:

$$\text{Voltage} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}} \quad V = \frac{J}{C}$$

$$\text{Current} = \text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \quad a = \frac{C}{s}$$

$$\text{Resistance} = \text{Ohm} = \frac{\text{Volt}}{\text{Ampere}} = \frac{\text{Joule} \times \text{second}}{\text{Coulomb}^2} \quad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = \text{Siemens or Mho} = \frac{\text{Ampere}}{\text{Volt}} = \frac{C^2}{J \cdot s}$$

Problem 2.3

Solution:

Known quantities:
Battery nominal rate of 100 A-h.

Find:
a) Charge potentially derived from the battery
b) Electrons contained in that charge.

Assumptions:
Battery fully charged.

Analysis:

a)

$$100A \times 1hr = \left(100\frac{C}{s}\right)(1hr)\left(3600\frac{s}{hr}\right) = 360000\ C$$

b)

charge on electron: $-1.602 \times 10^{-19}\ C$

no. of electrons =

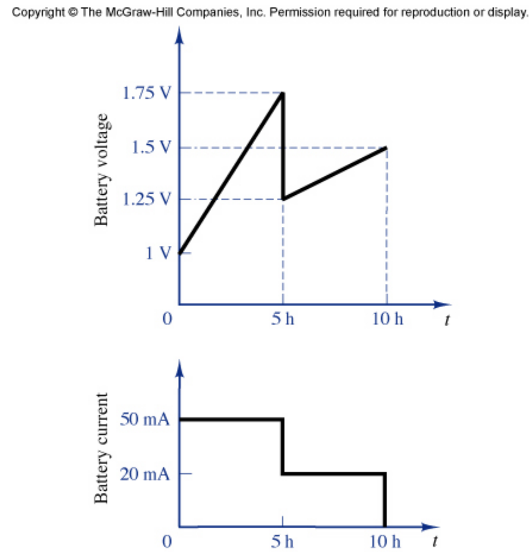
$$\frac{360 \times 10^3 C}{1.602 \times 10^{-19} C} = 224.7 \times 10^{22}$$

Problem 2.4

Solution:

Known quantities:

Two-rate change charge cycle shown in Figure P2.4.



Find:

a) The charge transferred to the battery
b) The energy transferred to the battery.

Analysis:

a) To find the charge delivered to the battery during the charge cycle, we examine the charge-current relationship:

$$i = \frac{dq}{dt} \quad or \quad dq = i \cdot dt$$

thus:

$$Q = \int_{t_0}^{t_1} i(t)dt$$

$$Q = \int_0^{5hrs} 50mAdt + \int_{5hrs}^{10hrs} 20mAdt$$

$$= \int_0^{18000s} 0.05dt + \int_{18000}^{36000s} 0.02dt$$

$$= 900 + 360$$

$$= 1260C$$

b) To find the energy transferred to the battery, we examine the energy relationship

$$p = \frac{dw}{dt} \quad or \quad dw = p(t)dt$$

$$w = \int_{t_0}^{t_1} p(t)dt = \int_{t_0}^{t_1} v(t)i(t)dt$$

observing that the energy delivered to the battery is the integral of the power over the charge cycle. Thus,

$$w = \int_0^{18000} 0.05(1 + \frac{0.75t}{18000})\, dt + \int_{18000}^{36000} 0.02(1 + \frac{0.25t}{18000})\, dt$$

$$= (0.05t + \frac{0.75}{36000}t^2)\Big|_0^{18000} + (0.02t + \frac{0.25}{36000}t^2)\Big|_{18000}^{36000}$$

$$\boxed{w = 1732.5\ \text{J}}$$

Problem 2.5

Solution:

Known quantities:

Rated voltage of the battery; rated capacity of the battery.

Find:

a) The rated chemical energy stored in the battery
b) The total charge that can be supplied at the rated voltage.

Analysis:

a)

$$\Delta V \equiv \frac{\Delta PE_c}{\Delta Q} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\text{Chemical energy} = \Delta PE_c = \Delta V \cdot \Delta Q = \Delta V \cdot \left(I \cdot \Delta t\right)$$

$$= 12\ V\ 350\ \text{A} - \text{hr}\ 3600\ \frac{\text{s}}{\text{hr}} = 15.12\ \text{MJ}.$$

As the battery discharges, the voltage will decrease below the rated voltage. The remaining chemical energy stored in the battery is less useful or not useful.

b) ΔQ is the total charge passing through the battery and gaining 12 J/C of electrical energy.

$$\Delta Q = I \cdot \Delta t = 350\ a\ hr = 350\ \frac{\text{C}}{\text{s}}\ \text{hr} \cdot 3600\ \frac{\text{s}}{\text{hr}} = 1.26\ \text{MC}.$$

Problem 2.6

Solution:

Known quantities:

Resistance of external circuit.

Find:

a) Current supplied by an ideal voltage source
b) Voltage supplied by an ideal current source.

Assumptions:

Ideal voltage and current sources.

Analysis:

a) An ideal voltage source produces a constant voltage at or below its rated current. Current is determined by the power required by the external circuit (modeled as R).

$$I = \frac{V_S}{R} P = V_S \cdot I$$

b) An ideal current source produces a constant current at or below its rated voltage. Voltage is determined by the power demanded by the external circuit (modeled as R).

$$V = I_S \cdot R \quad P = V \cdot I_S$$

A real source will overheat and, perhaps, burn up if its rated power is exceeded.

Problem 2.7

Solution:

Known quantities:

Rated discharge current of the battery; rated voltage of the battery; rated discharge time of the battery.

Find:

a) Energy stored in the battery when fully recharging
b) Energy stored in the battery after discharging

Analysis:

a) $$\text{Energy} = \text{Power} \times \text{time} = (1A)(12V)(120\text{hr})\left(\frac{60\,\text{min}}{\text{hr}}\right)\left(\frac{60\,\text{sec}}{\text{min}}\right)$$

$$\boxed{\text{w} = 5.184 \times 10^{6}\ \text{J}}$$

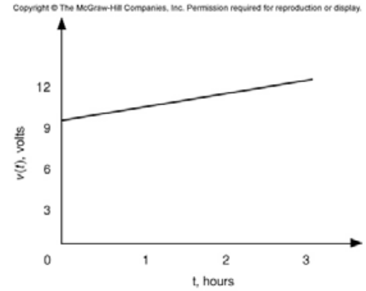
b) Assume that 150 W is the combined power rating of both lights; then,

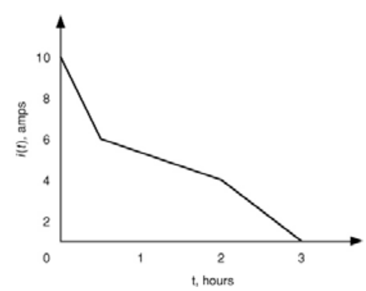
$$\text{w}_{\text{used}} = (150W)(8hrs)\left(\frac{3600\,\text{sec}}{hr}\right) = 4.32 \times 10^{6}\ \text{J}$$

$$\boxed{w_{stored} = w - w_{used} = 864 \times 10^{3}\ \text{J}}$$

Problem 2.8

Solution:





Known quantities:

Recharging current and recharging voltage

Find:

a) Total transferred charge
b) Total transferred energy

Analysis:

a)
$$\text{Q} = \text{area under the current - time curve} = \int Idt$$
$$= \frac{1}{2}(4)(30)(60) + 6(30)(60) + \frac{1}{2}(2)(90)(60) + 4(90)(60) + \frac{1}{2}(4)(60)(60) = 48{,}600\ \text{C}$$

$$\boxed{\text{Q} = 48{,}600\ \text{C}}$$

b) $$\frac{\text{dw}}{\text{dt}} = \text{p} \quad \text{so} \quad w = \int pdt = \int vidt$$

$$\text{v} = 9 + \frac{3}{10800}\text{t} \quad \text{V}, \quad 0 \le \text{t} \le 10800\ \text{s}$$

$$\text{i}_1 = 10 - \frac{4}{1800}\text{t} \quad \text{A}, \quad 0 \le \text{t} \le 1800\ \text{s}$$

$$\text{i}_2 = 6 - \frac{2}{5400}\text{t} \quad \text{A}, \quad 1800 \le \text{t} \le 7200\ \text{s}$$

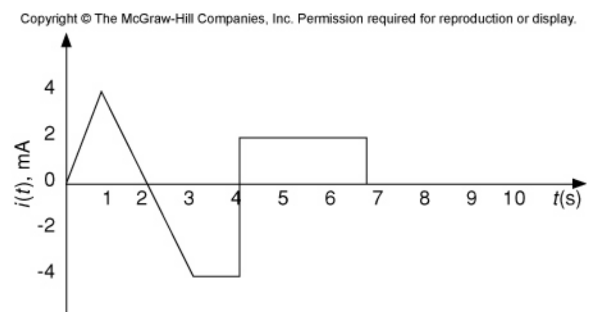
$$\text{i}_3 = 12 - \frac{4}{3600}\text{t} \quad \text{A}, \quad 7200 \le \text{t} \le 10800\ \text{s}$$

where $i = i_1 + i_2 + i_3$

Therefore,

$$w = \int_0^{1800} vi_1 dt + \int_{1800}^{7200} vi_2 dt + \int_{7200}^{10800} vi_3 dt$$
$$= \left(90t + \frac{t^2}{720} - \frac{t^2}{100} - \frac{t^3}{4.86 \times 10^6} \right)\Bigg|_0^{1800}$$
$$+ \left(60t + \frac{t^2}{1080} - \frac{t^2}{600} - \frac{t^3}{29.16 \times 10^6} \right)\Bigg|_{1800}^{7200}$$
$$+ \left(108t + \frac{t^2}{600} - \frac{t^2}{200} - \frac{t^3}{9.72 \times 10^6} \right)\Bigg|_{7200}^{10800}$$
$$= 132.9 \times 10^3 + 380.8 \times 10^3 - 105.4 \times 10^3 + 648 \times 10^3 - 566.4 \times 10^3$$
$$\boxed{Energy = 489.9\ kJ}$$

Problem 2.9



Solution:

Known quantities:

Current-time curve

Find:

a) Amount of charge during 1st second
b) Amount of charge for 2 to 10 seconds
c) Sketch charge-time curve

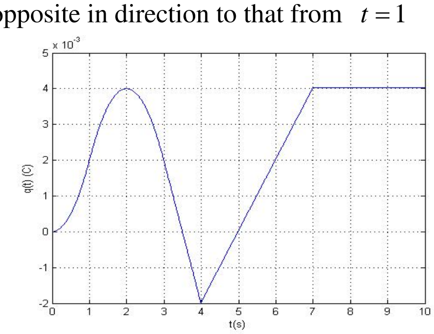
Analysis:

a) $i = \frac{4 \times 10^{-3} t}{1}$

$$Q_1 = \int_0^1 i dt = \int_0^1 4 \times 10^{-3} t dt = 4 \times 10^{-3} \frac{t^2}{2}\Bigg|_0^1 = 2 \times 10^{-3} \frac{\text{amp}}{\text{sec}} = 2 \times 10^{-3} \text{ Coulombs}$$

b) The charge transferred from $t = 1$ to $t = 2$ is the same as from $t = 0$ to $t = 1$. $Q_2 = 4 \times 10^{-3}$ Coulombs

The charge transferred from $t = 2$ to $t = 3$ is the same in magnitude and opposite in direction to that from $t = 1$ to $t = 2$. $Q_3 = 2 \times 10^{-3}$ Coulombs



$t = 4$

$$Q_4 = 2 \times 10^{-3} - \int_3^4 4 \times 10^{-3} dt = 2 \times 10^{-3} - 4 \times 10^{-3} = -2 \times 10^{-3} \text{ Coulombs}$$

$t = 5, 6, 7$

$$Q_5 = -2 \times 10^{-3} + \int_4^5 2 \times 10^{-3} dt = 0$$

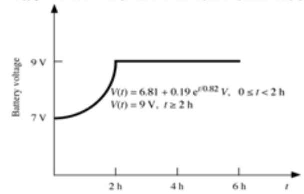
$$Q_6 = 0 + \int_5^6 2 \times 10^{-3} dt = 2 \times 10^{-3} \text{ Coulombs}$$

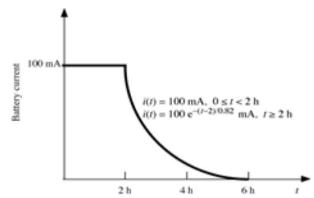
$$Q_7 = 2 \times 10^{-3} + \int_6^7 2 \times 10^{-3} dt = 4 \times 10^{-3} \text{ Coulombs}$$

$t = 8, 9, 10 s$

$$Q = 4 \times 10^{-3} \text{Coulombs}$$

Problem 2.10





Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging

Find:

a) Total transferred charge
b) Total transferred energy

Analysis:

a) $100mA = 0.1A$

$$Q \ = \ \text{area under the current - time curve} = \int Idt = (0.1)(2)(3600) + (3600)\int_2^6 0.1e^{-(t-2)/0.82}dt \ = 1{,}011\ \text{C}$$

$$\boxed{Q \ = \ 1{,}011\ \text{C}}$$

b) $\dfrac{dw}{dt} = P$ so

$$w = \int Pdt = \int vidt = (3600)\int_0^2 vidt + (3600)\int_2^4 vidt = (3600)\int_0^2 \left(6.81 + 1.89e^{t/0.82}\right)(0.1)dt + (3600)\int_2^4 9\left(0.1e^{-(t-2)/0.82}\right)dt = 8{,}114\text{J}$$

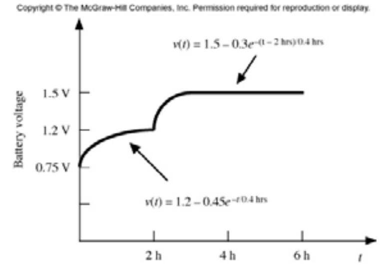
$$\boxed{\text{Energy} = 8{,}114\text{J}}$$

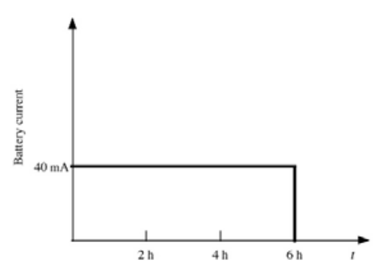
Problem 2.11

Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging





Find:

c) Total transferred charge
d) Total transferred energy

Analysis:

a) $40mA = 0.04A$

$$Q \ = \ \text{area under the current - time curve} = \int Idt = (0.04)(6)(3600) \ = 864\ \text{C}$$

$$\boxed{\text{Q} = 864\ \text{C}}$$

b) $\dfrac{\text{dw}}{\text{dt}} = \text{P}$ so

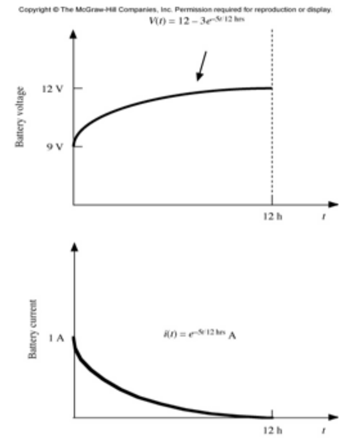
$$w = \int Pdt = \int vidt = (3600)\int_0^2 vidt + (3600)\int_2^4 vidt$$

$$= (3600)\int_0^2 \left(1.2 - 0.45e^{-t/0.4}\right)(0.04)dt + (3600)\int_2^4 \left(1.5 - 0.3e^{-(t-2)/0.4}\right)(0.04)dt$$

$$= 1{,}167J$$

$$\boxed{Energy \ = 1{,}167J}$$

Problem 2.12



Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging

Find:

e) Total transferred charge
f) Total transferred energy

Analysis:

a)

$$Q = \text{area under the current - time curve} = \int I dt = (3600)\int_0^{12} e^{-5t/12} dt = 8{,}564\,C$$

$$\boxed{Q = 8{,}564\,C}$$

b) $\dfrac{dw}{dt} = P$ so

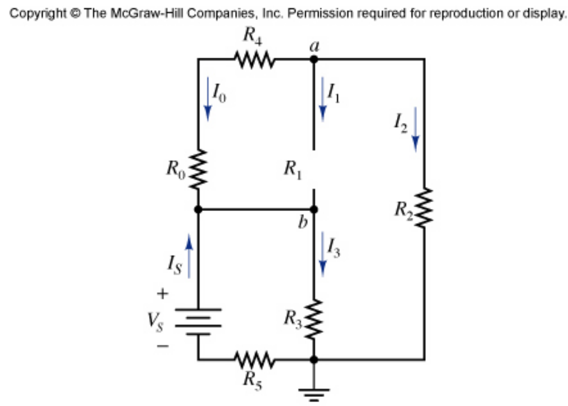
$$w = \int P dt = \int vi dt = (3600)\int_0^{2}\left(12 - 3e^{-5t/12}\right)\left(e^{-5t/12}\right)dt$$

$$= 8{,}986 J$$

$$\boxed{\text{Energy } = 8{,}986 J}$$

Section 2.2, 2.3 KCL, KVL

Problem 2.13



Solution:

Known quantities:

Circuit shown in Figure P2.13 with currents $I_0 = -2$ A, $I_1 = -4$ A, $I_S = 8$ A, and voltage source $V_S = 12$ V.

Find:

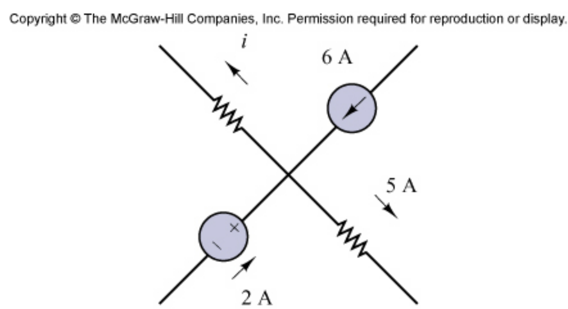
The unknown currents.

Analysis:

Applying KCL to node (a) and node (b):

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_2 = -(I_0 + I_1) = 6 \text{ A} \\ I_3 = I_0 + I_S + I_1 = 2 \text{ A} \end{cases}$$

Problem 2.14



Solution:

Known quantities:

Circuit shown in Figure P2.14.

Find:

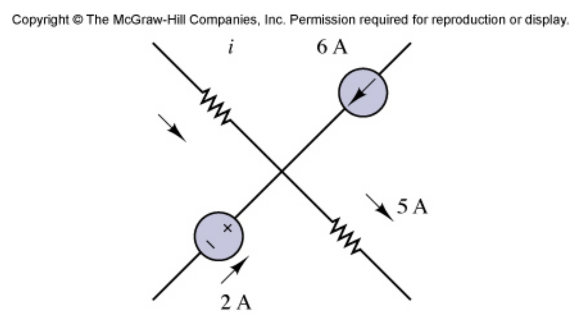
The unknown currents.

Analysis:

Applying KCL at the node: $-i + 2 + 6 - 5 = 0$

thus i = 3 A which means that a 3-A current is leaving the node.

Problem 2.15



Solution:

Known quantities:

Circuit shown in Figure P2.15.

Find:

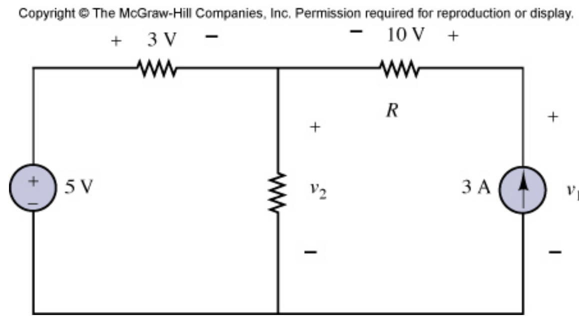
The unknown currents.

Analysis:

Applying KCL at the node: $i + 6 - 5 + 2 = 0$

thus i = -3 A which means that a 3-A current is leaving the node.

Problem 2.16



Solution:

Known quantities:

Circuit shown in Figure P2.16.

Find:

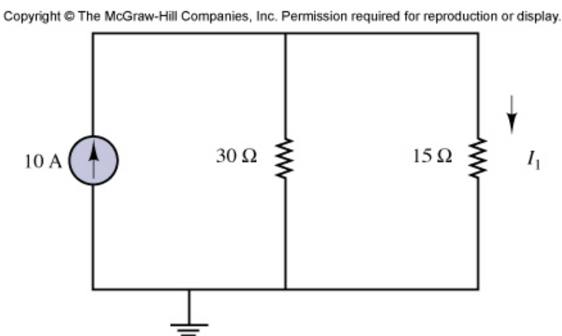
Voltages v_1 and v_2

Analysis:

Applying KVL:

$$-5+3+v_2=0 \Rightarrow v_2=2\ V$$
$$-5+3-10+v_1=0 \Rightarrow v_1=12\ V$$

Problem 2.17



Solution:

Known quantities:

Circuit shown in Figure P2.17.

Find:

Current I_1

Analysis:

Let us refer to the current (down) through the 30Ω resistor as I_2.

Applying KCL, we have

$$I_1+I_2=10A \qquad \text{(Eq.1)}$$

Also, applying KVL and Ohm's law, we have

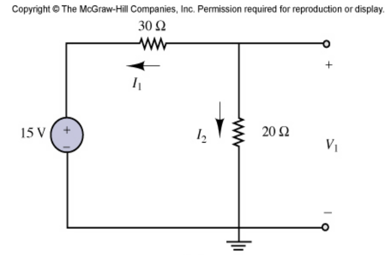
$$15I_1-30I_2=0 \qquad \text{(Eq.2)}$$

Solving Eq.1 and Eq.2, we obtain

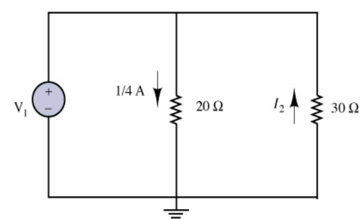
$$I_1=\frac{20}{3}\ A \quad \text{and} \quad I_2=\frac{10}{3}\ A$$

Section 2.4 Sign Convention

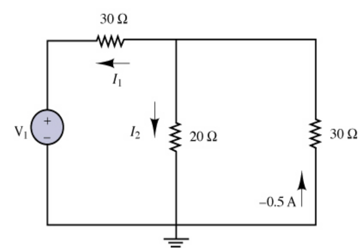
Problem 2.18



(a)



(b)



(c)

Solution:

Known quantities:

Circuit shown in Figure P2.18.

Find:

Voltages and currents in every figure.

Analysis:

(a) Using $I = \frac{15}{30+20}$ (clockwise current) : $I_1 = -0.3A; I_2 = 0.3A; V_1 = 6V$

(b) The voltage across the 20 Ω resistor is $\frac{20}{4} = 5\text{V}$; since the current flows from top to bottom, the polarity of this voltage is positive on top. Then it follows that $V_1 = 5V$ and $\text{I}_2 = \frac{5}{30} = \text{-0.167A}$

(the negative sign follows from the direction of I_2 in the drawing).

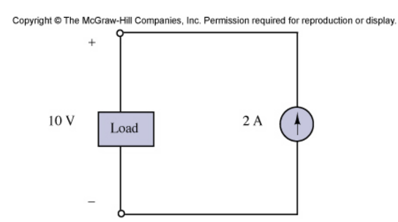
(c) Since -0.5A pointing upward is the same current as 0.5A pointing downward, the voltage across the 30 Ω resistor is

$V_{30\Omega} = 15V$ (positive on top); and $\text{I}_2 = \frac{15}{20} = 0.75\text{A}$,

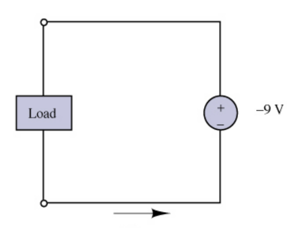
since $V_{30\Omega}$ is also the voltage across the 20 Ω resistor. Finally,

$I_1 = \text{-}(I_2 + 0.5) = \text{-}1.25A$, and $V_1 = \text{-}30\, I_1 + 15 = 52.5V$

Problem 2.19



(a)



(b)

Solution:

Known quantities:

Circuit shown in Figure P2.19.

Find:

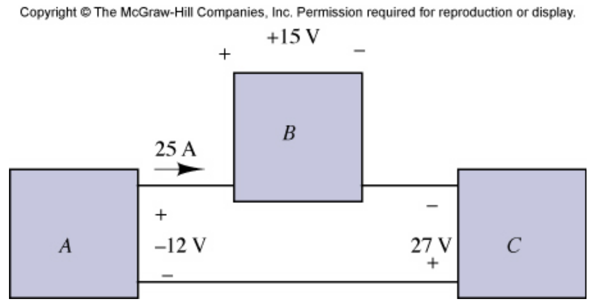
Power delivered by each source.

Analysis:

(a) Power delivered by source to load = power absorbed by load = $2 \times 10 = 20W$

(b) $P = (\text{-}9) \times 4 = \text{-}36\ W$; the source is actually absorbing power, thus the "load" must be a source!

Problem 2.20



Solution:

Known quantities:

Circuit shown in Figure P2.20.

Find:

Determine power dissipated or supplied for each power source.

Analysis:

Element A:

$$P = -vi = -(-12V)(25A) = 300W \quad \text{(dissipating)}$$

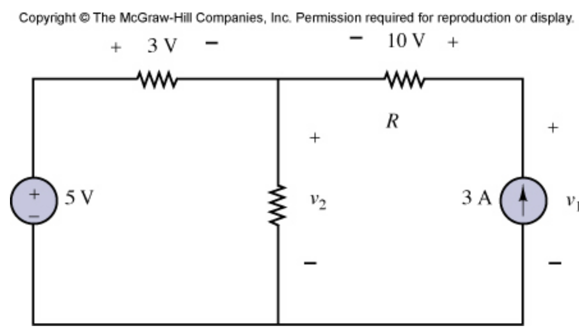
Element B:

$$P = vi = (15V)(25A) = 375W \quad \text{(dissipating)}$$

Element C:

$$P = vi = (27V)(25A) = 675W \quad \text{(supplying)}$$

Problem 2.21



Solution:

Known quantities:

Circuit shown in Figure P2.21.

Find:

Power absorbed by resistant R and power delivered by current source.

Analysis:

Power absorbed by $R = (10V)(3A) = 30W$

From Problem 2.16, $v_1 = 12\ V$. Therefore,

Power delivered by the current source = $(12V)(3A) = 36W$

Problem 2.22

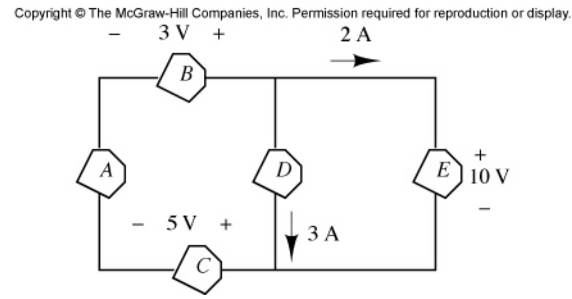
Solution:

Known quantities:

Circuit shown in Figure P2.22.

Find:

a) Determine power absorbed or power delivered
b) Testify power conservation



Analysis:

By KCL, the current through element B is 5A, to the right.

By KVL, $-v_a - 3 + 10 + 5 = 0$.

Therefore, the voltage across element A is

$v_a = 12V$ (positive at the top).

A supplies $(12V)(5A) = 60W$

B supplies $(3V)(5A) = 15W$

C absorbs $(5V)(5A) = 25W$

D absorbs $(10V)(3A) = 30W$

E absorbs $(10V)(2A) = 20W$

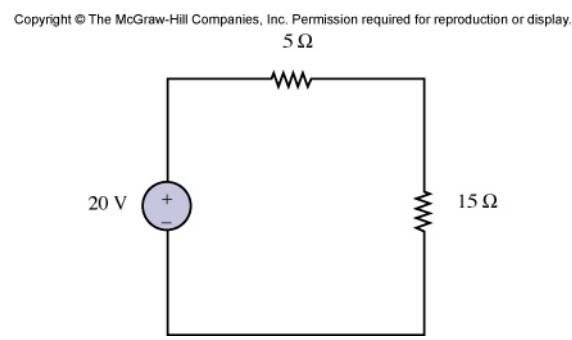
Total power supplied = $60W + 15W = 75W$

Total power absorbed = $25W + 30W + 20W = 75W$

Tot. power supplied = Tot. power absorbed

$\therefore$ conservation of power is satisfied.

Problem 2.23



Solution:

Known quantities:

Circuit shown in Figure P2.23.

Find:

Power absorbed by the 5Ω resistance.

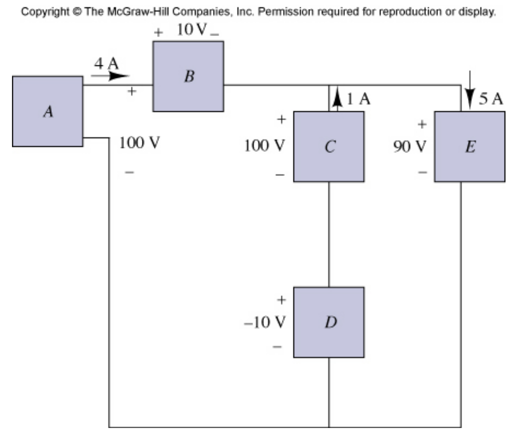
Analysis:

The current flowing clockwise in the series circuit is $i = \frac{20V}{20\Omega} = 1A$

The voltage across the 5 Ω resistor, positive on the left, is $v_{5\Omega} = (1A)(5\Omega) = 5V$

Therefore, $P_{5\Omega} = (5V)(1A) = 5\ W$

Problem 2.24



Solution:

Known quantities:

Circuit shown in Figure P2.24.

Find:

Determine power absorbed or power delivered and corresponding amount.

Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A supplies $(100V)(4A) = 400$ W

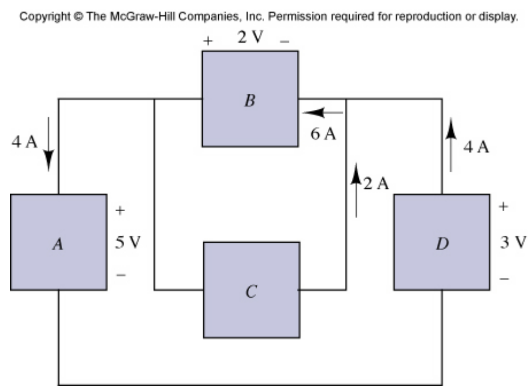
B absorbs $(10V)(4A) = 40$ W

C supplies $(100V)(1A) = 100W$

D supplies $(-10V)(1A) = -10W$, i.e absorbs $10W$

E absorbs $(90V)(5A) = 450W$

Problem 2.25



Solution:

Known quantities:

Circuit shown in Figure P2.25.

Find:

Determine power absorbed or power delivered and corresponding amount.

Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A absorbs $(5V)(4A) = 20W$

B supplies $(2V)(6A) = 12W$

D supplies $(3V)(4A) = 12W$

Since conservation of power is satisfied, Tot. power supplied = Tot. power absorbed

Total power supplied = $12\text{W} + 12\text{W} = 24\text{W}$

$\therefore$ C absorbs $24W - 20W = 4W$

Problem 2.26

Solution:

Known quantities:

Current absorbed by the heater; voltage at which the current is supplied; cost of the energy.

Find:

a) Power consumption
b) Energy dissipated in 24 hr.
c) Cost of the Energy

Assumptions:

The heater works for 24 hours continuously.

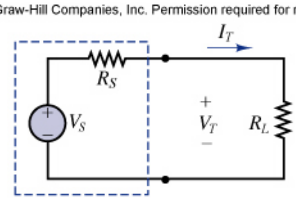
Analysis:

a) $P = VI = 110\ V\left(23\ A\right) = 2.53\times 10^{3}\ \frac{J}{A}\frac{A}{s} = 2.53\ \text{KW}$

b) $W = Pt = 2.53\times 10^{3}\ \frac{\text{J}}{\text{s}} \times 24\ \text{hr} \times 3600\frac{\text{s}}{\text{hr}} = 218.6\ \text{MJ}$

c) $\text{Cost} = (\text{Rate})\times W = 6\frac{\text{cents}}{\text{kW}-\text{hr}}\left(2.53\ \text{kW}\right)\left(24\ \text{hr}\right) = 364.3\ \text{cents} = \$3.64$

Problem 2.27



Nonideal source

Solution:

Known quantities:

Circuit shown in Figure P2.27 with voltage source, $V_s = 12V$; internal resistance of the source, $R_s = 5k\Omega$; and resistance of the load, $R_L = 7k\Omega$.

Find:

The terminal voltage of the source; the power supplied to the circuit, the efficiency of the circuit.

Assumptions:

Assume that the only loss is due to the internal resistance of the source.

Analysis:

$$KVL:\quad -V_S + I_T R_S + V_T = 0 \qquad OL:\quad V_T = I_T R_L \quad \therefore \quad I_T = \frac{V_T}{R_L}$$

$$-V_S + \frac{V_T}{R_L}R_S + V_T = 0$$

$$V_T = \frac{V_S}{1+\frac{R_S}{R_L}} = \frac{12\ V}{1+\frac{5\ k\Omega}{7\ k\Omega}} = 7\ V \quad or \quad VD: V_T = \frac{V_S R_L}{R_S + R_L} = \frac{12\ V\ 7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 7\ V.$$

$$P_L = \frac{V_R^2}{R_L} = \frac{V_T^2}{R_L} = \frac{\left(7\ V\right)^2}{7\times 10^3\ \frac{\text{V}}{\text{A}}} = 7\ \text{mW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{R_L}}{P_{R_S} + P_{R_L}} = \frac{\cancel{I_T^2}R_L}{\cancel{I_T^2}R_S + \cancel{I_T^2}R_L} = \frac{7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 0.5833 \quad or \quad 58.33\%.$$

Problem 2.28

Solution:

Known quantities:
Headlights connected in parallel to a 24-V automotive battery; power absorbed by each headlight.

Find:
Resistance of each headlight; total resistance seen by the battery.

Analysis:

Headlight no. 1:

$$P = v \times i = 100 \text{ W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{100} = \frac{576}{100} = 5.76\ \Omega$$

Headlight no. 2:

$$P = v \times i = 75 \text{ W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

The total resistance is given by the parallel combination:

$$\frac{1}{R_{TOTAL}} = \frac{1}{5.76\ \Omega} + \frac{1}{7.68\ \Omega} \quad \text{or } R_{TOTAL} = 3.29\ \Omega$$

Problem 2.29

Solution:

Known quantities:
Headlights and 24-V automotive battery of problem 2.13 with 2 15-W taillights added in parallel; power absorbed by each headlight; power absorbed by each taillight.

Find:
Equivalent resistance seen by the battery.

Analysis:

The resistance corresponding to a 75-W headlight is:

$$R_{75W} = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

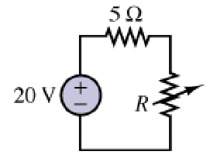
For each 15-W tail light we compute the resistance:

$$R_{15W} = \frac{v^2}{15} = \frac{576}{15} = 38.4\ \Omega$$

Therefore, the total resistance is computed as:

$$\frac{1}{R_{TOTAL}} = \frac{1}{7.68\Omega} + \frac{1}{7.68\Omega} + \frac{1}{38.4\Omega} + \frac{1}{38.4\Omega} \quad \text{or} \quad R_{TOTAL} = 3.2\ \Omega$$

Problem 2.30



Solution:

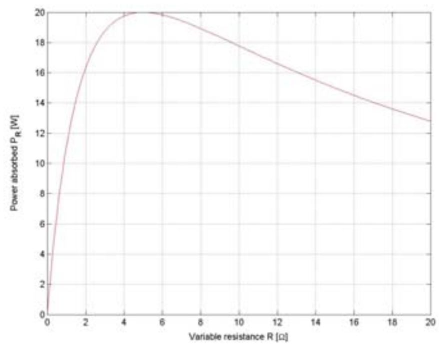
Known quantities:

Circuit shown in Figure P2.30 with voltage source, $V_s = 20V$; and resistor, $R_o = 5\Omega$.

Find:

The power absorbed by variable resistor R (ranging from 0 to 20 Ω).

Analysis:



The current flowing clockwise in the series circuit is:

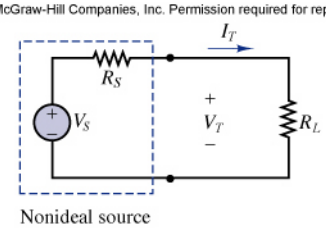
$$i = \frac{20}{5+R}$$

The voltage across the variable resistor R, positive on the left, is:

$$v_R = Ri = \frac{20R}{R+5}$$

Therefore, $P_R = v_R i = \left(\frac{20}{5+R}\right)^2 R$

Problem 2.31



Solution:

Known quantities:

Circuit shown in Figure P2.31 with source voltage, $V_s = 12V$; internal resistance of the source, $R_s = 0.3\Omega$.
Current, I_T =0, 5, 10, 20, 30 A.

Find:

a) The power supplied by the ideal source as a function of current
b) The power dissipated by the nonideal source as a function of current
c) The power supplied by the source to the circuit
d) Plot the terminal voltage and power supplied to the circuit as a function of current

Assumptions:

There are no other losses except that on Rs.

Analysis:

a) Ps = power supplied by the source $= V_S I_S = V_S I_T$.

b) Rs = equivalent resistance for internal losses

$$P_{loss} = I_T^2 R_S$$

c) V_T = voltage at the battery terminals:

$$VD: \quad V_T = V_S - R_S I_T$$

P_0 = power supplied to the circuit (R_L in this case) $= I_T V_T$.

Conservation of energy:

$$P_S = P_{loss} + P_0 .$$

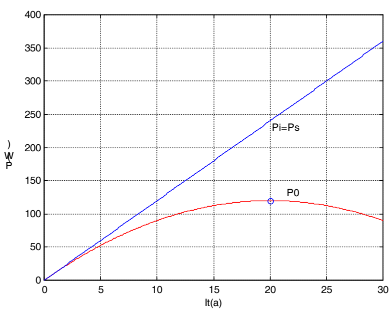
$I_T(A)$	$P_S(W)$	$P_{loss}(W)$	$V_T(V)$	$P_0(W)$
0	0	0	0	0
2	30	1.875	11.4	28.13
5	60	7.5	10.5	52.5
10	120	30	9	90
20	240	120	6	120
30	360	270	3	90

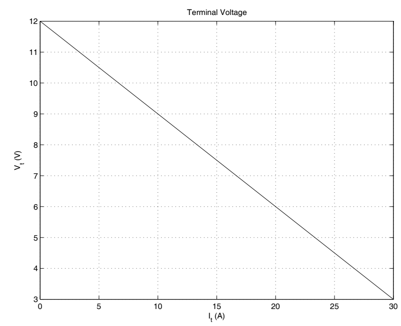
Note that the power supplied to the circuit is maximum when $I_T = 20a$.

$$R_L = \frac{P_0}{I_T^2} = \frac{120\ Va}{(20\ a)^2} = 30\ m\frac{V}{a} = 30\ m\Omega$$

$$R_S = \frac{P_{loss}}{I_T^2} = \frac{120\ Va}{(20\ a)^2} = 30\ m\Omega$$

$$R_L = R_S$$





Problem 2.32

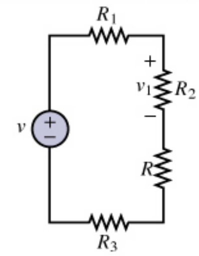
Solution:

Known quantities:

Circuit shown in Figure P2.32 if the power delivered by the source is 40 mW; the voltage v = v_1 /4; and $R_1 = 8k\Omega, R_2 = 10k\Omega, R_3 = 12k\Omega$

Find:

The resistance *R*, the current *i* and the two voltages *v* and v_1



Analysis:

$$P = v \cdot i = 40\ mW \quad \text{(eq. 1)}$$

$$v_1 = R_2 \cdot i = 10000 \cdot i = \frac{v}{4} \quad \text{(eq. 2)}$$

From eq.1 and eq.2, we obtain:

i = 1.0 mA and v = 40 V.

Applying KVL for the loop:

$$-v + 8000i + 10000i + Ri + 12000i = 0 \quad \text{or,} \quad 0.001R = 10$$

Therefore,

$R = 10k\Omega$ and $v_1 = 10V$.

Problem 2.33

Solution:

Known quantities:

Rated power; rated optical power; operating life; rated operating voltage; open-circuit resistance of the filament.

Find:

a) The resistance of the filament in operation
b) The efficiency of the bulb.

Analysis:

a)

$$P = VI \quad \therefore I = \frac{P_R}{V_R} = \frac{60 \text{ VA}}{115 \text{ V}} = 521.7 \text{ mA}$$

$$\text{OL:} \quad R = \frac{V}{I} = \frac{V_R}{I} = \frac{115 \text{ V}}{521.7 \text{ mA}} = 220.4 \ \Omega$$

b)

Efficiency is defined as the ratio of the useful power dissipated by or supplied by the load to the total power supplied by the source. In this case, the useful power supplied by the load is the optical power. From any handbook containing equivalent units: 680 lumens=1 W

$$P_{o,out} = \text{ Optical Power Out } = 820 \text{ lum} \frac{\text{W}}{680 \text{ lum}} = 1.206 \text{ W}$$

$$\eta = \text{ efficiency } = \frac{P_{o,out}}{P_R} = \frac{1.206 \text{ W}}{60 \text{ W}} = 0.02009 = 2.009 \ \%.$$

Problem 2.34

Solution:

Known quantities:

Rated power; rated voltage of a light bulb.

Find:

The power dissipated by a series of three light bulbs connected to the nominal voltage.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

$$\text{Ohm's Law:} \quad P = IV_B = I^2 R_B = \frac{V_B^2}{R_B} \qquad V_B = V_S = 110 \text{ V} \qquad R_B = \frac{V_B^2}{P} = \frac{(110 \text{ V})^2}{100 \text{ VA}} = 121 \ \Omega$$

Connected in series and assuming the resistance of each bulb remains the same as when connected individually:

$$\text{KVL:} \quad -V_S + V_{B1} + V_{B2} + V_{B3} = 0 \qquad \text{OL:} \quad -V_S + IR_{B3} + IR_{B2} + IR_{B1} = 0$$

$$I = \frac{V_S}{R_{B1} + R_{B2} + R_{B3}} = \frac{110 \text{ V}}{121 + 121 + 121 \text{ V}/A} = 303 \text{ mA}$$

$$P_{B1} = I^2 R_{B1} = (303 \text{ mA})^2 (121 \text{ V}/A) = 11.11 \text{ W} = \frac{1}{9} 100 \text{ W}.$$

Problem 2.35

Solution:

Known quantities:

Rated power and rated voltage of the two light bulbs.

Find:

The power dissipated by the series of the two light bulbs.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

Ohm's Law: $P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$

$$V_B = V_S = 110 \text{ V}$$

$$R_{60} = \frac{V_B^2}{P_{60}} = \frac{(110 \text{ V})^2}{60 \text{ V}A} = 201.7\,\Omega$$

$$R_{100} = \frac{V_B^2}{P_{100}} = \frac{(110 \text{ V})^2}{100 \text{ V}A} = 121\,\Omega$$

When connected in series and assuming the resistance of each bulb remains the same as when connected individually:

KVL: $-V_S + V_{B60} + V_{B100} = 0$

OL: $-V_S + IR_{B60} + IR_{B100} = 0$

$$I = \frac{V_S}{R_{B60} + R_{B100}} = \frac{110\ V}{201.7 + 121\ \frac{\text{V}}{\text{A}}} = 340.9 \text{ m}A$$

$$P_{B60} = I^2 R_{B60} = (340.9 \text{ m}A)^2 \left(201.7\ \frac{\text{V}}{\text{A}}\right) = 23.44 \text{ W}$$

$$P_{B100} = I^2 R_{B100} = (340.9 \text{ m}A)^2 \left(121\ \frac{\text{V}}{\text{A}}\right) = 14.06 \text{ W}$$

Notes: 1.It's strange but it's true that a 60 W bulb connected in series with a 100 W bulb will dissipate more power than the 100 W bulb. 2. If the power dissipated by the filament in a bulb decreases, the temperature at which the filament operates and therefore its resistance will decrease. This made the assumption about the resistance necessary.

Problem 2.36

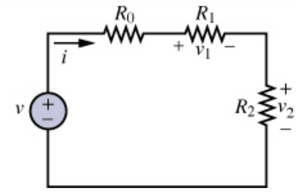
Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.36 with source voltage, $v = 24V$; and resistances, $R_o = 8\Omega, R_1 = 10\Omega, R_2 = 2\Omega$.

Find:

a) The equivalent resistance seen by the source
b) The current i
c) The power delivered by the source
d) The voltages v_1 and v_2
e) The minimum power rating required for R_1



Analysis:

a) The equivalent resistance seen by the source is $R_{eq} = R_0 + R_1 + R_2 = 8 + 10 + 2 = 20\Omega$

b) Applying KVL: $V - R_{eq} i = 0$, therefore $i = \dfrac{V}{R_{eq}} = \dfrac{24\text{V}}{20\Omega} = 1.2\text{A}$

c) $P_{source} = Vi = 24V \cdot 1.2A = 28.8\ W$

d) Applying Ohm's law: $v_1 = R_1 i = 10\Omega \cdot 1.2A = 12\ V$, and $v_2 = R_2 i = 2\Omega \cdot 1.2A = 2.4\ V$

e) $P_1 = R_1 i^2 = 10\Omega \cdot (1.2A)^2 = 14.4\ W$, therefore the minimum power rating for R_1 is 16 W.

Problem 2.37

Solution:

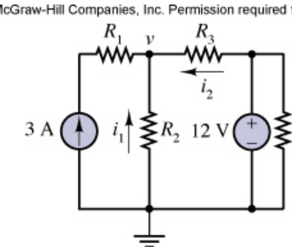
Known quantities:

Schematic of the circuit shown in Figure P2.37 with resistors, $R_1 = 25\Omega, R_2 = 10\Omega, R_3 = 5\Omega, R_4 = 7\Omega$.

Find:

a) The currents i_1 and i_2
b) The power delivered by the 3-A current source and the 12-V voltage source
c) The total power dissipated by the circuit.

Analysis:



a) KCL at node 1 requires that:

$$\frac{v_1}{R_2} + \frac{v_1 - 12\text{ V}}{R_3} - 3\text{ A} = 0$$

Solving for v_1 we have

$$v_1 = 3\frac{(4 + R_3)R_2}{R_2 + R_3} = 18\text{ V}$$

Therefore,

$$i_1 = -\frac{v_1}{R_2} = -\frac{18}{10} = -1.8\text{ A}$$

$$i_2 = \frac{12 - v_1}{R_3} = -\frac{6}{5} = -1.2\text{ A}$$

b) The power delivered by the 3-A source is:

$$P_{3\text{-}A} = (v_{3\text{-}A})(3)$$

Thus, we can compute the voltage across the 3-A source as

$$v_{3\text{-}A} = 3R_1 + v_1 = 3 \cdot 25 + 18 = 93 \text{ V}$$

Thus,

$$P_{3\text{-}A} = (93)(3) = 279 \text{ W}.$$

Similarly, the power supplied by the 12-V source is:

$$P_{12\text{-}V} = (12)(I_{12\text{-}V})$$

We have $I_{12\text{-}V} = \dfrac{12}{R_4} + i_2 = 514.3$ mA, thus:

$$P_{12\text{-}V} = (12)(I_{12\text{-}V}) = 6.17 \text{ W}$$

c) Since the power dissipated equals the total power supplied:

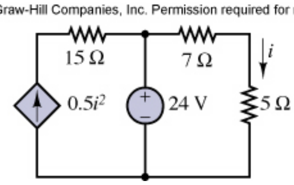
$P_{diss} = P_{3\text{-}A} + P_{12\text{-}V} = 279 + 6.17 = 285.17$ W

Problem 2.38

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.38.



Find:

The power delivered by the dependent source.

Analysis:

$$i = \frac{24V}{(7+5)\Omega} = \frac{24}{12} \text{ A} = 2 \text{ A}$$

$$i_{source} = 0.5i^2 = 0.5 \cdot (4) = 2 \text{ A}$$

The voltage across the dependent source (+ ref. taken at the top) can be found by KVL:

$$-v_D + (2A)(15\Omega) + 24V = 0 \quad \Rightarrow \quad v_D = 54 \text{ V}$$

Therefore, the power delivered by the dependent source is

$P_D = v_D i_{source} = 54 \cdot 2 = 108$ W.

Problem 2.39

Solution:

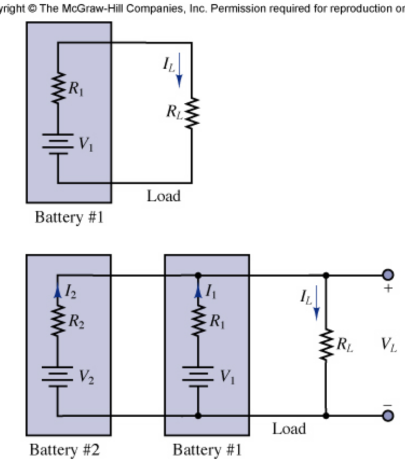
Known quantities:

Schematic of the circuit in Figure P2.39.

Find:

a) If $V_1 = 12.0V, R_1 = 0.15\Omega, R_L = 2.55\Omega$, the load current and the power dissipated by the load
b) If a second battery is connected in parallel with battery 1with $V_2 = 12.0V, R_2 = 0.28\Omega$, determine the variations in the load current and in the power dissipated by the load due to the parallel connection with a second battery.

Analysis:



a) $$I_L = \frac{V_1}{R_1 + R_L} = \frac{12}{0.15 + 2.55} = \frac{12}{2.7} = 4.44 \text{ A}$$

$$P_{Load} = I_L^2 R_L = 50.4 \text{ W.}$$

b) with another source in the circuit we must find the new power dissipated by the load. To do so, we write KVL twice using mesh currents to obtain 2 equations in 2 unknowns:

$$\begin{cases} I_2 R_2 + V_1 - V_2 - I_1 R_1 = 0 \\ (I_1 + I_2) R_L + I_2 R_2 = V_2 \end{cases} \Rightarrow \begin{cases} 0.28 \cdot I_2 - 0.15 \cdot I_1 = 0 \\ 2.55 \cdot (I_1 + I_2) + 0.28 \cdot I_2 = 12 \end{cases}$$

Solving the above equations gives us:

$$I_1 = 2.95 \text{ A}, \quad I_2 = 1.58 \text{ A} \quad \Rightarrow \quad I_L = I_1 + I_2 = 4.53 \text{ A}$$

$$\Rightarrow \quad P_{Load} = I_L^2 R_L = 52.33 \text{W}$$

This is an increase of 1%.

Problem 2.40

Solution:

Known quantities:

Open-circuit voltage at the terminals of the power source is 50.8 V; voltage drop with a 10-W load attached is 49 V.

Find:

a) The voltage and the internal resistance of the source
b) The voltage at its terminals with a 15-Ω load resistor attached
c) The current that can be derived from the source under short-circuit conditions.

Analysis:

(a) $\frac{(49V)^2}{R_L} = 10W \Rightarrow R_L = 240\Omega \quad v_s = 50.8V$, the open-circuit voltage is

$$\frac{R_L}{R_S + R_L} v_S = 49 \Rightarrow \frac{240}{R_S + 240} 50.8 = 49 \qquad \Rightarrow R_S = \frac{(240)(50.8)}{49} - 240 = 8.82\Omega$$

(b) $$v = \frac{R_L}{R_S + R_L} v_S = \frac{15}{8.82 + 15} 50.8 = 32.0V$$

(c) $$i_{CC}(R_L = 0) = \frac{v_S}{R_S} = \frac{50.8}{8.82} = 5.76 \text{ A}$$

Problem 2.41

Solution:

Known quantities:

Voltage of the heater, maximum and minimum power dissipation; number of coils, schematics of the configurations.

Find:

a) The resistance of each coil
b) The power dissipation of each of the other two possible arrangements.

Analysis:

(a) For the parallel connection, $P = 2000$ W. Therefore,

$$2000 = \frac{(220)^2}{R_1} + \frac{(220)^2}{R_2} = (220)^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or,

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{5}{121}.$$

For the series connection, $P = 300$ W. Therefore, $300 = \frac{(220)^2}{R_1 + R_2}$ or, $\frac{1}{R_1 + R_2} = \frac{3}{484}$.

Solving, we find that $R_1 = 131.6\Omega$ and $R_2 = 29.7\Omega$.

(b) the power dissipated by R_1 alone is:

$$P_{R_1} = \frac{(220)^2}{R_1} = 368W$$

and the power dissipated by R_2 alone is

$$P_{R_2} = \frac{(220)^2}{R_2} = 1631W.$$

Section 2.5, 2.6 Resistance and Ohm's Law

Problem 2.42

Solution:

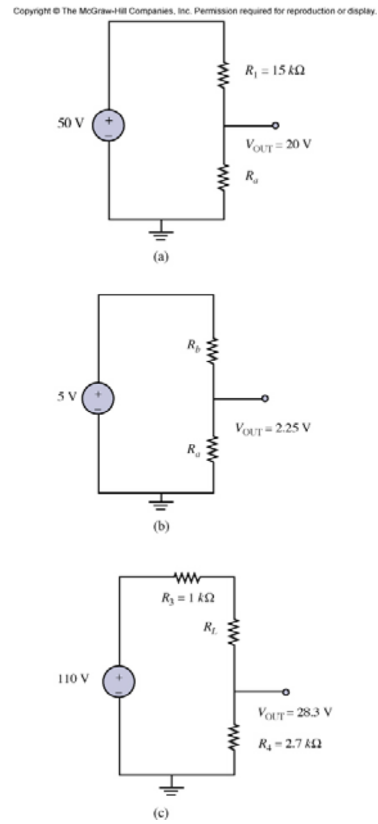
Known quantities:

Circuits of Figure 2.42.

Find:

Values of resistance and power rating

Analysis:



(a) $$20 = \frac{R_a}{R_a + 15{,}000}(50)$$

$$R_a\,(50 - 20) = 20(15)\times 10^3$$

$$R_a = 10\text{k}\Omega$$

$$Pa = I^2 R = \left(\frac{50}{25000}\right)^2 (10{,}000) = 40 \text{ mW}$$

$$P_{R_a} = \frac{1}{8}\text{W}$$

$$P_1 = I^2 R = 60\ mW$$

$$P_{R_1} = \frac{1}{8}\text{W}$$

(b) $$2.25 = 5\times\left(\frac{270}{270 + R_b}\right)$$

$$R_b = 330\Omega$$

$$P_{R_b} = \frac{1}{8}\text{W}$$

$$P_{R_2} = \frac{1}{8}\text{W}$$

c) $$28.3 = 110\times\left(\frac{2.7\times 10^3}{2.7\times 10^3 + 1\times 10^3 + R_L}\right)$$

$$R_L = 6.8k\Omega$$

$$P_{R_L} = 1\text{W}$$

$$P_{R_3} = \frac{1}{8}\text{W}$$

$$P_{R_4} = \frac{1}{2}\text{W}$$

Problem 2.43

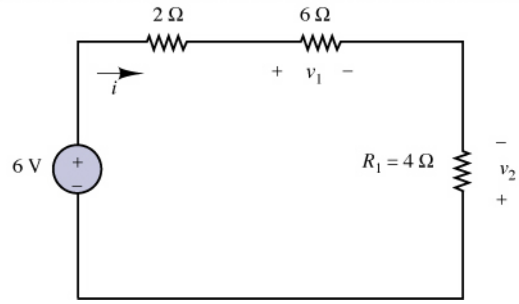
Solution:

Known quantities:

Circuit of Figure 2.43.

Find:

a) equivalent resistance
b) current
c) power delivered
d) voltages
e) minimum power rating for R_1



Analysis:

a) The equivalent resistance seen by the source is

$$R = 2 + 6 + 4 = 12\Omega$$

b) Applying KVL:

$$-6 + 12i = 0$$

Therefore, $i = 0.5$ A .

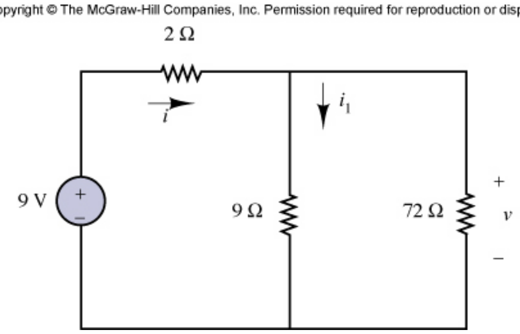
c) $P = vi = 6 \times 0.5 = 3$ W

d) Applying Ohm's law:

$$v_1 = 6i = 3 \text{ V} \quad \text{and} \quad v_2 = -4i = -2 \text{ V} .$$

e) $P_{R_1 \min} = i^2 R_1 = 1$ W .

Problem 2.44



Solution:

Known quantities:

Circuits of Figure 2.44.

Find:

Equivalent resistance and i, i_1, v .

Analysis:

$R_{EQ} = 2 + (9 \,||\, 72) = 10\ \Omega$. Therefore,

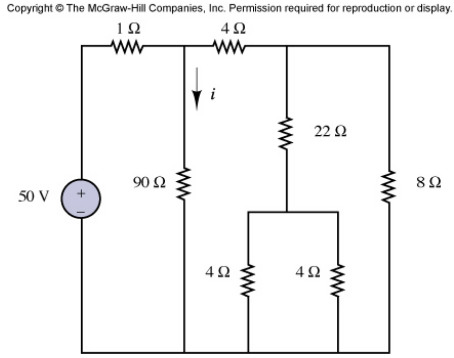
$$i = \frac{9}{10} = 0.9 \text{ A}$$

By the current divider rule:

$$i_1 = \frac{72}{72 + 9}(0.9) = \frac{72}{81}(0.9) = 0.8 \text{ A} .$$

Also, since the 9 Ωand 72 Ω resistors are in parallel, we can conclude that

$$v = 9i_1 = 7.2 \text{ V}$$



Problem 2.45

Solution:

Known quantities:

Circuits of Figure 2.45.

Find:

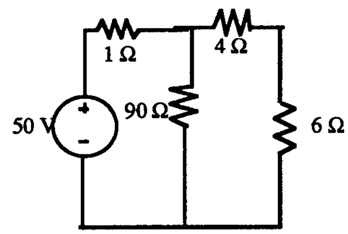
Equivalent resistance and current i

Analysis:

Step1: $(4\|4)+22=24\ \Omega$

Step 2: $24\|8\ =6\ \Omega$

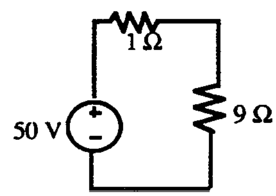
Therefore, the equivalent circuit is as shown in the figure:



Further, $(4+6)\|90=9\ \text{W}$

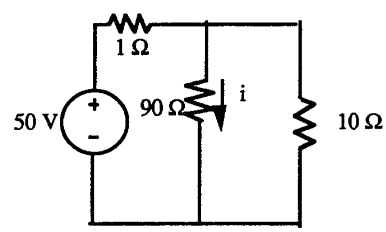
The new equivalent circuit is shown below.

Thus, $R_{total}\ =10\Omega$.

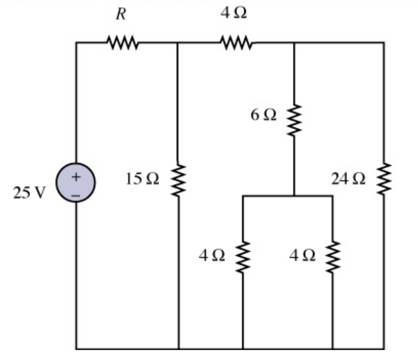


We can now find the current i by the current divider rule as follows:

$$i=\left(\frac{10}{10\,+\,90}\right)(5)\ =\ 0.5\text{ A}$$



Problem 2.46



Solution:

Known quantities:

Circuits of Figure 2.46.

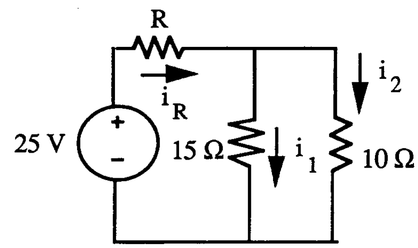
Find:

Resistance *R*

Analysis:

Combining the elements to the right of the 15 Ω resistor, we compute

$$R_{eq} = ((4\|4)+6)\|24 + 4 = 10\ \Omega .$$



The power dissipated by the 15-Ω resistor is

$$P_{15\Omega} = \frac{v^2}{15} = 15\ \text{W} \quad ,$$

therefore, $v_{15\Omega} = 15$ V and $i_1 = 1$ A.

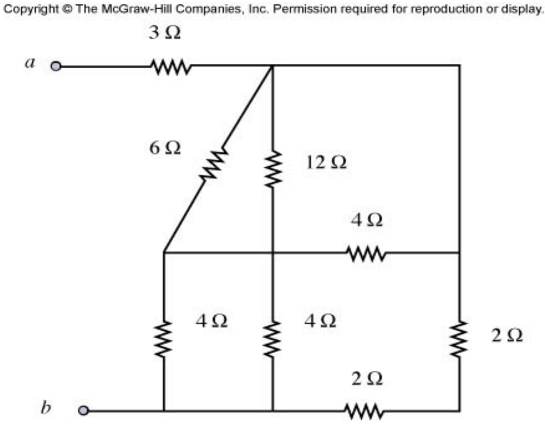
Using the current divider rule: $i_2 = \frac{15}{10}(i_1) = 1.5\ A$.

Applying KCL, we can find i_R: $i_R = i_1 + i_2 = 2.5$ A .

Using KVL: $25 + 2.5R + 15 = 0$

Therefore, R = 4Ω.

Problem 2.47



Solution:

Known quantities:

Circuits of Figure 2.47.

Find:

Equivalent resistance.

Analysis:

$$2\Omega + 2\Omega = 4\Omega$$

$$6\Omega \| 12\Omega = 4\Omega$$

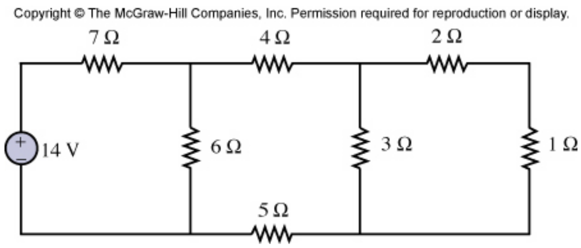
$$4\Omega \| 4\Omega = 2\Omega$$

$$4\Omega \| 4\Omega = 2\Omega$$

$$2\Omega + 2\Omega = 4\Omega$$

$$R_{eq} = 3\Omega + 4\Omega \| 4\Omega = 5\Omega$$

Problem 2.48



Solution:

Known quantities:

Circuits of Figure 2.48.

Find:

a) Equivalent resistance
b) Power delivered.

Analysis:

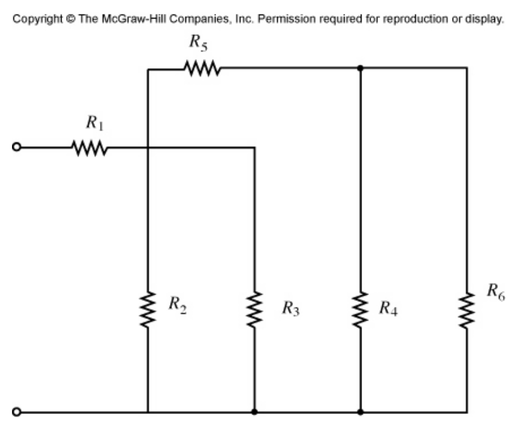
(a)

$$2\Omega + 1\Omega = 3\Omega$$
$$3\Omega \| 3\Omega = 1.5\Omega$$
$$4\Omega + 1.5\Omega + 5\Omega = 10.5\Omega$$
$$10.5\Omega \| 6\Omega = 3.818\Omega$$
$$R_{eq} = 3.818\Omega + 7\Omega = 10.818\Omega$$

(b)

$$I = \frac{14\text{V}}{10.818\Omega} = 1.29\text{A}$$
$$P = (14\text{V})(1.29\text{A}) = 18.06\text{W}$$

Problem 2.49



Solution:

Known quantities:

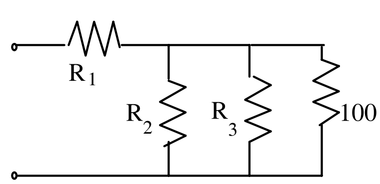
Circuits of Figure 2.49.

Find:

Equivalent resistance.

Analysis:

$(R_6 \| R_4) + R_5 = (1{,}000\,\Omega \| 100\,\Omega) + 9.1\,\Omega \approx 100\,\Omega$, resulting in the circuit shown below.



Therefore, the equivalent resistance is
$R_{eq} = (100 \| R_3 \| R_2) + R_1 = (100 \| 100 \| 1000) + 5 = 52.\ 6\ \Omega$

Problem 2. 50

Solution:

Known quantities:

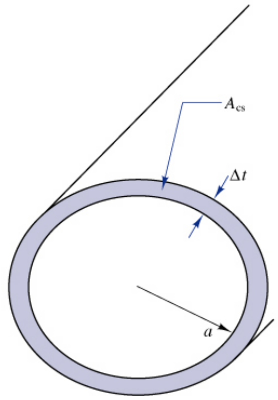
Figure P2.50. Diameter of the cylindrical substrate; length of the substrate; conductivity of the carbon.

Find:

The thickness of the carbon film required for a resistance R of 33 kΩ.

Assumptions:

Assume the thickness of the film to be much smaller than the radius
Neglect the end surface of the cylinder.



Analysis:

$$R = \frac{d}{\sigma \cdot A} \cong \frac{d}{\sigma \cdot 2\pi a \cdot \Delta t}$$

$$\Delta t = \frac{d}{R \cdot 2\pi a \cdot \sigma} = \frac{9 \cdot 10^{-3}\ m}{33 \cdot 10^{3}\ \Omega \cdot 2.9 \cdot 10^{6}\ \frac{S}{m} \cdot 2\pi \cdot 1 \cdot 10^{-3}\ m}$$

Problem 2.51

Solution:

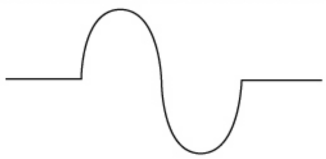
Known quantities:

Figure P2.51. The constants A and k; the open-circuit resistance.

Find:

The rated current at which the fuse blows, showing that this happens at:

$$I = \frac{1}{\sqrt{AkR_0}}.$$



Fuse

Assumptions:

Here the resistance of the fuse is given by:

$$R = R_0\left[1 + A(T - T_0)\right]$$

where T_0, room temperature, is assumed to be 25°C.
We assume that:

$$T - T_0 = kP$$

where P is the power dissipated by the resistor (fuse).

Analysis:

$$R = R_0(1 + A \cdot \Delta T) = R_0(1 + AkP) = R_0(1 + AkI^2 R)$$

$$R - R_0 AkI^2 R = R_0$$

$$R = \frac{R_0}{1 - R_0 AkI^2} \to \infty \quad \text{when} \quad I - R_0 AkI^2 \to 0$$

$$I = \frac{1}{\sqrt{AkR_0}} = \left(0.7\,\frac{m}{°C}\ 0.35\,\frac{°C}{Va}\ 0.11\,\frac{V}{a}\right)^{-\frac{1}{2}} = 6.09\ \text{A}.$$

Problem 2.52

Solution:

Known quantities:

Circuit shown in Figure P2.52 with voltage source, $V_s = 10V$ and resistors, $R_1 = 20\Omega, R_2 = 40\Omega, R_3 = 10\Omega, R_4 = R_5 = R_6 = 15\Omega$.

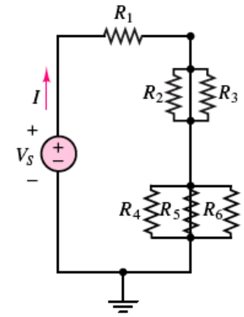


Figure P2.52

Find:

The current in the 15-Ω resistors.

Analysis:

Since the 3 resistors must have equal currents,

$$I_{15\Omega} = \frac{1}{3}\cdot I$$

and,

$$I = \frac{V_S}{R_1 + R_2 \parallel R_3 + R_4 \parallel R_5 \parallel R_6} = \frac{10}{20+8+5} = \frac{10}{33} = 303 \text{ mA}$$

Therefore, $I_{15\Omega} = \frac{10}{99} = 101 \text{ mA}$

Problem 2.53

Solution:

Known quantities:

Schematic of the circuit in Figure P2.13 with currents $I_0 = -2\text{A}$, $I_1 = -4\text{ A}$, $I_S = 8\text{A}$, voltage source $V_S = 12\text{V}$, and resistance $R_0 = 2\ \Omega$.

Find:

The unknown resistances R_1, R_2, R_3, R_4 and R_5.

Assumption:

In order to solve the problem we need to make further assumptions on the value of the resistors. For example, we may assume that $R_4 = \frac{2}{3}R_1$ and $R_2 = \frac{1}{3}R_1$.

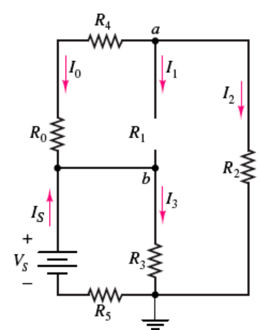


Figure P2.13

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4} = \frac{v_a - v_b}{2 + \frac{2}{3}R_1} = -2$$

$$I_1 = \frac{v_a - v_b}{R_1} = -4$$

$$I_2 = \frac{v_a}{R_2} = \frac{v_a}{\frac{1}{3}R_1} = 6$$

$$I_3 = \frac{v_b}{R_3} = 2$$

$$I_S = \frac{V_S - v_b}{R_5} = \frac{12 - v_b}{R_5} = 8$$

Solving the system we obtain:
$v_a = 3\text{ V}$, $v_b = 9\text{ V}$, $R_1 = 1.5\ \Omega$, $R_2 = 0.5\ \Omega$, $R_3 = 4.5\ \Omega$, $R_4 = 1\ \Omega$ and $R_5 = 0.375\ \Omega$.

Problem 2.54

Solution:

Known quantities:

Schematic of the circuit in Figure P2.13 with resistors $R_1 = 2\Omega$, $R_2 = 5\Omega$, $R_3 = 4\Omega$, $R_4 = 1\Omega$, $R_5 = 3\Omega$, voltage source $V_S = 54$ V, and current $I_2 = 4$ A.

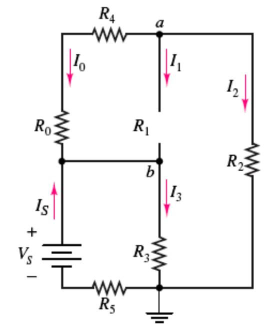


Figure P2.13

Find:

The unknown currents I_0, I_1, I_3, I_S and the resistor R_0.

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4}$$

$$I_1 = \frac{v_a - v_b}{R_1}$$

$$I_2 = \frac{v_a}{R_2} = 4 \quad \Rightarrow \quad v_a = 4 \cdot 5 = 20\text{V}$$

$$I_3 = \frac{v_b}{R_3}$$

$$I_S = \frac{V_S - v_b}{R_5}$$

Applying KCL to node (a) and (b) :

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{20 - v_b}{R_0 + 1} + \dfrac{20 - v_b}{2} + 4 = 0 \\ \dfrac{20 - v_b}{R_0 + 1} + \dfrac{54 - v_b}{3} + \dfrac{20 - v_b}{2} - \dfrac{v_b}{4} = 0 \end{cases}$$

Solving the system we obtain: $v_b = 24$ V and $R_0 = 1\ \Omega$.

Problem 2.55

Solution:

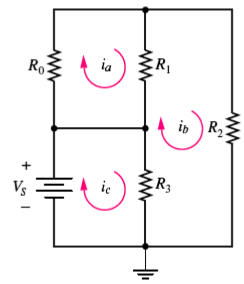


Figure P2.55

Known quantities:

Schematic of the circuit shown in Figure P2.55 with resistors
$R_0 = 2\Omega, R_1 = 1\Omega, R_2 = 4/3\Omega, R_3 = 6\Omega$ and voltage source $V_S = 12\,V$.

Find:

a) The mesh currents i_a, i_b, i_c
b) The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ V_S = (i_c - i_b)R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + (i_a - i_b) = 0 \\ (i_a - i_b) - \frac{4}{3} i_b + 6(i_c - i_b) = 0 \\ 6(i_c - i_b) = 12 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2\text{ A} \\ i_b = 6\text{ A} \\ i_c = 8\text{ A} \end{cases} \Rightarrow \begin{cases} I_{R_0} = i_a = 2\text{ A} & \text{(positive in the direction of } i_a) \\ I_{R_1} = i_b - i_a = 4\text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_2} = i_b = 6\text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_3} = i_c - i_b = 2\text{ A} & \text{(positive in the direction of } i_c) \end{cases}$$

Problem 2.56

NOTE: Typo in Problem Statement for units of R_3. It's Ω, not A.

Solution:

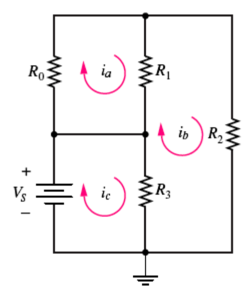


Figure P2.55

Known quantities:

Schematic of the circuit shown in Figure P2.55 with resistors
$R_0 = 2\Omega, R_1 = 2\Omega, R_2 = 5\Omega, R_3 = 4\Omega$ and voltage source $V_S = 24\,V$.

Find:

a) The mesh currents i_a, i_b, i_c
b) The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ V_S = (i_c - i_b)R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + 2(i_a - i_b) = 0 \\ 2(i_a - i_b) - 5i_b + 4(i_c - i_b) = 0 \\ 4(i_c - i_b) = 24 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2 \text{ A} \\ i_b = 4 \text{ A} \\ i_c = 10 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 4 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1(i_b - i_a) = 4 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 20 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3(i_c - i_b) = 24 \text{ V} & (\oplus \text{ up}) \end{cases}$$

Problem 2.57

NOTE: Typo in Problem Statement for units of R_3. It's Ω, not A.

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.55 with resistors $R_0 = 1\Omega, R_1 = 3\Omega, R_2 = 2\Omega, R_3 = 4\Omega$ and of the current source $I_S = 12\,\text{A}$.

Find:

The voltage across each resistance.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ i_c = I_S \end{cases} \Rightarrow \begin{cases} i_a + 3(i_a - i_b) = 0 \\ 3(i_a - i_b) - 2i_b - 4i_b + 48 = 0 \\ i_c = 12 \text{ A} \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = \dfrac{16}{3} \text{ A} \\ i_b = \dfrac{64}{9} \text{ A} \\ i_c = 12 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 5.33 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1(i_b - i_a) = 5.33 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 14.22 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3(i_c - i_b) = 19.55 \text{ V} & (\oplus \text{ up}) \end{cases}$$

Problem 2.58

Solution:

Known quantities:

Schematic of voltage divider network shown of Figure P2.58.

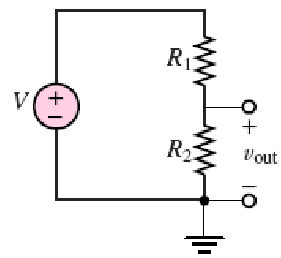


Figure P2.58

Find:

a) The worst-case output voltages for ± 10 percent tolerance
b) The worst-case output voltages for ± 5 percent tolerance

Analysis:

a) 10% worst case: low voltage

$R_2 = 4500\ \Omega$, $R_1 = 5500\ \Omega$

$$v_{OUT,MIN} = \frac{4500}{4500+5500}5 = 2.25V$$

10% worst case: high voltage

$R_2 = 5500\ \Omega$, $R_1 = 4500\ \Omega$

$$v_{OUT,MAX} = \frac{5500}{4500+5500}5 = 2.75V$$

b) 5% worst case: low voltage

$R_2 = 4750\ \Omega$, $R_1 = 5250\ \Omega$

$$v_{OUT,MIN} = \frac{4750}{4750+5250}5 = 2.375V$$

5% worst case: high voltage

$R_2 = 5250\ \Omega$, $R_1 = 4750\ \Omega$

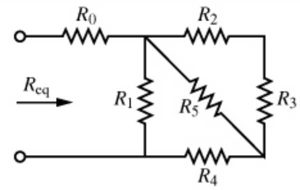
$$v_{OUT,MAX} = \frac{5250}{5250+4750}5 = 2.625V$$

Problem 2.59

Solution:

Known quantities:

Schematic of the circuit shown in figure P2.59 with resistances,
$R_0 = 4\Omega, R_1 = 12\Omega, R_2 = 8\Omega, R_3 = 2\Omega, R_4 = 16\Omega, R_5 = 5\Omega$.

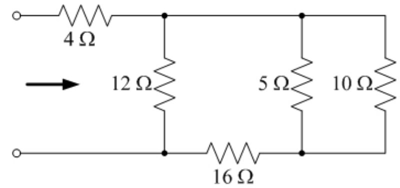


Find:

The equivalent resistance of the circuit.

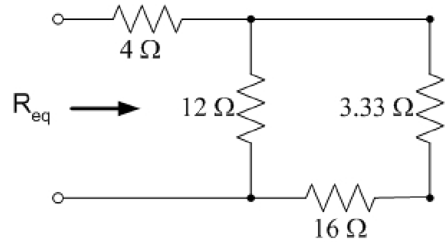
Analysis:

Starting from the right side, we combine the two resistors in series:

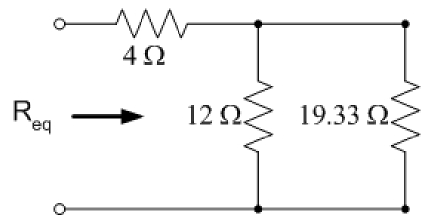


Then, we can combine the two parallel resistors, namely the 5 Ω resistor and 10 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{5} + \frac{1}{10} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = \frac{10}{3}\ \Omega$$

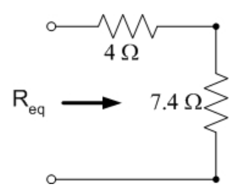


Then, we can combine the two resistors in series, namely the 3.33 Ω and the 16 Ω resistor:



Then, we can combine the two parallel resistors, namely the 12 Ω resistor and 19.33 Ω resistor:

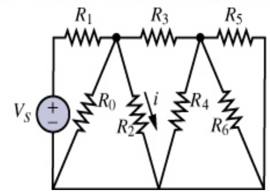
$$\frac{1}{R_{parallel}} = \frac{1}{12} + \frac{1}{19.33} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 7.4\ \Omega$$



Therefore, $R_{eq} = 4 + 7.4 = 11.4\ \Omega$.

Problem 2.60

Solution:



Known quantities:

Schematic of the circuit shown in Figure P2.60 with source voltage, $V_s = 12V$; and resistances, $R_0 = 4\Omega, R_1 = 2\Omega, R_2 = 50\Omega, R_3 = 8\Omega, R_4 = 10\Omega, R_5 = 12\Omega, R_6 = 6\Omega$.

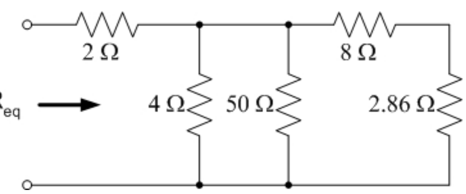
Find:

The equivalent resistance of the circuit seen by the source; the current i through the resistance R_2.

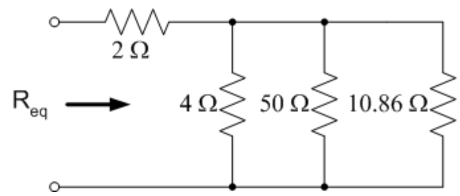
Analysis:

Starting from the right side, we can combine the three parallel resistors, namely the 10 Ω resistor, the 12 Ω resistor and the 6 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{10} + \frac{1}{12} + \frac{1}{6} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = \frac{20}{7}\,\Omega$$

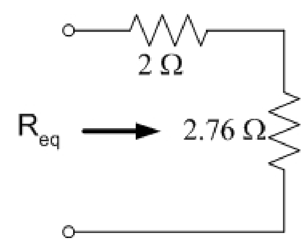


Then, we can combine the two resistors in series, namely the 8 Ω and the 2.86 Ω resistor:



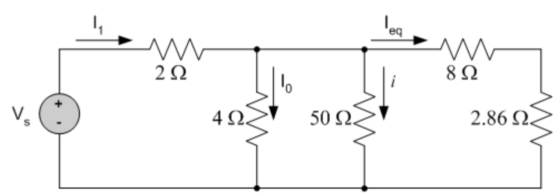
Then, we can combine the three parallel resistors, namely the 4 Ω resistor, the 50 Ω resistor and the 10.86 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{4} + \frac{1}{50} + \frac{1}{10.86} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 2.76\,\Omega$$



Therefore, $R_{eq} = 2 + 2.76 = 4.76\ \Omega$.

We can apply KVL and KCL to the equivalent circuit:



$$\begin{cases} V_S - 2I_1 - 4I_0 = 0 \\ I_1 = I_0 + i + I_{eq} \\ 4I_0 = 50i = 10.86 I_{eq} \end{cases} \Rightarrow \begin{cases} I_0 = 12.5i \\ I_1 = 6 - 25i \\ I_{eq} = 4.604i \\ i = I_1 - I_0 - I_{eq} \end{cases} \Rightarrow$$

$i = 140$ mA

Problem 2.61

Solution:

Known quantities:

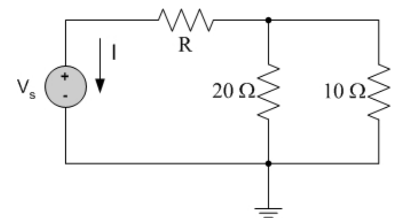
Schematic of the circuit shown in Figure P2.61 with source voltage, $V_s = 50V$; resistances, $R_1 = 20\Omega, R_2 = 5\Omega, R_3 = 2\Omega, R_4 = 8\Omega, R_5 = 8\Omega, R_6 = 30\Omega$; and power absorbed by the 20-Ω resistor.

Find:

The resistance R.

Analysis:

Starting from the right side, we can replace resistors R_i (i=2..6) with a single equivalent resistors:



$$R_{eq} = R_2 + \left(R_3 + \left(R_4 \parallel R_5\right)\right) \parallel R_6 = 10\ \Omega$$

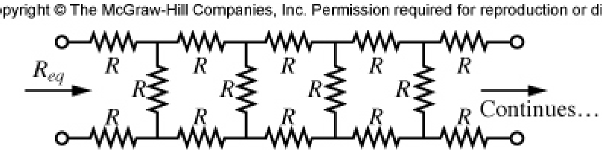
The same voltage appears across both R_1 and R_{eq} and, therefore, these element are in parallel. Applying the voltage divider rule:

$$V_{R_1} = \frac{R_1 \parallel R_{eq}}{R + R_1 \parallel R_{eq}} V_S = \frac{1000}{3R + 20}$$

The power absorbed by the 20-Ω resistor is:

$$P_{20\Omega} = \frac{\left(V_{R_1}\right)^2}{R_1} = \frac{1}{20}\left(\frac{1000}{3R+20}\right)^2 = \frac{50000}{\left(3R+20\right)^2} = 20 \quad \Rightarrow \quad R = 10\ \Omega$$

Problem 2.62



Solution:

Known quantities:

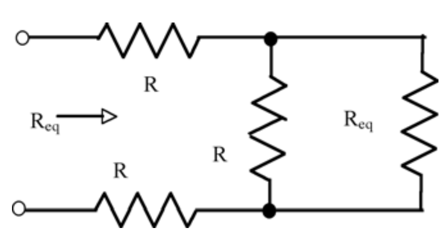
Schematic of the circuit shown in Figure P2.62.

Find:

The equivalent resistance R_{eq} of the infinite network of resistors.

Analysis:

We can see the infinite network of resistors as the equivalent to the circuit in the picture:



Therefore,

$$R_{eq} = R + \left(R \parallel R_{eq}\right) + R = 2R + \frac{RR_{eq}}{R + R_{eq}} \quad \Rightarrow \quad R_{eq} = \left(1 + \sqrt{3}\right)R$$

Problem 2.63

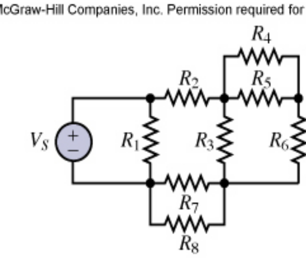
Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.63 with source voltage, $V_s = 110V$; and resistances, $R_1 = 90\Omega, R_2 = 50\Omega, R_3 = 40\Omega, R_4 = 20\Omega, R_5 = 30\Omega, R_6 = 10\Omega, R_7 = 60\Omega, R_8 = 80\Omega$.

Find:

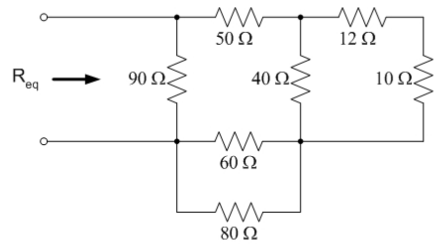
a) The equivalent resistance of the circuit seen by the source.
b) The current through and the power absorbed by the resistance 90-Ω resistance.



Analysis:

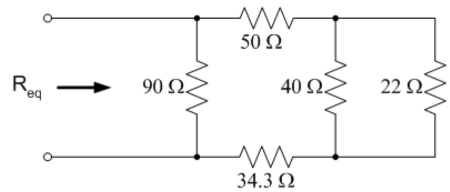
a) Starting from the right side, we can combine the two parallel resistors, namely the 20 Ω resistor and the 30 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{20} + \frac{1}{30} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 12\,\Omega$$



Then we can combine the two parallel resistors in the bottom, namely the 60 Ω resistor and the 80 Ω, and the two resistor in series:

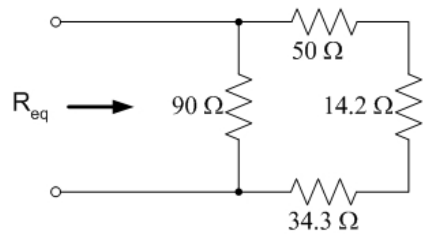
$$\frac{1}{R_{parallel}} = \frac{1}{60} + \frac{1}{80} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 34.3\,\Omega$$



Then we can combine the two parallel resistors on the right, namely the 40 Ω resistor and the 22 Ω:

$$\frac{1}{R_{parallel}} = \frac{1}{40} + \frac{1}{22} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 14.2\,\Omega$$

Therefore, $\frac{1}{R_{eq}} = \frac{1}{90} + \frac{1}{(50 + 14.2 + 34.3)} \left(\Omega^{-1}\right) \Rightarrow R_{eq} = 47\,\Omega$.



b) The current through and the power absorbed by the 90-Ω resistor are:

$$I_{90\Omega} = \frac{V_S}{R_1} = \frac{110}{90} = 1.22 \text{ A}$$

$$P_{90\Omega} = \frac{(V_S)^2}{R_1} = \frac{110^2}{90} = 134.4 \text{ W}$$

Problem 2.64

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.64.

Find:

The equivalent resistance at terminals a,b in the case that terminals c,d are a) open b) shorted; the same for terminals c,d with respect to terminals a,b.

Analysis:

With terminals c-d open, R_{eq}= $(360+540)\|(180+540)\Omega = 400\Omega$,

with terminals c-d shorted, R_{eq}= $(360\|180)+(540\|540)\Omega = 390\Omega$,

with terminals a-b open, R_{eq}= $(540+540)\|(360+180)\Omega = 360\Omega$,

with terminals a-b shorted, R_{eq}= $(360\|540)+(180\|540)\Omega = 351\Omega$.

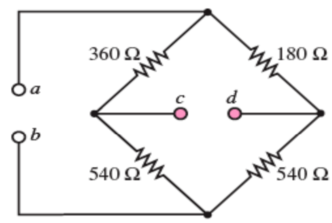


Figure P2.64

Problem 2.65

Solution:

Known quantities:

Layout of the site shown in Figure P2.65; characteristics of the cables; rated voltage of the generator; range of voltages and currents absorbed by the engine at full load.

Find:

The minimum AWG gauge conductors which must be used in a rubber insulated cable.

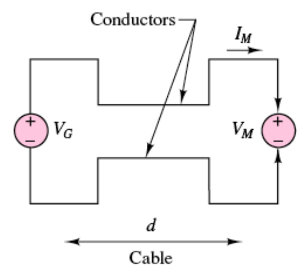


Figure P2.65

Analysis:

The cable must meet two requirements:

1. The conductor current rating must be greater than the rated current of the motor at full load. This requires AWG #14.
2. The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$KVL: \quad -V_G + V_{RC1} + V_{M-Min} + V_{RC2} = 0$$

$$-V_G + I_{M-FL}R_{C1} + V_{M-Min} + I_{M-FL}R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_G - V_{M-Min}}{I_{M-FL}} =$$

$$= \frac{110\ V - 105\ V}{7.103\ A} = 703.9\ m\Omega$$

$$R_{Max} = \frac{R_{C1}}{d} = \frac{R_{C2}}{d} = \frac{\frac{1}{2}[703.9\ m\Omega]}{150\ m} = 2.346\ m\frac{\Omega}{m}$$

Therefore, AWG #8 or larger wire must be used.

Problem 2.66

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.66 with resistances, $R_1 = 2.2k\Omega, R_2 = 18k\Omega$, $R_3 = 220k\Omega, R_4 = 3.3k\Omega$.

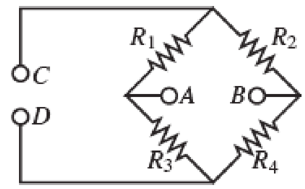


Figure P2.66

Find:

The equivalent resistance between A and B.

Analysis:

Shorting nodes C and D creates a single node to which all four resistors are connected.

$$R_{eq1} = R_1 \| R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{[2.2\ K\Omega][4.7\ K\Omega]}{2.2 + 4.7\ K\Omega} = 1.499\ K\Omega$$

$$R_{eq2} = R_2 \| R_4 = \frac{R_2 R_4}{R_2 + R_4} = \frac{[18\ K\Omega][3.3\ K\Omega]}{18 + 3.3\ K\Omega} = 2.789\ K\Omega$$

$$R_{eq} = R_{eq1} + R_{eq2} = 1.499 + 2.789\ K\Omega = 4.288\ K\Omega$$

Problem 2.67

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.67 with source voltage, $V_s = 12V$; and resistances, $R_1 = 11k\Omega, R_2 = 220k\Omega, R_3 = 6.8k\Omega, R_4 = 0.22m\Omega$

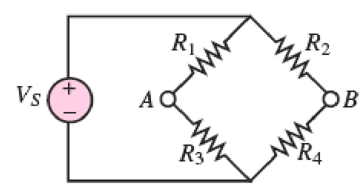


Figure P2.67

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.
SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A GUESS AT THIS POINT.
Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD: \quad V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[12\ V][6.8\ k\Omega]}{11 + 6.8\ k\Omega} = 4.584\ V$$

$$VD: \quad V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[12\ V]\left[0.22 \times 10^{-6}\ k\Omega\right]}{\left(220 + 0.22 \times 10^{-6}\right)k\Omega} = 1.20 \times 10^{-8}\ V \approx 0$$

$$KVL: \quad -V_{R3} + V_{AB} + V_{R4} = 0 \therefore V_{AB} = V_{R3} - V_{R4} = 4.584V$$

The voltage is negative indicating that the polarity of V_{AB} is opposite of that specified.

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.68

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.67 with source voltage, $V_s = 5V$; and resistances, $R_1 = 2.2k\Omega, R_2 = 18k\Omega, R_3 = 4.7k\Omega, R_4 = 3.3k\Omega$

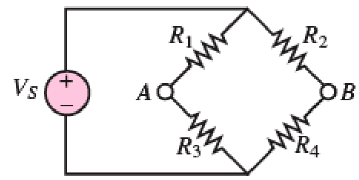


Figure P2.67

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.
SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A GUESS AT THIS POINT.
Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD: V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[5V][4.7K\Omega]}{2.2K\Omega + 4.7K\Omega} = 3.406V$$

$$VD: V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[5V][3.3K\Omega]}{18K\Omega + 3.3K\Omega} = 0.775V$$

$$KVL: -V_{R3} + V_{AB} + V_{R4} = 0 \Rightarrow V_{AB} = V_{R3} - V_{R4} = 2.631V$$

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.69

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.51 with source voltage, $V_s = 12V$; and resistances, $R_1 = 1.7m\Omega, R_2 = 3k\Omega, R_3 = 10k\Omega$.

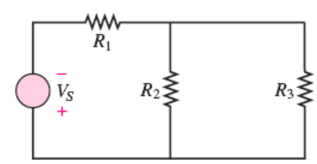


Figure P2.69

Find:

The voltage across the resistance R_3.

Analysis:

The same voltage appears across both R_2 and R_3 and, therefore, these element are in parallel. Applying the voltage divider rule:

$$V_{R_3} = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} V_S = \frac{2.3k\Omega}{1.7m\Omega + 2.3k\Omega} 12 \text{ V} = 11.999991 \text{ V} \quad (\oplus \text{ down})$$

Note that since $R_1 << R_2 \| R_3$, then $V_{R_3} \cong V_S$.

Sections 2.7, 2.8: Practical Sources and Measuring Devices

Problem 2.70

Solution:

Known quantities:

Parameters $R_0 = 300\ \Omega$ (resistance at temperature $T_0 = 298$ K), and $\beta = -0.01\ \mathrm{K}^{-1}$, value of the second resistor.

Find:

a) Plot $R_{th}(T)$ versus T in the range $350 \le T \le 750[°\mathrm{K}]$
b) The equivalent resistance of the parallel connection with the 250-Ω resistor; plot $R_{eq}(T)$ versus T in the range $350 \le T \le 750[°\mathrm{K}]$ for this case on the same plot as part a.

Assumptions:

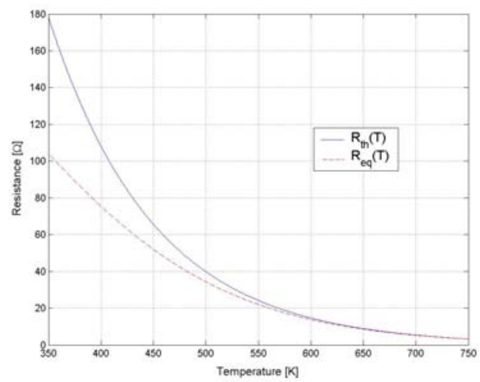
$$R_{th}(T) = R_0 e^{-\beta(T-T_0)}.$$

Analysis:

a) $R_{th}(T) = 300e^{-0.01\cdot(T-298)}$

b) $R_{eq}(T) = R_{th}(T) \,\|\, 250\Omega = \dfrac{1500\ e^{-0.01\ (T-298)}}{5+6\ e^{-0.01\ (T-298)}}$

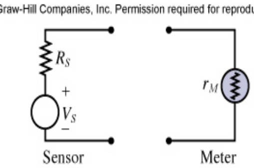
The two plots are shown below.

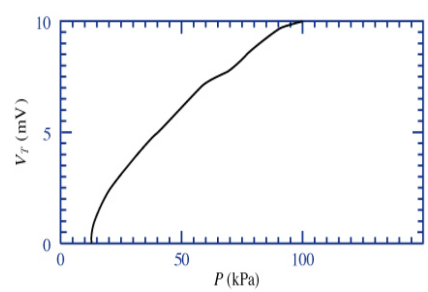


In the above plot, the solid line is for the thermistor alone; the dashed line is for the thermistor-resistor combination.

Problem 2.71

Solution:





Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

a) The circuit required to indicate the pressure measured by a sensor
b) The value of each component of the circuit; the linear range
c) The maximum pressure that can accurately be measured.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.71

Analysis:

a) A series resistor to drop excess voltage is required.
b) At full scale, meter:

$$I_{\hat{m}FS} = 10\mu A$$

$$r_{\hat{m}} = 200\Omega$$

$$0.L.: \quad V_{\hat{m}FS} = I_{\hat{m}FS} r_{\hat{m}} = 2\ mV.$$

at full scale, sensor (from characteristics):

$$P_{FS} = 100 \quad kPa$$
$$V_{TFS} = 9.5 \quad mV$$
$$KVL: \quad -V_{TFS} + V_{RFS} + V_{\hat{m}FS} = 0$$
$$V_{RFS} = V_{TFS} - V_{\hat{m}FS} = 9.5\, mV - 2\, mV = 7.5\, mV$$
$$I_{RFS} = I_{TFS} = I_{\hat{m}FS} = 10 \mu A$$

Ohm law: $R = \dfrac{V_{RFS}}{I_{RFS}} = \dfrac{7.5\, mV}{10\, \mu A} = 750\,\Omega$.

c) from sensor characteristic: 30 kPa –110 kPa.

Problem 2.72

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.72; voltage at terminals with switch open and closed for fresh battery; same voltages for the same battery after 1 year.

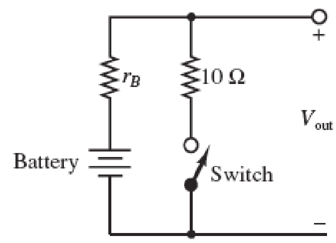


Figure P2.72

Find:

The internal resistance of the battery in each case.

Analysis:

a)

$$V_{out} = \left(\frac{10}{10 + r_B}\right) V_{oc}$$

$$r_B = 10\left(\frac{V_{oc}}{V_{out}} - 1\right) = 10\left(\frac{2.28}{2.27} - 1\right) = 0.044\Omega$$

b)

$$r_B = 10\left(\frac{V_{oc}}{V_{out}} - 1\right) = 10\left(\frac{2.2}{0.31} - 1\right) = 60.97\Omega$$

Problem 2.73

Solution:

Known quantities:

Ammeter shown in Figure P2.73; Current for full-scale deflection; desired full scale values.

Find:

Value of the resistors required for the given full scale ranges.

Analysis:

We desire R_1, R_2, R_3 such that $I_a = 30\ \mu A$ for I = 10 mA, 100 mA, and 1 A, respectively. We use conductances to simplify the arithmetic:

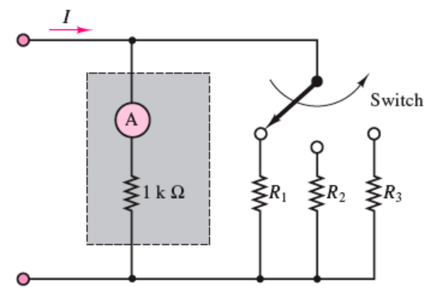


Figure P2.73

$$G_a = \frac{1}{R_a} = \frac{1}{1000}\ S$$

$$G_{1,2,3} = \frac{1}{R_{1,2,3}}$$

By the current divider rule:

$$I_a = \frac{G_a}{G_a + G_x} I$$

or:

$$G_x = G_a\left(\frac{I}{I_a}\right) - G_a \text{ or } \frac{1}{G_x} = \frac{1}{G_a}\left(\frac{I_a}{I - I_a}\right)$$

$$R_x = R_a\left(\frac{I_a}{I - I_a}\right).$$

We can construct the following table:

x	*I*	R_x *(Approx.)*
1	10^{-2} A	3 Ω
2	10^{-1} A	0.3 Ω
3	10^{0} A	0.03 Ω

Problem 2.74

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.74; for part b: value of R_p and current displayed on the ammeter.

Find:

The current *i*; the internal resistance of the meter.

Assumptions:

$$r_a << 50\ k\Omega$$

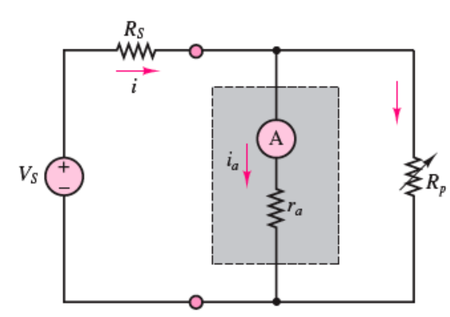


Figure P2.74

Analysis:

a) Assuming that $r_a << 50\ k\Omega$

$$i \approx \frac{V_s}{R_s} = \frac{12}{50000} = 240\ \mu A$$

b) With the same assumption as in part a)

$$i_{meter} = 150 \cdot (10)^{-6} = \frac{R_p}{r_a + R_p} i$$

or:

$$150 \cdot (10)^{-6} = \frac{15}{r_a + 15} 240 \cdot 10^{-6} .$$

Therefore, $r_a = 9\ \Omega$.

Problem 2.75

Solution:

Known quantities:

Voltage read at the meter; schematic of the circuit shown in Figure P2.75 with source voltage, $V_s = 12V$ and source resistance, $R_s = 25k\Omega$.

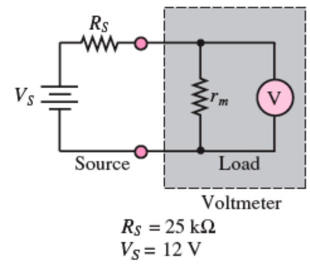


Figure P2.75

Find:

The internal resistance of the voltmeter.

Analysis:

Using the voltage divider rule:

$$V = 11.81 = \frac{r_m}{r_m + R_s} (12)$$

Therefore, $r_m = 1.55\ M\Omega$.

Problem 2.76

Solution:

Known quantities:

Circuit shown in Figure P2.75 with source voltage, $V_S = 24V$; and ratios between R_s and r_m.

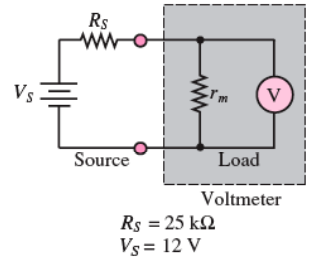


Figure P2.75

Find:

The meter reads in the various cases.

Analysis:

By voltage division:

$$V = \frac{r_m}{r_m + R_s}(24)$$

R_S	V
$0.2\,r_m$	20 V
$0.4\,r_m$	17.14 V
$0.6\,r_m$	15 V
$1.2\,r_m$	10.91 V
$4\,r_m$	4.8 V
$6\,r_m$	3.43 V
$10\,r_m$	2.18 V

For a voltmeter, we always desire $r_m >> R_s$.

Problem 2.77

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.77, values of the components.

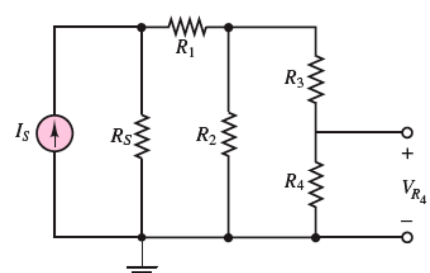


Figure P2.77

Find:

The voltage across R_4 with and without the voltmeter for the following values:

a) $R_4 = 100\Omega$
b) $R_4 = 1k\Omega$
c) $R_4 = 10k\Omega$
d) $R_4 = 100k\Omega$.

Assumptions:

The voltmeter behavior is modeled as that of an ideal voltmeter in parallel with a 120- kΩ resistor.

Analysis:

We develop first an expression for V_{R_4} in terms of R_4. Next, using current division:

$$\begin{cases} I_{R_1} = I_S\left(\dfrac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + R_4)}\right) \\ I_{R_4} = I_{R_1}\left(\dfrac{R_2}{R_2 + R_3 + R_4}\right) \end{cases}$$

Therefore,

$$\begin{aligned} I_{R_4} &= I_S\left(\frac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right) \\ V_{R_4} &= I_{R_4}R_4 \\ &= I_S\left(\frac{R_S R_4}{R_S + R_1 + R_2 \parallel (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right) \\ &= \frac{66000 \cdot R_4}{R_4 + 2.1352\cdot 10^6} \end{aligned}$$

Without the voltmeter:

a) $V_{R_4} = 3.08$ V

b) $V_{R_4} = 30.47$ V

c) $V_{R_4} = 269.91$ V

d) $V_{R_4} = 1260.7$ V.

Now we must find the voltage drop across R_4 with a 120-kΩ resistor across R_4. This is the voltage that the voltmeter will read.

$$\begin{aligned} I_{R_4} &= I_S\left(\frac{R_S}{R_S + R_1 + R_2 \parallel (R_3 + (R_4 \parallel 120k\Omega))}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + (R_4 \parallel 120k\Omega)}\right) \\ V_{R_4} &= I_{R_4}R_4 \\ &= I_S I_S\left(\frac{R_S R_4}{R_S + R_1 + R_2 \parallel (R_3 + (R_4 \parallel 120k\Omega))}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + (R_4 \parallel 120k\Omega)}\right) \\ &= 82.5\frac{(120000 + R_4)\cdot R_4}{7319R_4 + 320.28\cdot 10^6} \end{aligned}$$

With the voltmeter:

a) $V_{R_4} = 3.08$ V b) $V_{R_4} = 30.47$ V

c) $V_{R_4} = 272.57$ V d) $V_{R_4} = 1724.99$ V.

Problem 2.78

Solution:

Known quantities:

Schematic of the circuit shown in Figure P2.78, value of the components.

Find:

The current through R_5 both with and without the ammeter, for the following values of the resistor R_5:

a) $R_5 = 1k\Omega$

b) $R_5 = 100\Omega$

c) $R_5 = 10\Omega$

d) $R_5 = 1\Omega$.

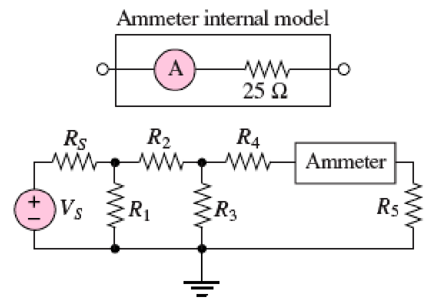


Figure P2.78

Analysis:

First we should find an expression for the current through R_5 in terms of R_5 and the meter resistance, R_m. By the voltage divider rule we have:

$$V_{R_3} = \frac{R_3 \| (R_4 + R_5 + R_m) V_S}{R_3 \| (R_4 + R_5 + R_m) + R_2 + (R_1 \| R_S)}$$

And $$I_{R_3} = \frac{V_{R_3}}{R_4 + R_5 + R_m}$$

Therefore,

$$I_{R_3} = \frac{R_3 \| (R_4 + R_5 + R_m) V_S}{R_3 \| (R_4 + R_5 + R_m) + R_2 + (R_1 \| R_S)} \cdot \frac{1}{R_4 + R_5 + R_m} = \frac{5904}{208350 + 373 \cdot (R_m + R_S)}$$

Using the above equation will give us the following table:

	with meter in circuit	without meter in circuit
a	10.15 mA	9.99 mA
b	24.03 mA	23.15 mA
c	27.84 mA	26.67 mA
d	28.29 mA	27.08 mA

Problem 2.79

Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P2.79, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for Strain gauge is 2

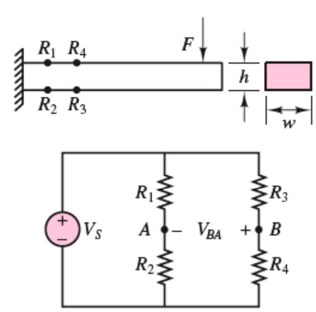


Figure P2.79

Analysis:

R_1 and R_2 are in series; R_3 and R_4 are in series.

Voltage Division: $V_{R_2} = \dfrac{V_S R_2}{R_1 + R_2} = \dfrac{V_S (R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \dfrac{V_S (R_0 - \Delta R)}{2R_0}$

Voltage Division: $V_{R_4} = \dfrac{V_S R_4}{R_3 + R_4} = \dfrac{V_S (R_0 - \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \dfrac{V_S (R_0 + \Delta R)}{2R_0}$

KVL: $-V_{R_2} - V_{BA} + V_{R_4} = 0$

$$V_{BA} = V_{R_4} - V_{R_2} = \frac{V_S (R_0 + \Delta R)}{2R_0} - \frac{V_S (R_0 - \Delta R)}{2R_0} = \frac{V_S 2\Delta R}{2R_0} = V_S GF\varepsilon = \frac{V_S 2 * 6 * LF}{wh^2 Y}$$

assuming GF=2 for aluminum.

$$F = \frac{V_{BA} wh^2 y}{V_s 12L} = \frac{0.050\ V\ (0.025\ m)(0.100\ m)^2\ 69 \times 10^9\ \dfrac{N}{m^2}}{12\ V(12)\ 0.3\ m} = 19.97\ kN.$$

Problem 2.80

Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P2.80, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for Strain gauge is 2

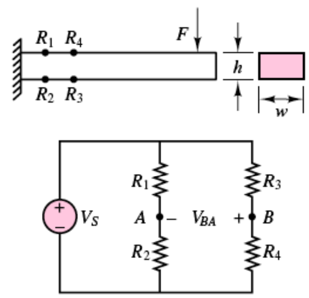


Figure P2.80

Analysis:

R_1 and R_2 are in series; R_3 and R_4 are in series.

VD: $V_{R_2} = \dfrac{V_S R_2}{R_1 + R_2} = \dfrac{V_S (R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \dfrac{V_S (R_0 - \Delta R)}{2R_0}$

VD: $V_{R_4} = \dfrac{V_S R_4}{R_3 + R_4} = \dfrac{V_S (R_0 - \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \dfrac{V_S (R_0 + \Delta R)}{2R_0}$

KVL: $-V_{R_2} - V_{BA} + V_{R_4} = 0$

$$V_{BA} = V_{R_4} - V_{R_2} = \frac{V_S (R_0 + \Delta R)}{2R_0} - \frac{V_S (R_0 - \Delta R)}{2R_0} = \frac{V_S 2\Delta R}{2R_0} = V_S GF\varepsilon = \frac{V_S 2 * 6 * LF}{wh^2 y}$$

Assuming GF=2 for aluminum.

$$F = 1.3 \times 10^6\ \mathrm{N} = \frac{V_{BA} wh^2 y}{V_s 12L} = \frac{V_{BA} (0.03\ \mathrm{m})(0.07\ \mathrm{m})^2\, 200 \times 10^9\ \dfrac{N}{m^2}}{24V(12)1.7m}$$

$$V_{BA} = \frac{1.3 \times 10^6\ \mathrm{N} \times 24V(12)1.7m}{(0.03\ \mathrm{m})(0.07\ \mathrm{m})^2\, 200 \times 10^9\ \dfrac{N}{m^2}} = 21.6\ \mathrm{mV}$$