1) **3.57** For the circuit shown in Fig. P3.57, the switch opens at time $t = 0$ s. Find $v(t)$ and $i(t)$ for all time.

2) **3.58** For the circuit shown in Fig. P3.57, change the value of the capacitor to $\frac{3}{5}$ F. For the resulting circuit, the switch opens at time $t = 0$ s. Find $v(t)$ and $i(t)$ for all time.

3) **3.59** For the circuit shown in Fig. P3.57, change the value of the capacitor to 3 F. For the resulting circuit, the switch opens at time $t = 0$ s. Find $v(t)$ and $i(t)$ for all time.

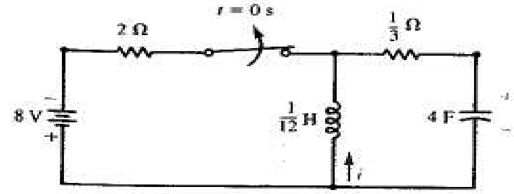
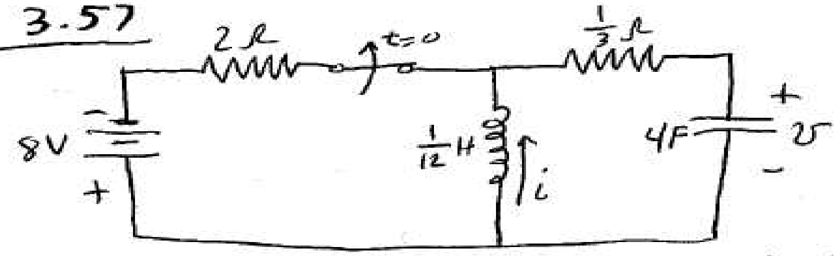


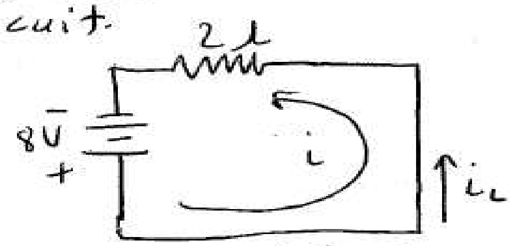
Fig. P3.57

① FoEE 3.57

15 pts

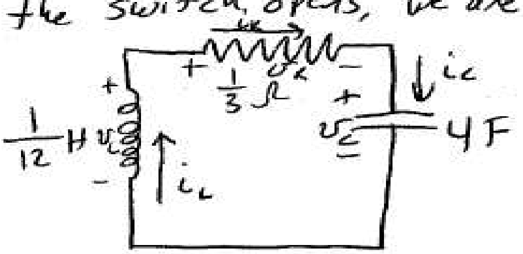


At $t < 0$, it's a DC circuit where the inductor acts as a short circuit and the capacitor is an open circuit.



So $i_L = i = 4A$

As soon as the switch opens, we are left with this circuit.



Here, $i_L = i_R = i_c$

$v_L = v_R + v_c$

We also know:

$i_L(0^+) = 4A$ (instant. change)

$v_c(0^+) = 0V$ (inst. change)

$v_R(0^+) = \frac{1}{3}\cdot 4 = \frac{4}{3}v$ (Ohm's Law)

$v_L(0^+) = \frac{4}{3}v$ (KVL)

$i_c(0^+) = 4A$ (KCL)

$i_R(0^+) = 4A$ (KCL)

Now, what happens after 0^+:

We need to determine if this is overdamped, under-damped or critically damped. Remember, the 2nd order differential equation that satisfies the series RLC circuit is:

$$\frac{d^2v}{dt} + \underbrace{\frac{R}{L}}_{2\alpha}\frac{dv}{dt} + \underbrace{\frac{1}{LC}}_{\omega_n^2}v = 0$$

$$\alpha = \frac{\frac{1}{3}}{\frac{1}{12}\cdot 2} = 2$$

$$\omega_n^2 = \frac{1}{\frac{1}{12}\cdot 4} = 3$$

So, since $\alpha^2 = 4 > \omega_n^2$ this is the "overdamped" case.

$\therefore$ $s_1 = -\alpha - \sqrt{\alpha^2 - \omega_n^2} = -2 - \sqrt{4-3} = -3$

$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_n^2} = -2 + \sqrt{4-3} = -1$

And, our expression for $v_c(t)$ is:

$$v_c(t) = A_1 e^{-3t} + A_2 e^{-t}$$

Now, to solve for A_1 & A_2 we need initial conditions

$$v_c(0) = A_1 + A_2 = 0 \quad (1)$$

$$\frac{dv_c(t)}{dt} = -3A_1 e^{-3t} - A_2 e^{-t}$$

$$i_c(t) = C\frac{dv_c(t)}{dt} = 4\cdot\left[-3A_1 e^{-3t} - A_2 e^{-t}\right]$$

$$i_c(0) = -12A_1 - 4A_2 = 4A \quad (2)$$

(1) & (2) combine to yield: $-8A_1 = 4 \Rightarrow A_1 = -\frac{1}{2}$

$\therefore A_2 = \frac{1}{2}$

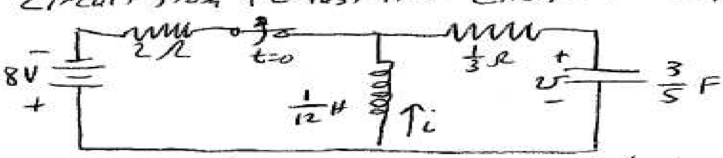
$$\boxed{\begin{aligned} &\text{So, } v_c(t) = -\tfrac{1}{2}e^{-3t} + \tfrac{1}{2}e^{-t} \text{ Volts} \\ &i_c(t) = 4\cdot\tfrac{3}{2}e^{-3t} - 4\tfrac{1}{2}e^{-t} = 6e^{-3t} - 2e^{-t} \text{ Amps} \end{aligned}}$$ for $t \geq 0$

$v_c(t) = 0V$ for $t < 0s$
$i_c(t) = 4A$

2) FoEE 3.58

10 pts.

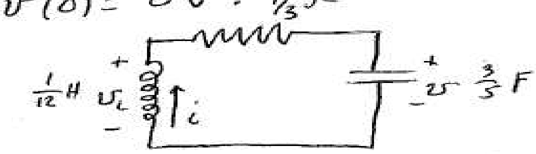
Here we revisit the circuit from the last HW except the capacitor is now $\frac{3}{5}$ F.



Note, the value of the capacitor has no impact on it's behavior at DC, so, the values at $t=0^-$ are the same as in HW 4: #5.

So, $i(0^-) = 4A$ and $v(0) = 0V$.

For $t \geq 0$ we have



$$\alpha = \frac{R}{2L} = \frac{1/3}{2\cdot 1/12} = 2$$

$$\omega_n^2 = \frac{1}{LC} = \frac{1}{\frac{1}{12}\cdot\frac{3}{5}} = 20$$

Since $\alpha^2 < \omega_n^2$ we have an underdamped case

So, $\omega_d = \sqrt{\omega_n^2 - \alpha^2} = 4$ rad/s

So, $v(t) = e^{-2t}(B_1 \cos 4t + B_2 \sin 4t)$

Using boundary conditions: $v(0) = B_1 = 0$.

$\therefore v(t) = e^{-2t}(B_2 \sin 4t)$

$i(t) = C\frac{dv}{dt} = \frac{3}{5}B_2\left[e^{-2t}4\cos 4t - 2e^{-2t}\sin 4t\right]$

$i(0) = \frac{3}{5}B_2[4 - 0] = 4A \quad \therefore B_2 = 5/3$

So, $v(t) = \frac{5}{3}e^{-2t}\sin 4t \; u(t)$ V

$$i(t) = \begin{cases} 4\text{ A} & t < 0 \\ e^{-2t}(4\cos 4t - 2\sin 4t)\text{ A} & \text{for } t \geq 0 \end{cases}$$

3) FoEE 3.59 Here we have the same circuit as in #2, except

10 pts.

$C = 3F$. So, $v(0^-) = 0$, $i(0^-) = 4A$.

$\alpha = 2$ and $\omega_n^2 = \frac{1}{\frac{1}{12}\cdot 3} = 4$.

So $\alpha^2 = \omega_n^2$ $\therefore$ This is a critically damped case.

So, $v(t) = A_1 t e^{-2t} + A_2 e^{-2t}$

Using boundary conditions:

$v(0) = A_1 (0) e^{-2\cdot 0} + A_2 e^{-2\cdot 0} = A_2 = 0$.

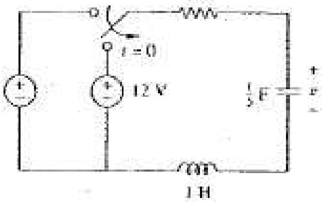
$\therefore v(t) = A_1 t e^{-2t}$

$i(t) = 3\frac{dv}{dt} = 3\cdot A_1 t(-2e^{-2t}) + 3A_1 e^{-2t}$

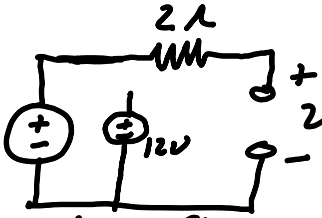
$i(0) = 3A_1 = 4$ $\therefore A_1 = 4/3$

$$\boxed{\begin{aligned} &\text{So, } v(t) = \tfrac{4}{3} t e^{-2t} u(t) \\ &i(t) = \begin{cases} -8te^{-2t} + 4e^{-2t}\ A & \text{for } t \geq 0 \\ 4\ A & \text{for } t < 0 \end{cases} \end{aligned}}$$

4. (15 points) Determine v(t) for t>0 for the following circuit:



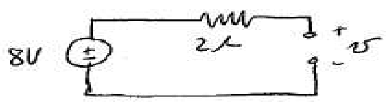
Initial condition:



Since no current can flow $v_c = 8V$

So $v(0^-) = v(0^+) = 8V$

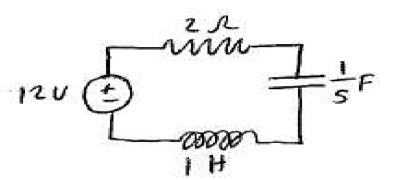
First, we find $v(0)$ by looking at the DC circuit:



$\therefore v(0) = 8V$

$i(0) = 0A$ ← No current flows through an open circuit.

After the switch closes we have the charging case:



$$\frac{d^2v}{dt} + \underbrace{\frac{R}{L}}_{2\alpha = 2}\frac{dv}{dt} + \underbrace{\frac{1}{LC}}_{\omega_n^2 = 5} v = \underbrace{\frac{V}{LC}}_{60}$$

$\therefore \alpha = 1$

Since $\alpha^2 < \omega_n^2$ this is underdamped.

Using the charging solution, we have.

$v(t) = 12 + e^{-t}(A_1 \cos 2t + A_2 \sin 2t)$

$v(0) = 12 + A_1 = 8V$ $\therefore A_1 = -4$

$i(t) = C\frac{dv}{dt} = \frac{1}{5}\Big[e^{-t}(-2A_1 \sin 2t + 2A_2 \cos 2t)$
$+ -e^{-t}(A_1 \cos 2t + A_2 \sin 2t)\Big]$

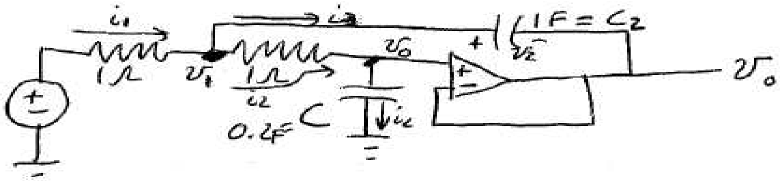
So, $i(0) = \frac{1}{5}(2A_2 - A_1) = 0A$

$= \frac{1}{5}(2A_2 + 4) = 0$ $\therefore A_2 = -2$

$$\boxed{\therefore v(t) = 12 + e^{-t}(-4\cos 2t - 2\sin 2t)\ V \text{ for } t > 0}$$

5 F.o.E.E 3.82

15 pts



After t=0:

$$i_2 = i_c \Rightarrow \frac{v_1 - v_o}{1\Omega} = C\frac{dv_o}{dt} \Rightarrow v_1 = v_o + C\frac{dv_o}{dt}$$

$$2 - v_1 = 2 - v_o - C\frac{dv_o}{dt}$$

$$i_1 = \frac{2 - v_1}{1\Omega}$$

$$i_3 = C_2\frac{dv_2}{dt} = (1F)\frac{d(v_1 - v_o)}{dt}$$

Combining: $i_1 = i_3 + i_2$

$$2 - v_1 = \frac{d(v_1 - v_o)}{dt} + v_1 - v_o$$

$$2 - v_o - C\frac{dv_o}{dt} = \frac{d\left(C\frac{dv_o}{dt}\right)}{dt} + C\frac{dv_o}{dt}$$

$$2 - v_o - C\frac{dv_o}{dt} = C\frac{d^2v_o}{dt^2} + C\frac{dv_o}{dt}$$

$$\therefore \frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + \frac{v_o}{C} = \frac{2}{C}$$

$C = 0.2F$ $\boxed{\text{So, } \frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + 5v_o = 10}$

6. (5 points) 4.5 b

4.5 Find the rectangular form of the sum $\mathbf{A}_1 + \mathbf{A}_2$ for $\mathbf{A}_1$ and $\mathbf{A}_2$ given in Problem 4.3.

4.3 Find the rectangular form of the product $\mathbf{A}_1\mathbf{A}_2$ given that: (a) $\mathbf{A}_1 = 3e^{j30°}$, $\mathbf{A}_2 = 4e^{j60°}$; (b) $\mathbf{A}_1 = 3e^{j30°}$, $\mathbf{A}_2 = 4e^{-j30°}$; (c) $\mathbf{A}_1 = 5e^{-j60°}$, $\mathbf{A}_2 = 2e^{j120°}$; (d) $\mathbf{A}_1 = 4e^{j45°}$, $\mathbf{A}_2 = 2e^{-j30°}$.

$$A_1 = 3e^{j30°} = 3\cos 30 + j3\sin 30° = 3\frac{\sqrt{3}}{2} + \frac{j3}{2}$$

$$A_2 = 4e^{-j30°} = 4\frac{\sqrt{3}}{2} - \frac{j4}{2}$$

$$\boxed{A = \frac{7\sqrt{3}}{2} - \frac{j}{2}}$$

Optional problems

1. (? points) FoEE 3.60

3.60 For the circuit shown in Fig. P3.60, the switch opens at time $t = 0$ s. Find $i(t)$ and $v(t)$ for all time. (See p. 184.)

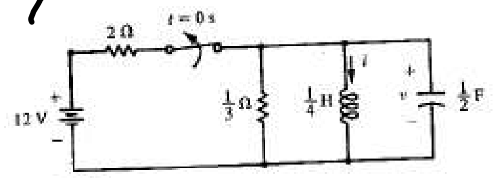
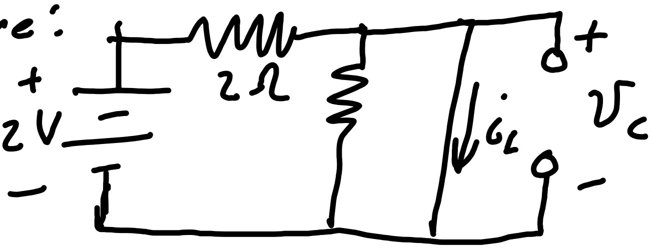


Fig. P3.60

The first thing we can do is find out the initial conditions by looking at $t=0^-$ and using instantaneous change rules.

Here:



At $t=0^-$ we use DC substitutions and replace the inductor with a short and the capacitor with an open.

So, $i_L(0^-) = \frac{12}{2} = 6A$

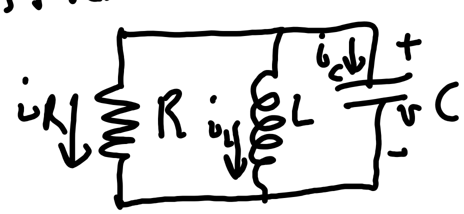
$v_c(0^-) = 0\,V$

These initial conditions are independent of the R, L, C values on the right hand side of the circuit and will hold for 3.60, 61 and 62.

And $i_L(0^+) = i_L(0^-) = 6A$

$v_c(0^+) = v_c(0^-) = 0\,V.$

After $t=0^-$ ... for $t>0$ we have:



$v_c = v_L = v_R$ & $i_R + i_L + i_c = 0$

So, $\frac{v_c}{R} + i_L + C\frac{dv_c}{dt} = 0$

or since $v_c = v_L = L\frac{di_L}{dt}$

$$CL\frac{d^2 i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = 0$$

Now we have things in a standard form and can solve: $2\alpha = \frac{1}{RC}$ so $\alpha = \frac{1}{2RC}$ $\quad \omega_n = \sqrt{\frac{1}{LC}}$

For this problem: $R = \frac{1}{3}\Omega$, $L = \frac{1}{4}H$, $C = \frac{1}{2}F$

So, $\alpha = 3$ and $\omega_n = \sqrt{8}$

Since $\alpha^2 > \omega_n^2$ this is <u>OVERDAMPED</u>

Our roots are: $-3 \pm \sqrt{\alpha^2 - \omega_n^2} = -3 \pm \sqrt{9-8} = -2, -4$

So, our solution is of the form:

$i_L(t) = A_1 e^{-2t} + A_2 e^{-4t}$ with two init. conditions $i_L(0^+) = 6A$, $v_L(0^+) = 0V$

$$v_L(t) = L\frac{d}{dt}\left(A_1 e^{-2t} + A_2 e^{-4t}\right)$$
$$= \frac{1}{4}\left(-2A_1 e^{-2t} - 4A_2 e^{-4t}\right)$$

So, $A_1 + A_2 = 6$ and $-\frac{1}{2}A_1 - A_2 = 0$

So, $A_1 = 12$, $A_2 = -6$

This results in the solution:

$$\boxed{i_L(t) = \begin{cases} 6\,A & \text{for } t < 0 \\ \left(12e^{-2t} - 6e^{-4t}\right)A & t \geq 0 \end{cases} \quad \text{and} \quad v(t) = \begin{cases} 0\,V & t < 0 \\ \left(-6e^{-2t} + 6e^{-4t}\right)V & t \geq 0 \end{cases}}$$

2. (10 points) FoEE 3.61

3.61 For the circuit shown in Fig. P3.60, change the value of the resistor to $\frac{1}{4}\,\Omega$. For the resulting circuit, the switch opens at time $t = 0$ s. Find $i(t)$ and $v(t)$ for all time. (See p. 184.)

Here, the initial analysis is exactly the same. It differs in our solution to the new 2nd order D.E.

Here $\alpha = \frac{1}{2RC} = 2$ $\quad \omega_n = \sqrt{8}$

Since $\alpha < \omega_n$ this is <u>UNDERDAMPED</u>.

So, our solution is of the form: $B_1 \cos\omega_d t + j B_2 \sin\omega_d t$

where $\omega_d = \sqrt{\omega_n^2 - \alpha^2} = \sqrt{8-4} = 2$

So, $i_L(t) = e^{-2t}(B_1 \cos 2t + B_2 \sin 2t)$
$i_L(0) = e^0(B_1 \cdot 1 + B_2 \cdot 0) = B_1 = 6\,A$ ← from init. conditions

$$v_L(t) = \frac{1}{4}\left[e^{-2t}\left(-2(\underbrace{6}_{B_1})\sin 2t + 2B_2 \cos 2t\right) + -2e^{-2t}(\underbrace{6}_{B_1}\cos 2t + B_2 \sin 2t)\right]$$

$$v_L(t) = \frac{1}{4}e^{-2t}\left[(2B_2 - 12)\cos 2t + (-2B_2 - 12)\sin 2t\right]$$

$$v_L(0) = \frac{1}{4}\left[(2B_2 - 12)\right] = 0 \quad \therefore \underline{B_2 = 6}$$

$$\text{so, } i_L(t) = \begin{cases} 6\,A & t < 0 \\ e^{-2t}\left[6\cos 2t + 6\sin 2t\right] A & t \geq 0 \end{cases}$$

$$v_L(t) = \begin{cases} 0\ V & t < 0 \\ -6e^{-2t}\sin 2t\ V & t \geq 0 \end{cases}$$

3. (10 points) FoEE 3.62

3.62 For the circuit shown in Fig. P3.60, change the value of the inductor to $\frac{2}{9}$ H. For the resulting circuit, the switch opens at time $t = 0$ s. Find $v(t)$ and $i(t)$ for all time. (See p. 184.)

Again, initial analysis is the same. Now $\alpha = \frac{1}{2RC} = 3$ and

$\omega_n = \sqrt{\frac{1}{LC}} = 3$ rad/s

So, this is critically damped and our solution is of the form:

$$i_L(t) = (A_1 t e^{-3t} + A_2 e^{-3t})$$

$$i_L(0) = A_2 = 6$$

$$\text{so, } i_L(t) = A_1 t e^{-3t} + 6e^{-3t}$$

$$v_L(t) = \frac{2}{9}\left[A_1 e^{-3t} + A_1 t(-3)e^{-3t} - 18e^{-3t}\right]$$

$$= \frac{2}{9}\left[-3A_1 t + (A_1 - 18)\right]e^{-3t}$$

$$v_L(0) = \frac{2}{9}\left[A_1 - 18\right] = 0 \quad \text{so, } A_1 = 18$$

$$\text{So, } i_L(t) = \begin{cases} 6\,A & t < 0 \\ 18te^{-3t} + 6e^{-3t} & t \geq 0 \end{cases}$$

$$v_L(t) = \begin{cases} 0\ V & t < 0 \\ -12te^{-3t}\ V & t \geq 0 \end{cases}$$