E84 Home Work 6

1. The resistance R of a circuit is a real value which can be measured by a multimeter. However, the impedance Z of a component in the circuit is complex which cannot be measured directly. Instead, one can use an oscilloscope to find sinusoidal voltage $\overline{v(t)}$ across and current $\overline{i(t)}$ through the component, and then obtain the impedance as the ratio between the complex representations of the voltage and current. Suppose we find:

$$v(t) = 12cos(1000t - 30^\circ), \quad i(t) = 6\,cos(1000t + 15^\circ)$$

   Find the impedance (both resistance and reactance) and the admittance (both conductance and susceptance) of the circuit.

   Solution:

   Represent voltage and current in complex forms:

$$v(t) = Re[12e^{j(1000t-30^\circ)}], \quad i(t) = Re[6e^{j(1000t+15^\circ)}]$$

$$Z = 2e^{-j45^\circ} = 2\angle -45^\circ = \sqrt{2} - j\sqrt{2}$$

$$R = \sqrt{2}, \quad X = -\sqrt{2}.$$

$$Y = 1/Z = G + jB = G + jB, \; G = R/(R^2 + X^2) = \sqrt{2}/4,$$

$$\overline{B = -X/\sqrt{R^2 + X^2} = \sqrt{2}/4}$$

2. A voltage $v(t) = 120\sqrt{2}cos(1000t + 90^\circ)V$ (volt) is applied to a resistor $R = 15\Omega$, a capacitor $C = 83.3\mu F$ and an inductor $L = 30\,mH$ connected in parallel. Find the over all steady

state current $i = i_R + i_C + i_L$ by phasor method.

Solution:

Express input voltage as a phasor $\dot{V} = 120\angle 90^\circ$. Then
$\dot{I}_R = \dot{V}/R = 120\angle 90^\circ/15 = 0 + j8A$

$\dot{I}_C = \dot{V}/Z_C = j\omega C\dot{V} = (0.00833\angle 90^\circ)(120\angle 90^\circ) = 10\angle 180^\circ = -10 + j0A$

$\dot{I}_L = \dot{V}/Z_L = \dot{V}/j\omega L = (120\angle 90^\circ)/(30\angle 90^\circ) = 4 + j0A$.

By KCL, we have
$\dot{I} = \dot{I}_R + \dot{I}_C + \dot{I}_L = (0 - 10 + 4) + j(8 + 0 + 0) = -6 + j8 = 10\angle 127^\circ A$

$i(t) = 10\sqrt{2}\, cos(1000t + 127^\circ)A$

3. A voltage $v(t) = 12\sqrt{2}\cos 5000t$ (volt V) is applied to a circuit composed of two branches in parallel. One branch has a capacitor $C = 10\mu F$, while the other has a resistor $R = 20\Omega$ and an inductor $L = 3mH$ in series. Using phasor method, find the impedances Z_C and Z_{RL} of the two branches, and then the overall combined impedance Z_{all} of the circuit. Then find the steady state current $i(t)$ through the circuit.

Solution:

$Z_R = R = 20 + j0 = 20\angle 0^\circ\Omega$, $Z_L = j\omega L = j5000 \times 0.003 = 15\angle 90^\circ\Omega$,
$Z_C = 1/j\omega C = -j/5000 \times 10^{-5} = -j20 = 20\angle -90^\circ\Omega$

$Z_{RL} = Z_R + Z_L = 20 + j15 = 25\angle 37^\circ\Omega$

$Z_{all} = Z_C//Z_{RL} = Z_C Z_{RL}/(Z_C + Z_{RL}) = 25\angle 37^\circ\ 20\angle -90^\circ/(20 + j15 - j20) = 500\angle -53^\circ/20.6\angle -14^\circ = 24.3\angle -39^\circ = 18.9 - j15.3\Omega$

$\dot{I} = \dot{V}/Z = 12\angle 0^\circ/24.3\angle -39^\circ = 0.49\angle 39^\circ A$

$i(t) = 0.49\sqrt{2}cos(5000t + 39^\circ)A$

4. Solve the problem above again but this time use the admittances $Y_C = 1/Z_C$, $Y_{RL} = 1/Z_{RL}$, $Y_{all} = 1/Z_{all}$ (instead of the impedances Z_C, Z_{RL}, Z_{all}). Recall that Ohm's law becomes $\dot{I} = \dot{V}/Z = \dot{V}Y$. (Make sure all impedances you found in previous problem are correct before you find the admittances as their reciprocals.)

   Solution:

   $Y_C = 1/Z_C = 1/20\angle 0^\circ = 0.05\angle 90^\circ = 0 + j0.05\ S$

   $Y_{RL} = 1/Z_{RL} = 1/25\angle 37^\circ = 0.04\angle{-37^\circ} = 0.032 - j0.024\ S$

   $Y_{all} = Y_C + Y_{RL} = 0 + j0.05 + 0.032 - j0.024 = 0.032 + j0.026 = 0.04\angle 39^\circ$

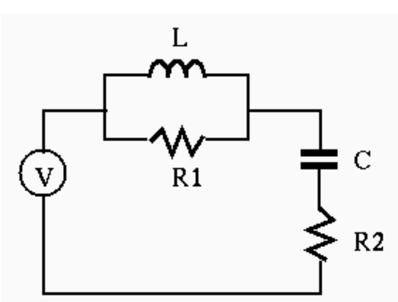
   $I_C = Y_C V = 0.05\angle 90^\circ \times 12\angle 0^\circ = 0.6\angle 90^\circ A$

   $I_{RL} = Y_{RL} V = 0.04\angle{-37^\circ} \times 12\angle 0^\circ = 0.48\angle{-37^\circ} A$

   $I = I_C + I_{RL} = 0 + j0.6 + 0.384 - j0.288 = 0.384 + j0.312 = 0.49\angle 39^\circ$

   Alternatively, $I = Y_{all} V = 0.041\angle 39^\circ \times 12\angle 0^\circ = 0.49\angle 39^\circ$

5. Find the output voltage $v_{out}(t)$ across the right most branch containing R_2 and C, when $\omega = 0$ and $\omega \to \infty$ and the input $v_{in}(t) = V = 10\ cos(\omega t)$, assuming $R_1 = 100\Omega$, $R_2 = 100\Omega$, $C = 10\mu F$ and $L = 10\ mH$.



   Solution

When $\omega = 0$, the inductor is short circuit, and the capacitor is open circuit, $\underline{v_{out}(t) = v_{in}(t)}$. When $\omega = 0$, the inductor is open circuit, and the capacitor is short circuit, $\underline{v_{out}(t) = v_{in}(t)/2 = 5\ cos(\omega_0 t)}$.

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