Chapter 8: Operational Amplifiers – Instructor Notes

Chapter 8 introduces the notion of integrated circuit electronics through the most common building block of electronic instrumentation, the operational amplifier. This is, in practice, the area of modern electronics that is most likely to be encountered by a practicing non-electrical engineer. Thus, the aim of the chapter is to present a fairly complete functional description of the operational amplifier, including a discussion of the principal limitations of the op-amp and of the effects of these limitations on the performance of op-amp circuits employed in measuring instruments and in signal conditioning circuits. **The material presented in this chapter lends itself particularly well to a series of laboratory experiments** (see for example[1]), which can be tied to the lecture material quite readily.

After a brief introduction, in which ideal amplifier characteristics are discussed, open- and closed- loop models of the op-amp are presented in section 8.2; the use of these models is illustrated by application of the basic circuit analysis methods of Chapters 2 and 3. **Thus, the Instructor who deems it appropriate can cover the first two sections in conjunction with the circuit analysis material of Chapters 2 and 3**. A brief, intuitive discussion of feedback is also presented to explain some of the properties of the op-amp in a closed-loop configuration. The closed-loop models include a fairly detailed introduction to the inverting, non-inverting and differential amplifier circuits; however, the ultimate aim of this section is to ensure that the student is capable of recognizing each of these three configurations, so as to be able to quickly determine the closed loop gain of practical amplifier circuits, summarized in Table 8.1 (p. 422). Section 8.2 contains various practical examples, introducing op-amp circuits used in practical instruments, such as the summing amplifier (p. 418), the voltage follower (p. 420)), a differential amplifier (*Focus on Measurements: Electrocardiogram (EKG) Amplifier*, pp. 422-424), the instrumentation amplifier (pp. 424-425), the level shifter (p.426), a new example on the use of op-amps in an analog proportional temperature controller (pp. 427-431) and a sensor calibration circuit (*Focus on Measurements: Sensor calibration circuit,* pp. 431-433). Two features will assist the instructor in introducing practical design considerations: the box *Practical Op-Amp Design Considerations* (p. 434) illustrates some standard design procedures, providing an introduction to a later section on op-amp limitations; the box *Focus on Methodology: Using Op-amp Data Sheets* (pp. 434-436) illustrates the use of device data sheets for two common op-amps. The use of Device Data Sheets is introduced in this chapter for the first time.

In a survey course, these first two sections might be sufficient to introduce operational amplifiers.

Section 8.3 presents the idea of active filters; this material can also be covered quite effectively together with the frequency response material of Chapter 6 to reinforce these concepts. Section 8.4 discusses integrator and differentiator circuits, and presents a new example (an extension of Example 8.5) in which the use of an op-amp integrator in a PI controller is demonstrated in the temperature control problem of Example 8.8 (pp. 444-447). A second application of the op-amp integrator is presented in the charge amplifier (*Focus on Measurements: Charge Amplifiers,* pp. 447-448). The latter example is of particular relevance to the non-electrical engineer, since charge amplifiers are used to amplify the output of piezo-electric transducers in the measurement of strain, force, torque and pressure (for additional material on piezo-electric transducers, see, for example[2]). A brief section (8.5) is also provided on analog computers, since these devices are still used in control system design and evaluation.

Coverage of sections 8.4 and 8.5 is not required to complete section 8.6.

The last section of the chapter, 8.6, is devoted to a discussion of the principal performance limits of the operational amplifier. Since the student will not be prepared to fully comprehend the reason for the saturation, limited bandwidth, limited slew rate, and other shortcomings of practical op-amps, the section focuses on describing the effects of these limitations, and on identifying the relevant parameters on the data sheets of typical op-amps. Thus, the student is trained to recognize these limits, and to include them in the design of practical amplifier circuits. Since some of these limitations are critical even in low frequency applications, it is easy (and extremely useful) to supplement this material with laboratory exercises. The box *Focus on Methodology: Using Op-amp Data Sheets – Comparison of LM 741 and LMC 6061,* pp. 441-446) further reinforces the value of data sheets in realizing viable designs.

The homework problems present a variety of interesting problems at varying levels of difficulty; many of these problems extend the ideas presented in the text, and present practical extensions of the circuits

---

[1] Rizzoni, G., A Practical Introduction to Electronic Instrumentation, 3rd Ed. Kendall-Hunt, 1998

[2] Doebelin E. O., Measurement Systems, McGraw-Hill, Fourth Edition, 1987.

discussed in the examples. In a one-semester course, Chapter 8 can serve as a very effective capstone to a first course in circuit analysis and electronics by stimulating curiosity towards integrated circuit electronics, and by motivating the student to pursue further study in electronics or instrumentation.

The 5th Edition of this book includes 19 new problems; some of the 4th Edition problems were removed, increasing the end-of-chapter problem count from 74 to 91.

Learning Objectives

1. Understand the properties of ideal amplifiers, and the concepts of gain, input impedance, and output impedance. *Section 8.1.*
2. Understand the difference between open-loop and closed-loop op-amp configuration, and compute the gain (or complete the design of) simple inverting, non-inverting, summing and differential amplifiers using ideal op-amp analysis. Analyze more advanced op-amp circuits, using ideal op-amp analysis, and identify important performance parameters in op-amp data sheets. *Section 8.2.*
3. Analyze and design simple active filters. Analyze and design ideal integrator and differentiator circuits. *Sections 8.3 and 8.4.*
4. Understand the structure and behavior of analog computers, and design analog computer circuits to solve simple differential equations. *Section 8.5.*
5. Understand the principal physical limitations of an op-amp. *Section 8.6.*

Section 8.1: Ideal Amplifiers

Problem 8.1

Solution:

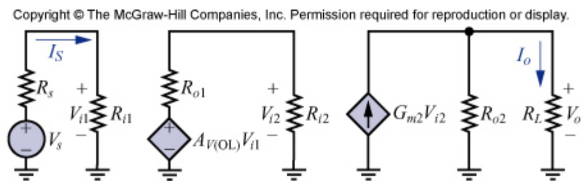
Known quantities:

For the circuit shown in Figure P8.1:

$G = \frac{P_o}{P_S}$ $\quad R_s = 0.6\,\text{k}\Omega$ $\quad R_L = 0.6\,\text{k}\Omega$

$R_{i1} = 3\,\text{k}\Omega$ $\quad R_{i2} = 3\,\text{k}\Omega$ $\quad R_{o1} = 2\,\text{k}\Omega$ $\quad R_{o2} = 2\,\text{k}\Omega$

$A_{vo1} = 100$ $\quad G_{M2} = 350\,\text{m}\Omega^{-1}$ $\quad V_{i2} = V_{I2}$ $\quad V_{i1} = V_{I1}$



Find:

The power gain $G = P_o/P_S$, in dB.

Analysis:

Starting from the last stage and going backward, we get

$$V_O = G_{M2} V_{I2} \frac{R_{o2} R_L}{R_{o2}+R_L} = 161.5\, V_{I2} \quad \Rightarrow \quad P_0 = V_0 I_0 = \frac{V_0^2}{R_L} = 43.49\, V_{I2}^2$$

$$V_{I2} = A_{v01} V_{I1} \frac{R_{i2}}{R_{i2} + R_{01}} = 60\, V_{I1} \quad \Rightarrow \quad P_0 = 1.566 \cdot 10^5\, V_{I1}^2$$

$$V_{I1} = V_S \frac{R_{i1}}{R_{i1} + R_S} = 0.833\, V_S \ \Rightarrow \ P_0 = 1.0875 \cdot 10^5\, V_S^2$$

$$I_S = V_S \frac{1}{R_{i1} + R_S} = 2.778 \cdot 10^{-4}\, V_S \Rightarrow P_S = V_S I_S = 2.778 \cdot 10^{-4} V_S^2$$

$$G = \frac{P_0}{P_S} = \frac{1.0875 \cdot 10^5}{2.778 \cdot 10^{-4}} = 3.915 \cdot 10^8 = 20\,\text{dB}\,\text{Log}_{10}[\,3.915 \cdot 10^8\,] = 171.85\,\text{dB}$$

Problem 8.2

Solution:

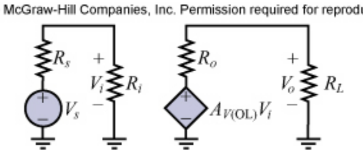
Known quantities:

The temperature sensor shown in Figure P8.2 produces a no load (i.e., sensor current = 0) voltage:

$$v_S = V_{so} \cos \omega t \qquad R_s = 400\ \Omega \qquad V_{so} = 500\ \text{mV} \qquad \omega = 6.28\ \text{k}\frac{\text{rad}}{\text{s}}$$

The temperature is monitored on a display (the load) with a vertical line of light emitting diodes. Normal conditions are indicated when a string of the bottommost diodes 2 cm in length are on. This requires that a voltage be supplied to the display input terminals where: $R_L = 12\ \text{k}\Omega \qquad v_o = V_o \cos \omega t \qquad V_o = 6\ \text{V}$.

The signal from the sensor must therefore be amplified. Therefore, a voltage amplifier is connected between the sensor and CRT with: $R_i = 2\ \text{k}\Omega \qquad R_o = 3\ \text{k}\Omega$.



Find:

The required no load gain of the amplifier.

Analysis:

The overall loaded voltage gain, using the amplitudes of the sensor voltage and the specified CRT voltage must be:

$$A_V = \frac{V_o}{V_{so}} = \frac{6\ \text{V}}{500\ \text{mV}} = 12$$

An expression for the overall voltage gain can also be obtained using two voltage divider relationships:

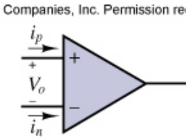
$$V_o = A_{vo} V_{io} \frac{R_L}{R_o + R_L} = A_{vo} [\, V_{so} \frac{R_i}{R_i + R_s} \,] \frac{R_L}{R_o + R_L}$$

$$A_{vo} = \frac{V_o}{V_{so}} = \frac{A_v [\, R_s + R_i \,] [\, R_o + R_L \,]}{R_i R_L} = \frac{[12][0.4 + 2][3 + 12]}{[2][12]} = 18$$

The loss in gain due to the two voltage divisions or "loading" is characteristic of all practical amplifiers. Ideally, there is no reduction in gain due to loading. This would require an ideal signal source with a source resistance equal to zero and a load resistance equal to infinity.

Problem 8.3

Solution:



Known quantities:

Figure P8.3.

Find:

What approximations are usually made about the voltages and currents shown for the ideal operational amplifier [op-amp] model.

Analysis:

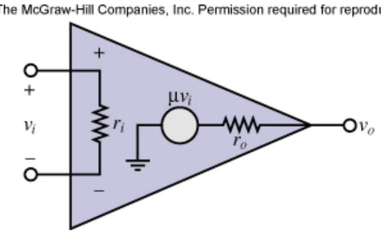
$$i_P \approx 0 \qquad i_N \approx 0 \qquad v_D \approx 0\ .$$

Problem 8.4

Solution:

Known quantities:

Figure P8.4.



Find:

What approximations are usually made about the circuit components and parameters shown for the ideal operational amplifier [op-amp] model.

Analysis:

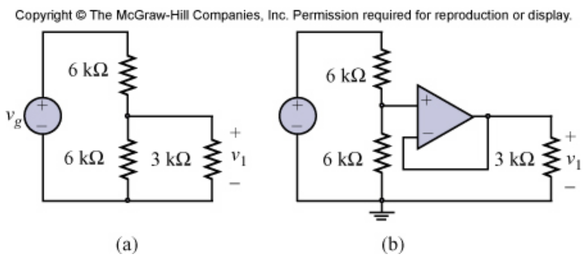
$r_i \approx \infty \qquad \mu \approx \infty \qquad r_o \approx 0$.

Section 8.2: The Operational Amplifier

Problem 8.5

Solution:

Known quantities:
The circuit of Figure P8.5.



Find:
V_1.

Analysis:
(a) Using nodal analysis we can see that in the circuit of Figure P8.5(a)

$$\frac{V_1}{3}+\frac{V_1}{6}+\frac{V_1-V_g}{6}=0 \text{ or } V_1=\frac{V_g}{4}$$

(b) For the circuit of Figure P8.5(b), $V_{6k\Omega}+V_{6k\Omega}=V_g$

and the non-inverting terminal voltage is $V_{6k\Omega}=V_g/2$

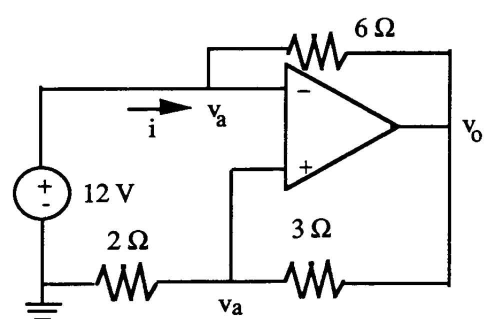
Since the circuit shown is a non-inverting amplifier with unity gain, the output voltage, V_1, is equal to the non-inverting terminal voltage: $V_1=V_g/2$

Problem 8.6

Note: The +/- terminals of the op amp are inverted in Figure P8.6

Solution:

Known quantities:
The circuit of Figure P8.6.



Find:
Current i.

Analysis:
Using nodal analysis for the circuit shown:

$$V_a=12\text{ V}$$
$$(1/2+1/3)V_a-V_o\cdot 1/3=0$$
$$\text{or}\quad V_o=30\text{ V}$$
$$\text{Thus}\quad i=\frac{12-30}{6}=-3\text{ A}$$

Problem 8.7

Note: The +/- terminals of the op amp are inverted in Figure P8.7.

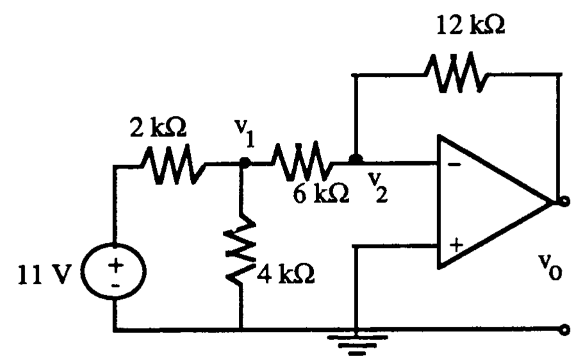
Solution:

Known quantities:

The circuit of Figure P8.7.

Find:

Voltage V_o.



Analysis:

Looking at the figure, we can immediately see that $V_2 = 0$ V. Using nodal analysis:

$$\left(\frac{1}{4000}+\frac{1}{2000}+\frac{1}{6000}\right)V_1-\frac{1}{2000}11=0$$

$$-\left(\frac{1}{6000}\right)V_1-\left(\frac{1}{12000}\right)V_o=0$$

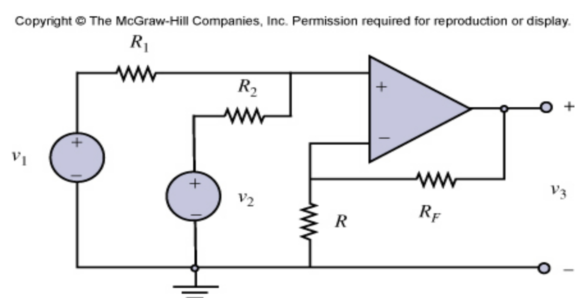
Solving for V_1 and V_o,

$V_1 = 6$ V and $V_o = -12$ V

Problem 8.8

Note: The +/- terminals of the op amp are inverted in Figure P8.8.

Solution:



Known quantities:

The circuit of Figure P8.8.

Find:

Show this circuit is a noninverting summing amplifier.

Analysis:

Applying KCL at the inverting terminal:

$$v_3 = \left(1+\frac{R_f}{R}\right)v^-$$

Applying KCL at the noninverting terminal:

$$\left(\frac{1}{R_2}+\frac{1}{R_1}\right)v^+ - \frac{1}{R_2}v_2 - \frac{1}{R_1}v_1 = 0 \quad \text{or} \quad v^+ = \frac{R_1}{R_1+R_2}v_2 + \frac{R_2}{R_1+R_2}v_1$$

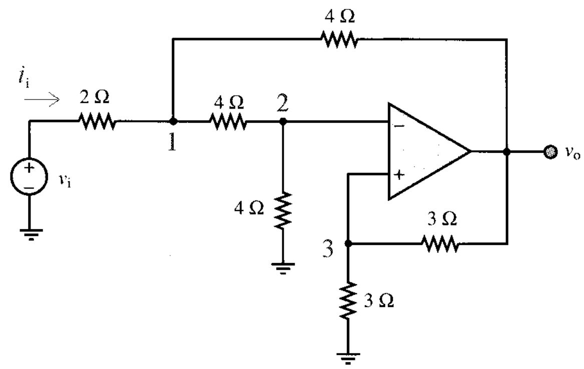
therefore, $v_3 = \left(1+\dfrac{R_f}{R}\right)\left(\dfrac{R_1}{R_1+R_2}v_2 + \dfrac{R_2}{R_1+R_2}v_1\right)$

and the circuit does indeed compute the weighted sum of the inputs.

Problem 8.9

Note: The +/- terminals of the op amp are inverted in Figure P8.9.

Solution:



Known quantities:

The circuit of Figure P8.9.

Find:

The overall gain $A_v = v_o/v_i$; input conductance $G_{in} = i_i/i_o$

Analysis:

Applying KVL at node 1:

$$\frac{v_1 - v_i}{2} + \frac{v_1 - v_2}{4} + \frac{v_1 - v_o}{4} = 0$$

Or $-2v_i + 4v_1 - v_2 = v_o$ (1)

Again, at node 2:

$$\frac{v_2 - v_1}{4} + \frac{v_2}{4} = 0$$

Or $2v_2 = v_1$ (2)

Similarly, at node 3:

$$\frac{v_2}{3} + \frac{v_2 - v_o}{3} = 0$$

Or $v_2 = \frac{1}{2}v_o$ (3)

Substituting (3) into (2):

$v_o = v_1$

Substituting (2) and (3) into (3):

$$-2v_i = v_o + \frac{1}{2}v_o - 8\left(\frac{1}{2}v_o\right)$$

Or $\frac{v_o}{v_i} = \frac{4}{5}$

$\therefore A_v = \frac{4}{5}$

Now, since

$$i_i = \frac{v_i - v_1}{2} = \frac{v_i - v_o}{2} = \frac{v_i - \frac{4}{5}v_i}{2} = \frac{1}{8}v_i$$

then

$$G = \frac{i_i}{v_i} = \frac{1}{8}S$$

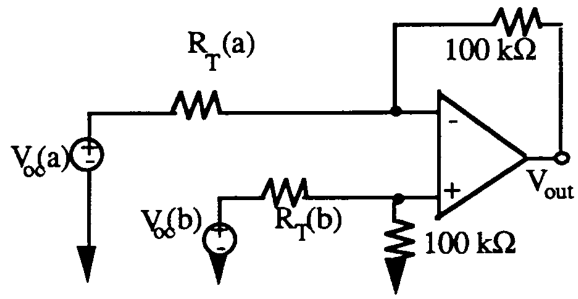
Problem 8.10

Note: The +/- terminals of the op amp are inverted in Figure P8.10.

Solution:

Known quantities:

The circuit of Figure P8.10: $\Delta R = \alpha(\pm \Delta T)$



Find:

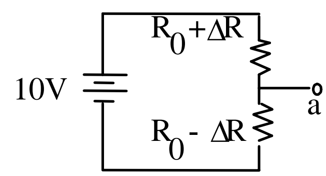
a) Thevenin equivalent seen from point a and b.

b) Expression for $v_{out}(\Delta T)$, if $|\Delta R| = K\Delta T$.

Analysis:

(a) The circuit can be reduced to the following form using Thèvenin equivalent circuits:

The equivalent circuit seen at the inverting node (node a) is:



$$R_o - |\Delta R| = R_o(1 - \frac{|\Delta R|}{R_o})$$

$$R_o + |\Delta R| = R_o(1 + \frac{|\Delta R|}{R_o})$$

The Thèvenin (open circuit) voltage is:

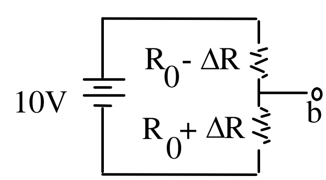
$$V_{OC(a)} = \frac{R_o(1 - \frac{|\Delta R|}{R_o})V_S}{R_o(1 + \frac{|\Delta R|}{R_o} + 1 - \frac{|\Delta R|}{R_o})} = \frac{(1 + \frac{|\Delta R|}{R_o})V_S}{2}$$

and the Thèvenin resistance is:

$$R_{T(a)} = R_o(1 + \frac{|\Delta R|}{R_o}) \,||\, R_o(1 + \frac{|\Delta R|}{R_o}) = \frac{R_o}{2}(1 - (\frac{|\Delta R|}{R_o})^2)$$

The equivalent circuit seen at the non-inverting node (node b) is:

The Thèvenin (open circuit) voltage is:



$$V_{OC(b)} = \frac{(1 + \frac{|\Delta R|}{R_o})V_S}{2}$$

$$R_{T(b)} = \frac{R_o}{2}(1 - (\frac{|\Delta R|}{R_o}))$$

Assuming $|\Delta R|^2 << R_o$

$$V_{OC(a)} = (1 - \frac{|\Delta R|}{R_o}) \cdot 5 \quad \text{V}$$

$$R_{T(a)} = 500 \quad \Omega$$

$$V_{OC(b)} = (1 + \frac{|\Delta R|}{R_o}) \cdot 5 \quad \text{V}$$

$R_{T(b)} = 500 \;\; \Omega$

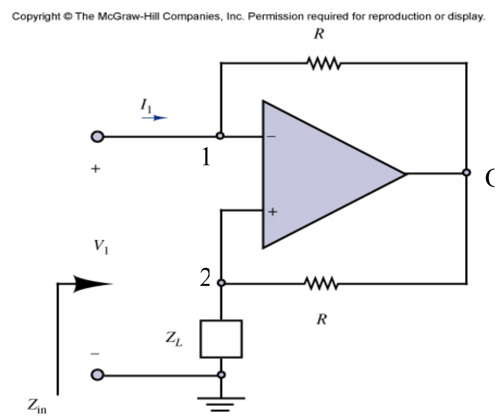
(b) For a difference amplifier:

$$V_{out} = \frac{R_f}{R_s}(V_{OC(b)} - V_{OC(a)})$$
$$= \frac{100^5}{500}(V_{OC(b)} - V_{OC(a)})$$
$$= 200 \cdot 5 \cdot (1 + \frac{|\Delta R|}{R_o} - (1 - \frac{|\Delta R|}{R_o}))$$
$$= 2|\Delta R|$$

But if $|\Delta R| = K\Delta T$, then

$V_{out} = 2K\Delta T$

Problem 8.11



Solution:

Known quantities:

The circuit of Figure 8.11.

Find:

The impedance of $Z_{in} = V_1 / I_1$

Assumptions:

Ideal op-amp open-loop voltage gain A is very big.

Analysis:

Applying KVL to node 1,

$V_o = V_1 - RI_1$ (1)

Applying KVL to node 2,

$\frac{V_2}{Z_L} = \frac{V_o - V_2}{R}$ (2)

At the same time, $V^+ = V^- \;\Rightarrow\; V_2 = V_1$ (3)

From (2) and (3), we have:

$\frac{V_1}{Z_L} = \frac{V_o - V_1}{R}$ (4)

From (1) and (4), we have:

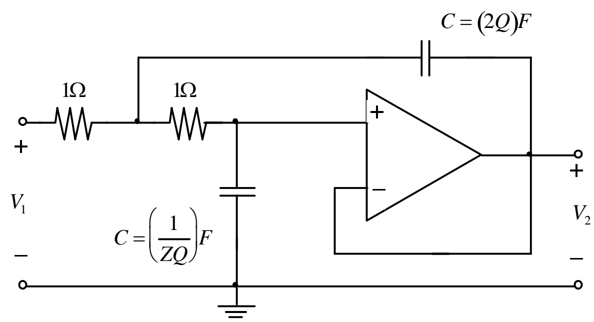
$\frac{V_1}{Z_L} = -I_1 \Rightarrow Z_{in} = \frac{V_1}{I_1} = -Z_L$ (5)

So,

a. when $Z_L = R, \;\; \Rightarrow Z_{in} = -R$
b. when $Z_L = 1/j\omega C, \;\; \Rightarrow Z_{in} = -1/j\omega C$

Problem 8.12

Solution:



Known quantities:

The circuit of Figure P8.12. C_1=2Q, C_2=1/2Q.

Find:

Determine the gain function V_{out}/V_{in}

Analysis:

Applying KVL at node 1:

$$\frac{V_{in}-V_1}{1}+\frac{V_{out}-V_1}{Z_1}=\frac{V_1-V_2}{1} \qquad (1)$$

Again, at node 2:

$$\frac{V_1-V_2}{1}=\frac{V_2}{Z_2} \quad \Rightarrow V_1=(1+1/Z_2)V_2 \quad (2)$$

And easy to get:

$$V_{out}=V_2 \qquad (3)$$

Substituting (3) and (3) into (1):

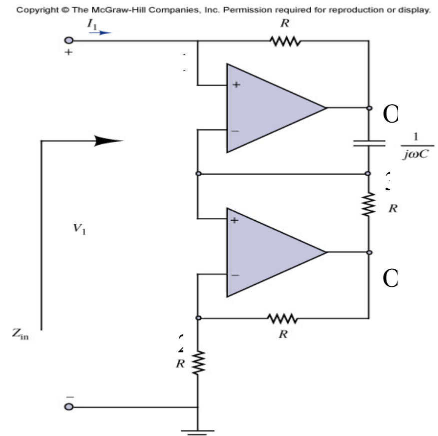
$$\frac{V_{out}}{V_{in}}=\frac{Z_1Z_2}{Z_1Z_2+2Z_1+1}$$

Note that $Z_1=1/j\omega 2Q, Z_2=2Q/j\omega$

We have $\dfrac{V_{out}}{V_{in}}=\dfrac{1}{(j\omega)^2+j\omega/Q+1}$

Problem 8.13

Solution:



Known quantities:

The circuit of P8.13

Find:

$$Z_{in} = \frac{V_1}{I_1}$$

Analysis:

We know that the voltage at point 2 is equal to V_1. Thus, the current through from point 2 is V_1/R. Hence the current from O2 to point 2 is also V_1/R. Hence the voltage at O2 is $2V_1$.

Since the voltage at point 3 is V_1, the current from O2 to point 3 is V_1/R. Bu thit must also be the current from point 3 to O1; hence the voltage at O1 is $V_1 + (V_1/R)(1/j\omega C)$. The voltage from O1 to point 1 is therefore $(V_1/R)(1/j\omega C)$. The current through the top resistor R is therefore $(V_1/R)(1/j\omega C)/R$. Thus, the current I_1 must be equal and opposite:

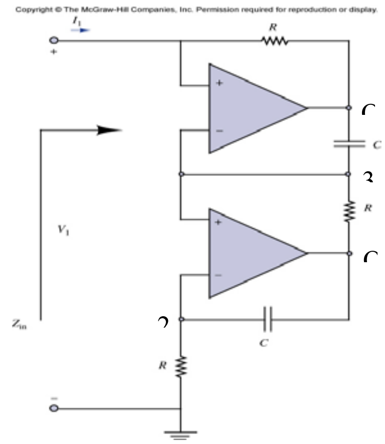
$I_1 = -(V_1/R)(1/j\omega C)/R$

And the impedance is

$$Z_{in} = \frac{V_1}{I_1} = -j\omega CR^2$$

Hence, this circuit behaves as a negative inductance!

Problem 8.14



Solution:

Known quantities:

The circuit of P8.14

Find:

$$Z_{in} = \frac{V_1}{I_1}$$

Analysis:

We know that the voltage at point 2 is equal to V_1. Thus, the current through from point 2 to ground is V_1/R. Hence the current from O2 to point 2 is also V_1/R. Hence the voltage at O2 is $V_1 + V_1/(j\omega RC)$.

Since the voltage at point 3 is V_1, the current from O2 to point 3 is $(V_1 + V_1/(j\omega RC) - V_1) = V_1/(j\omega R^2C)$. Bu this must also be the current from point 3 to O1; hence the voltage at O1 is $V_1 + (1/j\omega C)V_1/(j\omega R^2C) = V_1 + V_1/(-\omega^2R^2C^2)$ The voltage from O1 to point 1 is therefore $V_1/(-\omega^2R^2C^2)$, The current through the top resistor R is therefore $V_1/(-\omega^2R^3C^2)$. Thus, the current I_1 must be equal and opposite:

$I_1 = V_1/(\omega^2R^3C^2)$.

And the impedance is

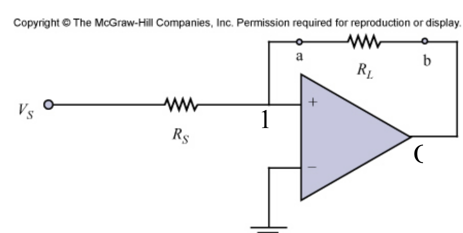
$$Z_{in} = \frac{V_1}{I_1} = \omega^2C^2R^3$$

Hence, this circuit behaves as a frequency-dependent resistor!

Problem 8.15

Note: The +/- terminals of the op amp are inverted in Figure P8.15.

Solution:



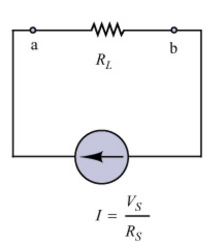
Known quantities:

The circuit of Figure P8.15. open-loop gain A.

Find:

Show that I_{R_L} is independent on R_L.

Analysis:



Applying KVL at node 1, we have

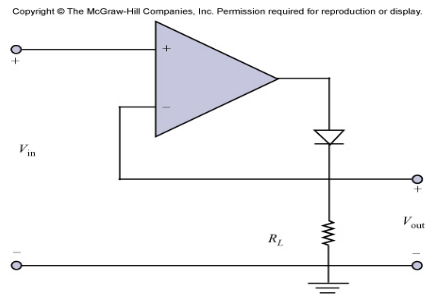
$$\frac{V_s - V^+}{R_s} = \frac{V^+ - V_o}{R_L} \qquad (1)$$

Because $V^- = 0 \Rightarrow A(V^+ - V^-) = AV^+ = V_o \qquad (2)$

With (1) and (2), we have

$V^+ = \dfrac{V_s}{(1-A)\,R_s/R_L+1} \approx 0$, So $I_{R_L} = \dfrac{V_s - V^+}{R_s} = \dfrac{V_s}{R_S}$, it's independent on R_L, and is constant.

Problem 8.16



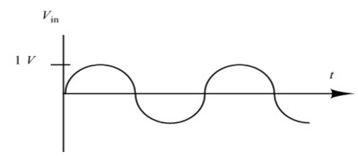
Solution:

Known quantities:

The circuit of P8.16.

Find:

Determine the output signal for the given input $V_{in}(t)$



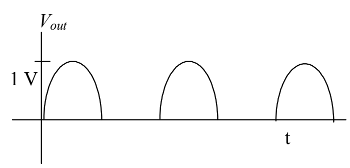
Analysis:

$$\left.\begin{array}{l} V^- \approx V^+ = V_{in} \\ I^- \approx 0 \end{array}\right\} \Rightarrow \ if \quad V_{in} > 0 \qquad V_{out} = V^- = V_{in}$$

The output will follow the input;

When $V_{in} < 0$, the diode should be "on", but the current through the resistor would want to flow upwards, which is counter to the diode-on condition. Hence $V_{out} = 0$

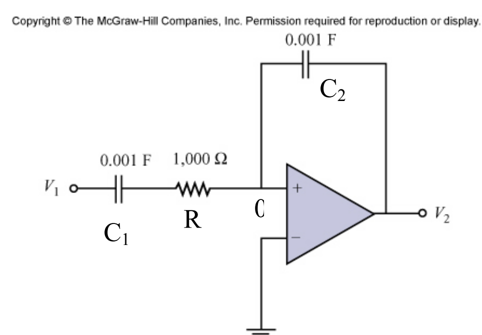
The V_{out} waveform is shown on the right.



Problem 8.17

Note: The +/- terminals of the op amp are inverted in Figure P8.17.

Solution:



Known quantities:

The circuit of Figure P8.17.

Find:

$\dfrac{V_2}{V_1}$.

Analysis:

Applying KVL at node 0:

$$\frac{V_1 - V_0}{R + Z_1} = \frac{V_0 - V_2}{Z_2} \qquad (1)$$

And $V_2 = A(V^+ - V^-) = AV_0 \quad (2)$

From (1) and (2), $\dfrac{V_2}{V_1} = \dfrac{1}{\dfrac{R+Z_1}{Z_2}(\dfrac{1}{A}-1)+\dfrac{1}{A}}$

For a ideal op-amp, A>>1, so $\dfrac{V_2}{V_1} = -\dfrac{Z_2}{R+Z_1} = -\dfrac{\dfrac{C_1}{C_2}}{RC_1 j\omega + 1} = -\dfrac{1}{s+1}$

Problem 8.18

Solution:

Known quantities:

T and *N*.

Find:

Design an approximate time delay circuit for T=1 and N=4 in Euler's definition $\left[\dfrac{1}{\dfrac{sT}{N}+1}\right]^N$.

Analysis:

According the solutions of Problem 8.17, we have

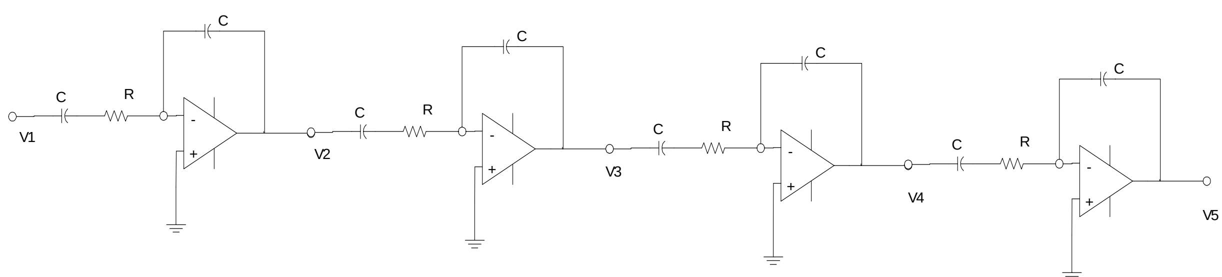
$$\frac{V_2}{V_1} = -\frac{\dfrac{C_1}{C_2}}{RC_1 s + 1}$$

Let $\dfrac{C_1}{C_2} = 1$, $RC_1 = 1/4$,and if we cascade 4 such op-amps with the output of each one be the input of the next one,

We can have the final output be :

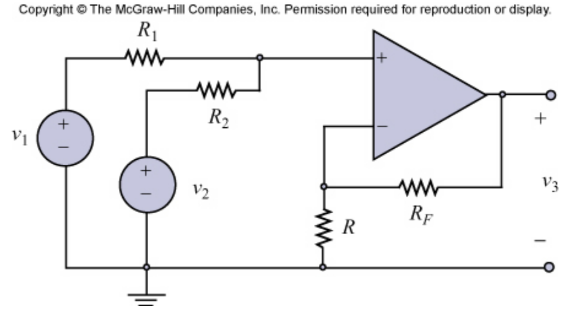
$$\frac{V_5}{V_1} = \frac{V_5}{V_4}\cdot\frac{V_4}{V_3}\cdot\frac{V_3}{V_2}\cdot\frac{V_2}{V_1} = \left(-\frac{1}{\dfrac{s}{4}+1}\right)\cdot\left(-\frac{1}{\dfrac{s}{4}+1}\right)\cdot\left(-\frac{1}{\dfrac{s}{4}+1}\right)\cdot\left(-\frac{1}{\dfrac{s}{4}+1}\right) = \left[\frac{1}{\dfrac{s}{4}+1}\right]^4$$

We can use $C_1 = C_2 = 0.001$ F, $R = 250\,\Omega$



Problem 8.19

Note: The +/- terminals of the op amp are inverted in Figure P8.19.



Solution:

Known quantities:
The circuit of Figure P8.19.

Find:
Show that this circuit is a noninverting summer.

Analysis:

Applying KCL at the inverting terminal: $V_3 = (1+\frac{R_f}{R})V^-$

Applying KCL at the noninverting terminal:

$$(\frac{1}{R_2}+\frac{1}{R_1})V^+ - \frac{V_2}{R_2} - \frac{V_1}{R_1} = 0$$

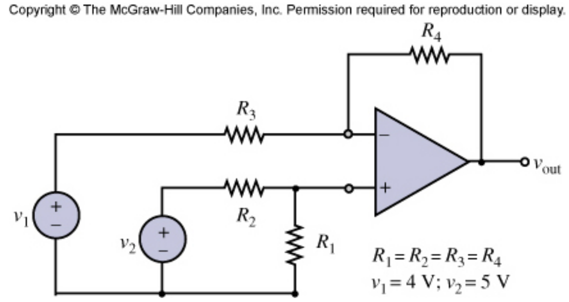
or $$V^+ = \frac{R_1}{R_1+R_2}V_2 + \frac{R_2}{R_1+R_2}V_1$$

therefore,

$$V_3 = (1+\frac{R_f}{R})(\frac{R_1}{R_1+R_2}V_2 + \frac{R_2}{R_1+R_2}V_1)$$

and the circuit does indeed compute the weighted sum of the inputs.

Problem 8.20



Solution:

Known quantities:
The circuit of Figure P8.20.

Find:
The voltage v and the current i.

Analysis:

The circuit is shown on the right:

Applying nodal analysis to V^+, $\frac{V_2 - V^+}{R_2} = \frac{V^+}{R_1}$ So $V^+ = \frac{V_2}{2} = 2.5$ V

Applying nodal analysis to V^-,

$$\frac{V_1 - V^-}{R_3} = \frac{V^- - V_{out}}{R_4}$$ and $V^+ = V^-$, So $V_{out} = V_1 - 2V^- = 1\,\text{V}$

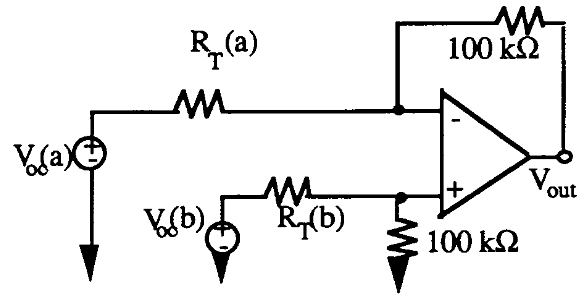
Problem 8.21

Note: The +/- terminals of the op amp are inverted in Figure P8.21.

Solution:

Known quantities:

The circuit of Figure P8.21: $\Delta R = \alpha(\pm\Delta T)$



Find:

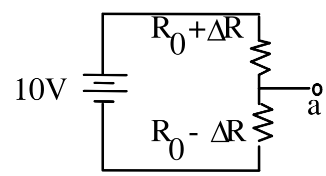
c) Thevenin equivalent seen from point a and b.

d) Expression for $v_{out}(\Delta T)$, if $|\Delta R| = K\Delta T$.

Analysis:

(a) The circuit can be reduced to the following form using Thèvenin equivalent circuits:

The equivalent circuit seen at the inverting node (node a) is:



$$R_o - |\Delta R| = R_o(1 - \frac{|\Delta R|}{R_o})$$

$$R_o + |\Delta R| = R_o(1 + \frac{|\Delta R|}{R_o})$$

The Thèvenin (open circuit) voltage is:

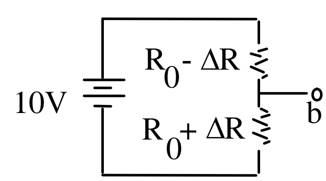
$$V_{OC(a)} = \frac{R_o(1 - \frac{|\Delta R|}{R_o})V_S}{R_o(1 + \frac{|\Delta R|}{R_o} + 1 - \frac{|\Delta R|}{R_o})} = \frac{(1 + \frac{|\Delta R|}{R_o})V_S}{2}$$

and the Thèvenin resistance is:

$$R_{T(a)} = R_o(1 + \frac{|\Delta R|}{R_o}) \,||\, R_o(1 + \frac{|\Delta R|}{R_o}) = \frac{R_o}{2}(1 - (\frac{|\Delta R|}{R_o})^2)$$

The equivalent circuit seen at the non-inverting node (node b) is:

The Thèvenin (open circuit) voltage is:



$$V_{OC(b)} = \frac{(1 + \frac{|\Delta R|}{R_o})V_S}{2}$$

$$R_{T(b)} = \frac{R_o}{2}(1 - (\frac{|\Delta R|}{R_o}))$$

Assuming $|\Delta R|^2 << R_o$

$$V_{OC(a)} = (1 - \frac{|\Delta R|}{R_o}) \cdot 5 \quad \text{V}$$

$$R_{T(a)} = 500 \quad \Omega$$

$$V_{OC(b)} = (1+\frac{|\Delta R|}{R_o})\cdot 5 \quad \text{V}$$

$$R_{T(b)} = 500 \quad \Omega$$

(b) For a difference amplifier:

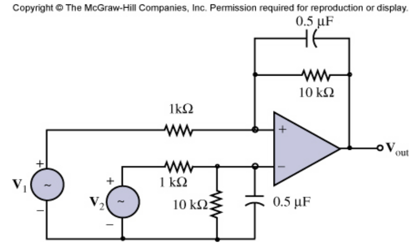
$$V_{out} = \frac{R_f}{R_s}(V_{OC(b)} - V_{OC(a)})$$
$$= \frac{100^5}{500}(V_{OC(b)} - V_{OC(a)})$$
$$= 200\cdot 5\cdot(1+\frac{|\Delta R|}{R_o} - (1-\frac{|\Delta R|}{R_o}))$$
$$= 2|\Delta R|$$

But if $|\Delta R| = K\Delta T$, then

$$V_{out} = 2K\Delta T$$

Problem 8.22

Note: The +/- terminals of the op amp are inverted in Figure P8.22.



Solution:

Known quantities:

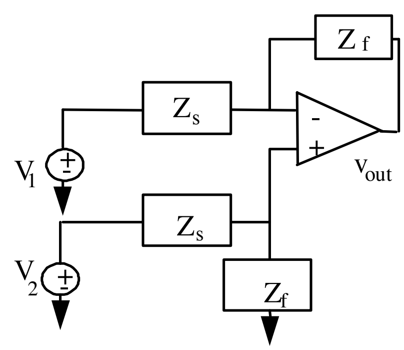
The circuit of Figure P8.22.

Find:

The peak amplitude and phase shift of $v_{out}(t)$.

Analysis:

Replacing the components with impedances Z_f and Z_S yields the circuit shown below.



Recognizing that the circuit is a differential amplifier we can directly write:

$$v_{out}(t) = \frac{Z_f}{Z_s}(V_1 - V_2)$$

$$Z_s = 1000 \ \Omega \qquad Z_f = \frac{10000}{1+j\omega 5\cdot 10^{-3}}$$

$$V_{out} = \frac{10}{1+j\omega 5\cdot 10^{-3}}(V_1 - V_2)$$

(a) Evaluating this voltage at ω=1,000, we have $v_{out}(t) = 1.961\cos(1000t - 78.7^\circ)$

Thus, the peak value is 1.961 V.

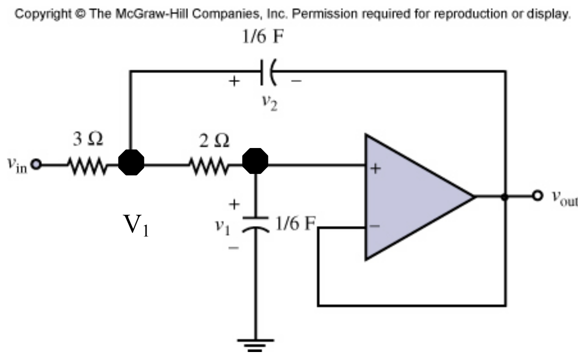
(b) The phase shift of $v_{out(t)}$ is $\phi = -\tan^{-1}(\frac{1000(5\cdot 10^{-3})}{1}) = -78.7$

Problem 8.23

Solution:

Known quantities:
The circuit of Figure P8.23.

Find:
The gain of the circuit.



Analysis:

Using nodal analysis at the two nodes shown in the figure,

$$\left(\frac{1}{2}+\frac{1}{3}+\frac{j\omega}{6}\right)V_1-\frac{1}{3}V_{in}-\frac{j\omega}{6}V_{out}=0$$

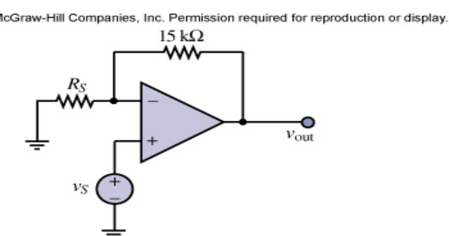
$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{j\omega}{6}\right)V_{out}=0$$

Therefore,

$$\frac{V_{out}}{V_{in}}=-\frac{6}{\omega^2-j5\omega-6}$$

Problem 8.24

Solution:



Known quantities:
The circuit shown in Figure P8.24.

Find:
The resistor R_s that will accomplish the nominal gain requirement, and state what the maximum and minimum values of R_s can be. Will a *standard* 5 percent tolerance resistor be adequate to satisfy this requirement?

Analysis:
For a non-inverting amplifier the voltage gain is given by:

$$A_v=\frac{v_{out}}{v_{in}}=\frac{R_f+R_s}{R_s}=\frac{R_f}{R_s}+1 \quad ; \quad A_{vnom}=16=\frac{R_f}{R_s}+1 \quad \Rightarrow \quad R_s=\frac{R_f}{16-1}=1\text{k}\Omega$$

To find the maximum and minimum R_S we note that $R_s \propto \frac{1}{A_v}$, so to find the maximum R_S we consider the minimum A_v: $R_{s\max}=\frac{15\text{k}\Omega}{16(1-0.02)-1}=1.02\text{k}\Omega$.

Conversely, to find the minimum R_S we consider the maximum A_v: $R_{s\min}=\frac{15\text{k}\Omega}{16(1+0.02)-1}=980\Omega$.

Since a standard 5% tolerance 1-kΩ resistor has resistance *950 < R < 1050*, a standard resistor will not suffice in this application.

Problem 8.25

Solution:

Known quantities:

The values of the two 10 percent tolerance resistors used in an inverting amplifier:
$R_F = 33\,\text{k}\Omega$; $R_S = 1.2\,\text{k}\Omega$.

Find:

a. The nominal gain of the amplifier.
b. The maximum value of $|A_v|$.
c. The minimum value of $|A_v|$.

Analysis:

a. The gain of the inverting amplifier is: $A_v = -\frac{R_f}{R_s} = \frac{-33}{1.2} = -27.5$.

b. First we note that the gain of the amplifier is proportional to R_f and inversely proportional to R_S. This tells us that to find the maximum gain of the amplifier we consider the maximum R_f and the minimum R_S.

$$|A_v|_{\max} = \frac{R_{f\max}}{R_{S\min}} = \frac{33 + 0.1(33)}{1.2 - 0.1(1.2)} = 33.6 .$$

c. To find $|A_v|_{\min}$ we consider the opposite case: $|A_v|_{\min} = \frac{R_{f\min}}{R_{S\max}} = \frac{33 - 0.1(33)}{1.2 + 0.1(1.2)} = 22.5$.

Problem 8.26

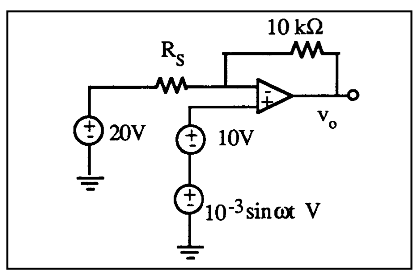
Solution:

Known quantities:

For the circuit shown in Figure P8.26, let $v_1(t) = 10 + 10^{-3}\sin(\omega t)\,\text{V}$, $R_F = 10\,\text{k}\Omega$, $V_{batt} = 20\,\text{V}$.

Find:

a. The value of R_S such that no DC voltage appears at the output.
b. The corresponding value of $v_{out}(t)$.



Analysis:

a. The circuit may be modeled as shown:

Applying the principle of superposition:

For the 20-V source:

$$v_o\big|_{20} = \frac{-10\text{k}\Omega}{R_S}(20)$$

For the 10-V source: $v_o\big|_{10} = \left(\frac{10\text{k}\Omega}{R_S} + 1\right)(10)$

The total DC output is: $v_o\big|_{DC} = v_o\big|_{20} + v_o\big|_{10} = -\frac{10,000}{R_S}(20) + \left(\frac{10,000}{R_S} + 1\right)(10) = 0$

Solving for R_S $\frac{10,000}{R_S}(10-20) = -10 \Rightarrow R_S = 10\,\text{k}\Omega$.

b. Since we have already determined R_S such that the DC component of the output will be zero, we can simply treat the amplifier as if the AC source were the only source present. Therefore,

$$v_o(t) = 0.001\sin(\omega t)\cdot\left(\frac{R_f}{R_S} + 1\right) = 0.001\sin(\omega t)\cdot(1+1) = 2\cdot 10^{-3}\sin(\omega t)\,\text{V}.$$

Problem 8.27

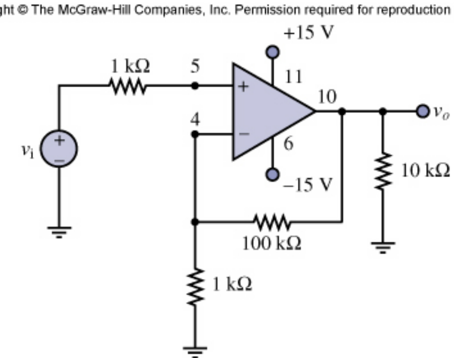
Solution:

Known quantities:

For the circuit shown in Figure P8.27, let:

$R = 2\,\text{M}\Omega \quad A_{V(OL)} = 200,000 \quad R_0 = 50\Omega$

$R_S = 1\text{k}\Omega \quad R_1 = 1\text{k}\Omega \quad R_2 = 100\text{k}\Omega \quad R_{LOAD} = 10\text{k}\Omega$



Find:

The gain $A_V = \frac{v_0}{v_i}$

Analysis:

The op-amp has a very large input resistance, a very large "open loop gain" [$A_{V(OL)}$], and a very small output resistance. Therefore, it can be modeled with small error as an ideal op-amp.
The amplifier shown in Figure P8.10 is a noninverting amplifier, so we have

$$A_V = 1 + \frac{R_2}{R_1} = 1 + \frac{100\cdot 10^3}{1\cdot 10^3} = 101$$

Problem 8.28

Solution:

Known quantities:

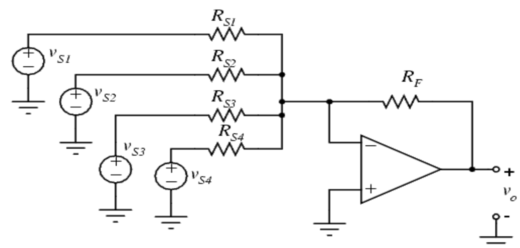
$v_{out}(t) = -(2\sin\omega_1 t + 4\sin\omega_2 t + 8\sin\omega_3 t + 16\sin\omega_4 t)$ V, $R_F = 5$ kΩ .

Find:

Design an inverting summing amplifier to obtain $v_{out}(t)$ and determine the require source resistors.

Analysis:

The inverting summing amplifier is shown in the following figure.



By superposition and by selecting

$R_{Si} = \dfrac{R_F}{2^i}$, $v_{Si} = \sin\omega_i t, i = 1\ldots 4$, the output voltage is

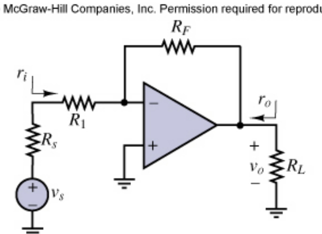
$$v_{out} = -\sum_{i=1}^{4}\frac{R_F}{R_{Si}}v_{Si} = -\sum_{i=1}^{4}2^i v_{Si} = -\sum_{i=1}^{4}2^i \sin\omega_i t$$

that coincides with the desired output voltage.
So, the required source resistors are

$$R_{S1} = \frac{R_F}{2} = 2.5\text{k}\Omega, R_{S2} = \frac{R_F}{4} = 1.25\text{k}\Omega, R_{S3} = \frac{R_F}{8} = 625\Omega, R_{S4} = \frac{R_F}{16} = 312.5\Omega$$

Problem 8.29

Solution:



Known quantities:

For the circuit shown in Figure P8.29:

$r_i = 2$ MΩ $\mu = 200{,}000$ $r_0 = 25\Omega$

$R_S = 2.2\text{k}\Omega$ $R_1 = 1\text{k}\Omega$ $R_F = 8.7\text{k}\Omega$ $R_L = 20\Omega$

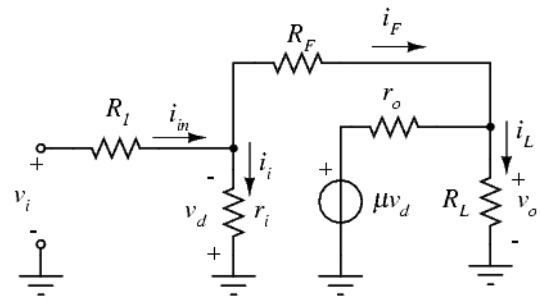
Find:

a. An expression for the input resistance v_i/i_i including the effects of the op-amp.
b. The value of the input resistance in including the effects of the op-amp.
c. The value of the input resistance with ideal op-amp.

Analysis:

a. The circuit in Figure P8.29 can be modeled as in the following figure
where

$v_i = -v_d + R_1 i_i$,
$v_d = -r_i(i_i - i_F)$,
$i_F = -\dfrac{v_d + v_0}{R_F}$,
$v_0 = \mu v_d + r_0(i_F - \dfrac{v_0}{R_L})$.



Substituting the expression for v_d in all the other equations, we obtain

$v_i = (R_1 + r_i)i_i - r_i i_F$,

$i_F = \dfrac{r_i i_i - v_0}{R_F + r_i}$,

$v_0 (1 + \dfrac{r_0}{R_L}) = -\mu r_i i_i + (\mu r_i + r_0) i_F$.

By solving the second equation above for v_0 and substituting in the third equation

$v_i = (R_1 + r_i)i_i - r_i i_F$,

$$i_F = \frac{\left(1 + \frac{r_0}{R_L} + \mu\right) r_i}{\left(1 + \frac{r_0}{R_L}\right)(R_F + r_i) + \mu r_i + r_0} i_i$$

and finally,

$$\frac{v_i}{i_i} = (R_1 + r_i) - \frac{\left(1 + \frac{r_0}{R_L} + \mu\right) r_i^2}{\left(1 + \frac{r_0}{R_L}\right)(R_F + r_i) + \mu r_i + r_0}$$

b. The value of the input resistance is

$$\frac{v_i}{i_i} = (R_1 + r_i) - \frac{\left(1 + \frac{r_0}{R_L} + \mu\right) r_i^2}{\left(1 + \frac{r_0}{R_L}\right)(R_F + r_i) + \mu r_i + r_0} = 2.001 \cdot 10^6 - \frac{\left(1 + \frac{25}{20} + 2 \cdot 10^5\right) 4 \cdot 10^{12}}{\left(1 + \frac{25}{20}\right) 2.0087 \cdot 10^6 + 4 \cdot 10^{11} + 25} =$$

$= 1.0001 \text{k}\Omega$

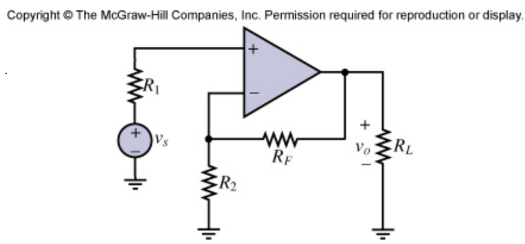
c. In the ideal case:

$\dfrac{v_i}{i_i} = R_1 = 1\text{k}\Omega$

Problem 8.30

Solution:

Known quantities:



For the circuit shown in Figure P8.30:

$v_S(t) = 0.02 + 10^{-3}\cos(\omega t)\text{V}$, $R_F = 220\,\text{k}\Omega$, $R_1 = 47\text{k}\Omega$, .

.

Find:

a) expression for the output voltage.
b) The corresponding value of $v_o(t)$.

Analysis:

a) the circuit is a noninverting amplifier, then

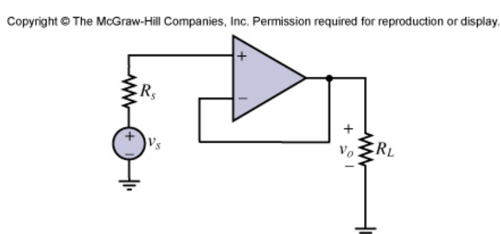
$$v_0 = \left(1 + \frac{R_F}{R_2}\right) v_S$$

b) $v_0 = \left(1 + \dfrac{220}{1.8}\right) v_S = 2.464 + 0.1232\cos(\omega t) \quad \text{V}$

Problem 8.31

Solution:

Known quantities:



For the circuit shown in Figure P8.31:

$v_S(t) = 0.05 + 30 \cdot 10^{-3}\cos(\omega t)\text{V}$, $R_S = 50\Omega$, $R_L = 200\Omega$.

Find:

The output voltage v_o.

Analysis:

It is a particular case of a noninverting amplifier where $v_0 = v_S$.

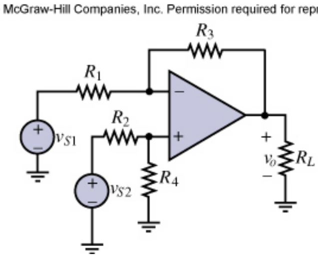
Problem 8.32

Solution:

Known quantities:

For the circuit shown in Figure P8.32:

$$v_{S1}(t) = 2.9 \cdot 10^{-3} \cos(\omega t)\text{V} \quad , \quad R_1 = 1\text{k}\Omega \quad , \quad R_2 = 3.3\text{k}\Omega$$
$$v_{S2}(t) = 3.1 \cdot 10^{-3} \cos(\omega t)\text{V} \quad , \quad R_3 = 10\text{k}\Omega \quad , \quad R_4 = 18\text{k}\Omega \;.$$



Find:

The output voltage v_o and a numerical value.

Analysis:

By using superposition,

$$v_0 = -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4}\left(1 + \frac{R_3}{R_1}\right) v_{S2} = \left(1 + \frac{10}{1}\right)\frac{18}{18 + 3.3} 3.1 \cdot 10^{-3} \cos(\omega t) - 2.9 \cdot 10^{-2} \cos(\omega t) =$$
$$= -1.83 \cdot 10^{-3} \cos(\omega t) \quad \text{V}$$

Problem 8.33

Solution:

Known quantities:

For the circuit shown in Figure P8.32:

$$v_{S1} = 5\,\text{mV} \quad , \quad R_1 = 1\text{k}\Omega \quad , \quad R_2 = 15\text{k}\Omega$$
$$v_{S2} = 7\,\text{mV} \quad , \quad R_3 = 72\text{k}\Omega \quad , \quad R_4 = 47\text{k}\Omega$$

Find:

The output voltage v_o analytically and numerically.

Analysis:

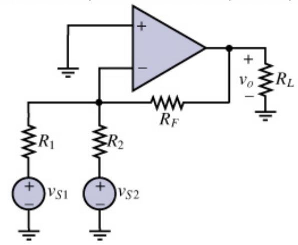
From the solution of Problem 8.32,

$$v_0 = -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4}\left(1 + \frac{R_3}{R_1}\right) v_{S2} = (1 + 72)\frac{47}{15 + 47} 7 \cdot 10^{-3} - 3.60 \cdot 10^{-1} = 27.37\text{mV}$$

Problem 8.34

Solution:

Known quantities:



If, in the circuit shown in Figure P8.34:

$$v_{S1} = v_{S2} = 7\text{ mV} \qquad R_1 = 850\,\Omega \qquad R_2 = 1.5\text{ k}\Omega \qquad R_F = 2.2\text{ k}\Omega$$

$$\text{Op Amp: } \textit{Motorola MC1741C}$$

$$r_i = 2\text{ M}\Omega \qquad \mu = 200{,}000 \qquad r_o = 25\,\Omega$$

Find:

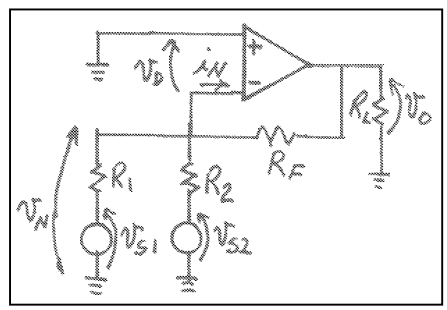
a) The output voltage.
b) The voltage gain for the two input signals.

Analysis:

a) The op amp has a very large input resistance, a very large "open loop gain" [μ], and a very small output resistance. Therefore, it can be modeled with small error as an ideal op amp with:

$$KVL: \; v_D + v_N = 0$$

$$v_D \approx 0 \qquad \Rightarrow \qquad v_N \approx 0$$



$$KCL:$$

$$i_N + \frac{v_N - v_{S2}}{R_2} + \frac{v_N - v_{S1}}{R_1} + \frac{v_N - v_O}{R_F} = 0$$

$$v_N \approx 0 \qquad i_N \approx 0$$

$$\Rightarrow v_O = -\frac{R_F}{R_1}v_{S1} - \frac{R_F}{R_2}v_{S2} = \left[-\frac{2.2}{0.85}\right][7\text{ mV}] + \left[-\frac{2.2}{1.5}\right][7\text{ mV}]$$

$$= [-2.588][7\text{ mV}] + [-1.467][7\text{ mV}] = -28.38\text{ mV}$$

b) Using the results above: $A_{V1} = -2.588$ $A_{V2} = -1.467$.
Note: The output voltage and gain are not dependent on either the op amp parameters or the load resistance. This result is extremely important in the majority of applications where amplification of a signal is required.

Problem 8.35

Solution:

Known quantities:
For the circuit shown in Figure P8.32:

$v_{S1} = kT_1 \quad , \quad R_1 = 11\text{k}\Omega \quad , \quad R_2 = 27\text{k}\Omega$

$v_{S2} = kT_2 \quad , \quad R_3 = 33\text{k}\Omega \quad , \quad R_4 = 68\text{k}\Omega$

$T_1 = 35\ ^\circ\text{C} \quad , \quad T_2 = 100\ ^\circ\text{C} \quad , \quad k = 50\text{mV}/^\circ\text{C}$

Find:
a) The output voltage.
b) The conditions required for the output voltage to depend only on the difference between the two temperatures.

Analysis:
a) From the solution of Problem 8.32,

$$v_0 = -\frac{R_3}{R_1}kT_1 + \frac{R_4}{R_2+R_4}\left(1+\frac{R_3}{R_1}\right)kT_2 = \left(1+\frac{33}{11}\right)\frac{68}{68+27}5 - \frac{33}{11}1.75 =$$

$= 9.065\text{ V}$

b) For $R_3=R_4$, $R_1=R_2= k\,R_3$, the output voltage is

$$v_0 = -\frac{R_3}{R_1}kT_1 + \frac{R_4}{R_2+R_4}\left(1+\frac{R_3}{R_1}\right)kT_2 = -\frac{1}{k}kT_1 + \frac{R_3}{R_1+R_3}\left(\frac{R_1+R_3}{R_1}\right)kT_2 = T_2 - T_1$$

Problem 8.36

Solution:

Known quantities:

Find:
In a differential amplifier, if: $A_{v1} = -20 \quad A_{v2} = +22$, derive expressions for and then determine the value of the common and differential mode gains.

Analysis:
There are several ways to do this. Using superposition:

$$If: v_{S\text{-}D} = 0: \; v_O = v_{O\text{-}C} + v_{O\text{-}D} = v_{O\text{-}C} \qquad \Rightarrow \qquad A_{v\text{-}C} = \frac{v_{O\text{-}C}}{v_{S\text{-}C}}$$

$$If: v_{S\text{-}C} = 0: \; v_O = v_{O\text{-}C} + v_{O\text{-}D} = v_{O\text{-}D} \qquad \Rightarrow \qquad A_{v\text{-}D} = \frac{v_{O\text{-}D}}{v_{S\text{-}D}}$$

Assume that signal source #2 is connected to the non-inverting input of the op-amp. The common-mode output voltage can be obtained using the gains given above but assuming the signal voltages have only a common-mode component:

$$v_O = v_{S1} A_{v1} + v_{S2} A_{v2} \qquad v_{S1} = v_{S\text{-}C} - \frac{1}{2} v_{S\text{-}D} \qquad v_{S2} = v_{S\text{-}C} + \frac{1}{2} v_{S\text{-}D}$$

$$Let: \; v_{S\text{-}D} = 0$$

$$v_{O\text{-}C} = v_{S\text{-}C} A_{v1} + v_{S\text{-}C} A_{v2} = v_{S\text{-}C} [A_{v1} + A_{v2}] = v_{S\text{-}C} A_{v\text{-}C}$$

$$\Rightarrow \quad A_{v\text{-}C} = A_{v1} + A_{v2} = [-20] + [+22] = 2$$

The difference-mode output voltage can be obtained using the gains given but assuming the signal voltage have only a difference-mode component:

$$Let: \; v_{S\text{-}C} = 0$$

$$v_{O\text{-}D} = [-\frac{1}{2} v_{S\text{-}D}] A_{v1} + [\frac{1}{2} v_{S\text{-}D}] A_{v2} = v_{S\text{-}D} \frac{1}{2} [-A_{v1} + A_{v2}] = v_{S\text{-}D} A_{v\text{-}D}$$

$$A_{v\text{-}D} = \frac{1}{2} [A_{v2} - A_{v1}] = \frac{1}{2} ([+22] - [-20]) = +21$$

Note: If signal source #1 were connected to the non-inverting input of the op amp, then the difference mode gain would be the negative of that obtained above.

Problem 8.37

Solution:

Known quantities:

For the circuit shown in Figure P8.32:

$v_{S1} = 1.3\text{V}$, $R_1 = R_2 = 4.7\text{k}\Omega$

$v_{S2} = 1.9\text{V}$, $R_3 = R_4 = 10\text{k}\Omega$

Find:

a) The output voltage.
b) The common-mode component of the output voltage.
c) The differential-mode component of the output voltage.

Analysis:

a) From the solution of Problem 8.17,

$$v_0 = -\frac{R_3}{R_1} v_{S1} + \frac{R_4}{R_2 + R_4}\left(1 + \frac{R_3}{R_1}\right) v_{S2} = \left(1 + \frac{10000}{4700}\right)\frac{10000}{4700 + 10000} 1.9 - \frac{10000}{4700} 1.3 = 1.28\text{V}$$

b) The common-mode component is zero.
c) The differential-mode component is v_0.

Problem 8.38

Solution:

Known quantities:

For the circuit shown in Figure P8.32:

$v_{S1,2} = A + BP_{1,2} \quad , \quad A = 0.3 \quad , \quad B = 0.7\,\text{V/psi}$

$R_1 = R_2 = 4.7\text{k}\Omega \quad , \quad R_3 = R_4 = 10\text{k}\Omega \quad , \quad R_L = 1.8\text{k}\Omega$

$P_1 = 6\text{kPa} \quad , \quad P_2 = 5\text{kPa}$

Find:

a) The common-mode input voltage.
b) The differential-mode input voltage.

Analysis:

a) The common-mode input voltage is

$$v_{in+} = \frac{v_{S2} + v_{S1}}{2} = A + B\frac{P_1 + P_2}{2} = 0.3 + 0.7 \cdot 1.4504 \cdot 10^{-4}\frac{11 \cdot 10^3}{2} = 0.858\text{V}$$

b) The differential-mode input voltage is

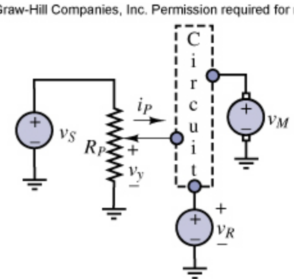
$$v_{in-} = v_{S2} - v_{S1} = B(P_2 - P_1) = -0.7 \cdot 1.4504 \cdot 10^{-4} \cdot 10^3 = -0.1015\text{V}$$

Problem 8.39

Solution:

Known quantities:

A linear potentiometer [variable resistor] R_p is used to sense and give a signal voltage v_Y proportional to the current y position of an x-y plotter. A reference signal v_R is supplied by the software controlling the plotter. The difference between these voltages must be amplified and supplied to a motor. The motor turns and changes the position of the pen and the position of the "pot" until the signal voltage is equal to the reference voltage [indicating the pen is in the desired position] and the motor voltage = 0. For proper operation the motor voltage must be 10 times the difference between the signal and reference voltage. For rotation in the proper direction, the motor voltage must be negative with respect to the signal voltage for the polarities shown. An additional requirement is that $i_P = 0$ to avoid "loading" the pot and causing an erroneous signal voltage.



Find:

a) Design an op amp circuit which will achieve the specifications given. Redraw the circuit shown in Figure P8.39 replacing the box [drawn with dotted lines] with your circuit. Be sure to show how the signal voltage and output voltage are connected in your circuit.
b) Determine the value of each component in your circuit. The op amp is an MC1741.

Analysis:

a) The output voltage to the motor must be dependent on the difference between two input voltages. A difference amp is required. The signal voltage must be connected to the inverting input of the amplifier. However, the feedback path is also connected to the inverting input and this will caused loading of the input circuit, ie, cause an input current. This can be corrected by adding an isolation stage [or voltage follower] between the input circuit and the inverting input of the 2nd stage. The isolation stage must have a gain = 1 and an input current = 0. The difference amplifier, the second stage, must give an output voltage:

$$v_0\Big|_{10} = \left(\frac{10\,\text{k}\Omega}{R_S}+1\right)(10)$$

$$v_M = 10\,[\,v_R - v_Y\,] = [\,10\,]\,v_R + [\,-10\,]\,v_Y$$

$$\Rightarrow \quad A_{vr} = 10 \qquad A_{vy} = -10$$

The circuit configuration shown will satisfy these specifications.

b) In this first approximation analysis, assume the op amps can be modeled as ideal op amps:

$$\Rightarrow \quad v_D \approx 0 \qquad i_P = i_N \approx 0$$

Consider the first or isolation or voltage follower stage:

$$Ideal: \Rightarrow i_P \approx 0 \quad KVL: \; -v_Y + v_D + v_{O1} = 0 \quad v_D \approx 0 \quad \Rightarrow \quad v_{O1} = v_Y$$

$$KVL: \; -v_N - v_D + v_P = 0 \quad v_D \approx 0 \quad \Rightarrow \quad v_N = v_P$$

$$KCL: \; \frac{v_N - v_{O1}}{R_1} + \frac{v_N - v_M}{R_2} + i_N = 0 \qquad i_N \approx 0 \quad \Rightarrow \quad v_N = \frac{\dfrac{v_{O1}}{R_1} + \dfrac{v_M}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} \; \frac{R_1 R_2}{R_1 R_2}$$

$$KCL: \; \frac{v_P - v_R}{R_3} + \frac{v_P - 0}{R_4} + i_P = 0 \qquad i_P \approx 0 \quad \Rightarrow \quad v_P = \frac{\dfrac{v_R}{R_3}}{\dfrac{1}{R_3} + \dfrac{1}{R_4}} \; \frac{R_3 R_4}{R_3 R_4}$$

The second stage:

$$v_{O1} = v_Y \quad v_N = v_P \qquad \Rightarrow \quad \frac{v_{O1} R_2}{R_2 + R_1} + \frac{v_M R_1}{R_2 + R_1} = \frac{v_R R_4}{R_4 + R_3}$$

$$v_M = v_Y\,[\,-\frac{R_2}{R_1}\,] + v_R \frac{R_4\,[\,R_2 + R_1\,]}{R_1\,[\,R_4 + R_3\,]} = v_Y A_{vy} + v_R A_{vr}$$

$$A_{vy} = -\frac{R_2}{R_1} = -10 \qquad Choose: \; R_2 = 100\,\text{k}\Omega \quad R_1 = 10\,\text{k}\Omega$$

$$A_{vr} = \frac{R_4\,[\,R_2 + R_1\,]}{R_1\,[\,R_4 + R_3\,]} = 10 \qquad Choose: \; R_3 = 10\,\text{k}\Omega \quad R_4 = 100\,\text{k}\Omega$$

The resistances chosen are standard values and there is some commonality in the choices. Moderately large values were chosen to reduce currents and the resistor power ratings. Cost of the resistors will be determined primarily by power rating and tolerance.

Problem 8.40

Solution:

Known quantities:

For the circuit shown in Figure P8.32:

$v_{S1} = 13\text{mV} \quad , \quad R_1 = 1\text{k}\Omega \quad , \quad R_2 = 13\text{k}\Omega$

$v_{S2} = 19\,\text{mV} \quad , \quad R_3 = 81\text{k}\Omega \quad , \quad R_4 = 56\text{k}\Omega$

Find:

The output voltage v_o.

Analysis:

From the solution of Problem 8.32,

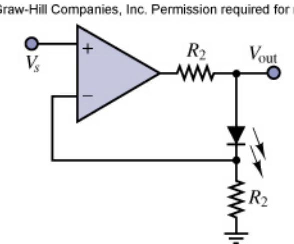
$$v_0 = \frac{R_4}{R_2 + R_4}\left(1 + \frac{R_3}{R_1}\right)v_{S2} - \frac{R_3}{R_1}v_{S1} = (1+81)\frac{56}{56+13}19\cdot 10^{-3} - 81\cdot 13\cdot 10^{-3} = 0.211\text{ V}$$

Problem 8.41

Solution:

Known quantities:

The circuit shown in Figure P8.41.



Find:

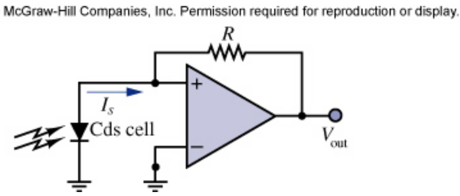
Show that the current I_{out} through the light-emitting diode is proportional to the source voltage V_S as long as :
$V_S > 0$.

Analysis:

Assume the op amp is ideal: $\left.\begin{array}{l} V^- \approx V^+ = V_S \\ I^- \approx 0 \end{array}\right\} \Rightarrow \; if \quad V_S > 0 \qquad I_{out} = \dfrac{V^-}{R_2} = \dfrac{V_S}{R_2}.$

Problem 8.42

Note: The terminals in the op amp of Figure P8.42 are inverted. Also, in reality, CdS cells are photoresistors, so a realistic circuit should include a voltage source to generate the source current. Also, the symbol of a CdS cell is different.



Solution:

Known quantities:

The circuit shown in Figure P8.42.

Find:

Show that the voltage V_{out} is proportional to the current generated by the CdS solar cell. Show that the transimpedance of the circuit V_{out}/I_s is $-R$.

Analysis:

Assuming an ideal op-amp : $v^+ \cong v^- = 0 \Rightarrow V_{out} = -RI_S$

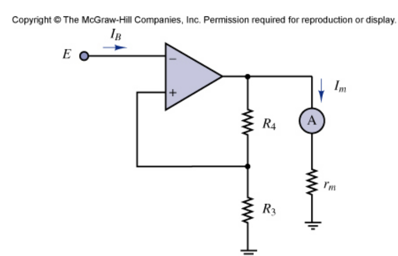
The transimpedance is given by $R_{trans} = \dfrac{V_{out}}{I_S} = -R$

Problem 8.43

Solution:

Known quantities:

The op-amp voltmeter circuit shown in Figure P8.43 is required to measure a maximum input of $E = 20$ mV. The op-amp input current is $I_B = 0.2$ μA, and the meter circuit has $I_m = 100$ μA full-scale deflection and $r_m = 10$ kΩ.



Find:

Determine suitable values for R_3 and R_4.

Analysis:

$V_{out} = I_m r_m = (100\mu\text{A})(10\text{k}\Omega) = 1\text{V}$

From KCL at the inverting input, $\dfrac{E}{R_3} + \dfrac{E - V_{out}}{R_4} = 0$ or $\dfrac{V_{out}}{E} = \dfrac{R_4 + R_3}{R_3} = 1 + \dfrac{R_4}{R_3}$.

Then, $\dfrac{R_4}{R_3} = \dfrac{V_{out}}{E} - 1 = \dfrac{1}{20\times10^{-3}} - 1 = 49$. Now, choose R_3 and R_4 such that $I_B = \dfrac{E}{R_3 \| (R_4 + r_m)} \le 0.2\mu$.

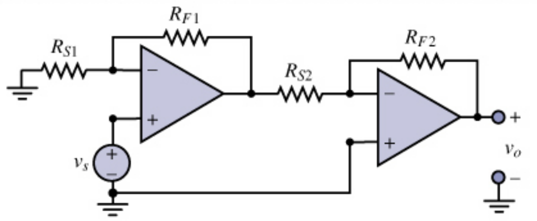
At the limit, $\dfrac{20\times10^{-3}}{R_3 \| (49R_3 + 10\times10^3)} = 0.2\times10^{-6}$. Solving for R_3, we have $R_3 \approx 102\text{k}\Omega$.

Therefore, $R_4 \approx 5\text{M}\Omega$.

Problem 8.44

Solution:

Known quantities:
Circuit in Figure P8.44.



Find:
The output voltage v_o.

Analysis:
The circuit is a cascade of a noninverting op-amp with an inverting op-amp. Assuming ideal op-amps, the input-output voltage gain is equal to the product of the single gains, therefore

$$v_0 = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right)v_S$$

Problem 8.45

Solution:

Known quantities:
Circuit in Figure P8.44.

Find:
Select appropriate components using standard 5% resistor values to obtain a gain of magnitude approximately equal to 1,000. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors have the nominal value.

Analysis:
From the solution of Problem 8.44, the gain is given by

$$A_V = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right)$$

From Table 2.2, if we select

$$R_{S1} = 1.8\text{k}\Omega \quad , \quad R_{S2} = 1\text{k}\Omega$$
$$R_{F1} = 8.2\text{k}\Omega \quad , \quad R_{F2} = 180\text{k}\Omega$$

we obtain

$$A_V = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right) = -\frac{180}{1}\left(\frac{1.8+8.2}{1.8}\right) = -1000$$

So, we can obtain a nominal error equal to zero.

Problem 8.46

Solution:

Known quantities:
Circuit in Figure P8.44.

Find:
Same as in Problem 8.45, but use the ±5% tolerance range to compute the possible range of gains for this amplifier.

Analysis:
From the solution of Problem 8.44, the gain is given by

$$A_V = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right)$$

From Table 2.2, if we select $\begin{matrix} R_{S1n} = 1.8\text{k}\Omega & , & R_{S2n} = 1\text{k}\Omega \\ R_{F1n} = 8.2\text{k}\Omega & , & R_{F2n} = 180\text{k}\Omega \end{matrix}$

we obtain

$$A_{Vn} = -\frac{R_{F2n}}{R_{S2n}}\left(1+\frac{R_{F1n}}{R_{S1n}}\right) = -\frac{180}{1}\left(\frac{1.8+8.2}{1.8}\right) = -1000$$

So, we can obtain a nominal error equal to zero.
The maximum gain is given by

$$A_{V^+} = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right) = -\frac{R_{F2n}(1+0.05)}{R_{S2n}(1-0.05)}\left(1+\frac{R_{F1n}(1+0.05)}{R_{S1n}(1-0.05)}\right) = -1200$$

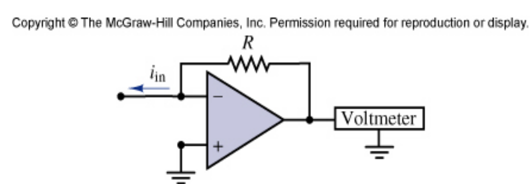
while the minimum gain is

$$A_{V^-} = -\frac{R_{F2}}{R_{S2}}\left(1+\frac{R_{F1}}{R_{S1}}\right) = -\frac{R_{F2n}(1-0.05)}{R_{S2n}(1+0.05)}\left(1+\frac{R_{F1n}(1-0.05)}{R_{S1n}(1+0.05)}\right) = -834$$

Problem 8.47

Solution:

Known quantities:
For the circuit in Figure P8.47, $\begin{matrix} R_L = 20\text{k}\Omega & , & v_0 = 0 \div 10\text{ V} \\ i_{in_{\max}} = 1\text{mA} & & \end{matrix}$



Find:
The resistance R such that $i_{in_{\max}} = 1\text{mA}$.

Analysis:

$$i_{in} = \frac{v_0}{R} \Rightarrow R = \frac{v_{0_{\max}}}{i_{in_{\max}}} = \frac{10}{10^{-3}} = 10\text{ k}\Omega$$

Problem 8.48

Solution:

Known quantities:
Circuit in Figure P8.30.

Find:
Select appropriate components using standard 5% resistor values to obtain a gain of magnitude approximately equal to 200. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors have the nominal value.

Analysis:
From the solution of Problem 8.30, the gain is given by $A_V = \left(1 + \dfrac{R_F}{R_2}\right)$

From Table 2.2, if we select $\begin{matrix} R_2 = 33\Omega \\ R_F = 6.8\text{k}\Omega \end{matrix}$, we obtain $A_{Vn} = \left(1 + \dfrac{R_F}{R_2}\right) = 1 + \dfrac{6800}{33} = 207$

The error in the gain is $\varepsilon_{A_V} = \dfrac{207 - 200}{200} = 3.5\%$

Problem 8.49

Solution:

Known quantities:
Circuit in Figure P8.30.

Find:
Same as in Problem 8.48, but use the ±5% tolerance range to compute the possible range of gains for this amplifier.

Analysis:
From the solution of Problem 8.48, the gain is given by $A_V = \left(1 + \dfrac{R_F}{R_2}\right)$

From Table 2.2, if we select $\begin{matrix} R_{2n} = 33\Omega \\ R_{Fn} = 6.8\text{k}\Omega \end{matrix}$

we obtain $A_{Vn} = \left(1 + \dfrac{R_{Fn}}{R_{2n}}\right) = 1 + \dfrac{6800}{33} = 207$

The maximum gain is given by $A_{V^+} = \left(1 + \dfrac{R_{Fn}(1+0.05)}{R_{2n}(1-0.05)}\right) = 1 + \dfrac{6800 \cdot 1.05}{33 * 0.95} = 228.75$

while the minimum gain is $A_{V^-} = \left(1 + \dfrac{R_{Fn}(1-0.05)}{R_{2n}(1+0.05)}\right) = 1 + \dfrac{6800 \cdot 0.95}{33 * 1.05} = 187.44$

Problem 8.50

Solution:

Known quantities:
For the circuit in Figure P8.32,
$R_1 = R_2 \quad , \quad R_3 = R_4$

Find:
Select appropriate components using standard 1% resistor values to obtain a differential amplifier gain of magnitude approximately equal to 100. How closely can you approximate the gain? Compute the error in the gain assuming that the resistors the have nominal value.

Analysis:
From the solution of Problem 8.32, the gain is given by

$$A_V = \frac{R_3}{R_1}$$

From Table 2.2, if we select
$R_1 = 1\text{k}\Omega \quad , \quad R_3 = 100\text{k}\Omega$
we obtain

$$A_V = \frac{R_3}{R_1} = 100$$

and the error for the gain with nominal values is zero.

Problem 8.51

Solution:

Known quantities:
For the circuit in Figure P8.32,
$R_1 = R_2 \quad , \quad R_3 = R_4$

Find:
Same as in Problem 8.50, but use the ±1% tolerance range to compute the possible range of gains for this amplifier.

Analysis:
From the solution of Problem 8.50

$$A_V = \frac{R_3}{R_1} \Rightarrow A_{V\,\max} = \frac{100(1+0.01)}{1(1-0.01)} = 102$$

$$A_V = \frac{R_3}{R_1} \Rightarrow A_{V\,\min} = \frac{100(1-0.01)}{1(1+0.01)} = 98$$

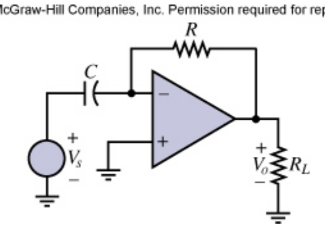
Section 8.3: Active Filters

Problem 8.52

Solution:

Known quantities:

For the circuit shown in Figure P8.52: $C = 1\ \mu\text{F} \quad R = 10\,\text{k}\Omega \quad R_L = 1\,\text{k}\Omega$.

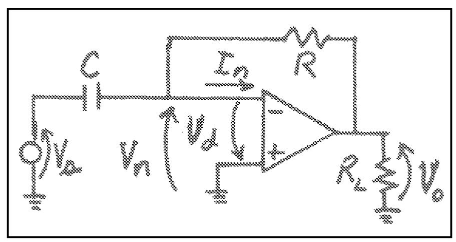


Find:

a) The gain [in dB] in the pass band.
b) The cutoff frequency.
c) If this is a low or high pass filter.

Analysis:

a) Assume the op-amp is ideal. Determine the transfer function in the form:



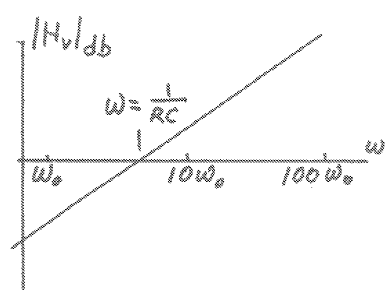
$$H_V[j\omega] = \frac{V_o[j\omega]}{V_s[j\omega]} = H_o \frac{1}{1 + j\, f[\omega]}$$

$$KVL: \ -V_n - V_d = 0 \quad V_d \approx 0 \qquad \Rightarrow V_n \approx 0$$

$$KCL: \ \frac{V_n - V_s}{\dfrac{1}{j\omega C}} + I_n + \frac{V_n - V_o}{R} = 0$$

$$I_n \approx 0 \quad V_n \approx 0 \quad \Rightarrow H_V[j\omega] = \frac{V_o[j\omega]}{V_s[j\omega]} = j\omega RC$$

This is not in the standard form desired but is the best that can be done. There are no cutoff frequencies and no clearly defined pass band. The gain [i.e., the magnitude of the transfer function] and output voltage increases continuously with frequency, at least until the output voltage tries to exceed the DC supply voltages and clipping occurs. In a normal high pass filter, the gain will increase with frequency until the cutoff frequency is reached above which the gain remains constant.



b) There is no cutoff frequency.
c) This filter is best called a high pass filter; however, since the output will be clipped and severely distorted above some frequency, it is not a particularly good high pass filter. It could even be called a terrible filter with few redeeming graces.

Problem 8.53

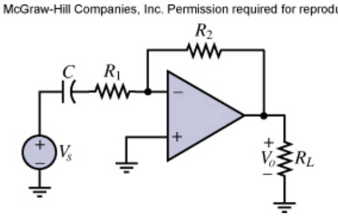
Solution:

Known quantities:

For the circuit shown in Figure P8.53: $C = 1\ \mu\text{F}$ $R_1 = 1.8\,\text{k}\Omega$ $R_2 = 8.2\text{k}\Omega$ $R_L = 333\Omega$.

Find:

a) Whether the circuit is a low- or high-pass filter.
b) The gain V_0/V_S in decibel in the passband.
c) The cutoff frequency.



Analysis:

a) From Figure 8.25, it results that the amplifier in Figure P8.53 is a high-pass filter.

In fact, the output voltage is $\mathbf{V}_0(j\omega) = -\dfrac{j\omega CR_2}{1 + j\omega CR_1}\mathbf{V}_S(j\omega)$

b) $\lim\limits_{\omega\to\infty}\left|\dfrac{\mathbf{V}_0(j\omega)}{\mathbf{V}_S(j\omega)}\right|_{dB} = 20\,\text{Log}\dfrac{R_2}{R_1} = 20\,\text{Log}\dfrac{8.2}{1.8} = 13.17\text{ dB}$

c) $\omega_0 = \dfrac{1}{CR_1} = \dfrac{1}{1\cdot 10^{-6}\cdot 1.8\cdot 10^3} = 5555\text{ rad/s}$

Problem 8.54

Solution:

Known quantities:

For the circuit shown in Figure P8.53: $C = 200\text{ pF}$ $R_1 = 10\,\text{k}\Omega$ $R_2 = 220\text{k}\Omega$ $R_L = 1\text{k}\Omega$.

Find:

a) Whether the circuit is a low- or high-pass filter.
b) The gain V_0/V_S in decibel in the passband.
c) The cutoff frequency.

Analysis:

a) From Figure 8.25, it results that the amplifier in Figure P8.53 is a high-pass filter.

In fact, the output voltage is $\mathbf{V}_0(j\omega) = -\dfrac{j\omega CR_2}{1 + j\omega CR_1}\mathbf{V}_S(j\omega)$

b) $\lim\limits_{\omega\to\infty}\left|\dfrac{\mathbf{V}_0(j\omega)}{\mathbf{V}_S(j\omega)}\right|_{dB} = 20\,\text{Log}\dfrac{R_2}{R_1} = 20\,\text{Log}\dfrac{220}{10} = 26.84\text{ dB}$

c) $\omega_0 = \dfrac{1}{CR_1} = \dfrac{1}{200\cdot 10^{-12}\cdot 10\cdot 10^3} = 5\cdot 10^5\text{ rad/s}$

Problem 8.55

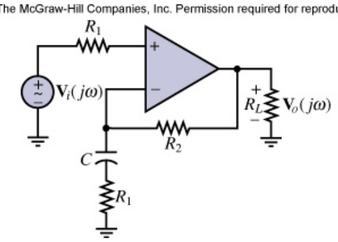
Solution:

Known quantities:

For the circuit shown in Figure P8.55: $C = 100$ pF $R_1 = 4.7\,\mathrm{k}\Omega$ $R_2 = 68\mathrm{k}\Omega$ $R_L = 220\mathrm{k}\Omega$.

Find:

Determine the cutoff frequencies and the magnitude of the voltage frequency response function at very low and at very high frequencies.



Analysis:

The output voltage in the frequency domain is

$$\mathbf{V}_0(j\omega) = \left(1 + \frac{R_2}{\dfrac{1}{j\omega C} + R_1}\right)\mathbf{V}_i(j\omega) = \frac{1 + j\omega C(R_1 + R_2)}{1 + j\omega CR_1}\mathbf{V}_i(j\omega)$$

The circuit pass all frequencies, with a higher gain at higher than at lower frequencies. The cutoff frequencies are

$$\omega_1 = \frac{1}{CR_1} = \frac{1}{100 \cdot 10^{-12} \cdot 4.7 \cdot 10^3} = 2.127 \cdot 10^6 \,\text{rad/s}$$

$$\omega_2 = \frac{1}{C(R_1 + R_2)} = \frac{1}{100 \cdot 10^{-12} \cdot 72.7 \cdot 10^3} = 1.375 \cdot 10^5 \,\text{rad/s}$$

For high frequencies we have

$$A_\infty = \lim_{\omega \to \infty}\left|\frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)}\right| = 1 + \frac{R_2}{R_1} = 1 + \frac{68}{4.7} = 15.46$$

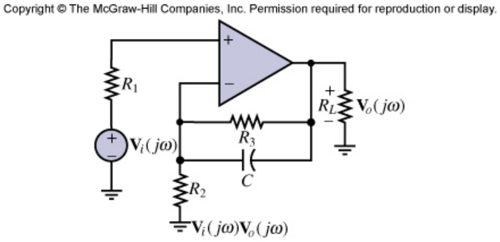
At low frequencies

$$A_0 = \lim_{\omega \to 0}\left|\frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)}\right| = 1$$

Problem 8.56

Solution:

Known quantities:



For the circuit shown in Figure P8.56:

$C = 20$ nF $\quad R_1 = 1\text{k}\Omega \quad R_2 = 4.7\text{k}\Omega \quad R_3 = 80\text{k}\Omega$.

Find:

a) An expression for $H_\upsilon(j\omega) = \dfrac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)}$.

b) The cutoff frequencies.

c) The passband gain.

d) The Bode plot.

Analysis:

a) The frequency response is

$$H_\upsilon(j\omega) = \frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)} = 1 + \frac{R_3 \| \dfrac{1}{j\omega C}}{R_2} = \frac{R_2 + R_3 + j\omega C R_2 R_3}{R_2 + j\omega C R_2 R_3} = \left(1 + \frac{R_3}{R_2}\right) \frac{1 + j\omega C \dfrac{R_2 R_3}{R_2 + R_3}}{1 + j\omega C R_3}$$

b) The cutoff frequencies are

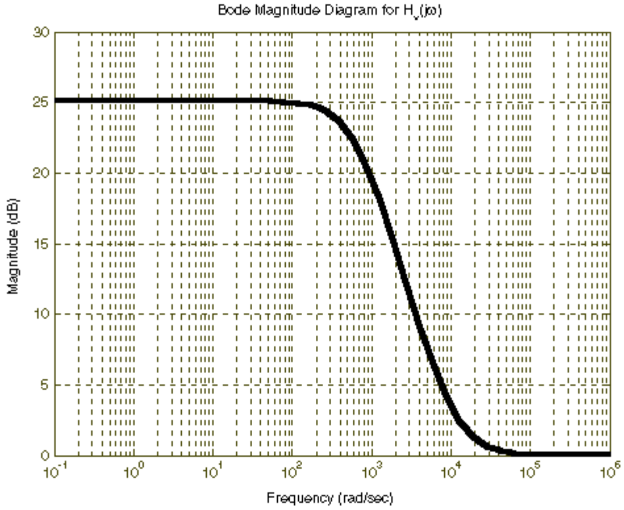
$$\omega_1 = \frac{1}{CR_3} = \frac{1}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3} = 625 \text{ rad/s}$$

$$\omega_2 = \frac{(R_2 + R_3)}{C R_2 R_3} = \frac{84.7 \cdot 10^3}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3 \cdot 4.7 \cdot 10^3} = 11263 \text{ rad/s}$$

c) The passband gain is obtained by evaluating the frequency response at low frequencies,

$$A_0 = \lim_{\omega \to 0} \left| H_\upsilon(j\omega) \right| = 1 + \frac{R_3}{R_2} = 1 + \frac{80}{4.7} = 18$$

d) The magnitude Bode plot for the given amplifier is as shown.



Problem 8.57

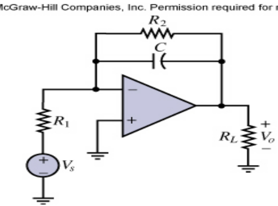
Solution:

Known quantities:

For the circuit shown in Figure P8.57: $C = 0.47\ \mu\text{F}$ $\quad R_1 = 9.1\text{k}\Omega \quad R_2 = 22\text{k}\Omega \quad R_L = 2.2\text{k}\Omega$.

Find:

a) Whether the circuit is a low- or high-pass filter.
b) An expression in standard form for the voltage transfer function.
c) The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.



Analysis:

a).From Figure 8.22, it results that the amplifier in Figure P8.57 is a low-pass filter.In fact, the output voltage is

$$\mathbf{V}_0(j\omega) = -\frac{R_2}{R_1}\frac{1}{1 + j\omega CR_2}\mathbf{V}_S(j\omega)$$

b). $H_\upsilon(j\omega) = \dfrac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)} = -\dfrac{R_2}{R_1}\dfrac{1}{1 + j\omega CR_2}$

c) The gain in decibel is obtained by evaluating $|H_\upsilon(j\omega)|$ at ω=0, $|H_\upsilon(j0)|_{dB} = 20\,\text{Log}\dfrac{R_2}{R_1} = 20\,\text{Log}\dfrac{22}{9.1} = 7.66$ dB .

The cutoff frequency is $\omega_0 = \dfrac{1}{CR_2} = \dfrac{1}{0.47 \cdot 10^{-6} \cdot 22 \cdot 10^3} = 96.71$ rad/s

---

Problem 8.58

Solution:

Known quantities:

For the circuit shown in Figure P8.57: $C = 0.47$ nF $\quad R_1 = 2.2\,\text{k}\Omega \quad R_2 = 68\text{k}\Omega \quad R_L = 1\text{k}\Omega$.

Find:

a) An expression in standard form for the voltage frequency response function.
b) The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

Analysis:

a) From Figure 8.22, it results that the amplifier in Figure P8.570 is a low-pass filter.

The voltage frequency response function is $H_\upsilon(j\omega) = \dfrac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)} = -\dfrac{R_2}{R_1}\dfrac{1}{1 + j\omega CR_2}$

b)The gain in decibel is obtained by evaluating $|H_\upsilon(j\omega)|$ at ω=0, $|H_\upsilon(j0)|_{dB} = 20\,\text{Log}\dfrac{R_2}{R_1} = 20\,\text{Log}\dfrac{68}{2.2} = 29.8$ dB .

The cutoff frequency is $\omega_0 = \dfrac{1}{CR_2} = \dfrac{1}{0.47 \cdot 10^{-9} \cdot 68 \cdot 10^3} = 31289$ rad/s

---

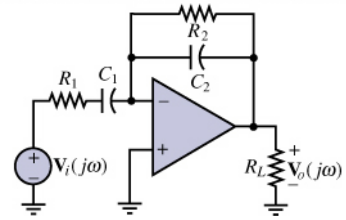
Problem 8.59

Solution:

Known quantities:

For the circuit shown in Figure P8.59:

$C_1 = C_2 = 0.1\ \mu\text{F} \quad R_1 = R_2 = 10\,\text{k}\Omega$.



Find:

a) The passband gain.
b) The resonant frequency.
c) The cutoff frequencies.
   a) The circuit Q.
   b) The Bode plot.

Analysis:

a) From Figure 8.27, we have

$$A_{BP}(j\omega) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{j\omega C_1 R_2}{(1+j\omega C_1R_1)(1+j\omega C_2R_2)} = -\frac{j\omega C_1 R_1}{(1+j\omega C_1R_1)^2} = -\frac{j\omega C_1R_1}{(j\omega)^2(C_1R_1)^2 + 2j\omega C_1R_1 + 1}$$

The magnitude of the frequency response is $|A_{BP}(j\omega)| = \dfrac{\omega C_1 R_1}{1+\omega^2 (C_1R_1)^2}$

The passband gain is the maximum over ω of the magnitude of the frequency response, i.e. $\left|A_{BP}(j\frac{1}{C_1R_1})\right| = \frac{1}{2}$

b) The resonant frequency is $\omega_n = \dfrac{1}{R_1C_1} = 1000$ rad/s

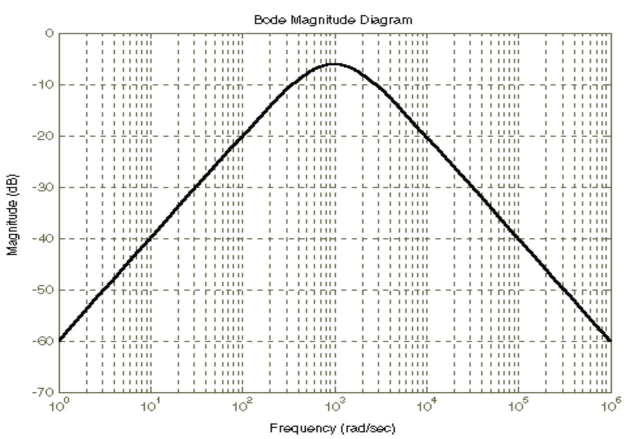
c) The cutoff frequencies are obtained by solving with respect to ω the equation

$$|A_{BP}(j\omega)| = \frac{\omega C_1R_1}{1+\omega^2(C_1R_1)^2} = \frac{1}{2\sqrt{2}} \Rightarrow \omega^2 - 2\sqrt{2}\omega_n\omega + \omega_n^2 = 0 \Rightarrow \omega_{1,2} = \omega_n(\sqrt{2}\pm 1)$$

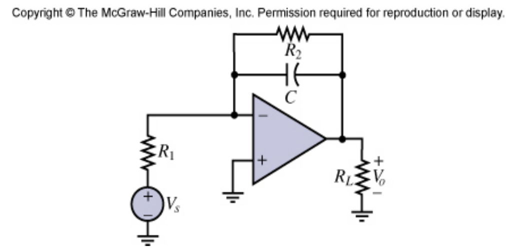
$\omega_1 = 2414$ rad/s $\quad \omega_2 = 414$ rad/s

d) In this case ζ=1, which implies $Q = \dfrac{1}{2\zeta} = \dfrac{1}{2}$

e) The Bode plot of the frequency response is as shown.



Problem 8.60



Solution:

Known quantities:

For the circuit shown in Figure P8.60:

$C = 0.47$ nF $\quad R_1 = 220\Omega \quad R_2 = 68\text{k}\Omega \quad R_L = 1\text{k}\Omega$.

Find:

a) An expression in standard form for the voltage frequency response function.
b) The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

Analysis:

a).The voltage frequency response function is $H_\upsilon(j\omega) = \dfrac{\mathbf{V}_O(j\omega)}{\mathbf{V}_i(j\omega)} = -\dfrac{R_2}{R_1}\dfrac{1}{1+j\omega CR_2}$

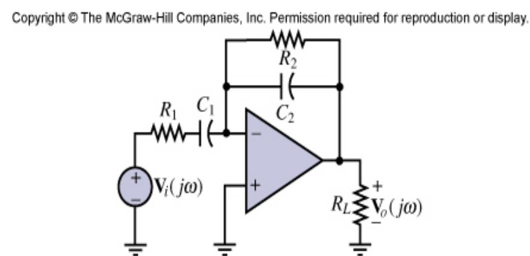
b).The gain in decibel is obtained by evaluating $|H_\upsilon(j\omega)|$ at ω=0, i.e.

$$|H_\upsilon(j0)|_{dB} = 20\,\text{Log}\frac{R_2}{R_1} = 20\,\text{Log}\frac{68000}{220} = 49.8 \text{ dB} .$$

The cutoff frequency is $\omega_0 = \dfrac{1}{CR_2} = \dfrac{1}{0.47\cdot 10^{-9}\cdot 68\cdot 10^3} = 31289$ rad/s

---

Problem 8.61



Solution:

Known quantities:

For the circuit shown in Figure P8.61:

$C_1 = 2.2\ \mu\text{F} \quad C_2 = 1\text{ nF} \quad R_1 = 2.2\text{k}\Omega \quad R_2 = 100\text{k}\Omega$.

Find:

Determine the passband gain.

Analysis:

The voltage frequency response is

$$A_{BP}(j\omega) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{j\omega C_1 R_2}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_2)} \Rightarrow |A_{BP}(j\omega)| = \frac{\omega C_1 R_2}{\sqrt{1+\omega^2 (C_1R_1)^2}\sqrt{1+\omega^2 (C_2R_2)^2}}$$

The cutoff frequencies

are $\omega_1 = \dfrac{1}{C_1R_1} = \dfrac{1}{2.2\cdot 10^{-6}\cdot 2.2\cdot 10^3} = 206.6$ rad/s $\quad \omega_2 = \dfrac{1}{C_2\,R_2} = \dfrac{1}{1\cdot 10^{-9}\cdot 100\cdot 10^3} = 10000$ rad/s

The passband gain can be calculated approximately by evaluating the magnitude of the frequency response at frequencies greater than ω_1 and smaller than ω_2, i.e.

$$\omega_1 << \omega << \omega_2 \Rightarrow 1+\omega^2 (C_1R_1)^2 \approx \omega^2 (C_1R_1)^2,\ 1+\omega^2 (C_2R_2)^2 \approx 1 \quad |A_{BP}| \cong \frac{\omega C_1 R_2}{\sqrt{\omega^2 (C_1R_1)^2}} = \frac{R_2}{R_1} = \frac{100}{2.2} = 45.45$$

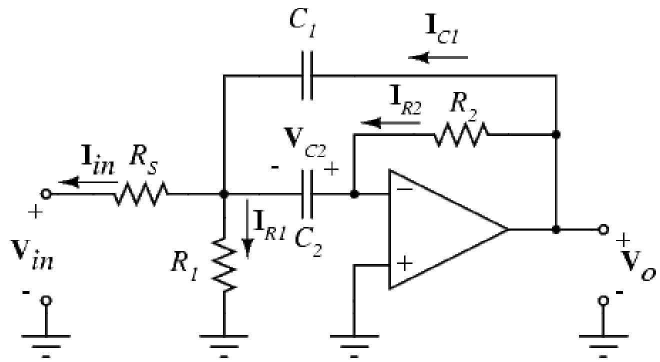
Problem 8.62

Solution:

Known quantities:

For the circuit shown in Figure P8.62, let

$C = C_1 = C_2 = 220\ \mu\text{F} \quad R = R_1 = R_2 = R_S = 2\text{k}\Omega$.



Find:

Determine the frequency response.

Analysis:

With reference to the Figure shown below, we have

$$\mathbf{I}_{R2}(j\omega) = \frac{1}{R_2}\mathbf{V}_O(j\omega) \Rightarrow \mathbf{V}_{C2}(j\omega) = \frac{1}{j\omega C_2}\mathbf{I}_{R2}(j\omega) = \frac{1}{j\omega C_2 R_2}\mathbf{V}_O(j\omega),$$

$$\mathbf{I}_{R1}(j\omega) = -\frac{1}{R_1}\mathbf{V}_{C2}(j\omega) = -\frac{1}{j\omega C_2 R_2 R_1}\mathbf{V}_O(j\omega),$$

$$\mathbf{I}_{C1}(j\omega) = \frac{\left(R_2 + \frac{1}{j\omega C_2}\right)\frac{1}{R_2}\mathbf{V}_O(j\omega)}{\frac{1}{j\omega C_1}} = \left(1 + \frac{1}{j\omega C_2 R_2}\right) j\omega C_1\, \mathbf{V}_O(j\omega),$$

$$\mathbf{I}_{in}(j\omega) = \mathbf{I}_{C1}(j\omega) + \mathbf{I}_{R2}(j\omega) - \mathbf{I}_{R1}(j\omega) = \left(j\omega C_1 + \frac{C_1}{C_2 R_2} + \frac{1}{R_2} + \frac{1}{j\omega C_2 R_2 R_1}\right)\mathbf{V}_O(j\omega) =$$

$$= \left(\frac{2}{R} + j\omega C + \frac{1}{j\omega C R^2}\right)\mathbf{V}_O(j\omega),$$

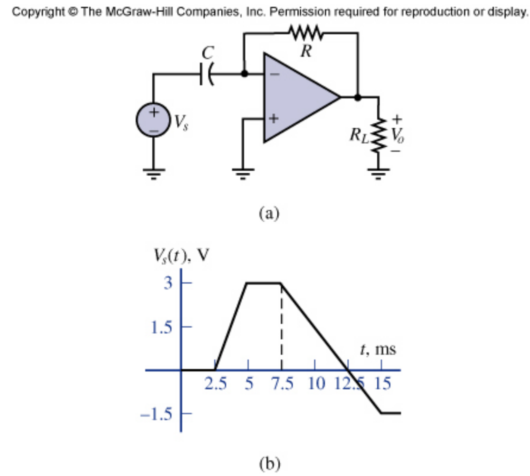
$$\mathbf{V}_{in}(j\omega) = -\mathbf{V}_{C2}(j\omega) - R_S \mathbf{I}_{in}(j\omega) = -\frac{1}{j\omega CR}\mathbf{V}_O(j\omega) - \left(2 + j\omega CR + \frac{1}{j\omega CR}\right)\mathbf{V}_O(j\omega) =$$

$$= -\left(2 + j\omega CR + \frac{2}{j\omega CR}\right)\mathbf{V}_O(j\omega) \Rightarrow$$

$$H_\upsilon(j\omega) = \frac{\mathbf{V}_O(j\omega)}{\mathbf{V}_{in}(j\omega)} = -\frac{1}{2 + j\omega CR + \frac{2}{j\omega CR}} = -\frac{j\omega CR}{(j\omega)^2(CR)^2 + 2j\omega CR + 2}$$

Problem 8.63

Solution:



Known quantities:

The inverting amplifier shown in Figure P8.63.

Find:

a) The frequency response of the circuit.
b) If $R_1 = R_2 = 100$ kΩ and $C = 0.1$ μF, compute the attenuation in dB at $\omega = 1{,}000$ rad/s.
c) Compute gain and phase at $\omega = 2{,}500$ rad/s.
d) Find range of frequencies over which the attenuation is less than 1 dB.

Analysis:

a) Applying KCL at the inverting terminal:

$$\frac{v_{OUT}}{v_{IN}} = -\frac{R_2 + \frac{1}{j\omega C}}{R_1} = -\frac{1 + j\omega R_2 C}{j\omega R_1 C}$$

b) Gain = 0.043 dB
c) Gain = 0.007 dB ; Phase = 177.71°
d) To find the desired frequency range we need to solve the equation:

$$\left|\frac{1 + j\omega R_2 C}{j\omega R_1 C}\right| < 0.8913 \text{ since } 20\log_{10}(0.8913) = -1\text{dB}.$$

This yields a quadratic equation in ω, which can be solved to find $\omega > 196.5$ rad/s.

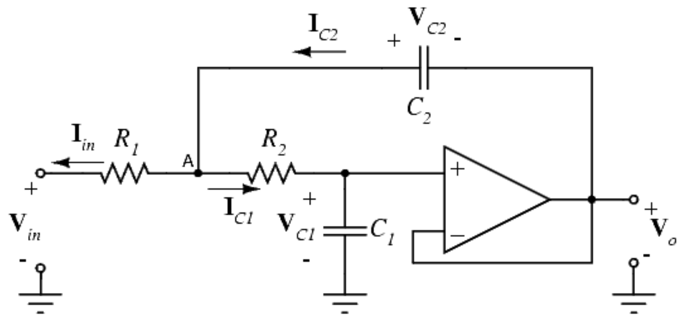
Problem 8.64

Solution:

Known quantities:

For the circuit shown in Figure P8.64, let

$C = C_1 = C_2 = 100\ \mu\text{F} \quad R_1 = 3\text{k}\Omega \quad R_2 = 2\text{k}\Omega$.



Find:

Determine an expression for the gain.

Analysis:

With reference to the Figure shown above, we have

$$\mathbf{V}_{C1}(j\omega) = \mathbf{V}_O(j\omega) \Rightarrow \mathbf{I}_{C1}(j\omega) = j\omega C_1 \mathbf{V}_O(j\omega),$$

$$\mathbf{V}_A(j\omega) = R_2\mathbf{I}_{C1}(j\omega) + \mathbf{V}_O(j\omega) = (1 + j\omega C_1R_2)\mathbf{V}_O(j\omega),$$

$$\mathbf{I}_{C2}(j\omega) = j\omega C_2(\mathbf{V}_O(j\omega) - \mathbf{V}_A(j\omega)) = -(j\omega)^2 C_1C_2R_2\,\mathbf{V}_O(j\omega),$$

$$\mathbf{V}_{in}(j\omega) = \mathbf{V}_A(j\omega) - R_1(\mathbf{I}_{C2}(j\omega) - \mathbf{I}_{C1}(j\omega)) = \left(1 + j\omega C_1(R_1 + R_2) + (j\omega)^2 C_1C_2R_1R_2\right)\mathbf{V}_O(j\omega)$$

And finally, the expression for the gain is

$$A_v(j\omega) = \frac{\mathbf{V}_O(j\omega)}{\mathbf{V}_{in}(j\omega)} = \frac{1}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 C^2 R_1 R_2} = \frac{1}{(1 + j\omega CR_1)(1 + j\omega CR_2)}$$

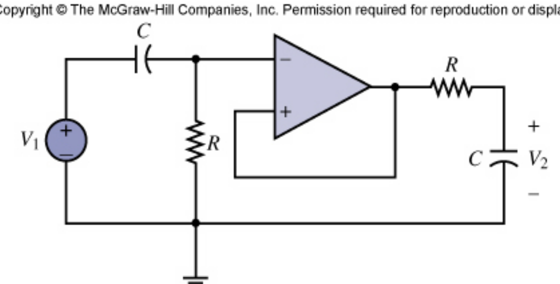
Problem 8.65

Note: the signs on the op amp terminals in Figure P8.65 are inverted

Solution:

Known quantities:

The circuit shown in Figure P8.65.



Find:

Sketch the amplitude response of V_2 / V_1, indicating the half-power frequencies. Assume the op-amp is ideal.

Analysis:

From KCL at the inverting input:

$$\frac{V_x - V_1}{Z_C} + \frac{V_x}{R} = 0 \qquad V_x\left(j\omega C + \frac{1}{R}\right) = j\omega C V_1 \qquad V_x\left(1 + \frac{1}{j\omega RC}\right) = V_1 \qquad V_x = V_1\left(\frac{1}{1 + \frac{1}{j\omega RC}}\right)$$

Similarly, from KCL at the output of the op-amp:

$$\frac{V_2 - V_x}{R} + \frac{V_2}{Z_C} = 0 \qquad V_2\left(\frac{1}{R} + j\omega C\right) = \frac{V_x}{R} \qquad V_x = V_2(1 + j\omega RC)$$

Combining the above results, we find $\frac{V_2}{V_1} = \frac{j\omega RC}{(1 + j\omega RC)^2}$ or $\left|\frac{V_2}{V_1}\right| = \frac{\omega RC}{1 + (\omega RC)^2}$

This function has the form of a band-pass filter, with maximum value determined as follows:

$$G = \left|\frac{V_2}{V_1}\right| = \frac{\omega RC}{1 + (\omega RC)^2} \qquad \frac{dG}{d\omega} = \frac{\left[1 + (\omega RC)^2\right]RC - \omega RC\left[2\omega (RC)^2\right]}{\left[1 + (\omega RC)^2\right]^2}$$

Setting the derivative equal to zero and solving for the center frequency,

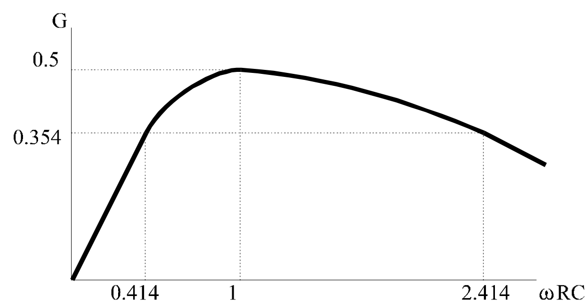
$$RC + \omega^2 R^3 C^3 - 2\omega^2 R^3 C^3 = 0 \qquad 1 - \omega^2 R^2 C^2 = 0 \qquad \omega = \frac{1}{RC}$$

Then $G_{\max} = \frac{1}{1+1} = \frac{1}{2}$, and the half-power frequencies are given by:

$$\frac{\omega RC}{1 + (\omega RC)^2} = \frac{1}{\sqrt{2}} G_{\max} = \frac{1}{\sqrt{2}}\frac{1}{2} \qquad \omega^2 R^2 C^2 + 1 = 2\sqrt{2}\omega RC \qquad R^2 C^2 \omega^2 - 2\sqrt{2} RC\omega + 1 = 0$$

$$\omega = \frac{2\sqrt{2}RC \pm \sqrt{8R^2C^2 - 4R^2C^2}}{2R^2C^2} = \frac{\sqrt{2} \pm 1}{RC}$$

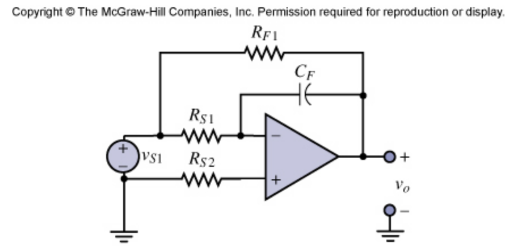
The curve is sketched below.



Problem 8.66

Solution:

Known quantities:



The circuit shown in Figure P8.66.

Find:

Determine an analytical expression for the output voltage for the circuit shown in Figure P8.66. What kind of filter does this circuit implement?

Analysis:

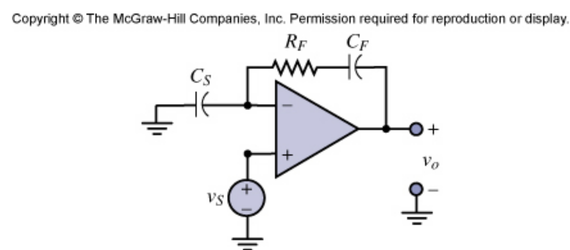
The resistance R_{F1} does not influence the output voltage, so $\mathbf{V}_O(j\omega) = -\frac{\frac{1}{j\omega C_F}}{R_{S1}}\mathbf{V}_{S1}(j\omega) = -\frac{1}{j\omega C_F R_{S1}}\mathbf{V}_{S1}(j\omega)$

The circuit is an integrator (low-pass filter).

Problem 8.67

Solution:

Known quantities:



The circuit shown in Figure P8.67.

Find:

Determine an analytical expression for the output voltage for the circuit shown in Figure P8.67. What kind of filter does this circuit implement?

Analysis:

Figure P8.67 shows a noninverting amplifier, so the output voltage is given by

$$\mathbf{V}_O(j\omega) = \left(1 + \frac{R_F + \frac{1}{j\omega C_F}}{\frac{1}{j\omega C_S}}\right)\mathbf{V}_S(j\omega) = \left(1 + \frac{C_S}{C_F} + j\omega C_S R_F\right)\mathbf{V}_S(j\omega)$$

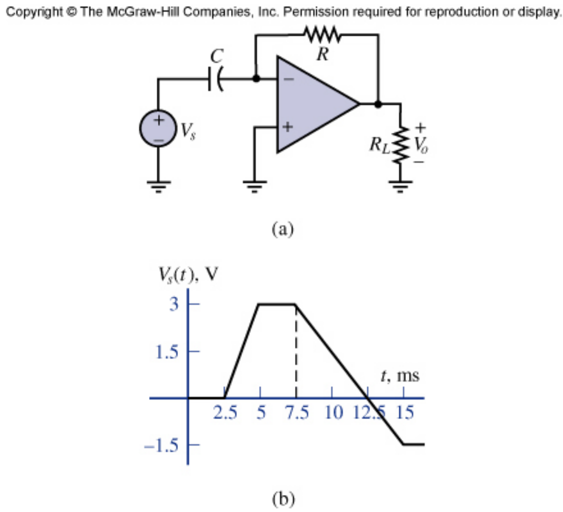
The filter is clearly a high-pass filter.

Sections 8.4, 8.5: Integrator and Differentiator Circuits, Analog Computers

Problem 8.68

Solution:

Known quantities:
Figure P8.68.



Find:
If: $C = 1\ \mu F$ $R = 10\,k\Omega$ $R_L = 1\,k\Omega$, determine an expression for and plot the output voltage as a function of time.

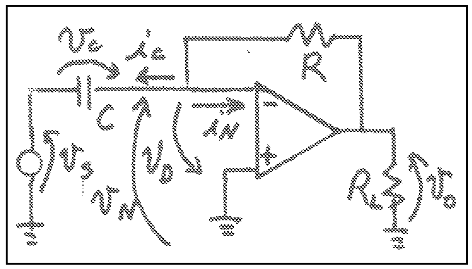
Analysis:

$$KVL: \ -v_N - v_D = 0 \qquad v_D \approx 0 \ \Rightarrow \ v_N \approx 0$$

$$i_C = C\frac{dv_C}{dt} = C\frac{d[\ v_N - v_S\ ]}{dt}$$

$$KCL: \ C\frac{d[\ v_N - v_S\ ]}{dt} + i_N + \frac{v_N - v_O}{R} = 0$$

$$i_N \approx 0 \quad v_N \approx 0 \quad \Rightarrow \quad v_O = -RC\frac{dv_S}{dt}$$



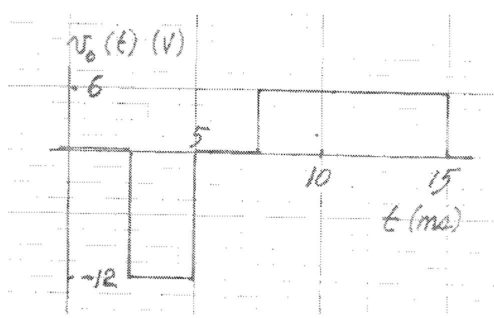
The derivative is the slope of the curve for the source voltage, which is zero for:
t < 2.5 ms, 5 ms < t < 7.5 ms, and t > 15 ms

For 2.5 ms < t < 5 ms:

$$v_O = [10\cdot 10^3][1\cdot 10^{-6}]\frac{3-0}{5\cdot 10^{-3} - 2.5\cdot 10^{-3}} = -12\ \text{V}$$

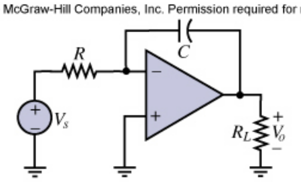
For 7.5 ms < t < 15 ms:

$$v_O = -10\cdot 10^{-3}\frac{[-1.5]-[+3]}{15\cdot 10^{-3} - 7.5\cdot 10^{-3}} = +6\ \text{V}$$

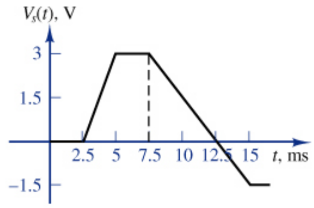


Problem 8.69

Solution:



(a)



(b)

Known quantities:

Figure P8.69(a) and Figure P8.69(b).

Find:

If: $C = 1\ \mu\text{F} \quad R = 10\,\text{k}\Omega \quad R_L = 1\,\text{k}\Omega$

a) An expression for the output voltage.

b) The value of the output voltage at t = 5, 7.5, 12.5, 15, and 20 ms and a plot of the output voltage as a function of time.

Analysis:

a) As usual, assume the op amp is ideal so:

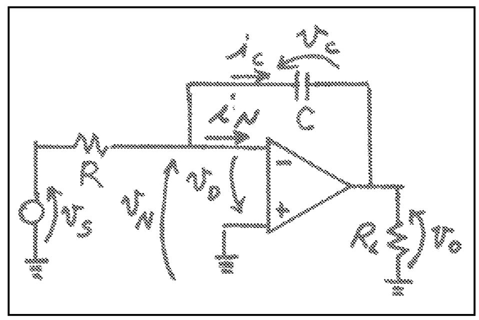
$$KVL: \ -v_N - v_D = 0 \qquad v_D \approx 0 \qquad \Rightarrow \qquad v_N \approx 0$$

$$i_C = C\frac{d[v_N - v_O]}{dt}$$

$$KCL: \ \frac{v_N - v_S}{R} + i_N + C\frac{d[v_N - v_O]}{dt} = 0$$

$$v_N \approx 0 \quad i_N \approx 0 \qquad \Rightarrow \qquad d\,v_O = -\frac{1}{RC}v_S\,dt$$

Integrating: $$v_O[t_f] = v_O[t_i] - \frac{1}{RC}\int_{t_i}^{t_f} v_S\,dt$$



b) Integrating gives the area under a curve. Recall area of triangle = 1/2[base x height] and area of rectangle = base x height.

Integrating the source voltage when it is constant gives an output voltage which is a linear function of time. Integrating the source voltage when it is a linear function of time gives an output voltage which is a quadratic function of time.

$$\frac{1}{RC} = \frac{1}{[10\cdot 10^3][1\cdot 10^{-6}]} = 100\,\frac{\text{rad}}{\text{s}}$$

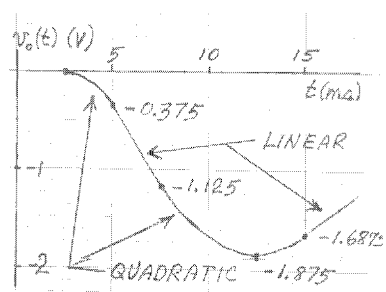
$$v_O[5\text{ms}] = 0\text{ V} - [100]\frac{1}{2}[2.5\cdot 10^{-3}][3\text{ V}] = -375\text{ mV}$$

$$v_O[7.5\text{ms}] = -375\cdot 10^{-3} - [100][2.5\cdot 10^{-3}][3] = -1125\text{ mV}$$

$$v_O[12.5\text{ms}] = -1124\cdot 10^{-3} - [100]\frac{1}{2}[5\cdot 10^{-3}][3] = -1875\text{ mV}$$

$$v_O[15\text{ms}] = -1875\cdot 10^{-3} - [100]\frac{1}{2}[2.5\cdot 10^{-3}][-1.5] = -1687.5\text{ mV}$$

$$v_O[20\text{ms}] = -1687.5\cdot 10^{-3} - [100][5\cdot 10^{-3}][-1.5] = -937.5\text{ mV}$$

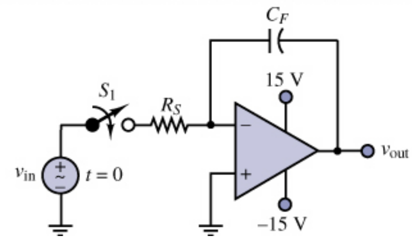


Problem 8.70

Solution:

Known quantities:

In the circuit shown in Figure P8.70, the capacitor is initially uncharged, and the source voltage is:
$v_{in}(t) = 10 \cdot 10^{-3} + \sin(2{,}000\pi t)\,\text{V}$.



Find:

a) At $t = 0$, the switch S_1 is closed. How long does it take before clipping occurs at the output if $R_S = 10\,\text{k}\Omega$ and $C_F = 0.008\ \mu\text{F}$?

b) At what times does the integration of the DC input cause the op-amp to saturate fully?

Analysis:

a)
$$v_{out} = -\frac{1}{R_S C_F}\int v_{in}(t)dt = -\frac{1}{R_S C_F}\int_0^\tau [0.01 + \sin(2000\pi t)]dt$$
$$= -\frac{1}{R_S C_F}\int_0^\tau 0.01dt - \frac{1}{R_S C_F}\int_0^\tau \sin(2000\pi t)dt$$

The peak amplitude of the AC portion of the output is: $v_p = \frac{1}{R_S C_F}\left(\frac{1}{2000\pi}\right) = 1.989\,\text{V}$

The output will begin to clip when: $v_0(DC) - v_p = -15\,\text{V}$ so we need to find at what time the condition: $-\frac{1}{R_S C_F}\int_0^\tau 0.01dt = -13\,\text{V}$ is satisfied. The answer is found below:

$$-\frac{1}{R_S C_F}0.01\tau = -13 \quad \Rightarrow \quad \tau = \frac{13 R_S C_F}{0.01} = 104\,\text{ms}$$

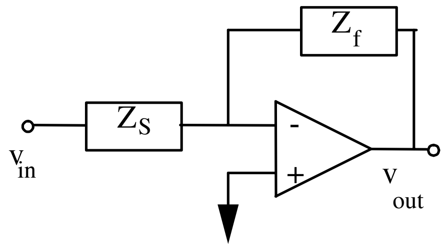
b) Using the results obtained in part a.: $\tau = \frac{15 R_S C_F}{0.01} = 120\,\text{ms}$

Problem 8.71

Solution:

Known quantities:

The circuit shown in Figure 8.22.



Find:

a) If $R_S = 10\,\text{k}\Omega$, $R_F = 2\,\text{M}\Omega$, $C_F = 0.008\ \mu\text{F}$, and $v_S(t) = 10 + \sin(2{,}000\pi t)\,\text{V}$, find v_{out} using phasor analysis.

b) Repeat part a if $R_F = 200\,\text{k}\Omega$, and if $R_F = 20\,\text{k}\Omega$.

c) Compare the time constants with the period of the waveform for part *a* and *b*. What can you say about the time constant and the ability of the circuit to integrate?

Analysis:

a) Replacing the circuit elements with the corresponding impedances: $Z_f = -\dfrac{R_f}{1 + j\omega R_f C_f}$ $\quad Z_S = R_S$

For the signal component at $\omega = 2{,}000\pi$:

$$v_{out} = -\frac{R_f}{R_S}\left(\frac{1}{1 + j\omega R_f C_f}\right)v_{in} = -200\left(\frac{1}{1 + j\omega/62.5}\right)v_{in}$$

$$= v_{in}\frac{200}{\sqrt{1 + \left(\omega/62.5\right)^2}}\angle\left(180^o - \arctan\left(\omega/62.5\right)\right) = 1.9839\angle 90.57^o\ \text{V}$$

For the signal component at $\omega = 0$ (DC): $v_{out} = -200 v_{in} = -2{,}000\,\text{V}$. Thus,

$v_{out}(t) = -2000 + 1.9839\sin\left(2{,}000\pi t + 90.57^o\right)\text{V} \approx -2000 + 2\cos(2{,}000\pi t)\,\text{V}$.

b) $\underline{R_F = 200\,\text{k}\Omega}$

For the signal component at $\omega = 2{,}000\pi$: $v_{out} = -\dfrac{R_f}{R_S}\left(\dfrac{1}{1 + j\omega R_f C_f}\right)v_{in} = 1.9797\angle 95.68^o\ \text{V}$

For the signal component at $\omega = 0$ (DC): $v_{out} = -20 v_{in} = -200\,\text{V}$. Thus,

$v_{out}(t) = -200 + 1.9797\sin\left(2{,}000\pi t + 95.68^o\right)\text{V} \approx -200 + 2\cos(2{,}000\pi t)\,\text{V}$

$\underline{R_F = 20\,\text{k}\Omega}$

For the signal component at $\omega = 2{,}000\pi$: $v_{out} = -\dfrac{R_f}{R_S}\left(\dfrac{1}{1 + j\omega R_f C_f}\right)v_{in} = 1.41\angle 134.8^o\ \text{V}$

For the signal component at $\omega = 0$ (DC): $v_{out} = -2 v_{in} = -20\,\text{V}$. Thus,

$v_{out}(t) = -20 + 1.41\sin\left(2{,}000\pi t + 135^o\right)\text{V} \approx -20 + 1.41\cos\left(2{,}000\pi t + 45^o\right)\text{V}$.

c) In order to have an ideal integrator, it is desirable to have τ >> T.

R_f	τ	T
2 MΩ	16 ms	1 ms
200 kΩ	1.6 ms	1 ms
20 kΩ	0.16 ms	1 ms

Problem 8.72

Solution:

Known quantities:

For the circuit of Figure 8.27, assume an ideal op-amp with $v_S(t) = 10 \cdot 10^{-3}\sin(2{,}000\pi t)\,\text{V}$, $C_S = 100\ \mu\text{F}$, $C_F = 0.008\ \mu\text{F}$, $R_F = 2\,\text{M}\Omega$, and $R_S = 10\,\text{k}\Omega$.

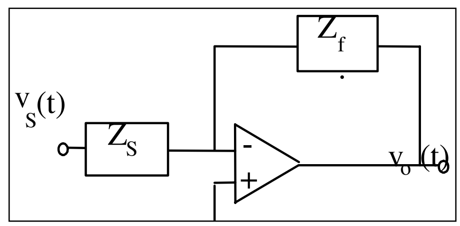
Find:

a) The frequency response, $\frac{v_0}{v_S}(j\omega)$.

b) Use superposition to find the actual output voltage (remember that DC = 0 Hz).

Analysis:

a)



$$Z_S = R_S + \frac{1}{j\omega C_S} \quad Z_f = \frac{R_f}{1 + R_f C_f j\omega}$$

$$\frac{v_o}{v_S} = -\frac{Z_f}{Z_S} = -\frac{\frac{R_f}{1 + j\omega R_f C_f}}{R_S + \frac{1}{j\omega C_S}} = -\frac{j\omega R_f C_S}{(j\omega R_S C_S + 1)(j\omega R_f C_f + 1)}$$

$$\frac{v_o}{v_S}(j\omega) = -\frac{\frac{j\omega}{5 \cdot 10^{-3}}}{(j\omega + 1)\left(\frac{j\omega}{62.5} + 1\right)}$$

b) $v_o = v_S \frac{v_o}{v_S}(j\omega)$. By superposition, $\quad v_o\big|_{10\,\text{mV}} = v_S \frac{j0 v_S}{1+1} = 0\,\text{V}$

$$v_o\big|_{\omega = 2000\pi} = \frac{j1.257 \cdot 10^6 v_S}{(1 + j6283)(1 + j100)} = 20 \cdot 10^{-3} \angle -89.43^o\ \text{V}$$

$$v_o(t) = 20 \cdot 10^{-3}\sin\left(2000\pi t - 89.43^o\right)\text{V}$$

We can say that the practical differentiator is a good approximation of the ideal differentiator.

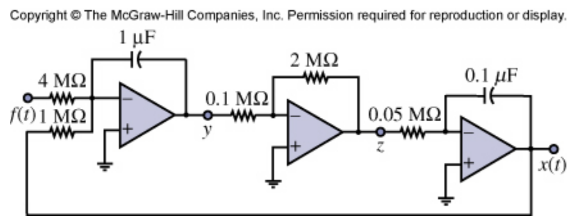
Problem 8.73

Solution:

Known quantities:

Find:

Derive the differential equation corresponding to the analog computer simulation circuit of Figure P8.73.



Analysis:

$x(t) = -200\int z\,dt$ or $z = -\frac{1}{200}\frac{dx}{dt}$. Also, $z = -20y$.

Therefore, $y = \frac{1}{4000}\frac{dx}{dt}$. Also, $y = \int\left(4f(t) + x(t)\right)dt$ or

$$\frac{dy}{dt} = 4f(t) + x(t).$$

Therefore $\frac{1}{4000}\frac{d^2x}{dt^2} = 4f(t) + x(t)$ or $\frac{d^2x}{dt^2} - 4000x(t) - 16000f(t) = 0$.

Problem 8.74

Solution:

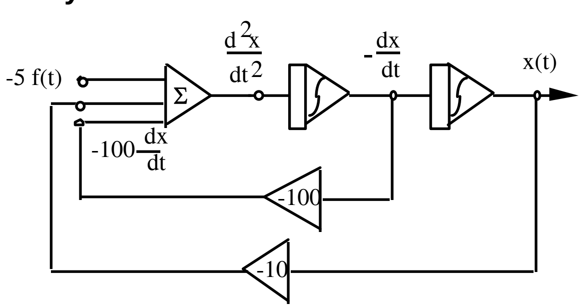
Known quantities:

Find:

Construct the analog computer simulation corresponding to the following differential equation:

$$\frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 10x = -5f(t).$$

Analysis:



Section 8.6: Physical Limitations of Operational Amplifiers

Problem 8.75

Solution:

Known quantities:

For the circuit shown in Figure 8.8: $R_S = R_F = 2.2\text{k}\Omega$.

Find:

Find the error introduced in the output voltage if the op-amp has an input offset voltage of 2 mV.
Assume zero bias currents and that the offset voltage appears as in Figure 8.50.

Analysis:

By superposition, the error is given by

$$\Delta v_O = \left(1 + \frac{R_F}{R_S}\right) V_{Offset} = 4\text{mV}$$

Problem 8.76

Solution:

Known quantities:

For the circuit shown in Figure 8.8: $R_S = R_F = 2.2\text{k}\Omega$.

Find:

Repeat Problem 8.75 assuming that in addition to the input offset voltage, the op-amp has an input bias current of 1 μA. Assume that the bias currents appear as in Figure 8.51.

Analysis:

By superposition, the effect of the bias currents on the output is

$$v_+ = -RI_{B+} = v_-$$

$$\frac{\Delta v_{O,IB} - v_-}{R_F} - \frac{v_-}{R_S} = I_{B-} \quad \Rightarrow \quad \Delta v_{O,IB} = R_F\left(I_{B-} - R\left(\frac{1}{R_F} + \frac{1}{R_S}\right) I_{B+}\right)$$

If $R = R_F \parallel R_S$ then $\Delta v_{O,IB} = R_F\left(I_{B-} - I_{B+}\right) = -R_F I_{OS} = -2200 \cdot 1 \cdot 10^{-6} = -2.2\text{mV}$

The effect of the bias voltage is

$$\Delta v_{O,VB} = \left(1 + \frac{R_F}{R_S}\right) V_{Offset} = 4\text{mV}$$

So, the total error on the output voltage is

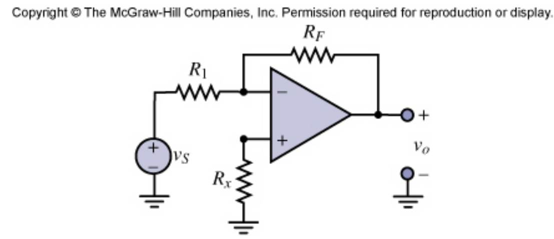
$$\Delta v_O = \Delta v_{O,VB} + \Delta v_{O,IB} = 1.8\text{mV}$$

Problem 8.77

Solution:

Known quantities:

The circuit shown in Figure P8.77.
The input bias currents are equal and the input bias voltage is zero.



Find:

The value of R_x that eliminates the effect of the bias currents in the output voltage.

Analysis:

We can write

$$v_+ = -R_x I_{B+} = v_- \quad , \quad I_{B+} = I_{B-}$$

$$\frac{\Delta v_{O,IB} - v_-}{R_F} - \frac{v_-}{R_1} = I_{B-} \quad \Rightarrow \quad \Delta v_{O,IB} = R_F\left(I_{B-} - R_x\left(\frac{1}{R_F} + \frac{1}{R_1}\right)I_{B+}\right)$$

By selecting $R_x = R_F \parallel R_1$, a zero output voltage error is obtained.

Problem 8.78

Solution:

Known quantities:

For the circuit shown in Figure P8.77: $R_F = 3.3\text{k}\Omega \quad R_1 = 1\text{k}\Omega \quad v_S = 1.5\sin(\omega t)$ V

Find:

The highest-frequency input that can be used without exceeding the slew rate limit of 1V/μs.

Analysis:

The maximum slope of a sinusoidal signal at the output of the amplifier is

$$S_0 = |A| \cdot \omega = \frac{R_F}{R_1}\omega = 10^6\ \frac{\text{V}}{\text{s}} \Rightarrow \omega_{\max} = \frac{1}{3.3}10^6 = 3.03 \cdot 10^5 \text{ rad/s}$$

Problem 8.79

Solution:

Known quantities:

The Bode plot shown in Figure 8.47: $A_0 = 10^6 \quad \omega_0 = 10\pi$ rad/s

Find:

The approximate bandwidth of a circuit that uses the op-amp with a closed loop gain of A_1 =75 and A_2 =350.

Analysis:

The product of gain and bandwidth in any given op-amp is constant, so

$$\omega_1 = \frac{A_0\omega_0}{A_1} = \frac{10^7\pi}{75} = 4.186 \cdot 10^5 \text{ rad/s}$$

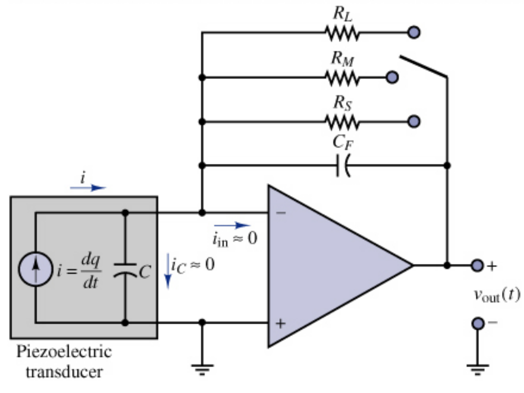
$$\omega_2 = \frac{A_0\omega_0}{A_2} = \frac{10^7\pi}{350} = 8.971 \cdot 10^4 \text{ rad/s}$$

Problem 8.80

Solution:

Known quantities:

For the practical charge amplifier circuit shown in Figure P8.80, the user is provided with a choice of three time constants - $\tau_{long} = R_L C_F$, $\tau_{medium} = R_M C_F$, $\tau_{short} = R_S C_F$, which can be selected by means of a switch. Assume that $R_L = 10\,\text{M}\Omega$, $R_M = 1\,\text{M}\Omega$, $R_S = 0.1\,\text{M}\Omega$, and $C_F = 0.1\ \mu\text{F}$.



Find:

Analyze the frequency response of the practical charge amplifier for each case, and determine the lowest input frequency that can be amplified without excessive distortion for each case. Can this circuit amplify a DC signal?

Analysis:

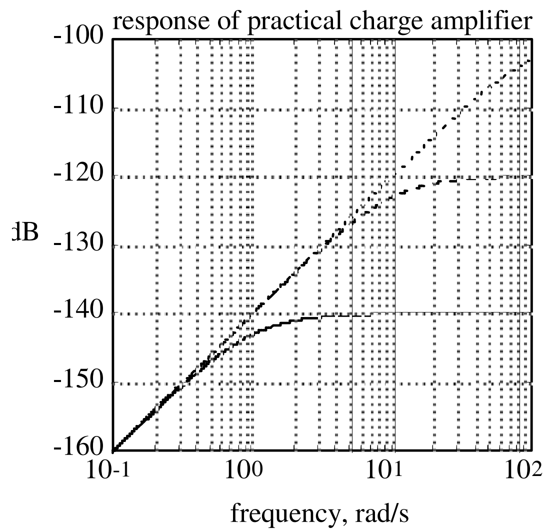
Applying KCL at the inverting terminal:

$$i + \frac{V_{out}}{R + \frac{1}{j\omega C}} = 0$$

or, $$\frac{V_{out}}{i} = -\frac{j\omega C}{1 + j\omega RC}$$

This response is clearly that of a high-pass filter, therefore the charge amplifier will never be able to amplify a DC signal.

The low end of the (magnitude) frequency response is plotted below for the three time constants. The figure illustrates how as the time constant decreases the cut-off frequency moves to the right (solid line: R = 10 MΩ; dashed line: R =1 MΩ; dotted line: R = 0.1 MΩ).
From the frequency response plot one can approximate the minimum useful frequency for distortionless response to be (nominally) 1 Hz for the 10 MΩ case, 10 Hz for the 1 MΩ case, and 100 Hz for the 0.1 MΩ case.



Problem 8.81

Solution:

Known quantities:

Find:

Consider a differential amplifier. We would desire the common-mode output to be less than 1% of the differential-mode output. Find the minimum dB common-mode rejection ratio ($CMRR$) that fulfills this requirement if the differential mode gain A_{dm} = 1,000. Let

$$v_1 = \sin(2{,}000\pi t) + 0.1\sin(120\pi)\text{V}$$

$$v_2 = \sin\left(2{,}000\pi t + 180^o\right) + 0.1\sin(120\pi)\text{V}$$

$$v_0 = A_{dm}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

Analysis:

We first determine which is the common mode and which is the differential mode signal:

$$v_1 - v_2 = 2\sin(2{,}000\pi t)$$

$$\frac{v_1 + v_2}{2} = 0.1\sin(120\pi t)$$

Therefore, $v_{out} = A_{dm} 2\sin(2000\pi t) + A_{cm} 2\sin(120\pi t)$

Since we desire the common mode output to be less than 1% of the differential mode output, we require:
$A_{cm}(0.1) \le 0.01(2)$ or $A_{cm} \le 0.2$.

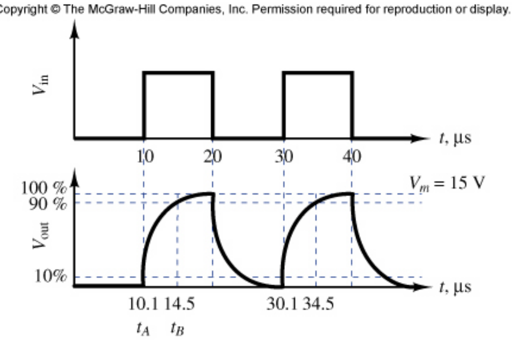
$CMRR = \dfrac{A_{dif}}{A_{cm}}$ So $CMRR_{\min} = \dfrac{1000}{0.2} = 5000 = 74$ dB .

Problem 8.82

Solution:

Known quantities:

As indicated in Figure P8.82, the rise time, t_r, of the output waveform is defined as the time it takes for the waveform to increase from 10% to 90% of its final value, i.e., $t_r \equiv t_b - t_a = -\tau(\ln 0.1 - \ln 0.9) = 2.2\tau$, where τ is the circuit time constant.



Find:

Estimate the slew rate for the op-amp.

Analysis:

$$\left|\frac{dv_{out}}{dt}\right|_{\max} = \frac{V_m(0.9-0.1)}{(14.5-10.1)\times 10^{-6}} = \frac{15\times 0.8}{4.4}\frac{\text{V}}{\mu\text{s}} \approx 2.73\frac{\text{V}}{\mu\text{s}}.$$

Therefore, the slew rate is approximately $2.73\frac{\text{V}}{\mu\text{s}}$.

Problem 8.83

Solution:

Known quantities:

Find:

Consider an inverting amplifier with open-loop gain 10^5. With reference to Equation 8.18,

a) If $R_S = 10\,\text{k}\Omega$ and $R_F = 1\,\text{M}\Omega$, find the voltage gain $A_{V(CL)}$.

b) Repeat part a if $R_S = 10\,\text{k}\Omega$ and $R_F = 10\,\text{M}\Omega$.

c) Repeat part a if $R_S = 10\,\text{k}\Omega$ and $R_F = 100\,\text{M}\Omega$.

d) Using the resistors values of part c, find $A_{V(CL)}$ if $A_{V(OL)} \to \infty$.

Analysis:

$$A_{CL} = -\frac{R_F}{R_S}\frac{1}{1+\dfrac{R_F+R_S}{R_S A_{OL}}}$$

a) A_{CL} = - 99.899

b) A_{CL} = - 990

c) A_{CL} = -9091

d) As $A_{OL} \to \infty$, $A_{CL} = -10{,}000$.

Problem 8.84

Solution:

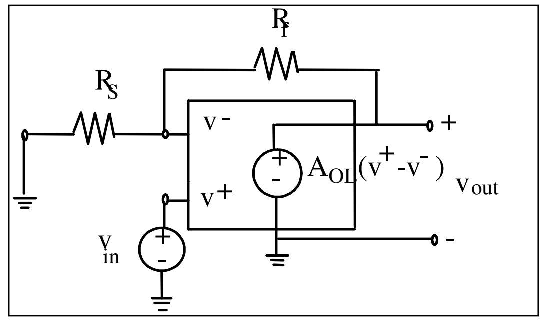
Known quantities:

Figure P8.84.

Find:

a) If the op-amp shown in Figure P8.84 has an open-loop gain of 45 X 10^5, find the closed-loop gain for $R_S = R_F = 7.5\,\mathrm{k}\Omega$.

b) Repeat part a if $R_F = 5R_S = 37.5\,\mathrm{k}\Omega$.



Analysis:

a)

$v_{in} = v^+$ and

$$v_0 = A_{OL}\left(v_{in} - v^-\right) \quad \Rightarrow \quad v^- = -\left(\frac{v_0}{A_{OL}} - v_{in}\right).$$

Writing KCL at v^-: $\dfrac{v^- - 0}{R_S} + \dfrac{v^- - v_0}{R_F} = 0$.

Substituting, $$\frac{\frac{-v_0}{A_{OL}} + v_{in}}{R_S} + \frac{\frac{-v_0}{A_{OL}} + v_{in}}{R_F} = \frac{v_0}{R_F} \quad \Rightarrow \quad v_0\left(-\frac{1}{A_{OL}R_S} - \frac{1}{A_{OL}R_F} - \frac{1}{R_F}\right) = -v_{in}\left(\frac{1}{R_S} + \frac{1}{R_F}\right)$$

$$\frac{v_0}{v_{in}} = \frac{-\left(\frac{1}{R_S} + \frac{1}{R_F}\right)}{-\frac{1}{A_{OL}R_S} - \frac{1}{A_{OL}R_F} - \frac{1}{R_F}} = A_{OL}R_S R_F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

where: $$K_1 = R_F R_S + R_S^2 + R_S A_{OL} R_F$$ $$K_2 = R_F R_S + R_F^2 + R_S A_{OL} R_F$$ $$\rightarrow \quad \frac{v_0}{v_{in}} = \frac{R_F}{R_S}\left(\frac{1}{\frac{R_S + R_F}{A_{OL}R_S} + 1}\right) + \left(\frac{1}{\frac{R_S + R_F}{A_{OL}R_S} + 1}\right)$$

For the conditions of part a we obtain:

$$A_{CL} = \frac{1}{2/45 \times 10^5 + 1} + \frac{1}{2/45 \times 10^5 + 1} = 1.999$$

b) $$A_{CL} = 5\frac{1}{6/45 \times 10^5 + 1} + \frac{1}{6/45 \times 10^5 + 1} = 5.999.$$

Problem 8.85

Solution:

Known quantities:

Find:
Given the unity-gain bandwidth for an ideal op-amp equal to 5.0 MHz, find the voltage gain at frequency of f = 500 kHz.

Analysis:

$$A_0\omega_0 = K = A_1\omega_1 \Rightarrow K = 1\times 2\pi \times 5.0\text{MHz} = 10\pi\times 10^6 \quad \text{So } A_1 = \frac{K}{\omega_1} = \frac{10\pi\times 10^6}{2\pi\times 500} = 10{,}000$$

Problem 8.86

Solution:

Known quantities:

Find:
Determine the relationship between a finite and frequency-dependent open-loop gain $A_{V(OL)}(\omega)$ and the closed-loop gain $A_{V(CL)}(\omega)$ of an inverting amplifier as a function of frequency. Plot $A_{V(CL)}$ versus ω.

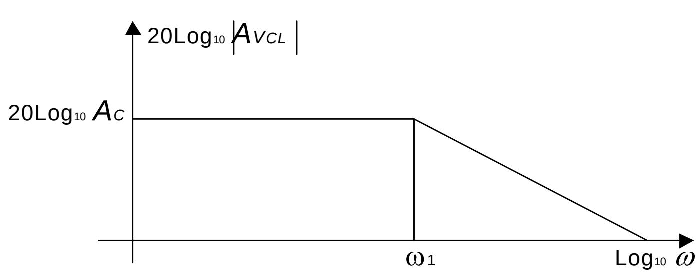
Analysis:
As shown in Equation 8.84, if we consider a real op-amp:

$$\frac{A_{V(OL)}(\omega)}{A_{V(CL)}(\omega)} = \frac{A_0 \Big/ \left(1+ j\omega/\omega_0\right)}{A_C \Big/ \left(1+ j\omega/\omega_1\right)}$$

, where $A_C = -R_F/R_S$, and A_0 is the low-frequency open-loop gain.

If we choose $A_0 = 10^6$, and $\omega_0 = 10\pi$, and since $A_0\omega_0 = A_C\omega_1$: $\omega_1 = \dfrac{10^6\cdot 10\pi}{R_F/R_S}$.

Therefore:



Problem 8.87

Solution:

Known quantities:

Find:

A sinusoidal sound (pressure) wave impinges upon a condenser microphone of sensitivity S (mV / Pa). The voltage output of the microphone v_S is amplified by two cascaded inverting amplifiers to produce an amplified signal v_0. Determine the peak amplitude of the sound wave (in dB) if v_0 = 5 V_{RMS}. Estimate the maximum peak magnitude of the sound wave in order that v_0 not contain any saturation effects of the op-amps.

Analysis:

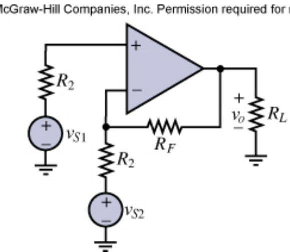
$$\left.\begin{array}{l} p(t) = P_0 \sin(\omega t)\,\text{Pa} \quad ; \quad v_S(t) = S\cdot p(t)\,\text{mV} \\ v_0(t) = \dfrac{A_1A_2}{1000} v_S(t)\,\text{V} = \dfrac{A_1A_2}{1000} S\cdot P_0 \sin(\omega t)\,\text{V} \\ v_0 = 5\ \text{V}_{\text{RMS}} \end{array}\right\} \Rightarrow \quad A_1A_2\cdot S\cdot P_0 = 5000\ \text{mV} \quad \Rightarrow$$

$$\Rightarrow \quad P_0 = 20\log_{10}\frac{5000}{A_1A_2\cdot S}\ \text{dB}$$

$$\frac{A_1A_2\cdot S\cdot P_0}{1000} \le 12\ \text{V} \quad \Rightarrow \quad P_0 \le 12{,}000\cdot A_1A_2\cdot S\ \text{Pa}\,.$$

Problem 8.88

Solution:



Known quantities:

If, in the circuit shown in Figure P8.88

$$v_{S1} = 2.8 + 0.01\cos(\omega t)\text{V} \quad ; \quad v_{S2} = 3.5 - 0.007\cos(\omega t)\text{V}$$

$$A_{v1} = -13 \quad ; \quad A_{v2} = 10 \quad ; \quad \omega = 4\frac{\text{krad}}{\text{s}}$$

Find:

a) The common and difference mode input signals.
b) The common and differential mode gains.
c) The common and difference mode components of the output voltage.
d) The total output voltage.
e) The common mode rejection ratio.

Analysis:

a)

$$v_{S\text{-}C} = \frac{1}{2}[\,v_{S1} + v_{S2}\,] = \frac{1}{2}(\,[\,2.8\ \text{V} + 10\ \text{mV}\cos\omega t\,] + [\,3.5\ \text{V} - 7\ \text{mV}\cos\omega t\,]\,) = $$
$$= 3.15\ \text{V} + 1.5\ \text{mV}\cos\omega t$$

$$v_{S\text{-}D} = v_{S1} - v_{S2} = [\,2.8\ \text{V} + 10\ \text{mV}\cos\omega t\,] - [\,3.5\ \text{V} - 7\ \text{mV}\cos\omega t\,] =$$
$$= -0.7\ \text{V} + 17\ \text{mV}\cos\omega t$$

Note that the expression for the differential input voltage (and the differential gain below) depends on which of the two sources (in this case, v_{S1}) is connected to the non-inverting input.

b) $A_{vc} = A_{v1} + A_{v2} = -3 \qquad A_{vd} = \frac{1}{2}[\, A_{v1} - A_{v2} \,] = -11.5$

c)
$$v_{O\text{-}C} = v_{S\text{-}C} A_{vc} = [\,3.15\text{ V} + 1.5\text{ mV}\cos\omega t\,][\,-3\,] = -9.450\text{ V} - 4.5\text{ mV}\cos\omega t$$
$$v_{O\text{-}D} = v_{S\text{-}D} A_{vd} = [\,-0.7\text{ V} + 17\text{ mV}\cos\omega t\,][\,-11.5\,] = 8.050\text{ V} - 195.5\text{ mV}\cos\omega t$$

d) $v_O = v_{O\text{-}C} + v_{O\text{-}D} = -1.4\text{ V} - 200.0\text{ mV}\cos\omega t$

e) $CMRR = 20\text{ dB}\,\text{Log}_{10}\left[\frac{|A_{vc}|}{|A_{vd}|}\right] = 20\text{ dB}\,\text{Log}_{10}\left[\frac{3}{11.5}\right] = -11.67\text{ dB}$.

Problem 8.89

Solution:

Known quantities:

If, in the circuit shown in Figure P8.88:

$$v_{S1} = 3.5 + 0.01\cos\omega t\text{ V} \qquad v_{S2} = 3.5 - 0.01\cos\omega t$$
$$A_{vc} = 10\text{ dB} \quad A_{vd} = 20\text{ dB} \qquad \omega = 4\ \frac{\text{krad}}{\text{s}}$$

Find:

a) The common and differential mode input voltages.
b) The voltage gains for v_{S1} and v_{S2}.
c) The common mode component and differential mode components of the output voltage.
d) The common mode rejection ratio [*CMRR*] in dB.

Analysis:

a)
$$v_{S\text{-}C} = \frac{1}{2}[\, v_{S1} + v_{S2} \,] = \frac{1}{2}\big([\,3.5\text{ V} + 10\text{ mV}\cos\omega t\,] + [\,3.5\text{ V} - 10\text{ mV}\cos\omega t\,]\big) = 3.5\text{ V}$$
$$v_{S\text{-}D} = v_{S1} - v_{S2} = [\,3.5\text{ V} + 10\text{ mV}\cos\omega t\,] - [\,3.5\text{ V} - 10\text{ mV}\cos\omega t\,] = 20\text{ mV}\cos\omega t$$

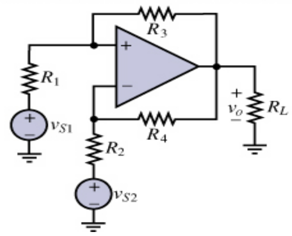
Note the expression for the difference mode voltage depends on which signal [in this case, v_{S1}] is connected to the non-inverting input. This is also true for the expression for the differential gain below.

b)
$$A_{vc} = 10^{\frac{10\text{ dB}}{20\text{ dB}}} = 3.162 \qquad A_{vd} = 10^{\frac{20\text{ dB}}{20\text{ dB}}} = 10$$
$$A_{vc} = A_{v1} + A_{v2} \quad \Rightarrow \quad A_{v1} = \frac{1}{2}A_{vc} + A_{vd} = \frac{1}{2}[\,3.162\,] + 10 = 11.581$$
$$A_{vd} = \frac{1}{2}[\, A_{v1} - A_{v2} \,] \quad \Rightarrow \quad A_{v2} = \frac{1}{2}A_{vc} - A_{vd} = \frac{1}{2}[\,3.162\,] - 10 = -8.419$$

c)
$$v_{\text{O-C}} = v_{\text{S-C}} A_{\text{vc}} = [\,3.5\text{ V}\,][\,3.162\,] = 11.07\text{ V}$$
$$v_{O\text{-}D} = v_{S\text{-}D} A_{vd} = [\,20\text{ mV}\cos\omega t\,][\,10\,] = 200\text{ mV}\cos\omega t$$

d) $CMRR = 20\text{ dB}\,\text{Log}_{10}\left[\left|\frac{A_{vc}}{A_{vd}}\right|\right] = 20\text{ dB}\,\text{Log}_{10}\left[\frac{3.162}{10}\right] = -10\text{ dB}$.

Problem 8.90



Solution:

Known quantities:

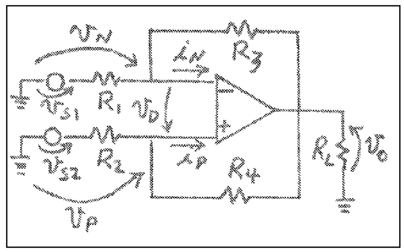
If, in the circuit shown in Figure P8.90, the two voltage sources are temperature sensors with T = Temperature [Kelvin] and:

$$v_{S1} = kT_1 \qquad v_{S2} = kT_2$$

$$Where: \; k = 120\,\mu\frac{\text{V}}{\text{K}} \qquad R_1 = R_3 = R_4 = 5\,\text{k}\Omega \quad R_2 = 3\,\text{k}\Omega \quad R_L = 600\,\Omega$$

Find:

a) The voltage gains for the two input voltages.
b) The common mode and differential mode input voltage.
c) The common mode and difference mode gains.
d) The common mode component and the differential mode component of the output voltage.
e) The common mode rejection ratio [*CMRR*] in dB.



Analysis:

a)

Assume the op amp is ideal.:
$$KVL: \; -v_N - v_D + v_P = 0$$
$$v_D \approx 0 \qquad \Rightarrow \qquad v_N = v_P$$

$$KCL: \; i_N + \frac{v_N - v_{S1}}{R_1} + \frac{v_N - v_O}{R_3} = 0 \qquad i_N \approx 0: \Rightarrow \qquad v_N = \frac{\dfrac{v_{S1}}{R_1} + \dfrac{v_O}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_3}} \frac{R_1 R_3}{R_1 R_3}$$

$$KCL: \; i_P + \frac{v_P - v_{S2}}{R_2} + \frac{v_P - v_O}{R_4} = 0 \qquad i_P \approx 0: \Rightarrow \qquad v_P = \frac{\dfrac{v_{S2}}{R_2} + \dfrac{v_O}{R_4}}{\dfrac{1}{R_2} + \dfrac{1}{R_4}} \frac{R_2 R_4}{R_2 R_4}$$

Equating: $$\frac{v_{S1} R_3 + v_O R_1}{R_3 + R_1} = \frac{v_{S2} R_4 + v_O R_2}{R_4 + R_2}$$

$$v_O = \frac{v_{S2}\dfrac{R_4}{R_4 + R_2} + v_{S1}\left[-\dfrac{R_3}{R_3 + R_1}\right]}{\dfrac{R_1}{R_3 + R_1} - \dfrac{R_2}{R_4 + R_2}} \frac{[R_4 + R_2][R_3 + R_1]}{[R_4 + R_2][R_3 + R_1]}$$

$$= v_{S2}\frac{R_4[R_3 + R_1]}{R_1[R_4 + R_2] - R_2[R_3 + R_1]} + v_{S1}\left[-\frac{R_3[R_4 + R_2]}{R_1[R_4 + R_2] - R_2[R_3 + R_1]}\right]$$

$$A_{v2} = \frac{R_4[R_3 + R_1]}{R_1[R_4 + R_2] - R_2[R_3 + R_1]} = \frac{[5][10]}{[5][8] - [3][10]} = 5$$

$$A_{v1} = \frac{-R_3[R_4 + R_2]}{R_1[R_4 + R_2] - R_2[R_3 + R_1]} = \frac{-[5][8]}{[5][8] - [3][10]} = -4$$

b)

$$v_{S1} = k\,T_1 = [120\,\mu\frac{\mathrm{V}}{\mathrm{K}}][310\,\mathrm{K}] = 37.20\,\mathrm{mV}$$

$$v_{S2} = k\,T_2 = [120\,\mu\frac{\mathrm{V}}{\mathrm{K}}][335\,\mathrm{K}] = 40.20\,\mathrm{mV}$$

$$v_{S\text{-}C} = \frac{1}{2}[\,v_{S1} + v_{S2}\,] = \frac{1}{2}[\,37.20\,\mathrm{mV} + 40.20\,\mathrm{mV}\,] = 38.70\,\mathrm{mV}$$

$$v_{S\text{-}D} = v_{S2} - v_{S1} = 40.20\,\mathrm{mV} - 37.2\,\mathrm{mV} = +3\,\mathrm{mV}$$

Note that the expression for the differential mode voltage (and the differential mode gain below) depends on Source #2 being connected to the non-inverting input.

c)

$$v_O = v_{S2}\,A_{v2} + v_{S1}\,A_{S1} = [\,v_{S\text{-}C} + \frac{1}{2}v_{S\text{-}D}\,]\,A_{v2} + [\,v_{S\text{-}C} - \frac{1}{2}v_{S\text{-}D}\,]\,A_{v1}$$

$$= v_{S\text{-}C}\,[\,A_{v2} + A_{v1}] + v_{S\text{-}D}\frac{1}{2}[\,A_{v2} - A_{v1}] = v_{S\text{-}C}\,A_{v\text{-}c} + v_{S\text{-}D}\,A_{vd}$$

$$\Rightarrow \begin{cases} A_{vc} = A_{v2} + A_{v1} = 5 + [-4] = 1 \\ A_{vd} = \frac{1}{2}[\,A_{v2} - A_{v1}\,] = \frac{1}{2}[\,5 - (-4)\,] = 4.5 \end{cases}$$

d)

$$v_{O\text{-}C} = v_{S\text{-}C}\,A_{vc} = [\,38.70\,\mathrm{mV}\,][\,1\,] = 38.70\,\mathrm{mV}$$

$$v_{O\text{-}D} = v_{S\text{-}D}\,A_{vd} = [\,3\,\mathrm{mV}\,][\,4.5\,] = 13.5\,\mathrm{mV}$$

e) An ideal difference amplifier would eliminate all common mode output. This did not happen here. A figure of merit for a differential amplifier is the Common Mode Rejection Ratio [*CMRR*]:

$$CMRR = 20\,\mathrm{dB}\,\mathrm{Log}_{10}[\,\frac{A_{vc}}{A_{vd}}\,] = 20\,\mathrm{dB}\,\mathrm{Log}_{10}[\,\frac{1}{4.5}\,] = -13.06\,\mathrm{dB}\,.$$

Problem 8.91

Solution:

Known quantities:

If, for the differential amplifier shown in Figure P8.90:

$v_{S1} = 13\text{ mV} \quad v_{S2} = 9\text{ mV} \quad v_0 = v_{0C} + v_{0D}$

$v_{0C} = 33\text{ mV} \quad v_{0D} = 18\text{ V}$

Find:

a) The common mode gain.
b) The differential mode gain.
c) The common mode rejection ratio in dB.

Analysis:

a)

$$v_{SC} = \frac{1}{2}[v_{S1} + v_{S2}] = \frac{1}{2}[13\text{ mV} + 9\text{ mV}] = 11\text{ mV}$$

$$\Rightarrow A_{VC} = \frac{v_{OC}}{v_{SC}} = \frac{33\text{ mV}}{11\text{ mV}} = 3$$

b)

$$v_{SD} = v_{S2} - v_{S1} = 9\text{ mV} - 13\text{ mV} = -4\text{ mV}$$

$$\Rightarrow A_{VD} = \frac{v_{OD}}{v_{SD}} = \frac{18\text{ V}}{-4\text{ mV}} = -4500$$

c) $$CMRR = 20\text{ dB Log}_{10}\left[\frac{|A_{VC}|}{|A_{VD}|}\right] = -63.52\text{ dB}.$$