Chapter 17: Introduction to Digital Communications – Instructor Notes

Digital communication devices have become ubiquitous in today's world and warrant further study by engineering students from all disciplines. Chapter 17, *Introduction to Digital Communications*, builds on the analog communication concepts introduced in Chapter 16, though many concepts in Chapter 17 are self-contained. Chapter 17 provides several real-world examples to solidify the student's understanding and to connect abstract ideas to reality. The instructor should refer often to figure 17.2 when discussing each part of the digital communications system so that the student can see how each component of a digital communication system fits into the big picture. The contents of the chapter are outlined in the following paragraph.

Section 17.1 describes a typical digital communications system through a block-diagrammatic approach. Section 17.2 introduces elements from probability theory necessary to understand bit-error rate calculations in later sections. Sections 17.3 through 17.6 describe in detail the blocks of the typical digital communication system shown in Figure 17.2, including the ideas of pulse-coded modulation and quantization. Advanced concepts are reviewed in Section 17.7, including the following: time-division multiple access (TDMA), frequency-division multiple access (FDMA), and code-division multiple access (CDMA). Section 17.8 covers the basic elements of digital data transmission, including a brief survey of the IEEE 488 and RS-232 standards and an introduction to USB and CAN protocols.

Chapter 17 includes 23 homework problems, most of which are extensions of the examples presented in the text. Many of the homework problems require the use of Matlab™. The instructor can find the required files on the course website.

Learning Objectives

1. Understand the basic units of a digital communications system. *Section 17.1*
2. Understand basic probability theory and solve problems relating to digital communications. *Section 17.2*.
3. Understand and use formulas relating to pulse-coded modulation and quantization to calculate solutions to homework problems. *Section 17.3*
4. Compare techniques of source coding and design a Huffman source code. Understand source information entropy and apply this knowledge to homework problems. *Section 17.4*
5. Determine waveforms that are ideal for digital communications. Understand the probability of error in a digital baseband system. Simulate a simple digital baseband communications system. *Section 17.5*
6. Understand channel capacity—the fundamental limit on the rate of reliable data transmission. Understand how coding can help one achieve this capacity. Calculate the capacity of practical systems. Compare the bit error rate (BER) of coded and uncoded systems. *Section 17.6*
7. Contrast different schemes for multiple-user access to shared radio-frequency resources: time-division multiple access (TDMA), frequency-division multiple access (FDMA), and code-division multiple access (CDMA). Understand the basics of spread spectrum and ultra-wideband communications. *Section 17.7*
8. Compare specific schemes for wireline data transmission in digital instruments: IEEE 488 Bus, RS-232, CAN, Ethernet, and USB. Learn the definitions of serial and parallel transmission, handshaking, simplex, half- and full-duplex communications. *Section 17.8*

Section 17.2: Introductory Probability

Problem 17.1

Solution:

Known quantities:

The complementary error function is $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$.

Find:

The Q-function in terms of the complementary error function. Note that the Q-function is defined as
$\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}\,dt$. The Q-functions measures the tail probability of a Normal Gaussian random variable.

Analysis:

Substituting $s = t\sqrt{2}$ into the expression for the complementary error function, we have

$$\mathrm{erfc}(x) = \sqrt{\frac{2}{\pi}}\int_{x\sqrt{2}}^\infty e^{-\frac{s^2}{2}}\,ds .$$

It is now easy to see that

$$\mathrm{Q}(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right).$$

Section 17.3: Pulse Code Modulation

Problem 17.2

Solution:

Known quantities:

Bandwidth of voice signal is 3,400 Hz. Bit rate of channel is 35,000 bits/sec.

Find:

The sample rate and maximum number of bits for quantization.

Assumptions:

Each sample is quantized independently of other samples. It is possible to achieve better performance when jointly quantizing several samples. This is called *vector quantization*.

Analysis:

From Nyquist sampling theory, the sample rate must be at least 6,800 samples per second (twice the highest frequency). We find the number of bits per sample by dividing the bit rate of the channel by the sample rate:

Bits per sample = $\dfrac{35000\,\dfrac{\text{bits}}{\text{sec}}}{6800\,\dfrac{\text{samples}}{\text{sec}}} = 5.15\,\dfrac{\text{bits}}{\text{sample}}$. Since integer numbers of bits must be sent, we choose the greatest integer less than the result, which is **5 bits/sample**.

Section 17.4: Source Coding

Problem 17.3

Solution:

Known quantities:

Probabilities:
P[Red] = 3/4
P[Green] = 1/8
P[Blue] = 1/16
P[Yellow] = 1/16

Code:
Red → 00
Green → 01
Blue → 10
Yellow → 11
(leftmost bit sent first)

Find:

a) average length of the code
b) The observed sequence of colors from: (00000001000000110000000000000100)
c) Devise a new code and find its average length.
d) Encode the sequence: {Red, Red, Red, Green, Red, Red, Red, Yellow, Red, Red, Red, Red, Red, Red, Green, Red}. Is there an improvement?

Analysis:

a) The average length obviously will be 2 because all the codewords are of length two. However, we can more rigorously see this by performing the following analysis. Denote the function L(*color*) as the number of bits needed to encode the observed color. The average length can be written:

$$\mathrm{L(Red)P[Red]+L(Green)P[Green]+L(Blue)P[Blue]+L(Yellow)P[Yellow]}$$
$$=2\cdot\frac{3}{4}+2\cdot\frac{1}{8}+2\cdot\frac{1}{16}+2\cdot\frac{1}{16}$$
$$=2$$

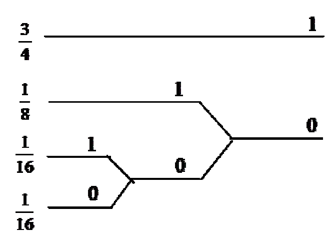
Thus, the average number of bits per codeword, i.e., average length, is two.

b) The sequence is decoded by inverting the encoding process. If we see a 00, then we declare Red, etc. The observed colors are: (00, 00, 00, 01, 00, 00, 00, 11, 00, 00, 00, 00, 00, 00, 01, 00)
{Red, Red, Red, Green, Red, Red, Red, Yellow, Red, Red, Red, Red, Red, Red, Green, Red}

c) We will design a Huffman code to decrease the average length of the code. First we compute the entropy of the observations to find the theoretical minimum average length.

$$H(X)=-\frac{3}{4}\log\left(\frac{3}{4}\right)-\frac{1}{8}\log\left(\frac{1}{8}\right)-\frac{1}{16}\log\left(\frac{1}{16}\right)-\frac{1}{16}\log\left(\frac{1}{16}\right)$$
$$=1.2 \quad \text{bits}$$

We follow the procedure in example 17.1 to design the code:



Reading from the diagram, we have our code:

Red → 1
Green → 01
Blue → 001
Yellow → 000

The average length is

$$L(\text{Red})P[\text{Red}]+L(\text{Green})P[\text{Green}]+L(\text{Blue})P[\text{Blue}]+L(\text{Yellow})P[\text{Yellow}]$$
$$=1\cdot\frac{3}{4}+2\cdot\frac{1}{8}+3\cdot\frac{1}{16}+3\cdot\frac{1}{16}=1.375 \quad \text{bits}$$

This code is on average just 0.175 bits longer than the best we could do!

d) The given sequence is encoded as:
(11101111000111111011)
and takes 20 bits. We note that this is the same sequence that was decoded in part b), which took 32 bits. We have a savings of 12 bits, or a compression ratio of 20/32 = 0.62! We conclude that devising a more efficient code can produce a large savings in the number of bits needed to convey information.

Problem 17.4

Solution:

Known quantities:

Two stereo signals are sampled with a 16-bit ADC at 44.1 kHz.

Find:

a) Output signal to quantization noise ratio for a sinusoidal input signal.
b) Playback bit-rate assuming a 50% overhead of extra bits.
c) Number of bits recorded for an hour-worth of music.
d) Number of bits required to record a 1000 page, 50 lines per page, 15 words per line, 6 letters per word, 7 bits per word history textbook. How many textbooks can be stored on a CD?

Analysis:

a) From equation 17.7, the SQNR is $6\cdot b=96 \quad \text{dB}$, very impressive.

b) Playback bit rate per stereo signal is

$$16\frac{\text{bits}}{\text{sample}}\cdot 44{,}100\frac{\text{samples}}{\text{sec}}\cdot 1.5=1{,}058{,}400\frac{\text{bits}}{\text{sec}}.$$

Doubling the answer gives the overall bit rate:

$$2{,}116{,}800\frac{\text{bits}}{\text{sec}}$$

c) Bits per hour is:

$$2{,}116{,}800\frac{\text{bits}}{\text{sec}}\cdot 3600\frac{\text{sec}}{\text{hr}}=7{,}620{,}480{,}000\frac{\text{bits}}{\text{hr}}$$

d) The number of bits in the 1000-page textbook is

$$1000\text{pages}\cdot 50\frac{\text{lines}}{\text{page}}\cdot 15\frac{\text{words}}{\text{line}}\cdot 6\frac{\text{letters}}{\text{word}}\cdot 7\frac{\text{bits}}{\text{letter}}=31{,}500{,}000 \quad \text{bits}$$

Thus, the number of books that can fit on a CD is approximately

$$\frac{7{,}620{,}480{,}000}{31{,}500{,}000}=241.2 \ \text{books}.$$

Problem 17.5

Solution:

Known quantities:

Sinusoidal signal has maximum amplitude of $a = 2$. Quantization noise ratio must be at least 10 dB

Find:

Minimum number of quantization bits per sample needed.

Assumptions:

The power in the sinusoidal signal is $P_S = \frac{a^2}{2}$, where $a = 2$ is the sinusoidal signal's amplitude.

Analysis:

According to Section 17.3, the resolution of the quantizer is $\Delta = \frac{v_{\max} - v_{\min}}{2^b} = \frac{2a}{2^b}$,

where $a = 2$ and b is the number of quantization bits. The maximum average quantization noise power is therefore $P_{qn} = \left(\frac{\Delta}{2}\right)^2 = \frac{a^2}{2^{2b}}$. The power in the sinusoidal signal is $P_S = \frac{a^2}{2}$, hence, the signal power to quantization noise power is $\frac{P_S}{P_{qn}} = 2^{2b-1}$. Putting the result into decibels, we have $SQNR_{dB,\min} \le (2b-1) \cdot 10 \log_{10} 2$. We now set $SQNR_{dB,\min} = 10$ and solve for b, to find $b \ge 2.166$. Hence, the minimum number of bits needed is **3 bits**.

Problem 17.6

Solution:

Known quantities:

Each HDTV frame is 1920 by 1080 pixels. Each pixel has 16 brightness levels. 30 frames per second.

Find:

The information rate of the TV source.

Analysis:

If we think of each pixel as being a symbol, then the TV source is emitting $r = 1920 \cdot 1080 \frac{\text{symbols}}{\text{frame}} \cdot 30 \frac{\text{frames}}{\text{sec}} = 62{,}208{,}000 \frac{\text{symbols}}{\text{sec}}$. Each symbol has an entropy of $H(X) = -\sum_{n=1}^{16} p_n \log_2 p_n = -\sum_{n=1}^{16} \frac{1}{16} \log_2 \frac{1}{16} = \log_2 16 = 4 \frac{\text{bits}}{\text{symbol}}$. Hence the total information rate of the TV signal is $rH(x) = 62{,}208{,}000 \frac{\text{symbols}}{\text{sec}} 4 \frac{\text{bits}}{\text{symbol}} = 248{,}832{,}000 \frac{\text{bits}}{\text{sec}}$.

Section 17.5: Digital Baseband Modulation

Problem 17.7

Solution:

Known quantities:

Baud rate is 1,000,000 symbols per second. Message set has 256 waveforms

Find:

The bit rate of the system.

Analysis:

The number of bits per symbol B is the base-2 logarithm of the number of message waveforms,

$B = \log_2(256) = 8\,\frac{\text{bits}}{\text{symbol}}$. Thus, the overall bit rate is $R = 8\,\frac{\text{bits}}{\text{symbol}} \cdot 1{,}000{,}000\,\frac{\text{symbols}}{\text{sec}} = 8{,}000{,}000\,\frac{\text{bits}}{\text{sec}}$.

Problem 17.8

Solution:

Known quantities:

Signal waveforms given in Figure P17.8.

Find:

1) Average energy of the waveform sets.
2) RMS distance between the waveforms.
3) Which set of waveforms would you choose for binary signaling? Why?

Assumptions:

We assume that each waveform is equally likely to be sent.

Analysis:

1)

The average energy is calculated as $E = P[m_0]\int_0^1 |m_0(t)|^2\,dt + P[m_1]\int_0^1 |m_1(t)|^2\,dt$, where $P[m_i]$ is the probability of sending waveform *i*, $i \in \{0,1\}$. According to our assumptions, $P[m_0] = P[m_1] = \frac{1}{2}$, so the average energies of the waveform sets are:

a) $E = \frac{1}{2}\int_0^{1/2} \left|\sqrt{2}\right|^2 dt + \frac{1}{2}\int_{1/2}^{1} \left|\sqrt{2}\right|^2 dt = 1$

b) $E = \frac{1}{2}\int_0^1 |1|^2\,dt + \frac{1}{2}\int_0^1 |-1|^2\,dt = 1$

c) $E = \frac{1}{2}\left(\int_0^{1/3} |1|^2\,dt + \int_{1/3}^{2/3} |-1|^2\,dt + \int_{2/3}^{1} |1|^2\,dt\right) + \frac{1}{2}\int_0^1 \left|2\sqrt{3}t - \sqrt{3}\right|^2 dt = 1$

Thus, they all have the same average energy.

2)

The RMS distance is calculated as $d = \sqrt{\int_0^1 \left| m_0(t) - m_1(t) \right|^2 dt}$.

a) $d = \sqrt{\int_0^{1/2} \left| \sqrt{2} - 0 \right|^2 dt + \int_{1/2}^{1} \left| 0 - \sqrt{2} \right|^2 dt} = \sqrt{2}$.

b) $d = \sqrt{\int_0^1 \left| 1 - (-1) \right|^2 dt} = 2$.

c) $d = \sqrt{\int_0^{1/3} \left| 1 - (2\sqrt{3}t - \sqrt{3}) \right|^2 dt + \int_{1/3}^{2/3} \left| -1 - (2\sqrt{3}t - \sqrt{3}) \right|^2 dt + \int_{2/3}^{1} \left| 1 - (2\sqrt{3}t - \sqrt{3}) \right|^2 dt} = \sqrt{2}$

3) We should choose waveform set b) because according to the probability of error, given by $Q\left(\frac{d}{2kT}\right)$ as shown in problem 17.9. The larger *d* is, the smaller the probability of error for any given values of *kT*.

Section 17.6: Channel Coding

Problem 17.9

Solution:

The solution to this problem is given by the following MATLAB code:

```
% Matlab Code for problem 17.9
% Values of SNR (dB): 10*log10(Eb/(kT));
SNRdb = [0:10] ;
% Initialize the number of bits per trial
numBits = 1e6;
% Initialize the probability of error vector
Pe = zeros(1,length(SNRdb));
% Iterate through all the SNR's
for snrLoop = 1:length(SNRdb)
    disp(['Currently processing SNR ',num2str(SNRdb(snrLoop)),'.'])
    % Assume kT = 1 and find the relative value of Eb
    kT = 1 ;
    Eb = kT*10.^(SNRdb(snrLoop)/10) ;

    % continue while the number of errors are less than 100
    trials = 0 ;
    numErrors = 0;
    keepGoing = 1;
    while keepGoing
        trials = trials + 1 ;
        disp(['   Trial number ',num2str(trials),'.'])
        % Generate some random bits
        bits = rand(numBits,1) > 1/2 ;
        % Map bits to binary symbols: 0 -> 1, 1 -> -1
        syms = -sqrt(Eb)*(2*bits - 1) ;
        % generate Gaussian noise with variance of kT/2
        noise = sqrt(kT/2)*randn(numBits,1) ;
        % Create the received signal by adding the noise
        r = syms + noise ;
        % Detect the bits
        rbits = (-sign(r) + 1)/2 ;
        % Count the number of errors
        numErrors = numErrors + sum(rbits ~= bits);
        % See if 100 bit errors have occured
        if numErrors > 100
            keepGoing = 0 ;
        end
    end
    % Calculate the bit error probability
    Pe(snrLoop) = numErrors/(trials*numBits) ;
end
figure(1)
% Plot the emperical results
semilogy(SNRdb,Pe,'rx')
% Overlay the theoretical results
hold on
SNR = 10.^(SNRdb/10);  % Linear SNR
semilogy(SNRdb,1/2*erfc(sqrt(SNR)))
grid on
xlabel('SNR (dB)')
       ylabel('Probability of Bit Error')
```

Problem 17.10

Solution:

Known quantities:

Distance between sites is 2 km. Measured SNR (P/kT) is 20 dB.

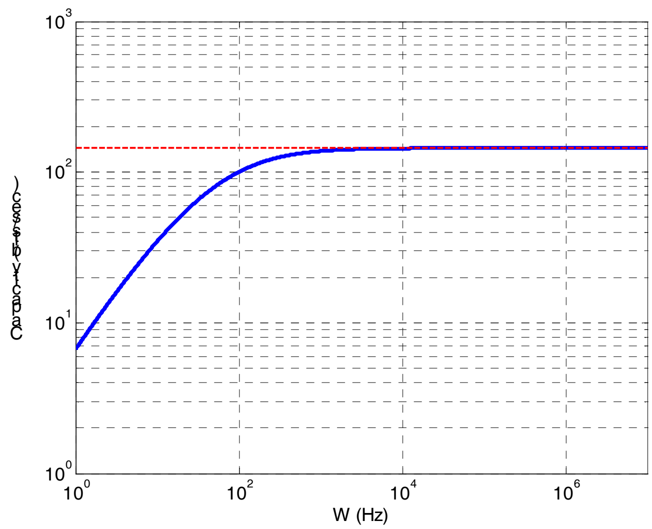
Find:

a) Minimum bandwidth needed to support a 1 Mbps link.
b) Suppose there is a bandwidth limitation of 5MHz, can a 5 Mbps link be supported?

Analysis:

a)
The channel capacity expression given in equation 17.18 states that the maximum bit rate for reliable communications is

$$C = W\log_2\left(1+\frac{P}{kTW}\right) \text{ bits/sec.}$$



We are given $\frac{P}{kT} = 100$, so we plot C as a function of W, as shown in the figure to the right. Note that the capacity never exceeds 300 bits/second; thus, a 1 Mbps bit rate is never able to be supported with the given SNR. This can be made rigorous by taking the limit as W goes to inifity

$$C_\infty = \lim_{W\to\infty} W\log_2\left(1+\frac{P}{kTW}\right)$$

$$= \lim_{W\to\infty} W\log_e\left(1+\frac{P}{kTW}\right)\log_2(e)$$

$$= \frac{P}{kT}\log_2(e)$$

In the figure, the limit is shown as a dashed line, where that P/kT = 100. Note that this maximum is far less than 1 Mbps.

b) The link cannot be supported for the same reasons as found in part a).

Problem 17.11

Solution:

Known quantities:

Voice channel with bandwidth of 3.4 kHz. Signal is sampled at 1.5 times the Nyquist rate. Each sample is quantized into 256 equally likely levels.

Find:

a) Bit rate of the information source.
b) Can you transmit this source over a channel with 10 kHz and 20 dB SNR?
c) What is minimum SNR needed to transmit

Analysis:

a) The bit rate of the information source is the number of samples per second times the number of bits per sample. The Nyquist sampling rate is twice the bandwidth: 6.8 kHz.

$$R = 1.5 \times 6{,}800 \frac{\text{samples}}{\text{sec}} \times 8 \frac{\text{bits}}{\text{sample}} = 81{,}600 \frac{\text{bits}}{\text{sec}}.$$

b) The channel capacity according to equation 17.18 is

$$C = W \log_2\left(1 + \frac{P}{kTW}\right) = 10{,}000 \log_2\left(1 + \frac{100}{1000}\right) = 143.6 \frac{\text{bits}}{\text{sec}}.$$

This capacity is far below the rate needed to convey the source's information.

c) We solve for the SNR

$$C = W \log_2\left(1 + \frac{P}{kTW}\right)$$

$$\frac{C}{W} = \log_2\left(1 + \frac{P}{kTW}\right)$$

$$2^{\frac{C}{W}} = 1 + \frac{P}{kTW}$$

$$\frac{P}{kT} = W\left(2^{\frac{C}{W}} - 1\right)$$

Now, plugging in the values we have

$$\frac{P}{kT} = W\left(2^{\frac{C}{W}} - 1\right)$$

$$= 10{,}000\left(2^{\frac{81{,}600}{10{,}000}} - 1\right)$$

$$= 2{,}850{,}000$$

$$= 64.5 \quad \text{dB}$$

Problem 17.12

Solution:

Known quantities:

The crossover probability of the binary symmetric channel is *b*. The bit is repeated *n* times where $n = 2m+1$.

Find:

a) Probability of bit error.
b) If $b = 0.05$, find P_e when n = 3, 5, 7.

Assumptions:

We assume that the 0's and 1's are equally-likely to be transmitted.

Analysis:

a) We denote the probability of error given that a 1 was transmitted by $P_{e|1}$. An error occurs when $m + 1$ or more zeros are observed at the receiver. This would require $m + 1$ or more crossovers, each with probability of *b*. The probability of not crossing over is $1 - b$. The errors could happen in at any bit-position and in any order, so we need to count the number of combinations for each number of crossovers:

$$P_{e|1} = \binom{n}{m+1} b^{m+1}(1-b)^m + \binom{n}{m+2} b^{m+2}(1-b)^{m-1} + \cdots + \binom{n}{2m+1} b^{2m+1}(1-b)^0 .$$

Since the channel-errors are symmetric, it is easy to see that the probability of error given that a 0 was transmitted is the same as if a 1 were transmitted, i.e., $P_{e|1} = P_{e|0}$. The total probability of error is

$$\begin{aligned} P_e &= P[1 \text{ was transmitted}] \times P_{e|1} + P[0 \text{ was transmitted}] \times P_{e|0} \\ &= \frac{1}{2} \times P_{e|1} + \frac{1}{2} \times P_{e|0} \\ &= \frac{1}{2} \times 2P_{e|1} \\ &= P_{e|1} \end{aligned}$$

where we have used the assumption that P[1 was transmitted] = P[0 was transmitted] = 1/2. So the total probability of error is

$$P_e = \binom{n}{m+1} b^{m+1}(1-b)^m + \binom{n}{m+2} b^{m+2}(1-b)^{m-1} + \cdots + \binom{n}{2m+1} b^{2m+1}(1-b)^0$$

b) The probability of error for $b = 0.05$ and $n = 3$ is:

$$P_e = \binom{3}{2} b^2(1-b)^1 + \binom{3}{3} b^3(1-b)^0 = 0.0073.$$

The probability of error for $b = 0.05$ and $n = 5$ is:

$$P_e = \binom{5}{3} b^3(1-b)^2 + \binom{5}{4} b^4(1-b)^1 + \binom{5}{5} b^5(1-b)^0 = 0.0012.$$

The probability of error for $b = 0.05$ and $n = 5$ is:

$$P_e = \binom{7}{4} b^4(1-b)^3 + \binom{7}{5} b^5(1-b)^2 + \binom{7}{6} b^6(1-b)^1 + \binom{7}{7} b^7(1-b)^0 = 0.00019.$$

Problem 17.13

Solution:

Known quantities:

Figure P17.13, which shows a plot of bit error rate versus signal-to-noise ratio.

Find:

a) For uncoded transmission, find SNR needed to achieve a BER of 10-5.
b) For coded transmission, find SNR needed to achieve a BER of 10-5.
c) What is the coding gain?

Analysis:

a) Figure P17.13 indicates that the SNR must be approximately 9.3 dB.
b) Figure P17.13 indicates that the SNR must be approximately 7.6 dB.
c) The coding gain is the difference between the uncoded and coded case: 9.3 – 7.6 = 1.7 dB.

Problem 17.14

Solution:

Known quantities:

The target maximum number of errors per 125 kbyte file is 100. Define the additive Gaussian noise intensity as $\frac{N_o}{2} = \frac{kT}{2} = 10^{-3}$ W/Hz. Each bit is encoded as a 1 V pulse of duration T seconds. The bit-error-rate for uncoded systems is $Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$.

Find:

The shortest bit duration T (largest bit rate) so that the bit-error-rate is less than 100 errors per 125 kbyte file.

Assumptions:

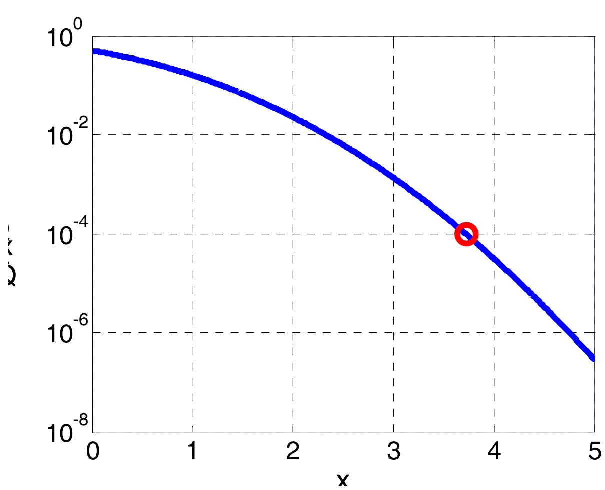
We assume that the voltage is across a 1 Ohm resister so that the instantaneous power is $V(t)^2$ and the bit energy is $E_b = \int_0^T V^2(t)dt = \int_0^T 1dt = T$. We also assume there is no channel coding.

Analysis:

As given in the problem, the error rate for uncoded transmission is $P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$. Our assumptions state that $E_b = T$; hence, the probability of error can be written in terms of the bit duration

$$P_e = Q\left(\sqrt{\frac{2T}{N_o}}\right).$$

We now solve for T in the previous expression:



$T = \left(Q^{-1}(P_e)\right)^2 \frac{N_o}{2}$.

All the terms on the right hand side are given in the problem statement, but the expression for the inverse of the Q-function cannot be put in closed form. To find $Q^{-1}(P_e)$, we plot $Q(x)$ (see the figure) and find the value of x such that $Q(x) = P_e = 10^{-4}$. This can be done by the evaluating the following MATLAB code:

```
% Matlab Code for problem 17.14

% Calculate the probability of error
Pe = 100/(125e3*8) ;  % 100 errors in 125 kbytes

% Calculate the Q-function: Q(x) = 1/2*erfc(x/sqrt(2)).
x = linspace(0,5,1000) ;  % Linear SNR
Q = 1/2*erfc(x/sqrt(2)) ;

% Find the index where Q(x) is nearest Pe
[dummy,index] = find(Q < Pe);

% Print value of x-axis where Q(x) is nearest Pe
QinvPe = x(index(1))

% Make a plot
figure(1)
semilogy(x,Q);
hold on
semilogy(x(index(1)),Q(index(1)),'ro');
grid on
xlabel('x')
ylabel('Q(x)')
hold off
```

The results are the following

```
>> plotFunction

QinvPe =

    3.7237
```

We can now calculate *T*:

$T = \left(Q^{-1}(P_e)\right)^2 \frac{N_o}{2} = (3.7237)^2 \times 10^{-3} = 0.0139$

Thus the bit duration should be no less than T = 0.0139 sec/bit, so the bit rate is no greater than **1/T = 72.1 bits/sec.**

Section 17.7: Multiuser Channel Access

Problem 17.15

Solution:

Known quantities:

Voice signals have bandwidth of 3.4 kHz and sampled at 8 kHz with an 8-bit quantizer. Each frame of the T-1 line consists of 24 voice channel samples plus a single bit for synchronization.

Find:

a) Duration of each bit.
b) Transmission rate.

Analysis:

a) The number of bits per frame is $24\frac{\text{samples}}{\text{T1 Frame}}\times 8\frac{\text{bits}}{\text{sample}}+1\frac{\text{synch bit}}{\text{T1 Frame}}=193\frac{\text{bits}}{\text{T1 Frame}}$. The duration of a T1 frame is the period between samples: $\frac{1}{8{,}000}\frac{\text{sec}}{\text{T1 Frame}}$. Thus the duration of a single bit is

$$\frac{1}{193}\frac{\text{T1 Frames}}{\text{bit}}\times\frac{1}{8{,}000}\frac{\text{sec}}{\text{T1 Frame}}=6.48\times 10^{-7}\frac{\text{sec}}{\text{bit}}.$$

b) The bit transmission rate is the reciprocal of the bit duration: $\frac{1}{6.48\times 10^{-7}}\frac{\text{bits}}{\text{sec}}=1{,}544{,}000\frac{\text{bits}}{\text{sec}}=1.544\quad\text{Mbps}$

Section 17.8: Data Transmission in Digital Instruments

Problem 17.16

Solution:

Known quantities:
An ASCII (hex) encoded message.

Find:
Decode the message.

Analysis:
Decoded message is: `ASCII decoding is easy!`

Problem 17.17

Solution:

Known quantities:
An ASCII (binary) encoded message.

Find:
Decode the message.

Analysis:
Decoded message:

T	h	i	s
i	m	e	–
n	g		p
	i	s	
c	o	n	s
r	o	b	l
a		t	
u	m	i	
e	m	.	

Problem 17.18

Solution:

Known quantities:
Some decimal numbers.

Find:
The ASCII form for the numbers.

Analysis:

Decimal	ASCII
12	31 32
345.2	33 34 35 2E 32
43.5	34 33 2E 35

Problem 17.19

Solution:

Known quantities:

Some words.

Find:

The ASCII form for the words.

Analysis:

a) 44 69 67 69 74 61 6C
b) 43 6F 6D 70 75 74 65 72
c) 41 73 63 69 69
d) 41 53 43 49 49

Problem 17.20

Solution:

Find:

Explain why data transmission over long distances is usually done via a serial scheme rather than parallel.

Analysis:

Serial data transmission requires only a single data path. Parallel requires 16 (or more, depending on word length), and would, therefore, be much more expensive.

Problem 17.21

Solution:

Known quantities:

The on-board memory of an automated data-logging, 16 K-words, that samples the variable of interest once every 5 min.

Find:

How often must the data be downloaded and the memory cleared in order to avoid losing any data.

Analysis:

$$\text{Longest possible delay} = 16 \cdot 1024 \cdot 5 = 81920 \text{ min} =$$

$$= 81920\text{min} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \approx 56.9 \text{ days}$$

Problem 17.22

Solution:

Find:

Explain why three wires are required for the handshaking technique employed by IEEE 488 bus system.

Analysis:

Three lines are used for handshaking in the IEEE 488 bus to accomplish the following functions: One line is used to declare the bus ready to accept data; another line to declare that data has been accepted, and a third one to declare that the data was indeed valid.

Problem 17.23

Solution:

Known quantities:

The information held in a CD-ROM, 650 MB. The CD-ROM are packaged 50 per box, and 100 boxes are shipped. The distance for the trip is 2,500 miles and the airplane speed is 400 mi/hr.

Find:

The transmission rate between the two cities in bits/s.

Analysis:

$$\frac{650 \cdot 1024 \cdot 1024 \ \dfrac{\mathbf{bytes}}{\mathbf{CD}} \cdot 50 \ \dfrac{\mathbf{CDs}}{\mathbf{box}} \cdot 100 \ \mathbf{boxes} \cdot 400 \ \dfrac{\mathbf{mi}}{\mathbf{hr}}}{2500 \ \mathbf{mi}} = 5.4525952 \cdot 10^{11} \ \frac{\mathbf{bytes}}{\mathbf{hr}}$$

$$5.4525952 \cdot 10^{11} \frac{bytes}{hr} \cdot 8 \frac{bits}{byte} \cdot \frac{1hr}{3600sec} \approx 1.21 \cdot 10^{9} \frac{bits}{s}$$

or approximately $1.13 \frac{Gbits}{s}$.
