Chapter 7: AC Power – Instructor Notes

Chapter 7 surveys all important aspects of electric power. **Coverage of Chapter 7 can take place immediately following Chapter 4, or as part of a later course on energy systems or electric machines**. The material in this chapter will be of particular importance to Aerospace, Civil, Industrial, Marine and Mechanical engineers, who are concerned with the utilization of electric power.

The chapter permits very flexible coverage, with sections 7.1 and 7.2 describing basic single-phase AC power ideas. A survey course might only use this introductory material. The next two sections discuss transformers and three-phase power. Two descriptive sections are also provided to introduce the ideas of residential wiring, grounding and safety, and the generation and distribution of AC power. **These sections can be covered independent of the transformer and three-phase material.**

The section *Focus on Measurements: The wattmeter* provides a practical look at the measurement of power, while *Focus on Measurements: Power factor* proposes a more applied look at the problem of improving the power factor of an industrial load.

The homework problems present a few simple applications in addition to the usual exercises meant to reinforce the understanding of the fundamentals. Problems 7.18-21 and 7.33 present a variety of power factor correction problems. Problem 7.34 illustrates the billing penalties incurred when electric loads have insufficient power factors (this problem is based on actual data supplied by Detroit Edison). Problem 7.35 is a more advanced problem related to a high-speed train. Two advanced problems (7.50, 7.51) discuss transformer test methods; these problems may be suitable in a second course in energy systems.

Those instructors who plan to integrate the three-phase material into a course on power systems and electric machines, will find that **most of the problems in this section (7.58 – 7.77) can be assigned in conjunction with the material covered in Chapter 17**, as part of a more in-depth look at three-phase power systems.

The 5th Edition of this book includes 18 new problems; some of the 4th Edition problems were removed, increasing the end-of-chapter problem count from 59 to 77.

Learning Objectives for Chapter 7

1. Understand the meaning of instantaneous and average power, master AC power notation, and compute average power for AC circuits. Compute the power factor for a complex load. *Section 7.1*
2. Learn complex power notation; compute apparent, real and reactive power for complex loads. Draw the power triangle, and compujte the capacitor size required to perform power factor correction on a load. *Section 7.2*.
3. Analyze the ideal transformer; compute primary and secondary currents and voltages and turns ratios. Calculate reflected sources and impedances across ideal transformares. Understand maximum power transfer. *Section 7.3*.
4. Learn three-phase AC power notation; compute load currents and voltages for balanced wye and delta loads. *Section 7.4.*
5. Understand the basic principles of residential electrical wiring and of electrical safety. *Sections 7.5, 7.6.*

Section 7.1: Power in AC Circuits

FOCUS ON METHODOLOGY

COMPLEX POWER CALCULATION FOR A SINGLE LOAD

1. Compute the load voltage and current in rms phasor form, using the AC circuit analysis methods presented in Chapter 4 and converting peak amplitude to rms values.

   $\tilde{\mathrm{V}} = \tilde{V}\angle\theta_V$

   $\tilde{\mathrm{I}} = \tilde{I}\angle\theta_I$

2. Compute the complex power $S = \tilde{\mathrm{V}}\tilde{\mathrm{I}}^*$ and set $\mathrm{Re}\, S = P_{\mathrm{av}}$, $\mathrm{Im}\, S = Q$.
3. Draw the power triangle, as shown in Figure 7.11.
4. If Q is negative, the load is capacitive; if positive, the load is reactive.
5. Compute the apparent power $|S|$ in volt-amperes.

FOCUS ON METHODOLOGY

COMPLEX POWER CALCULATION FOR POWER FACTOR CORRECTION

1. Compute the load voltage and current in rms phasor form, using the AC circuit analysis methods presented in Chapter 4 and converting peak amplitude to rms values.
2. Compute the complex power $S = \tilde{\mathrm{V}}\tilde{\mathrm{I}}^*$ and set $\mathrm{Re}\, S = P_{\mathrm{av}}$, $\mathrm{Im}\, S = Q$.
3. Draw the power triangle, for example, as shown in Figure 7.17.
4. Compute the power factor of the load $pf = \cos(\theta)$.
5. If the reactive power of the original load is positive (inductive load), then the power factor can be brought to unity by connecting a parallel capacitor across the load, such that $Q_C = -1/\omega C = -Q$, where Q is the reactance of the inductive load.

Problem 7.1

Solution:

Known quantities:

Resistance value, $R = 30\ \Omega$, and the voltage across the soldering iron, $\tilde{V} = 117\mathrm{V}$.

Find:

The power dissipated in the soldering iron.

Analysis:

The power dissipated in the soldering iron is:

$$P = \frac{\tilde{V}^2}{R} = \frac{117^2}{30} = 456.3\mathrm{W}$$

Problem 7.2

Solution:

Known quantities:

Rated power, $P = 1000\,\text{W}$, and the voltage across the heating element, $\tilde{V} = 240\text{V}$.

Find:

The resistance of the heating element.

Analysis:

The power dissipated in the electric heater is:

$$R = \frac{\tilde{V}^2}{P} = \frac{240^2}{1000} = 57.6\ \Omega$$

Problem 7.3

Solution:

Known quantities:

Resistance value, $R = 50\ \ \Omega$ of the resistor.

Find:

The power dissipated in the resistor if the current source connected to the resistor is:

a) $i(t) = 5\cos(50t)$ A

b) $i(t) = 5\cos(50t - 45°)$ A

c) $i(t) = 5\cos(50t) - 2\cos(50t - 50°)$ A

d) $i(t) = 5\cos(50t) - 2$ A

Analysis:

The average power can be expressed as: $P_{av} = \frac{1}{2} I^2 R$

a) $P_{av} = \frac{5^2 \cdot 50}{2} = 625\,\text{W}$

b) $P_{av} = \frac{5^2 \cdot 50}{2} = 625\,\text{W}$

c) By using phasor techniques:

$$\mathbf{I} = 5\angle 0° - 2\angle -50° = 5 - 1.2856 + j1.5321 = 3.7144 + j1.5321 = 4.0180\angle 22.41°\ \text{A}$$

Then, the instantaneous current can be expressed as:

$$i(t) = 4.0180\cos(100t + 22.41°)\,\text{A}$$

Therefore, the average power is: $P_{av} = \frac{4.0180^2 \cdot 50}{2} = 403.6\,\text{W}$

d) The instantaneous voltage can be expressed as:

$$v(t) = Ri(t) = 250\cos(50t) - 100\,\text{V}$$

Then, the instantaneous power can be written as:

$$p(t) = v(t) \cdot i(t) = [250\cos(50t) - 100] \cdot [5\cos(50t) - 2]$$

$$= 1250\cos^2(50t) - 1000\cos(50t) + 200 = 625 + 625\cos(100t) - 1000\cos(50t) + 200\,\text{W}$$

Therefore, the average power is:

$$P_{av} = 625 + 200 = 825\,\text{W}$$

Problem 7.4

Solution:

Known quantities:

The current values.

Find:

The rms value of each of the following currents.

a) $\cos 450t + 2\cos 450t$
b) $\cos 5t + \sin 5t$
c) $\cos 450t + 2$
d) $\cos 5t + \cos(5t + \pi/3)$
e) $\cos 200t + \cos 400t$

Analysis:

The rms current can be expressed as:

$I_{rms} = \tilde{I} = \dfrac{I}{\sqrt{2}}$ if the current is periodic or if the current can be converted to a phasor quantity.

Otherwise, the rms current must be calculated using integration techniques.

a) Summing the common cosine terms leads to

$I = 3\cos 450t$

$$\tilde{I} = \frac{3}{\sqrt{2}} = 2.1213\,\mathrm{A}$$

b) Using phasor analysis:

$I = \cos 5t + \cos(5t - 90°) = 1\angle 0° + 1\angle -90° = 1 - j = \sqrt{2}\angle -45°\ \mathrm{A}$

$$\tilde{I} = \frac{\sqrt{2}}{\sqrt{2}} = 1\,\mathrm{A}$$

c) Since the second term is not periodic, integration techniques must be used:

$$\tilde{I} = \sqrt{\frac{2\pi}{450}\int_0^{\frac{2\pi}{450}} (\cos 450t + 2)^2\,dt} = \sqrt{\frac{2\pi}{450}\int_0^{\frac{2\pi}{450}} (\cos^2 450t + 4\cos 450t + 4)\,dt} =$$

$$\tilde{I} = \sqrt{\frac{2\pi}{450}\int_0^{\frac{2\pi}{450}} (\cos^2 450t + 4\cos 450t + 4)\,dt} = \sqrt{\frac{2\pi}{450}\int_0^{\frac{2\pi}{450}} \left(\frac{1}{2} + \frac{1}{2}\cos 900t + 4\cos 450t + 4\right)dt} =$$

$$\tilde{I} = \sqrt{\frac{2\pi}{450}\,\frac{450}{2\pi}\left[\frac{9}{4} - \frac{1}{1800}\sin 900t - \frac{4}{450}\sin 450t\right]} = \frac{3}{\sqrt{2}} = 2.1213\,\mathrm{A}$$

d) Using phasor analysis:

$I = 1\angle 0° + 1\angle 60° = 1 + 0.5 + j0.866 = 1.732\angle 30°\ \mathrm{A}$

$$\tilde{I} = \frac{1.732}{\sqrt{2}} = 1.225\,\mathrm{A}$$

e) Can't use phasor analysis because phasor analysis does not work for different frequencies. Must integrate as in part c:

$$\tilde{I} = \frac{2}{\sqrt{2}} = 1.414\ \mathrm{A}$$

Problem 7.5

Solution:

Known quantities:

The current rms value, 4 A, the voltage source rms value, 110 V, the lag between the current and the voltage, 60°.

Find:

The power dissipated by the circuit and the power factor

Analysis:

The average power drawn by the circuit is:

$$P = \frac{VI}{2}\cos(\theta) = \frac{110\sqrt{2}\cdot 4\sqrt{2}}{2}\cos(60°) = 220\,\text{W}$$

The power factor is:

$$pf = \cos(60°) = 0.5$$

Problem 7.6

Solution:

Known quantities:

The voltage source rms value, 120 V, the source frequency, 60 Hz, the power consumption, 1.2 kW, and the power factor, 0.8.

Find:

a) The rms current.
b) The phase angle.
c) The impedance.
d) The resistance.

Analysis:

a) The power is expressed as:

$$P = \tilde{V}\tilde{I}\cos(\theta)$$

Thus, the rms current is:

$$\tilde{I} = \frac{P}{\tilde{V}\cos(\theta)} = \frac{1200}{120\cdot 0.8} = 12.5\,\text{A}$$

b) The power factor is:

$$pf = \cos(\theta)$$

Thus, the phase angle θ is:

$$\theta = \cos^{-1}(0.8) = 36.87°$$

c) The impedance Z is:

$$Z = \frac{\tilde{V}\angle\theta}{\tilde{I}} = \frac{120}{12.5} = 9.6\angle 36.87^\circ\ \Omega$$

d) The resistance R is:

$$R = Z\cos(\theta) = 7.68\,\Omega$$

Problem 7.7

Solution:

Known quantities:

The rms values of the supply voltage and current, 110 V and 14 A, the power requirement, 1 kW, the machine efficiency, 90%, and the power factor, 0.8.

Find:

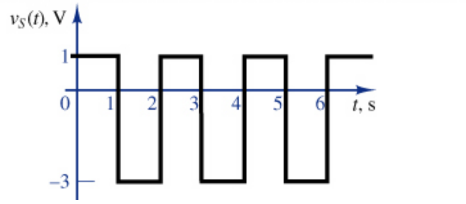
The AC machine efficiency.

Analysis:

The efficiency is:

$$\eta_{motor} = \frac{Mechanical\ Power}{Electrical\ Power} = \frac{1\,\mathrm{kW}/0.9}{\tilde{V}\tilde{I}\cos(\theta)} = \frac{1111\,\mathrm{W}}{1232\,\mathrm{W}} = 0.9$$

Problem 7.8



Solution:

Known quantities:

The waveform of a voltage source shown in Figure P7.8.

Find:

a) The steady DC voltage that would cause the same heating effect across a resistance.
b) The average current supplied to a 10-Ω resistor connected across the voltage source.
c) The average power supplied to a 1-Ω resistor connected across the voltage source.

Analysis:

a) $V_{DC} = \tilde{V} = \sqrt{\frac{(1\cdot 1)+(9\cdot 1)}{2}} = \sqrt{\frac{10}{2}} = 2.24\mathrm{V}$

b) $I_{av} = \frac{(1\cdot 1)+(-3\cdot 1)}{2\cdot 10} = \frac{-2}{20} = -0.1\,\mathrm{A}$

c) $P_{av} = \frac{\tilde{V}^2}{R} = \frac{2}{1} = 5\mathrm{W}$

Problem 7.9

Solution:

Known quantities:
The waveform of a current source.

Find:
The average power delivered to the resistor.

Analysis:

The average power is $P_{av} = \frac{1}{2} I^2 R \cos(\theta)$

In this case, load is only a resistor, $\theta = 0$

(a) $P_{av} = \frac{4^2 \times 100}{2} = 800$ W

(b) $P_{av} = \frac{4^2 \times 100}{2} = 800$ W

(c) By using phasor techniques,we have

$I = 4\angle 0° - 3\angle\text{-}50° = 2.0716 + j2.298 = 3.094\angle 47.967°$

$i(t) = 3.094\cos(100t + 47.967°)$ $\quad P_{av} = \frac{3.094^2 \times 100}{2} = 479.64$ W

(d) $v(t) = Ri(t) = 400\cos 100t - 300$

$p(t) = v(t)i(t) = (400\cos 100t - 300)(4\cos 100t - 3)$

$= 800 + 800\cos 200t\text{-}2400\cos 100t + 900$

Therefore, the average power is

$P_{av} = 800 + 900 = 1700$ W

Problem 7.10

Solution:

Known quantities:
The waveform of periodic currents.

Find:
The rms value.

Analysis:

(a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} = 1.414$

(b) Using phasor analysis, we have $I = 1\angle 0° + 1\angle\text{-}90° = 1 - j = \sqrt{2}\angle\text{-}45°$ **so** $i(t) = \sqrt{2}\cos(2t - 45°)$

Therefore, $I_{rms} = 1$ A

(c) $\sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$

(d) $I = 1\angle 0° + 1\angle 135° = 1 - 0.707 + j0.707 = 0.765\angle 67.55°$; $\quad I_{rms} = 0.5412$

(e) The minimum common period is 2π; thus the current must be squared, then integrated from 0 to 2π.

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (\cos 2t + \cos 3t)^2\, dt} = 1 .$$

Section 7.2: Complex power

Problem 7.11

Solution:

Known quantities:
The waveform of current and voltage.

Find:
Power dissipated by the circuit and the power factor.

Analysis:

The average power drawn by the circuit is
$$P = \frac{VI}{2}\cos(\theta) = \frac{220\sqrt{2} \times 10\sqrt{2}}{2}\cos(60°) = 1100 \quad \text{W}$$

The power factor is
$pf = \cos\ 60° = 0.5$ lagging

Problem 7.12

Solution:

Known quantities:
The waveform of current and voltage.

Find:
a) The power factor
b) The phase angle
c) The impedance
d) The resistance.

Analysis:

(a) The power factor is

$$pf = \cos\theta = \frac{P}{V_{rms}I_{rms}} = \frac{800}{12 \times 120} = 0.56$$

(b) The phase angle θ is

$$\theta = \arccos(0.56) = 56.25°$$

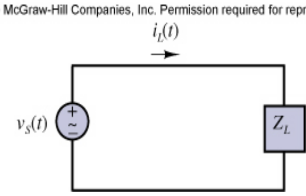
(c) The impedance Z is

$$Z = |Z|\angle\theta = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle 0°}{12\angle -56.25°} = 10\angle 56.25° = 5.56 + j8.31 \qquad \Omega$$

(d) Obviously, the resistance is $5.56\,\Omega$

Problem 7.13

Solution:



Known quantities:
Circuit as shown in Figure P7.13 and values of current and voltages.

Find:
Average power, the reactive power and the complex power.

Analysis:
a) $P = \frac{650}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos(10°) = 6401.25 \text{ W}$

$Q = \frac{650}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \sin(10°) = 1128.7 \text{ VAR}$ $S = \frac{650}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \angle 10° = 6500 \angle 10° \text{ VA}$

b) Use the same calculation as shown above, we can have

$P = 4599.3 \text{ W}$ $Q = 4599.3 \text{ VAR}$ $S = 6504.4 \angle 45° \text{ VA}$

c) $P = 60 \text{ W}$ $Q = 857.91 \text{ VAR}$ $S = 860 \angle 86° \text{ VA}$

d) $P = 401.22 \text{ W}$ $Q = 260.56 \text{ VAR}$ $S = 478.4 \angle 33° \text{ VA}$

Problem 7.14

Solution:

Known quantities:
Circuit as shown in Figure P7.13 and values of current and voltages.

Find:
The power factor for the load and state whether it is leading or lagging.

Analysis:
a) $pf = \cos(\theta_v - \theta_i) = \cos(-32°) = 0.848$ leading

b) $pf = \cos(\theta_v - \theta_i) = \cos(7°) = 0.9925$ lagging

c) $pf = \cos(\theta_v - \theta_i) = \cos(-85°) = 0.08716$ leading

d) $\theta = \arctan\frac{16}{48} = 18.43°$ $pf = \cos(\theta_v - \theta_i) = \cos(18.43°) = 0.9487$ lagging

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Problem 7.15

Solution:

Known quantities:
Circuit as shown in Figure P7.13 and values of current and voltages or power factor.

Find:
Whether it is leading or lagging.

Analysis:
a) Since it is leading, it is capacitive

b) Since it is leading, it is capacitive

c) $i_L(t) = 4.2\sin(\omega t) = 4.2\cos(\omega t - 90°)$ Since it is lagging, it is inductive

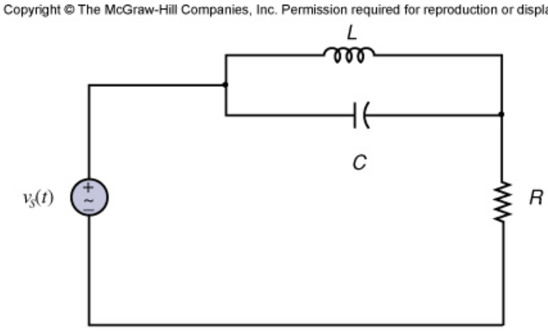
d) Since both current and voltage have the same phase, it is resistive

Problem 7.16

Solution:

Known quantities:

Circuit as shown in Figure P7.16 and values of voltages.



$C = 265\ \mu\text{F}$ $\quad L = 25.55\ mH$ $\quad R = 10\ \Omega$

Find:

The instantaneous real and reactive power.

Analysis:

(a) $jX_L = j\omega L = j377 \times 25.55 \times 10^{-3} = j9.63\ \ \Omega$

$$jX_C = \frac{1}{j\varpi C} = \frac{1}{j377 \times 265 \times 10^{-6}} = -j10.01\ \ \Omega$$

$jX_L || jX_C = j255.64\ \ \Omega$

The equivalent impedance Z is

$$Z = jX_L \,||\, jX_C + R = 10 + j255.64 = 255.83\angle 87.76°$$

The current in the circuit is

$$I = \frac{V_S}{Z} = \frac{120\angle 0}{255.5\angle 87.8} = 0.47\angle\text{-}87.76°$$

The real power P is

$$P = I^2 R = 0.47^2 \times 10 = 2.20\ \ \text{W}$$

The reactive power Q is

$$Q = I^2 X\ = 0.47^2 \times 255.64 = 56.24\ \text{VAR}$$

(b) $jX_L = j\omega L = j314 \times 25.55 \times 10^{-3} = j8.02\ \ \Omega$

$$jX_C = \frac{1}{j\varpi C} = \frac{1}{j314 \times 265 \times 10^{-6}} = -j12.02\ \ \Omega$$

$jX_L || jX_C = j24.13\ \ \Omega$

The equivalent impedance Z is

$$Z = jX_L \,||\, jX_C + R = 10 + j24.13 = 26.12\angle 67.49°$$

The current in the circuit is

$$I = \frac{V_S}{Z} = \frac{120\angle 0}{26\angle 67.38} = 4.59\angle\text{-}67.49°$$

The real power P is

$$P = I^2 R = 4.59^2 \times 10 = 211.01\ \ \text{W}$$

The reactive power Q is

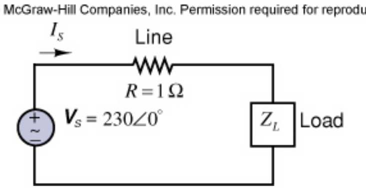
$$Q = I^2 X\ = 4.59^2 \times 24.13 = 509.25\ \ \text{VAR}$$

Problem 7.17

Solution:

Known quantities:

Circuit as shown in Figure P7.17.



Find:

a) the average power delivered to the load
b) the average power absorbed by the line
c) the apparent power supplied by the generator
d) the power factor of the load
e) the power factor of line plus load

Analysis:

$$I_S(?) = \frac{V_S}{R+Z_L} = \frac{230}{1+10+j3} = \frac{230}{11.4\angle 15.26^\circ} = 20.18\angle\text{-}15.26^\circ\ \text{ A}$$

a) The average power delivered to the load: $P_L = I_S^2 R_{ZL} = 20.182 \times 10 = 4072.32\ \text{ W}$

b) The average power absorbed by the line

$$P_{line} = I_S^2 R = 20.182 \times 1 = 407.23\ \text{ W}$$

c) The apparent power supplied by the generator is:

$$S = V_S I_S^* = 230 \times 20.18\angle 15.26^\circ = 4641.86\angle 15.26^\circ\ \text{ VA}$$

$$|S| = 4641.86\ \text{ VA}$$

d) The load impedance angle

$$\theta = \arctan\frac{3}{10} = 16.7^\circ$$

∴ The power factor of the load is

$$pf = \cos(16.7^\circ) = 0.9578 \quad \text{lagging}$$

e) The power factor of the line plus load is:

$$pf = \cos(15.26^\circ) = 0.9647 \quad \text{lagging}$$

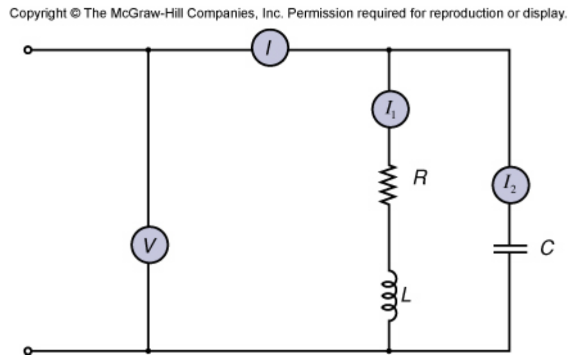
Problem 7.18

Solution:

Known quantities:
Circuit as shown in Figure P7.18.

Find:
The required capacitance.

Analysis:



The magnitude of the current I is

$$I_S = \frac{P}{V_S \cos\theta} = \frac{220}{200 \times 0.8} = 1.375 \text{ A}$$

Therefore, $I_S = 1.375\angle\text{-}36.87° \text{ A}$

The load impedance is $Z = \dfrac{V_S}{I_S} = \dfrac{200}{1.375\angle\text{-}36.87°} = 145.45\angle 36.87° = 116.36 + j87.27\ \Omega$

The reactive power Q_L is $Q_L = V_S I_S \sin\theta = 200 \times 1.375 \times 0.6 = 165 \text{ VAR}$

The reactance is $X_C = \dfrac{V_S^2}{Q_C} = \dfrac{200^2}{165} = 242.42\ \Omega$

The required capacitor is $C = \dfrac{1}{\omega X_C} = \dfrac{1}{375 \times 242.42} = 11\ \mu\text{F}$

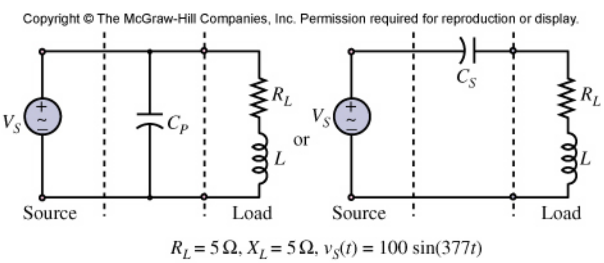
Problem 7.19

Solution:

Known quantities:
Circuit as shown in Figure P7.19.

Find:
The value of capacitor when circuit is at unity power factor.

Analysis:



(a) The source current in the parallel circuit is

$$I_S = \frac{100/\sqrt{2}}{5 + j5} = 10\angle\text{-}45° \text{ A}$$

The reactive power in the inductor is $Q_L = I_S^2 X_L = 10^2 \times 5 = 500 \text{ VAR}$

Thus, the capacitive reactance required to cancel the reactive power in the inductor is $X_C = \dfrac{V_S^2}{Q_L} = 10$

The required capacitor is $C = \dfrac{1}{377 X_C} = 265.3\ \mu\text{F}$

(b) In the series circuit, we can cancel the inductive reactance by setting $j\omega L + \dfrac{1}{j\omega C} = 0$,

resulting in $C = \dfrac{1}{\omega^2 L} = \dfrac{1}{\omega X_L} = \dfrac{1}{377 \times 5} = 530.5\ \mu\text{F}$

Problem 7.20

Solution:

Known quantities:

Find:
The value of capacitor to make the correction.

Analysis:

$$I_{old} = \frac{1000}{(120)(0.8)} = 10.4\angle -36.9° \quad \text{A}$$

$$I_{new} = \frac{1000}{(120)(0.95)} = 8.77\angle -18.2° \quad \text{A}$$

$$Q_{old} = 1000\tan 36.9° = 750 \quad \text{VAR}$$

$$Q_{new} = 1000\tan 18.2° = 328.8 \quad \text{VAR}$$

$$Q_C = 750 - 328.8 = 421.2 \text{ VAR}$$

$$\mathrm{X_C} = \frac{120^2}{Q_C} = 34.2 \ \Omega$$

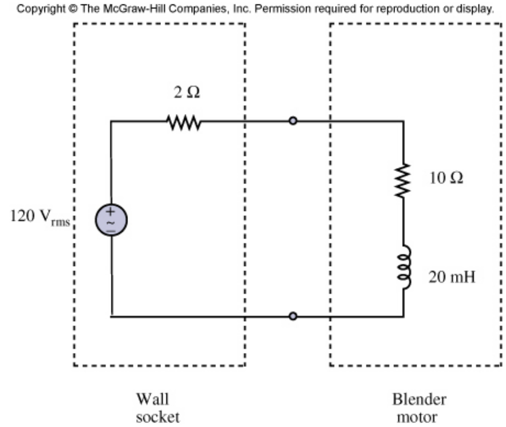
$$C = \frac{1}{(377)(34.2)} = 77.6 \ \mu\text{F}$$

Problem 7.21

Solution:

Known quantities:
Circuit as shown in Figure P7.21.

Find:
a) The average power dissipated in the load
b) Motor's power factor
c) What value of capacitor will change the power factor to 0.9 (lagging).



Analysis:

The current is

$$I = \frac{120}{12 + j377\times 20\times 10^{-3}} = \frac{120}{14.17\angle 32.14°} = 8.47\angle -32.14° \quad \text{A}$$

(a) The average power dissipated in the load is $P_{av} = I^2R = 8.47^2 \times 10 = 717.4$ W

(b) The power factor of the motor is $pf = \cos\ 32.14° = 0.847$ lagging

(c) $\theta = \cos^{-1} 0.9 = 25.84°$

$S_{NEW}\angle 25.84° = 717.4 \text{ W} + j(Q_L - Q_C)$ $\quad S_{NEW} = 797.1$ $\quad Q_{NEW} = 347.4$

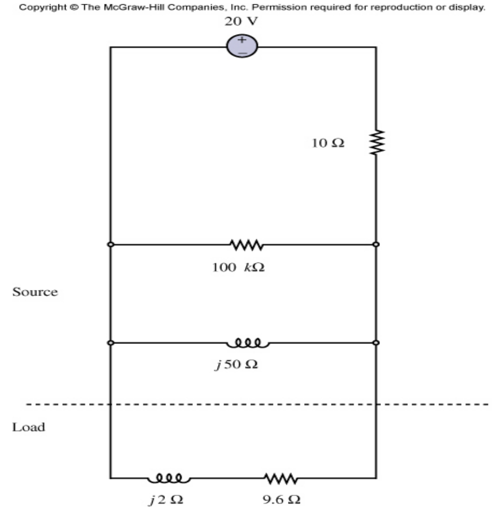
$Q_L = 450.7$ VAR $\quad Q_C = 103.3$ VAR

$$Q_C = \frac{V^2}{X_C} = \frac{120^2}{X_C} = 103.3$$

$X_C = 139.4\ \Omega$ $\qquad C = \dfrac{1}{\omega X_C} = 19\mu\text{F}$

Problem 7.22

Note: The voltage source is sinusoidal, with frequency 60 Hz, and its polarity is such that the current from the voltage source flows into the 10-Ω resistor.



Solution:

Known quantities:

Circuit as shown in Figure P7.22.

Find:

a) Thevenin equivalent circuit for the source
b) Power dissipated by the load resistor
c) What value of load impedance would permit maximum power transfer.

Analysis:

(a) $Z_T = 10 \,||\, 100\times 10^3 \,||\, j50 \approx \dfrac{10\times j50}{10+j50} \approx 9.6 + j2 \;\; \Omega$

$$V_T = \frac{100\times 10^3 \,||\, j50}{100\times 10^3 \,||\, j50 + 10} V_S = \frac{j5\times 10^6}{10^6 + j500 + j5\times 10^6} V_S = 19.22 + j3.84 = 19.6\angle 11.31° \text{ V}$$

(b) The load current can be computed from the Thèvenin equivalent as follows:

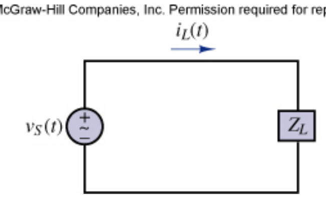
$$I = \frac{19.6\angle 11.31°}{2\times 9.6 + j4} = 1\angle\text{-}0.46° \text{ A}$$

The power dissipated by the load resistor is

$$P_L = 9.6 \text{ W}$$

(c) For maximum power transfer, we must have $Z_L = Z_T^* = (9.6 + j2)^* = 9.6 - j2 \; \Omega$

Problem 7.23



Solution:

Known quantities:

The current and the voltage values. In the circuit of Figure P7.23.

Find:

The average power, the reactive power and the complex power.

Analysis:

a) $P = \tilde{V}\tilde{I}\cos(\theta) = \dfrac{450}{\sqrt{2}}\cdot\dfrac{50}{\sqrt{2}}\cdot\cos(20°) = 21140 \text{ W}$ $Q = \tilde{V}\tilde{I}\sin(\theta) = \dfrac{450}{\sqrt{2}}\cdot\dfrac{50}{\sqrt{2}}\cdot\sin(20°) = 7696 \text{ VAR}$

$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 11250\angle 20° \text{ VA}$

b) $P = \tilde{V}\tilde{I}\cos(\theta) = 140\cdot 5.85\cdot\cos(-30°) = 709.3 \text{ W}$

$Q = \tilde{V}\tilde{I}\sin(\theta) = 140\cdot 5.85\cdot\sin(-30°) = -409.5 \text{ VAR}$ $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 819\angle -30° \text{ VA}$

c) $P = \tilde{V}\tilde{I}\cos(\theta) = 50\cdot 19.2\cdot\cos(-45.8°) = 668.8 \text{ W}$

$Q = \tilde{V}\tilde{I}\sin(\theta) = 50\cdot 19.2\cdot\sin(-45.8°) = -688.7 \text{ VAR}$ $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 960\angle -45.8° \text{ VA}$

d) $P = \tilde{V}\tilde{I}\cos(\theta) = 740\cdot 10.8\cdot\cos(-85.9° + 45°) = 6040.8 \text{ W}$

$Q = \tilde{V}\,\tilde{I}\sin(\theta) = 740\cdot 10.8\cdot\sin(-85.9° + 45°) = -5232.7 \text{ VAR}$ $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = 7992\angle -40.9° \text{ VA}$

Problem 7.24

Solution:

Known quantities:
The current and the voltage values or the impedance.

Find:
The power factor and state if it is leading or lagging.

Analysis:
a) $pf = \cos(\theta_i - \theta_v) = \cos(21.2°) = 0.932$ Leading

b) $pf = \cos(\theta_i - \theta_v) = \cos(-40.6°) = 0.759$ Lagging

c) $i_L(t) = 48.7\sin(\omega t + 2.74) = 48.7\sin(\omega t + 2.74 - 90°)$

$pf = \cos(\theta_i - \theta_v) = \cos(67°) = 0.391$ Leading

d) $\theta = \tan^{-1}\left(\frac{8}{12}\right) = 33.7° \Rightarrow pf = \cos(\theta_i - \theta_v) = \cos(-33.7°) = 0.832$ Lagging

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Problem 7.25

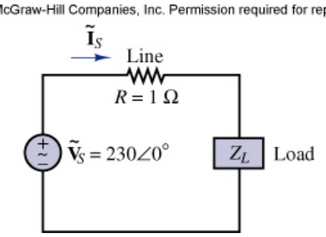
Solution:

Known quantities:
The power factor or the values of the current and the voltage.

Find:
The kind of the load (capacitive or inductive).

Analysis:
a) Capacitive.
b) Capacitive.
c) Since $i_L(t) = 1.8\cos(\omega t - 90°)$, Inductive.
d) Since the phase difference is zero, Resistive.

Problem 7.26



Solution:

Known quantities:
Circuit shown in Figure P7.26, the values of the resistance, $R = 4\,\Omega$, the capacitance, $C = 1/18\,\text{F}$, the inductance, $L = 2\,\text{H}$, and the voltage source.

Find
The real and reactive power supplied by the following sources.

a) $v_S(t) = 10\cos(3t)\,\text{V}$

b) $v_S(t) = 10\cos(9t)\,\text{V}$

Analysis:
a) $\omega = 3,\ Z_T = j6 - j6 + 4 = 4 + j0\,\Omega,\ \tilde{I} = \dfrac{10}{\sqrt{2}\cdot 4} = 1.77\,\text{A} \qquad P = \tilde{I}^2 R = 12.5\,\text{W},\ Q = \tilde{I}^2 X = 0\,\text{VAR}$

b) $\omega = 9,\ Z_T = j18 - j2 + 4 = 4 + j16\,\Omega,\ \tilde{I} = \dfrac{10}{\sqrt{2}\cdot 16.5} = 0.42\,\text{A}$

$P = \tilde{I}^2 R = 0.7\,\text{W},\ Q = \tilde{I}^2 X = 2.82\,\text{VAR}$

Problem 7.27

Solution:

Known quantities:

Circuit shown in Figure P7.27, the values of the resistances, $R_1 = 8\,\Omega$, $R_2 = 6\,\Omega$, the reactances, $X_C = -12$, $X_L = 6$, and the voltage sources, $\tilde{\mathbf{V}}_{S1} = 36\angle -\pi/3\,\text{V}$, $\tilde{\mathbf{V}}_{S2} = 24\angle 0.644\text{V}$.

Find:

a).The active and reactive current for each source b). The total real power.

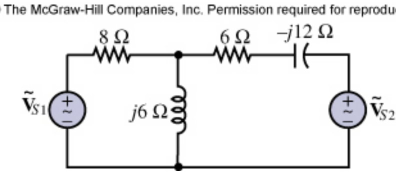
Analysis:

a) From Figure P7.13:

$$\mathbf{V}_{S1} = R_1\mathbf{I}_1 + jX_L(\mathbf{I}_1 - \mathbf{I}_2) = (8 + j6)\mathbf{I}_1 - j6\mathbf{I}_2$$

$$-\mathbf{V}_{S2} = -jX_L(\mathbf{I}_1 - \mathbf{I}_2) + R_2\mathbf{I}_2 + X_C\mathbf{I}_2 = -j6\mathbf{I}_1 + (6 - j6)\mathbf{I}_2$$

Substituting the values for the voltages sources gives:



$$\begin{cases} 18 - j31.2 = (8 + j6)\mathbf{I}_1 - j6\mathbf{I}_2 \\ -19.2 - j14.4 = -j6\mathbf{I}_1 + (6 - j6)\mathbf{I}_2 \end{cases}$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$ yields:

$$\begin{cases} \mathbf{I}_1 = 0.398 - j3.38\,\text{A} \\ \mathbf{I}_2 = 1.091 - j0.911\,\text{A} \end{cases}$$

Therefore, the active and reactive currents for each source are:

$$\begin{cases} \mathbf{I}_{A1} = 0.398\,\text{A} \\ \mathbf{I}_{A2} = 1.091\,\text{A} \end{cases} \text{ and } \begin{cases} \mathbf{I}_{R1} = 3.38\,\text{A} \\ \mathbf{I}_{R2} = 0.91\,\text{A} \end{cases}$$

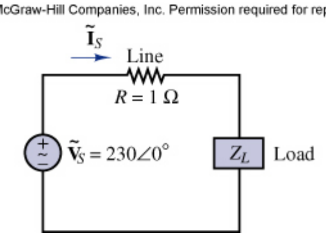
b) $P = R_2\mathbf{I}_2^2 + R_1\mathbf{I}_1^2 = 6 \cdot 1.421^2 + 8 \cdot 3.403^2 = 105\,\text{W}$

Problem 7.28

Solution:

Known quantities:

Circuit shown in Figure P7.28, the values of the resistors, $R_L = 25\,\Omega$, $R = 1\,\Omega$, the capacitor, $C = 0.1\,\mu\text{F}$, the voltage source, $\tilde{\mathbf{V}}_S = 230\,\text{V}$, and the frequency, $f = 60\,\text{Hz}$.



Find:

a) The source power factor.
b) The current $\mathbf{I}_S$.
c) The apparent power delivered to the load.
d) The apparent power supplied by the source.
e) The power factor of the load.

Analysis:

a) $pf_{source} = \dfrac{R}{Z} = \dfrac{R_{line} + R_{load}}{\sqrt{(R_{line} + R_{load})^2 + X_C^2}} = \dfrac{26}{\sqrt{676 + 7.036e^8}} = 0.00098$ Leading

b) $\tilde{I}_S = \dfrac{\tilde{V}_S}{Z} = \dfrac{230}{26525\angle -90^\circ} = 8.67m\,\text{A}$. Therefore, $\mathbf{I}_S = 8.67\angle -90^\circ\,m\text{A}$

c) $S_{load} = \sqrt{P_{load}^2 + Q_{load}^2} = \tilde{I}_S^2\sqrt{R_{load}^{\;2} + X_C^2} = 1.994\text{ VA}$

d) $S_{source} = \tilde{I}_S\tilde{V}_S = 1.994\text{ VA}$

e) $pf_{load} = \dfrac{R_{load}}{Z_{load}} = \dfrac{25}{26525} = 0.00094$

Problem 7.29

Solution:

Known quantities:

Circuit shown in Figure P7.28, the e values of the resistors, $R_L = 25\,\Omega$, $R = 1\,\Omega$, the inductor, $L = 0.1\,\text{H}$, the voltage source, $\tilde{\mathbf{V}}_S = 230\,\text{V}$, and the frequency, $f = 60\,\text{Hz}$.

Find:

a) The apparent power supplied by the source.
b) The apparent power delivered to the load.
c) The power factor of the load.

Analysis:

a) $$S = \frac{\tilde{V}^2}{Z} = \frac{\tilde{V}^2}{\sqrt{(R_{line} + R_{load})^2 + X_L^2}} = \frac{230^2}{\sqrt{26^2 + 37.7^2}} = 1.155\,\text{kVA}$$

b) $$S_{load} = \tilde{I}_S^2 Z_{load} = \frac{\tilde{V}^2}{Z^2}\sqrt{R_{load}^2 + X_L^2} = \left(\frac{230}{45.8}\right)^2 \sqrt{25^2 + 37.7^2} = 1.141\,\text{kVA}$$

c) $$pf_{load} = \frac{R_{load}}{Z_{load}} = \frac{25}{45.2} = 0.55$$

Problem 7.30

Solution:

Known quantities:

Circuit shown in Figure P7.28, the values of the resistors, $R_L = 25\,\Omega$, $R = 1\,\Omega$, the capacitor, $C = 0.1\,\text{mF}$, the inductor, $L = 70.35\,\text{mH}$, the voltage source, $\tilde{\mathbf{V}}_S = 230\,\text{V}$, and the frequency, $f = 60\,\text{Hz}$.

Find:

a) The apparent power delivered to the load.
b) The real power supplied by the source.
c) The power factor of the load.

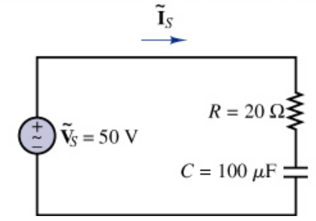
Analysis:

a) $$S = \frac{\tilde{V}^2}{Z} = \frac{\tilde{V}^2}{\sqrt{(R_{line} + R_{load})^2 + (X_L - X_C)^2}} = \frac{230^2}{\sqrt{26^2 + (26.5 - 26.5)^2}} = 2.03\,\text{kVA}$$

b) $$X_C = X_L \;\Rightarrow\; P = \frac{\tilde{V}^2}{R} = \frac{230^2}{25+1} = 2.03\,\text{kW}$$

c) $$X_C = X_L \;\Rightarrow\; pf = 1$$

Problem 7.31



Solution:

Known quantities:

Circuit shown in Figure P7.31, the values of the resistor, $R = 20?$, the capacitor, $C = 0.1\,\text{mF}$, the voltage source, $\tilde{\mathbf{V}}_S = 50\,\text{V}$.

Find:

The apparent power, the real power, and the reactive power; draw the power triangle.

Analysis:

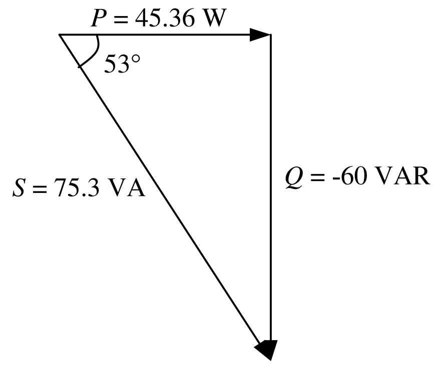
$$S = \frac{\tilde{V}^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{50^2}{\sqrt{20^2 + 26.5^2}} = \frac{2500}{33.2} = 75.3\,\text{VA}$$

$$P = S \cdot \cos(\theta) = S \cdot \frac{R}{Z} = 75.3\frac{20}{33.2} = 45.36\,\text{W}$$

$$Q = \sqrt{S^2 - P^2} = -60\,\text{VAR}$$

For the power triangle, $\theta = \cos^{-1}\left(\frac{R}{Z}\right) = 53°$

Therefore, the power triangle can be drawn as shown in the figure:



Problem 7.32

Solution:

Known quantities:

Circuit shown in Figure P7.31, the values of the resistor, $R = 20\,\Omega$, the capacitor, $C = 0.1\,\text{mF}$, the voltage source, $\tilde{\mathbf{V}}_S = 50\,\text{V}$.

Find:

The apparent power, the real power, and the reactive power, in the cases of $f = 50$ and 0 Hz.

Analysis:

For the frequency of 0 Hz,

$$S_{0Hz} = 0,\ P_{0Hz} = 0,\ Q_{0Hz} = 0$$

For the frequency of 50 Hz,

$$S_{50Hz} = \frac{\tilde{V}^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{2500}{37.2} = 67.15\,\text{VA}$$

$$P_{50Hz} = S \cdot \cos(\theta) = S \cdot \frac{R}{Z} = 67.15\frac{20}{37.2} = 36\,\text{W}$$

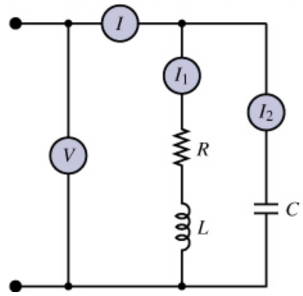
$$Q_{50Hz} = \sqrt{S^2 - P^2} = 56.7\,\text{VA}$$

Problem 7.33

Solution:

Known quantities:

A single-phased motor connected across a 220-V source at 50 Hz as shown in Figure P7.33, power factor *pf* = 1.0, I = 20 A, and I_1 = 25 A.



Find:

The capacitance required to give a unity power factor when connected in parallel with the load.

Analysis:

The magnitude of the current $\tilde{\mathbf{I}}_2$ is:

$$\tilde{I}_2 = \sqrt{\tilde{I}_1^2 - \tilde{I}^2} = \sqrt{625 - 400} = 15\,\text{A}$$

The voltage source can be expressed as:

$$\tilde{V} = \tilde{I}_2 \cdot X_C$$

Therefore, the required capacitor is:

$$C = \frac{\tilde{I}_2}{\tilde{V} \cdot ?} = \frac{15}{220 \cdot 314} = 217\,?\text{F}$$

Problem 7.34

Solution:

Known quantities:

The currents and voltages required by an air-conditioner, a freezer, a refrigerator, and their power factors.

Find:

The power to be supplied by an emergency generator to run all the appliances.

Analysis:

In this problem we will use the following equations:

$$P = \tilde{I}\,\tilde{V}\cos(\theta),\quad Q = \tilde{I}\,\tilde{V}\sin(\theta),\quad pf = \cos(\theta)$$

The real and reactive power used by the air conditioner are:

$$P_1 = 9.6 \cdot 120 \cdot 0.9 = 1036.8\ \mathbf{W},\quad Q_1 = 9.6 \cdot 120 \cdot \sin\left(\cos^{-1}(0.9)\right) = 502.15\ \mathbf{VAR}$$

The real and reactive power used by the freezer are:

$$P_2 = 4.2 \cdot 120 \cdot 0.87 = 438.48\ \mathbf{W},\quad Q_2 = 4.2 \cdot 120 \cdot \sin\left(\cos^{-1}(0.87)\right) = 248.5\ \mathbf{VAR}$$

The real and reactive power used by the refrigerator are:

$$P_3 = 3.5 \cdot 120 \cdot 0.8 = 336\ \mathbf{W},\quad Q_3 = 3.5 \cdot 120 \cdot \sin\left(\cos^{-1}(0.8)\right) = 252\ \mathbf{VAR}$$

The total real and reactive power P are:

$$P = P_1 + P_2 + P_3 = 1811.28\ \mathbf{W},\quad Q = Q_1 + Q_2 + Q_3 = 1002.65\ \mathbf{VAR}$$

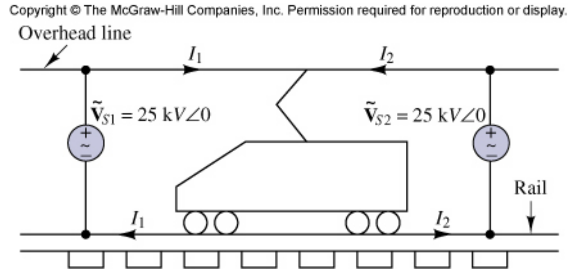
Therefore, the following power must be supplied:

$$S = P + jQ = 1811.28 + j1002.65\ \mathbf{VA} = 2070.3\angle 28.97^o\ \mathbf{VA}$$

Problem 7.35

Solution:

Known quantities:



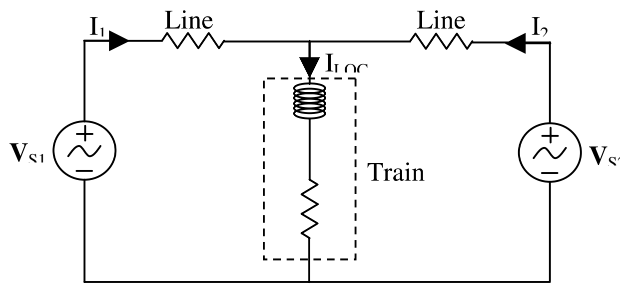
The schematics of the power supply module consisting of two 25-kV single-phase power stations shown in Figure P7.35, the power consumption by the train, the DC power supply at a low speed operation, the average power factor in AC operation, the over-head line equivalent specific resistance, and negligible rail resistance.

Find:

a) The equivalent circuit.
b) The locomotive current in the condition of a 10% voltage drop.
c) The reactive power.
d) The supplied real power, over-head line losses, and the maximum distance between two power station supplied in the condition of a 10% voltage drop when the train is located at the half distance between the stations.
e) Over-head line losses in the condition of a 10% voltage drop when the train is located at the half distance between the stations, assuming *pf* = 1 (The French TGV is designed with the state of art power compensation system).
f) The maximum distance between the two power station supplied in the condition of a 10% percent voltage drop when the train is located at the half distance between the stations, assuming the DC (1.5 kV) operation at a quarter power.

Analysis:

a) The equivalent circuit is shown in the figure.



b) The locomotive current for the 10% voltage drop is:

$$\tilde{I}_{LOC} = \tilde{I}_1 + \tilde{I}_2 = 2\tilde{I}_1 = 2\tilde{I}_2 = \frac{P_{LOC}}{\left(\tilde{V}_S - 10\%\right)\cos(\theta)} = \frac{11\,\text{MW}}{22.5\,\text{kV}\cdot 0.8} = 611\,\text{A}$$

c) The reactive power is:

$$Q = \sqrt{S_{LOC}^2 - P_{LOC}^2} = \sqrt{\left(\tilde{V}_S - 10\%\right)^2 I_{LOC}^2 - P_{LOC}^2} = \sqrt{(13.75\,\text{MVA})^2 - (11\,\text{MW})^2} = 8.25\,\text{MVAR}$$

d) The supplied real power is:

$$P = \sqrt{S^2 - Q^2} = \sqrt{\left(\tilde{V}_S \cdot \tilde{I}\right)^2 - Q^2} = 12.85\,\text{MW}$$

The over-head line power loss is:

$$P_{Line} = -P_{LOC} + \sqrt{\left(\tilde{V}_S \cdot \tilde{I}\right)^2 - Q^2} = -11\text{MW} + \sqrt{(15.27\text{MVA})^2 - (8.25\text{MVAR})^2} = 1.85\text{MW}$$

The maximum distance between the two power stations is:

$$R_{Line} \parallel R_{Line} = \frac{P_{Line}}{\tilde{I}_{LOC}^2} = 5\,\Omega \;\Rightarrow\; \text{Distance}_{\max} = \frac{2R_{Line}}{0.2\,\Omega/\text{km}} = 100\,\text{km}$$

e) The over-head line power loss is: $P_{LOC} = 10\% \cdot \tilde{V}_S \cdot I_{LOC} \cdot \cos(\theta) = 2500\,\text{V} \cdot 489\,\text{A} = 1.22\,\text{MW}$

f) $I_{LOC} = \dfrac{0.25 P_{LOC}}{90\% \cdot V_{S,DC}} = \dfrac{2.75\,\text{MW}}{1350\,\text{V}} = 2037\,\text{A}$ $\quad P_{Line} = 10\% \cdot V_{S,DC} \cdot I_{LOC} = 305\,\text{kW}$ $\quad R_{Line} \parallel R_{Line} = \dfrac{P_{Line}}{I_{LOC}^2} = 0.0735\,\Omega$

The maximum distance between the two power stations is:

$$\text{Distance}_{\max} = \frac{2R_{Line}}{0.2\,\Omega/\text{km}} = 1.5\,\text{km}$$

Problem 7.36

Solution:

Known quantities:
One hundred 40-W lamps supplied by a 120-V and 60-Hz source, the power factor of 0.65, the penalty at billing, and the average prices of the power supply and the capacitors.

Find:
Number of days of operation for which the penalty billing covers the price of the power factor correction capacitor.

Analysis:

The capacitor value for $pf = 0.85$ is:

$$C = \frac{\tilde{I}_C}{\tilde{V}_C \cdot \omega} = \frac{\left(\tilde{I}_{X,0.65} - \tilde{I}_{X,0.85}\right)}{\omega \cdot V_C} = \frac{\sqrt{\left(\frac{P}{\tilde{V} \cdot 0.65}\right)^2 - \left(\frac{P}{V}\right)^2} - \sqrt{\left(\frac{P}{\tilde{V} \cdot 0.85}\right)^2 - \left(\frac{P}{V}\right)^2}}{\omega \cdot V_C}$$

$$= \frac{\sqrt{51.3^2 - 33.3^2} - \sqrt{39.2^2 - 33.3^2}}{337 \cdot 120} = \frac{19}{377 \cdot 120} = 420\ \mu\text{F}$$

Therefore, the number of days of operation for which the penalty billing covers the price of the power factor correction capacitor is:

$$\text{Number of Days} = \frac{420\ \mu\text{F} \cdot 50 \frac{\$}{\text{mF}}}{\frac{4\text{ kW}}{4} \cdot 1\text{ hr} \cdot 0.01 \frac{\$}{\text{kW}} \cdot 24 \frac{\text{hr}}{\text{day}}} = 88\ \text{Days}$$

Problem 7.37

Solution:

Known quantities:
Reference to the problem 7.36, and the network current decreasing with the power factor correction.

Find:
a) The capacitor value for the unity power factor.
b) The maximum number of lamps that can be installed supplementary without changing the cable network if a local compensation capacitor is used.

Analysis:
a) $\left|\tilde{\mathbf{I}}_C\right| = \left|\tilde{\mathbf{I}}_L\right|$

$$C = \frac{\left|\tilde{\mathbf{I}}_L\right|}{\tilde{V}\omega} = \frac{\sqrt{\left(\frac{P}{\tilde{V} \cdot 0.65}\right)^2 - \left(\frac{P}{\tilde{V}}\right)^2}}{\tilde{V}\omega} = \frac{\sqrt{51.3^2 - 33.3^2}}{377 \cdot 120} = 862\ \mu\text{F}$$

b) Initial cable network has: $\tilde{I} = \frac{P}{\tilde{V}\cos(\theta)} = \frac{4000}{120 \cdot 0.65} = 51.3\text{ A}$

One lamp current for pf = 1 is: $I_{Lamp} = \frac{P_{Lamp}}{\tilde{V}} = \frac{40}{120} = 0.333\text{ A}$

The total number of lamps $= \frac{\tilde{I}}{\tilde{I}_{Lamp}} = 154$

Therefore, the number of supplementary lamps $= 154 - 100 = 54$

Problem 7.38

Solution:

Known quantities:

The voltage and the current supplied by a source, $\tilde{\mathbf{V}}_S = 7\angle 50°\text{ V}$, $\tilde{\mathbf{I}}_S = 13\angle -20°\text{ A}$.

Find:

a) The power supplied by the source which is dissipated as heat or work in the load
b) The power stored in reactive components in the load.
c) Determine if the circuit is an inductive or a capacitive load.

Analysis:

a) $S = \tilde{\mathbf{V}}_S \tilde{\mathbf{I}}_S^* = (7\angle 50°\text{ V})(13\angle -20°\text{ A})^* = 91\angle 70°\text{ VA} = 31.12 + j85.51\text{ VA}$

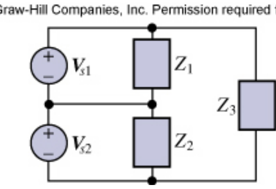
$P_{av} = 31.12\text{ W}$

b) $Q = 85.51\text{VAR}$

c) $\theta = (\theta_I - \theta_V) = -70° \Rightarrow pf = \cos(\theta) = 0.342$ Lagging

The load is inductive.

Problem 7.39



Solution:

Known quantities:

Circuit shown in Figure P7.39, the values of the voltages and all the impedances.

Find:

The total average power, the real power dissipated and the reactive power stored in each of the impedances.

Analysis:

$$S_1 = \frac{\tilde{\mathbf{V}}_{S1}}{Z_1^*} = \frac{\left(170/\sqrt{2}\text{ V}\right)^2}{0.7\angle -30°\Omega} = 20.643\angle 30°\text{kVA} = 17.88 + j10.32\text{kVA} = P_{av1} + jQ_1$$

$$S_2 = \frac{\tilde{\mathbf{V}}_{S2}}{Z_2^*} = \frac{\left(170/\sqrt{2}\text{ V}\right)^2}{1.5\angle -7°\Omega} = 9.633\angle 7°\text{ kVA} = 9.56 + j1.17\text{ kVA} = P_{av2} + jQ_2$$

$$S_3 = \frac{\left(\tilde{\mathbf{V}}_{S1} + \tilde{\mathbf{V}}_{S2}\right)^2}{Z_3^*} = \frac{(170\angle 0° + 170\angle 90°\text{ V})^2}{0.3 - j0.4\,\Omega} = \frac{(240.42\angle 45°\text{ V})^2}{0.5\angle -53.13°\Omega} = 57.8\angle 53.13°\text{ kVA} = 34.68 - j46.24\text{ kVA} = P_{av3} + jQ_3$$

Problem 7.40

Solution:

Known quantities:

The voltage and the current supplied by a source, $\tilde{\mathbf{V}}_S = 170\angle -9° \text{ V}$, $\tilde{\mathbf{I}}_S = 13\angle 16° \text{ A}$.

Find:

a) The power supplied by the source which is dissipated as heat or work in the load
b) The power stored in reactive components in the load.
c) Determine if the circuit is an inductive or a capacitive load.

Analysis:

a) $S = \tilde{\mathbf{V}}_S \tilde{\mathbf{I}}_S^* = (170\angle -9° \text{ V})(13\angle 16° \text{ A})^* = 2210\angle -25° \text{ VA} = 2003 - j934.0 \text{ VA} \quad P_{av} = 2003 \text{ W}$

b) $Q = -934 \text{ VAR}$

c) $\theta = (\theta_I - \theta_V) = 25° \Rightarrow pf = \cos(\theta) = 0.906$ Leading

The load is capacitive.

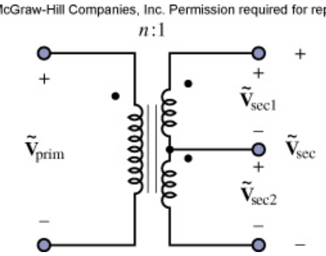
Section 7.3: Transformers

Problem 7.41

Solution:

Known quantities:

Circuit shown in Figure P7.41, 3ach secondary connected to 5-kW resistive load, the primary connected to 120-V rms.



Find:

a) Primary power.
b) Primary current.

Analysis:

a) $P_{prim} = P_{\sec 1} + P_{\sec 2} = 10\,\text{kW}$

b) $\tilde{I}_{prim} = \dfrac{P_{prim}}{\tilde{V}} = \dfrac{10000}{120} = 83.3\,\text{A}$

$\tilde{\mathbf{I}}_{prim} = 83.3\angle 0\,\text{A}$

Problem 7.42

Solution:

Known quantities:

Circuit shown in Figure P7.41, the ratio between the secondary and the primary, $\dfrac{\tilde{\mathbf{V}}_{\sec}}{\tilde{\mathbf{V}}_{prim}} = n$ and

$\tilde{\mathbf{V}}_{\sec 1} = \tilde{\mathbf{V}}_{\sec 2} = \dfrac{1}{2}\tilde{\mathbf{V}}_{\sec}$.

Find:

a) V_{sec} and V_{sec1} if V_{prim} = 220 V rms and n = 11.
b) n if V_{prim} = 110 V rms and V_{sec2} = 5 V rms.

Analysis:

a) $V_{\sec} = \dfrac{V_{prim}}{n} = \dfrac{220}{11} = 20\,\text{V}$

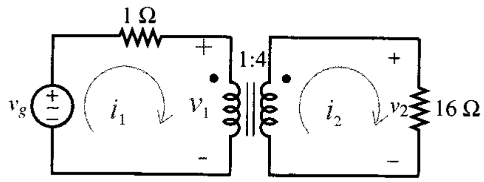
$V_{\sec 1} = \dfrac{V_{\sec}}{2} = 10\,\text{V}$

b) $n = \dfrac{V_{prim}}{2 \cdot V_{\sec 2}} = \dfrac{110}{2 \cdot 5} = 11$

Problem 7.43

Solution:

Known quantities:
The circuit shown in Figure P7.43 and v_g = 120 V rms.



Find:
a) The total resistance seen by the voltage source.
b) The primary current.
c) The primary power.

Analysis:
From the circuit shown on the right hand side:

$$v_g = i_1 R_1 + v_1 \qquad i_2 = v_2 / R_2$$

$$v_2 = n v_1 \qquad i_1 = n i_2$$

a) $R_{tot} = \dfrac{v_g}{i_1} = \dfrac{R_1 i_1 + v_1}{i_1} = R_1 + \dfrac{R_2}{n^2} = 2\,\Omega$

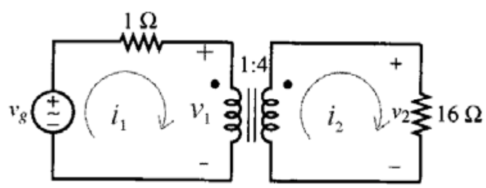
b) $\tilde{I}_1 = \dfrac{v_g}{R_{tot}} = \dfrac{120}{2} = 60\,\text{A}$

c) $P_1 = \tilde{I}_1 \cdot \tilde{V}_1 = 7.2\,\text{kW}$

Problem 7.44

Solution:

Known quantities:
The circuit shown in Figure P7.43 and v_g = 120 V rms.



Find:
a) The secondary current.
b) The installation efficiency P_{load}/P_{source}.
c) The value of the load resistance which can absorb the maximum power from the given source.

Analysis:
From the circuit shown on the right hand side:

a) $I_2 = \dfrac{I_1}{4} = \dfrac{60}{4} = 15\,\text{A}$

b) $\eta = \dfrac{P_{load}}{P_{source}} = \dfrac{I_2^2 \cdot R_2}{v_g \cdot I_1} = \dfrac{225 \cdot 16}{120 \cdot 60} = \dfrac{3.6\,\text{kW}}{7.2\,\text{kW}} = 0.5$

c) For the maximum power transfer:

$$R_{prim} = R'_{\sec}$$

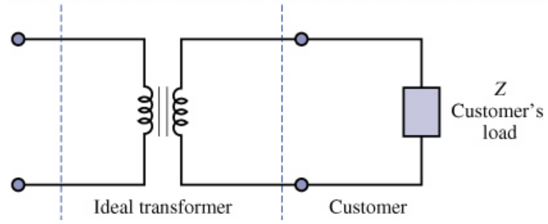
$$R_1 = \frac{1}{n^2} R_{load} \;\Rightarrow\; R_{load} = R_1 \cdot n^2 = 1\,\Omega \cdot 16 = 16\,\Omega$$

Problem 7.45

Solution:

Known quantities:

Circuit shown in Figure P7.45, the voltage and the power that a transformer is rated to deliver to a customer, $\tilde{V}_1 = 380\,\text{V}$, $P_{in} = 460\,\text{kW}$.



Find:

a) The current that the transformer supply to the customer.
b) The maximum power that the customer can receive if the load is purely resistive.
c) The maximum power that the customer can receive if the power factor is 0.8, lagging.
d) The maximum power that the customer can receive if the power factor is 0.7, lagging.
e) The minimum power factor to operate if the customer requires 300 kW.

Analysis:

a) From $S_{in} = \tilde{V}_1\tilde{I}_1 = \tilde{V}_2\tilde{I}_2 = S_{out}$:

$$I_1 = \frac{S_{in}}{\tilde{V}_1} = \frac{460 \cdot 1000}{380} = 1.21\,\text{kA}$$

b) For an ideal transformer:

$$P_{out} = P_{in}\cos(\theta) = \tilde{V}_1\tilde{I}_1\cos(\theta)$$

For $\cos(\theta) = 1$:

$$P_{out} = \tilde{V}_1\tilde{I}_1 = 460\,\text{kW}$$

c) For $\cos(\theta) = 0.8$, the maximum power is:

$$P_{out} = \tilde{V}_1\tilde{I}_1\cos(\theta) = 368\,\text{kW}$$

d) For $\cos(\theta) = 0.7$, the maximum power is:

$$P_{out} = \tilde{V}_1\tilde{I}_1\cos(\theta) = 322\,\text{kW}$$

e) For $P_{out} = 300\,\textbf{kW}$, the minimum power factor is:

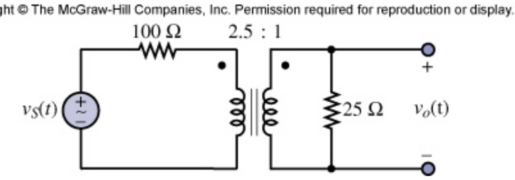
$$\cos(\theta) = \frac{P_{out}}{P_{in}} = \frac{300\,\text{kW}}{460\,\text{kW}} = 0.65$$

Problem 7.46

Solution:

Known quantities:

Circuit shown in Figure P7.46, the voltage, $v_S(t) = 294\cos(377t)\,\text{V}$, the resistances in a circuit containing a transformer and the ratio $n = \dfrac{v_0(t)}{v_S(t)} = \dfrac{1}{2.5}$.



Find:

a) Primary current.
b) $v_0(t)$.
c) Secondary power.
d) The installation efficiency P_{load}/P_{source}.

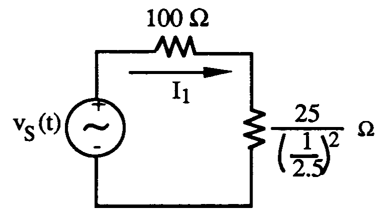
Analysis:

a) The primary circuit is described in the figure.
The primary current is:

$$\tilde{V}_S = \tilde{I}_1(R_1 + R'_2) = \tilde{I}_1\left(R_1 + n^2 R_2\right)$$

$$\tilde{I}_1 = \frac{\tilde{V}_1}{R_1 + n^2 R_2} = \frac{294}{\sqrt{2}} \cdot \frac{1}{100 + 6.25 \cdot 25} = 0.82\,\text{A}$$

$$i(t) = 1.144\cos(377t)\,\text{A}$$



b) The output voltage is:

$$\tilde{V}_0 = \frac{\tilde{V}_1}{n} = \frac{\tilde{V}_S - R_1\tilde{I}_1}{n} = \frac{208 - 100 \cdot 0.82}{2.5} = 50.4\,\text{V}$$

$$v_0(t) = 71\cos(377t)\,\text{V}$$

c) For the secondary power, if $pf = 1$:

$$P_2 = \tilde{I}_2\tilde{V}_0 = \tilde{I}_1 \cdot \tilde{V}_0 \cdot n = 0.82 \cdot 50.4 \cdot 2.5 = 103.3\,\text{W}$$

d) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{P_2}{\tilde{V}_S \cdot \tilde{I}_1} = \frac{103.3\,\text{W}}{208\,\text{V} \cdot 0.82\,\text{A}} = 0.6$$

Problem 7.47

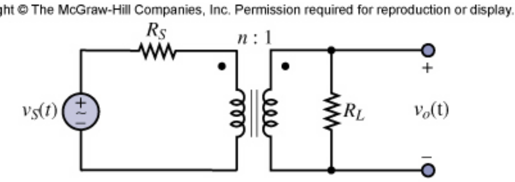
Solution:

Known quantities:

Circuit shown in Figure P7.47, the resistances, $R_S = 1800\,\Omega$, $R_L = 8\,\Omega$.

Find:

The turn's ratio that will provide the maximum power transfer to the load.



R_S = 1,800 Ω R_L = 8 Ω

Analysis:

From Equation (7.41) for the reflected source impedance circuit, we have: $R_{S_{eq}} = N^2 R_S$

Therefore, the power is maximized if:

$$R_{S_{eq}} = R_L \;\Rightarrow\; N = 1/n = \sqrt{R_L/R_S} = 0.067 \quad \therefore n = 15$$

Problem 7.48

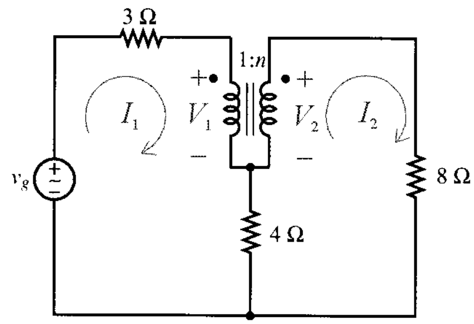
Solution:

Known quantities:

The voltage source and the resistances in the circuit shown in Figure P7.48.

Find:

a) Maximum power dissipated by the load.
b) Maximum power absorbing from the source.
c) The installation efficiency.



Analysis:

All the impedances are resistances, and therefore it is possible to consider the modules of voltages and currents.

a) To maximize the power delivered to the 8-Ω resistance, n must be selected to maximize the load current $I_2 > 0$.

Note that $V_1 = \frac{1}{n} V_2$ and $I_1 = nI_2$.

KVL Mesh 1:

$$v_g = 3I_1 + \frac{1}{n}V_2 + 4(I_1 - I_2) = 3nI_2 + \frac{1}{n}V_2 + 4(nI_2 - I_2)$$

KVL Mesh 2:

$$V_2 = 8I_2 + 4(I_2 - I_1) = 8I_2 + 4(I_2 - nI_2)$$

Rearranging the two mesh equations:

$$\begin{cases}(7n-4)I_2 + \frac{1}{n}V_2 = v_g \\ (12-4n)I_2 = V_2\end{cases} \Rightarrow (7n-4)I_2 + \frac{1}{n}(12-4n)I_2 = v_g$$

$$I_2 = \frac{n}{7n^2 - 8n + 12} v_g$$

$$\frac{d}{dn} I_2 = \frac{(7n^2 - 8n + 12) - n(14n - 8)}{(7n^2 - 8n + 12)^2} V_g = \frac{-7n^2 + 12}{(7n^2 - 8n + 12)^2} v_g$$

For the maximum value of the load current I_2, $\frac{dI_2}{dn} = 0$:

$$-7n^2 + 12 = 0 \Rightarrow n = \sqrt{\frac{12}{7}} = 1.31$$

The maximum load current is:

$$I_2 = \frac{n}{7n^2 - 8n + 12} v_g = 0.122 v_g = 13.47\,\text{A}$$

The maximum power dissipated by the load is:

$$P_{load} = R_{load} I_2^2 = 2.45\,\text{kW}$$

b) The maximum power absorbing from the source is:

$$P_{source} = P_{load} + 3\Omega \cdot I_1^2 + 4\,\Omega \cdot (I_1 - I_2)^2$$

$$P_{source} = P_{load} + 3\Omega \cdot nI_2^2 + 4\,\Omega \cdot (nI_2 - I_2)^2$$

$$P_{source} = 2540\,\text{W} + 934\,\text{W} + 70\,\text{W} = 3.54\,\text{kW}$$

c) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{2.45\,\text{kW}}{3.54\,\text{kW}} = 0.7$$

Problem 7.49

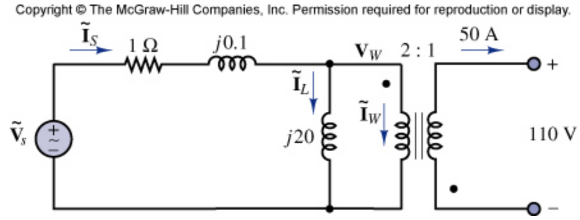
Solution:

Known quantities:

The current and the voltage delivered by the transformer, and the circuit of the transformer shown in Figure P7.49.

Find:

The efficiency of the installation.



Analysis:

$$\tilde{V}_W = n\tilde{V}_{\text{sec}} = 2\tilde{V}_{\text{sec}} = 220\,\text{V rms}$$

$$\tilde{I}_W = \frac{1}{n}\tilde{I}_{\text{sec}} = \frac{1}{2}\tilde{I}_{\text{sec}} = 25\,\text{A rms}$$

$$\tilde{I}_L = \frac{\tilde{V}_W}{j20} = \frac{220}{20}\angle -90° = 11\angle -90°\,\text{A rms}$$

Since the currents are exactly 90° out of phase, the current is the square root of the sum of the magnitudes squared:

$$\tilde{I}_S = \sqrt{\tilde{I}_L^2 + \tilde{I}_W^2} = 27.3\,\text{A rms}$$

$$P_S = 1\Omega \cdot \tilde{I}_S^2 + P_{\text{sec}} = 745\,\text{W} + 5.5\,\text{kW} = 6.245\,\text{kW}$$

Therefore, the efficiency of the installation is:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{P_{\text{sec}}}{P_{source}} = \frac{5.5\,\text{kW}}{6.245\,\text{kW}} = 0.88$$

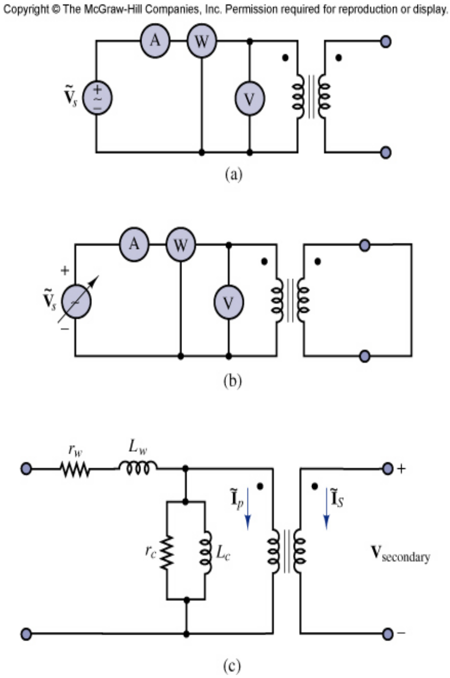
Problem 7.50

Solution:

Known quantities:

The model for the circuit of a transformer shown in Figure P7.50 and the results of two tests performed at $\omega = 377\ rad/s$:

1. Open-circuit test: $\tilde{V}_{oc} = 241\,\text{V}$, $\tilde{I}_{oc} = 0.95\,\text{A}$, $P_{oc} = 32\,\text{W}$.
2. Short-circuit test: $\tilde{V}_{sc} = 5\,\text{V}$, $\tilde{I}_{sc} = 5.25\,\text{A}$, $P_{sc} = 26\,\text{W}$.



Find:

The value of the impedances in the equivalent circuit.

Analysis:

The power factor during the open circuit test is:

$$pf_{oc} = \cos(\theta_{oc}) = \frac{P_{oc}}{\tilde{V}_{oc}\tilde{I}_{oc}} = 0.1398 \quad \text{Lagging}$$

The excitation admittance is given by:

$$Y_c = \frac{\tilde{I}_{oc}}{\tilde{V}_{oc}}\angle\cos^{-1}(pf_{oc}) = \frac{0.95}{241}\angle -81.96°\,\text{S} = 0.0005511 - j0.003903\text{S}$$

$$\begin{cases} R_c = 1.8\text{k}\Omega \\ X_c = 256.2\Omega \end{cases} \Rightarrow \begin{cases} R_c = 1.8\text{k}\Omega \\ L_c = \dfrac{X_c}{\omega} = 0.68\text{H} \end{cases}$$

The power factor during the short circuit test is: $pf_{sc} = \cos(\theta_{sc}) = \dfrac{P_{sc}}{\tilde{V}_{sc}\tilde{I}_{sc}} = 0.9905 \quad \text{Leading}$

The series impedance is given by: $Z_W = \dfrac{\tilde{V}_{sc}}{\tilde{I}_{sc}}\angle\cos^{-1}(pf_{sc}) = \dfrac{5}{5.25}\angle 7.914°\,\Omega = 0.9476 + j0.1311\Omega$

$$\begin{cases} R_w = 0.9476\Omega \\ X_w = 0.1311 \end{cases} \Rightarrow \begin{cases} R_w = 0.9476\Omega \\ L_w = \dfrac{X_w}{\omega} = 0.348\text{mH} \end{cases}$$

Problem 7.51

Solution:

Known quantities:

The model for the circuit shown in Figure P7.50 of a $460\,\text{kVA}$ transformer and the results of two tests performed at $f = 60\,\text{Hz}$:

1. Open-circuit test: $\tilde{V}_{oc} = 4600\,\text{V}$, $\tilde{I}_{oc} = 0.7\,\text{A}$, $P_{oc} = 200\,\text{W}$.
2. Short-circuit test: $\tilde{V}_{sc} = 5.2\,\text{V}$, $P_{sc} = 50\,\text{W}$.

Find:

The value of the impedances in the equivalent circuit.

Analysis:

The power factor during th e open circuit test is:

$$pf_{oc} = \cos(\theta_{oc}) = \frac{P_{oc}}{\tilde{V}_{oc}\tilde{I}_{oc}} = 0.062 \quad \text{Lagging}$$

The excitation admittance is given by:

$$Y_c = \frac{\tilde{I}_{oc}}{\tilde{V}_{oc}} \angle \cos^{-1}(pf_{oc}) = \frac{0.7}{4600} \angle -86.45°\,\text{S} = 9.4 \cdot 10^{-6} - j0.152 \cdot 10^{-3}\,\text{S}$$

$$\begin{cases} R_c = 106.38\,\text{k?} \\ X_c = 6.58\,\text{k}\Omega \end{cases} \Rightarrow \begin{cases} R_c = 106.38\,\text{k}\Omega \\ L_c = \dfrac{X_c}{\omega} = 17.46\,\text{H} \end{cases}$$

The power factor during the short circuit test:

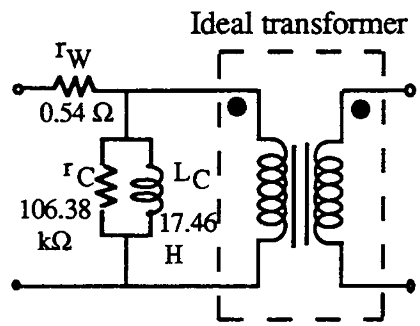
$$pf_{sc} = \cos(\theta_{sc}) = \frac{P_{sc}}{S_{sc}} \approx 1$$

since it is a high power transformer.

The series impedance has therefore imaginary part ≈ 0:

$$Z_w = \frac{\tilde{V}_{sc}^2}{P_{sc}} \angle \cos^{-1}(pf_{sc}) = 0.54\,\Omega$$

$$\begin{cases} R_w = 0.54\,\Omega \\ X_w = 0 \end{cases} \Rightarrow \begin{cases} R_w = 0.54\,\Omega \\ L_w = \dfrac{X_w}{\omega} = 0 \end{cases}$$



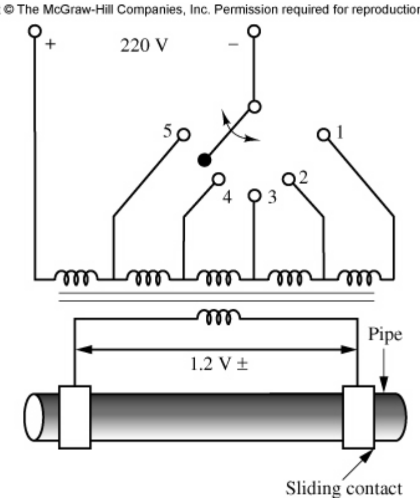
Therefore, the equivalent circuit is shown besides.

Problem 7.52

Solution:

Known quantities:

Known quantities:
Circuit shown in Figure P7.52 of the single-phase transformer with the high voltage regulation from five different slots in the primary winding, the secondary voltage regulation in the range of 10%, and the number of turns in the secondary coil.



Find:
The number of turns for each slot.

Analysis:
The secondary voltages are:

$$\tilde{V}_{21} = \tilde{V}_{23} - 0.12\text{ V} = 1.08\text{ V rms}$$
$$\tilde{V}_{22} = \tilde{V}_{23} - 0.06\text{ V} = 1.14\text{ V rms}$$
$$\tilde{V}_{23} = 1.20\text{ V rms}$$
$$\tilde{V}_{24} = \tilde{V}_{23} + 0.06\text{ V} = 1.26\text{ V rms}$$
$$\tilde{V}_{25} = \tilde{V}_{23} + 0.12\text{ V} = 1.32\text{ V rms}$$

Therefore, the number of turns for each slot is:

$$n_{prim} = \frac{\tilde{V}_{prim}}{\tilde{V}_{\sec}} \cdot n_{\sec} \Rightarrow \begin{cases} n_{11} = 203.7 \times 2 \cong 408\text{ turns} \\ n_{12} = 192.9 \times 2 \cong 386\text{ turns} \\ n_{13} = 183.3 \times 2 \cong 367\text{ turns} \\ n_{14} = 174.6 \times 2 \cong 349\text{ turns} \\ n_{15} = 166.6 \times 2 \cong 333\text{ turns} \end{cases}$$

Problem 7.53

Solution:

Known quantities:
Figure P7.52. The pipe's resistance = 0.0002 Ω, the secondary resistance = 0.00005 Ω, the primary current = 28.8 A, and $pf = 0.91$.

Find:
a) The slot number.
b) The secondary reactance.
c) The installation efficiency.

Analysis:
a) The secondary current is: $P_{prim} = P_{\sec} \Rightarrow \tilde{V}_{prim} \cdot \tilde{I}_{prim} \cdot \cos(\theta) = R_{\sec} \cdot \tilde{I}^2_{\sec}$

$$I_{\sec} = \sqrt{\frac{\tilde{V}_{prim} \cdot \tilde{I}_{prim} \cdot \cos(\theta)}{R_{\sec}}} = \sqrt{\frac{220 \cdot 28.8 \cdot 0.91}{0.00025}} = 4800\text{ A}$$

Therefore, the slot number is: $N = \frac{I_{\sec}}{I_{prim}} = \frac{4800}{28.8} = 166.6 \Rightarrow$ Slot Number 5

b) The secondary reactance is: $pf_{prim} = pf_{\sec} = \frac{R_{\sec}}{\sqrt{R^2_{\sec} + X^2_{\sec}}}$

$$X_{\sec} = R_{\sec}\sqrt{\frac{1}{pf^2} - 1} = 0.00025\sqrt{\frac{1}{0.91^2} - 1} = 114\ \mu\Omega$$

c) The installation efficiency is:

$$\eta = \frac{P_{load}}{P_{prim}} = \frac{R_{load} \cdot I_{\sec}^2}{R_{\sec} \cdot I_{\sec}^2} = \frac{R_{load}}{R_{\sec}} = \frac{200\ \mu\Omega}{250\ \mu\Omega} = 0.8$$

Problem 7.54

Solution:

Known quantities:

A single-phase transformer converting 6 kV to 230 V with 0.95 efficiency, the *pf* of 0.8, and the primary apparent power of 30 KVA.

Find:

a) The secondary current.
b) The transformer's ratio.

Analysis:

a) The secondary current is:

$$P_{\sec} = P_{prim} \cdot \eta = S_{prim} \cdot \cos(\theta) \cdot \eta = 30 \cdot 0.8 \cdot 0.95 = 22.8\,\text{kW}$$

$$\tilde{I}_{\sec} = \frac{P_{\sec}}{\tilde{V}_{\sec} \cdot \cos(\theta)} = 124\ \text{A}$$

b) The primary current is:

$$\tilde{I}_{prim} = \frac{S_{prim}}{\tilde{V}_{prim}} = \frac{30000}{6000} = 5\,\text{A}$$

Therefore, the transformer's ratio is:

$$\frac{1}{N} = \frac{\tilde{I}_{\sec}}{\tilde{I}_{prim}} = 24.8 \Rightarrow N = 0.04$$

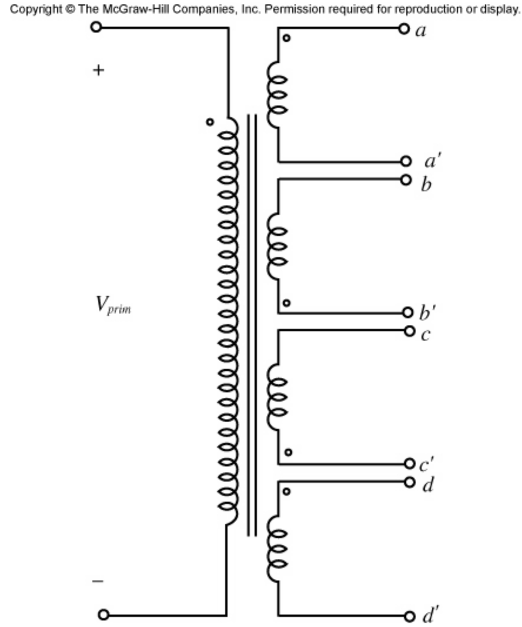
Problem 7.55

Solution:

Known quantities:
A transformer shown in Figure P7.55 with several sets of windings.

Find:
The connections that can construct the desired voltage sources.

Analysis:

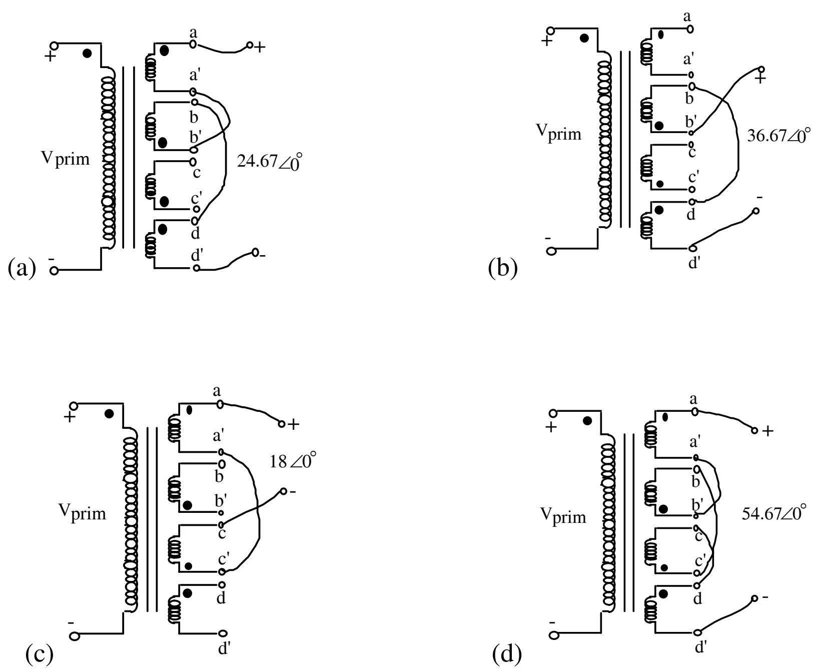


The transformer direct output voltages are:

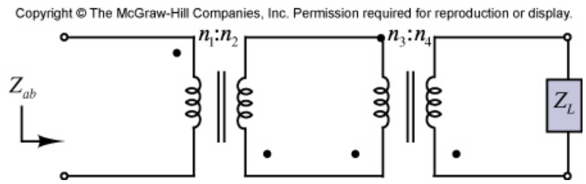
$$V_{aa'} = \frac{120}{15} = 8 \text{ V}; \quad V_{bb'} = \frac{120}{4} = 30 \text{ V}$$

$$V_{cc'} = \frac{120}{12} = 10 \text{ V}; \quad V_{dd'} = \frac{120}{18} = 6.67 \text{ V}$$

The connections required to obtain the desired voltages are shown below.



Problem 7.56



Solution:

Known quantities:
A transformer shown in Figure P7.56 with several sets of windings.

Find:
The connections that can construct the desired voltage sources.

Analysis:
From $\frac{475}{267} = 1.779$, we have

$$\left(\frac{n_4}{n_3}\right)^2\left(\frac{n_2}{n_1}\right)^2 = 1.779$$

That is

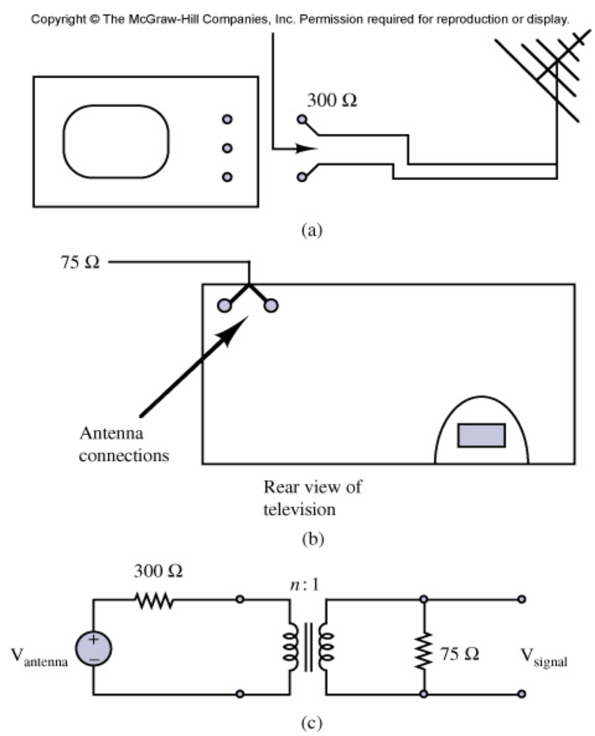
$$\left(\frac{n_4}{n_3}\right)\left(\frac{n_2}{n_1}\right) = 1.334$$

From 2:1 and 2:3, we have

$$\frac{2}{1}\times\frac{2}{3} = 2\times 0.667 = 1.334$$

Therefore, we can choose the transformers with the turns ratio of 2:1 and 2:3 to match this impedance.

Problem 7.57



Solution:

Known quantities:
A transformer shown in Figure P7.57.

Find:
The turns ratio needed to obtain maximum power transfer.

Analysis:

From Equation (5.48), we have

$$R_L = N^2 R_S$$

Therefore, the required turns ratio is

$$N = \frac{1}{n} = \sqrt{\frac{R_L}{R_S}} = \sqrt{\frac{75}{300}} = 0.5$$

$$n = 2$$

Section 7.4: Three-Phase Power

Problem 7.58

Solution:

Known quantities:
The magnitude of the phase voltage of a three-phase wye system, 220 V rms.

Find:
The expression of each phase in both polar and rectangular coordinates.

Analysis:
The phase voltages in polar form are:

$$\tilde{\mathbf{V}}_{an} = 220\angle 0^o \text{ V}, \quad \tilde{\mathbf{V}}_{bn} = 220\angle -120° \text{ V}, \quad \tilde{\mathbf{V}}_{cn} = 220\angle 120° \text{ V}$$

The rectangular forms are:

$$\tilde{\mathbf{V}}_{an} = 220 \text{ V}, \quad \tilde{\mathbf{V}}_{bn} = -110 - j190.52 \text{ V}, \quad \tilde{\mathbf{V}}_{cn} = -110 + j190.5 \text{ V}$$

The line voltages in polar form are:

$$\tilde{\mathbf{V}}_{ab} = \sqrt{3}\tilde{\mathbf{V}}_{an}\angle 30° = 380\angle 30° \text{ V}$$

$$\tilde{\mathbf{V}}_{bc} = 380\angle -90° \text{ V}$$

$$\tilde{\mathbf{V}}_{ca} = 380\angle 150° \text{ V}$$

The line voltages in rectangular form are:

$$\tilde{\mathbf{V}}_{ab} = 329 + j190 \text{ V}, \quad \tilde{\mathbf{V}}_{bc} = -j380 \text{ V}, \quad \tilde{\mathbf{V}}_{ca} = -329 + j190 \text{ V}$$

Problem 7.59

Solution:

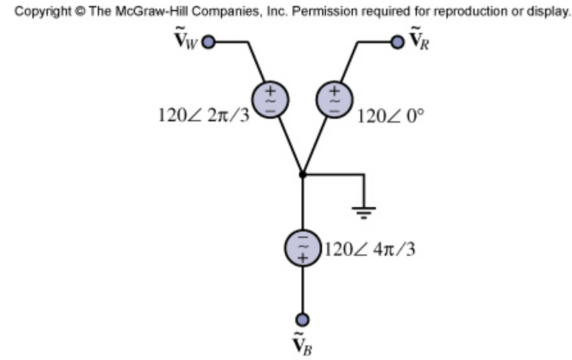
Known quantities:
The phase currents, $\tilde{\mathbf{I}}_{an} = 10\angle 0$, $\tilde{\mathbf{I}}_{bn} = 12\angle 150°$, $\tilde{\mathbf{I}}_{cn} = 8\angle 165°$.

Find:
The current in the neutral wire.

Analysis:
The neutral current is:

$$\tilde{\mathbf{I}}_n = \tilde{\mathbf{I}}_{an} + \tilde{\mathbf{I}}_{bn} + \tilde{\mathbf{I}}_{cn} = 5\angle 0° + 12\angle 150° + 8\angle 165° = -13.11 + j8.07 = 15.39\angle 148.4° \text{ A}$$

Problem 7.60



Solution:

Known quantities:

Circuit shown in Figure P7.60, the voltage sources,
$\tilde{\mathbf{V}}_R = 120\angle 0°\text{ V}$, $\tilde{\mathbf{V}}_W = 120\angle 120°\text{V}$, $\tilde{\mathbf{V}}_B = 120\angle 240°\text{ V}$.

Find:

a) The voltages, $\tilde{\mathbf{V}}_{RW}, \tilde{\mathbf{V}}_{WB}, \tilde{\mathbf{V}}_{BR}$.
b) The voltages, $\tilde{\mathbf{V}}_{RW}, \tilde{\mathbf{V}}_{WB}, \tilde{\mathbf{V}}_{BR}$, using $\tilde{\mathbf{V}}_{xy} = \tilde{\mathbf{V}}_x\sqrt{3}\angle -30°$.
c) Compare the results obtained in a and b.

Analysis:

a) $\tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R - \tilde{\mathbf{V}}_W = 120\angle 0° - 120\angle 120° = 120 + 60 - j103.92 = 207.8\angle -30°\text{ V}$

$\tilde{\mathbf{V}}_{WB} = \tilde{\mathbf{V}}_W - \tilde{\mathbf{V}}_B = 120\angle 120° - 120\angle 240° = -60 + j103.92 + 60 - j103.92 = 207.8\angle 90°\text{ V}$

$\tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B - \tilde{\mathbf{V}}_R = 120\angle 240° - 120\angle 0° = -60 - j103.92 - 120 = 207.8\angle -150°\text{ V}$

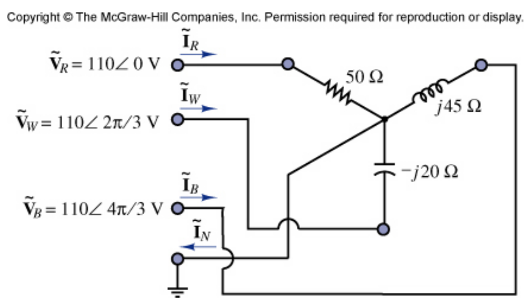
b) $\tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R\sqrt{3}\angle -30° = 120\sqrt{3}\angle -30° = 207.8\angle -30°\text{ V}$

$\tilde{\mathbf{V}}_{WB} = \tilde{\mathbf{V}}_W\sqrt{3}\angle -30° = 120\angle 120°\sqrt{3}\angle -30° = 207.8\angle 90°\text{ V}$

$\tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B\sqrt{3}\angle -30° = 120\angle 240°\sqrt{3}\angle -30° = 207.8\angle 210°\text{ V} = 207.8\angle -150°\text{ V}$

c) The two calculations are identical to the ones above.

Problem 7.61



Solution:

Known quantities:

Circuit shown in Figure P7.61, the voltage sources,
$\tilde{\mathbf{V}}_R = 110\angle 0°\text{ V}$, $\tilde{\mathbf{V}}_W = 110\angle 120°\text{ V}$, $\tilde{\mathbf{V}}_B = 110\angle 240°\text{ V}$,
and the three loads, $Z_R = 50\Omega$, $Z_W = -j20\Omega$, $Z_B = j45\Omega$.

Find:

a) The current in the neutral wire.
b) The real power.

Analysis:

a) $\tilde{\mathbf{I}}_R = \dfrac{\tilde{\mathbf{V}}_R}{Z_R} = \dfrac{110\angle 0°}{50} = 2.2\angle 0°\text{ A}$

$$\tilde{\mathbf{I}}_B = \frac{\tilde{\mathbf{V}}_B}{Z_B} = \frac{110\angle 240°}{j45} = 2.44\angle 150°\text{ A}$$

$$\tilde{\mathbf{I}}_W = \frac{\tilde{\mathbf{V}}_W}{Z_W} = \frac{110\angle 120°}{-j20} = 5.5\angle 210°\text{ A}$$

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 2.2 + 5.5\angle 210° + 2.44\angle 150° = 4.92\angle -161.9°\text{ A}$$

b) $P = R\cdot\tilde{\mathbf{I}}_R^2 = 50\cdot 2.2^2 = 242\text{W}$

Problem 7.62

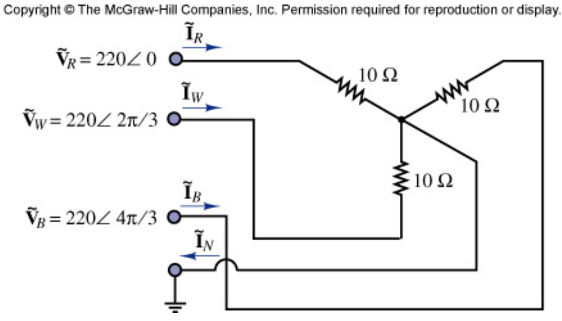
Solution:

Known quantities:

Circuit shown in Figure P7.62, the voltage sources, $\tilde{\mathbf{V}}_R = 220\angle 0°\ \text{V}$, $\tilde{\mathbf{V}}_W = 220\angle 120°\ \text{V}$, $\tilde{\mathbf{V}}_B = 220\angle 240°\ \text{V}$, and the impedances, $R_W = R_B = R_R = 10\,\Omega$.

Find:

a) The current in the neutral wire.
b) The real power.



Analysis:

a)
$$\tilde{\mathbf{I}}_R = \frac{\tilde{\mathbf{V}}_R}{R_R} = \frac{220\angle 0°}{10} = 22\angle 0°\ \text{A}$$
$$\tilde{\mathbf{I}}_W = \frac{\tilde{\mathbf{V}}_W}{R_W} = \frac{220\angle 120°}{10} = 22\angle 120°\ \text{A}$$
$$\tilde{\mathbf{I}}_B = \frac{\tilde{\mathbf{V}}_B}{R_B} = \frac{220\angle 240°}{10} = 22\angle 240°\ \text{A}$$

Therefore, the current in the neutral wire is:
$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 0\ \text{A}$$

b) The real power is:
$$P = \tilde{\mathbf{I}}_R^2 \cdot R + \tilde{\mathbf{I}}_W^2 \cdot R + \tilde{\mathbf{I}}_B^2 \cdot R = 3R \cdot \tilde{\mathbf{I}}_R^2 = 3\frac{\tilde{\mathbf{V}}_R^2}{R} = 3\frac{220^2}{10} = 14.52\ \text{kW}$$

Problem 7.63

Solution:

Known quantities:

A three-phase electric oven with a phase resistance of 10 Ω, connected at 3 × 380 V AC.

Find:

a) The current flowing through the resistors in Y and Δ connections.
b) The power of the oven in Y and Δ connections.

Analysis:

a) In Y-connection:
$$\tilde{I}_{RY} = \frac{\tilde{V}_{phase}}{R} = \frac{380/\sqrt{3}}{10} = \frac{220}{10} = 22\ \text{A}$$

In Δ-connection:
$$\tilde{I}_{RD} = \frac{\tilde{V}_{line}}{R} = \frac{380}{30} = 12.7\ \text{A}$$

b) In Y-connection:
$$P = \sqrt{3} \cdot \tilde{V}_{line} \cdot \tilde{I}_{line} = \sqrt{3} \cdot \tilde{V}_{line} \cdot \tilde{I}_{RY} = \sqrt{3} \cdot 380 \cdot 22 = 14.5\ \text{kW}$$

In Δ-connection:
$$P = \sqrt{3} \cdot \tilde{V}_{line} \cdot \tilde{I}_{line} = \sqrt{3} \cdot \tilde{V}_{line} \cdot \tilde{I}_{RD} = \sqrt{3} \cdot 380 \cdot 12.7 = 8.36\ \text{kW}$$

Problem 7.64

Solution:

Known quantities:
Apparent power of 50 kVA and supplied voltage of 380 V for a synchronous generator.

Find:
The phase currents, the active powers, and the reactive powers if:
a) The power factor is 0.85.
b) The power factor is 1.

Analysis:
a) For the power factor of 0.85:

$$S = \sqrt{3}\tilde{V}\tilde{I} \;\Rightarrow\; \tilde{I} = \frac{S}{\sqrt{3}\tilde{V}} = \frac{50000}{\sqrt{3}\cdot 380} = 76\,\text{A}$$

$$P = S\cdot\cos(\theta) = 50000\cdot 0.85 = 42.5\,\text{kW}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{50^2 - 42.5^2} = 26.3\,\text{kVAR}$$

b) For the power factor of 1.00:

$$S = P \;\Rightarrow\; \tilde{I} = 76\,\text{A}$$

$$P = S\cdot\cos(\theta) = 50000\cdot 1.00 = 50.0\,\text{kW}$$

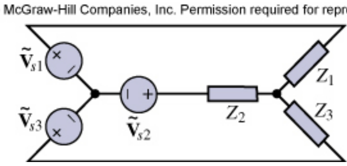
$$Q = \sqrt{S^2 - P^2} = 0$$

Problem 7.65

Solution:

Known quantities:
Circuit shown in Figure P7.65, the voltage sources, $v_{s1}(t) = 170\cos(\omega t)\,\text{V}$, $v_{s2}(t) = 170\cos(\omega t + 120°)\,\text{V}$, $v_{s3}(t) = 170\cos(\omega t - 120°)\,\text{V}$, and the impedances, $Z_1 = 0.5\angle 20°\,\Omega$, $Z_2 = 0.35\angle 0°\,\Omega$, $Z_3 = 1.7\angle -90°\,\Omega$, the frequency, $f = 60\,\text{Hz}$.

Find:
The current through Z_1, using:
a) Loop/mesh analysis.
b) Node analysis.
c) Superposition.



Analysis:
a) Applying KVL in the upper mesh:

$$\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s1} + \tilde{\mathbf{I}}_1 Z_1 + \left(\tilde{\mathbf{I}}_1 - \tilde{\mathbf{I}}_2\right)Z_2 = 0 \;\Rightarrow\; \tilde{\mathbf{I}}_1(Z_1 + Z_2) + \tilde{\mathbf{I}}_2(-Z_2) = \tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s2}$$

Applying KVL in the lower mesh:

$$\tilde{\mathbf{V}}_{s3} - \tilde{\mathbf{V}}_{s2} + \left(\tilde{\mathbf{I}}_2 - \tilde{\mathbf{I}}_1\right)Z_2 + \tilde{\mathbf{I}}_2 Z_3 = 0 \;\Rightarrow\; \tilde{\mathbf{I}}_1(-Z_2) + \tilde{\mathbf{I}}_2(Z_2 + Z_3) = \tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s3}$$

For each mesh equation:

$$\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s2} = 170\angle 0° - 170\angle 120° = 170 - (-85 + j147) = 294\angle -30°\,\text{V}$$

$$\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s3} = 170\angle 120° - 170\angle -120° = (-85 + j147) - (-85 - j147) = 294\angle 90°\,\text{V}$$

$$Z_1 + Z_2 = 0.47 + j0.171 + 0.35 = 0.838\angle 11.8°\,\Omega$$

$$Z_2 + Z_3 = 0.35 - j1.7 = 1.74\angle -78.4°\,\Omega$$

Therefore, the current through Z_1 is:

$$\tilde{\mathbf{I}}_1 = \frac{\begin{vmatrix} \tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s2} & -Z_2 \\ \tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s3} & Z_2 + Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}} = \frac{\begin{vmatrix} 294\angle -30° & -0.35\angle 0° \\ 294\angle 90° & 1.74\angle -78.4° \end{vmatrix}}{\begin{vmatrix} 0.838\angle 11.8° & -0.35\angle 0° \\ -0.35\angle 0° & 1.74\angle -78.4° \end{vmatrix}}$$

$$= \frac{512\angle -108.4° + 103\angle 90°}{1.46\angle -66.6° - 0.123\angle 0°} = \frac{415.9\angle -112.9°}{1.416\angle -71.2°} = 293\angle -41.8° \text{A}$$

b) Choose the ground at the center of the three voltage source, and let *a* be the center of the three loads. The voltage between the node *a* and the ground is unknown.

Applying KCL at the node *a*:

$$\frac{\tilde{\mathbf{V}}_a - \tilde{\mathbf{V}}_{s1}}{Z_1} + \frac{\tilde{\mathbf{V}}_a - \tilde{\mathbf{V}}_{s2}}{Z_2} + \frac{\tilde{\mathbf{V}}_a - \tilde{\mathbf{V}}_{s3}}{Z_3} = 0$$

Rearranging the equation:

$$\tilde{\mathbf{V}}_a = \frac{\frac{\tilde{\mathbf{V}}_{s1}}{Z_1} + \frac{\tilde{\mathbf{V}}_{s2}}{Z_2} + \frac{\tilde{\mathbf{V}}_{s3}}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{170\angle 0°}{0.5\angle 20°} + \frac{170\angle 120°}{0.35\angle 10°} + \frac{170\angle -120°}{1.7\angle -90°}}{\frac{1}{0.5\angle 20°} + \frac{1}{0.35\angle 10°} + \frac{1}{1.7\angle -90°}}$$

$$= \frac{340\angle -20° + 486\angle 120° + 100\angle 330°}{2\angle -20° + 2.86\angle 0° + 0.59\angle 90°} = \frac{303\angle 57.3°}{4.74\angle -1.2°} = 63.9\angle 58.5° \text{ V}$$

Applying KVL, the current through Z_1 is:

$$\tilde{\mathbf{V}}_a - \tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{I}}_1 Z_1 = 0 \Rightarrow \tilde{\mathbf{I}}_1 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_a}{Z_1} = \frac{170\angle 0° - 63.9\angle 58.5°}{0.5\angle 20°} = 293\angle -41.8° \text{ A}$$

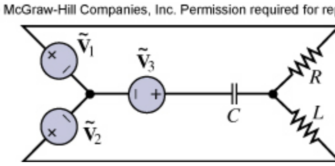
c) Superposition is not the method of choice in this case.

Problem 7.66

Solution:

Known quantities:

Circuit shown in Figure P7.66, the voltage sources, $v_1(t) = 170\cos(\omega t)\text{V}$, $v_2(t) = 170\cos(\omega t + 120°)\text{V}$, $v_3(t) = 170\cos(\omega t - 120°)\text{V}$, and the impedances, $R = 100\,\Omega$, $C = 0.47\,\mu\text{F}$, $L = 100\,\text{mH}$, the frequency, $f = 400\,\text{Hz}$.



Find:

The current through *R*.

Analysis:

For each impedance:

$$Z_1 = 100\,\Omega = 100\angle 0°\,\Omega$$

$$Z_2 = -j\frac{1}{\omega C} = -j\frac{1}{2\pi f \cdot C} = -j846.6\,\Omega = 846.6\angle -90°\,\Omega$$

$$Z_3 = j\omega L = -j2\pi f \cdot L = j251.3\,\Omega = 251.3\angle 90°\,\Omega$$

Applying KVL in the upper mesh:

$$\tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_1 + \tilde{\mathbf{I}}_1 Z_1 + (\tilde{\mathbf{I}}_1 - \tilde{\mathbf{I}}_2)Z_2 = 0 \Rightarrow \tilde{\mathbf{I}}_1(Z_1 + Z_2) + \tilde{\mathbf{I}}_2(-Z_2) = \tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2$$

Applying KVL in the lower mesh:

$$\tilde{\mathbf{V}}_3 - \tilde{\mathbf{V}}_2 + (\tilde{\mathbf{I}}_2 - \tilde{\mathbf{I}}_1)Z_2 + \tilde{\mathbf{I}}_2 Z_3 = 0 \Rightarrow \tilde{\mathbf{I}}_1(-Z_2) + \tilde{\mathbf{I}}_2(Z_2 + Z_3) = \tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3$$

For each mesh equation:

$$\tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2 = 170\angle 0° - 170\angle 120° = 170 - (-85 + j147) = 294\angle -30° \text{V}$$

$$\tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3 = 170\angle 120° - 170\angle -120° = (-85 + j147) - (-85 - j147) = 294\angle 90° \text{ V}$$

$$Z_1 + Z_2 = 100 - j846.6 = 852.5\angle -83.3°\,\Omega$$

$$Z_2 + Z_3 = -j846.6 + j251.3 = 595.3\angle -90°\,\Omega$$

Therefore, the current through R is:

$$\tilde{\mathbf{I}}_1 = \frac{\begin{vmatrix} \tilde{\mathbf{V}}_1 - \tilde{\mathbf{V}}_2 & -Z_2 \\ \tilde{\mathbf{V}}_2 - \tilde{\mathbf{V}}_3 & Z_2 + Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}} = \frac{\begin{vmatrix} 294\angle -30° & -846.6\angle -90° \\ 294\angle 90° & 595.3\angle -90° \end{vmatrix}}{\begin{vmatrix} 852.5\angle -83.3^o & -846.6\angle -90^o \\ -846.6\angle -90^o & 595.3\angle -90^o \end{vmatrix}}$$

$$= \frac{175.0\cdot 10^3\angle -120° + 248.9\cdot 10^3\angle 0°}{507.5\cdot 10^3\angle -173.3° - 716.7\cdot 10^3\angle -180°} = \frac{221.4\angle -43.2°}{220.8\angle -15.6°}$$

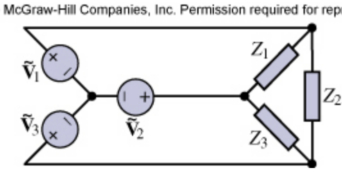
$$= 1.003\angle -27.6°\text{ A}$$

Problem 7.67

Solution:

Known quantities:

Circuit shown in Figure P7.67, the voltage sources, $v_1(t) = 170\cos(\omega t)\,\text{V}$, $v_2(t) = 170\cos(\omega t + 120°)\,\text{V}$, $v_3(t) = 170\cos(\omega t - 120°)\,\text{V}$, and the impedances, $Z_1 = 3\angle 0°\,\Omega$, $Z_2 = 7\angle 90°\,\Omega$, $Z_3 = 0 - j11\,\Omega$, the frequency, $f = 60\,\text{Hz}$.



Find:

The currents, $\tilde{\mathbf{I}}_1, \tilde{\mathbf{I}}_2, \tilde{\mathbf{I}}_3$.

Analysis:

Applying KVL in the upper mesh:

$$\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s1} + \tilde{\mathbf{I}}_1 Z_1 = 0$$

$$\tilde{\mathbf{I}}_1 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s2}}{Z_1} = \frac{170\angle 0° - 170\angle 120°}{3\angle 0°} = \frac{294\angle -30°}{3\angle 0°} = 98.1\angle -30°\text{ A}$$

Applying KVL in the right-side mesh:

$$\tilde{\mathbf{V}}_{s3} - \tilde{\mathbf{V}}_{s1} + \tilde{\mathbf{I}}_2 Z_2 = 0$$

$$\tilde{\mathbf{I}}_2 = \frac{\tilde{\mathbf{V}}_{s1} - \tilde{\mathbf{V}}_{s3}}{Z_2} = \frac{170\angle 0° - 170\angle -120°}{7\angle 90°} = \frac{294\angle 30°}{7\angle 90°} = 42.1\angle -60°\text{ A}$$

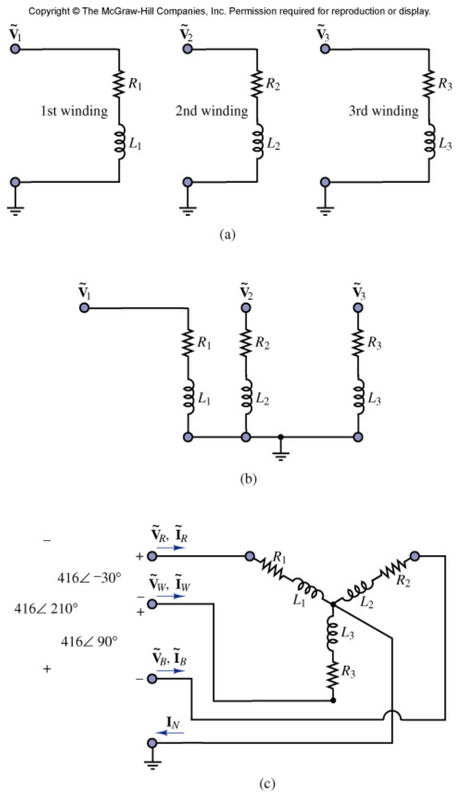
Applying KVL in the lower mesh:

$$\tilde{\mathbf{V}}_{s3} - \tilde{\mathbf{V}}_{s2} + \tilde{\mathbf{I}}_3 Z_3 = 0$$

$$\tilde{\mathbf{I}}_3 = \frac{\tilde{\mathbf{V}}_{s2} - \tilde{\mathbf{V}}_{s3}}{Z_3} = \frac{170\angle 120° - 170\angle -120°}{11\angle -90°} = \frac{294\angle 90°}{11\angle -90°} = 26.8\angle -180°\text{ A}$$

Problem 7.68

Solution:



Known quantities:

Circuit shown in Figure P7.68, the voltage sources, $\tilde{\mathbf{V}}_{RW} = 416\angle -30°\text{ V}$, $\tilde{\mathbf{V}}_{WB} = 416\angle 210°\text{ V}$, $\tilde{\mathbf{V}}_{BR} = 416\angle 90°\text{ V}$, and the impedances, $R_1 = R_2 = R_3 = 40\,\Omega$, $L_1 = L_2 = L_3 = 5\,\text{mH}$. The frequency of each of the sources, $f = 60\,\text{Hz}$.

Find:

The currents, $\tilde{\mathbf{I}}_W$, $\tilde{\mathbf{I}}_B$, $\tilde{\mathbf{I}}_R$, $\tilde{\mathbf{I}}_N$.

Analysis:

The line voltages are:

$\tilde{\mathbf{V}}_{RW} = 416\angle -30°\text{ V}$, $\tilde{\mathbf{V}}_{WB} = 416\angle 210°\text{ V}$, $\tilde{\mathbf{V}}_{BR} = 416\angle 90\text{ V}$

The phase voltages are:

$\tilde{\mathbf{V}}_R = 240\angle 0°\text{ V}$, $\tilde{\mathbf{V}}_W = 240\angle 120°\text{ V}$, $\tilde{\mathbf{V}}_B = 240\angle -120°\text{ V}$

The currents are:

$$\tilde{\mathbf{I}}_R = \frac{\tilde{\mathbf{V}}_R}{Z_1} = \frac{\tilde{\mathbf{V}}_R}{R_1 + j\omega L_1} = \frac{\tilde{\mathbf{V}}_R}{R_1 + j2\pi f L_1} = \frac{240\angle 0°}{40 + j377\cdot 5\cdot 10^{-3}} = 6\angle -2.7°\text{ A}$$

$$\tilde{\mathbf{I}}_W = \frac{\tilde{\mathbf{V}}_W}{Z_2} = \frac{240\angle 120°}{40 + j377\cdot 5\cdot 10^{-3}} = 6\angle 117.3°\text{ A}$$

$$\tilde{\mathbf{I}}_B = \frac{\tilde{\mathbf{V}}_B}{Z_3} = \frac{240\angle -120°}{40 + j377\cdot 5\cdot 10^{-3}} = 6\angle -122.7°\text{ A}$$

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_R + \tilde{\mathbf{I}}_W + \tilde{\mathbf{I}}_B = 0$$

Problem 7.69

Solution:

Known quantities:

Circuit shown in Figure P7.67, the voltage sources, $\tilde{\mathbf{V}}_{RW} = 416\angle -30°\text{ V}$, $\tilde{\mathbf{V}}_{WB} = 416\angle 210°\text{ V}$, $\tilde{\mathbf{V}}_{BR} = 416\angle 90°\text{ V}$, and the impedances, $R_1 = R_2 = R_3 = 40\,\Omega$, $L_1 = L_2 = L_3 = 5\,\text{mH}$. The frequency of each of the sources, $f = 60\,\text{Hz}$.

Find:

a) The power delivered to the motor.
b) The motor's power factor.
c) The reason for which it is common in industrial practice not to connect the ground lead to motors of this type.

Analysis:

a) The power delivered to the motor is:
$P = 3\tilde{V}_R\tilde{I}_R\cos(\theta) = 3\cdot 240\cdot 6\cdot\cos(-2.7°) = 4315\,\text{W}$

b) The motor's power factor is:
$pf = \cos(-2.7°) = 0.9988$ Lagging

c) The circuit is balanced and no neutral current flows; thus the connection is unnecessary.

Problem 7.70

Solution:

Known quantities:

A three-phase induction motor designed not only for Y connection operation in general but also for Δ connection at the nominal Y voltage for a short time operation.

Find:

The ratio between the powers.

Analysis:

The power for Y connection operation is:

$$P_Y = 3 \cdot \tilde{\mathbf{V}}_{phase_{MOTOR}} \cdot \tilde{\mathbf{I}}_{line_Y} = 3 \cdot \frac{\tilde{\mathbf{V}}_{line}}{\sqrt{3}} \cdot \tilde{\mathbf{I}}_{line_Y} = \sqrt{3} \cdot \tilde{\mathbf{V}}_{line} \cdot \tilde{\mathbf{I}}_{line_Y}$$

The power for Δ connection operation is:

$$P_\Delta = 3 \cdot \tilde{\mathbf{V}}_{line} \cdot \tilde{\mathbf{I}}_{phase_{MOTOR}} = 3 \cdot \tilde{\mathbf{V}}_{line} \cdot \frac{\tilde{\mathbf{V}}_{line}}{Z_{phase_{MOTOR}}}$$

$$= 3 \cdot \tilde{\mathbf{V}}_{line} \cdot \sqrt{3} \cdot \frac{\tilde{\mathbf{V}}_{phase_{MOTOR}}}{Z_{phase_{MOTOR}}} = 3 \cdot \tilde{\mathbf{V}}_{line} \cdot \sqrt{3} \cdot \tilde{\mathbf{I}}_{line_Y} = 3 \cdot P_Y$$

Therefore, the ratio between the powers is: $\text{Ratio} = \frac{P_\Delta}{P_Y} = 3$

Problem 7.71

Solution:

Known quantities:

The voltage source at 220 V rms of a residential four-wire system supplying power to the single-phase appliances; ten 75-W bulbs on the 1st phase, one 750-W vacuum cleaner with *pf* = 0.87 on the 2nd phase, ten 40-W lamps with *pf* = 0.64 on the 3rd phase.

Find:

a) The current in the neutral wire.
b) The real, reactive, and apparent power for each phase.

Analysis:

a) The current in the neutral wire is:

$$\tilde{\mathbf{I}}_A = \frac{P_A}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_A}{\tilde{\mathbf{V}}_{line}} = \frac{\sqrt{3} \cdot 10 \cdot 75}{380} = 3.4\angle 0 \text{ A}$$

$$\tilde{\mathbf{I}}_B = \frac{P_B}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_B}{\tilde{\mathbf{V}}_{line} \cdot \cos(\theta)} = \frac{\sqrt{3} \cdot 750}{380 \cdot 0.87} = 3.92\angle -150° \text{ A} = -3.4 - j1.95 \text{ A}$$

$$\tilde{\mathbf{I}}_C = \frac{P_C}{\tilde{\mathbf{V}}_{phase}} = \frac{\sqrt{3} \cdot P_C}{\tilde{\mathbf{V}}_{line} \cdot \cos(\theta)} = \frac{\sqrt{3} \cdot 10 \cdot 40}{380 \cdot 0.64} = 2.8\angle -290° \text{ A} = 0.95 + j2.63 \text{ A}$$

$$\tilde{\mathbf{I}}_N = \tilde{\mathbf{I}}_A + \tilde{\mathbf{I}}_B + \tilde{\mathbf{I}}_C = 0.95 - j0.68 \text{ A} \Rightarrow \tilde{I}_N = 1.16 \text{ A}$$

b) The real, reactive, and apparent powers for the 1st phase are: $S_A = P_A = 750 \text{ W}, Q_A = 0$

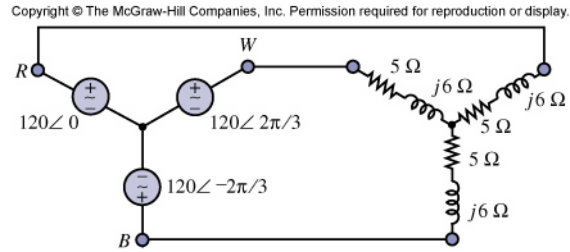
The real, reactive, and apparent powers for the 2nd phase are:

$$P_B = 750 \text{ W}, S_B = \frac{P_B}{\cos(\theta_B)} = \frac{750}{0.87} = 862 \text{ VA}, Q_B = S_B \cdot \sin(\theta_B) = 431 \text{ VAR}$$

The real, reactive, and apparent powers for the 3rd phase are:

$$P_C = 400 \text{ W}, S_C = \frac{P_C}{\cos(\theta_C)} = \frac{400}{0.64} = 625 \text{ VA}, Q_C = S_C \cdot \sin(\theta_C) = 478 \text{ VAR}$$

Problem 7.72



Solution:

Known quantities:

Circuit shown in Figure P7.72, the voltage sources, $\tilde{\mathbf{V}}_R = 120\angle 0°\,\mathrm{V}$, $\tilde{\mathbf{V}}_W = 120\angle 120°\,\mathrm{V}$, $\tilde{\mathbf{V}}_B = 120\angle 240°\,\mathrm{V}$, and the impedances, $R_1 = R_2 = R_3 = 5\Omega$, $jX_{L_1} = jX_{L_2} = jX_{L_3} = 6\Omega$ of the motor.

Find:

a) The total power supplied to the motor.
b) The power converted to mechanical energy if the motor is 80% efficient.
c) The power factor.
d) The risk for the company to face a power factor penalty if all the motors in the factory are similar to this one.

Analysis:

a) By virtue of the symmetry of the circuit, we can solve the problem by considering just one phase. The current $\tilde{\mathbf{I}}_R$ is:

$$\tilde{\mathbf{I}}_R = \frac{\tilde{\mathbf{V}}_R}{R_1 + jX_1} = \frac{120\angle 0°}{5 + j6} = 15.36\angle -50.19°\ \mathrm{A}$$

The total power supplied to the motor is:

$$P = 3\tilde{V}_R \tilde{I}_R \cos(\theta) = 3 \cdot 120 \cdot 15.36 \cdot \cos(50.19°) = 3541.3\,\mathrm{W}$$

b) The mechanical power is:

$$P_m = 0.8P = 2832.23\,\mathrm{W}$$

c) The power factor is:

$$pf = \cos(\theta) = 0.64$$

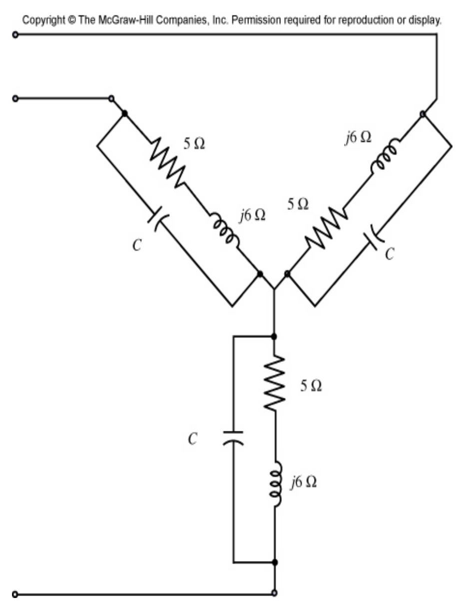
d) The company will face a 25% penalty.

Problem 7.73

Solution:

Known quantities:

Circuit shown in Figure P7.73.



Find:

a) What capacitance to achieve a unity power factor if the line frequency is 60 Hz.
b) What capacitance to achieve the 0.85 (lagging) power factor.

Analysis:

This problems can be solved on a per-phase basis, due to its symmetry.

(a) The reactive power per phase is

$$Q = 120 \times 15.36 \times \sin(50.19°) = 1415.89\ \ \mathrm{VAR}$$

To achieve a unit power factor, we need

$$X_C = \frac{V^2}{Q} = 10.17\ \ \Omega$$

The capacitance therefore is

$$C = \frac{1}{\omega X_C} = 260.8\ \ \mu\mathrm{F}$$

(b) If pf is 0.85, the impedance angle is

$$\theta = \cos^{-1}(0.85) = 31.79°$$

From the power triangle, the reactive power is

$$Q = P \times \tan(31.79°) = 120 \times 15.36 \times \cos(50.19°) \times 0.62 = 731.66 \text{ VAR}$$

Therefore, we can write

$$X_C = \frac{V^2}{Q} = 19.68 \ \Omega$$

The value of the capacitor is

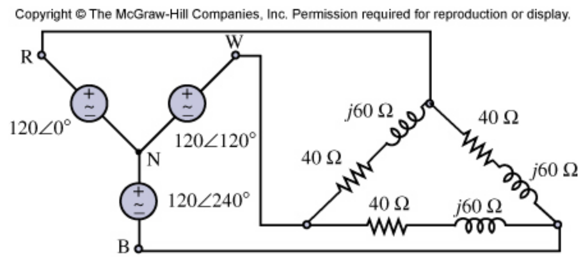
$$C = \frac{1}{\omega X_C} = 134.8 \ \mu\text{F}$$

Problem 7.74

Solution:

Known quantities:

Circuit shown in Figure P7.74.



Find:

The power factor.

Analysis:

$$\tilde{V}_L = 120\sqrt{3}$$

$$S = 3\frac{\tilde{V}_L^2}{Z_\Delta^*} = 3\frac{\left(120\sqrt{3}\right)^2}{(40 + j60)^*} = \frac{129{,}600}{40 - j60} \cdot \frac{40 + j60}{40 + j60}$$

$$= \frac{129{,}600}{5200}(40 + j60) = 996.9 + j1{,}495.4 \text{ VA}$$

$$= 1{,}797.2\angle 56.31° \text{ VA}$$

Apparent Power $= |S| = 1{,}797.2$ VA

$$P = \mathrm{R_e}\{\mathbf{S}\} = 996.9 \text{ W}$$

$$Q = \mathrm{I_m}\{\mathbf{S}\} = 1{,}495 \text{ VAR}$$

Power Factor $= pf = \cos(56.31°) = 0.555$ lagging

Problem 7.75

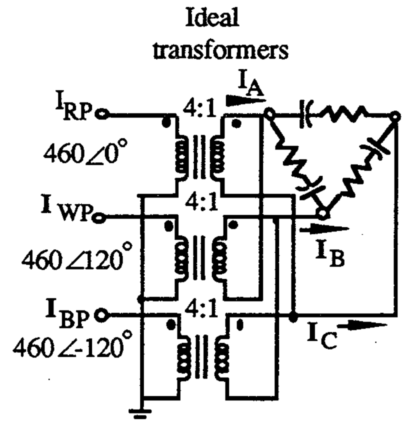
Solution:

Known quantities:

Circuit shown in Figure P7.75.

Find:

Currents $I_{RP}, I_{WP}, I_{BP}, I_A, I_B$ *and* I_C.



Analysis:

One method of solving the problem is to convert the wye load to an equivalent delta load. The circuit is shown below.

The delta load is

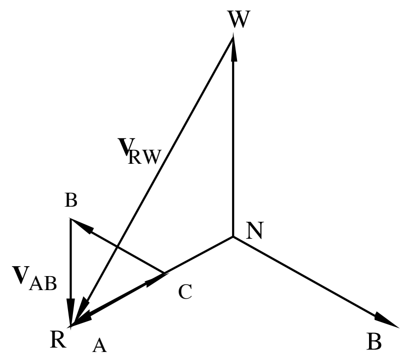
$$Z_\Delta = 3Z_Y = 30 - j21\ \Omega = 36.6\angle -35°$$

The line-to-line voltage of the secondary side is $\dfrac{460}{4} = 115\ V$

For this connection of the secondary side of the transformer, the line voltage of the secondary is in phase with the primary (even though it's a delta connection) the phasor diagram is shown below.

Thus, the primary line-to-line voltage is:

$$V_{RW} = \sqrt{3}460\angle\text{-}30° = 796.74\angle\text{-}30° \quad \text{V}$$



and the secondary line-to-line voltage is

$$V_{AB} = 115\angle -60° \ \text{V}$$

$$V_{BC} = 115\angle 60° \ \text{V}$$

$$V_{CA} = 115\angle -180° \ \text{V}$$

The phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = 3.14\angle -25° \quad \text{A}$$

$$I_{BC} = 3.14\angle 95° \quad \text{A}$$

$$I_{CA} = 3.14\angle 215° \quad \text{A}$$

The line currents are

$$I_A = I_{AB} - I_{CA} = 3.14\angle -25° - 3.14\angle 215° = 5.42 + j0.47 = 5.44\angle 5° \ \text{A}$$

$$I_B = I_{BC} - I_{AB} = 5.44\angle 125° \ \text{A}$$

$$I_C = I_{CA} - I_{BC} = 5.44\angle -115° \ \text{A}$$

The currents on the primary side are

$$I_{RP} = 1.36\angle 5° \ \text{A}$$

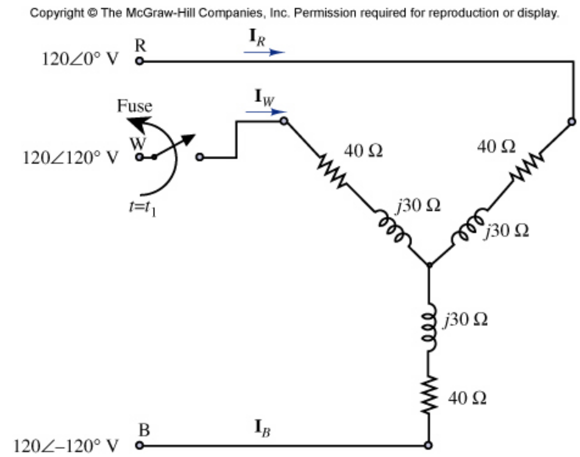
$$I_{WP} = 1.36\angle 125° \ \text{A}$$

$$I_{BP} = 1.36\angle\text{-}115° \ \text{A}$$

Problem 7.76

Solution:

Known quantities:
Circuit shown in Figure P7.76.



Find:
The currents I_R, I_W, I_B and the power dissipated by the motor.

Analysis:
(a) For $t < t_1$, the line current $\mathbf{I}_R$ is

$$I_R = \frac{120\angle 0°}{40 + j30} = 2.4\angle -36.87° \text{ A}$$

The circuit is symmetrical for $t < t_1$, therefore

$I_W = 2.4\angle 83.13°$ A

$I_B = 2.4\angle -156.87°$ A

The total power dissipated in the motor is

$P = 3 \times 120 \times 2.4 \cos 36.87° = 691.2$ W

(b) The initial conditions at $t = t_1$ can be found as

$i_R(t_1) = 2.4\cos(377t_1 - 36.87°)\ A$

$i_W(t_1) = 2.4\cos(377t_1 + 83.13°)\ A$

$i_B(t_1) = 2.4\cos(377t_1 - 156.87°)\ A$

The steady state currents are

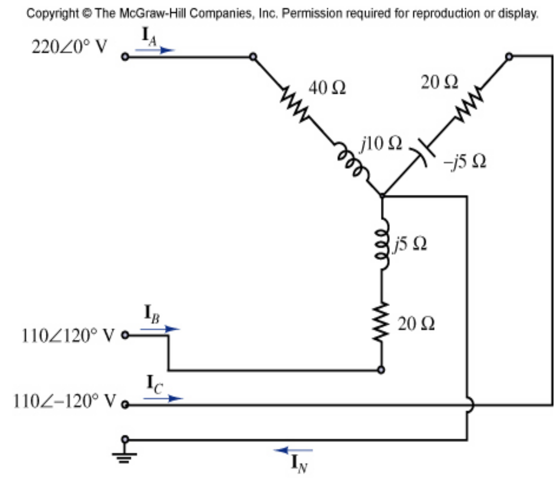
$$I_R = \frac{120 - 120\angle -120°}{80 + j60} = \frac{207.8\angle 30°}{100\angle 36.87°} = 2.078\angle -6.37° \text{ A}$$

$I_W = 0$ A

$I_B = -I_R = 2.078\angle 173.63°$ A

$P = 2 \times 1.2^2 \times 40 = 115.2$ W

Problem 7.77



Solution:

Known quantities:
Circuit shown in Figure P7.77.

Find:
The currents $I_{A,}I_B, I_C, I_N$ and the real power dissipated by the load.

Analysis:

(a) $I_A = \dfrac{220\angle 0°}{40 + j10} = 5.3\angle\text{-}14°\ \text{A}$

$$I_B = \frac{110\angle 120°}{20 + j5} = 5.3\angle 106°\ \text{A}$$

$$I_C = \frac{110\angle\text{-}120°}{20 - j5} = 5.3\angle\text{-}106°\ \text{A}$$

The neutral current is

$$I_N = I_A + I_B + I_C = 2.22 - j1.28 = 2.56\angle\text{-}30°\ \text{A}$$

(b) The real power in phase A is

$$P_A = 220 \times 5.3 \cos 14° = 1131.4\ \text{W}$$

The real power in phase B is

$$P_B = 110 \times 5.3 \cos(120° - 106°) = 565.68\ \text{W}$$

The real power in phase C is

$$P_C = 110 \times 5.3 \cos(-120° + 106°) = 565.68\ \text{W}$$

The total real power dissipated in the load is

$$P = P_A + P_B + P_C = 2262.8\ \text{W}$$
