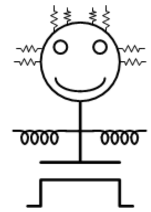
E84: Introduction to Electrical Engineering



Lab 10 Solutions

Grading Rubric

	1	time spent
Warm-up		
1: Twin-T		
	3	Correct transfer function
	2	Correct w0 and Q
	2	Bode plot
2: Sallen-Key		
	3	Correct transfer function
	2	Bode plot
Lab		
	2	Correct component values for twin-T
	2	Reasonable component values for Sallen-key
	2	MultiSim Bode plots confirming component values meet specs
	1	Monte Carlo results for 60 Hz notch meeting specs across variation
	1	Photo of circuit
	4	Measurement results consistent with simulations
Extra credit		
	3	test results connected to ECG board

Warm-Up

1)

$$\frac{V_{in}-V_A}{R}+\frac{V_{out}-V_A}{R}+\left(\alpha V_{out}-V_A\right)\left(2sC\right)=0$$

$$\left(V_{in}-V_B\right)sC+\left(V_{out}-V_B\right)sC+\frac{\left(\alpha V_{out}-V_B\right)}{R/2}=0$$

$$\frac{V_{out}-V_A}{R}+\left(V_{out}-V_B\right)sC=0$$

Simplify these:

$$V_A = \frac{V_{in} + V_{out}\left(1+2\alpha sRC\right)}{2\left(1+sRC\right)}$$

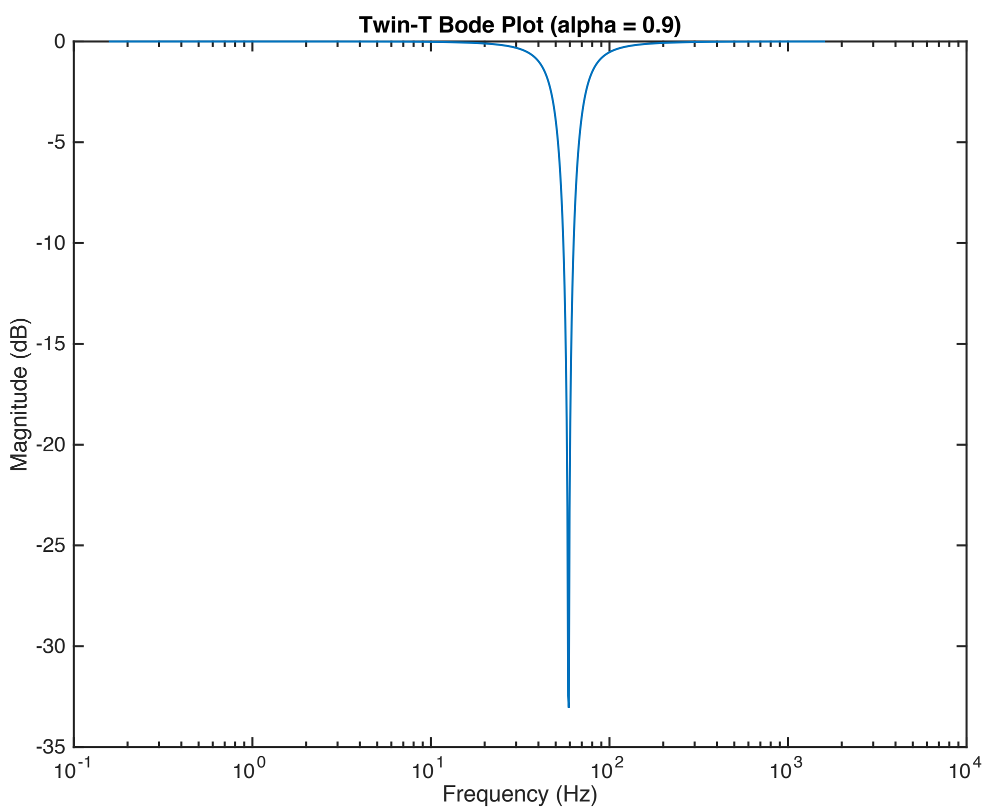
$$V_B = \frac{V_{in} sRC + V_{out}\left(2\alpha + sRC\right)}{2\left(1+sRC\right)}$$

$$V_{out}\left(1+sRC\right) = V_A + V_B sRC$$

Then substitute VA and VB into the Vout equation and simplify

$$\frac{V_{out}}{V_{in}} = \frac{1+\left(sRC\right)^2}{1+4sRC\left(1-\alpha\right)+\left(sRC\right)^2} = \frac{s^2+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

with $\omega_0 = 1/RC$ and $Q = 1/4(1-\alpha)$. This is a narrower band-stop filter with a magnitude of 1 at high and low frequency but 0 at ω_0. The quality factor and hence bandwidth can be tuned by adjusting α via the potentiometer. The Bode plot is shown below.



2) By KCL at the two intermediate nodes and by approximating that V+ = V- for an ideal op-amp, we get the node equations.

$$\frac{b-x}{R_5}+\frac{c-x}{R_6}+j\omega C_6\left(c-x\right)=0$$

$$\frac{x-c}{R_6}-j\omega C_5 c=0$$

Solving, we find a 2nd order low pass transfer function:

$$x=c\left[1+j\omega C_5 R_6\right]$$

$$b=x\left[1+\frac{R_5}{R_6}+j\omega C_6 R_5\right]-c\left[\frac{R_5}{R_6}+j\omega C_6 R_5\right]$$

$$=c\left\{\left(1+j\omega C_5 R_6\right)\left[1+\frac{R_5}{R_6}+j\omega C_6 R_5\right]-\left[\frac{R_5}{R_6}+j\omega C_6 R_5\right]\right\}$$

$$=c\left[1+j\omega C_5\left(R_5+R_6\right)+\left(j\omega\right)^2 R_5 C_5 R_6 C_6\right]$$

$$\frac{c}{b}=\frac{1}{1+j\omega C_5\left(R_5+R_6\right)+\left(j\omega\right)^2 R_5 C_5 R_6 C_6}=\frac{1}{1+2\zeta\frac{j\omega}{\omega_0}+\left(\frac{j\omega}{\omega_0}\right)^2}$$

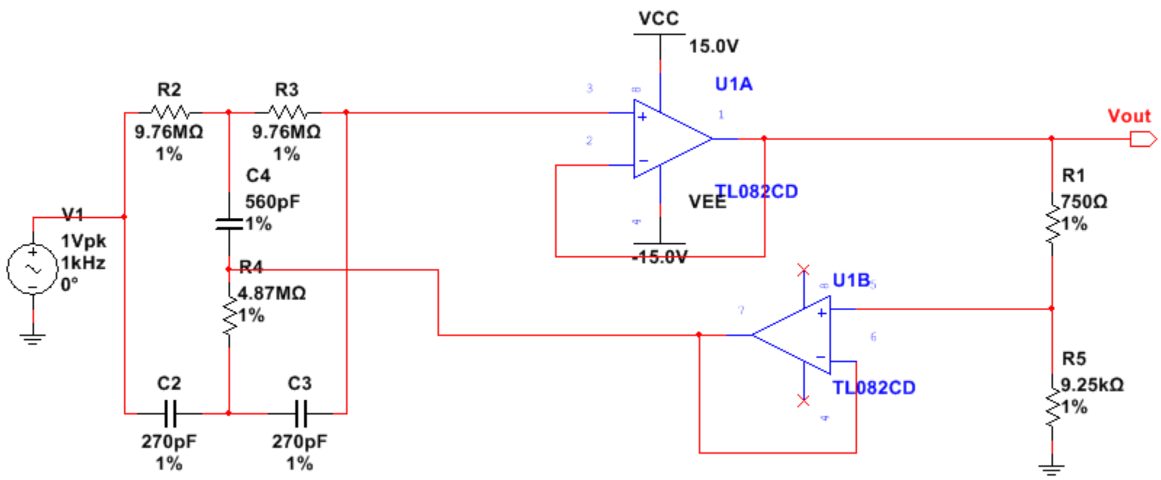
with

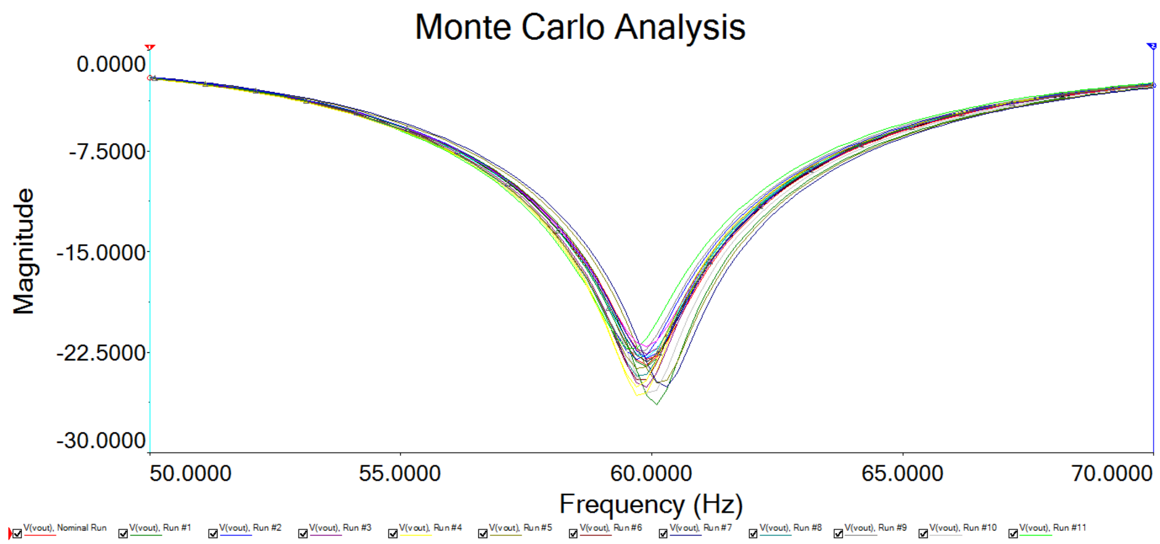
$$\omega_0=\frac{1}{\sqrt{R_5 C_5 R_6 C_6}};\zeta=\frac{C_5\left(R_5+R_6\right)}{2}\omega_0$$

Lab

For the twin-T filter, C is specified as 270 pF. To achieve $\omega_0 = 2\pi(60)$, $R = 1/(C\ \omega_0) = 9.8\ M\Omega$. Hence, based on parts available from DigiKey in 0805 packages with 1% tolerances, choose R2 = R3 = 9.76M, R4 = 4.87M, C2 = C3 = 270 pF, C4 = 560 pF. This gives an actual center frequency of $1/(2\pi RC)$ = 60.4 Hz, neglecting the small error of C4 being slightly more than twice C2.

Alpha can be tuned using simulation. The following figure shows a Multisim schematic of the circuit with component tolerances. Run a 20-point Monte Carlo AC simulation to understand the impact of component variations and find that alpha = 0.925 gives attenuation of at least 20.8 dB at 60 Hz and less than 2.7 dB at 50 and 70 Hz.





Use two identical Sallen-Key low-pass anti-aliasing filters to achieve steep roll-off. Use a critically damped design so that the response in the pass-band is fairly flat.

To achieve -30 dB at 1 KHz, each filter must have a gain of -15 dB = 0.178 at this point. The second order term dominates here, so we solve $(\omega_0/\omega)^2 = 0.178$ for $\omega = 2.37\,\omega_0$. Choose ω = 800 Hz rather than 1 KHz to give some margin, meaning ω_0 = 800 / 2.37 = 337 Hz, comfortably above the 140 Hz required corner frequency.

Choose $\zeta = 0.707$ for critical damping to get a crisp transition at the corner.

Using the formula from the warm-up, we can solve for component values to get the desired ω_0 and ζ.

$$\omega_0 = \frac{1}{\sqrt{R_5 R_6 C_5 C_6}};\zeta = \frac{C_6\left(R_5 + R_6\right)}{2}\omega_0$$

Choose R5 = R6 = R. Then substitute into these equations to find

$$\zeta = \frac{2RC_6}{2\sqrt{R^2C_5C_6}} = \sqrt{\frac{C_5}{C_6}} \triangleq \sqrt{\frac{1}{2}}$$

Hence C = C6 = 2C5 to obtain the desired damping.

Now substitute these component ratios into the natural frequency equation to find

$$\omega_0 = \frac{\sqrt{2}}{RC} \triangleq 2\pi \times 337$$

$$RC = \frac{1}{\pi\sqrt{2} \times 337} = 334\mu s$$

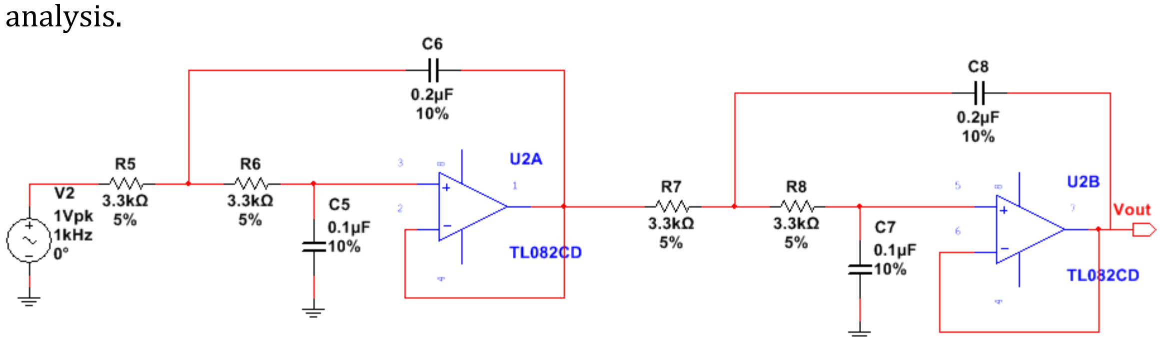
To get in a reasonable range, choose C = 0.2 uF and R = 3.3k. Substitute these back for component values R5 = R6 = 3.3k, C5 = 0.1 uF, C6 = 0.2 uF. These give ζ = 0.707, ω_0 = 2143 rad/s = 341 Hz .

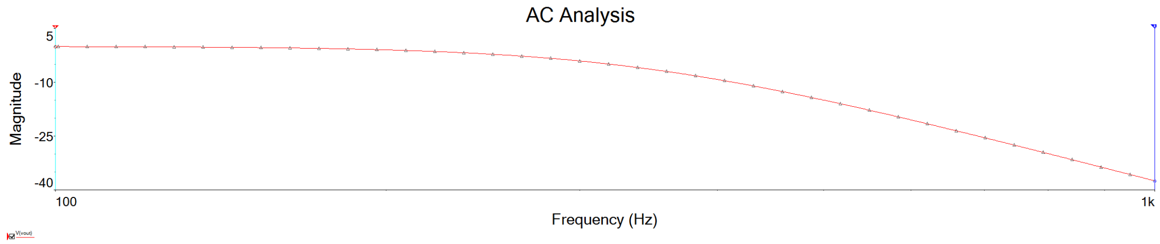
The frequency response function of the cascaded active filters is thus

$$\left[\frac{1}{1+2\zeta j\frac{\omega}{\omega_0}+\left(j\frac{\omega}{\omega_0}\right)^2}\right]^2$$

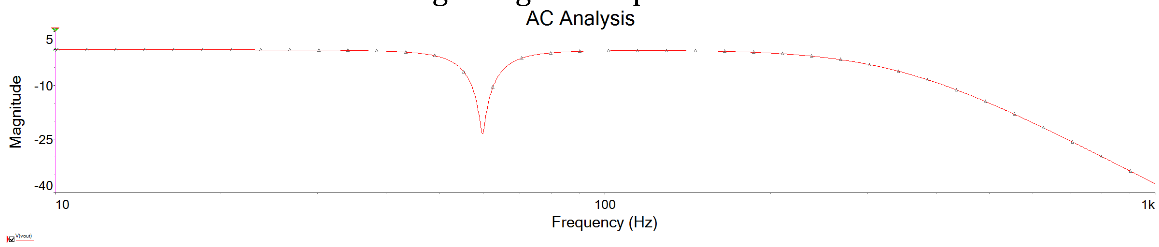
To confirm the specifications, the attenuation at 140 Hz is -0.24 dB and the attenuation at 1000 Hz is -37.5 dB, much better than the -3 and -30 dB requirements for passband and stopband.

These specs are also confirmed with Multisim simulation. Simulation shows attenuation of -0.24 dB at 140 Hz and -37.5 dB at 1000 Hz, in exact agreement with analysis.





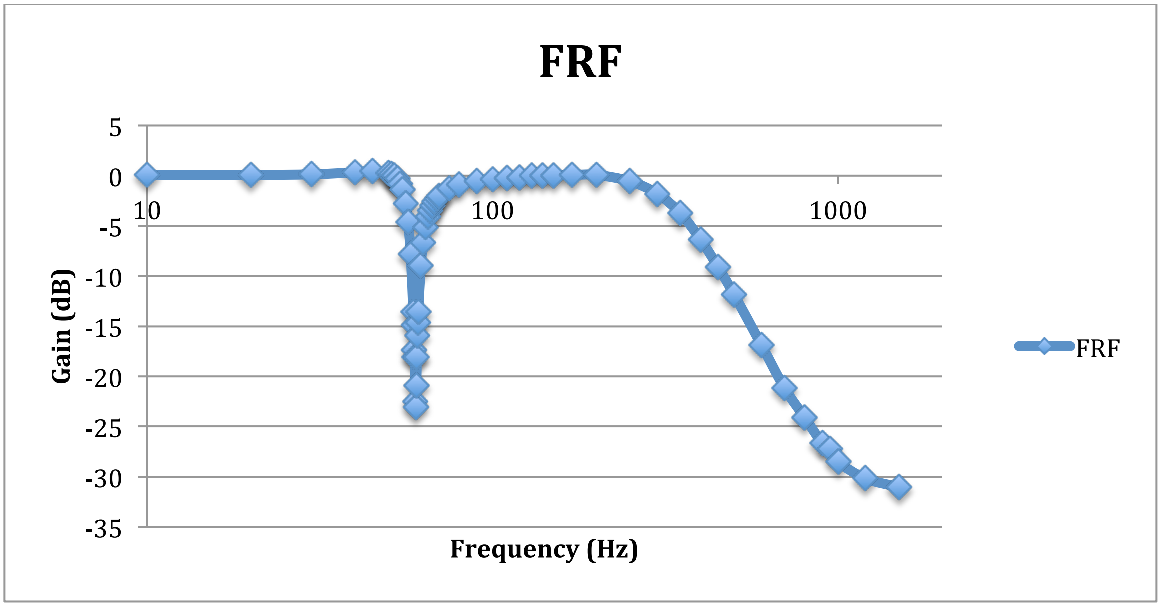
Putting these together gives the complete desired frequency response function with a notch at 60 Hz and antialiasing of higher frequencies.



The bill of materials for the filter components is given below.

Ref	**Component**	**Supplier**	**Part #**	**Cost**	**Quantity**	**Total**
R2-R3	SMT 0805 1% 9.76 MΩ	DigiKey	541-9.76MCCT-ND	0.39	2	
R4	SMT 0805 1% 4.87 MΩ	DigiKey	1276-5454-1-ND	0.10	1	
C2-C3	SMT 0805 1% 270 pF	DigiKey	399-9213-1-ND	1.05	2	
C4	SMT 0805 1% 560 pF	DigiKey	478-6047-1-ND	0.64	1	
C5, C7	Thru-hole 0.1 uF 10%				2	
C6, C8	Thru-hole 0.2 uF 10%				2	
R5-R8	Thru-hole 3.3k 5%				4	
U1-U2	TL082CD Dual Op-Amp SOIC	DigiKey	296-1284-5-ND	0.77	2	1.54
R1	10k Trim Potentiometer	DigiKey	A105870-ND	0.96	1	0.96
J1	5-pin female header	DigiKey	S6103-ND	0.62	1	0.62
C1p, C1m	SMT 0805 0.1 uF Cap	DigiKey	1276-1003-1-ND	0.10	2	0.20

The measured frequency response function is shown below. The 60 Hz notch is 23.1 dB deep at the selected potentiometer setting and is centered exactly on 60 Hz +/- 1 Hz. The attenuation at high frequency is 28.5 dB, somewhat less than the 37.5 dB expected, because the 2-layer board picks up some 60 Hz noise after the notch filter, and this noise sets a floor that prevents the Sallen-Key filter from dropping off as fast as expected.


FRF
Gain (dB)
5
0
-5
-10
-15
-20
-25
-30
-35
10
100
1000
Frequency (Hz)
FRF