$$\frac{1}{\infty} = 0$$

$$\frac{1}{0} = \infty$$

90°

$$-18 = 0$$

8

$$-10 = 8$$

$$x(t) = A\ \cos(\omega t + \phi),$$

$$x(t) = A \ \sin(\omega t + \phi + \pi/2)$$

A

ω

ϕ

$$0 \leq \phi \leq 2\pi$$

$$0^\circ \leq \phi \leq 360^\circ$$

$$\omega = 2\pi f = 2\pi/T$$

$$f = 1/T$$

T

$$\omega = 2\pi f = 2\pi/T, \quad f = 1/T = \omega/2\pi, \quad T = 1/f = 2\pi/\omega$$

$$\begin{cases} e^{j\theta} = \cos\theta + j\,\sin\theta \\ e^{-j\theta} = \cos\theta - j\,\sin\theta \end{cases} \qquad \begin{cases} \cos\theta = (e^{j\theta} + e^{-j\theta})/2 \\ \sin\theta = (e^{j\theta} - e^{-j\theta})/2j \end{cases}$$

$$A\ \cos(\omega t + \phi) = Re[A\ e^{j\omega t + \phi}], \quad A\ \sin(\omega t + \phi) = Im[A\ e^{j\omega t + \phi}]$$

$t = 2\ ms$

$i(t)$

I_{av}

$\mathcal{Q}$

$$I_{av}T = Q = \int_0^T i(t)dt,$$

$$I_{av} = \frac{1}{T} \int_0^T i(t)dt$$

$$V_{av} = \frac{1}{T} \int_0^T v(t)dt$$

$$i(t) = i(t + T)$$

$$v(t) = v(t + T)$$

$$i(t) = I_p \sin(\omega t) = I_p \sin(2\pi f t) = I_P \sin(2\pi t/T)$$

$$T = 1/f$$

$T/2$

I_{av}

=

$$\frac{1}{T/2}\int_0^{T/2} i(t)\;dt = \frac{2}{T}\int_0^{T/2}\;\; I_P\;\sin(2\pi t/T)\;dt = -\frac{2}{T}\,\frac{T I_P}{2\pi}\,\cos(2\pi t/T)\Big|_0^{T/2}$$

$$\frac{1}{\pi}\left[\cos(0) - \cos(\pi)\right] I_P = \frac{2}{\pi} I_P \; = 0.637\, I_P$$

I_{rms}

$$V_{rms}$$

$$W = \int_0^T p(t)dt$$

$$R \int_0^T i^2(t)dt = RI_{rms}^2 T$$

$$\frac{1}{R}\int_0^T v^2(t)dt = \frac{1}{R}V_{rms}^2 T$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt},$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$i(t) = I_p \cos(\omega t)$$

$$\cos^2\alpha = [1 + \cos(2\alpha)]/2$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)dt = \frac{I_p^2}{T}\int_0^T \cos^2(2\pi t/T)dt = \frac{I_p^2}{2T}\int_0^T \left[1+\cos(4\pi t/T)\right] dt = \frac{I_p^2}{2}$$

$$I_{rms} = \frac{I_p}{\sqrt{2}} = I_p \ 0.707$$

$$v(t) = V_p \cos(\omega t)$$

$$V_{rms} = \frac{1}{\sqrt{2}} = V_p \ 0.707$$

$$A\cos(\omega t+\phi)+B\cos(\omega t+\psi)$$

$$x(t) + y(t)$$

$$Re[\dot{X}e^{j\omega t}] + Re[\dot{Y}e^{j\omega t}] = Re[(\dot{X} + \dot{Y})e^{j\omega t}]$$

$$\dot{X} = Ae^{j\phi}$$

$$\dot{Y} = Be^{j\psi}$$

$$
\left\{ \begin{array}{l} x(t) = A\cos(\omega t + \phi) = Re[Ae^{j\phi}e^{j\omega t}] = Re[\dot{X}e^{j\omega t}] \\ y(t) = B\cos(\omega t + \psi) = Re[Be^{j\psi}e^{j\omega t}] = Re[\dot{Y}e^{j\omega t}] \end{array} \right.
$$

$$\dot{Y}e^{j\psi}$$

$$\dot{X} = Ae^{j\phi} = A\ \angle\phi, \qquad \dot{Y} = Be^{j\psi} = B\ \angle\psi$$

$$e^{j\omega t}$$

$$x(t) = Re[\dot{X}e^{j\omega t}] = Re[Ae^{j\phi}e^{j\omega t}] = A\cos(\omega t + \phi)$$

X

V

I

$$A_{rms} = A/\sqrt{2}$$

$$v_1(t) = 6\sqrt{2}\ \sin(\omega t)$$

$$v_2(t) = 12\sqrt{2}\ \sin(\omega t + \pi/2)$$

$$v_3(t) = 4\sqrt{2}\ \sin(\omega t - \pi/2)$$

$$v(t) = v_1(t) + v_2(t) + v_3(t) = 6\sqrt{2}\ \sin(\omega t) + 12\sqrt{2}\ \sin(\omega t + \pi/2) + 4\sqrt{2}\ \sin(\omega t - \pi/2)$$

V

$$\dot{V}_1 + \dot{V}_2 + \dot{V}_3 = \sqrt{2}(6\angle 0^\circ + 12\angle 90^\circ + 4\angle -90^\circ)$$

$$\sqrt{2}(6+j8) = 10\sqrt{2}\angle\left[\tan^{-1}(8/6)\right] = 10\sqrt{2}\angle\, 53.1^{\circ}$$

$$v(t) = 10\sqrt{2}\ sin(\omega t + 53.1^{\circ})$$

$$x(t) = A\cos(\omega t + \phi)$$

$x(t)$

$$A\cos(\omega t+\phi)=Re[Ae^{j\phi}\ e^{j\omega t}]=Re[\dot{X}e^{j\omega t}]$$

$$\left(\frac{A}{2}e^{j\phi}\right)e^{j\omega t}+\left(\frac{A}{2}e^{-j\phi}\right)e^{-j\omega t}=X_1e^{j\omega t}+X_{-1}e^{-j\omega t}$$

$$\dot{X} = Ae^{j\phi}, \qquad X_1 = \frac{A}{2}e^{j\phi}, \quad X_1 = \frac{A}{2}e^{-j\phi}$$

$$X_1 e^{j\omega t}$$

$$X_{-1}e^{-j\omega t}$$

$$\dot{X}e^{j\omega t}$$

$$Z = \frac{\dot{V}}{\dot{I}}, \qquad \dot{I} = \frac{\dot{V}}{Z}, \qquad \dot{V} = Z\dot{I}$$

$$\sum_k \dot{I}_k = 0$$

$$\sum_k \dot{V}_k = 0$$

$$y(t) = B\cos(\omega t + \psi)$$

$$x(t) = A\ \cos(\omega t + \phi) = Re[A\, e^{j(\omega t + \phi)}] = Re[A e^{j\phi} e^{j\omega t}] = Re[\dot{X} e^{j\omega t}]$$

$$\dot{X} = Ae^{j\phi} = Ae^{j\phi}$$

$x(t)$

$$\dot{Z} = \dot{X} \ * \ \dot{Y} = Ae^{j\phi} \ * \ Be^{j\psi}$$

$$z(t) = x(t) \ * \ y(t) = Re[\dot{Z}e^{j\omega t}]$$

$v(t)$

$$v(t) = V_p \cos(\omega t + \phi) = Re[V_p e^{j(\omega t + \phi)}], \qquad i(t) = I_p \cos(\omega t + \psi) = Re[I_p e^{j(\omega t + \psi)}]$$

$$= Z = \frac{V_p e^{j(\omega t+\phi)}}{I_p e^{j(\omega t+\psi)}} = \frac{V_p e^{j\phi}}{I_p e^{j\psi}} = \frac{\dot{V}}{\dot{I}} = \frac{\text{Phasor of voltage}}{\text{Phasor of current}}$$

$$V_p e^{j(\omega t+\phi)} = R I_p e^{j(\omega t+\psi)}$$

$$\psi = \phi$$

$$Z_R = \frac{V_p\, e^{j\phi}}{I_p\, e^{j\psi}} = \frac{V_p}{I_p} = R, \qquad |Z_R| = R, \qquad \angle Z_R = 0$$

$$V_p = |Z_R|\, I_p = R\, I_p, \qquad \angle\dot{V} = \angle\dot{I}$$

$$I_p e^{j(\omega t+\psi)} = C\frac{d}{dt}[V_p e^{j(\omega t+\phi)}] = j\omega C V_p e^{j(\omega t+\phi)}$$

$$Z_C = \frac{V_p e^{j\phi}}{I_p e^{j\psi}} = \frac{1}{j\omega C} = \frac{-j}{\omega C}, \qquad |Z_C| = \frac{1}{\omega C}, \qquad \angle Z_C = -\frac{\pi}{2}$$

$$\dot{V} = Z_C \dot{I} = -j\dot{I}/\omega C, \qquad V_p = I_p/\omega C, \qquad \angle\dot{V} = \angle\dot{I} - \frac{\pi}{2}$$

$$\angle Z_C = -\pi/2 = -90^\circ$$

$$V_p e^{j(\omega t+\phi)} = L\frac{d}{dt}[I_p e^{j(\omega t+\psi)}] = j\omega L I_p e^{j(\omega t+\psi)}$$

$$Z_L = \frac{V_p e^{j\phi}}{I_p e^{j\psi}} = j\omega L, \quad |Z_L| = \omega L, \quad \angle Z_L = \frac{\pi}{2}$$

$$\dot{V} = Z_L \dot{I} = j\omega L \dot{I}, \qquad V_p = \omega L I_p, \qquad \angle \dot{V} = \angle \dot{I} + \frac{\pi}{2}$$

$$\angle Z_L = \pi/2 = 90^\circ$$

E

C

L

$$\omega = 0$$

$$Z_C \to \infty$$

$$Z_L = 0$$

$$\omega \to \infty$$

$$Z_C \to 0$$

$$Z_L \to \infty$$

R

$$G = 1/R$$

Z

$$Y = 1/Z$$

G

B

$$Z = R + jX = |Z|e^{j\angle Z} = |Z|\angle Z$$

$$Re[Z] = R$$

$$Im[Z] = X$$

Ω

$$|Z| = \sqrt{R^2 + X^2}, \quad \angle Z = tan^{-1}\left(\frac{X}{R}\right)$$

$$Z_C = 1/j\omega C = -j/\omega C$$

$$Z_L = j\omega L$$

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} + j\frac{-X}{R^2 + X^2} = G + jB$$

$$G = Re[Y] = \frac{R}{R^2 + X^2} \neq \frac{1}{R}$$

$$B = Im[Y] = \frac{-X}{R^2 + X^2} \neq \frac{1}{X}$$

S

$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}, \qquad \angle Y = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}\left(\frac{-X}{R}\right) = -\angle Z$$

$$\begin{cases} \text{Impedance} & = & \text{Resistance} + j\ \text{Reactance} \\ \text{Admittance} & = & \text{Conductance} + j\ \text{Susceptance} \\ \text{Admittance} & = & 1/\text{Impedance} \end{cases}$$

$$Re[Y] = G$$

$$Im[Y] = B$$

Y

$$Y_R = \frac{1}{R}, \quad Y_L = \frac{1}{Z_L} = \frac{1}{j\omega L} = \frac{-j}{\omega L}, \quad Y_C = \frac{1}{Z_C} = \frac{1}{1/j\omega C} = j\omega C$$

$$Z_{total} = \frac{Z_1 \ Z_2}{Z_1 + Z_2}, \quad Y_{total} = Y_1 + Y_2$$

$$Z_{total} = Z_1 + Z_2, \quad Y_{total} = \frac{Y_1 \ Y_2}{Y_1 + Y_2}$$

$$v(t) = 28.3\ \cos(5000t + 45^\circ)\ V = 20\sqrt{2}\ \cos(5000t + 45^\circ)$$

$$v(t) = 28.3\ \cos(5000t + 45^\circ)\ V$$

$$\dot{V} = 28.3 \angle 45^{\circ}$$

$$\omega = 5000$$

$$Y_C = j\omega C = j\ 5000 \times 10^{-5} = j0.05 = 0.05\angle 90^\circ$$

$$Z_C = 1/Y_C = 1/j0.05 = -j20 = 20\angle{-90^\circ}$$

$$Z_L = j\omega L = j5000 \times 4 \times 10^{-3} = 0 + j20 = 20\angle 90^\circ$$

$$Y_L = 1/Z_L = -j0.05 = 0.05\angle{-90^\circ}$$

$$Z_R = R = 20 + j0 = 20\angle 0^\circ$$

$$Z_{RL} = Z_R + Z_L = 20 + j20 = 20\sqrt{2}\angle 45^\circ$$

$$Y_{RL} = 1/Z_{RL} = 1/20\sqrt{2}\angle 45^\circ = 0.025\sqrt{2}\angle{-45^\circ} = 0.025 - j0.025$$

$$Y_{load} = Y_C + Y_{RL} = 0.025 + j0.025 = 0.025\sqrt{2}\angle 45^\circ$$

$$\dot{I}_{load} = Y_{load}\dot{V} = (28.3\angle 45^\circ)\,(0.025\sqrt{2}\angle 45^\circ) = 1\,\angle 90^\circ = j$$

$$i_{load}(t) = Re[\dot{I}e^{j5000t}] = Re[(1\,e^{j\,90^\circ})e^{j5000t}] = \cos(5000t + 90^\circ)$$

$$\dot{I}_C = \dot{V} Y_C = (0.05\angle 90^\circ)(28.3\angle 45^\circ) = 1.41\angle 135^\circ = -1 + j$$

$$i_C(t) = Re[(1.41e^{j135^\circ})e^{j5000t}] = 1.41\ \cos(5000t + 135^\circ)$$

$$\dot{I}_{RL} = Y_{RL}\dot{V} = (0.025\sqrt{2}\angle{-45^\circ})(28.3\angle 45^\circ) = 1$$

$$i_{RL}(t) = Re[(0.707\, e^{j0})e^{j5000t}] = 0.707\ \cos(5000t)$$

$$\dot{V}_R = Z_R \dot{I}_{RL} = (20\angle 0^\circ)(1\ \angle 0^\circ) = 20\ \angle 0^\circ = 20 + j0$$

$$i_R(t) = 20\ \cos(5000t)$$

$$\dot{V}_L = Z_L \dot{I}_{RL} = (20\angle 90^\circ)(1\ \angle 0^\circ) = 20\ \angle 90^\circ = 0 + j20$$

$$v_L(t) = 20 \ \cos(5000t + 90^\circ)$$

$$\dot{V}_R + \dot{V}_L = 10\sqrt{2} + j10\sqrt{2} = 20\angle 45^\circ = \dot{V}$$

$$\dot{I}_C + \dot{I}_{RL} = 0.5\sqrt{2} - 0.5\sqrt{2} + j0.5\sqrt{2} = 0.5\sqrt{2}\angle 90^\circ = \dot{I}_{load}$$

$$i(t) = 17\ \cos(1000t + 90^\circ) = 12\sqrt{2}\cos(1000t + 90^\circ)$$

$R = 18\Omega$

$$C = 83.3\mu F = 83.3 \times 10^{-6} F$$

$$L = 30mH = 30 \times 10^{-3}H$$

$$\dot{I} = 12\sqrt{2}\ \angle 90^\circ = j\ 12\sqrt{2}$$

$$\omega = 1000$$

$$Z_R = R = 18, \quad Z_C = 1/j\omega C = 12\angle{-90^\circ} = 0 - j12, \quad Z_L = j\omega L = 30\angle 90^\circ = 0 + j30$$

$$Z_{total} = Z_R + Z_C + Z_L = 18 + j(-12 + 30) = 18 + j18 = 18\sqrt{2}\angle 45^\circ$$

$$\dot{V}_{total} = \dot{I} Z_{total} = (12\sqrt{2}\ \angle 90^\circ)\ (18\sqrt{2} \angle 45^\circ) = 432\ \angle 135^\circ$$

$$v(t) = 432\ \cos(1000t + 135^\circ)$$

$$\dot{V}_R$$

$$\dot{I}Z_R = 216\sqrt{2}\ j = 305.5\ \angle 90^\circ$$

$$\dot{V}_C$$

$$\dot{I} Z_C = 144\sqrt{2}\ \angle 0^\circ = 203.6$$

$$\dot{V}_L$$

$$\dot{I}Z_L = 360\sqrt{2}\ \angle 180^\circ = 510\ \angle 180^\circ$$

$$v_R(t)$$

$$305.5\ \cos(1000t + \pi/2)$$

$v_C(t)$

$$203.6\ \cos(1000t)$$

$v_L(t)$

$$509\ \cos(1000t + \pi)$$

$$\dot{V}_R$$

$\dot{V}_C$

$$\dot{V}_L$$

$$\dot{V}_{total} = \dot{V}_R + \dot{V}_C + \dot{V}_L = 216\sqrt{2}\angle 90^\circ + 216\sqrt{2}\angle 180^\circ = 432\angle 135^\circ$$

$$v_S(t) = A\cos(\omega t) = A\cos(10^6 t)$$

$$R_1 = R_2 = R_3 = R = 10\Omega$$

$$L = 10\,\mu H$$

v_2

R_2

$v_s(t)$

$$i_s(t)$$

$$v_C(t)$$

$$v_L(t)$$

$$v_{cd} = v_{RC} = V_{RL}$$

v_{ab}

$$\dot{V}_2 = \dot{I}_2 R_2$$

$\pi/2$

$$\dot{V}_3 = \dot{I}_3 R_3$$

$$Z_{RL}(\omega)||Z_{RC}(\omega) = \frac{(R + jX_L)(R + jX_C)}{R + jX_L + R + jX_C} = \frac{R^2 - X_L X_C + j(X_L + X_C)}{2R + j(X_L + X_C)}$$

$$X_C + X_L = 0$$

$$-1/j\omega C = j\omega L = j10$$

$$C = 10^{-7} = 0.1\mu F$$

$$Z_{RL} = 10 + j\,10$$

$$Z_{RC} = 10 - j\,10$$

$$|Z_{RL}| = |Z_{RC}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$$

$$Z_{RL}(\omega)||Z_{RC}(\omega) = \frac{(10+j10)(10-j10)}{10+j10+10-j10} = 10$$

$$|R_2| = |Z_C| = 10\Omega$$

$$|\dot{V}_C| = |\dot{V}_R| = 10V$$

$$|\dot{V}_{cd}| = |\dot{V}_{RC}| = |\dot{V}_{RL}| = 10\sqrt{2}$$

$$|\dot{V}_{ab}| = 10\sqrt{2}$$

$$|\dot{I}_{RC}| = \frac{|\dot{V}_{RC}|}{|Z_{RC}|} = \frac{10}{10} = 1, \qquad |\dot{I}_{RL}| = \frac{|\dot{V}_{RL}|}{|Z_{RL}|} = \frac{10}{10} = 1$$

$$|\dot{I}_s| = |\dot{I}_{RL} + \dot{I}_{RC}| = \sqrt{2}$$

R_1

$$V_1 = RI_s = 10\sqrt{2}$$

$$\dot{V}_s = \dot{V}_1 + \dot{V}_{cd} = 10\sqrt{2} + 10\sqrt{2} = 20\sqrt{2}$$

$$A = \sqrt{2}V_s = 40\,V$$

$$t = 0$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$v_C(0) = V_0$$

$$v_R(t)$$

$$v_R(t) + v_C(t) = R\,i(t) + v_C(t) = RC\frac{d}{dt}v_c(t) + v_c(t) = \tau\frac{d}{dt}v_C(t) + v_C(t) = v(t)$$

$$\tau = RC$$

$$[RC] = \frac{[V]}{[I]}\frac{[Q]}{[V]} = \frac{[Q]}{[I]} = [T]$$

$$v_R(t) + v_L(t) = R\, i(t) + L\frac{d}{dt}i(t) = v(t)$$

$$\tau \frac{d}{dt} i(t) + i(t) = \frac{v(t)}{R}$$

$$\tau = L/R$$

$$\frac{[L]}{[R]} = \frac{[V][T]}{[I]}\frac{1}{[R]} = \frac{[R][T]}{[R]} = [T]$$

$$v_R$$

v_C

$y(t)$

$$\tau \frac{d}{dt} y(t) + y(t) = x(t)$$

τ

$$v_s(t) = \cos(\omega t)$$

$$\tau \frac{d}{dt} v_C(t) + v_C(t) = v_s(t) = \left\{ \begin{array}{c} 0 \\ 1 \\ \cos(\omega t) \end{array} \right.$$

$$v_s(t) = 0$$

$$\tau \frac{d}{dt} v_C(t) + v_C(t) = 0, \qquad v_C(0) = V_0$$

$$v_C(0) = V_0 \neq 0$$

$$v_C(t) = Ae^{st},$$

$$\frac{d}{dt}v_C(t) = sAe^{st}$$

$$\tau s A e^{st} + A e^{st} = (\tau s + 1) A e^{st} = 0$$

$$Ae^{st} = 0$$

$$\tau s + 1 = 0,$$

$$s = -\frac{1}{\tau}$$

$$v_C(t) = Ae^{-t/\tau}$$

$t \geq 0$

$$v_C(0) = Ae^{-t/\tau}\bigg|_{t=0} = Ae^0 = V_0,$$

$$A = V_0$$

$$v_C(t) = V_0 e^{-t/\tau}$$

$t \to \infty$

$$\lim_{t\to\infty} V_0 e^{-t/\tau} = 0$$

$$i(t) = C\frac{d}{dt}\, v_C(t) = C\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -V_0\frac{C}{\tau}e^{-t/\tau} = -\frac{V_0}{R}e^{-t/\tau}$$

$$v_R(t) = i(t)R = -V_0 e^{-t/\tau} = -v_C(t)$$

$$v_C(t) + v_R(t) = v_s(t) = 0$$

$$v(t) = V_0 e^{-t/\tau} = e^{-t\tau}$$

$$V_0 = 1$$

$$\frac{d}{dt}v(t)\big|_{t=0} = -\frac{1}{\tau}e^{-t/\tau}\bigg|_{t=0} = -\frac{1}{\tau}$$

$$\tau \dot{v}_C(t) + v_C(t) = v_s(t) = V_S$$

$$v_C(t) = c$$

$$c = V_s$$

$$v_C(t) = V_s$$

$$\tau\dot{v}_C(t) + v_C(t) = v_s(t) = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$v_C(t) = A\, e^{j\omega t}, \qquad \frac{d}{dt} v_C(t) = j\omega A\, e^{j\omega t}$$

$$j\omega\tau A e^{j\omega t} + A e^{j\omega t} = (j\omega\tau + 1)A\ e^{j\omega t} = e^{j\omega t}$$

$$A = \frac{1}{j\omega\tau + 1} = |A|e^{-j\phi},$$

$$|A| = \frac{1}{\sqrt{\omega^2\tau^2 + 1}}, \qquad \phi = \angle A = tan^{-1}\omega\tau$$

$$A\ e^{j\omega t} = \frac{1}{\sqrt{\omega^2\tau^2+1}} e^{j(\omega t-\phi)}$$

$$v_s(t) = \cos(\omega t) = Re(e^{j\omega t})$$

$$v_C(t) = Re\left[\frac{1}{\sqrt{\omega^2\tau^2+1}}e^{j(\omega t-\phi)}\right] = \frac{1}{\sqrt{\omega^2\tau^2+1}}\cos(\omega t-\phi)$$

$$v(t) = \cos(\omega t)$$

$$V_C = V \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega\tau + 1} = \frac{1}{\sqrt{\omega^2\tau^2 + 1}} e^{-j\phi}$$

$$\phi = \tan^{-1} \omega\tau$$

$$v_C(t) = Re[V_C e^{j\omega t}] = Re\left[\frac{1}{\sqrt{\omega^2\tau^2+1}} e^{j(\omega t - \phi)}\right] = \frac{1}{\sqrt{\omega^2\tau^2+1}} \cos(\omega t - \phi)$$

$$v_L(t) + v_R(t) = L\frac{d}{dt}i(t) + R\, i(t) = v(t) = \cos(\omega t)$$

$$L\frac{d}{dt}i(t) + Ri(t) = e^{j\omega t}$$

$$i(t) = A\ e^{j\omega t}, \qquad \frac{d}{dt} i(t) = j\omega A\ e^{j\omega t}$$

$$j\omega L A e^{j\omega t} + R A e^{j\omega t} = (R + j\omega L) A \, e^{j\omega t} = e^{j\omega t}$$

$$A = \frac{1}{R + j\omega L} = |A|e^{-j\phi},$$

$$|A| = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}, \qquad \phi = \angle A = tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$A\ e^{j\omega t} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} e^{j(\omega t - \phi)}$$

$$v(t) = \cos \omega t$$

$$i(t) = Re\left[\frac{1}{\sqrt{R^2+\omega^2 L^2}} e^{j(\omega t-\phi)}\right] = \frac{1}{\sqrt{R^2+\omega^2 L^2}}\cos(\omega t-\phi)$$

$$Z = Z_R + Z_L = R + j\omega L = |Z|e^{j\angle Z} = \sqrt{R^2 + \omega^2 L^2}\ e^{j\phi}$$

$$\phi = tan^{-1}(\omega L/R)$$

$$I = \frac{1}{\sqrt{R^2 + \omega^2 L^2}\ e^{j\phi}} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} e^{-j\phi}$$

$$i(t) = Re\left[\frac{1}{\sqrt{R^2+\omega^2L^2}}e^{-j\phi)}e^{j\omega t}\right] = \frac{1}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t-\phi)$$

$$H = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{Z}$$

$$|H| = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}, \qquad \angle H = -\phi$$

$$i(t) = |H|\cos(\omega t + \angle H) = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \phi)$$

+

$$\tau \frac{d}{dt} v_c(t) + v_c(t) = v_s(t) = V_s$$

$v(t) \neq 0$

$t > 0$

$$v(t) = V_s$$

$$v(t) = V_s u(t) = \begin{cases} V_s & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$v'_C(t) = Ae^{-t/\tau}$$

$$v_C(t)\Big|_{t=0} = v_C(0) = V_0$$

$$v_C''(t) = v_C(t)|_{t\to\infty} = V_s$$

V_s

$$V_c = V_s \frac{Z_c}{R + Z_c} = V_s \frac{1/j\omega C}{R + 1/j\omega C} = V_s$$

$$Z_c = 1/j\omega C = \infty$$

$$v_C(t) = v'_C(t) + v''_C(t) = V_s + Ae^{-t/\tau}$$

$$v_C(0) = V_0 = v_C(t)\Big|_{t=0} = V_s + Ae^0 = V_s + A,$$

$$A = V_0 - V_s$$

$$v_C(t) = V_s + Ae^{-t/\tau} = V_s + (V_0 - V_s)e^{-t/\tau}$$

$$v_C(0) = V_0 = 0$$

$$v_C(t) = V_s + (-V_s)e^{-t/\tau} = V_s(1 - e^{-t/\tau})$$

$$i(t) = C\frac{d}{dt}\,v_C(t) = -\frac{C}{\tau}(V_0 - V_s)e^{-t/\tau} = \frac{V_s - V_0}{R}e^{-t/\tau}$$

$$v_R(t) = R\, i(t) = (V_s - V_0)e^{-t/\tau}$$

$$v_C(t) + v_R(t) = V_s + (V_0 - V_s)e^{-t/\tau} + (V_s - V_0)e^{-t/\tau} = V_s$$

$$V_0 = 2$$

$$V_s = 1$$

$$V_0 > V_s$$

$$V_s = 2$$

$$V_0 < V_s$$

$$v_C(0) = V_0 > V_s$$

$V_s > 0$

$$V_0 = 2V$$

$$V_s = 1V$$

$-1V$

$0V$

$$V_S - V_0$$

$$v_R(t) = 0$$

$t < 0$

$$v_R(t) = V_s - v_c(0) = V_s - V_0$$

$$t = 0^+$$

$$v_C(t) + v_R(t)$$

V_0

$$V_S$$

$$v_C(t) = \frac{1}{C}\int i(t)dt, \qquad i_C(t) = \frac{1}{L}\int v(t)dtt$$

$$v_C(0) = 0$$

$$W_R$$

$$w_C$$

$$v_C(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$

$$v_C(t) = v_C(0) = V_0$$

$$v_C(t) = v_C(\infty) = V_s$$

$$f(t) = f(\infty) + [f(0) - f(\infty)]e^{-t/\tau}$$

$$f(\infty)$$

$f(0)$

$$f(t)$$

$f(0_{-})$

$$f(0_+)$$

$$f(0_-) \neq f(0_+)$$

$$f(t) = f(\infty) + f(0) - f(\infty) = f(0)$$

$$f(t) = f(\infty)$$

$0 < t < \infty$

$$f(0) - f(\infty)$$

$$V_{in} = 2\,V$$

$$R_1 = R_2 = 2\,k\Omega$$

$$C = 10^{-6}\, F = 1\, \mu F$$

$v_C(\infty)$

$$v_C(\infty) = V_s \frac{R_2}{R_1 + R_2} = \frac{V_s}{2} = 1\,V$$

$$R = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} = 1\ k\Omega$$

$$\tau = RC = 1000 \times 10^{-6} = 10^{-3}\ sec$$

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau} = 1 + (0-1)e^{-t/10^{-3}} = 1 - e^{-1000\,t}$$

$$v_C(0) = \left[1 - e^{-1000\,t}\right]_{t=0} = 0, \qquad v_C(\infty) = \left[1 - e^{-1000\,t}\right]_{t=\infty} = 1$$

$$V_0 = 10V$$

$$R_1 = R_2 = 2K\Omega$$

$$R_3 = 5K\Omega$$

$$C = 0.5\mu F$$

c

$$\frac{V_0 - v_c(t)}{R_1} = C\dot{v}_c(t) + \frac{v_c(t)}{R_2}$$

$$\frac{R_1 R_2}{R_1 + R_2} C\dot{v}_c(t) + v_c(t) = \tau\dot{v}_c(t) + v_c(t) = V_0 \frac{R_2}{R_1 + +R_2}$$

$$\tau = C\frac{R_1 R_2}{R_1 + R_2} = C(R_1||R_2) = 0.5 \times 10^{-6} \times 10^3 = 5 \times 10^{-4}$$

$$v_c(0) = -V_0/2$$

$$v_h(t) = Ae^{-t/\tau}$$

$$v_p(t) = V_0/2$$

$$v_c(t) = v_h(t) + v_p(t) = Ae^{-t/\tau} + \frac{V_0}{2}$$

$$v_c(t)$$

$$v_c(0)$$

$$v_c(t)\bigg|_{t=0} = V_s + Ae^{-t/\tau}\bigg|_{t=0} = \frac{V_0}{2} + A = v_c(0) = -\frac{V_0}{2}$$

$$A = -V_0$$

$$v_c(t) = Ae^{-t/\tau} + \frac{V_0}{2} = \frac{V_0}{2} - V_0 e^{-t/\tau} = 5 - 10\, e^{-2000\, t}$$

$v_C(0)$

$$v_C(0_-) = -5V$$

$$R = 0$$

$$\tau = RC = 0$$

$$v_C(0_+) = v_C(0_-) = -5\,V$$

$$t = 0_+$$

V_d

$10\,V$

$0\ V$

V_c

$5\,V$

$-5\,V$

$$v_C(\infty) = V_{R_2} = V_0 \frac{R_2}{R_1 + R_2} = \frac{V_0}{2} = 5V$$

$$\tau = RC = 1000 \times 0.5 \times 10^{-6} = 5 \times 10^{-4}$$

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-t/\tau} = 5 + (-5 - 5)e^{-t/0.0005} = 5 - 10e^{-2000t}\ (V)$$

$$v_C(0) = \left[5 - 10e^{-2000t}\right]_{t=0} = -5, \qquad v_C(\infty) = \left[5 - 10e^{-2000t}\right]_{t=\infty} = 5$$

$i_C(t)$

$$i_C(t)$$

$$C\frac{dv_C(t)}{dt} = C\frac{d}{dt}(5 - 10e^{-t/\tau}) = C\,\frac{10e^{-t/\tau}}{\tau} = 10e^{-2000t}\ (mA)$$

$$\frac{10e^{-t/\tau}}{R} = 10e^{-2000t}\ (mA)$$

$v_1(t)$

$v_2(t)$

$$v_2(t) = v_C(t) = 5 - 10e^{-2000t} \ (V)$$

$$v_1(t) = V_0 - v_C(t) = 10 - (5 - 10e^{-2000t}) = 5 + 10e^{-2000t}$$

$i_1(t)$

$$i_2(t)$$

$$i_1(t) = \frac{v_1(t)}{R_1} = 2.5 + 5e^{-2000t}\ (mA)$$

$$i_2(t) = \frac{v_C(t)}{R_2} = 2.5 - 5e^{-2000t}\ (mA)$$

$$i_C(t) = i_1(t) - i_2(t) = 10e^{-2000t}\ (mA)$$

$$v_1(\infty) = v_2(\infty) = 5\ V$$

$v_1(0_+)$

$v_2(0_+)$

$$v_1(0_-) = v_2(0_-) = 5\ V$$

$$v_1(0_+) = 15V$$

$$v_2(0_+) = -5V$$

$$\tau = RC = 5 \times 10^{-4}$$

$$v_1(t) = v_1(\infty) + [v_1(0_+) - v_1(\infty)]e^{-t/\tau} = 5 + (15 - 5)e^{-t/\tau} = 5 + 10e^{-t/\tau}\ V$$

$$v_2(t) = v_2(\infty) + [v_2(0_+) - v_2(\infty)]e^{-t/\tau} = 5 + (-5 - 5)e^{-t/\tau} = 5 - 10e^{-t/\tau} \ V$$

$$v_1(t) + v_2(t) = V_0 = 10V$$

$$i_1(t) = \frac{v_1(t)}{R_1} = 2.5 + 5e^{-t/\tau}\ mA$$

$$i_2(t) = \frac{v_2(t)}{R_2} = 2.5 - 5e^{-t/\tau}\ mA$$

$$i_C(t) = i_1(t) - i_2(t) = 10e^{-t/\tau}\ mA$$

$$v_2(t) = V_c = -5V$$

V_b

$$v(t) = V_s \cos(\omega t + \psi)$$

$$v_R(t) + v_C(t) = \tau \frac{d}{dt} v_c(t) + v_c(t) = \tau \dot{v}_c(t) + v_c(t) = V_s \cos(\omega t + \psi)$$

$$v_p(t)$$

$V_s \cos(\omega t + \psi)$

$$V_s e^{j(\omega t+\psi)}$$

$$y_p(t) = Be^{j\omega t}$$

$$\dot{y}_p(t) = j\omega B e^{j\omega t}$$

$$\tau\dot{v}_c(t) + v_c(t) = j\omega\tau Be^{j\omega t} + Be^{j\omega t} = (j\omega\tau + 1)Be^{j\omega t} = V_s e^{j\omega t} e^{j\psi}$$

$$B = \frac{V_s e^{j\psi}}{j\omega\tau + 1}$$

$$y_p(t)$$

$$y_p(t) = Be^{j\omega t} = \frac{V_s e^{j\psi}}{j\omega\tau + 1} e^{j\omega t} = \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}} e^{j(\omega t + \psi - \phi)}$$

$$V_s \cos(\omega t + \psi) = Re(V_s e^{j(\omega t + \psi)})$$

$$y_p(t) = Re\left[\frac{V_s e^{j(\omega t+\psi-\phi)}}{j\omega\tau+1}\right] = \frac{V_s}{\sqrt{(\omega\tau)^2+1}}\cos((\omega t+\psi-\phi)$$

$$v(t) = V_s \cos(\omega t) = Re[\dot{V} e^{j\omega t}]$$

$$\dot{V} = V_s \; e^{j\psi}$$

$$\dot{V}H(\omega) = \dot{V}\frac{Z_C}{R + Z_C} = \dot{V}\frac{1/j\omega C}{R + 1/j\omega C} = \frac{\dot{V}}{j\omega\tau + 1} = \frac{\dot{V}}{\sqrt{(\omega\tau)^2 + 1}\ e^{j\phi}}$$

$$\frac{\dot{V}\ e^{-j\phi}}{\sqrt{(\omega\tau)^2+1}} = \frac{V_s e^{j\psi}\ e^{-j\phi}}{\sqrt{(\omega\tau)^2+1}} = \frac{V_s e^{j(\psi-\phi)}}{\sqrt{(\omega\tau)^2+1}}$$

$H(\omega)$

$$H(\omega) = \frac{Z_C}{Z_C + Z_R} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{\sqrt{1 + (\omega\tau)^2}} e^{-j\phi}$$

$v_p(t)$

$$Re\left[\dot{V}_C e^{j\omega t}\right] = Re\left[\frac{V_s e^{j(\psi-\phi)} e^{j\omega t}}{\sqrt{(\omega\tau)^2+1}}\right] = \frac{V_s}{\sqrt{(\omega\tau)^2+1}}\cos(\omega t+\psi-\phi)$$

$$|H(\omega)|V_s \cos(\omega t + \psi - \angle H(\omega))$$

$$v(t) = V_S \cos(\omega t + \psi)$$

$$|H(\omega)| = 1/\sqrt{(\omega\tau)^2 + 1}$$

$$\angle H(\omega) = -\phi$$

$$v_C(t) = v_h(t) + v_p(t) = \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}} \cos(\omega t + \psi - \phi) + Ae^{-t/\tau}$$

$$v_C(t)\Big|_{t=0} = v_C(0) = V_0 = \frac{V_s}{\sqrt{(\omega\tau)^2+1}}\cos(\psi-\phi) + A$$

$$A = V_0 - \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}} \cos(\psi - \phi)$$

$$v_C(t) = \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}} \cos(\omega t + \psi - \phi) + \left[V_0 - \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}} \cos(\psi - \phi) \right] e^{-t/\tau}$$

$$f(t) = f_\infty(t) + [f(0) - f_\infty(0)]e^{-t/\tau}$$

$$f_\infty(t)$$

$$f_\infty(0) = f_\infty(t)\Big|_{t=0}$$

t

$$V_0 = 0$$

$$v_C(t) = \frac{V_s}{\sqrt{(\omega\tau)^2 + 1}}[\cos(\omega t + \psi - \phi) - \cos(\psi - \phi)e^{-t/\tau}]$$

$\psi - \phi$

$$\psi - \phi = \pm 90^\circ$$

$$\cos(\psi - \phi) = 0$$

$$\psi - \phi = 0^\circ$$

180°

$$\cos(\psi - \phi) = \pm 1$$

0°

$$R = 20\Omega$$

$$L = 0.3H$$

$120V$

$60Hz$

$$\psi = 10^\circ$$

$$v(t) = 120\sqrt{2}\ \cos(6.28 \times 60t + 10^\circ)$$

$$i(0) = 0$$

$$Z = R + j\omega L = 20 + j6.28 \times 60 \times 0.3 = 20 + j113 = 114.8\angle 80^\circ$$

$i_{\infty}(t)$

$$\dot{V} = 120\angle 10^\circ, \quad \dot{I} = \frac{\dot{V}}{Z} = \frac{120\angle 10^\circ}{114.8\angle 80^\circ} = 1.05\angle{-70^\circ}$$

$$i_{\infty}(t) = 1.05\sqrt{2}\ \cos(6.28 \times 60t - 70^{\circ})$$

$$i_\infty(0) = 1.05\sqrt{2}\ \cos(-70^\circ) = 1.05\sqrt{2} \times 0.342 = 0.51$$

$$\tau = L/R = 0.3/20 = 0.015$$

$$i(t) = i_\infty(t) + [i(0) - i_\infty(0)]e^{-t/\tau} = 1.05\sqrt{2}\ \cos(6.28 \times 60t - 70^\circ) - 0.51e^{-t/0.015}$$

$$v_R(t) + v_L(t) + v_C(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = v_s(t)$$

$$i''(t) + \frac{R}{L}\, i'(t) + \frac{1}{LC}\, i(t) = \frac{1}{L}\dot{v}_s(t)$$

$$i(t) = C dv_C(t)/dt$$

$$v_R(t) = R\; i(t) = R\,C\frac{d\,v_C(t)}{dt}, \qquad v_L(t) = L\frac{d}{dt}i(t) = L\frac{d}{dt}\left[C\frac{d\,v_C(t)}{dt}\right] = LC\frac{d^2\,v_C(t)}{dt^2}$$

$$v_C''(t) + \frac{R}{L} v_C'(t) + \frac{1}{LC} v_C(t) = \frac{1}{LC} v_s(t)$$

$$i_R(t) + i_C(t) + i_L(t) = \frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)\,dt = i_s(t)$$

$$v''(t) + \frac{1}{RC}\,v(t) + \frac{1}{LC}\,v(t) = \frac{1}{C}\dot{i}_a(t)$$

$$\frac{v_s - v}{R} = C\frac{dv}{dt} + \frac{1}{L}\int v \, dt, \qquad \frac{1}{L}\int (v_s - v) dt = \frac{v}{R} + C\frac{dv}{dt}, \qquad C\frac{d}{dt}(v_s - v) = \frac{v}{R} + \frac{1}{L}\int v \, dt$$

$$v'' + \frac{1}{RC}v' + \frac{1}{LC}v = \frac{\dot{v}_s}{RC}, \qquad v'' + \frac{1}{RC}v' + \frac{1}{LC}v = \frac{1}{LC}v_s, \qquad v'' + \frac{1}{RC}v' + \frac{1}{LC}v = v_s''$$

$$\left[\frac{R}{L}\right] = \frac{[V][/[I]}{[VT]/[I]} = \frac{1}{[T]}, \quad \left[\frac{1}{RC}\right] = \frac{[I]}{[V]}\frac{[V]}{[I][T]} = \frac{1}{[T]}$$

$$\left[\frac{1}{LC}\right] = \frac{1}{[VT]/[I]\ [IT]/[V]} = \frac{1}{[T]^2}$$

$$y''(t) + 2\zeta\omega_n\, y'(t) + \omega_n^2\, y(t) = x(t)$$

ζ

ω_n

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\frac{R}{L} = 2\zeta_s \omega_n = 2\zeta_s \frac{1}{\sqrt{LC}},$$

$$\zeta_s = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\frac{1}{RC} = 2\zeta_p \omega_n = 2\zeta_p \frac{1}{\sqrt{LC}},$$

$$\zeta_p = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$\zeta_p \zeta_s = \frac{1}{4}, \qquad \zeta_p = \frac{1}{4\zeta_s}, \qquad \zeta_s = \frac{1}{4\zeta_p}$$

$$\left[\sqrt{\frac{L}{C}}\right] = \sqrt{\frac{[Henry]}{[Farad]}} = \sqrt{\frac{[Volt]\ [second]}{[Ampere]}\frac{[Volt]}{[second]\ [Ampere]}} = \frac{[Volt]}{[Ampere]} = [Ohm]$$

$$\left[\sqrt{LC}\right] = \sqrt{[Henry]\ [Farad]} = \sqrt{\frac{[Volt]\ [second]}{[Ampere]}\frac{[Ampere]\ [second]}{[Volt]}} = [second]$$

ζ_s

ζ_p

$$\omega_n = 1/\sqrt{LC}$$

$1/[second]$

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) = |Z|e^{j\angle Z}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \begin{cases} \infty & \omega = 0 \;\; (Z_C = \infty, \text{ capacitive}) \\ R & \omega = 1/\sqrt{LC} \;\; (Z_C + Z_L = 0, \text{ resistive}) \\ \infty & \omega \to \infty \;\; (Z_L = \infty, \text{ inductive}) \end{cases}$$

$$\angle Z = tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = \left\{ \begin{array}{rl} -90^\circ & \omega = 0 \;\; (Z_L = 0, \text{ capacitive}) \\ 0^\circ & \omega = 1/\sqrt{LC} \;\; (Z_C + Z_L = 0, \text{ resistive}) \\ 90^\circ & \omega \rightarrow \infty \;\; (Z_C = 0, \text{ inductive}) \end{array} \right.$$

$$Z = Z_R + Z_C + Z_L$$

$$\omega L = 1/omegaC$$

$$\omega = 1/\sqrt{LC} = \omega_n$$

$$Z_L = j\omega_n L = j\sqrt{\frac{L}{C}}, \qquad Z_C = \frac{1}{j\omega_n C} = -j\sqrt{\frac{L}{C}}, \qquad Z_L + Z_C = 0$$

$$Z = Z_R + Z_C + Z_L = Z_R = R$$

$$\dot{I} = \dot{V}/Z = \dot{V}/R$$

$$\angle Z = \angle R = 0$$

I

V

Q_s

$$\frac{\text{Magnitude of inductor/capacitor impedance at } \omega_n}{\text{Resistance}}$$

$$\frac{|Z_L|}{R} = \frac{\omega_n L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2\zeta_s}$$

$$\frac{|Z_C|}{R} = \frac{1}{\omega_n C R} = \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2\zeta_s}$$

$$\omega = \omega_n = 1/\sqrt{LC}$$

$$\dot{V}_R = \dot{I} Z_R = \frac{\dot{V}}{R}\ R = \dot{V}$$

$$\dot{V}_L = \dot{I} Z_L = \frac{\dot{V}}{R} \, j\omega_n L = j\frac{\dot{V}}{R} \, \frac{L}{\sqrt{LC}} = j \, \dot{V} \frac{1}{R} \sqrt{\frac{L}{C}} = jQ_s \dot{V}$$

$$\dot{V}_C = \dot{I} Z_C = \frac{\dot{V}}{R}\ \frac{1}{j\omega_n C} = -j\,\frac{\dot{V}}{R}\ \frac{\sqrt{LC}}{C} = -j\,\dot{V}\ \frac{1}{R}\sqrt{\frac{L}{C}} = -jQ_s\dot{V}$$

$$\dot{V}_R = \dot{V}$$

Q_s

$$|\dot{V}_L| = |\dot{V}_C| = Q_s \dot{V}$$

$$\angle V_L = -\angle V_C$$

$$\dot{V}_L = -\dot{V}_C$$

$$V_L$$

V_C

$$R = 5\Omega$$

$$L = 4\ mH$$

$C = 0.1\ \mu F$

$$\omega_n = 1/\sqrt{LC} = 1/\sqrt{4 \times 10^{-3} \times 10^{-7}} = 5 \times 10^4$$

$$Q_s = \frac{\omega_n L}{R} = \frac{(5 \times 10^4) \times (4 \times 10^{-3})}{5} = 40$$

$$Q_s = \frac{1}{\omega_n CR} = \frac{1}{(5 \times 10^4) \times 10^{-7} \times 5} = 40$$

$$V_{rms} = 10V$$

$$I = V/R = 10/5 = 2A$$

$$\dot{V}_R = R\dot{I} = V = 10V$$

$$\dot{V}_L = j\omega L\dot{I} = j5 \times 10^4 \times 4 \times 10^{-3} \times 2 = j400V = jQ_sV$$

$$\dot{V}_C = \dot{I}/j\omega_n C = -j2/(5 \times 10^4 \times 0.1 \times 10^{-6}) = -j400V = -jQ_s V$$

$$10V$$

$$Y = G + j\omega C + \frac{1}{j\omega L} = G + j\left(\omega C - \frac{1}{\omega L}\right) = |Y|e^{j\angle Y}$$

$$|Y| = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}, \quad \angle Y = tan^{-1}\left(\frac{\omega C - 1/\omega L}{G}\right)$$

$$\omega C = 1/\omega L$$

$$Y = Y_R + Y_C + Y_L = G + j\left(\omega C - \frac{1}{\omega L}\right) = G$$

$$\dot{V} = \dot{I}/Y = \dot{I}/G$$

$$\angle Y = 0$$

Q_p

$$Q_p$$

$$\frac{\text{Magnitude of inductor/capacitor susceptance at } \omega_n}{\text{Conductance}}$$

$$\frac{|Y_L|}{G} = \frac{1}{\omega_n L G} = \frac{\sqrt{LC}}{LG} = R\sqrt{\frac{C}{L}} = \frac{1}{2\zeta_p}$$

$$\frac{|Y_C|}{G} = \frac{\omega_n C}{G} = \frac{C}{\sqrt{LC}G} = R\sqrt{\frac{C}{L}} = \frac{1}{2\zeta_p}$$

$$\frac{1}{Q_s} = Q_p, \qquad \frac{1}{Q_p} = Q_s$$

$$\dot{V} = \dot{I}/Y = \dot{I}/G = \dot{I}R$$

$$\dot{I}_R = \dot{V}Y_R = \dot{V}G = \frac{\dot{I}}{G}G = \dot{I},$$

$$\dot{I}_C = \dot{V} Y_C = \dot{I} R \; j\omega_n C = j \dot{I} R \frac{C}{\sqrt{LC}} = j \dot{I} R \; \sqrt{\frac{C}{L}} = j Q \dot{I}$$

$$\dot{I}_L = \dot{V} Y_L = \dot{I} R \, \frac{1}{j\omega_n L} = -j\dot{I} R \frac{\sqrt{LC}}{L} = -j\dot{I} R \, \sqrt{\frac{C}{L}} = -jQ\dot{I}$$

$$\dot{I}_R = \dot{I}$$

$$|\dot{I}_L| = |\dot{I}_C| = Q_p \dot{I}$$

$$\angle I_L = -\angle I_C$$

$$\dot{I}_L = -\dot{I}_C$$

$$I_L$$

I_C

$\zeta < 1$	$Q > 0.5$	under damped
$\zeta = 1$	$Q = 0.5$	critically damped
$\zeta > 1$	$Q < 0.5$	over damped

$$Q = 2\pi \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

ωL

$$1/\omega C$$

$$T = 1/f = 2\pi/\omega$$

$$W_L = \int_0^T i(t)\ v(t)dt = \int_0^T i(t)\ L\frac{di(t)}{dt}dt = L\int_0^{I_p} i\ di = \frac{1}{2}LI_p^2 = LI_{rms}^2$$

$$W_C = \int_0^T v(t)\ i(t) dt = \int_0^T v(t)\, C \frac{dv(t)}{dt} dt = C \int_0^{V_p} v\ dv = \frac{1}{2} C V_p^2 = C V_{rms}^2$$

$$I_p = \sqrt{2} I_{rms}$$

$$V_p = \sqrt{2} V_{rms}$$

$$|Z_C| = |Z_L|$$

$$|V_C| = |V_L|$$

$$W_L = LI_{rms}^2 = L\left(\frac{V_{rms}}{\omega_n L}\right)^2 = L\frac{V_{rms}^2 LC}{L^2} = CV_{rms}^2 = W_C$$

$$W_L = W_C$$

$$T = 2\pi/\omega_n = 1/f_n$$

$$W_R = T\ P_R = T\ I_{rms}^2 R$$

$$Q = 2\pi \frac{W_L}{W_R} = 2\pi \frac{LI_{rms}^2}{TI_{rms}^2 R} = 2\pi f_n \frac{L}{R} = \frac{\omega_n L}{R} = \frac{1}{\omega_n C R}$$

$$Q_p = \frac{1}{2\zeta_p},$$

$$Q_s = \frac{1}{2\zeta_s}$$

$$y''(t) + 2\zeta\omega_n y'(t) + \omega_n^2 y(t) = x(t) = 0$$

$$x(t) = 0$$

$$y(t) = Ae^{st}$$

$$\dot{y}(t) = sAe^{st},\ y''(t) = s^2Ae^{st}$$

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)Ae^{st} = 0$$

$$Ae^{st} \neq 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

A

$$
s_{1,2} = \begin{cases} \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n & |\zeta| > 1 \\ -\zeta\omega_n = \pm\omega_n & |\zeta| = 1 \\ \left(-\zeta \pm j\sqrt{1 - \zeta^2}\right)\omega_n = \omega_n e^{\mp j\phi} & |\zeta| < 1 \end{cases}
$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$s_{1,2}$

$$\Delta = (2\zeta\omega_n)^2 - 4\omega_n^2 = 4\omega_n^2(\zeta^2 - 1) \begin{cases} \geq 0 & |\zeta| \geq 1 \\ < 0 & |\zeta| < 1 \end{cases}$$

$$-\infty$$

∞

s_1

s_2

$$\zeta \geq 0$$

$$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

C_1

C_2

$$y(0)$$

$$y'(0)$$

$$\dot{y}(0) = 0$$

$$y(0) = y_0$$

$y(0)$

$$y_h(t)\Big|_{t=0} = C_1 + C_2 = y_0$$

$$\dot{y}(0)$$

$$\dot{y}_h(t)\Big|_{t=0} = s_1 C_1 + s_2 C_2 = 0$$

$$C_1 = \frac{s_2}{s_2 - s_1} y_0, \qquad C_2 = \frac{s_1}{s_1 - s_2} y_0$$

$$y_h(t) = y_0 \left[\frac{s_2 e^{s_1 t}}{s_2 - s_1} - \frac{s_1 e^{s_2 t}}{s_2 - s_1} \right] = \frac{y_0}{s_2 - s_1} (s_2 e^{s_1 t} - s_1 e^{s_2 t})$$

$$\zeta > 1$$

$y_h(t)$

$$y_0 \left[\frac{s_2 e^{s_1 t}}{s_2 - s_1} + \frac{s_1 e^{s_2 t}}{s_1 - s_2} \right]$$

$$y_0 \left[\frac{-\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t} - \frac{-\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} \right]$$

$$y_h = y_0$$

$$\zeta = 1$$

$$s_1 = s_2 = -\omega_n = s$$

$$C_1 e^{st} + C_2 t e^{st} = C_1 e^{-\omega_n t} + C_2 t e^{-\omega_n t}$$

$$\dot{y}_h(t)$$

$$\frac{d}{dt}[C_1e^{st} + C_2te^{st}] = C_1se^{st} + C_2(e^{st} + ste^{st})$$

$$C_1 = y_0$$

$$sC_1 + C_2 = 0$$

$$C_1 = y_0, \qquad C_2 = y_0$$

$$y_h(t) = C_1 e^{st} + C_2 t e^{st} = y_0 \left[e^{-\omega_n t} + \omega_n t e^{-\omega_n t} \right]$$

$$0 \leq \zeta < 1$$

$$s_{1,2} = (-\zeta \pm j\sqrt{1-\zeta^2})\omega_n = -\zeta\omega_n \pm j\omega_d,$$

$$s_1 - s_2 = 2j\omega_d$$

ω_d

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$y_0 \left[\frac{(-\zeta - j\sqrt{1-\zeta^2})\omega_n}{-2j\omega_n\sqrt{1-\zeta^2}} e^{(-\zeta\omega_n + j\omega_d)t} + \frac{(-\zeta + j\sqrt{1-\zeta^2})\omega_n}{-2j\omega_n\sqrt{1-\zeta^2}} e^{(-\zeta\omega_n - j\omega_d)t} \right]$$

$$y_0 \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \left(\frac{e^{j\phi} e^{j\omega_d t} - e^{-j\phi} e^{-j\omega_d t}}{2j} \right) = y_0 \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right), \qquad \zeta + j\sqrt{1-\zeta^2} = e^{j\phi}, \qquad \zeta - j\sqrt{1-\zeta^2} = e^{-j\phi}$$

$$\zeta = 0$$

$$s_1 = j\omega_n, \qquad s_2 = -j\omega_n$$

$$y_h(t) = y_0 \left[\frac{s_2 e^{s_1 t}}{s_2 - s_1} + \frac{s_1 e^{s_2 t}}{s_1 - s_2} \right] = y_0 \left(\frac{e^{j\omega_n} + e^{-j\omega_n}}{2} \right) = y_0 \cos(\omega_n t)$$

$$\lim_{\zeta \leftarrow 0} \left(y_0 \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi) \right) = y_0 \sin(\omega_n t + \frac{\pi}{2}) = y_0 \cos(\omega_n t)$$

$$y''(t) + 2\zeta\omega_n y'(t) + \omega_n^2 y(t) = x(t)$$

$$x(t) = u(t)$$

$$y(0) = \dot{y}(0) = 0$$

$$y_p(t) = C$$

$$y_p'(t) = y_p''(t) = 0$$

$$y_p(t) = C = 1/\omega_n^2$$

$$y_{ss}(t) = y_p(t) = y(\infty) = \frac{1}{\omega_n^2}$$

$$y(t) = y_h(t) + y_p(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \frac{1}{\omega_n^2}$$

$$y(t)\Big|_{t=0} = C_1 + C_2 + \frac{1}{\omega_n^2} = 0$$

$$\dot{y}(t)\Big|_{t=0} = C_1 s_1 + C_2 s_2 = 0$$

$$C_1 = \frac{s_2}{\omega_n^2(s_1 - s_2)} = \frac{-s_2}{\omega_n^2(s_2 - s_1)}, \qquad C_2 = \frac{s_1}{\omega_n^2(s_2 - s_1)}$$

$$y(t) = \frac{1}{\omega_n^2} \left[1 - \left(\frac{s_2 e^{s_1 t}}{s_2 - s_1} - \frac{s_1 e^{s_2 t}}{s_2 - s_1} \right) \right] = \frac{1}{\omega_n^2} \left(1 - \frac{s_2 e^{s_1 t} - s_1 e^{s_2 t}}{s_2 - s_1} \right)$$

$$\Delta = 4\omega_n^2(\zeta^2 - 1)$$

$$0 < \zeta < 1$$

$$\Delta < 0$$

$$s_{1,2} = \omega_n \left(-\zeta \pm j\sqrt{1-\zeta^2}\right) = -\zeta\omega_n \pm j\omega_d,$$

$$s_2 - s_1 = -2j\omega_d$$

$y(t)$

$$\frac{1}{\omega_n^2}\left[1-\left(\frac{s_2}{s_2-s_1}e^{s_1 t}-\frac{s_1}{s_2-s_1}e^{s_2 t}\right)\right]$$

$$\frac{1}{\omega_n^2}\left[1-\left(\frac{\zeta+j\sqrt{1-\zeta^2}}{2j\sqrt{1-\zeta^2}}e^{(-\zeta\omega_n+j\omega_d)t}-\frac{\zeta-j\sqrt{1-\zeta^2}}{2j\sqrt{1-\zeta^2}}e^{(-\zeta\omega_n-j\omega_d)t}\right)\right]$$

$$\frac{1}{\omega_n^2}\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\frac{\zeta+j\sqrt{1-\zeta^2}}{2j}e^{j\omega_d t}-\frac{\zeta-j\sqrt{1-\zeta^2}}{2j}e^{-j\omega_d t}\right)\right]$$

$$\frac{1}{\omega_n^2}\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\frac{e^{j\phi}e^{j\omega_d t}-e^{-j\phi}e^{-j\omega_d t}}{2j}\right)\right]$$

$$\frac{1}{\omega_n^2}\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)\right]$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right),$$

$$\sin\phi = \sqrt{1-\zeta^2}, \quad \cos\phi = \zeta$$

$$y(t) = \frac{1}{\omega_n^2}\left[1 - \sin(\omega_n t + \pi/2)\right] = \frac{1}{\omega_n^2}\left[1 - \cos(\omega_n t)\right]$$

$$f_n = 1, \omega_n = 2\pi$$

$\zeta < 1$

$$y(0) = 0$$

$$y(t) = v_C(t)$$

$$x(t) = V u(t)$$

$$A \sin \omega_n t + B \cos \omega_n t$$

$$y(t) = V + A\sin(\omega_n t) + B\cos(\omega_n t), \qquad t > 0$$

$$y(0) = V + B = 0$$

$$\dot{y}(0) = A\omega_n = 0$$

$$y(t) = V + A\sin(\omega_n t) + B\cos(\omega_n t) = V[1 - \cos(\omega_n t)]$$

$$y(0) = V$$

$$\dot{y} = 0$$

$$y(0) = V + B = V$$

$$A = B = 0$$

$$y(t) = V + A\sin(\omega_n t) + B\cos(\omega_n t) = V$$

$$x(t) = \begin{cases} 1 & 0 \le t < t_0 \\ 0 & \text{else} \end{cases}$$

$$x(t) = u(t) - u(t - t_0)$$

$$y(t) = [1 - \cos(\omega_n t)]u(t) - [1 - \cos(\omega_n (t - t_0))]u(t - t_0)$$

$t > t_0$

$$y(t) = \cos(\omega_n(t - t_0)) - \cos(\omega_n t)$$

$$t_0 = T = 2\pi/\omega_n$$

$$[1-\cos(\omega_n t)]u(t)-[1-\cos(\omega_n(t-T))]u(t-T)$$

$$\begin{cases} 1-\cos(\omega_n t) & 0<t<T \\ 0 & \text{else} \end{cases}$$

$$t_0 = T/2 = \pi/\omega_n$$

$$[1-\cos(\omega_n t)]u(t)-[1-\cos(\omega_n(t-T/2))]u(t-T/2)$$

$$[1 - \cos(\omega_n t)]u(t) - [1 + \cos(\omega_n t)]u(t - T/2)$$

$$\begin{cases} 1 - \cos(\omega_n t) & 0 < t < T/2 \\ -2\cos(\omega_n t) & t > T/2 \end{cases}$$

$t > T/2$

$h(t)$

$$x(t) = \delta(t) = \lim_{t_0 \to 0} \begin{cases} 1/t_0 & 0 \leq t < t_0 \\ 0 & \text{else} \end{cases}$$

$$t_0 \to 0$$

$$\cos(\omega_n t_0) \approx 1$$

$$\sin(\omega_n t_0) \approx \omega_n t_0$$

$$\cos(\omega_n(t-t_0)) = \cos(\omega_n t)\ \cos(\omega_n t_0) + \sin(\omega_n t)\ \sin(\omega_n t_0) \approx \cos(\omega_n t) + \sin(\omega_n t)\, \omega_n t_0$$

$$y(t) = \cos(\omega_n(t - t_0)) - \cos(\omega_n t)$$

$$y(t) = \omega_n \sin(\omega_n t)$$

$$\delta(t)/\omega_n$$

$$\sin(\omega_n t)$$

$$u(t - T/4)$$

$$[1-\cos(\omega_n(t-T/4))]u(t-T/4) = [1-\cos(\omega_n t-\pi/2)]u(t-T/4) = [1-\sin(\omega_n t)]u(t-T/4)$$

$$y(t) = \sin(\omega_n t)u(t) + [1 - \sin(\omega_n t)]u(t - T/4) = \begin{cases} \sin(\omega_n t) & 0 < t < T/4 \\ 1 & t > T/4 \end{cases}$$

1

$$t = T/4$$

$t > T/4$

$$v = 1$$

$$x(t) = [u(t) - u(t - T/2)]/2$$

$$u(t)/2$$

$$\frac{1}{2}[1 - \cos(\omega_n t)]u(t)$$

$$u(t - T/2)/2$$

$$\frac{1}{2}[1-\cos(\omega_n(t-T/2))]u(t-T/2)=\frac{1}{2}[1-\cos(\omega_n t-\pi)]u(t-T/2)=\frac{1}{2}[1+\cos(\omega_n t)]u(t-T/2)$$

$$y(t) = \begin{cases} [1 - \cos\omega_n t]/2 & 0 < t < T/2 \\ 1 & t > T/2 \end{cases}$$

$$y(t) = 1$$

t_0

$$V > 0$$

$$(1-a)V < 0$$

$$x(t) = \begin{cases} V & (0 < t < t_0/2) \\ (1-a)V & (t_0/2 < t < t_0) \end{cases} = V\left[u(t) - a u(t - t_0/2)\right]$$

$$t_0/2 < t < t_0$$

$$y(t) = V\left[1 - \cos(\omega_n t) - a[1 - \cos(\omega_n(t - t_0/2))]\right]$$

$$x(t) = 1$$

$$t = t_0$$

$$y(t_0) = 1$$

$$\dot{y}(t_0) = 0$$

$$\left.\frac{dy(t)}{dt}\right|_{t=t_0} = V\omega_n \left[\sin(\omega_n t) - a\sin(\omega_n (t - t_0/2))\right]_{t=t_0} = V\omega_n \left[\sin(\omega_n t_0) - a\sin(\omega_n t_0/2)\right] = 0$$

$$\sin(\omega_n t_0) = 2\sin(\omega_n t_0/2)\cos(\omega_n t_0/2)$$

$$2\cos(\omega_n t_0/2) = a$$

$$y(t_0) = V\left[1 - \cos(\omega_n t_0) - 2\cos(\omega_n t_0/2)[1 - \cos(\omega_n (t_0/2))]\right] = 2V(1 - \cos(\omega_n t_0/2) = 4V\sin^2(\omega_n t_0/4) = 1$$

$$2\cos^2(\omega_n t_0/2) = 1 + \cos(\omega_n t_0)$$

$$2\sin^2(\omega_n t_0/4) = 1 - \cos(\omega_n t_0/2)$$

$$V = \frac{1}{4\sin^2(\omega_n t_0/4)}$$

$$x(t+T) = x(t) = \begin{cases} 1 & 0 < t < T_0 \\ 0 & T_0 < t < T \end{cases}$$

T_0

$$T_1 = T - T_0$$

$$y(t)\Big|_{t=0} = V_0$$

$$y(t) = 1 + (V_0 - 1)e^{-t/\tau}, \qquad (0 < t < T_0)$$

$$y(t)\Big|_{t=T_0} = y(T_0) = 1 + (V_0 - 1)e^{-T_0/\tau} = V_1$$

$$y(T_0) = V_1$$

$$y(t) = 0 + (V_1 - 0)e^{-(t-T_0)/\tau} = V_1 e^{-(t-T_0)/\tau}, \qquad (T_0 < t < T)$$

$$y(T) = y(0) = V_1 e^{-(T-T_0)/\tau} = V_1 e^{-T_1/\tau} = V_0$$

V_1

$$\begin{cases} V_0 = V_1 e^{-T_1/\tau} \\ V_1 = 1 + (V_0 - 1) e^{-T_0/\tau} \end{cases}$$

$$V_1 = \frac{1 - e^{-T_0/\tau}}{1 - e^{-T/\tau}}, \qquad V_0 = \frac{e^{-T_1/\tau} - e^{-T/\tau}}{1 - e^{-T/\tau}}$$

$$T_0 \gg \tau$$

$$T_1 \gg \tau$$

$$V_1 = 1$$

$T_0 \ll \tau$

$$T_1 \ll \tau$$

$$e^{-x} \approx 1 - x$$

$$V_1 = \frac{1 - e^{-T_0/\tau}}{1 - e^{-T/\tau}} \approx \frac{T_0}{T}, \qquad V_0 = \frac{e^{-T_1/\tau} - e^{-T/\tau}}{1 - e^{-T/\tau}} \approx \frac{T - T_1}{T} = \frac{T_0}{T}$$

$$T_0 = T_1 = T/2$$

$$1 - e^{-T/\tau} = (1 + e^{-T_0/\tau})(1 - e^{-T_0/\tau})$$

$$V_1 = \frac{1 - e^{-T_0/\tau}}{1 - e^{-T/\tau}} = \frac{1}{1 + e^{-T_0/\tau}}, \qquad V_0 = \frac{e^{-T_1/\tau} - e^{-T/\tau}}{1 - e^{-T/\tau}} = \frac{e^{-T_0/\tau}}{1 + e^{-T_0/\tau}}, \qquad V_0 + V_1 = 1$$

$$T_0 = T_1 = T/2 = 1$$

$\tau = 0.5$

$2T_0$

$$x(t+2T_0) = x(t) = \begin{cases} 1 & 0 < t < T_0 \\ 0 & T_0 < t < 2T_0 \end{cases}$$

$$x(t) = u(t) - u(t - T_0) + u(t - 2T_0) - u(t - 3T_0) + u(t - 4T_0) - \cdots$$

$$y(t) = 1 - \cos(\omega_n t)$$

$$y(t) = 1 - \cos \omega_n t, \qquad (0 < t < T_0)$$

$$y(t) = (1 - \cos\omega_n t) - (1 - \cos\omega_n(t - T_0)) = -\cos\omega_n t + \cos\omega_n(t - T_0), \qquad (T_0 < t < 2T_0)$$

$$(1-\cos\omega_n t)-(1-\cos\omega_n(t-T_0))+(1-\cos\omega_n(t-2T_0))$$

$$1 - \cos\omega_n(t - 2T_0) + \cos\omega_n(t - T_0) - \cos\omega_n t, \qquad (2T_0 < t < 3T_0)$$

$$(1-\cos\omega_n t)-(1-\cos\omega_n(t-T_0))+(1-\cos\omega_n(t-2T_0))-(1-\cos\omega_n t(t-3T_0))$$

$$- \cos \omega_n t + \cos \omega_n (t - T_0) - \cos \omega_n (t - 2T_0) + \cos \omega_n (t - 3T_0), \qquad (3T_0 < t < 4T_0)$$

$$T_0 = T = 2\pi/\omega_n$$

$$\cos \omega_n (t - kT_0) = \cos \omega_n t$$

k

$$y(t) = 1 - \cos \omega_n t, \qquad 2kT_0 < t < (2k+1)T_0$$

$$y(t) = 0, \qquad (2k-1)T_0 < t < 2kT_0$$

$$1 - \cos(\omega_n t)$$

$$T_0 = T/2 = \pi/\omega_n$$

$$T = 2\pi/\omega_n$$

$$y(t) = \cos\omega_n(t - T/2) - \cos\omega_n t = -2\cos\omega_n t, \qquad (T_0 < t < 2T_0)$$

$$y(t) = 1 - \cos\omega_n(t-T) + \cos\omega_n(t-T/2) - \cos\omega_n t = 1 - 3\cos\omega_n t, \qquad (2T_0 < t < 3T_0)$$

$$y(t) = \cos\omega_n(t - T/2) - \cos\omega_n t + \cos\omega_n(t - 3T/2) - \cos\omega_n(t - T) = -4\cos\omega_n t, \qquad (3T_0 < t < 4T_0)$$

$$100 = 10^2$$

$$10\ \mu s = 10^{-5}\ sec$$

$$v_s(t) = 10^{-3}\delta(t)$$

$$i_{L1} + i_C + i_{L2} = 0$$

$$i_{L1} = \frac{1}{L_1} \int_{-\infty}^{t} (v_C - v_S) d\tau, \quad i_{L2} = \frac{1}{L_2} \int_{-\infty}^{t} (v_C - v_R) d\tau, \quad i_C = C \frac{dv_C}{dt}$$

$$\frac{d}{dt}(i_{L1} + i_{L2} + i_{L2}) = \frac{v_C - v_S}{L_1} + \frac{v_C - v_R}{L_2} + \frac{d}{dt}i_C = 0$$

$$i_{L2} + i_R = 0$$

$$i_{L2} = \frac{1}{L_2} \int_{-\infty}^{t} (v_R - v_C) d\tau, \quad i_R = \frac{v_R}{R}$$

$$\frac{d}{dt}(i_{L2} + i_{L2}) = \frac{v_R - v_C}{L_2} + \frac{\dot{v}_R}{R} = 0$$

$$\frac{d}{dt}\begin{bmatrix} v_R \\ v_C \\ i_C \end{bmatrix} = \begin{bmatrix} -R/L_2 & R/L_2 & 0 \\ 0 & 0 & 1/C \\ 1/L_2 & -(1/L_1 + 1/L_2) & 0 \end{bmatrix} \begin{bmatrix} v_R \\ v_C \\ i_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v_s/L_1 \end{bmatrix}$$

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{x}$$

$$\mathbf{y} = e^{\mathbf{A}t}\mathbf{y}_0 + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{x}(\tau)\, d\tau$$

$$\mathbf{y}(0) = \mathbf{0}$$

$$\mathbf{x}(t) = [0,\ 0,\ 10^{-3}\delta(t)/L_1]^T$$

$$v_R = [1\ 0\ 0]\mathbf{y} = e^{\mathbf{A}t} \begin{bmatrix} 0 \\ 0 \\ 10^{-3}/L_1 \end{bmatrix}$$

$$e^{\mathbf{A}t} = \mathbf{V}e^{\mathbf{\Lambda}t}\mathbf{V}^{-1}$$

$$\mathbf{\Lambda} = diag(\lambda_1,\ \lambda_2,\ \lambda_3)$$

$$\mathbf{V} = [\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3]$$

$$\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i, \quad (i = 1, 2, 3)$$

$$\mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$$

$$\begin{cases} \frac{V_C - V_s}{sL} + \frac{V_C}{1/sC} + \frac{V_C - V_R}{sL} = 0 \\ \frac{V_R - V_C}{sL} + \frac{V_R}{R} = 0 \end{cases}$$

V_R

$$V_R = V_C \frac{R}{R + sL}$$

$$[(R + sL)(1 + LCs^2) + sL]V_C(s) = (R + sL)V_s(s)$$

$$V_C(s) = \frac{R + sL}{(R + sL)(1 + LCs^2) + sL} V_s$$

$$V_R(s) = V_C(s)\frac{R}{R+sL} = \frac{R}{(R+sL)(1+LCs^2)+sL}V_s$$

$$v_R(t) = \mathcal{L}^{-1}[\ V_R(s)\ ]$$

$$X(\omega)$$

$$Y(\omega)$$

$\dot{x}$

$\dot{Y}$

$$e^{j\omega}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\dot{Y}}{\dot{X}}$$

$v_0(t)$

$$\omega_c = 1/\tau$$

$$\dot{V}_R = \dot{V}_0 \frac{R}{R + 1/j\omega C} = \dot{V}_0 \frac{j\omega RC}{j\omega RC + 1} = \dot{V}_0 \frac{j\omega\tau}{j\omega\tau + 1}$$

$$\dot{V}_C = \dot{V}_0 \frac{1/j\omega_c}{R + 1/j\omega C} = \dot{V}_0 \frac{1}{1 + j\omega RC} = \dot{V}_0 \frac{1}{1 + j\omega\tau}$$

$$H_R(\omega) = \frac{\dot{V}_R}{\dot{V}_0} = \frac{j\omega\tau}{j\omega\tau + 1}$$

$$H_C(\omega) = \frac{\dot{V}_C}{\dot{V}_0} = \frac{1}{1 + j\omega\tau}$$

H_R

H_C

$$\omega_c = 1/\tau = 1/RC$$

$$1/\sqrt{2}$$

$$|H_R(\omega)| = \begin{cases} 0 & \omega = 0 \\ 1/\sqrt{2} & \omega = \omega_c = 1/\tau \\ 1 & \omega = \infty \end{cases} \qquad |H_C(\omega)| = \begin{cases} 1 & \omega = 0 \\ 1/\sqrt{2} & \omega = \omega_c = 1/\tau \\ 0 & \omega = \infty \end{cases}$$

$$\dot{V}_R = \dot{V}_0 \frac{R}{R + j\omega L} = \dot{V}_0 \frac{1}{1 + j\omega L/R} = \dot{V}_0 \frac{1}{1 + j\omega\tau}$$

$$\dot{V}_L = \dot{V}_0 \frac{j\omega L}{R + j\omega L} = \dot{V}_0 \frac{j\omega}{R/L + j\omega} = \dot{V}_0 \frac{j\omega}{j\omega + 1/\tau} = \dot{V}_0 \frac{j\omega\tau}{j\omega\tau + 1}$$

$$H_R(\omega) = \frac{\dot{V}_R}{\dot{V}_0} = \frac{1}{j\omega\tau + 1}$$

$$H_L(\omega) = \frac{\dot{V}_L}{\dot{V}_0} = \frac{j\omega\tau}{j\omega\tau + 1}$$

$$|H_R(\omega)| = \begin{cases} 0 & \omega = \infty \\ 1/\sqrt{2} & \omega = \omega_c = 1/\tau \\ 1 & \omega = 0 \end{cases}$$

$$\omega_c = 1/\tau = L/R$$

$$|H(\omega_c)| = 1/\sqrt{2}$$

$$20\log_{10}|H(\omega)|$$

$$20\log_{10}(1/\sqrt{2}) = -20\log_{10}\sqrt{2} = -10\log_{10} 2 = -3$$

ω_r

$$|H(\omega)|$$

$$\omega = \omega_r$$

$$\omega = \omega_n$$

$$\omega = \infty$$

$$\dot{V}_R = \dot{V}_0 \, \frac{Z_R}{Z_R + Z_C + Z_L} = \dot{V}_0 \, \frac{R}{R + 1/j\omega C + j\omega L} = \dot{V}_0 \, \frac{R}{R + j(\omega L - 1/\omega C)}$$

$H_R(\omega)$

$$\frac{R}{R + j\omega L + 1/j\omega C} = \frac{R\, j\omega/L}{(R + j\omega L + 1/j\omega C)\, j\omega/L}$$

$$\frac{j\omega R/L}{j\omega R/L + (j\omega)^2 + 1/LC} = \frac{j\omega 2\zeta\omega_n}{(j\omega)^2 + j\omega 2\zeta\omega_n + \omega_n^2}$$

$$\omega_n L - 1/\omega_n C$$

$$H_R(\omega_n) = 1$$

$$\omega_r = \omega_n = 1/\sqrt{LC}$$

$$\omega \neq \omega_n$$

$$|H_R(\omega)| < |H_R(\omega_n)|$$

$$|H_R(0)| = 0$$

$$|H_R(\omega_n)| = |H_R(\omega_r)| = 1$$

$$|H_R(\infty)| = 0$$

$$\triangle\omega = \omega_2 - \omega_1$$

$$\omega_1 < \omega_r$$

$$\omega_2 > \omega_r$$

$$|H(\omega_1)| = |H(\omega_2)| = \frac{1}{\sqrt{2}} = 0.707,$$

$$|H(\omega_1)|^2 = |H(\omega_2)|^2 = \frac{1}{2}$$

$$H_R(\omega) = \frac{R}{R + j\omega C + 1/j\omega C} = \frac{1}{1 + j(\omega L/R - 1/\omega RC)}$$

$$|H_R(\omega)| = 1/\sqrt{2}$$

$$\frac{\omega L}{R} - \frac{1}{\omega C R} = \pm 1$$

$\omega R/L$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = \omega^2 \pm 2\zeta\omega_n\omega - \omega_n^2 = 0$$

$$\omega = \frac{1}{2}\left(\mp 2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 + 4\omega_n^2}\right) = \left(\mp\zeta \pm \sqrt{\zeta^2 + 1}\right)\omega_n$$

ω_1

ω_2

$$\omega_{1,2} = \left(\sqrt{1+\zeta^2} \mp \zeta\right)\omega_n$$

$$\triangle\omega = \omega_2 - \omega_1 = 2\zeta\omega_n = \frac{\omega_n}{Q} = \frac{R}{L}$$

$$y''(t) + 2\zeta\omega_n\, y'(t) + \omega_n^2\, y(t) = y''(t) + \triangle\omega\, y'(t) + \omega_n^2\, y(t) = x(t)$$

$$\triangle\omega = \omega_n/Q$$

$$\Delta\omega$$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} = \omega_n \sqrt{1 + \zeta^2} \ > \ \omega_n$$

$$\omega_r = \omega_n$$

$$\omega_2 - \omega_n > \omega_n - \omega_1$$

$$Q = 1/2\zeta \gg 1$$

$Q > 10$

$\zeta < 0.05$

$$\sqrt{1+\zeta^2} = \sqrt{1+(1/2Q)^2} \approx 1$$

$$\omega_0 \approx \omega_n$$

$$\omega_{1,2} \approx \omega_n \mp \frac{\triangle\omega}{2} = \omega_n \mp \frac{2\zeta\omega_n}{2} = \omega_n(1 \mp \zeta)$$

$$H_{bp}(\omega) = \frac{j\omega 2\zeta\omega_n}{(j\omega)^2 + j\omega 2\zeta\omega_n + \omega_n^2} = \frac{j\omega\triangle\omega}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2} = \frac{j\omega\triangle\omega}{\triangle\omega j\omega + \omega_n^2 - \omega^2}$$

$$H_L(\omega)$$

$$H_{hp}(\omega) = \frac{j\omega L}{R + j\omega L + 1/j\omega C} = \frac{j\omega L\ j\omega/L}{(R + j\omega L + 1/j\omega C)\ j\omega/L}$$

$$\frac{(j\omega)^2}{(j\omega)^2 + j\omega 2\zeta\omega_n + \omega_n^2} = \frac{(j\omega)^2}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2} = \frac{-\omega^2}{\triangle\omega j\omega + \omega_n^2 - \omega^2}$$

$$|H(0)| = 0$$

$$|H(\omega_n)| = \omega_n L/R = Q$$

$$|H(\infty)| = 1$$

$$H_L(\omega)$$

$$\frac{d}{d\omega}|H_{hp}(\omega)|^2$$

$$\frac{d}{d\omega}\left[\frac{\omega^4}{(\omega_n^2-\omega^2)^2+4\zeta^2\omega^2\omega_n^2}\right]$$

$$\frac{4\omega^3[(\omega_n^2-\omega^2)^2+4\zeta^2\omega^2\omega_n^2]-\omega^4[4\omega(\omega^2-\omega_n^2)+8\zeta^2\omega\omega_n^2]}{[(\omega_n^2-\omega^2)^2+4\zeta^2\omega^2\omega_n^2]^2}=0$$

$$4\omega^3[(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega^2\omega_n^2] = 4\omega^5[\omega^2 - \omega_n^2 + 2\zeta^2\omega_n^2]$$

$$\omega_r = \frac{\omega_n}{\sqrt{1-2\zeta^2}}$$

$$|H_{lp}(\omega)|^2$$

$$\frac{\omega^4}{(2\zeta\omega_n\omega)^2 + (\omega_n^2 - \omega^2)^2} = \frac{\omega^4}{\omega^4 - (1 - 2\zeta^2)2\omega_n^2\omega^2 + \omega_n^4}$$

$\zeta < 1/\sqrt{2}$

$$\omega_r > \omega_n$$

$$|H_L(\omega)|$$

$$|H_{hp}(\omega_n)|^2 = \frac{1}{4\zeta^2} \;<\; |H_{hp}(\omega_r)|^2 = \frac{1}{4\zeta^2(1-\zeta^2)} \;>\; |H_{hp}(\infty)| = 1$$

$$\omega_r \approx \omega_n$$

$$\zeta = 0.05$$

$$\omega_r = 1.0025\,\omega_n$$

$$\zeta = 1/\sqrt{2}$$

$$\omega_r = \infty$$

$$|H_{hp}(\omega_n)|^2 = \frac{1}{2} \ < \ |H_{hp}(\omega_r)|^2 \ = \ 1 \ = \ |H_{lp}(\infty)|$$

$$|H_{hp}(\omega_n)| = 1/\sqrt{2}$$

$$20\log|H_{hp}(\omega_n)| = -3\,dB$$

$$\zeta > 1/\sqrt{2}$$

$$H_C(\omega)$$

$$H_{lp}(\omega) = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1/j\omega C \ j\omega/L}{(R + j\omega L + 1/j\omega C) \ j\omega/L}$$

$$\frac{\omega_n^2}{(j\omega)^2 + j\omega 2\zeta\omega_n + \omega_n^2} = \frac{\omega_n^2}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2} = \frac{\omega_n^2}{\triangle\omega j\omega + \omega_n^2 - \omega^2}$$

$$|H(0)| = 1$$

$$|H(\omega_n)| = 1/\omega_n CR = Q$$

$$|H(\infty)| = 0$$

$$\frac{d}{d\omega}\left|(j\omega)^2 + j\omega 2\zeta\omega_n + \omega^2\right|^2$$

$$\frac{d}{d\omega}[(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2]$$

$$-4\omega(\omega_n^2 - \omega^2) + 8\zeta^2\omega_n^2\omega = 0$$

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2} < \omega_n$$

$$|H_C(\omega)|^2 = \frac{\omega_n^4}{(2\zeta\omega_n\omega)^2 + (\omega_n^2 - \omega^2)^2} = \frac{\omega_n^4}{\omega^4 - (1 - 2\zeta^2)2\omega_n^2\omega^2 + \omega_n^4}$$

$$\zeta < 1/\sqrt{2} = 0.707$$

$$|H_C(\omega)|$$

$$|H_{lp}(0)| \ < \ |H_{lp}(\omega_r)|^2 = \frac{1}{4\zeta^2(1-\zeta^2)} \ > \ |H_{lp}(\omega_n)|^2 = \frac{1}{4\zeta^2}$$

$$\omega_r = 0.9975\,\omega_n$$

$$|H_{lp}(0)| \;=\; |H_{lp}(\omega_r)|^2 = 1 > |H_{lp}(\omega_n)|^2 = \frac{1}{2}$$

$$|H_{lp}(\omega_n)| = 1/\sqrt{2}$$

$$20\log|H_{lp}(\omega_n)| = -3\,dB$$

$H_C(\omega)$

$$H_{lp} = \frac{\omega_n^2}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2}$$

$$H_{hp} = \frac{(j\omega)^2}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2}$$

$$H_{bp} = \frac{\triangle\omega j\omega}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2}$$

$$H_{bs} = \frac{\omega_n^2 + (j\omega)^2}{(j\omega)^2 + \triangle\omega j\omega + \omega_n^2}$$

$$Z_{LC} = Z_C || Z_L = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{j\omega L / j\omega C}{j\omega L + 1/j\omega C} = \frac{j\omega L}{1 - \omega^2 LC} = \frac{j\omega L}{1 - (\omega/\omega_n)^2}$$

$$Z_{LC} = \infty$$

$$|Z_{LC}| = \begin{cases} 0 & \omega = 0 \\ \infty & \omega = \omega_n \\ 0 & \omega = \infty \end{cases}$$

$$H_{LC} = \frac{Z_{LC}}{Z_{LC} + R} = \frac{\frac{j\omega L}{1-(\omega/\omega_n)^2}}{\frac{j\omega L}{1-(\omega/\omega_n)^2} + R} = \frac{j\omega L}{j\omega L + R(1-(\omega/\omega_n)^2)} = \frac{1}{1 - j\,\frac{(1-(\omega/\omega_n)^2)}{\omega L/R}}$$

$$H_{LC} = 1$$

$$\omega > \omega_n$$

$$\omega < \omega_n$$

$$Z_{LC}$$

$$|H_{BP}| = \left| \frac{Z_C || Z_L}{R + Z_C || Z_L} \right| = \begin{cases} 0 & \omega \to 0 \\ 1 & \omega = \omega_n \\ 0 & \omega \to \infty \end{cases}$$

$$H_R = \frac{R}{Z_{LC} + R} = \frac{R}{\frac{j\omega L}{1-(\omega/\omega_n)^2} + R} = \frac{1}{j\,\frac{\omega L/R}{1-(\omega/\omega_n)^2} + 1}$$

$$H_R = 0$$

$$|H_{BS}| = \left| \frac{R}{R + Z_C || Z_L} \right| = \begin{cases} 1 & \omega \to 0 \\ 0 & \omega = \omega_n \\ 1 & \omega \to \infty \end{cases}$$

$$\omega_n = 1000$$

$$\Delta\omega = 100$$

$$\Delta\omega = \omega_2 - \omega_1$$

$\omega_1 < \omega_n$

$$\omega_2 > \omega_n$$

$$|H| = 1/\sqrt{2}$$

$$1 - \left(\frac{\omega}{\omega_n}\right)^2 = \pm\frac{\omega L}{R}$$

$$1 - \omega^2 LC = \pm \omega L/R, \qquad \omega^2 \pm \frac{1}{RC}\omega - \frac{1}{LC} = 0$$

$$\omega_{1,2} = \frac{1}{2}\left(\sqrt{\frac{1}{R^2C^2} + \frac{4}{LC}} \pm \frac{1}{RC}\right)$$

$$\Delta\omega = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$\frac{\omega_1 + \omega_2}{2} = \frac{1}{2}\sqrt{\frac{1}{R^2C^2} + \frac{4}{LC}} > \frac{1}{\sqrt{LC}} = \omega_n$$

$$\omega_n = \frac{1}{\sqrt{LC}} = 10^3, \qquad \Delta\omega = \frac{1}{RC} = 10^2$$

$$LC = 10^{-6}, \qquad RC = 10^{-2}$$

$$R = 100$$

$$C = 10^{-4}\ F$$

$$L = 10^{-2}\ H$$

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}}} = 10^3, \qquad \Delta\omega = \frac{1}{RC} = \frac{1}{10^{-2}} = 100$$

$$L = 80\mu H$$

$R = 8\Omega$

$$f = 400\ kHz$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad C = \frac{1}{\omega_n^2 L} = \frac{1}{(2\pi 400 \times 10^3)^2 \times 80 \times 10^{-6}} = 20nF$$

$$Q = \frac{\omega_n L}{R} = \frac{2\pi 400 \times 10^3 \times 80 \times 10^{-6}}{8} = 25.13$$

$$\triangle f = \frac{f_0}{Q} = \frac{400 \times 10^3}{25.13} = 15.9\ kHz$$

$$\triangle\omega = 2\pi\triangle f = \frac{\omega_n}{Q} = \frac{R}{L} = 10^5$$

$$L = 0.3\ mH, \quad R = 16\ \Omega$$

$$e = 2\ \mu V$$

$$f = 640\ kHz$$

$$X_L = \omega L = 2\pi f L = 2 \times 3.14 \times 640 \times 10^3 \times 0.3 \times 10^{-3} = 1206\ \Omega$$

$$Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1206}{16} \approx 75$$

$$\triangle f = \frac{f_0}{Q} = \frac{640}{75} \approx 8.53\ kHz$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 640\ kHz$$

$$C = \frac{1}{(2\pi f_0)^2 L} = 206 \times 10^{-12} F = 206\ pF$$

$$I_{rms} = e/R = 2 \times 10^{-6}/16 = 0.125 \times 10^{-6}\ A = 0.125\ \mu A$$

$$V_R = I_{rms}R = 0.125 \times 16 \times 10^{-6} = 2 \times 10^{-6}\ V = 0.002\ mV$$

$$V_C = V_L = I_{rms} X_L = 0.125 \times 10^{-6} \times 1206 = 150 \times 10^{-6}\ V = 150\ \mu V = 0.150\ mV = Q\, V_R$$

$$Q = 75$$

$$V_R = 2 \times 10^{-6}$$

$$Y(\omega)$$

$$\frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L + j\omega C(R^2 + \omega^2 L^2)}{R^2 + \omega^2 L^2}$$

$$\frac{1}{R^2+\omega^2 L^2}[R-j(\omega L-\omega C(R^2+\omega^2 L^2))]$$

$Re[Y(\omega)]$

$$Im[Y(\omega)]$$

$$|Y(\omega)|$$

$$\frac{d}{d\omega}\ |Y(\omega)| = 0$$

$$Q = \omega_n L / R$$

$$Q > 20$$

$$Im[Y(\omega)] = 0$$

$$\omega_n L = \omega_n C(R^2 + \omega_n^2 L^2), \quad \Longrightarrow \quad \omega_n = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$\frac{1}{LC} > (\frac{R}{L})^2,$$

$$R < \sqrt{\frac{L}{C}}$$

$$R \ll \sqrt{L/C}$$

$$\omega_n = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2} \approx \frac{1}{\sqrt{LC}}$$

ω_p

$$s(t)$$

$$c(t) = \cos(\omega_c t)$$

$$\omega_c = 2\pi f_C$$

$$x(t) = \cos(\omega_c t)\,[1 + k_a s(t)]$$

$$0 < k_a < 1$$

$$s(t) = \cos(\omega_s t)$$

$$x(t) = \cos(\omega_c t)[1 + \cos(\omega_s t)] = \cos(\omega_c t) + \frac{1}{2}[\cos(\omega_c + \omega_s)t + \cos(\omega_c - \omega_s)t]$$

ω_{max}

$$\triangle\omega = (\omega_c + \omega_{max}) - (\omega_c - \omega_{max}) = 2\omega_{max}$$

ω_c

$$x(t) = \cos(\omega_c t + k_p s(t))$$

k_p

$$x(t) = \cos(\omega_c t + k_p \cos(\omega_s t))$$

$$(-k_p,\ k_p)$$

$$\omega(t) = \omega_c + k_f s(t)$$

k_f

$$\omega(t) = d\phi(t)/dt$$

$$\phi(t) = \int_0^t \omega(\tau)d\tau = \int_0^t (\omega_c + k_f s(\tau))d\tau = \omega_c t + k_f \int_0^t s(\tau)d\tau$$

$$x(t) = \cos(\phi(t)) = \cos\left(\omega_c t + k_f \int_0^t s(\tau)d\tau\right)$$

$$x(t) = \cos\left(\omega_c t + k_f \int_0^t \cos(\omega_s \tau) d\tau\right) = \cos\left(\omega_c t + \frac{k_f}{\omega_s} \sin(\omega_s t)\right)$$

$$Z = R + jX = |Z|e^{j\angle Z} = |Z|e^{j\phi}, \qquad \left\{ \begin{array}{l} |Z| = \sqrt{R^2 + X^2} \\ \angle Z = \phi = \tan^{-1}(X/R) > 0 \end{array} \right.$$

$$X = \omega L > 0$$

$$\phi = \angle Z$$

$$v(t) = \sqrt{2}V_{rms}\cos(\omega t), \quad \dot{V} = \sqrt{2}V_{rms}\angle 0$$

$$\dot{I} = \frac{\dot{V}}{Z} = \frac{\sqrt{2}V_{rms}}{|Z|\angle\phi} = \sqrt{2}I_{rms}\angle(-\phi), \quad i(t) = \sqrt{2}I_{rms}\cos(\omega t - \phi)$$

$$I_{rms} = V_{rms}/|Z|$$

$p_{in}(t)$

$$v(t)\ i(t) = \sqrt{2} V_{rms} \cos(\omega t)\ \sqrt{2} I_{rms} \cos(\omega t - \phi)$$

$$2V_{rms}I_{rms}\ \cos(\omega t)\ [\cos(\omega t)\cos\phi + \sin(\omega t)\sin\phi]$$

$$V_{rms} I_{rms} \left[\cos\phi \ \left(2\cos^2(\omega t) \right) + \sin\phi \ \left(2\cos(\omega t)\sin(\omega t) \right) \right]$$

$$V_{rms}I_{rms}\ \ [\cos\phi\ p(t) + \sin\phi\ q(t)]$$

$$S \ \left[\cos\phi \ p(t) + \sin\phi \ q(t)\right] = P \ p(t) + Q \ q(t)$$

$$S = V_{rms} I_{rms}$$

$$P = S \cos \phi$$

$$Q = S \sin \phi$$

$$p(t) = 2\cos^2(\omega t) = 1 + \cos 2\omega t, \qquad q(t) = 2\cos(\omega t)\sin(\omega t) = \sin 2\omega t$$

$$\frac{1}{T}\int_T p(t)dt = \frac{1}{T}\int_T [1 + cos(2\omega t)]dt = 1 \qquad \frac{1}{T}\int_T q(t)dt = \frac{1}{T}\int_T \sin(2\omega t)dt = 0$$

$p(t)$

$q(t)$

$p_{in}(t)$

$$T = 2\pi/\omega$$

$$P_{average}$$

$$\frac{1}{T}\int_T p_{in}(t)dt = \frac{1}{T}\int_T \left[P\ p(t) + Q\ q(t)\right]dt$$

$$S \cos \phi \, \frac{1}{T} \int_T p(t) dt + S \sin \phi \, \frac{1}{T} \int_T q(t) dt$$

$$P\ 1 + Q\ 0 = P$$

$$\dot{V} = V_{rms}\angle 0$$

$$\overline{\dot{I}} = \overline{I_{rms}\angle(-\phi)} = I_{rms}\angle\phi$$

$$\dot{V}\overline{\dot{I}} = V_{rms}\angle 0 \ I_{rms}\angle\phi = V_{rms}I_{rms}\angle\phi = S(\cos\phi + j \ \sin\phi) = P + jQ$$

$$\dot{V} = Z\dot{I} = (R + jX)\dot{I}$$

$$\dot{V}\overline{\dot{I}} = Z\dot{I}\overline{\dot{I}} = (R + jX)\dot{I}\overline{\dot{I}} = RI_{rms}^2 + jXI_{rms}^2$$

$$\dot{V}\,\bar{I}$$

$$\begin{cases} P = I_{rms}^2 R = S\ \cos\phi \\ Q = I_{rms}^2 X = S\ \sin\phi \end{cases}$$

$$P = I_{rms}^2 R$$

$$Q = I_{rms}^2 X$$

$$\lambda \triangleq \cos\phi < 1$$

λ

P

$$C = 1/\omega L$$

$$j\omega L$$

$$1/j\omega C = -j\omega L$$

$$\dot{V}_L = jQ\dot{V}_R, \qquad \dot{V}_C = -jQ\dot{V}_R$$

$$R + j\omega L$$

$$\dot{V}_{RL} = \dot{V}_L + \dot{V}_R = jQ\dot{V} + \dot{V} = (1 + jQ)\dot{V}$$

$$Z_{total}$$

$$Z_{RL}||Z_C = (R + j\omega L) \mid\mid (1/j\omega C)$$

$$\frac{(R + j\omega L)/j\omega C}{(R + j\omega L) + 1/j\omega C} = \frac{R + j\omega L}{j\omega CR - \omega^2 LC + 1}$$

$$\phi = \angle Z_{total}$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right),$$

$$\frac{\omega L}{R} = \frac{\omega RC}{1 - \omega^2 LC}$$

$$L - \omega^2 L^2 C = R^2 C$$

$$C_p = \frac{L}{R^2 + \omega^2 L^2} = \frac{1}{R^2/L + \omega^2 L} < \frac{1}{\omega^2 L} = C_s$$

C_p

$$C_s = 1/\omega^2 L$$

$$\phi = 0$$

$$\lambda = \cos\phi = 1$$

$$\phi = \angle Z' = \tan^{-1}\left(\frac{\omega L}{R}\right) - \tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right) = \cos^{-1}\ 0.95 = 18.2^\circ$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) > \tan^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)$$

$$\frac{\omega L}{R} > \frac{\omega RC}{1 - \omega^2 LC}$$

$$C < \frac{L}{R^2 + \omega^2 L^2} = \frac{1}{R^2/L + \omega^2 L}$$

R_L

$$\dot{V}_0 = V_{rms}\angle 0^\circ$$

$$Z_0 = R_0 + jX_0$$

$$Z_L = R_L + jX_L$$

$$\dot{I} = \frac{\dot{V}_0}{Z_0 + Z_L} = \frac{\dot{V}_0}{(R_0 + R_L) + j(X_0 + X_L)}$$

$$|\dot{I}|^2 = I_{rms}^2 = \frac{|\dot{V}_0|^2}{|Z_0 + Z_L|^2} = \frac{V_{rms}^2}{(R_0 + R_L)^2 + (X_0 + X_L)^2}$$

$$P_L = I_{rms}^2 R_L = \frac{V_{rms}^2 R_L}{(R_0 + R_L)^2 + (X_0 + X_L)^2} = \frac{V_{rms}^2 R_L}{|Z_0 + Z_L|^2}$$

$$Z_0 + Z_L = (R_0 + R_L) + j(X_0 + X_L)$$

$$X_0 + X_L$$

$$X_0 = -X_L$$

$$R_0 + R_L > 0$$

P_L

$$P_L = I_{rms}^2 R_L = \frac{V_{rms}^2 R_L}{(R_0 + R_L)^2}$$

$$R_L = R_0$$

$$X_L = -X_0$$

$$Z_L = R_L + jX_l = R_0 - jX_0 = Z_0^*$$

$$Z_L$$

$$Z_0$$

$$P_L = \frac{V_{rms}^2 R_L}{(R_0 + R_L)^2} = \frac{V_{rms}^2}{4R_0}$$

$\dot{I}_L$

$$V_0 = 200V$$

$$I_0 = j\ 10A$$

$10\ \Omega$

$j\ 10\ \Omega$

$$-j\ 10\ \Omega$$

$$Z_N = Z_T$$

$$Z_{RL} = Z_L + Z_R = 10 + j10$$

$$Z_N = Z_T = Z_{RL}||Z_C = \frac{Z_{RC}Z_C}{Z_{RC}+Z_C} = \frac{-j10(10+j10)}{10+j10-j10} = 10(1-j) = R_0 + jX_0$$

$$R_0 = 10\ \Omega$$

$$X_0 = -10\ \Omega$$

$$\dot{V}_T = \dot{V}_{oc}$$

$$\dot{V}'_{oc} = V_0 \frac{Z_C}{Z_{RL} + Z_C} = 200 \frac{-j10}{(10 + j10) - j10} = -200j$$

$$\dot{V}''_{oc} = I_0 \ Z_{RL} || Z_C = j10 \frac{(10 + j10)(-j10)}{(10 + j10) - j10} = 100 + 100j$$

$$\dot{V}_T = \dot{V}_{oc} = \dot{V}'_{oc} + \dot{V}''_{oc} = -200j + 100 + 100j = 100(1-j) = 100\sqrt{2}\angle(-45^\circ)$$

$$\dot{I}_N = \dot{I}_{sc}$$

$$\dot{I}_N = I_0 + \frac{V_0}{Z_{RL}} = 10j + \frac{200}{10+10j} = 10j + \frac{20(1-j)}{(1+j)(1-j)} = 10$$

$$\dot{I}_N Z_N = 10 \times 10(1-j) = 100(1-j) = 100\sqrt{2}\angle(-45^\circ) = \dot{V}_T$$

$$Z_L = Z_T^* = Z_N^* = R_0 - jX_0 = 10(1 + j)$$

$$P_L = \frac{V_{rms}^2}{4R_0} = \frac{(100\sqrt{2})^2}{4 \times 10} = 500\ W$$

$$\dot{I} = \frac{\dot{V}}{2R_0} = \frac{100\sqrt{2}\angle(-45^\circ)}{20} = 5\sqrt{2}\angle(-45^\circ)$$

R_0

$$R_L \neq R_0$$

$$C_1 = C_2$$

$$-jX$$

jX

$$\omega = 2\pi f$$

$$X = \omega L = 1/\omega C$$

$$LC = 1/\omega^2$$

$$Z_L = -jX + jX || (R_L - jX) = -jX + \frac{jX(R_L - jX)}{R_L} = \frac{X^2}{R_L}$$

$$Z_L = jX + (-jX)||(R_L + jX) = jX + \frac{-jX(R_L + jX)}{R_L} = \frac{X^2}{R_L}$$

$$Z_L = \frac{X^2}{R_L} = R_0,$$

$$X = \sqrt{R_0 R_L}$$

$$P_L = V^2/4R_0$$

$$-j(X + X_L)$$

$$-X_L$$

X_L

$$R_L - jX$$

$$-j(X + X_0)$$

$$-jX_0$$

$$X_0$$

$$R_0 - jX$$

$$Z_L = R_0 + jX_0$$

$$Z_L = -j(X + X_0) + jX||(R_L - jX) = -j(X + X_0) + \frac{jX(R_L - jX)}{R_L} = \frac{X^2}{R_L} - jX_0 = R_0 - jX_0 = Z_0^*$$

$$\omega = 1/\sqrt{LC}$$

$$R_0 = 200\ \Omega$$

$R_L = 8\ \Omega$

$$P_L = I^2 R_L = \frac{V_0^2}{(R_0 + R_L)^2} R_L = V_0^2 \frac{8}{208^2}$$

$$P_0 = I^2 R_0 = \frac{V_0^2}{(R_0 + R_L)^2} R_0 = V_0^2 \frac{200}{208^2}$$

$$P_T = P_L + P_0 = \frac{V_0^2}{R_0||R_L} = V_0^2 \frac{208}{208^2}$$

$$P_L/P_T = 8/208 = 3.8\%$$

$$P_0/P_T = 200/208 = 96.2\%$$

$$Z_C = -jX$$

$$X_L = jX$$

$$X = \sqrt{R_0 R_L} = \sqrt{1600} = 40$$

$$R'_L = 200\ \Omega$$

$$P_L = V_0^2/4R_0 = V^2/800$$

$$P_T = V^2/2R_0 = V^2/400$$

$$P_1 = V_1 I_1 = P_2 = V_2 I_2,$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{n_1}{n_2} = r$$

$$r = n_1/n_2$$

$$V_2 = V_1 / r$$

$$I_2 = rI_1$$

$$R_L = \frac{V_2}{I_2} = \frac{V_1/r}{rI_1} = \frac{1}{r^2}\frac{V_1}{I_1} = \frac{R'_L}{r^2}$$

$$R'_L = V_1/I_1$$

$$R_L' = r^2 R_L$$

$$P_L = P_L'$$

$$R'_L$$

$$R_0 = R'_L = r^2 R_L,$$

$$r = \frac{n_1}{n_2} = \sqrt{\frac{R_0}{R_L}}$$

$$r = \sqrt{R_0/R_L} = \sqrt{200/8} = 5$$

$$R_L = 8$$

$$R_0 = 200$$

$$R_0 = 1000\Omega$$

$$R_L = 10\Omega$$

$$V = 12V$$

$$R'_L = r^2 \ R_L = 10 \ r^2$$

$$R'_L = 10\ r^2$$

$$10\ r^2 = 1000\Omega$$

$$r = n_1/n_2 = 10$$

$$P = \frac{V}{4R'_L} = 36/1000 = 0.036W$$

$$v(t) = V \ \cos(\omega t + \phi), \qquad \dot{V} = V_{rms} \angle \phi$$

$$i(t) = I \ \cos(\omega t + \psi), \qquad \dot{I} = I_{rms} \angle \psi$$

$$\sum_k \dot{V}_k = 0, \qquad \sum_k \dot{I}_k = 0$$

$$Z = \frac{\dot{V}}{\dot{I}} = \frac{V_{rms}}{I_{rms}} \angle(\phi - \psi)$$

$$Z = R + jX,$$

$+j$

$$Y = G + jB,$$

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} + j\ \frac{-X}{R^2 + X^2} = G + jB$$

$$f_0(t)$$

$$f_\infty(t)$$

$$f(t) = f(\infty) + [f(0) - f(\infty)]\, e^{-t/\tau},$$

$$f(t) = f_\infty(t) + [f_0) - f_\infty(0)]\, e^{-t/\tau}$$

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C}) = R + jX$$

$$X = \omega L - 1/\omega C = 0$$

$$Z = R$$

$$Q = \frac{\omega_n L}{R} = \frac{1}{\omega_n R C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\dot{V}_R = \dot{V}, \quad \dot{V}_L = jQ\dot{V}, \quad \dot{V}_C = -jQ\dot{V}$$

$$Y = G + j\omega C + \frac{1}{j\omega L} = G + j\left(\omega C - \frac{1}{\omega L}\right) = G + jB$$

$$B = \omega L - 1/\omega C = 0$$

$$Y = G$$

$$Q = \frac{\omega_n C}{G} = \frac{1}{\omega_n L G} = \frac{1}{G}\sqrt{\frac{C}{L}}$$

$$\dot{I}_R = \dot{I}, \quad \dot{I}_C = jQ\dot{I}, \quad \dot{I}_L = -jQ\dot{I}$$

$$1/\sqrt{2} = 0.707$$

$$\triangle\omega = \omega_2 - \omega_1 = \frac{\omega_n}{Q}$$

$$\dot{I} = \frac{\dot{V}}{Z} = \frac{V_{rms}\angle 0}{|Z|\angle\phi} = I_{rms}\angle{-\phi}$$

$$p(t) = v(t)i(t) = 2V_{rms}I_{rms}\cos(\omega t)\cos(\omega t - \phi)$$

$$P_{average} = \frac{1}{T}\int_T p(t)dt = V_{rms}I_{rms}\cos\phi$$

$$P = S\ \cos\phi = V_{rms} I_{rms} \cos\phi$$

$$Q = S \ \sin \phi = V_{rms} I_{rms} \sin \phi$$

$$0 < \lambda = \cos\phi < 1$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} V_1 \\ -I_2 \end{bmatrix}$$

$$\left[\begin{array}{c} V_1 \\ I_2 \end{array}\right] = \mathbf{H} \left[\begin{array}{c} I_1 \\ V_2 \end{array}\right]$$

$$I = 0$$

$$V = 0$$

$$Z_{12} = Z_{21}$$

$$Y_{12} = Y_{21}$$

$$\mathbf{Z}^{-1} = \mathbf{Y}$$

$$Z_{11} = Z_1 + Z_3, \qquad Z_{22} = Z_2 + Z_3, \qquad Z_{12} = Z_{21} = Z_3$$

$$Z_1 = Z_{11} - Z_{12}, \quad Z_2 = Z_{22} - Z_{21}, \quad Z_3 = Z_{12} = Z_{21}$$

$$Y_{11} = Y_1 + Y_3, \qquad Y_{22} = Y_2 + Y_3, \qquad Y_{12} = Y_{21} = -Y_3$$

$$Y_1 = Y_{11} + Y_{12}, \quad Y_2 = Y_{22} + Y_{21}, \quad Y_3 = -Y_{12} = -Y_{21}$$

$$\frac{V_2}{V_1} = \frac{I_2}{I_1} = \frac{n_1}{n_2}, \qquad \frac{Z_2}{Z_1} = \frac{n_2^2}{n_1^2}$$

$$\begin{cases} x_1(t) = A_1 \cos(\omega t + \phi_1) \\ x_2(t) = A_2 \cos(\omega t + \phi_2) \end{cases}$$

$$x(t) = x_1(t) + x_2(t) = A\cos(\omega t + \phi)$$

A_1

$$A_2$$

ϕ_1

ϕ_2

$$x_1(t) + x_2(t) = A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2)$$

$$A_1 \cos \omega t \cos \phi_1 - A_1 \sin \omega t \sin \phi_1 + A_2 \cos \omega t \cos \phi_2 - A_2 \sin \omega t \sin \phi_2$$

$$(A_1 \cos \phi_1 + A_2 \cos \phi_2) \cos \omega t - (A_1 \sin \phi_1 + A_2 \sin \phi_2) \sin \omega t$$

$$A \cos \phi \cos \omega t - A \sin \phi \sin \omega t$$

$$A_r \cos \omega t - A_j \sin \omega t$$

$$A \cos(\omega t + \phi)$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\left\{ \begin{array}{l} A_r = A\cos\phi = A_1\cos\phi_1 + A_2\cos\phi_2 \\ A_j = A\sin\phi = A_1\sin\phi_1 + A_2\sin\phi_2 \end{array} \right.$$

$$\begin{cases} A = \sqrt{A_r^2 + A_j^2} \\ \phi = \tan^{-1}(A_j/A_r) \end{cases}$$

$$(A_r\, A_j)$$

$(A,\, \phi)$

$$A_r + j\ A_j = Ae^{j\phi}$$

$$A_1 e^{j\phi_1} + A_2 e^{j\psi} = A_1\cos(\phi_1) + A_2\cos(\phi_2) + j[A_1\sin(\phi_1) + A_2\sin(\phi_2)] = A_r + j\ A_j$$

$$x(t) = Re[(A_r + j\ A_j)e^{j\omega t}] = Re\left[\sqrt{A_r^2 + A_j^2}e^{j\tan^{-1}(A_j/A_r)}e^{j\omega t}\right] = A\ \cos(\omega t + \phi)$$

$$z = x + jy$$

x

y

z

$$z = |z|\ e^{j\angle z}$$

$$z = |z| \angle z$$

$$\left| z \right|$$

$\angle z$

(x, y)

$$(|z|, \angle z)$$

$$\begin{cases} |z| = \sqrt{x^2 + y^2} & \text{magnitude} \\ \angle z = tan^{-1}(y/x) & \text{phase angle} \end{cases}$$

$$z = |z|\ e^{j\angle z} = |z|(\cos\angle z + j\ \sin\angle z) = x + jy$$

$$\begin{cases} x = |z| \ \cos \angle z & \text{real part} \\ y = |z| \ \sin \angle z & \text{imaginary part} \end{cases}$$

$$z = x + jy = |z|\, e^{j\angle z}$$

$$w = u + jv = |w|\, e^{j\psi}$$

$$z + w = (x + u) + j(y + v), \quad z - w = (x - u) + j(y - v)$$

$$z\ w = (x + jy)(u + jv) = (xu - yv) + j(xv + yu) = |z|\ |w|\ e^{j(\angle z + \angle w)}$$

$$\frac{z}{w} = \frac{x + jy}{u + jv} = \frac{(x + jy)(u - jv)}{(u + jv)(u - jv)} = \frac{(xu + yv) + j(yu - xv)}{u^2 + v^2} = \frac{|z|}{|w|}\ e^{j(\angle z - \angle w)}$$

$e^{j\alpha}$

$$z\ e^{j\angle z} = |z|\ e^{j(\angle z+\alpha)}$$

α

$\alpha > 0$

$\alpha < 0$

$$\alpha = \omega t$$

$$z = |z| \; e^{j(\angle z + \omega t)}$$

$$e^{j\pi/2} = j$$

$$e^{-j\pi/2} = -j$$

j

$$-j$$

$$z = x + jy = |z|e^{j\angle z}$$

$$z^* = x - jy = |z|e^{-j\angle z}$$

z^*

$$\sqrt{zz^*} = \sqrt{(x+jy)(x-jy)} = \sqrt{x^2+y^2} = |z|$$

$$|z^{-1}| = \frac{1}{|z|} = \frac{1}{\sqrt{x^2 + y^2}}, \quad \angle\left(z^{-1}\right) = \angle\left(\frac{1}{z}\right) = 0 - \angle z = -\angle z$$

$$w = u + jv = |w|\angle w = |w|e^{j\angle w}$$

$$z = x + jy = |z|\angle z = |z|e^{j\angle z}$$

$$\begin{cases} |w| = \sqrt{u^2 + v^2}, & \angle w = \tan^{-1}(v/v) \\ |z| = \sqrt{x^2 + y^2}, & \angle z = \tan^{-1}(y/x) \end{cases}$$

$$wz = (u + jv)(x + jy) = |w|e^{j\angle w}\ |z|e^{j\angle z}$$

$$|wz| = |w||z|, \qquad e^{j\angle w} e^{j\angle z} = e^{j(\angle w + \angle z)},$$

$$\angle(wz) = \angle w + \angle z$$

$$\frac{w}{z} = \frac{u + jv}{x + jy} = \frac{|w|e^{j\angle w}}{|z|e^{j\angle z}}$$

$$\left|\frac{w}{z}\right| = \frac{|w|}{|z|}, \qquad \frac{e^{j\angle w}}{e^{j\angle z}} = e^{j(\angle w - \angle z)},$$

$$\angle\left(\frac{w}{z}\right) = \angle w - \angle z$$

$$\log (ab) = \log a + \log b$$

$$\log\,(a/b) = \log a - \log b$$

$$\log\,(a^n) = n\ \log a$$

$$\log\left(a^{-n}\right) = -n\ \log a$$

v

i

$$R = v/i$$

$$Z = \frac{V}{I} = \frac{v_m e^{j\phi}}{i_m e^{j\psi}} = \frac{v_m}{i_m} e^{j(\phi - \psi)}$$

$$Ie^{j\omega t}$$

$$Ve^{j\omega t} = L\frac{d}{dt}[Ie^{j\omega t}] = j\omega LIe^{j\omega t}$$

$$Z_L = \frac{V}{I} = \frac{j\omega L I}{I} = j\omega L$$

$$Ve^{j\omega t}$$

$$Ie^{j\omega t} = C\frac{d}{dt}[Ve^{j\omega t}] = j\omega CVe^{j\omega t}$$

$$Z_C = \frac{V}{I} = \frac{V}{j\omega CV} = \frac{1}{j\omega C}$$

$$Z_R = \frac{V}{I} = \frac{v_m e^{j\phi}}{i_m e^{j\phi}} = \frac{v_m}{i_m} = R$$

$$P_{in} = 100\ mW$$

$$P_{out} = 10\ W$$

$$P_{out}/P_{in} = 100$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} \frac{10}{0.1} = \log_{10} 100 = 2 \ bel(B)$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} 100 = 2\ B = 20\ dB,$$

$$L_{dB} = 10\log_{10}\frac{P_{out}}{P_{in}} = 20\ dB$$

$$L_B = \log_{10} \frac{P_{out}}{P_{in}} = \log_{10} 1000 = 3\ B = 30\ dB,$$

$$L_{dB} = 10 \log_{10} \frac{P_{out}}{P_{in}} = 30\ dB$$

$$P_{in}$$

$$L_{dB} = 30\ dB$$

$$\frac{P_{out}}{P_{in}} = 10^{L_{dB}/10} = 10^{30/10} = 10^3,$$

$$P_{out} = 10^3\ P_{in} = 1,000\ P_{in}$$

W/m^2

$$10^{-12}\ W/m^2$$

10^{14}

$10^2\ W/m^2$

$$P = V^2/R = I^2R$$

$$E = mv^2/2$$

$$E = kx^2/2$$

$$V_{out}$$

V_{in}

$$L_{dB} = 10 \log_{10} \frac{V_{out}^2}{V_{in}^2} = 20 \log_{10} \frac{V_{out}}{V_{in}} \ dB$$

$$20 \log_{10} \frac{V_{out}}{V_{in}} = 20 \log_{10} \frac{1,000}{10} = 40 \ dB$$

$$20 \log_{10} \frac{V_{out}}{V_{in}} = 20 \log_{10} \frac{10,000}{10} = 60\ dB$$

$$L_{dB} = 60\ dB$$

$$\frac{V_{out}}{V_{in}} = 10^{L_{dB}/20} = 10^{60/20} = 10^3,$$

$$V_{out} = 10^3 V_{in} = 1,000 V_{in}$$

$\zeta < 0.2$

$$|H(j\omega)|$$

$$\omega = \omega_p \approx \omega_n$$

$$\omega_1 < \omega_n < \omega_2$$

$$|H(j\omega_1)|^2 = |H(j\omega_2)|^2 = \frac{1}{2}|H(j\omega_p)|^2$$

$$|H(j\omega_{1,2})| = 0.707\ |H(j\omega_p)|$$

$$20\log_{10}\left(\frac{|H(j\omega_{1,2})|}{|H(j\omega_p)|}\right) = 20\log_{10} 0.707 = -3.01\ dB \approx -3\ dB$$

$$H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)} = |H(j\omega)|\angle H(j\omega)$$

$$\angle H(j\omega)$$

$H(j\omega)$

$$10^{-\infty} = 0$$

$$Lm\ H(j\omega) = 20\ log_{10}|H(j\omega)|\ dB$$

10ω

$$\begin{cases} Lm(H_1H_2) = Lm\ H_1 + Lm\ H_2, & \angle(H_1H_2) = \angle H_1 + \angle H_2 \\ Lm(H_1/H_2) = Lm\ H_1 - Lm\ H_2, & \angle(H_1/H_2) = \angle H_1 - \angle H_2 \\ Lm\ H^n = n\ Lm\ H, & \angle H^n = n\ \angle H \\ Lm(1/H) = -Lm\ H, & \angle(1/H) = -\angle H \end{cases}$$

$(j\omega)$

$$e^{\pm j\omega\tau}$$

$$(1 + j\omega\tau)$$

$$(j\omega)^2 + 2\zeta\omega_n\omega j + \omega_n^2 = (\omega_n^2 - \omega^2) + j\,2\zeta\omega_n$$

$$H(j\omega) = \frac{N(j\omega)}{1 + j\omega\tau}$$

$$H(j\omega) = \frac{N(j\omega)}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{N(j\omega)}{(\omega_n^2 - \omega^2) + j\ (2\zeta\omega_n\omega)}$$

$$\begin{cases} \text{If } k > 0, & k = |k|e^{j0}, \;\; Lm\ k = 20\ \log_{10}|k|, \;\; \angle k = 0 \\ \text{If } k < 0, & k = -|k| = |k|e^{j\pi}, \;\; Lm\ k = 20\ \log_{10}|k|, \;\; \angle k = \pi \end{cases}$$

$$Lm\ e^{j\omega\tau} = 20\ \log_{10}|e^{j\omega\tau}| = 20\ \log_{10} 1 = 0, \quad \angle e^{j\omega\tau} = \pm\omega\tau$$

$$j\omega = \omega\ e^{j\pi/2}$$

$$Lm\ (j\omega) = 20\ log_{10}\omega\ dB, \qquad \angle(j\omega) = \frac{\pi}{2}$$

$$\omega = 1$$

$$Lm\ (1) = 20\ \log_{10} 1 = 0\ dB$$

$$Lm\ (j10\omega) = 20\ \log_{10} 10\omega = 20\ \log_{10} 10 + 20\ \log_{10} \omega = 20 + Lm(j\omega)$$

$j\omega$

$$(j\omega)^{\pm m} = \omega^{\pm m} e^{\pm jm\pi/2}$$

$$Lm(j\omega)^{\pm m} = \pm m \; Lm(j\omega), \qquad \angle(j\omega)^{\pm m} = \pm m\pi/2$$

$\pm 20m/dec$

$m = 2$

$$Lm\ (j\omega)^2 = 40\log_{10}\omega, \qquad \angle\ (j\omega)^2 = \pi$$

$j\omega\tau$

$$\omega\tau = 1$$

$$\omega = 1/\tau$$

$$1/j\omega = (j\omega)^{-1}$$

$$Lm\ (j\omega)^{-1} = -Lm\ (j\omega) = -20\ log_{10}\omega\ dB, \quad \angle\ (j\omega)^{-1} = -\angle(j\omega) = -\frac{\pi}{2}$$

$$1/j\omega$$

$$1 + j\omega\tau$$

$$1 + j\omega\tau = \sqrt{1 + (\omega\tau)^2}\; e^{j\tan^{-1}(\omega\tau)} = \sqrt{1 + (\omega\tau)^2}\;\angle\tan^{-1}(\omega\tau)$$

$$Lm(1 + j\omega\tau) = 20\ \log_{10} \sqrt{1 + (\omega\tau)^2} = 20\ \log_{10}(1 + (\omega\tau)^2)^{1/2} = 10\ \log_{10}(1 + (\omega\tau)^2)$$

$$\angle(1 + j\omega\tau) = \tan^{-1}(\omega\tau)$$

$$Lm(1+j) = 20\ \log_{10} \sqrt{1^2+1^2} = 20\ \log_{10} 0.707 \approx 3.01\ dB, \qquad \angle(1+j) = \frac{\pi}{4}$$

$$\omega\tau \ll 1$$

$$\omega\tau \leq 10$$

$$Lm(1 + j\omega\tau) \approx 10 \ \log_{10}(1) = 0, \qquad \angle(1 + j\omega\tau) \approx \angle(1) = 0$$

$$\omega\tau \gg 1$$

$$\omega\tau \geq 10$$

$$Lm(1 + j\omega\tau) \approx 20\ \log_{10}(\omega\tau), \quad \angle(1 + j\omega\tau) \approx \angle(j\omega\tau) = \frac{\pi}{2}$$

$$Lm(1 + j\omega\tau)$$

$\omega\tau < 1$

$$\omega\tau > 1$$

$$\angle(1 + j\omega\tau)$$

$$\omega\tau < 0.1$$

$$\omega\tau > 10$$

$45°/dec$

$$1/(1 + j\omega\tau) = (1 + j\omega\tau)^{-1}$$

$$Lm\ (1 + j\omega\tau)^{-1} = -Lm(1 + j\omega\tau) = -10\ \log_{10}(1 + (\omega\tau)^2)$$

$$\angle\,(1+j\omega\tau)^{-1} = -\angle(1+j\omega\tau) = -\tan^{-1}(\omega\tau)$$

$$1/(1 + j\omega\tau)$$

$$\omega_1 = 100$$

$$\omega_2 = 1000$$

$$H(j\omega) = \frac{1}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{1}{(\omega_n^2 - \omega^2) + 2\zeta\omega_n j\omega} = \frac{\frac{1}{\omega_n^2}}{1 - (\frac{\omega}{\omega_n})^2 + j\,2\zeta\,\frac{\omega}{\omega_n}}$$

$$\Delta = b^2 - 4ac = (2\zeta\omega_n)^2 - 4\omega_n^2 = 4\omega_n^2(\zeta^2 - 1) \geq 0$$

$$\zeta \geq 1$$

$$p_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1})\omega_n < 0$$

$$H(j\omega) = \frac{1}{(j\omega - p_1)(j\omega - p_2)} = \frac{1/p_1 p_2}{(j\omega/p_1 - 1)(j\omega/p_2 - 1)} = \frac{\tau_1}{1 + j\omega\tau_1} \ \frac{\tau_2}{1 + j\omega\tau_2} = H_1(j\omega)H_2(j\omega)$$

$$\tau_1 = -1/p_1 > 0$$

$\tau_2 = -1/p_2 > 0$

$$Lm\ (H_1 H_2) = Lm\ H_1 + Lm\ H_2, \quad \angle(H_1 H_2) = \angle H_1 + \angle H_2$$

$$\omega_{c1} = 1/\tau_1 = p_1$$

$$\omega_{c1} = 1/\tau_2 = p_2$$

$$20 \log_{10} \omega_n^- 2 = -40 \log_{10} \omega_n$$

$$|H(j\omega)| = [(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2]^{-1/2}$$

$$Lm\ H(j\omega) = 20\log_{10}|H(j\omega)| = -10\ \log_{10}[\ (1-(\frac{\omega}{\omega_n})^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2\ ]$$

$$\angle H(j\omega) = -\tan^{-1}\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}$$

$$\omega/\omega_n = 1$$

$$H(j\omega) = 1/j2\zeta = -j/2\zeta$$

$$Lm\ H(j\omega) = -20\ \log_{10} 2\zeta, \qquad \angle H(j\omega) = -\frac{\pi}{2}$$

$$\omega/\omega_n \ll 1$$

$$\omega \ll \omega_n$$

$$Lm\ H(j\omega) \approx -10\ \log_{10}(1) = 0, \qquad \angle H(j\omega) = 0^\circ$$

$$\omega/\omega_n \gg 1$$

$$Lm\ H(j\omega) \approx -10\ \log_{10}[\ (\frac{\omega}{\omega_n})^4] = -40\ \log_{10} \frac{\omega}{\omega_n}$$

$$\angle H(j\omega) \approx -\tan^{-1}(-2\zeta\omega_n/\omega) \approx -\tan^{-1}(-0) = -\pi = -180^\circ$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 4\zeta^2\frac{\omega^2}{\omega_n^2}}} = \frac{1}{\sqrt{(1-u)^2+4\zeta^2 u}}$$

$$u = (\omega/\omega_n)^2$$

$$u = 1$$

$$|H(j\omega_n)| = \frac{1}{2\zeta} = Q$$

u

$$\frac{d}{du}[u^2 + (4\zeta^2 - 2)u + 1] = 2u + 4\zeta^2 - 2 = 0$$

$$u = \frac{\omega^2}{\omega_n^2} = 1 - 2\zeta^2,$$

$$\omega = \omega_n \sqrt{1 - 2\zeta^2} < \omega_n$$

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}$$

$$|H(j\omega_p)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}} > \frac{1}{2\zeta} = |H(j\omega_n)|$$

$$\zeta^2 > 1/2$$

$\zeta > 0.707$

$$H_C(j\omega) = \frac{V_C}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega\tau + 1}$$

$$H_R(j\omega) = \frac{V_R}{V_{in}} = \frac{Z_R}{Z_R + Z_C} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{j\omega RC + 1} = \frac{j\omega\tau}{j\omega\tau + 1}$$

$$H_R(j\omega)$$

$$H_R(j\omega) = \frac{1}{j\omega\tau + 1} j\omega\tau$$

$$H_C(j\omega)$$

$$Lm\ H_R(j\omega) = 20\log_{10}\left|\frac{1}{j\omega\tau + 1}\right| + 20\log_{10}|j\omega\tau| = Lm\ H_C(j\omega) + 20\log_{10}(\omega\tau)$$

$$\omega = \omega_c = 1/\tau$$

$$\angle H_R(j\omega) = \angle\left(\frac{1}{j\omega\tau + 1}\right) + \angle j\omega\tau = \angle H_C(j\omega) + \frac{\pi}{2}$$

$\tau = 0.01$

$$\omega_C = 100$$

$\omega = \omega_c$

$$|H_R(j\omega)| = |H_C(j\omega)| = 1/\sqrt{2}$$

$$H_C(j\omega)$$

$$\frac{V_C}{V_{in}} = \frac{Z_C}{Z_L + Z_R + Z_C} = \frac{1/j\omega C}{j\omega L + R + 1/j\omega C} = \frac{1}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{1/LC}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = \frac{1}{(1 - \frac{\omega^2}{\omega_n^2}) + j2\zeta\frac{\omega}{\omega_n}}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$|H_C(j\omega)| = \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + 4\zeta^2 \frac{\omega^2}{\omega_n^2}}} = \frac{1}{\sqrt{(1-u)^2 + 4\zeta^2 u}}$$

$$u = \omega / \omega_n = 1$$

$$H_R(j\omega)$$

$$\frac{V_R}{V_{in}} = \frac{Z_R}{Z_L + Z_R + Z_C} = \frac{R}{j\omega L + R + 1/j\omega C} = \frac{j\omega RC}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{j\omega R/L}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{2\zeta\omega_n j\omega}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = H_C(j\omega)\ 2\zeta\omega_n\ j\omega$$

$$Lm\ H_R(j\omega) = Lm\ H_C(j\omega) + Lm\ (2\zeta\omega_n\ j\omega), \quad \angle H_R(j\omega) = \angle H_C(j\omega) + \angle(2\zeta\omega_n\ j\omega)$$

$$20 \log_{10} 2\zeta\omega_n^2$$

$$R + j(\omega L - 1/\omega C)$$

$$j\omega L = 1/j\omega C$$

$$|H_R(j\omega)|$$

$$H_L(j\omega)$$

$$\frac{V_L}{V_{in}} = \frac{Z_L}{Z_L + Z_R + Z_C} = \frac{j\omega L}{j\omega L + R + 1/j\omega C} = \frac{(j\omega)^2 LC}{(j\omega)^2 LC + j\omega RC + 1}$$

$$\frac{(j\omega)^2}{(j\omega)^2 + j\omega R/L + 1/LC} = \frac{(j\omega)^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2} = H_C(j\omega)\ (j\omega_n)^2$$

$$Lm\ H_L(j\omega) = Lm\ H_C(j\omega) + Lm\ (j\omega)^2, \quad \angle H_L(j\omega) = \angle H_C(j\omega) + \angle (j\omega)^2$$

$$20 \log_{10} \omega_n^2$$

$$\omega_n = 100$$

$$20 \log_{10}(2\zeta\omega_n^2) = 20 \log_{10} 1000 = 60$$

$$20 \log_{10}(\omega_n^2) = 20 \log_{10} 10,000 = 80$$

$$H(j\omega) = -\frac{Z_2(j\omega)}{Z_1(j\omega)} = -\frac{R_2||1/j\omega C_2}{R_1 + 1/j\omega C_1} = -\frac{R_2/(1 + j\omega R_2 C_2)}{(1 + j\omega R_1 C_1)/j\omega C_1} = -\frac{j\omega\tau_3}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

$$\tau_1 = R_1 C_1$$

$$\tau_2 = R_2 C_2$$

$$\tau_3 = R_2 C_1$$