# Chapter 18: Principles of Electromechanics – Instructor Notes

The last part of the book presents an introduction to electro-magneto-mechanical systems. **Some of the foundations needed for this material (AC power) were discussed in Chapter 7; the polyphase AC power material in Chapter 7 may be introduced prior to covering Chapter 17, or together with the AC machine material of Chapter 17.**

The emphasis in this chapter (and the next two) is on preparing the student for the use of electro-magneto-mechanical systems as practical actuators for industrial applications. Thus, more emphasis is placed on describing the performance characteristics of linear motion actuators and of rotating machines than on a description of their construction details. The material in Chapters 18-20 has been used by several instructors over the last many years in a second (quarter-length) course in system dynamics (System Dynamics and Electromechanics) designed for mechanical engineering juniors.

Section 18.1 reviews basic laws of electricity and magnetism, which should already be familiar to the student from an earlier Physics course. The box *Focus on Measurements: Linear Variable Differential Transformer* (pp. 915-916) presents an example related to sensors with a discussion of the LVDT as a position transducer. Section 18.2 discusses approximate linear magnetic circuits and the idea of reluctance, and introduces magnetic structures with air gaps and simple electro-magnets. The box *Focus on Methodology: Magnetic Structures and Magnetic Equivalent Circuits* (p. 924) summarizes the analysis methods used in this section. A magnetic reluctance position sensor is presented in *Focus on Measurements: Magnetic Reluctance Position Sensor* (pp. 931-932) and *Focus on Measurements: Voltage Calculation in Magnetic Reluctance Position Sensor* (pp. 932-934). The non-ideal properties of magnetic materials are presented in Section 18.3, where hysteresis, saturation, and eddy currents are discussed qualitatively. Section 18.4 introduces simple models for transformers; more advanced topics are presented in the homework problems.

Section 18.5 is devoted to the analysis of forces and motion in electro-magneto-mechanical structures characterized by linear motion. The boxes *Focus on Methodology: Analysis of Moving-Iron Electromechanical Transducers* (pp. 943-944) and *Focus on Methodology: Analysis of Moving-Coil Electromechanical Transducers* (p. 956) summarize the analysis methods used in this section. The author has found that it is pedagogically advantageous to introduce the *Bli* and *Blu* laws for linear motion devices before covering these concepts for rotating machines: the student can often visualize these ideas more clearly in the context of a loudspeaker or of a vibration shaker. Example 18.9 (pp. 944-945) analyzes the forces in a simple electromagnet, and Examples 18.10 (pp. 945-947) and 18.12 (p. 951) extend this concept to a solenoid and a relay. Example 18.11 (p. 947-948) ties the material presented in this chapter to the transient analysis topics of Chapter 5. Example 18.13 (pp. 956-959) performs a dynamic analysis of a loudspeaker, showing how the frequency response of a loudspeaker can be computed from an electromechanical analysis of its dynamics. Finally, The box *Focus on Measurements: Seismic Transducer* (pp. 959-960) presents the dynamic analysis of an electromechanical seismic transducer.

The homework problems are divided into four sections. The first reviews basic concepts in electricity and magnetism; the second presents basic and more advanced problems related to the concept of magnetic reluctance; the third offers some problems related to transformers. Section 5 contains a variety of applied problems related to electromechanical transducers, and is divided into two separate sections on moving-iron and moving-coil transducers; some of the problems in this last section emphasize dynamic analysis (18.32, 18.41-18.44, and 18.48-53) and are aimed at a somewhat more advanced audience.

## Learning Objectives

1. Review the basic principles of electricity and magnetism. *Section 18.1.*
2. Use the concepts of reluctance and magnetic circuit equivalents to compute magnetic flux and currents in simple magnetic structures. *Section 18.2.*
3. Understand the properties of magnetic materials and their effects on magnetic circuit models. *Section 18.3.*
4. Use magnetic circuit models to analyze transformers. *Section 18.4.*
5. Model and analyze force generation in electro-magneto-mechanical systems. Analyze moving iron transducers (electromagnets, solenoids, relays), and moving-coil transducers (electro-dynamic shakers, loudspeakers, seismic transducers. *Section 18.5.*

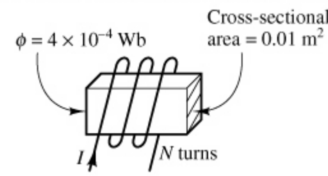
# Section 18.1: Electricity and Magnetism

## Problem 18. 1

### *Solution:*

#### Known quantities:
As shown in Figure P18.1.





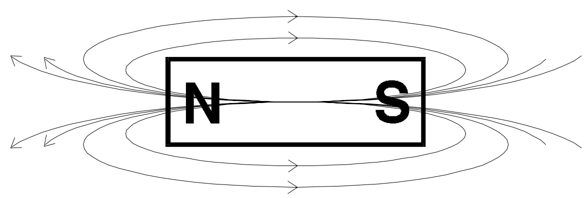

#### Find:
a) The flux density in the core.
b) Sketch the magnetic flux lines and indicate their direction.
c) The north and south poles of the magnet.

#### Assumptions:
None.

#### Analysis:
a)

$$B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{0.01} = 0.04\,T$$

b)
Viewed from the top:




c)
See above.

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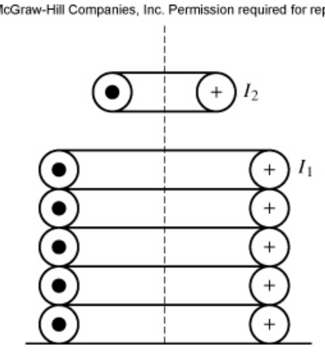
## Problem 18.2

### *Solution:*






#### Known quantities:
As shown in Figure P18.2.

#### Find:
If there is a resultant force on the single coil? If so, in what direction? Why?

#### Assumptions:
None.

#### Analysis:
Yes, the resultant force on the single coil is in the downward direction. If the coils are thought of as electromagnets, there is a north pole from the lower coil attracting a south pole from the upper coil.

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## Problem 18.3

### *Solution:*

#### Known quantities:

A LVDT is connected to a resistive load $R_L$.

#### Find:

The LVDT equations.

#### Assumptions:

None.

#### Analysis:

Assume both secondary windings have resistance $R_S$ and inductance $L_S$, if $M_S$ is the mutual coupling, we have:

$$V_1 = M_1 \frac{di}{dt} - (R_S i_L + L\frac{di_L}{dt}) + M_S \frac{di_L}{dt}$$

$$V_2 = M_2 \frac{di}{dt} + (R_S i_L + L\frac{di_L}{dt}) - M_S \frac{di_L}{dt}$$

$$\therefore V_{out} = V_1 - V_2$$

$$= (M_1 - M_2)\frac{di}{dt} - 2(R_S i_L + L\frac{di_L}{dt}) + 2M_S \frac{di_L}{dt}$$

$$i_L = \frac{V_{out}}{R_L}$$

$$\therefore (2(L - M_S)s + 1 + 2\frac{R_S}{R_L})V_{out} = (M_1 - M_2)sI_L$$

Therefore, the transfer function is:

$$\frac{V_{out}}{I_L} = \frac{(M_1 - M_2)s}{2(L - M_S)s + 1 + 2\frac{R_S}{R_L}}$$

# Problem 18.4

## *Solution:*

### Known quantities:

Equations of "Focus on Measurements: Linear Variable Differential Transformer", and the results of Problem 18.3.

### Find:

The frequency response of the LVDT and the range of frequencies for which the device will have maximum sensitivity for a given excitation.

### Assumptions:

None.

### Analysis:

We have $V_{ex} = L_p \dfrac{di}{dt} + R_p i$

$\therefore V_{ex}(s) = (L_p s + R_p) I(s)$

$V_{out}(s)(2(L-M)s + 1 + 2\dfrac{R_s}{R_L}) = M_s I(s)$

where $M = M_1 - M_2$. Therefore,

$$\frac{V_{out}(s)}{V_{ex}(s)} = \frac{M_s}{(L_p s + R_p)2(L-M)s + 1 + 2\dfrac{R_s}{R_L}}$$

To determine the maximum sensitivity of the output voltage to the excitation we could compute the derivative of $H(s) = \dfrac{V_{out}(s)}{V_{ex}(s)}$ with respect to s, set $\dfrac{\partial H(s)}{\partial s} = 0$, and solve for $s$. By setting $s = j\omega$, this procedure will yield the excitation frequency for which the sensitivity of the output is maximum. It may, however, be more useful to compute the frequency response $H(j\omega)$ numerically, to visualize the range of frequencies over which the sensitivity is acceptable.

# Problem 18.5

## *Solution:*

### Known quantities:

$i = \frac{\lambda}{0.5+\lambda}$

a.

$\lambda = 1V \cdot s$

b.

$R = 1\Omega$

$i(t) = 0.625 + 0.01\sin 400t\ A$

### Find:

a) The energy, coenergy, and incremental inductance.

b) The voltage across the terminals on the inductor.

### Assumptions:

None.

### Analysis:

a)For $\lambda = 1V \cdot s$:

The current is: $i = \frac{\lambda}{0.5+\lambda} = 0.667A$

The energy is :

$$W_m = \int_0^{1.0}\left(\frac{\lambda}{0.5+\lambda}\right)d\lambda = \int_0^{1.0}\left(1-\frac{0.5}{0.5+\lambda}\right)d\lambda = \left(\lambda - 0.5\ln|0.5+\lambda|\right)\Big|_0^{1.0}$$

$$W_m = 1.0 - 0.5\ln|0.5+1.0| - 0 + 0.5\ln|0.5| = 0.4507\ J$$

The coenergy is: $W'_m = i\lambda - W_m = 0.2163\ J$

The incremental inductance is:

$$L_\Delta = \frac{d\lambda}{di}\Big|_{\lambda=0.625}$$

$$\frac{di}{d\lambda} = \frac{0.5+\lambda-\lambda}{(0.5+\lambda)^2} \qquad \frac{d\lambda}{di} = \frac{(0.5+\lambda)^2}{0.5}$$

$$L_\Delta = \frac{d\lambda}{di}\Big|_{\lambda=0.625} = \frac{(0.5+\lambda)^2}{0.5}\Big|_{\lambda=0.625} = 4.5H$$

b)To compute the voltage, we must add the contribution of the voltage across the resistive part of the inductor plus that generated by the inductance:

$$V_L(t) = Ri(t) + L_\Delta\frac{di}{dt}$$

$$\frac{di}{dt} = 4\cos(400t)$$

$$V_L(t) = (1\Omega)(0.625 + 0.01\sin(400t)) + (4.5H)(4\cos(400t))$$

$$V_L(t) = 0.625 + 18\sin(400t + 90°)$$

It is important to observe that this is the inductor terminal voltage only for values of flux linkage in the neighborhood of $1V \cdot s$.

# Problem 18.6

## *Solution:*

### Known quantities:

$i = \frac{\lambda^2}{0.5 + \lambda^2}$

a) $\lambda = 1 V \cdot s$

b) $R = 1\Omega$

$i(t) = 0.625 + 0.01 \sin 400t \ A$

### Find:

a) The energy, coenergy, and incremental inductance.
b) The voltage across the terminals on the inductor.

### Assumptions:

None.

### Analysis:

a) For $\lambda = 1 V \cdot s$: The current is: $i = \frac{\lambda^2}{0.5 + \lambda^2} = 0.667 A$

The energy is : $W_m = \int_0^{1.0} \left( \frac{\lambda^2}{0.5 + \lambda^2} \right) d\lambda = \int_0^{1.0} \left( 1 - \frac{0.5}{0.5 + \lambda^2} \right) d\lambda$

Integral of form: $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c$

$$W_m = \int_0^{1.0} \left( 1 - \frac{0.5}{0.5 + \lambda^2} \right) d\lambda = \left( \lambda - \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left( \frac{\lambda}{\sqrt{0.5}} \right) \right) \Bigg|_0^{1.0}$$

$$W_m = 1.0 - \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left( \frac{1}{\sqrt{0.5}} \right) - 0 + \frac{0.5}{\sqrt{0.5}} \tan^{-1} \left( \frac{0}{\sqrt{0.5}} \right) = 0.3245 \ J$$

The coenergy is: $W_m' = i\lambda - W_m = 0.3425 \ J$

The incremental inductance is: $L_\Delta = \frac{d\lambda}{di} |_{\lambda = 0.625}$

$$\frac{di}{d\lambda} = \frac{2\lambda(0.5 + \lambda^2) - 2\lambda(\lambda^2)}{(0.5 + \lambda^2)^2} \qquad \frac{d\lambda}{di} = \frac{(0.5 + \lambda^2)^2}{\lambda}$$

$$L_\Delta = \frac{d\lambda}{di} |_{\lambda = 0.625} = \frac{(0.5 + \lambda^2)^2}{\lambda} |_{\lambda = 0.625} = 2.25 H$$

b) To compute the voltage, we must add the contribution of the voltage across the resistive part of the inductor plus that generated by the inductance:

$$V_L(t) = Ri(t) + L_\Delta \frac{di}{dt} \qquad \frac{di}{dt} = 4\cos(400t)$$

$$V_L(t) = (1\Omega)(0.625 + 0.01\sin(400t)) + (2.25H)(4\cos(400t))$$

$$V_L(t) = 0.625 + 9\sin(400t + 89.9°)$$

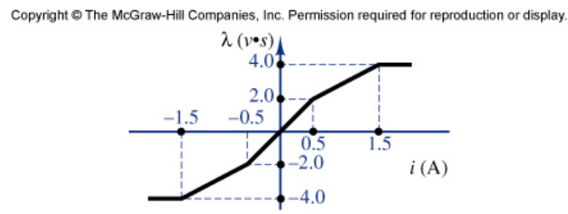
It is important to observe that this is the inductor terminal voltage only for values of flux linkage in the neighborhood of $1 V \cdot s$.

# Problem 18.7

## *Solution:*

### Known quantities:

Characteristic plot shown in Figure P18.7.




### Find:

a) The energy and the incremental inductance for $i = 1.0\,A$.

b) The voltage across the terminals of the inductor when $R = 2\,\Omega, i(t) = 0.5\sin 2\pi t$.

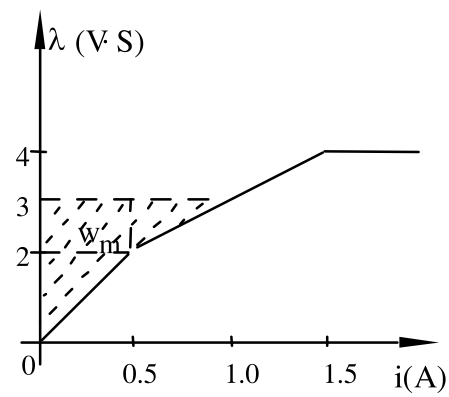
### Assumptions:

None.

### Analysis:

a)

The diagram is shown below:




$W_m$ is the area at the left of the curve as shown.

$$W_m = \frac{1}{2}\times 0.5\times 2 + 0.5\times 1 + \frac{1}{2}\times 0.5\times 1 = 1.25\,J$$

The incremental inductance is:

$$L_\Delta = \frac{\Delta\lambda}{\Delta i}\Big|_{i=1} = \frac{2}{1} = 2\,H$$

b)

For $i = 0.5\sin 2\pi t$ and $R = 2\,\Omega$:

$$V = Ri + \frac{d\lambda}{dt}$$

For $|i| < 0.5, \lambda = 4i$:

$$V_L(t) = \sin(2\pi t) + 4\times 0.5\times 2\pi\cos(2\pi t)$$
$$= \sin(2\pi t) + 4\pi\cos(2\pi t)$$

# Problem 18.8

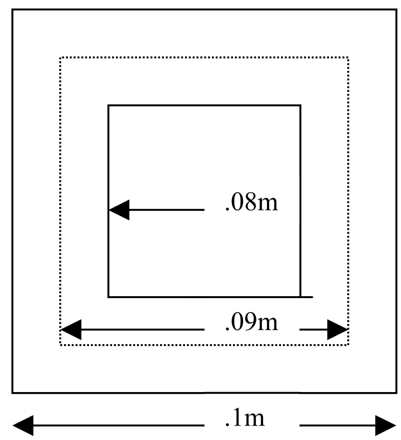
## *Solution:*

### Known quantities:
Structure of Figure 18.12
$A = 0.1 m^2$
$\mu_r = 2000$

### Find:
The reluctance of the structure.

### Assumptions:
Each leg is 0.1 m in length
Mean magnetic path runs through the exact center of the structure

### Analysis:
Calculation of mean path:
Using the assumption that the mean magnetic path runs through the exact center of the structure, and since the structure is square, the mean path is determined using the following figure:




$l_c = 4 \times 0.09 m = 0.36 m$

Calculation of Reluctance:

$$\Re = \frac{l_c}{\mu A} = \frac{l_c}{\mu_r \mu_o A} = \frac{0.36}{2000 \times 4\pi \times 10^{-7} \times 0.1} = 1432 A-turns/Wb$$

# Section 18.2: Magnetic Circuits

# Problem 18.9

## *Solution:*

### Known quantities:

a) $\phi = 4.2\times10^{-4}\ Wb, mmf = 400\ A\cdot t$ .

b) $l = 6\, in.$

### Find:

a) The reluctance of a magnetic circuit.

b) The magnetizing force in SI units.

### Assumptions:

None.

### Analysis:

a)

$$\Re = \frac{F}{\phi} = \frac{400}{4.2\times10^{-4}} = 9.52\times10^{5}\ \frac{A\cdot t}{W_b}$$

b)

$$H = \frac{F}{6\times0.0254\,\frac{m}{in}} = 2625\,\frac{A\cdot t}{m}$$

# Problem 18.10

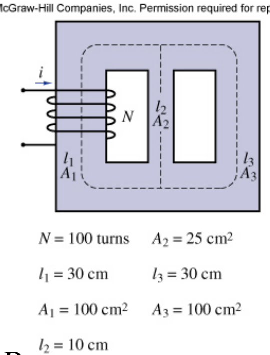
## *Solution:*

### Known quantities:
As shown in Figure P18.10.

### Find:
a) The reluctance values and show the magnetic circuit when $\mu = 3000\mu_0$.
b) The inductance of the device.
c) The new value of inductance when a gap of $0.1mm$ is cut in the arm of length $l_3$.
d) The limiting value of inductance when the gap is increased in size (length).

### Assumptions:
Neglect leakage flux and fringing effects.





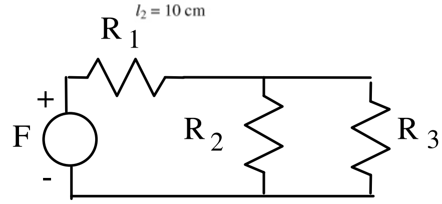

### Analysis:
a)

$$\Re_1 = \frac{l_1}{\mu A_1} = \frac{0.3}{(3000)4\pi \times 10^{-7}(0.01)} = 7.96 \times 10^3\, H^{-1}$$

$$\Re_2 = \frac{l_2}{\mu A_2} = \frac{0.1}{(4\pi \times 10^{-7})3000(25 \times 10^{-4})} = 10.671 \times 10^3\, H^{-1}$$

$$\Re_3 = \Re_1 = 7.96 \times 10^3\, H^{-1}$$

The circuit is shown in the right:




$$\Re_T = \Re_1 + \frac{\Re_2 \Re_3}{\Re_2 + \Re_3} = 12.51 \times 10^3\, H^{-1}$$

b)

$$L = \frac{N^2}{\Re_T} = \frac{100^2}{12.51 \times 10^3} = 0.8\, H$$

c)
We have

$$\Re_g = \frac{0.0001}{(4\pi \times 10^{-7})(100 \times 10^{-4})} = 7.96 \times 10^3\, H^{-1}$$

$\Re_g$ is in series with $\Re_3$ and thus:

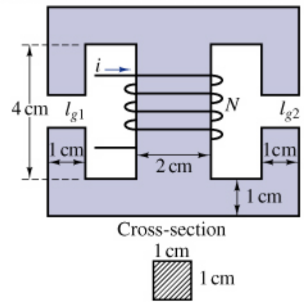
$$\Re_T = \Re_1 + \frac{\Re_2(\Re_3 + \Re_g)}{\Re_2 + \Re_3 + \Re_g} = 14.33 \times 10^3\, H^{-1} \qquad L = \frac{N^2}{\Re_T} = 0.7\, H$$

d)
As the gaps get longer, $\Re_g$ will get larger and as an extreme case the circuit is made of $\Re_1$ and $\Re_2$ in series, therefore:

$$\Re_T = 18.57 \times 10^3\, H^{-1} \qquad L = \frac{N^2}{\Re_T} = 0.54\, H$$

# Problem 18.11






## *Solution:*

### Known quantities:

$N = 1000\,turns, i = 0.2\,A, l_{g1} = 0.02\,cm, l_{g2} = 0.04cm.$

### Find:

The flux and flux density in each of the legs of the magnetic circuit.

### Assumptions:

Neglect fringing at the air gaps and any leakage fields. Assume the reluctance of the magnetic core to be negligible.

### Analysis:

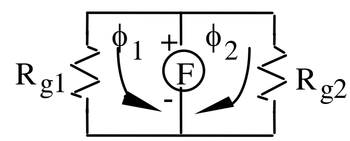
Calculate Reluctance in each air gap

$$\Re_{g1} = \frac{0.0002}{4\pi \times 10^{-7} \times (0.01)^2}$$

$$= 1.59 \times 10^6$$

$$\Re_{g2} = 2\Re_{g1} = 3.18 \times 10^6$$

Assume the reluctance of the material can be neglected when compared to the reluctance of the air gaps; the analogous circuit is shown below:




$$\phi_1 = \frac{Ni}{\Re_{g1}} = 1.26 \times 10^{-4}\ Wb$$

$$B_1 = \frac{\phi_1}{A} = 1.26\,Wb/m^2$$

$$\phi_2 = \frac{1}{2}\phi_1 = 0.63 \times 10^{-4}\ Wb$$

$$B_2 = \frac{1}{2}B_1 = 0.63\,Wb/m^2$$

# Problem 18.12

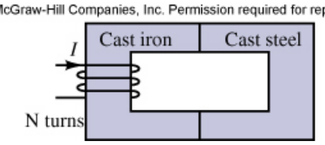
## *Solution:*

### Known quantities:

$\phi = 3\times10^{-4}\,Wb, l_{iron} = l_{steel} = 0.3\,m, A = 5\times10^{-4}\,m^2, N = 100\,turns.$

### Find:

The current needed to establish the flux.




### Assumptions:

None

### Analysis:

Reluctance for each material is calculated as follows:

$$\Re = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o}$$

The total reluctance of the structure is the sum of the reluctances for each material.

From Table 16.1:

Cast Iron: $\mu_r = 5195$

Cast Steel: $\mu_r = 1000$

Cast Iron:

$$\Re_{CI} = \frac{l_{CI}}{\mu_r \mu_o A} = \frac{0.3m}{(5195)(4\pi\times10^{-7})(5\times10^{-4}\,m^2)} = 9.1909\times10^4\,A-turns/Wb$$

Cast Steel:

$$\Re_{CS} = \frac{l_{CS}}{\mu_r \mu_o A} = \frac{0.3m}{(1000)(4\pi\times10^{-7})(5\times10^{-4}\,m^2)} = 4.7746\times10^5\,A-turns/Wb$$

Note: Cast Steel is less permeable than cast iron

Total Reluctance:

$$\Re_T = \Re_{CI} + \Re_{CS} = 9.1909\times10^4\,A-turns/Wb + 4.7746\times10^5\,A-turns/Wb =$$
$$\Re_T = 5.6937\times10^5\,A-turns/Wb$$

From $\phi = \dfrac{Ni}{\Re_T}$ , we can compute the current.

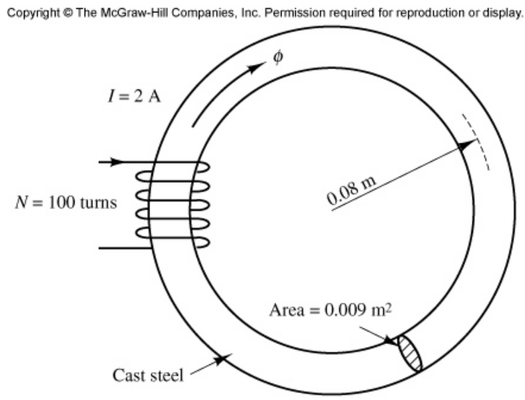
$$i = \frac{\phi R_T}{N} = \frac{(3\times10^{-4}Wb)(5.6937\times10^5\,A-turns/Wb)}{100turns} = 1.71\,A$$

# Problem 18.13

## *Solution:*

### Known quantities:

As shown in Figure P18.13.




### Find:

The magnetic flux $\phi$ .

### Assumptions:

None.

### Analysis:

$$I = 2\ A, r = 0.08\ m, N = 100, A_{cross} = 0.009\ m^2, \mu_r = 1000$$
$$l = 2\pi r = 0.50265\ m, \mu = \mu_r \mu_0$$
$$R = \frac{1}{\mu A_{cross}} = 4.44444 \times 10^4\ \frac{A}{Wb}$$
$$mmf = I \cdot N$$
$$\therefore \phi = \frac{mmf}{R} = 0.0045\ Wb$$

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# Problem 18.14

## *Solution:*

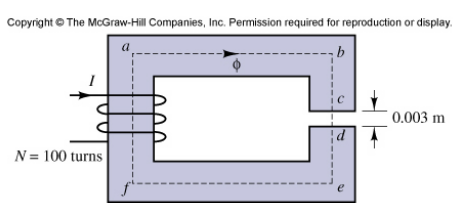
### Known quantities:

$\phi = 2.4 \times 10^{-4}\ Wb, l_{ab} = l_{ef} = 0.05\ m, l_{af} = l_{be} = 0.02\ m, l_{bc} = l_{dc}, A = 2 \times 10^{-4}\ m^2$.

The material is sheet steel.

### Find:

a) The current required to establish the flux.
b) Compare the mmf drop across the air gap to that across the rest of the magnetic circuit and discuss your results using the value of $\mu$ for each material.




### Assumptions:

None.

### Analysis:

a)
Assume the material is cast steel.

$$B = \frac{2.4\times10^{-4}}{2\times10^{-4}} = 12\,T$$

$$H_{CS} = 1400\,\frac{A\cdot t}{m}$$

$$H_{AG} = 9.55\times10^{5}\,\frac{A\cdot t}{m}$$

$$100I = 1400(0.1+0.02+0.017)+(9.55\times10^{5})(0.003)$$

$$\Rightarrow I = 30.6\,A$$

b)

$$F_{CS} = 191.8\,A\cdot t$$

$$F_{AG} = 2865\,A\cdot t$$

$$\mu_{CS} = 0.0009$$

Note: $\mu_{AG} = 0.00000126$

$$\frac{\mu_{CS}}{\mu_{AG}} \approx 700$$

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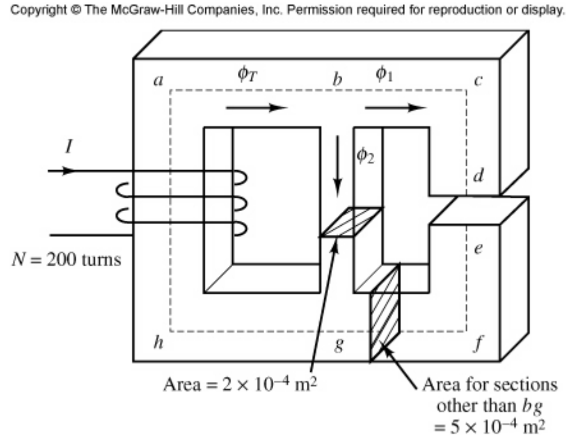
## Problem 18.15

### *Solution:*

#### Known quantities:

Magnet of Figure P18.15, $\phi = 2\times10^{-4}\,Wb, l_{ab} = l_{bg} = l_{gh} = l_{ha} = 0.2\,m, l_{bc} = l_{fg} = 0.1\,m, l_{cd} = l_{ef} = 0.099\,m.$ The material is sheet steel.






#### Find:

The value of $I$ required to establish the flux.

#### Assumptions:

None.

#### Analysis:

$$A_1 = 2\times10^{-4}\,m^2, A_2 = 5\times10^{-4}\,m^2$$

$$\mu_r = 4000$$

$$l_{gap} = l_{ha} - 2l_{cd} = 0.002\,m$$

$$R_{gap} = \frac{l_{gap}}{\mu_0 A_2} = 3.1831\times10^{6}\,\frac{A}{Wb}$$

$$mmf_{gap} = \phi_1 R_{gap} = 636.61977\,A$$

$$R_{ef} = R_{cd} = \frac{l_{cd}}{\mu A_2} = 3.93908\times10^{4}\,\frac{A}{Wb}$$

$$mmf_{cd} = \phi_1 R_{cd} = 7.87817\,A$$

$$R_{fg} = R_{bc} = \frac{l_{bc}}{\mu A_2} = 3.97887\times10^{4}\,\frac{A}{Wb}$$

$$mmf_{bc} = \phi_1 R_{bc} = 7.95775\,A$$

To find the mmf in the rightmost leg of the magnetic circuit,

$$R_{ef} = R_{cd} = \frac{l_{cd}}{\mu A_2} = 3.93908 \times 10^4 \ \frac{A}{Wb}$$

$$R_{ab} = \frac{l_{ab}}{\mu A_2} = 7.95775 \times 10^4 \ \frac{A}{Wb}$$

$$R_{series} = 3R_{ab} = 2.38732 \times 10^5 \ \frac{A}{Wb}$$

$$mmf_{series} = \phi_T R_{series} = 848.69641 \, A$$

$$mmf_{total} = mmf_{series} + mmf_{parallel} = 1.51799 \times 10^3 \ A$$

$$\therefore i = \frac{mmf_{total}}{N} = 7.59 \, A$$

$$mmf_{parallel} = mmf_{gap} + 2mmf_{cd} + 2mmf_{bc} = 66829161 \, A$$

$$R_{bg} = \frac{l_{bg}}{\mu A_1} = 1.98944 \times 10^5 \ \frac{A}{Wb}$$

$$\phi_2 = \frac{mmf_{parallel}}{R_{bg}} = 0.00336 \, Wb$$

$$\phi_T = \phi_1 + \phi_2 = 0.00356 \, Wb$$

---

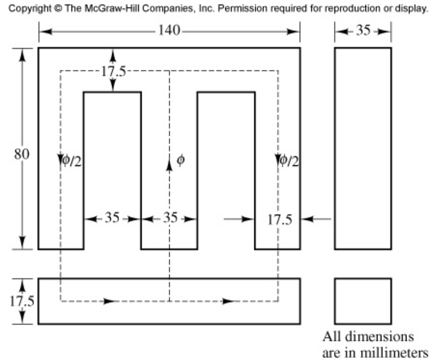
# Problem 18.16

## *Solution:*

### Known quantities:

Actuator of Figure P18.16, $N = 2000 \, turns, g = 10 \, mm, B = 1.2 \, T$ , the air gap is fixed.






### Find:

a) The coil current.
b) The energy stored in the air gaps.
c) The energy stored in the steel.

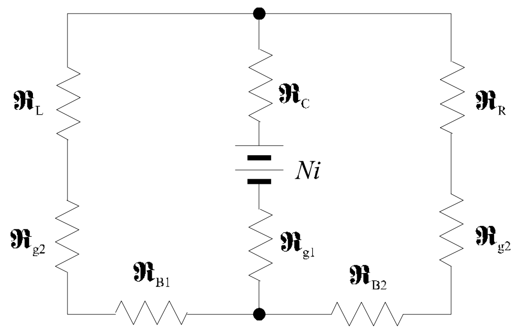
### Assumptions:

None.

### Analysis:

a)
The equivalent circuit is:

where:
From Table 16.1, the relative permeability of sheet steel is 4000.




$$\Re_L = \Re_R = \frac{80\times10^{-3}}{4000(4\pi\times10^{-7})(35\times17.5)\times10^{-6}} = 25{,}984\ A\cdot t/Wb$$

$$\Re_C = \frac{80\times10^{-3}}{4000(4\pi\times10^{-7})(35\times35)\times10^{-6}} = 12{,}992\ A\cdot t/Wb$$

$$\Re_{B1} = \Re_{B2} = \frac{61.25\times10^{-3}}{4000(4\pi\times10^{-7})(35\times17.5)\times10^{-6}} = 19{,}894\ A\cdot t/Wb$$

$$\Re_{g1} = \frac{10\times10^{-3}}{(4\pi\times10^{-7})(35\times35)\times10^{-6}} = 6{,}496{,}120\ A\cdot t/Wb$$

$$\Re_{g2} = \frac{10\times10^{-3}}{(4\pi\times10^{-7})(35\times17.5)\times10^{-6}} = 12{,}955{,}225\ A\cdot t/Wb$$

$$\Re_T = \Re_C + \Re_{g1} + \frac{\Re_R + \Re_{g2} + \Re_{B2}}{2} = 13.01\times10^6\ A\cdot t/Wb$$

$$\phi_T = BA = 1.2(35\times35)\times10^{-6} = 1.47\times10^{-3}\ Wb$$

$$2000I = \Re_T\phi_T \Rightarrow I = 9.56\ A$$

b)

$$H_{1g} = \frac{1.2}{4\pi\times10^{-7}}(10\times10^{-3}) = 9549.3\ A\cdot t$$

$$w_{g1} = \frac{1}{2}(1.47\times10^{-3})(9549.3) = 7.02\ J$$

$$w_{g2} = \frac{1}{2}\left(\frac{1.47\times10^{-3}}{2}\right)(9549.3) = 3.51\ J$$

$$w_g = w_{g1} + 2w_{g2} = 14.04\ j$$

c)

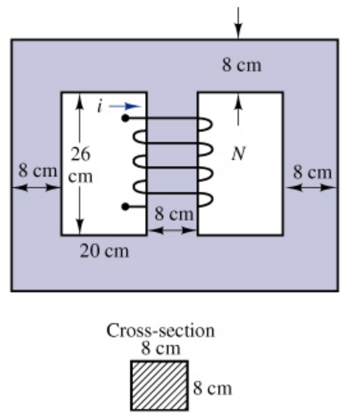
$$w_T = \frac{1}{2}(1.47\times10^{-3})(2000)(9.56) = 14.05\ J$$

$$w_{ST} = w_T - w_g = 0.01\ J$$

## Problem 18.17

### *Solution:*






### Known quantities:

$\mu_r = 2000, N = 100$.

### Find:

a) The current needed to produce $\phi = 0.4 Wb/m^2$ in the center leg.

b) The current needed to produce $\phi = 0.8 Wb/m^2$ in the center leg.

### Assumptions:

None.

### Analysis:

With $l_1 = 34\,cm, l_2 = l_3 = 90\,cm$ and $A = (8\times10^{-2})^2\ cm^2$, we compute:

$$\Re_1 = \frac{0.34}{2000\times 4\pi\times10^{-7}\times(8\times10^{-2})^2} = 2.114\times10^4\ H^{-1}$$

$$\Re_2 = 5.595\times10^4\ H^{-1} = \Re_3$$

$$\Re_T = \Re_1 + \frac{\Re_2\Re_3}{\Re_2+\Re_3} = 4.91\times10^4\ H^{-1}$$

$$\phi_T = 0.4\times(0.08)^2 = 2.56\times10^{-3}\ Wb$$

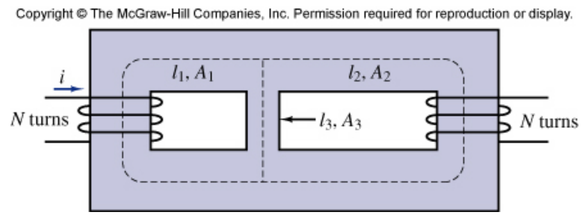
From $\phi_T = BA = \frac{Ni}{\Re_T}$, we have $i = \frac{BAR_T}{N}$.

a) $$i = \frac{2.56\times10^{-3}\times4.91\times10^4}{N} = \frac{125.7}{100}$$
$$= 1.257\ A$$

b)Since the current is directly proportional to B, the current will have to be doubled.

## Section 18.4: Transformers

## Problem 18.18




### *Solution:*

### Known quantities:

$N = 1000\,turns, l_1 = 16\,cm, A_1 = 4\,cm^2, l_2 = 22\,cm, A_2 = 4\,cm^2, l_3 = 5\,cm, l_3 = 2\,cm^2, \mu_r = 1500.$

### Find:

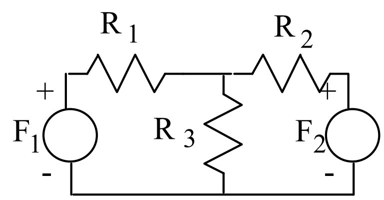
a) Construct the equivalent magnetic circuit and find the reluctance associated with each part of the circuit.
b) The self-inductance and mutual-inductance for the pair of coils.

### Assumptions:

None.

### Analysis:

a)
The analogous circuit is shown below:




The individual reluctances are:

$$\Re_1 = \frac{16\times10^{-2}}{4\pi\times10^{-7}\times1500\times4\times10^{-4}} = 2.12\times10^5\ H^{-1}$$

$$\Re_2 = \frac{22\times10^{-2}}{4\pi\times10^{-7}\times1500\times4\times10^{-4}} = 2.92\times10^5\ H^{-1}$$

$$\Re_3 = \frac{5\times10^{-2}}{4\pi\times10^{-7}\times1500\times2\times10^{-4}} = 1.33\times10^5\ H^{-1}$$

b)
The inductance can be computed as follows:

$$L_{m1} = \frac{N^2}{\Re_1} = 4.72\,H \quad L_{m2} = \frac{N^2}{\Re_2} = 3.43\,H \quad L_{m3} = \frac{N^2}{\Re_3} = 7.54\,H$$

$$let\ L_T = L_{m1} + L_{m2} + L_{m3} = 15.68\,H$$

$$L_m = \frac{L_{m1}L_{m2}}{L_T} = 1.03\,H = L_{12} = L_{21} = M$$

$$L_1 = \frac{L_{m1}L_{m3}}{L_T} = 2.27\,H$$

$$L_2 = \frac{L_{m2}L_{m3}}{L_T} = 1.65\,H$$

$$L_{11} = L_1 + L_m = 3.3\,H$$

$$L_{22} = L_2 + L_m = 2.68\,H$$

# Problem 18.19

## *Solution:*

### Known quantities:

A $300\,\Omega$ resistive load referred to the primary is $7500\,\Omega$.

$r_1 = 20\,\Omega, L_1 = 1.0\,mH, L_m = 25\,mH, r_2 = 20\,\Omega, L_2 = 1.0\,mH.$

### Find:

a) The turns ratio.

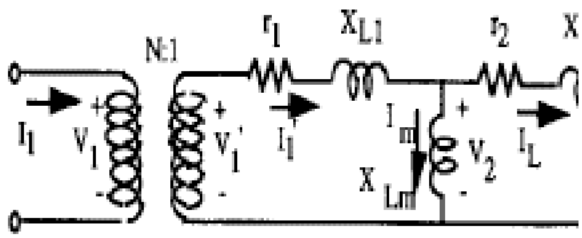
b) The input voltage, current, and power and the efficiency when this transformer is delivering $12\,W$ to the $300\,\Omega$ load at a frequency $f = 10{,}000/2\pi\ Hz$.

### Assumptions:

Core losses are negligible.

### Analysis:

The equivalent circuit is:




**a)**

From $7500 = N^2 \times 300$, we have $N = 5$.

b)

$$X_{L1} = 2\pi f L_1 = 10 = X_{L2}$$
$$X_{Lm} = 250$$

From $I_L^2 R_L = 12\,W, I_L^2 = 0.04$, we have:

$$I_L = 0.2\angle 0^\circ\ A$$
$$V_L = 60\angle 0^\circ\ V$$
$$V_2 = I_L(R_L + r_2 + jX_{L2}) = 64 + j2$$
$$= 64.03\angle 1.79^\circ\ V$$
$$I_m = 0.256\angle -88.21^\circ = 0.008 - j0.2559\ A$$
$$I_1' = I_m + I_L = 0.33\angle -50.9^\circ\ A$$
$$V_1' = I_1'(r_1 + jX_{L1}) + V_2$$
$$= 70.72 - j1.04 = 70.72\angle -0.84^\circ\ V$$
$$V_1 = NV_1' = 353.6\angle -0.84^\circ\ V$$
$$I_1 = \frac{1}{N} I_1' = 0.066\angle -50.9^\circ\ A$$
$$P_{in} = V_1 I_1 \cos\theta = 14.98\,W$$
$$\therefore efficiency = \eta = \frac{P_{out}}{P_{in}} = 80.1\%$$

# Problem 18.20

## *Solution:*

### Known quantities:

A $220/20V$ transformer has $50$ turns on its low-voltage side.

### Find:

a) The number of turns on its high side.
b) The turns ratio $\alpha$ when it is used as a step-down transformer.
c) The turns ratio $\alpha$ when it is used as a step-up transformer.

### Assumptions:

None.

### Analysis:

$$\alpha = 220/20 = 11$$

a) The primary has $N_P = 50 \times 11 = 550\, turns$
b) $\alpha = 11$ is a step-down transformer.
c) $\alpha = 1/11$ is a step-up transformer.

---

# Problem 18.21

## *Solution:*

### Known quantities:

The high-voltage side of the transformer has $750$ turns, and the low-voltage side $50$ turns. The high side is connected to a rated voltage of $120V$. A rated load of $40A$ is connected to the low side.

### Find:

a) The turns ratio.
b) The secondary voltage.
c) The resistance of the load.

### Assumptions:

No internal transformer impedance voltage drops

### Analysis:

a) $\alpha = \dfrac{750}{50} = 15$

b) $V_2 = \dfrac{1}{\alpha} V_1 = \dfrac{120}{15} = 8V$

c) $R_L = \dfrac{8}{40} = 0.2\,\Omega$

---

# Problem 18.22

## *Solution:*

### Known quantities:

A transformer is used to match an $8\,\Omega$ loudspeaker to a $500\,\Omega$ audio line.

### Find:

a) The turns ratio of the transformer.
b) The voltages at the primary and secondary terminals when $10W$ of audio power is delivered to the speaker.

### Assumptions:

The speaker is a resistive load and the transformer is ideal.

### Analysis:

a) From $\alpha^2 R_L = 500$, we have $\alpha = 7.91$

b) From $10 = \frac{V_2^2}{R_L}$, we have $V_2 = 8.94V$

c) $V_1 = \alpha V_2 = 70.7V$

---

# Problem 18.23

## *Solution:*

### Known quantities:

It is a step-down transformer. The high-voltage and low-voltage sides have $800$ turns and $100$ turns respectively. $240VAC$ voltage is applied to the high side. The impedance of the low side is $3\,\Omega$.

### Find:

a) The secondary voltage and current.
b) The primary current.
c) The primary input impedance from the ratio of primary voltage and current.
d) The primary input impedance.

### Assumptions:

None.

### Analysis:

We have $\alpha = \frac{N_1}{N_2} = 8$.

a) $V_2 = \frac{1}{\alpha} V_1 = 30V \quad I_2 = \frac{V_2}{R_L} = 10A$ b) $I_1 = \frac{1}{\alpha} I_2 = 1.25A$

c) $Z_{in} = \frac{240}{1.25} = 192$ d) $Z_{in} = \alpha^2 R_L = 192$

# Problem 18.24

## *Solution:*

### Known quantities:
It is a step-up transformer. All the others are the same as Problem 18.23.

### Find:
The transformer ration of the transformer.

### Assumptions:
None.

### Analysis:
$$\alpha = \frac{100}{800} = \frac{1}{8}$$

---

# Problem 18.25

## *Solution:*

### Known quantities:
It is a $2{,}300/240 - V, 60 - Hz, 4.6 - kVA$ transformer. It has an induced emf of $2.5V/turn$.

### Find:
a) The number of high-side turns $N_h$ and low-side turns $N_l$.
b) The rated current of the high-voltage side $I_h$.
c) The transformer ratio when the device is used as a step-up transformer.

### Assumptions:
It is an ideal transformer.

### Analysis:
a) $N_h = \dfrac{2300}{2.5} = 920\,turns \quad N_l = \dfrac{240}{2.5} = 96\,turns$

b) $I_h = \dfrac{4.6\times 10^3}{2300} = 2\,A$

c) $\alpha = \dfrac{N_l}{N_h} = 0.1044$

---

# Section 18.5: Electromechanical Transducers (a) Moving-Iron Transducers

## Problem 18.26

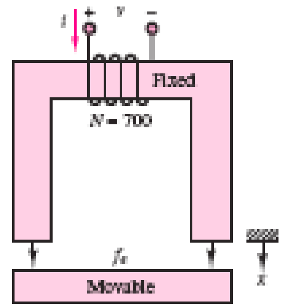



Figure 18.38

### *Solution:*

#### Known quantities:

Electromagnet of Example 18-9 (Figure 18.38) with electromagnet parameters.

#### Find:

The current required to keep the bar in place

#### Assumptions:

Air gap becomes zero and the iron reluctance cannot be neglected

#### Analysis:

a) To compute the current we need to derive an expression for the force in the air gap.
Without neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$\Re(x) = \frac{L}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}$$

where $L$ is the total length of the iron magnetic path (excluding the air gap).
Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\Re(x)} = Ni \frac{\mu_r \mu_0 A}{L + 2\mu_r x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2} \frac{d\Re(x)}{dx} = \frac{1}{2} \frac{(Ni\mu_r \mu_0 A)^2}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2} \frac{2}{\mu_0 A} = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2}$$

As $x$ approaches zero, we can calculate the force to be:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

and the current required to maintain the gravity force is:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2} = mg$$

$$i = \pm\sqrt{\frac{L^2 mg}{(N\mu_r)^2 \mu_0 A}} = \pm \frac{L}{N\mu_r} \sqrt{\frac{mg}{\mu_0 A}} = \pm \frac{l_1 + l_2}{N\mu_r} \sqrt{\frac{mg}{\mu_0 A}} = \pm \frac{0.8 + 0.4}{700 \times 10^4} \sqrt{\frac{10 \times 9.8}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}} = \pm 0.068 \text{ A}$$

# Problem 18.27

## *Solution:*

### Known quantities:

Electromagnet of Example 18-9 (Figure 18.38)

$N = 700 turns;\ f_{restore} = 8{,}900 N;\ A_{gap} = 0.01 m^2;\ L = 1m;\ \mu_r = 1000$

### Find:

a) The current required to keep the bar in place
b) Initial current to lift the magnet if the bar is initially 0.1 m away from the electromagnet

### Assumptions:

a) Air gap becomes zero and the iron reluctance cannot be neglected
b) Neglect the iron reluctance

### Analysis:

b) To compute the current we need to derive an expression for the force in the air gap.
Without neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$\Re(x) = \frac{L}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}$$

where $L$ is the total length of the iron magnetic path (excluding the air gap).
Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\Re(x)} = Ni\frac{\mu_r \mu_0 A}{L + 2\mu_r x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2}\frac{d\Re(x)}{dx} = \frac{1}{2}\frac{(Ni\mu_r\mu_0 A)^2}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2}\frac{2}{\mu_0 A} = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2}$$

As $x$ approaches zero, we can calculate the force to be: $|f(x=0)| = \dfrac{(Ni\mu_r)^2 \mu_0 A}{L^2}$

and the current required to maintain the given force is:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

$$i = \pm\sqrt{\frac{L^2|f(x=0)|}{(N\mu_r)^2 \mu_0 A}} = \pm\frac{L}{N\mu_r}\sqrt{\frac{|f(x=0)|}{\mu_0 A}}$$

Assuming that the total length of the magnetic path is L=1 m and that $\mu_r$ = 1,000, we can calculate a value for the current to be 1.20 A.

c) Since the bar is initially 0.1 m away from the structure, the reluctance of the air dominates the reluctance of the structure. The reluctance is calculated as: $\Re(x) = \dfrac{2x}{\mu_0 A}$

Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\Re(x)} = Ni\frac{\mu_0 A}{2x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2}\frac{d\Re(x)}{dx} = \frac{1}{2}\frac{(Ni\mu_0 A)^2}{4x^2}\frac{2}{\mu_0 A} = \frac{(Ni)^2\mu_0 A}{4x^2}$$

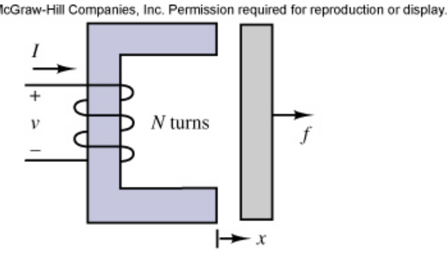
Finally, the current required is: $i = \pm\frac{2x}{N}\sqrt{\frac{|f|}{\mu_0 A}} = \pm\frac{2(0.1m)}{700}\sqrt{\frac{8900N}{4\pi\times10^{-7}\times0.01m^2}} = \pm240.3A$

Note that the holding current from part a is significantly smaller than the current required to lift the bar from the initial distance of 0.1 m.

---

## Problem 18.28






### *Solution:*

### Known quantities:

$\Re(x) = 7\times10^8(0.002+x)\,H^{-1}, N = 980\,t, R = 30\,\Omega, V_a = 120\,V$

### Find:

a) The energy stored in the magnetic field for $x = 0.005\,m$ .
b) The magnetic force for $x = 0.005\,m$ .

### Assumptions:

None.

### Analysis:

a)

We have $L(x) = \frac{N^2}{\Re(x)}$.

$$W_m^{'} = W_m = \frac{L(x)i^2}{2}$$

The current is:

$$I_{DC} = \frac{120}{30} = 4\,A$$

$$\therefore W_m = \frac{980^2\times4^2}{2\times7\times10^8\times0.007} = 1.568\,J$$

b)

$$f = -\frac{i^2}{2}\frac{N^2}{(\Re(x))^2}\frac{d\Re(x)}{dx} = -224\,N$$

The minus sign indicates that the force $f$ is in a direction opposite to that indicated in the figure.

---

# Problem 18.29

## *Solution:*

### Known quantities:
Solenoid of Example 18.10 (Figure 18.40)

### Find:
The best combination of current magnitude and wire diameter to reduce the volume of the solenoid coil.
Will this minimum volume result in the lowest possible resistance?
How does the power dissipation of the coil change with the wire gauge and current value?

### Assumptions:
Use of Copper wire in solenoid

### Analysis:
In order to access the effects of the wire diameter and current magnitude to the volume, resistance, and power dissipated, mathematical expressions need to be developed for each variable.

Volume:

$V_{coil} = l_{coil} A_{coil}$

The length of the coil is given by the circumference:

$l_{coil} = \pi N d_{coil}$

The area of the coil:

$$A_{coil} = \frac{\pi}{4} d_{coil}^2$$

$$V_{coil} = \pi N d_{coil} \frac{\pi}{4} d_{coil}^2 = \frac{\pi^2 N d_{coil}^3}{4}$$

From Example 18.10, the relationship between the current and the number of turns is given as:

$Ni = 56.4 A - turns$

Hence:

$$V_{coil} = \frac{(56.4)\pi^2 d_{coil}^3}{4i}$$

Resistance:

The resistance of the wire is given as:

$$R = \frac{\rho l_{coil}}{A_{coil}}$$

where $\rho$ is the resistivity of the wire, which is assumed to be copper with a $\rho = 1.725 \times 10^{-8}\ \Omega / m$

Using the previous derivations:

$$R = \frac{\rho \pi N d_{coil}}{\frac{\pi}{4} d_{coil}^2} = \frac{4\rho N}{d_{coil}} = \frac{(4)(56.4)\rho}{i d_{coil}}$$

Power:

The dissipated power is given by:

$$P = i^2 R = \frac{(4)(56.4) i \rho}{d_{coil}}$$

Using a chart for AWG wire gauge and current carrying capacity, a table can be developed relating the wire gauge and current carrying capacity to the volume, resistance, and power dissipated.

| **AWG Gauge** | **Wire Diameter [in]** | **Wire Diameter [m]** | **Current Capacity [A]** | **Required Number of Turns [turns]** | **Coil Volume [m$^3$]** | **Resistance [Ω]** | **Power Dissipated [W]** |
|---|---|---|---|---|---|---|---|
| 10 | 1.019E-01 | 2.588E-02 | 30.0 | 2 | 8.041E-05 | 5.012E-06 | 4.511E-03 |
| 12 | 8.081E-02 | 2.053E-02 | 20.0 | 3 | 6.017E-05 | 9.480E-06 | 3.792E-03 |
| 14 | 6.408E-02 | 1.628E-02 | 15.0 | 4 | 4.001E-05 | 1.594E-05 | 3.586E-03 |
| 16 | 5.082E-02 | 1.291E-02 | 10.0 | 6 | 2.993E-05 | 3.015E-05 | 3.015E-03 |
| 18 | 4.030E-02 | 1.024E-02 | 5.0 | 11 | 2.986E-05 | 7.603E-05 | 1.901E-03 |
| 20 | 3.196E-02 | 8.118E-03 | 3.3 | 17 | 2.256E-05 | 1.453E-04 | 1.582E-03 |
| 22 | 2.530E-02 | 6.426E-03 | 2.5 | 23 | 1.477E-05 | 2.422E-04 | 1.514E-03 |
| 24 | 2.010E-02 | 5.105E-03 | 1.25 | 45 | 1.481E-05 | 6.098E-04 | 9.528E-04 |
| 26 | 1.590E-02 | 4.039E-03 | 0.83 | 68 | 1.111E-05 | 1.168E-03 | 7.950E-04 |
| 28 | 1.260E-02 | 3.200E-03 | 0.63 | 90 | 7.299E-06 | 1.946E-03 | 7.600E-04 |
| 30 | 1.000E-02 | 2.540E-03 | 0.31 | 180 | 7.297E-06 | 4.903E-03 | 4.788E-04 |
| 32 | 8.000E-03 | 2.032E-03 | 0.21 | 273 | 5.661E-06 | 9.286E-03 | 3.950E-04 |
| 34 | 6.300E-03 | 1.600E-03 | 0.16 | 361 | 3.649E-06 | 1.556E-02 | 3.800E-04 |
| 36 | 5.000E-03 | 1.270E-03 | 7.81E-02 | 722 | 3.649E-06 | 3.922E-02 | 2.394E-04 |
| 38 | 4.000E-03 | 1.016E-03 | 5.16E-02 | 1094 | 2.831E-06 | 7.428E-02 | 1.975E-04 |
| 40 | 3.100E-03 | 7.874E-04 | 3.91E-02 | 1444 | 1.739E-06 | 1.265E-01 | 1.931E-04 |
| 42 | 2.500E-03 | 6.350E-04 | 1.95E-02 | 2888 | 1.824E-06 | 3.138E-01 | 1.197E-04 |
| 44 | 2.000E-03 | 5.080E-04 | 1.29E-02 | 4375 | 1.415E-06 | 5.943E-01 | 9.875E-05 |
| 46 | 1.600E-03 | 4.064E-04 | 9.77E-03 | 5775 | 9.565E-07 | 9.806E-01 | 9.351E-05 |
| 48 | 1.200E-03 | 3.048E-04 | 4.88E-03 | 11551 | 8.070E-07 | 2.615E+00 | 6.234E-05 |
| 50 | 1.000E-03 | 2.540E-04 | 3.22E-03 | 17501 | 7.076E-07 | 4.754E+00 | 4.938E-05 |

As the wire gauge increases, the wire diameter and current carrying capacity both decrease, and in turn the number of turns required increases, and the coil volume decreases. However, the resistance also increases with an increase in wire gauge. Hence, the minimum volume will not result in the lowest possible resistance. Finally, the power dissipation decreases with the increase of wire gauge due to decrease in current capacity.

# Problem 18.30

## *Solution:*

### Known quantities:

Solenoid of Example 18.10 (Figure 18.40)
$a = 0.01m$, $l_{gap} = 0.001m$, $K = 10N/m$

### Find:

$f, mmf$ using equation 18.46 and equation 18.30

### Assumptions:

The reluctance of the iron is negligible, Neglect fringing
At $x = 0$, the plunger is in the gap by an infinitesimal displacement $\varepsilon$

### Analysis:

From Example 18.10:

$$\Re(x) = \frac{2l_{gap}}{\mu_o ax}$$

Compute inductance if the magnetic circuit as a function of reluctance (equation 18.30):

$$L = \frac{N^2}{\Re(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

Compute stored magnetic energy:

$$W_m = \frac{1}{2}Li^2 = \frac{1}{2}\frac{N^2 i^2 \mu_o ax}{2l_{gap}}$$

Finally, use equation 16.46 to write the expression for the magnetic force:

$$f_e = -\frac{dW_m}{dx} = -\frac{N^2 i^2 \mu_o a}{4l_{gap}}$$

This matches the derivation from example 18.10 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force.
The calculation for the required $mmf$ is identical to example 18.10:

$$f_{gap} = kx = ka = (10N/m)\times(0.01m) = 0.1N$$

---

# Problem 18.31

## *Solution:*

### Known quantities:

Solenoid of Example 18.11 (Figure 18.40)
$a = 0.01m$, $l_{gap} = 0.001m$, $K = 10N/m$, $N = 1000 turns$, $V = 12V$, $R_{coil} = 5\Omega$

### Find:

Current and magnetic force response as a function of time using equation 18.46 and equation 18.30 in the derivation

### Assumptions:

The reluctance of the iron is negligible, Neglect fringing
The inductance of the solenoid is approximately constant and is equal to the midrange value (plunger displacement equal to $a/2$).

### Analysis:

From Example 18.11:

$$\Re_{gap}(x) = \frac{2l_{gap}}{\mu_o ax}$$

Compute inductance if the magnetic circuit as a function of reluctance (equation 18.30):

$$L = \frac{N^2}{\Re_{gap}(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

Compute stored magnetic energy:

$$W_m = \frac{1}{2} L i(t)^2 = \frac{1}{2} \frac{N^2 i(t)^2 \mu_o ax}{2l_{gap}}$$

Finally, use equation 16.46 to write the expression for the magnetic force:

$$f_{gap} = -\frac{dW_m}{dx} = -\frac{N^2 i(t)^2 \mu_o a}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 0.01}{4 \times 0.001} i(t)^2 = \pi i(t)^2$$

With the assumption that the inductance is constant with $x = a/2$:

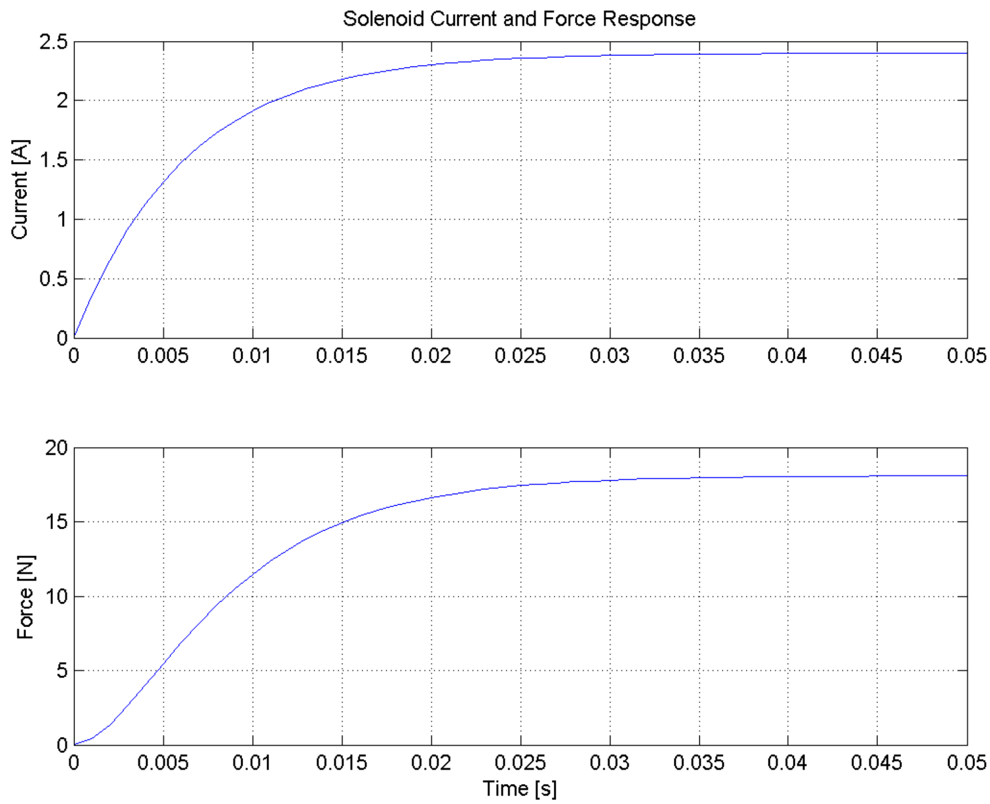
$$L \approx \frac{N^2}{\Re_{gap}(x)} = \frac{N^2 \mu_o a^2}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times (0.01)^2}{4 \times 0.001} = 31.4mH$$

From Example 18.11:

$$i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right) = \frac{12}{5}\left(1 - e^{-t/6.3\times 10^{-3}}\right) A$$

$$f_{gap} = \pi \left[\frac{12}{5}\left(1 - e^{-t/6.3\times 10^{-3}}\right)\right]^2$$

This matches the derivation from example 18.11 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force. The response curves are shown below:


Solenoid Current and Force Response
Current [A]
2.5
2
1.5
1
0.5
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Force [N]
20
15
10
5
0
Time [s]

# Problem 18.32

## *Solution:*

### Known quantities:

Solenoid of Example 18.11 (Figure 18.40)
$a = 0.01m$, $l_{gap} = 0.001m$, $K = 10N/m$, $N = 1000turns$, $V = 12V$, $R_{coil} = 5\Omega$
$m = 0.5kg$

### Find:

Generate a simulation program that accounts for the fact that the solenoid inductance is not constant, but is a function of plunger position
Compare graphically the current and force step responses of this system to the step response obtained in Example 18.11

### Assumptions:

The reluctance of the iron is negligible, Neglect fringing
Neglect damping on the plunger

### Analysis:

From Example 18.11:

$$\Re_{gap}(x) = \frac{2l_{gap}}{\mu_o ax}$$

The inductance is now a function of plunger position

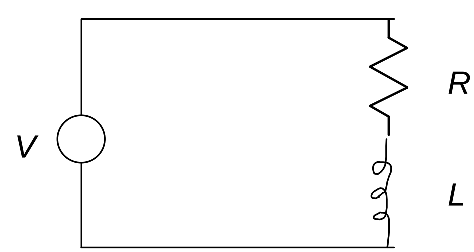
$$L = \frac{N^2}{\Re_{gap}(x)} = \frac{N^2 \mu_o ax}{2l_{gap}}$$

The differential equation for the current using the equivalent circuit shown below:

$$V(t) = Ri(t) + L\frac{di}{dt}$$

To find the equation of the force, compute stored magnetic energy as a function of current:

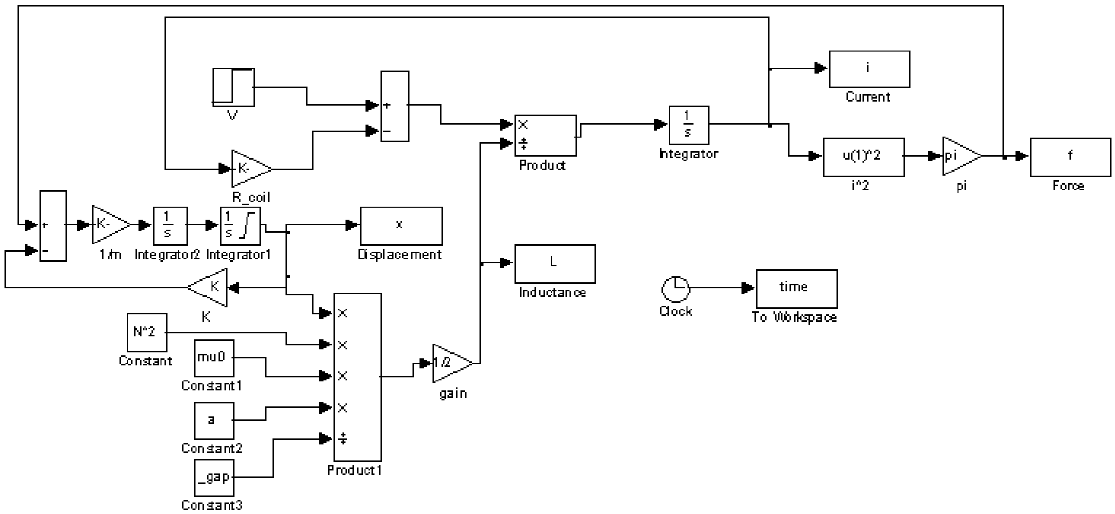
$$W_m = \frac{1}{2}Li(t)^2 = \frac{1}{2}\frac{N^2 i(t)^2 \mu_o ax}{2l_{gap}}$$



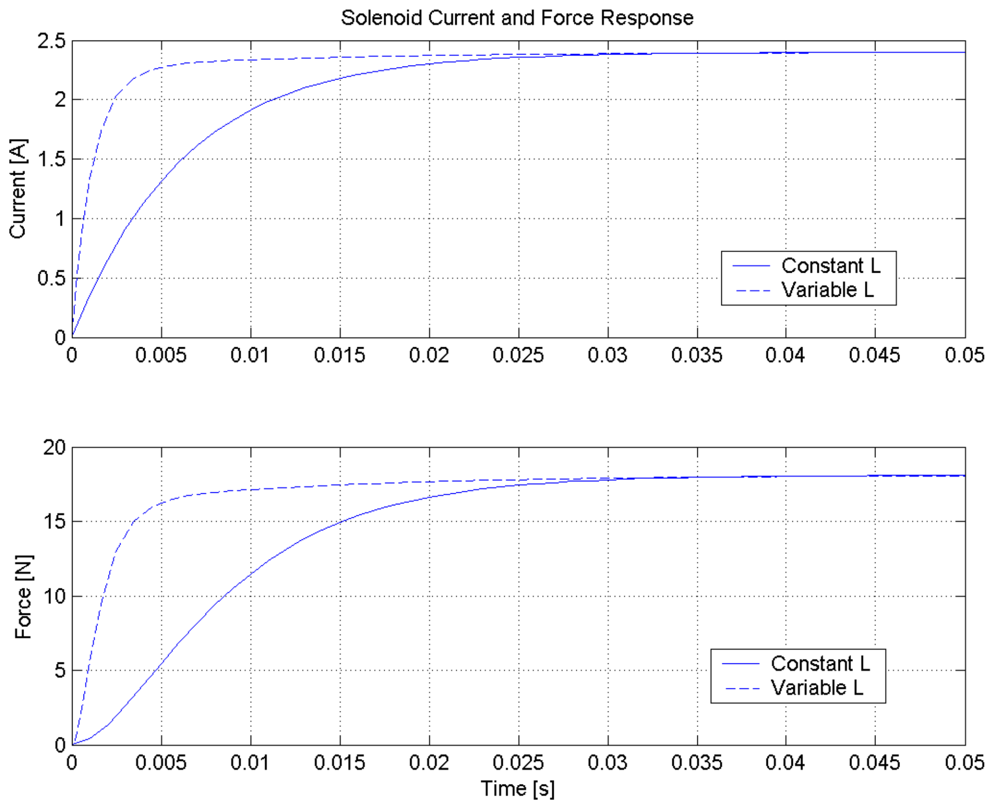

Finally, use equation 16.46 to write the expression for the magnetic force:

$$f_{gap} = -\frac{dW_m}{dx} = -\frac{N^2 i(t)^2 \mu_o a}{4l_{gap}} = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 0.01}{4 \times 0.001} i(t)^2 = \pi i(t)^2$$

This matches the derivation from example 18.11 using the relationship between the magnetic flux, the reluctance of the structure, and the magnetic force.
Simulink Block Diagram:




Comparison of Step Responses for Constant Inductance and Variable Inductance Systems:




Note the quicker response of the variable inductance system due to the smaller inductance initially. The larger constant inductance results in a delayed response.

# Problem 18.33

## *Solution:*

### Known quantities:

Relay of Example 18.12 (Figure 18.46)

$N = 10{,}000 turns;\ f_{restore} = 5N;\ A_{gap} = (0.01m)^2;\ x = 0.05m \mu_r = 1000$

### Find:

Required holding current to keep relay closed

### Assumptions:

Air gap becomes zero and the iron reluctance cannot be neglected

### Analysis:

To compute the current we need to derive an expression for the force in the air gap.
Without neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$\Re(x) = \frac{L}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}$$

where $L$ is the total length of the iron magnetic path (excluding the air gap).
Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\Re(x)} = Ni \frac{\mu_r \mu_0 A}{L + 2\mu_r x}$$

Then, the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2} \frac{d\Re(x)}{dx} = \frac{1}{2} \frac{(Ni\mu_r \mu_0 A)^2}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2} \frac{2}{\mu_0 A} = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2 + 4L\mu_r x + 4\mu_r^2 x^2}$$

As $x$ approaches zero, we can calculate the force to be:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

and the current required to maintain the given force is:

$$|f(x=0)| = \frac{(Ni\mu_r)^2 \mu_0 A}{L^2}$$

$$i = \pm\sqrt{\frac{L^2 |f(x=0)|}{(N\mu_r)^2 \mu_0 A}} = \pm \frac{L}{N\mu_r} \sqrt{\frac{|f(x=0)|}{\mu_0 A}}$$

The total length of the magnetic path:

$L = 0.05m + 0.05m + 0.10m + 0.10 = 0.30m$

The current:

$$i = \pm \frac{(0.30)}{(10000)(1000)} \sqrt{\frac{(5)}{(4\pi \times 10^{-7})(0.01)^2}} = \pm 0.0060A = \pm 6.0mA$$

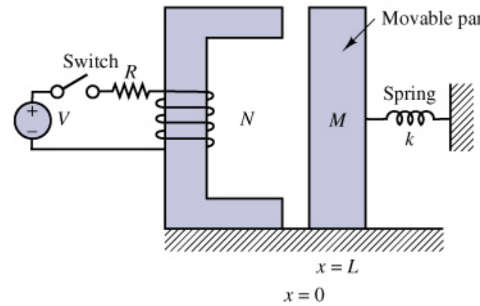
# Problem 18.34

## *Solution:*

### Known quantities:

Relay Circuit shown in Figure P18.34

$N = 500 turns; A_{gap} = 0.001 m^2; L = 0.02 m; k = 1000 N/m; R = 18\Omega$




### Find:

Minimum DC supply voltage *v* for which the relay will make contact when the electrical switch is closed

### Assumptions:

Neglect the iron reluctance

### Analysis:

The reluctance of the gap:

$$\Re_{gap}(x) = \frac{2x}{\mu_0 A_{gap}}$$

$$\frac{d\Re_{gap}(x)}{dx} = \frac{2}{\mu_0 A_{gap}}$$

Magnetic Flux:

$$\phi = \frac{Ni}{\Re(x)} = \frac{Ni\mu_0 A_{gap}}{2x}$$

Magnetic force:

$$f_{gap} = \frac{\phi^2}{2}\frac{d\Re(x)}{dx} = \left(\frac{Ni\mu_0 A_{gap}}{2x}\right)^2 \frac{1}{2}\frac{2}{\mu_0 A_{gap}} = \frac{(Ni)^2 \mu_0 A_{gap}}{4x^2}$$

The force that must be overcome is the spring force, $f_k$:

$$f_{gap} = f_k = kx$$

Equating the two force equations and solving for the current:

$$i = \pm\frac{2}{N}\sqrt{\frac{kx^3}{\mu_0 A_{gap}}}$$

The voltage is determined using Ohm's law, and $x = L$:

$$v = iR = \pm\frac{2R}{N}\sqrt{\frac{kx^3}{\mu_0 A_{gap}}} = \pm\frac{2(18)}{(500)}\sqrt{\frac{(1000)(0.02)^3}{4\pi\times10^{-7}(0.001)}} = 182.0V$$

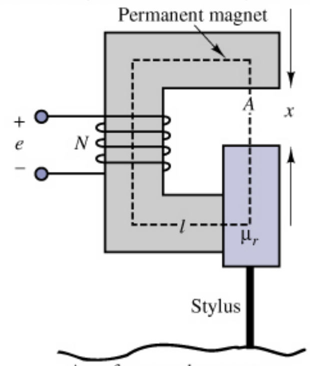
# Problem 18.35

## *Solution:*

### Known quantities:

The simplified representation of a surface roughness sensor shown in Figure P18.35





A surface roughness sensor

### Find:

Derive an expression for the displacement $x$ as a function of the various parameters of the magnetic circuit and of the measured emf $e$

### Assumptions:

The flux $\phi = \beta / \Re(x)$ where $\beta$ is a known constant

Frictionless contact between the moving plunger and the magnetic structure

The plunger is restrained to vertical motion only

### Analysis:

$$e = N\frac{d\phi}{dt};$$

$$\phi = \frac{\beta}{\Re(x)};$$

$$\Re(x) = \frac{L-x}{\mu_r \mu_0 A} + \frac{x}{\mu_0 A} = \frac{1}{\mu_r \mu_0 A}\left[L + (\mu_r - 1)x\right]$$

$$\phi = \frac{\beta}{\Re(x)} = \frac{\mu_r \mu_0 A}{\left[L + (\mu_r - 1)x\right]};$$

$$e = N\frac{d\phi}{dt} = N\frac{\partial\phi}{\partial x}\frac{dx}{dt} = -\frac{N\mu_r \mu_0 A\beta(\mu_r - 1)}{\left[L + (\mu_r - 1)x\right]^2}\frac{dx}{dt}$$

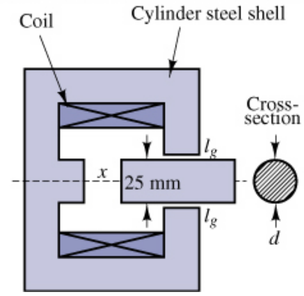
# Problem 18.36

## *Solution:*

### Known quantities:

As shown in Figure P18.36. The air gap between the shell and the plunger is uniform and equal to $1\,mm$. The diameter is $d = 25\,mm$. The exciting current is $7.5\,A$. $N = 200$.






### Find:

The force acting on the plunger when $x = 2\,mm$.

### Assumptions:

None.

### Analysis:

The cross-section area $A$ is:

$$A = \pi\left(\frac{25\times 10^{-3}}{2}\right)^2\ m^2$$

From this expression we can compute the variable reluctance of the air gap:

$$\Re_x = \frac{x}{4\pi\times 10^{-7}\times\frac{\pi(25\times 10^{-3})^2}{4}} = 1621\times 10^6\,x$$

and the resulting force is:

$$f = \frac{i^2}{2}\frac{N^2}{R_x^2}\frac{dR_x}{dx}$$

$$= \frac{7.5^2}{2}\frac{200^2}{(1621\times 10^6\,x)^2}\times 1621\times 10^6\ |_{x=2\times 10^{-3}}$$

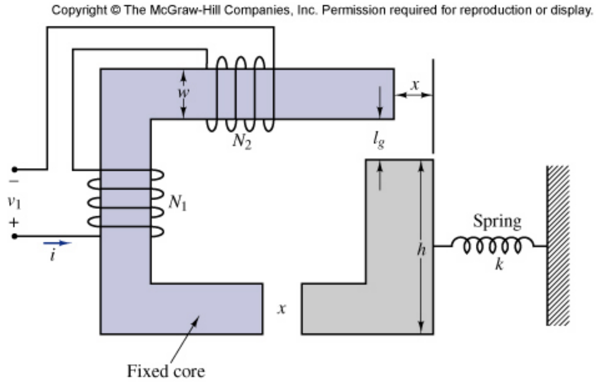
$$= 173.5\,N$$

# Problem 18.37

## *Solution:*

### Known quantities:

As shown in Figure P18.37, the double-excited electromechanical system moves horizontally. The cross section of the structure is $w \times w$.




### Find:

a) The reluctance of the magnetic circuit.
b) The magnetic energy stored in the air gap.
c) The force on the movable part as a function of its position.

### Assumptions:

Resistance, magnetic leakage and fringing are negligible. The permeability of the core is very large.

### Analysis:

a)
The reluctance of the magnetic circuit is:

$$\Re = \frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)}$$

b)
The magnetic energy stored in the air gap is:

$$W_m = \frac{(N_1 + N_2)^2 i^2}{2\Re}$$

c)
The magnetic force is:

$$f = \frac{i^2}{2} \frac{(N_1 + N_2)^2 \left( \frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)^2} \right)}{\left( \frac{x}{\mu_0 w^2} + \frac{l_g}{\mu_0 w(w-x)} \right)^2}$$

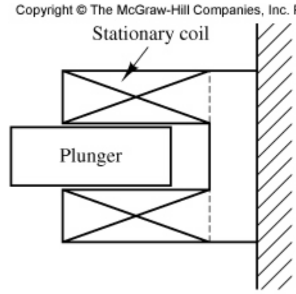
# Problem 18.38

## *Solution:*

### Known quantities:

The flux density in the cast steel pathway is $1.1T$. The diameter of the plunger is $10\,mm$.






### Find:

The force $f$ between the faces of the poles.

### Assumptions:

Air gap is negligible between walls and plunger
Since the pathway is cast steel, $\mu_r = 1000$ (from Table 18.1).

### Analysis:

Using Equation 18.50:

$$f = \frac{\phi^2}{2}\frac{d\Re(x)}{dx}$$

The flux is determined from the flux density and the area:

$$\phi = BA$$

Since the plunger is cylindrical and the air gap between the plunger and the coil is negligible, the reluctance is calculated as:

$$\Re(x) = \frac{x}{\mu A}$$

where x is the gap between the plunger and back wall of the solenoid. The reluctance of the cast steel pathway can be neglected due to the low reluctance of the structure.
The derivative of the reluctance is:

$$\frac{d\Re(x)}{dx} = \frac{1}{\mu A}$$

The area is the cross-sectional area of the plunger:

$$A = \frac{\pi}{4}d^2$$

Combining all of the equations into the force equation:

$$f = \frac{\phi^2}{2}\frac{d\Re(x)}{dx} = \frac{(BA)^2}{2}\frac{1}{\mu A} = \frac{\pi d^2 B^2}{(2)(4)}\frac{1}{\mu_o} = \frac{\pi(0.010)^2(1.1)^2}{8\left(4\pi\times 10^{-7}\right)} = 37.8N$$

# Problem 18.39

## *Solution:*

### Known quantities:

A force of 10,000 $N$ is required to support the weight. The cross-sectional area of the magnetic core is $0.01 m^2$. The coil has 1000 *turns*.

### Find:

The minimum current that can keep the weight from falling for $x = 1.0\,mm$.

### Assumptions:

Negligible reluctance for the steel parts and negligible fringing in the air gaps.

### Analysis:

The variable reluctance is given by: $\Re_T(x) = 2\Re(x) = \dfrac{2x}{4\pi\times10^{-7}(0.01)} = 159.15\times10^6 x$

The force is related to the reluctance by: $f = -10000 = -\dfrac{N^2 i^2}{2R_T^2(x)}\dfrac{d\Re_T(x)}{dx}$

Therefore, $i = \sqrt{3.18} = 1.784\,A$

---

# Problem 18.40

## *Solution:*

### Known quantities:

The $12-VDC$ control relay is made of sheet steel. Average length of the magnetic circuit is $12\,cm$. The average cross section of the magnetic circuit is $0.60\,cm^2$. The coil has 250 *turns* and carries $50\,mA$.

### Find:

a. The flux density $B$ in the magnetic circuit of the relay when the coil is energized.
b. The force $F$ exerted on the armature to close it when the coil is energized.

### Assumptions:

None.

### Analysis:

a. $NI = Hl \quad 250(250\times10^{-3}) = H(12\times10^{-2}) \Rightarrow H = 104.2$

from the curve, $B = 0.75\,T$.

b. $F = \dfrac{1}{2}\dfrac{AB^2}{\mu_0} = \dfrac{1}{2}\dfrac{(0.6\times10^{-4})(0.75)^2}{4\pi\times10^{-7}} = 13.4\,N$

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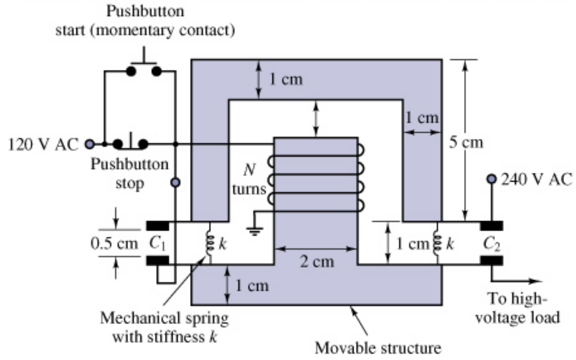
# Problem 18.41

## *Solution:*

### Known quantities:

As shown in Figure P18.41.




### Find:

The differential equations describing the system.

### Assumptions:

None.

### Analysis:

The equation for the electrical system is:

$$v = iR + L(x)\frac{di}{dt}$$

where: $L(x) = \dfrac{N^2}{\Re_T(x)} = \dfrac{N^2 \mu_0 A}{2x}$

The equation for the mechanical system is:

$$F_m = m\frac{d^2x}{dt^2} + kx$$

where $F_m$ is the magnetic pull force. To calculate this force we use the following equation:

$$F_m = -\frac{dW_m}{dx}$$

where $W_m$ is the energy stored in the magnetic field.

Let $F$ and $\Re$ be the magnetomotive force acting on the structure and its reluctance, respectively; then:

$$W_m = \frac{\phi^2 \Re(x)}{2} = \frac{F^2}{2\Re(x)} = \frac{N^2 i^2 \mu A}{4x} \qquad F_m = -\frac{dW_m}{dx} = \frac{N^2 i^2 \mu A}{4x^2}$$

Finally, the differential equations governing the system are:

$$v = iR + \frac{N^2 \mu_0 A}{2x}\frac{di}{dt}$$

$$m\frac{d^2x}{dt^2} + kx = \frac{N^2 i^2 \mu A}{4x^2}$$

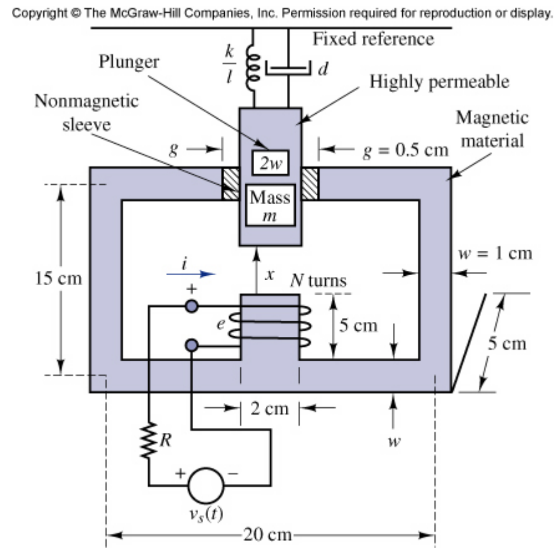
This system of equations could be solved using a numerical simulation, since it is nonlinear.

# Problem 18.42

## *Solution:*

### Known quantities:

As shown in Figure P18.42. The solenoid has a cross section of $5\,cm^2$.




### Find:

a. The force exerted on the plunger when the distance $x$ is $2\,cm$ and the current is $5\,A$. $N = 100\,turns$. Relative permeabilities of the magnetic material and the nonmagnetic sleeve are $2{,}000$ and $1$.

b. Develop a set of differential equations governing the behavior of the solenoid.

### Assumptions:

The fringing and leakage effects are negligible.

### Analysis:

a.

$$\Re_x = \frac{x}{\mu_0 A_x}$$

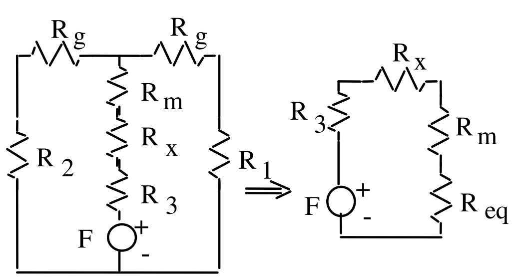
$$\Re_g = \frac{l_g}{\mu_0 A_g} = \frac{0.005}{(4\pi\times10^{-7})(5\times10^{-4})} = 7.96\times10^6 H^{-1}$$

$$\Re_1 = \Re_2 = \frac{l_1}{\mu_r\mu_0 A_g} = \frac{0.335}{2000(4\pi\times10^{-7})(5\times10^{-4})} = 2.67\times10^5\ H^{-1}$$

$$\Re_3 = \frac{l_3}{\mu_r\mu_0 A_x} = \frac{0.055}{2000(4\pi\times10^{-7})(10\times10^{-4})} = 2.19\times10^4\ H^{-1}$$

$$\Re_m = \frac{l_x}{\mu_r\mu_0 A_x} = \frac{l_x}{2000(4\pi\times10^{-7})(10\times10^{-4})}$$

$$= (3.98\times10^5)(0.095 - x)\,H^{-1}$$




The circuit is shown on the right:

$$\Re_{eq} = \frac{\Re_g + \Re_1}{2} = \frac{l_g}{2\mu_0 A_g} + \frac{l_1}{2\mu_r\mu_0 A} = 4.11\times10^6 H^{-1}$$

$$\Re_T = \Re_3 + \Re_x + \Re_m + \Re_{eq} = \frac{l_3}{\mu_r\mu_0 A_x} + \frac{x}{\mu_0 A_x} + \frac{(0.095 - x)}{\mu_r\mu_0 A_x} + \frac{l_g}{2\mu_0 A_g} + \frac{l_1}{2\mu_r\mu_0 A_g}$$

Since $A_x = 2A_g$:

$$\Re_T = \frac{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1}{2\mu_r \mu_0 A_g} =$$

$$= \frac{0.055 + 0.095 - x + 2000x + (0.005)(2000) + 0.335}{2(2000)(4\pi \times 10^{-7})(5 \times 10^{-4})} = \frac{10.485 + 1999x}{2.51 \times 10^{-6}} =$$

$$= 4.18 \times 10^6 + 7.96 \times 10^8 x$$

$$f_m = -\frac{\phi^2}{2}\frac{\partial \Re_T(x)}{\partial x} = -\frac{N^2 i^2}{2R_T^2}\frac{d\Re_T}{dx} = -\frac{N^2 i^2 (2\mu_r \mu_0 A_g)^2}{2(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2}\frac{(\mu_r - 1)}{2\mu_r \mu_0 A_g} =$$

$$= -\frac{\mu_r \mu_0 A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2} =$$

$$= -\frac{(2000)(4\pi \times 10^{-7})(5 \times 10^{-4})(100)^2 (5)^2 (2000 - 1)}{(0.055 + 0.095 - x + 2000x + (0.005)(2000) + 0.335)^2}$$

For $x = 0.02,\ f = -0.25N$ .

b.

Electrical subsystem:

$$v_s(t) = L(x)\frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem: $m\ddot{x}(t) = f_m(x) - d\frac{dx(t)}{dt} - \frac{k}{l}x(t)$

Reluctance: $\Re_T = \dfrac{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1}{2\mu_r \mu_0 A_g}$

Flux: $\phi = \dfrac{Ni(t)}{\Re_m(x)} = \dfrac{2\mu_r \mu_0 A_g Ni(t)}{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1}$

Magnetic force:

$$f_m(x) = \frac{\phi^2}{2}\frac{\partial \Re_T(x)}{\partial x} = -\frac{N^2 i^2}{2R_T^2}\frac{d\Re_T}{dx} = \frac{N^2 i^2 (2\mu_r \mu_0 A_g)^2}{2(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2}\frac{(\mu_r - 1)}{2\mu_r \mu_0 A_g} =$$

$$= \frac{\mu_r \mu_0 A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2}$$

Inductance: $L(x) = \dfrac{N^2}{\Re_T(x)} = \dfrac{N^2 2\mu_r \mu_0 A_g}{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1}$

Substituting the expressions for $f_m$ and *L(x)* in the two differential equations, we have the final answer.

Electrical subsystem: $v_s(t) = \left(\dfrac{N^2 2\mu_r \mu_0 A_g}{l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1}\right)\dfrac{di(t)}{dt} + Ri(t)$

Mechanical subsystem: $m\ddot{x}(t) = \dfrac{\mu_r \mu_0 A_g N^2 i^2 (\mu_r - 1)}{(l_3 + x\mu_r + (0.095 - x) + l_g \mu_r + l_1)^2} - d\dfrac{dx(t)}{dt} - \dfrac{k}{l}x(t)$

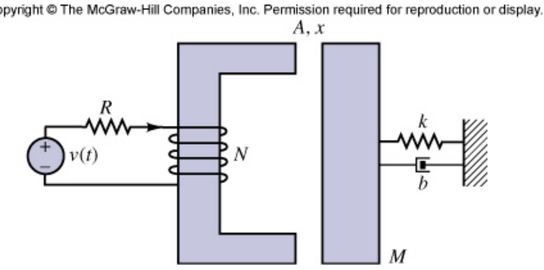
Note that these equations are very nonlinear!

# Problem 18.43

## *Solution:*

### Known quantities:

The relay shown in Figure P18.43




### Find:

Derive the differential equations (electrical and mechanical) for the relay

### Assumptions:

The inductance is a function of $x$
The iron reluctance is negligible

### Analysis:

Electrical subsystem:

$$v(t) = L(x)\frac{di(t)}{dt} + Ri(t)$$

Mechanical subsystem:

$$m\ddot{x}(t) = f_m(x) - b\frac{dx(t)}{dt} - kx(t)$$

Next, we calculate the magnetic force and inductance as functions of *x*.

Reluctance:

$$\Re(x) = \frac{2x}{\mu A}$$

Flux:

$$\phi = \frac{Ni(t)}{\Re(x)} = \frac{Ni(t)\mu A}{2x}$$

Magnetic force:

$$f_m(x) = \frac{\phi^2}{2}\frac{\partial \Re(x)}{\partial x} = \frac{N^2 i^2 \mu A}{4x^2}$$

Inductance:

$$L(x) = \frac{N^2}{\Re(x)} = \frac{N^2 \mu A}{2x}$$

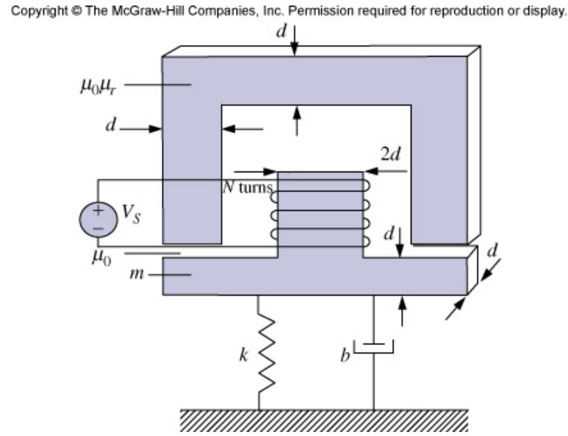
Substituting the expressions for $f_m$ and $L(x)$ in the two differential equations, we have the final answer.

Electrical subsystem: $v(t) = \frac{N^2 \mu A}{2x(t)}\frac{di(t)}{dt} + Ri(t)$

Mechanical subsystem: $m\ddot{x}(t) = \frac{N^2 i^2 \mu A}{4x(t)^2} - b\frac{dx(t)}{dt} - kx(t)$

Note that these equations are very nonlinear!

# Problem 18.44




## *Solution:*

### Known quantities:

The relay shown in Figure P18.44

### Find:

Derive the differential equations

### Assumptions:

The inductance is a function of $x$
The iron reluctance is negligible

### Analysis:

The equation for the electrical system is:

$$V_S = i(t)R + \frac{d(L(x)i(t))}{dt} = i(t)R + L(x)\frac{di(t)}{dt} + i(t)\frac{dL(x)}{dt} = i(t)R + L(x)\frac{di(t)}{dt} + i(t)\frac{\partial L(x)}{\partial x}\frac{dx(t)}{dt} \text{ where:}$$

$$L(x) = \frac{N^2}{\Re_T(x)} = \frac{N^2\mu_0 A}{2x}$$

$$\frac{\partial L(x)}{\partial x} = -\frac{N^2\mu_0 A}{2x^2}$$

The equation for the mechanical system is:

$$F_m = m\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + kx(t)$$

where $F_m$ is the magnetic pull force. To calculate this force we use the following equation:

$$F_m = -\frac{dW_m}{dx}$$

where $W_m$ is the energy stored in the magnetic field.

Let $F$ and $\Re$ be the magnetomotive force acting on the structure and its reluctance, respectively; then:

$$W_m = \frac{\phi^2\Re(x)}{2} = \frac{F^2}{2\Re(x)} = \frac{N^2 i^2 \mu A}{4x}$$

$$F_m = -\frac{dW_m}{dx} = -\frac{N^2 i^2 \mu A}{4x^2}$$

Finally, the differential equations governing the system are:

$$V_S(t) = Ri(t) + \frac{N^2\mu_0 A}{2x}\frac{di(t)}{dt} - i\frac{N^2\mu_0 A}{2x^2}\frac{dx(t)}{dt}$$

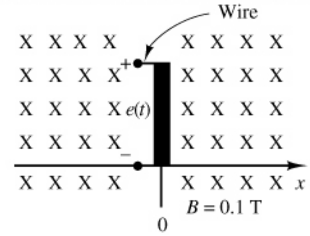
$$m\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + kx(t) = -\frac{N^2 i^2 \mu A}{4x^2}$$

This system of equations could be solved using a numerical simulation, since it is nonlinear.

## b) Moving-Coil Transducers

## Problem 18.45






### *Solution:*

#### Known quantities:
Length of the wire is $20\,cm$ ; Flux density is $0.1T$ ; The position of the wire is $x(t) = 0.1\sin 10t\, m$

#### Find:
The induced emf across the length of the wire as a function of time.

#### Assumptions:
None.

#### Analysis:
From $e = Blu$ , we have

$$e(t) = Bl\frac{dx}{dt} = 0.02\cos(10t)V$$

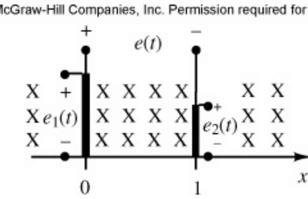
---

## Problem 18.46

### *Solution:*

#### Known quantities:
Emf: $e_1(t) = 0.02\cos 10t$ ; the length of the second wire: $0.1m$ ; the position of the second wire: $x(t) = 1 - 0.1\sin 10t$ .






#### Find:
The induced emf $e(t)$ defined by the difference in emf's $e_1(t)$ and $e_2(t)$ .

#### Assumptions:
None.

#### Analysis:
We have

$$e(t) = e_1(t) - e_2(t)$$
$$e_2(t) = (0.1)(0.1)(-1\cos 10t)V$$
$$e(t) = 0.02\cos(10t) + 0.01\cos(10t) = 0.03\cos(10t)V$$

---

## Problem 18.47

### *Solution:*

#### Known quantities:

$I = 4A, B = 0.3 Wb/m^2$.

#### Find:

The magnitude and direction of the force induced on the conducting bar.

#### Assumptions:

None.

#### Analysis:

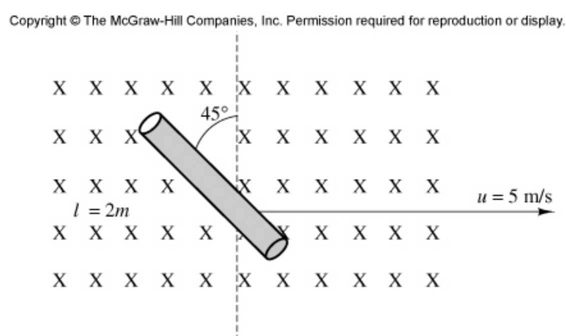
$f = Bli = 0.3 \times l \times 4 = 1.2l\ N$

Force will be to the left if current flows upward.

---

## Problem 18.48

### *Solution:*




#### Known quantities:

$B = 0.3 Wb/m^2$ .

#### Find:

The magnitude and direction of the induced voltage in the wire.

#### Assumptions:

None.

#### Analysis:

$e = Blu \cos 45^\circ = 2.83 V$

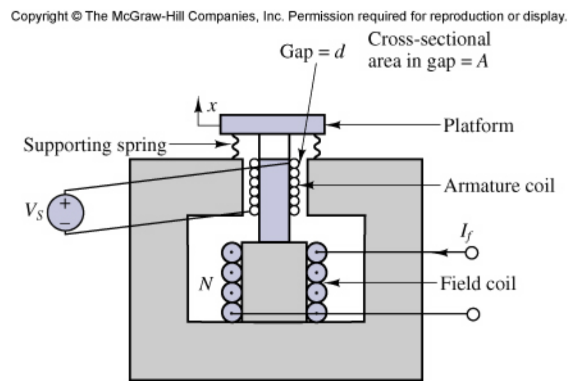
---

# Problem 18.49

## *Solution:*

### Known quantities:

The electrodynamic shaker shown in Figure P18.49
Shaker mass, $m$; air gap dimension, $d$; number of coil turns, $N$; spring parameter, $k$; armature resistance, $R$ and inductance, $L$




### Find:

a. Reluctance of the structure and Flux density B
b. Dynamic equations of the shaker
c. Transfer function and frequency response of the shaker velocity to input voltage $V_s$.

### Assumptions:

Iron reluctance is negligible. Neglect fringing. Assume no damping in this system.

### Analysis:

a. Reluctance of structure: $\Re = \dfrac{2d}{\mu_0 A}$

Compute flux density: $B = \dfrac{\phi}{A} = \dfrac{NI_f}{A\Re} = \dfrac{NI_f \mu_0}{2d}$

b. Electrical Subsystem, using KVL: $V_s = Ri + L\dfrac{di}{dt} + Bl\dfrac{dx}{dt}$

Mechanical Subsystem: $m\dfrac{d^2x}{dt^2} = Bli - kx$

c. Laplace Transform:

$$V_s(s) = (R + Ls)I(s) + BlsX(s)$$

$$0 = -BlI(s) + (ms^2 + k)X(s)$$

$$V_s(s) = \left(\frac{(R + Ls)(ms^2 + k)}{Bl} + Bls\right)X(s)$$

$$\frac{X(s)}{V_s(s)} = \frac{Bl}{mLs^3 + mRs^2 + (kL + B^2l^2)s + Rk}$$

$$\frac{U(s)}{V_s(s)} = \frac{SX(s)}{V_s(s)} = \frac{Bls}{mLs^3 + mRs^2 + (kL + B^2l^2)s + Rk}$$

Frequency Response: $s = j\varpi$

$$\frac{U(j\varpi)}{V_s(j\varpi)} = \frac{Blj\varpi}{mL(j\varpi)^3 + mR(j\varpi)^2 + (kL + B^2l^2)j\varpi + Rk}$$

$$\frac{U(j\varpi)}{V_s(j\varpi)} = \frac{Blj\varpi}{R(k - m\varpi^2) + j\left[(kL + B^2l^2)\varpi - mLw^3\right]}$$

# Problem 18.50

## *Solution:*

### Known quantities:

The electrodynamic shaker shown in Figure P18.49 is used to perform vibration testing of an electrical connector.
$B = 1,000 \text{ Wb/m}^2;\ l = 5 \text{ m};\ k = 1000 \text{ N/m};\ m = 1 \text{ kg};$
$b = 5 \text{ N-s/m};\ L = 0.8 \text{ H};\ R = 0.5\ \Omega;$
The test consists of shaking the connector at the frequency $\varpi = 2\pi \times 100 rad/s$

### Find:

The peak amplitude of sinusoidal voltage $V_s$ required to generate an acceleration of $5g(49 m/s^2)$ under the stated test conditions

### Assumptions:

Connector has negligible mass when compared to the platform.

### Analysis:

Applying KVL around the coil circuit: $L\frac{di}{dt} + Ri + Bl\frac{dx}{dt} = V_S$
Next, we apply Newton's Second Law:

$$-Bli + m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

To derive the frequency response, Laplace transform the two equations to obtain:

$$(sL + R)I(s) + BlsX(s) = V_S(s)$$

$$-BlI(s) + (ms^2 + bs + k)X(s) = 0$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for *U(s)* as a function of *V(s)*:

$$\begin{bmatrix} (sL + R) & Bls \\ -Bl & (ms^2 + bs + k) \end{bmatrix}\begin{bmatrix} I(s) \\ X(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

with solution

$$X(s) = \frac{\det\begin{bmatrix} (Ls + R) & V(s) \\ -Bl & 0 \end{bmatrix}}{\det\begin{bmatrix} (Ls + R) & Bls \\ -Bl & (ms^2 + bs + k) \end{bmatrix}} = \frac{BlV(s)}{(Ls + R)(ms^2 + bs + k) + (Bl)^2 s}$$

To obtain the acceleration response, we multiply the numerator by $s^2$:

$$\frac{s^2X(s)}{V(s)} = \frac{\ddot{X}(s)}{V(s)} = \frac{Bls^2}{(Lm)s^3 + (bL + Rm)s^2 + \left(bR + kL + (Bl)^2\right)s + (kR)}$$

$$\frac{\ddot{X}(j\omega)}{V(j\omega)} = \frac{-Bl\omega^2}{\left[kR - (bL + Rm)\omega^2\right] + j\left[\left(bR + kL + (Bl)^2\right)\omega - (Lm)\omega^3\right]}$$

The magnitude of this complex number, evaluated at $\omega = 2\pi \times 100$ is 0.62.
Thus, to obtain the desired acceleration (peak) of 49 m/s$^2$, we wish to have a peak voltage amplitude $|V_S| = 49/0.62 \approx$ 78 V.

# Problem 18.51

## *Solution:*

### Known quantities:
As described in Example 18.13.

### Find:
Derive and sketch the frequency response of the loudspeaker in the following two cases. Describe qualitatively how the loudspeaker frequency response changes as the spring stiffness $k$ increases and decreases. Find the limit of the frequency response and the kind of the speaker as $k$ approaches zero.

a. $k = 50{,}000\ N/m$.

b. $k = 5\times10^6\ N/m$.

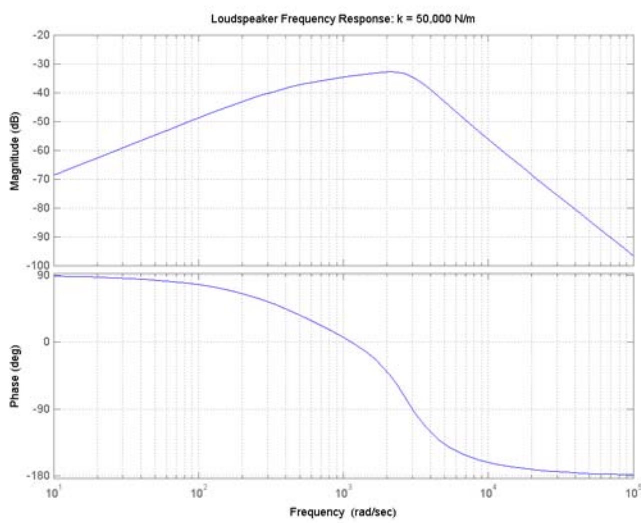
### Assumptions:
None.

### Analysis:
a.

For $k = 50{,}000\ N/m$, the transfer function is:

$$\frac{U}{V} = \frac{1.478\times10^5 s}{s^3 + 3075s^2 + 9\times10^6 s + 4\times10^9}$$

The magnitude frequency response is plotted below:



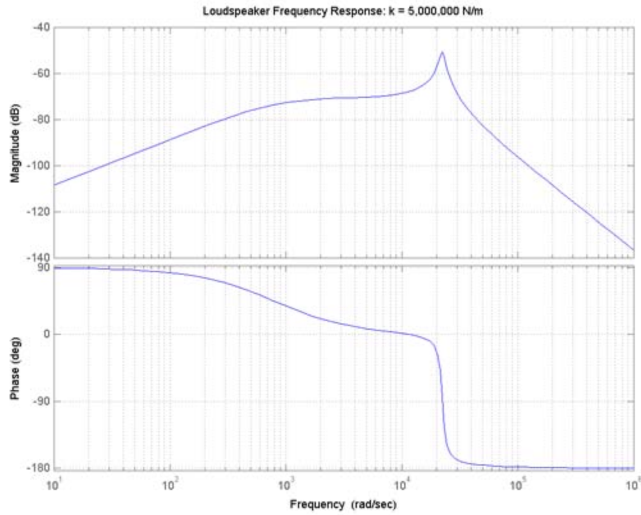

This response would correspond to a midrange speaker.

b.

For $k = 5\times10^6\ N/m$, the transfer function is :

$$\frac{U}{V} = \frac{1.478\times10^5 s}{s^3 + 3075s^2 + 5.04\times10^8 s + 4\times10^{11}}$$

The magnitude frequency response is plotted below:



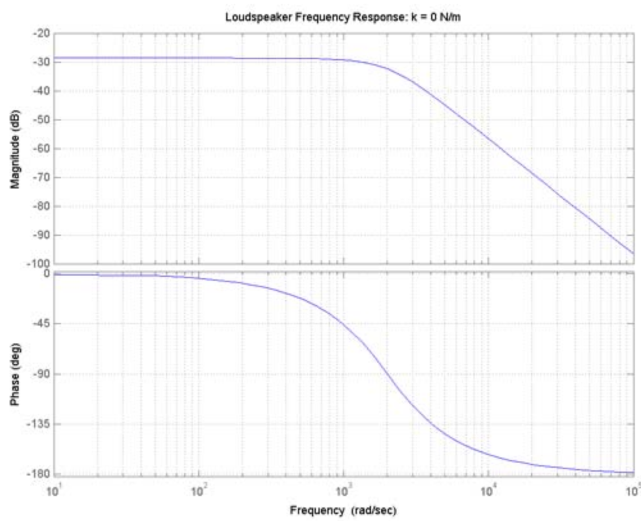

It should be apparent that this response would enhance the treble range, and is the response of a "tweeter".

c.

For $k = 0$, the transfer function is :

$$\frac{U}{V} = \frac{1.478\times10^5}{s^2 + 3075s + 4\times10^6}$$

The magnitude frequency response is plotted below:




In this case, the speaker acts as a "woofer", emphasizing the low frequency range. In practice, $k$ cannot be identically zero, so the actual response of a woofer would resemble that of a midrange speaker, shifted towards the lower frequencies.

# Problem 18.52

## *Solution:*

### Known quantities:

Loudspeaker of Example 18.13 (Figure 18.52)

$r_{coil} = 0.05m; L = 10mH; R = 8\Omega; N = 47; B = 1T;$

$m = 0.01kg; b = 22.75N - s/m; k = 5 \times 10^4 N/m$

### Find:

Modify the parameters of the loudspeaker (mass, damping, and spring rate), so as to obtain a loudspeaker with a bass response centered on 400 Hz.
Demonstrate that your design accomplishes the intended task, using frequency response plots.

### Assumptions:

None.

### Analysis:

From Example 18.13, the system transfer function is:

$$\frac{U(s)}{V(s)} = \frac{Bls}{(Lm)s^3 + (Rm + Lb)s^2 + \left[Rb + kL + (Bl)^2\right]s + kR}$$

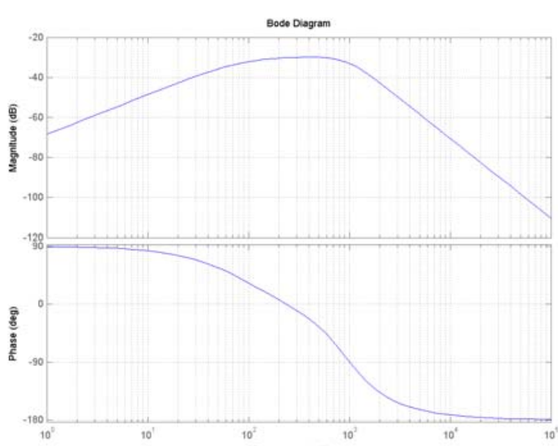
The frequency response is:

$$\frac{U(j\varpi)}{V(j\varpi)} = \frac{jBl\varpi}{kR - (Rm + Lb)\varpi^2 + j\left\{\left[Rb + kL + (Bl)^2\right]\varpi - (Lm)\varpi^3\right\}} \qquad \text{where: } l = 2\pi N r_{coil}$$

Using MATLAB, it is easy to adjust the mechanical parameters one-by-one in order to see each parameters' effect on the system frequency response.
Mass: Increasing the mass decreases the frequency response center frequency.
Damping: Increasing the damping also decreases the center frequency, but also widens the response bandwidth.
Spring Rate: Decreasing the spring rate also decreases the center frequency.
There are many possible combinations of mechanical parameters that could be substituted to generate the desired response.

One such set of parameters is: $m = 0.05kg; b = 30N - s/m; k = 5 \times 10^3 N/m$

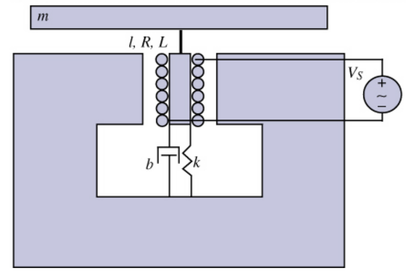
The frequency response for this system is given below, with the response centered over 400 Hz:



# Problem 18.53

## *Solution:*






### Known quantities:

Electrodynamic shaker as shown in Figure 18.53, with parameters.

### Find:

a) Write down dynamic equations and indicate system input and output.
b) Find the frequency response function of the table acceleration in response to the applied voltage.
c) Determine the peak amplitude of the sinusoidal voltage $V_S$ required to generate an acceleration of 5g ($49 \text{m/s}^2$).

### Assumptions:

None.

### Analysis:

a) Applying KVL around the coil circuit: $L\frac{di}{dt} + Ri + Bl\frac{dx}{dt} = V_S$

Next, we apply Newton's Second Law :

$$-Bli + m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

Voltage applied is the input of the system, while acceleration response of the shaker are the output of the system.

b) To derive the frequency response we Laplace transform the two equations to obtain:

$$(sL+R)I(s) + BlsX(s) = V_S(s)$$

$$-BlI(s) + (ms^2 + bs + k)X(s) = 0$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for *U(s)* as a function of *V(s)*:

$$\begin{bmatrix} (sL+R) & Bls \\ -Bl & (ms^2+bs+k) \end{bmatrix} \begin{bmatrix} I(s) \\ X(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

with solution

$$X(s) = \frac{\det\begin{bmatrix} (Ls+R) & V(s) \\ -Bl & 0 \end{bmatrix}}{\det\begin{bmatrix} (Ls+R) & Bls \\ -Bl & (ms^2+bs+k) \end{bmatrix}} = \frac{BlV(s)}{(Ls+R)(ms^2+bs+k) + (Bl)^2 s}$$

To obtain the acceleration response, we multiply the numerator by $s^2$:

$$\frac{s^2X(s)}{V(s)} = \frac{\ddot{X}(s)}{V(s)} = \frac{Bls^2}{(Lm)s^3 + (bL+Rm)s^2 + \left(bR+kL+(Bl)^2\right)s + (kR)}$$

$$\frac{\ddot{X}(j\omega)}{V(j\omega)} = \frac{-Bl\omega^2}{\left[kR-(bL+Rm)\omega^2\right] + j\left[\left(bR+kL+(Bl)^2\right)\omega - (Lm)\omega^3\right]}$$

c) The magnitude of this complex number, evaluated at $\omega = 2\pi \times 100$ is 0.62.
Thus, to obtain the desired acceleration (peak) of 49 $\text{m/s}^2$, we wish to have a peak voltage amplitude $|V_S| = 49/0.62 \approx 78$ V .