Handout #25
E84: Fall '07
11/19/07

1

5.27 For the impedance shown in Fig. P5.27, suppose that $R = 1\ \Omega$, $L = \frac{1}{5}$ H, and $C = 1$ F. Find the resonance frequency.

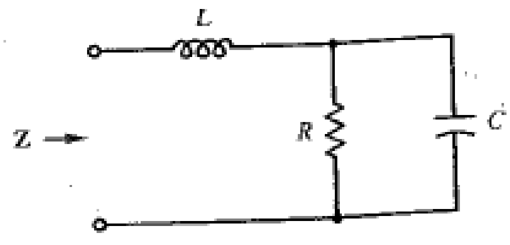
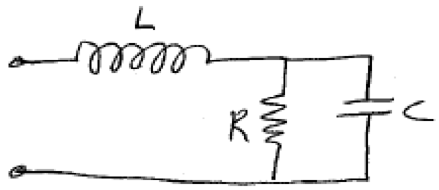


Fig. P5.27

FoEE 5.27 Find the resonant frequency of:



Resonant frequency is the frequency where the imaginary part of the impedance is 0.

So, $Z_R = R$

$Z_L = j\omega L$

$Z_C = \frac{-j}{\omega C}$

$$Z_{RC} = \frac{\frac{-Rj}{\omega C}}{R - j/\omega C} = \frac{-Rj}{\omega RC - j}$$

$$Z_T = \frac{-Rj}{\omega RC - j} + j\omega L\left[\frac{\omega RC - j}{\omega RC - j}\right]$$

$$= \frac{-Rj + j\omega^2 RLC + \omega L}{\omega RC - j}\left[\frac{\omega RC + j}{\omega RC + j}\right]$$

$$= \frac{-R^2\omega Cj + j\omega^3 R^2 LC^2 + \omega^2 RLC + R - \omega^2 RLC + j\omega L}{\omega^2 R^2 C^2 + 1}$$

So imaginary portion is

$$j\left(-\omega R^2 C + \omega^3 R^2 LC^2 + \omega L\right) = 0$$

Solve for $\omega \implies \omega^3 R^2 LC^2 = \omega(R^2 C - L)$

$$\omega^2 = \frac{R^2 C - L}{R^2 LC^2} = \frac{1}{LC} - \frac{1}{R^2 C^2}$$

$\omega = 0\ \frac{rad}{s}$ ... trivial case

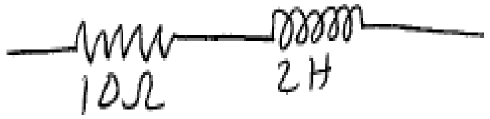
$\omega_r = 2\ \frac{rad}{s}$ $\Longleftarrow$ $\omega = \sqrt{\frac{1}{LC} - \frac{1}{R^2C^2}} = \sqrt{5-1} = 2\ \frac{rad}{s}$

(2)

5.35 A 10-Ω resistor and a 2-H inductor are connected in series, and $\omega = 50$ rad/s. (a) What is the Q of this series connection? (b) What parallel RL connection has the same admittance as the series connection? (c) What is the Q of this parallel connection?

(3) 5.35 in FoEE

(a)



$\omega = 50 \frac{rad}{s}$

$Z_R = 10\Omega$

$Z_L = j100\Omega$

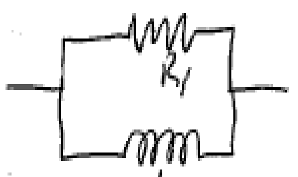
So, $R_s = 10\Omega$, $X_s = 100\Omega$

$$\boxed{\therefore Q = \frac{|X_s|}{R_s} = 10}$$

(b) $$Y = \frac{1}{Z} = \frac{1}{10+j100}\left(\frac{10-j100}{10-j100}\right)$$

$$= \frac{10-j100}{100+10000} = \frac{10}{10100} - \frac{j100}{10100} = \frac{1}{1010} - \frac{j}{101}$$

And,



$$Z_T = \frac{1}{\frac{1}{R_p} + \frac{1}{Z_{L_p}}}$$

So,

$$Y_T = \frac{1}{R_p} + \frac{1}{Z_{Lp}} = \frac{1}{R_p} + \frac{1}{j\omega L_p}$$

So, $\boxed{R_p = 1010\Omega}$

$$\frac{1}{j\omega L_p} = \frac{-j}{101}$$

$$\frac{1}{50 L_p} = \frac{1}{101} \Rightarrow \boxed{L_p = \frac{101}{50} = 2.02\,H}$$

(c) $$\boxed{Q = \frac{R_p}{|X_p|} = \frac{1010}{(2.02)(50)} = 10}$$

3

5.41 For the series RLC circuit shown in Fig. P5.15, suppose that $R = 2\ \Omega$, $L = \frac{1}{2}$ H, $C = 2$ F, and $v_1(t) = 20e^{-6t}\cos 3t$ V. Find (a) $v_R(t)$, (b) $v_L(t)$, and (c) $v_C(t)$.

FoEE 5.41 If $v_1(t) = 20e^{-6t}\cos 3t\ V$, then $Z_R = R = 2\,\Omega$

$Z_C = \frac{1}{sC} = \frac{1}{s2}\,\Omega$

$Z_L = sL = \frac{1}{2}s\,\Omega$

Since these are in series, a simple voltage divider will yield the requested voltages

a) $v_R(t) = \dfrac{Z_R}{Z_R + Z_L + Z_C} v_1(t) \Rightarrow \underline{V_R} = \dfrac{Z_R}{Z_R + Z_L + Z_C}\underline{V_1}$

$$\underline{V_R} = \frac{2}{2 + \frac{1}{2}s + \frac{1}{s2}}\underline{V_1} = \frac{2s}{\frac{1}{2}s^2 + 2s + \frac{1}{2}}\underline{V_1} = \frac{4s}{s^2 + 4s + 1}\underline{V_1}$$

Here, $s = -6 + j3$ So, $\underline{V_R} = \dfrac{4(-6+j3)}{(-6+j3)^2 + 4(-6+j3) + 1}\underline{V_1}$

$$= \frac{-24 + 12j}{36 - 36j - 9 - 24 + 12j + 1}\underline{V_1}$$

$$= \frac{-24 + 12j}{4 - 24j}\underline{V_1} = \frac{-6 + 3j}{1 - 6j}\underline{V_1}$$

$$= \frac{6.7\angle 153.4^\circ}{6.08\angle -80.54^\circ} 20\angle 0^\circ$$

$$= 22.04\angle 233.9^\circ$$

$\boxed{\text{So, } v_R(t) = 22.04\, e^{-6t}\cos(3t + 233.9^\circ)\ V}$

$\boxed{= 22.04\, e^{6t}\cos(3t - 126.1^\circ)\ V}$

b) $\underline{V_L} = \dfrac{Z_L}{Z_R + Z_L + Z_C}\underline{V_1} = \dfrac{\frac{1}{2}s}{\frac{1}{2}s^2 + 2s + \frac{1}{2}} \cdot s\,\underline{V_1} = \dfrac{s^2}{s^2 + 4s + 1}\underline{V_1}$

From math above we get $= \dfrac{(-6+j3)^2}{4 - 24j} = \dfrac{27 - 36j}{4 - 24j}\underline{V_1} = \dfrac{45\angle -53.1^\circ}{24.3\angle -80.5^\circ} 20\angle 0^\circ$

$\underline{V_L} = 37\angle 27.4^\circ$ $v_L(t) = 37e^{-6t}\cos(3t + 27.4^\circ)$ Volts

c) $\underline{V_C} = \dfrac{Z_C}{Z_R + Z_L + Z_C}\underline{V_1} = \dfrac{\frac{1}{2s} \cdot 2s}{s^2 + 4s + 1}\underline{V_1} = \dfrac{1}{4 - 24j}\underline{V_1} = \dfrac{1}{24.3\angle -80.5^\circ} 20\angle 0^\circ$

$\underline{V_C} = 0.82\angle 80.5^\circ$ $v_C(t) = 0.82\, e^{-6t}\cos(3t + 80.5^\circ)\ V$

4

5.45 For the series *RLC* circuit shown in Fig. P5.15, suppose that $R = 2\ \Omega$, $L = \frac{1}{2}$ H, and $C = 2$ F. Draw a pole-zero plot for (a) $\mathbf{I}/\mathbf{V}_1$, (b) $\mathbf{V}_R/\mathbf{V}_1$, (c) $\mathbf{V}_L/\mathbf{V}_1$, and (d) $\mathbf{V}_C/\mathbf{V}_1$.

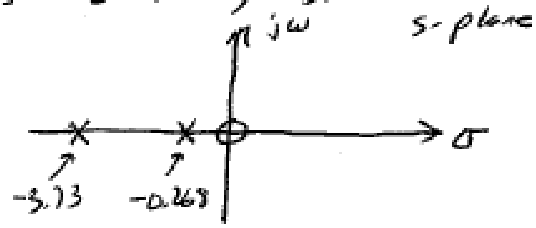
FoEE 5.45 $\quad R = 2\Omega,\ L = \frac{1}{2}H,\ C = 2F$

So, a) $$\frac{I}{V_1} = \frac{1}{Z_T} = \frac{1}{2 + \frac{1}{2}s + \frac{1}{2s}} = \frac{2s}{s^2 + 4s + 1}$$

zeroes: $s = 0$

poles: roots of $s^2 + 4s + 1 = 0$ (using quad. eqn.)

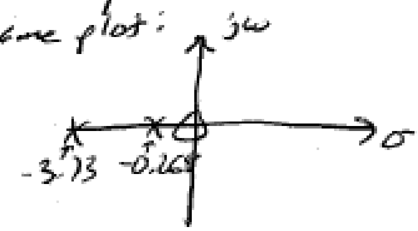
$s = -0.268,\ -3.73$



b) $$\frac{V_R}{V_1} = \frac{Z_R}{Z_T} = \frac{2}{2 + \frac{1}{2}s + \frac{1}{2s}} = \frac{4s}{s^2 + 4s + 1}$$

So, same zeroes and poles as in part (a).

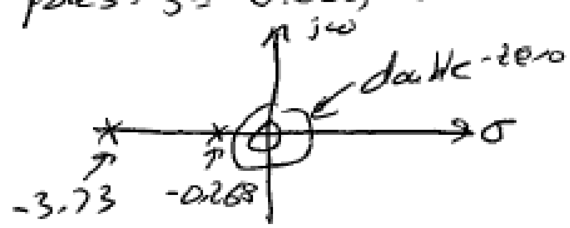
Same plot:



c) $$\frac{V_L}{V_1} = \frac{Z_L}{Z_T} = \frac{\frac{1}{2}s}{2 + \frac{1}{2}s + \frac{1}{2s}} = \frac{s^2}{s^2 + 4s + 1}$$

zeroes: $s = 0, 0$

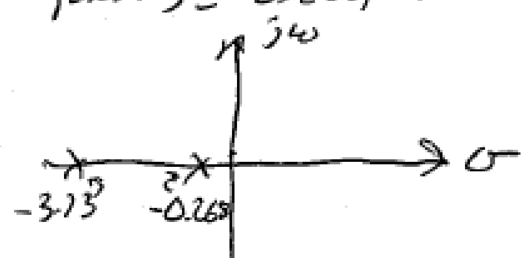
poles: $s = -0.268,\ -3.73$



d) $$\frac{V_C}{V_1} = \frac{Z_C}{Z_T} = \frac{\frac{1}{2s}}{2 + \frac{1}{2}s + \frac{1}{2s}} = \frac{1}{s^2 + 4s + 1}$$

No zeroes

poles: $s = -0.268,\ -3.73$



5.49 For the op-amp circuit shown in Fig. P5.49, draw the pole-zero plot of $H(s) = V_2/V_1$ for the case that C is (a) $\frac{1}{2}$ F, (b) 1 F, and (c) 2 F.

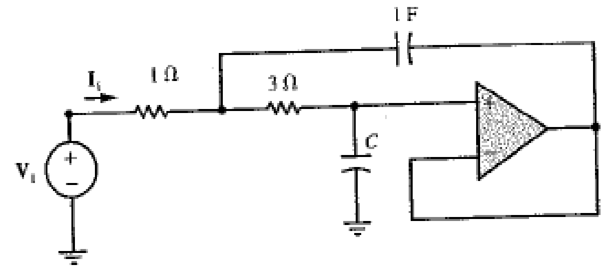


Fig. P5.49

FoEE 5.49

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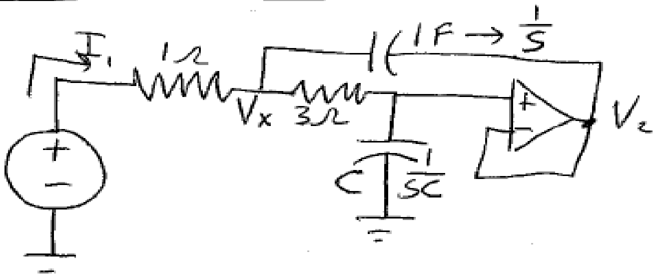


Need a pole-zero plot of $\frac{V_2}{V_1}$ ... so first KCL on V_x & V_+

(1) $$\frac{V_1 - V_x}{1} = \frac{V_x - V_2}{3} + \frac{V_x - V_2}{1/s}$$

$$V_1 - V_x = \frac{1}{3}V_x - \frac{1}{3}V_2 + sV_x - sV_2$$

$$V_1 - \left(\frac{4}{3} + s\right)V_x = \left(-\frac{1}{3} - s\right)V_2$$

(2) KCL on V_+: $$\frac{(V_x - V_2)}{3} = \frac{V_2}{\frac{1}{sC}} \implies \frac{V_x}{3} = \frac{V_2}{3} + V_2 sC$$

$$V_x = V_2 + 3sCV_2$$

So (1) becomes: $$V_1 - \left(\frac{4}{3} + s\right)(1 + 3sC)V_2 = \left(-\frac{1}{3} - s\right)V_2$$

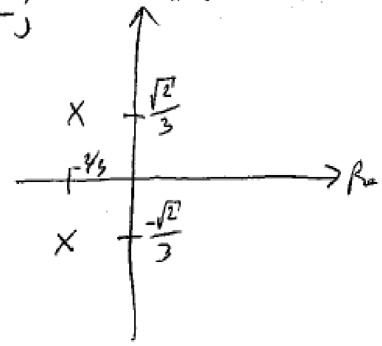
$$V_1 - \left(\frac{4}{3} + s + 4sC + 3s^2C\right)V_2 = \left(-\frac{1}{3} - s\right)V_2$$

$$V_1 = \left(+1 + 4sC + 3s^2C\right)V_2$$

$$\frac{V_2}{V_1} = \frac{1}{3s^2C + 4sC + 1}$$

So, for (a) $C = \frac{1}{2} \implies \frac{V_2}{V_1} = \frac{1}{\frac{3}{2}s^2 + 2s + 1}$

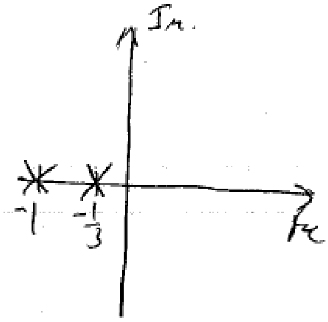
roots : poles $= -\frac{2}{3} + \frac{\sqrt{2}}{3}j, \; -\frac{2}{3} - \frac{\sqrt{2}}{3}j$



b) C=1F

$$\frac{V_L}{V_1} = \frac{1}{3s^2+4s+1} = \frac{1}{(3s+1)(s+1)}$$

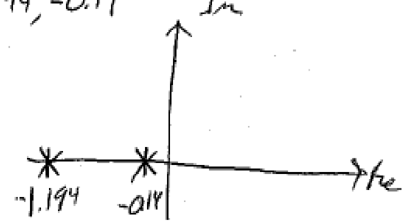
poles $= -\frac{1}{3}, -1$



c) C=2F

$$\frac{V_L}{V_1} = \frac{1}{6s^2+8s+1} \Rightarrow \text{poles} = -\frac{2}{3} \pm \frac{\sqrt{40}}{12}$$

$= -1.194, -0.14$



6

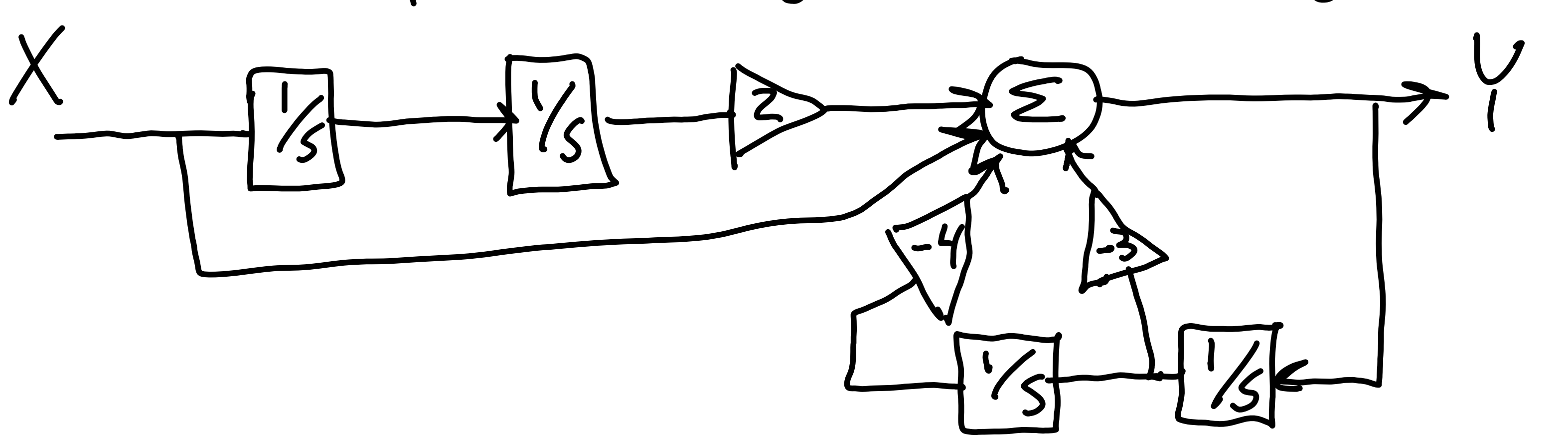
5.52 Use integrators, adders, and scalers to simulate the transfer function

$$H(s) = \frac{(s^2+2)}{(s^2+3s+4)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{(s^2+2)}{(s^2+3s+4)}$$

$$s^2X(s) + 2X(s) = Y(s)s^2 + Y(s)3s + Y(s)4$$

$$Y(s) = X(s) + \frac{2}{s^2}X(s) - \frac{3}{s}Y(s) - \frac{4}{s^2}Y(s)$$



7. (0 points) FoEE 5.17

5.17 For the circuit shown in Fig. P5.15, w
$R = 2\ \Omega$, $L = 1$ H, and $C = 1$ F, then

$$\mathbf{H}_C(j\omega) = \frac{\mathbf{V}_C}{\mathbf{V}_1} = \frac{1}{1 + j\omega 2 + (j\omega)^2}$$
$$= \left(\frac{1}{1+j\omega}\right)\left(\frac{1}{1+j\omega}\right)$$

Thus $\mathbf{H}_C(j\omega)$ can be expressed as the prod
$\mathbf{H}_C(j\omega) = \mathbf{H}_1(j\omega)\mathbf{H}_2(j\omega)$, where $\mathbf{H}_1(j\omega) = \mathbf{H}_2($
$= 1/(1 + j\omega)$. Use only the straight-line asymptc
to sketch the Bode plot—both the amplitude a
phase responses—for $\mathbf{H}_C(j\omega)$ by adding the Bc
plots for $\mathbf{H}_1(j\omega)$ and $\mathbf{H}_2(j\omega)$. What type of filter
this?

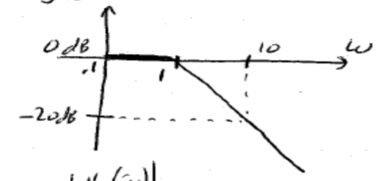
⑦ FoEE 5.17 : $H_1(j\omega) = H_2(j\omega) = \frac{1}{1+j\omega}$

$|H_1(j\omega)| = |H_2(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$

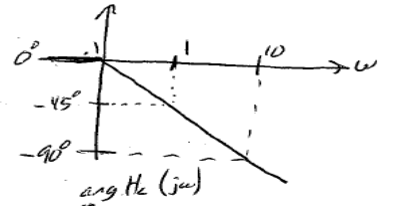
$\angle H_1(j\omega) = \angle H_2(j\omega) = -\tan^{-1}\omega$

So,

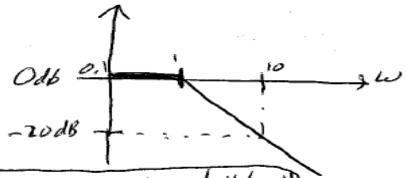
$20\log_{10}|H_1(j\omega)|$



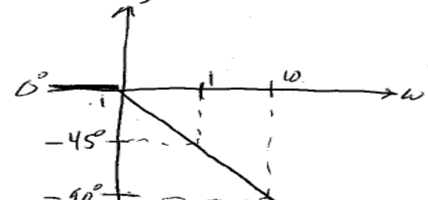
ang $H_1(j\omega)$



$20\log_{10}|H_2(j\omega)|$

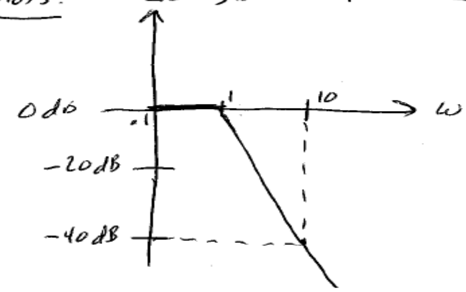


ang $H_2(j\omega)$

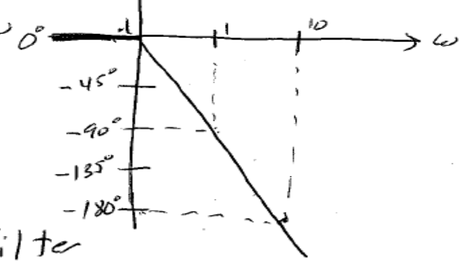


Adding plots:

$20\log_{10}|H(j\omega)|$



ang $H(j\omega)$

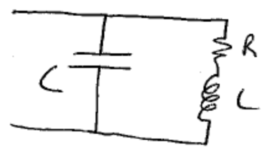


Low Pass Filter

8. (0 points) FoEE 5.38

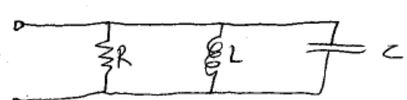
5.38 Consider the practical tank circuit shown in Fig. 5.17 on p. 285. Suppose that $R_s = 50\ \Omega$, $L = 50$ mH, and $C = 0.005\ \mu$F. Approximate this admittance by a parallel RLC connection. What is the quality factor of this parallel connection?

⑧ FoEE 5.38



$C = .005\ \mu F$
$R = 50\ \Omega$
$L = 50\ mH$

This is a high Q coil, so we can approximate this as:



where $R = \frac{L}{R_s C}$

See pgs. 285–286

So, $R = \frac{50\ mH}{50 \cdot (.005)\ \mu F}$

$\boxed{R = 200\ k\Omega}$

$$\boxed{Q = R\sqrt{\frac{C}{L}} = 200K\sqrt{\frac{.005\ \mu F}{50\ mH}} = 63.2}$$

9. (0 points) FoEE 5.51

5.51 Use integrators, adders, and scalers to simulate the transfer function

$$\mathbf{H}(s) = \frac{4s}{(s^2 + 2s + 3)}$$

FoEE 5.51

$$H(s) = \frac{Y}{X} = \frac{4s}{s^2+2s+3} \qquad (s^2+2s+3)Y = 4sX$$

$$s^2Y + 2sY + 3Y = 4sX$$

$$Y + \frac{2Y}{s} + \frac{3Y}{s^2} = \frac{4X}{s}$$

$$Y = \frac{4X}{s} - \frac{2}{s}Y - \frac{3}{s^2}Y$$