Chapter 16 Analog Communication Systems – Instructor Notes

Chapter 16 introduces a new section on communications systems. The section consists of two chapters, 16 and 17, of which chapter 16 is an evolution of the communications chapter (19) that had been available in electronic form since the third edition of this book. Chapter 16 has been revised and updated, and a new chapter on digital communications has been written by Dr. Michael Carr of the Ohio State University ElectroScience Laboratory. The addition of a new section has been prompted by requests from a number of instructors who feel that communications is a topic of growing importance, and that an overview of analog and digital communications should be part of the education of all engineers.

Chapter 16 builds on the basic system concepts built in Chapter 6, and **can** therefore **be covered immediately after Chapter 6,** if so desired, **offering the instructor yet another option to cap a course in basic circuits with a more advanced topic**.

Section 16.1 presents a brief introduction to communications systems, explaining the basic block diagram of an an alog communication system. Section 16.2 expands on the Fourier Series topic covered in Chapter 6 (pp. 289 – 296), introducing the Fourier Transform and exploring the concept of bandwidth. Sections 16.3 and 16.4 cover the concepts of AM and FM signals, respectively. Both sections contain numerous examples, and make significant use of Matlab™. The Matlab files used in the examples and homework problems are available to the instructor on the course website. The chapter closes with a brief section describing two practical applications: GPS and Sonar.

Chapter 16 includes 26 homework problems, most of which are extensions of the examples presented in the text. Many of the homework problems require the use of Matlab™. The instructor can find the required files on the course website.

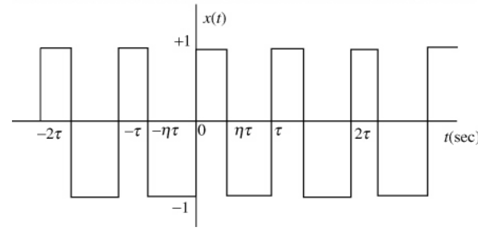
Learning Objectives

1. Become familiar with common types of communications systems in block diagram form. *Section 16.1.*
2. Learn how to perform spectral analysis of simple sugnals using analytical and computer-aided tools. *Section 16.2.*
3. Understand the principles of AM modulation and demodulation, and perform basic calculations and numerical computations on AM signals. *Section 16.3.*
4. Understand the principles of AM modulation and demodulation, and perform basic calculations and numerical computations on FM signals. *Section 16.4.*

Section 16.2: Fourier Series and Transform

Problem 16.1

Solution:



Known quantities:

Please note the following definition of the signal, which is slightly different from that in the text.

The square wave signal: $x(t) = \begin{cases} 1 & for \quad t \le \eta\tau \\ -1 & for \quad \eta\tau < t < \tau \end{cases}$, $\tau = \frac{1}{300}\text{sec}$,

$\eta = 50\%, 30\%$

Find:

a. The Fourier series coefficients for the square wave signal.
b. Frequency spectrum of the signal for the numerical values.

Analysis:

a. $\eta = 50\%$

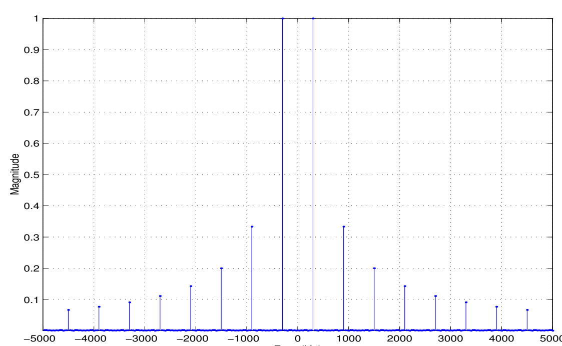
The wave signal is an odd function, so we need to compute only the b_n Fourier coefficients. The interval of integration $0 \le t \le \tau$ would be convenient.

$$a_0 = \frac{1}{\tau}\left[\int_0^{\frac{\tau}{2}} dt - \int_{\frac{\tau}{2}}^{\tau} dt\right] = 0$$

$$b_n = \frac{2}{\tau}\left[\int_0^{\frac{\tau}{2}} \sin(n\frac{2\pi}{\tau}t)dt - \int_{\frac{\tau}{2}}^{\tau} \sin(n\frac{2\pi}{\tau}t)dt\right] = \frac{2}{\tau}\left[\frac{-\cos(n\frac{2\pi}{\tau}t)}{\frac{2\pi n}{\tau}}\Big|_0^{\tau/2} + \frac{\cos(n\frac{2\pi}{\tau}t)}{\frac{2\pi n}{\tau}}\Big|_{\tau/2}^{\tau}\right] = \frac{2}{n\pi}[1-\cos(n\pi)]$$

$$= \begin{cases} \frac{4}{n\pi} & n: odd \\ 0 & n: even \end{cases} \qquad n \ne 0$$

$$\therefore x(t) = \frac{4}{\pi}\left[\sin\frac{2\pi t}{\tau} + \frac{1}{3}\sin\frac{6\pi t}{\tau} + \cdots\right]$$



b. $\eta = 30\%$

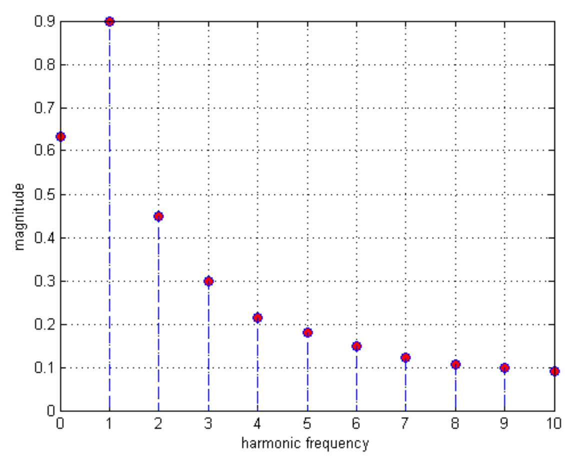
$$a_0 = \frac{1}{\tau}\left[\int_0^{\frac{\tau}{3}} dt - \int_{\frac{\tau}{3}}^{\tau} dt\right] = -\frac{1}{3}$$

$$a_n = \frac{2}{\tau}\left[\int_0^{\frac{\tau}{3}} \cos(n\frac{2\pi}{\tau}t)dt - \int_{\frac{\tau}{3}}^{\tau} \cos(n\frac{2\pi}{\tau}t)dt\right] = \frac{2}{\tau}\cdot\left[\sin(n\frac{2\pi}{\tau}t)\Big|_0^{\tau/3} - \sin(n\frac{2\pi}{\tau}t)\Big|_{\tau/3}^{\tau}\right]\Big/\frac{n2\pi}{\tau} = \frac{2}{\mathrm{n}\pi}\sin(\frac{2\pi n}{3})$$

$$b_n = \frac{2}{\tau}\left[\int_0^{\frac{\tau}{3}} \sin(n\frac{2\pi}{\tau}t)dt - \int_{\frac{\tau}{3}}^{\tau} \sin(n\frac{2\pi}{\tau}t)dt\right] = \frac{1}{n\pi}\left[-\cos(n\frac{2\pi}{\tau}t)\Big|_0^{\tau/3} + \cos(n\frac{2\pi}{\tau}t)\Big|_{\tau/3}^{\tau}\right] = \frac{-2}{n\pi}\cos(\frac{2n\pi}{3}) \quad n \neq 0$$

$$c_n = \sqrt{a_n^2 + b_n^2} = \sqrt{[\frac{2}{\mathrm{n}\pi}\sin(\frac{2\pi n}{3})]^2 + [\frac{-2}{n\pi}\cos(\frac{2n\pi}{3})]^2} = \frac{2\sqrt{2}}{n\pi}$$

n	a_n	b_n	c_n
0	0.55133	0.31831	0.633
1	-0.27566	0.15915	0.9
2	-0.0	-0.21221	0.45
3	0.13783	0.079577	0.3
4	-0.11027	0.063662	0.215
5	-0.0	-0.1061	0.18
6	0.078761	0.045473	0.15
7	-0.068916	0.039789	0.123
8	-0.0	-0.070736	0.1075
9	0.055133	0.031831	0.1
10	-0.050121	0.028937	0.09



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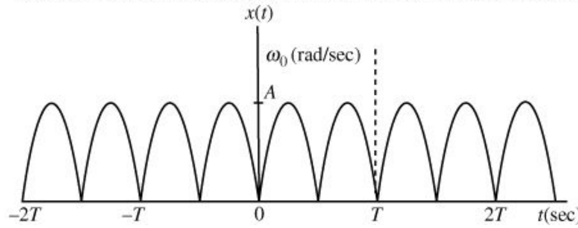
Problem 16.2

Solution:

Known quantities:

Functional form of a full-wave rectified sinusoidal signal of time period T sec, $x(t) = |\sin(\omega_o t)|$, and natural frequency $\omega_0 = 200\pi \dfrac{rad}{s}$.



Find:

a. The Fourier series coefficients.
b. Frequency spectrum of the signal.

Analysis:

The rectified sine wave signal is an even function. Hence, we need to compute only the a_n coefficients of the Fourier series.

$$a_0 = \frac{1}{T}\int_0^T |\sin(\omega_o t)|\,dt = \frac{\omega_o}{\pi}\int_0^{\pi/\omega_o} \sin(\omega_o t)dt = \frac{2}{\pi}$$

$$a_n = \frac{2}{2\pi/\omega_o}\int_0^{2\pi/\omega_o} |\sin(\omega_o t)|\cos(n\omega_o t)d(\omega_o t) = \frac{1}{\pi}\int_0^{2\pi} |\sin(\omega_o t)|\cos(n\omega_o t)d(\omega_o t)$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin(\omega_o t)\cos(n\omega_o t)d(\omega_o t) = \frac{1}{\pi}\int_0^{\pi} (\sin(1+n)\omega_o t + \sin(1-n)\omega_o t)d(\omega_o t)$$

$$= \frac{1}{\pi}\left[-\frac{\cos(1+n)\omega_o t}{n+1} - \frac{\cos(1-n)\omega_o t}{1-n}\right]_0^{\pi} = \begin{cases} -\dfrac{2}{\pi}\dfrac{2}{(n-1)(n+1)} & \text{n : even} \\ 0 & \text{n : odd} \end{cases}$$

$$b_n = 0$$

$$x(t) = -\frac{2}{\pi}\left[-1 + \frac{2}{3}\cos(2\omega_0 t) + \frac{2}{15}\cos(4\omega_0 t) + \frac{2}{35}\cos(6\omega_0 t)\cdots\right]$$

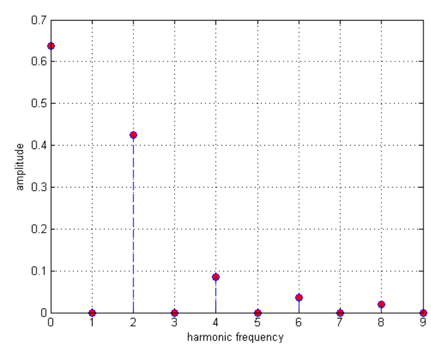
b. To compute the spectrum of the signal, we have

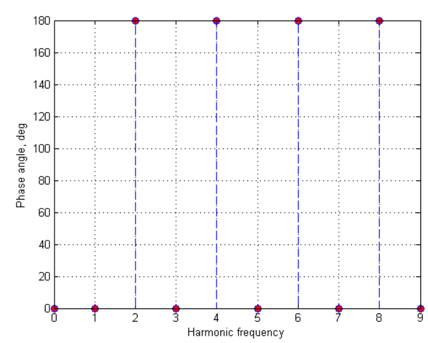
$$c_n = \sqrt{a_n^2 + b_n^2} = |a_n|$$

$$\text{phase } \psi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = \tan^{-1}(0)$$

$$\text{So } \psi_0 = 0, \psi_n = \begin{cases} \pi, n = 2,4,6,8,... \\ 0, n = 1,3,5,... \end{cases}$$

The spectrum of x(t) is shown on the right:



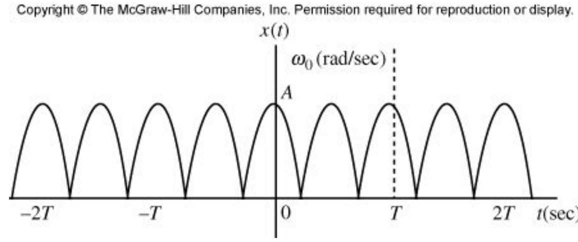


Problem 16.3

Solution:

Known quantities:

Functional form of a full-wave rectified cosine wave of time period T sec, $x(t) = |\cos(\omega_o t)|$, and natural frequency $\omega_0 = 150\pi \dfrac{rad}{s}$.



Find:

a. The Fourier series coefficients.
b. Frequency spectrum of the signal.

Analysis:

a. The rectified sine wave signal is an even function. Hence, we need to compute only the a_n coefficients of the Fourier series.

$$a_0 = \frac{1}{T}\int_0^T |\cos(\omega_o t)|\, dt = \frac{1}{2\pi}\int_0^{2\pi} |\cos(\omega_o t)|\, d(\omega_o t) = \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} \cos(\omega_o t)\, d(\omega_o t) = \frac{2}{\pi}$$

$$a_n = \frac{2}{2\pi/\omega_o}\int_0^{2\pi/\omega_o} |\cos(\omega_o t)| \cos(n\omega_o t)\, d(\omega_o t) = \frac{1}{\pi}\int_0^{2\pi} |\cos(\omega_o t)| \cos(n\omega_o t)\, d(\omega_o t)$$

$$= \frac{2}{\pi}\left(\int_0^{\pi/2} \cos(\omega_o t)\cos(n\omega_o t)\, d(\omega_o t) - \int_{\pi/2}^{\pi} \cos(\omega_o t)\cos(n\omega_o t)\, d(\omega_o t)\right)$$

$$= \frac{1}{\pi}\left(\int_0^{\pi/2} \left(\cos(1+n)\omega_o t + \cos(n-1)\omega_o t\right) d(\omega_o t) - \int_{\pi/2}^{\pi} \left(\cos(1+n)\omega_o t + \cos(n-1)\omega_o t\right) d(\omega_o t)\right)$$

$$= \frac{1}{\pi}\left[\frac{\sin(1+n)\omega_o t}{n+1} + \frac{\sin(n-1)\omega_o t}{n-1}\right]_0^{\pi/2} - \frac{1}{\pi}\left[\frac{\sin(1+n)\omega_o t}{n+1} + \frac{\sin(n-1)\omega_o t}{n-1}\right]_{\pi/2}^{\pi}$$

$$= \begin{cases} \dfrac{2}{\pi}\dfrac{2}{(n-1)(n+1)} & \text{n}: 2,6,10... \\ -\dfrac{2}{\pi}\dfrac{2}{(n-1)(n+1)} & \text{n}: 4,8,12... \\ 0 & \text{n}: \text{odd} \end{cases}$$

$$b_n = 0$$

, $$x(t) = \frac{2}{\pi}\left[1 + \frac{2}{3}\cos(2\omega_0 t) - \frac{2}{15}\cos(4\omega_0 t) + \frac{2}{35}\cos(6\omega_0 t) \cdots\right]$$

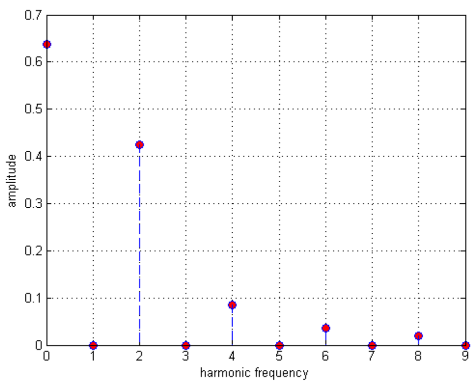
b. To compute the spectrum of the signal, we have

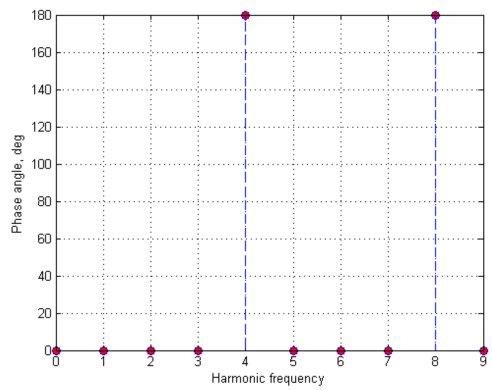
$$c_n = \sqrt{a_n^2 + b_n^2} = | a_n |$$

$$\text{phase } \psi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = \tan^{-1}(0)$$

$$\text{So } \psi_0 = 0, \psi_n = \begin{cases} 0, n = 1,3,5... \\ 0, n = 2,6,10... \\ \pi, n = 4,8,12 \end{cases}$$

The spectrum of x(t) is shown in the below:

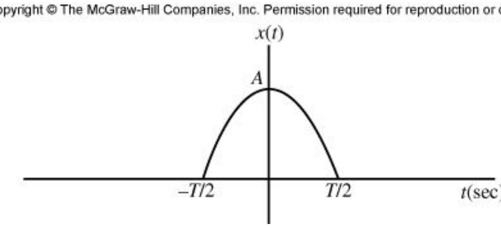




Note: Though the magnitude spectrum is the same with P16.2, the phase spectrum is different.

Problem 16.4

Solution:



Known quantities:

Functional form of a cosine burst as shown in Fig. 16.4 and

$$x(t) = \cos(\frac{\pi}{T}t)$$

mathematically defined as:

Find:

Fourier transform for the cosine burst.

Analysis:

We define this cosine burst mathematically as follows:

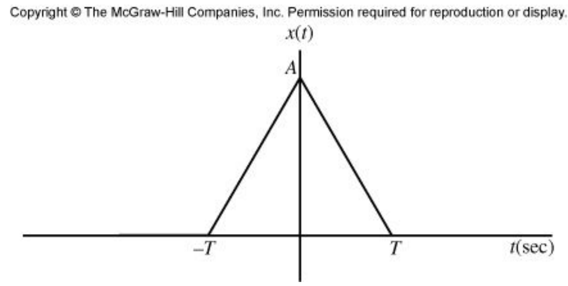
$$x(t) = \begin{cases} 1 \quad \text{for } | t | < T/2 \\ 0 \quad \text{for } | t | > T/2 \end{cases}$$

The Fourier transform of x(t) is :

$$X(\omega) = \int_{-T/2}^{T/2} \cos(\frac{\pi}{T}t)e^{-j\omega t}dt = \left[e^{-j\omega t} \frac{-j\omega \cos\frac{\pi}{T}t + \frac{\pi}{T}\sin\frac{\pi}{T}t}{(-j\omega)^2 + (\pi / T)^2} \right]_{-T/2}^{T/2} = \frac{2T\pi}{\pi^2 - (T\omega)^2}\cos\frac{T}{2}\omega$$

Problem 16.5

Solution:



Known quantities:

Functional form of a triangular pulse signal as shown in Fig. 16.5 and mathematically defined as:

$$x(t) = A\left[1-\frac{|t|}{T}\right]\left(u(t+T)-u(t-T)\right)$$

Find:

a. Fourier transform of the function.
b. Plot the frequency spectrum of the triangular pulse of period, $\mathrm{T}=0.01$ sec and amplitude, $\mathrm{A}=0.5$.

Analysis:

The mathematical equation for the triangular pulse can be split into a function defined over different periods as follows:

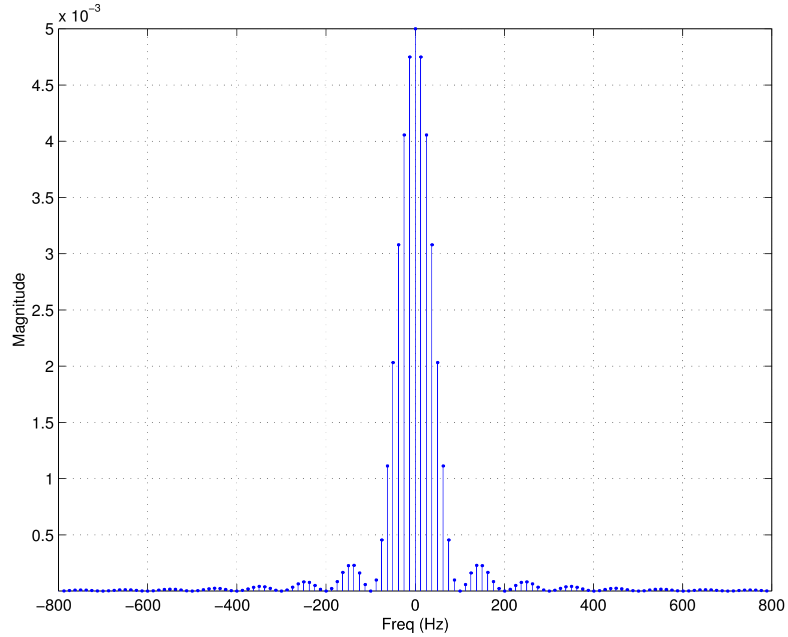
$$x(t) = \begin{cases} A\left[1+\dfrac{t}{T}\right] & \text{for} \quad -T \le t < 0 \\ A\left[1-\dfrac{t}{T}\right] & \text{for} \quad 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

The Fourier transform is defined as: $X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)dt$ and can be computed for the triangular pulse as follows:

$$\begin{aligned}
X(f) &= \int_{-T}^{T} A\left[1-\frac{|t|}{T}\right]\exp(-j2\pi ft)dt \\
&= \int_{-T}^{0} A\left[1+\frac{t}{T}\right]\exp(-j2\pi ft)dt + \int_{0}^{T} A\left[1-\frac{t}{T}\right]\exp(-j2\pi ft)dt \\
&= A\left\{\frac{j\exp(j\pi fT)}{2\pi^2 f^2 T}\left[\frac{\exp(-j\pi fT)-\exp(j\pi fT)}{2j}\right] + \frac{j\exp(-j\pi fT)}{2\pi^2 f^2 T}\left[\frac{\exp(j\pi fT)-\exp(-j\pi fT)}{2j}\right]\right\} \\
&= A\left\{\frac{j\exp(j\pi fT)}{2\pi^2 f^2 T}\left[-\sin(\pi fT)\right] + \frac{j\exp(-j\pi fT)}{2\pi^2 f^2 T}\left[\sin(\pi fT)\right]\right\} \\
&= \frac{A}{\pi^2 f^2 T}\sin(\pi fT)\left[\frac{-\exp(-j\pi fT)+\exp(j\pi fT)}{2j}\right] \\
&= \frac{AT}{(\pi fT)^2}\sin(\pi fT)\sin(\pi fT)
\end{aligned}$$

$$X(f) = AT\mathrm{sinc}^2(fT) \qquad \text{where } \mathrm{sinc(fT)} = \frac{\sin(\pi \mathrm{fT})}{\pi \mathrm{fT}}$$

b.



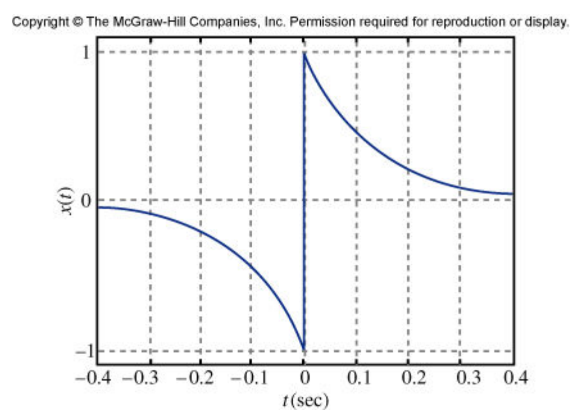
Problem 16.6

Solution:

Known quantities:

Functional form of a exponential pulse signal as shown in Fig. 16.6 and mathematically defined as:

$$x(t) = \begin{cases} (\exp)^{-at}, & \text{for } \quad t > 0 \\ 0, & \text{for } \quad t = 0 \\ -(\exp)^{at}, & \text{for } \quad t < 0 \end{cases}$$



Find:
Fourier transform for the exponential pulse.

Analysis:

a. We can formulate a compact notation for the pulse signal by using "signum function" which equals +1 for positive time and –1 for negative time. This function is defined as:

$$\mathrm{sgn}(t) = \begin{cases} 1, & \text{for } \quad t > 0 \\ 0, & \text{for } \quad t = 0 \\ -1, & \text{for } \quad t < 0 \end{cases}$$

The signal $x(t)$ can be written as:

$x(t) = \exp(-a\,|\,t\,|)\,\mathrm{sgn}(t)$

The Fourier transform is now calculated as follows:

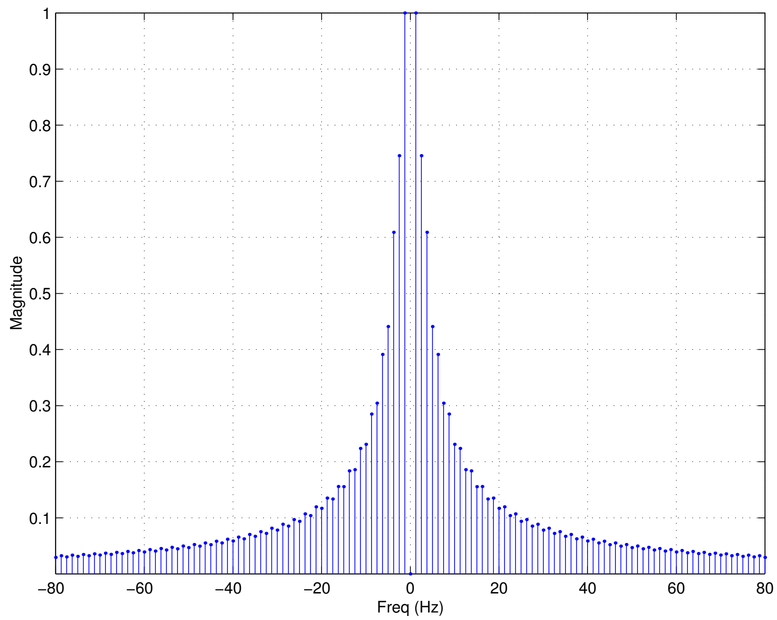
$$X(f) = \int_{-\infty}^{\infty} \exp(-a|t|)\,\mathrm{sgn}(t)\exp(-j2\pi ft)\,dt$$

$$= \int_{-\infty}^{0} -\exp((a - j2\pi f)t)\,dt + \int_{0}^{\infty} \exp(-(a + j2\pi f)t)\,dt$$

$$= -\frac{1}{a - j2\pi f} + \frac{1}{a + j2\pi f}$$

$$X(f) = \frac{-j4\pi f}{a^2 + 4\pi^2 f^2}$$

b.



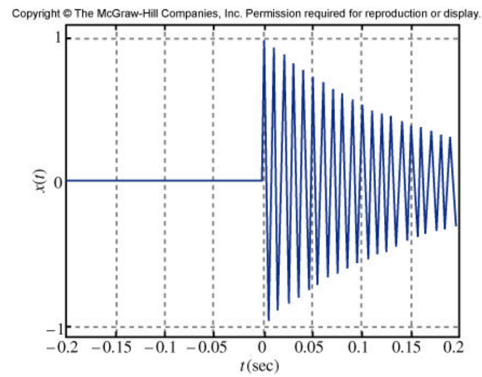
Problem 16.7

Solution:

Known quantities:

Functional form of a damped sinusoid signal as shown in Fig. 16.xx and mathematically defined as:

$x(t) = \exp(\text{-at})\cos(2\pi f_c t)u(t)$



Find:

Fourier transform for the damped sinusoid.

Analysis:

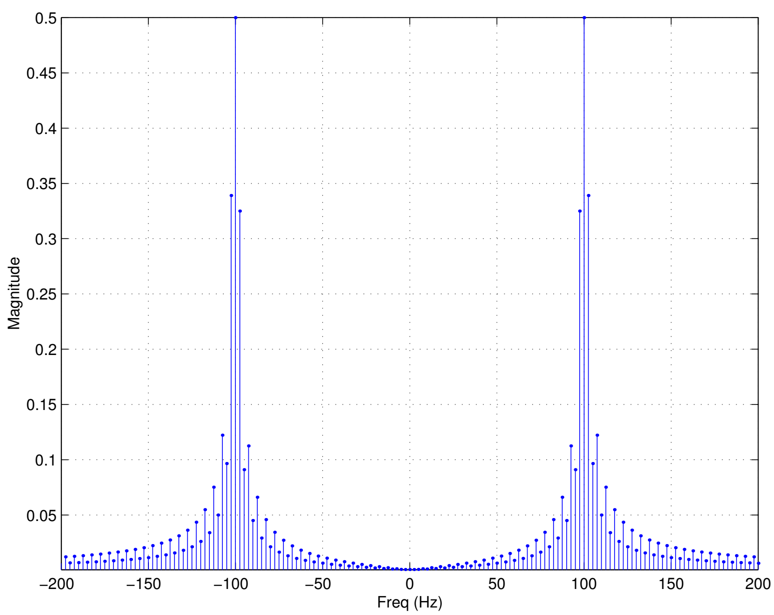
$$X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)dt$$

$$= \int_{-\infty}^{\infty} \exp(-at)\cos(2\pi f_c t)u(t)\exp(-j2\pi ft)dt$$

$$= \int_{0}^{\infty} \exp(-at)\frac{1}{2}\left[\exp(j2\pi f_c t) + \exp(-j2\pi f_c t)\right]\exp(-j2\pi ft)dt$$

$$= \frac{1}{2}\int_{0}^{\infty}\left\{\exp(-[a + j2\pi(f - f_c)]t) + \exp(-[a + j2\pi(f + f_c)]t)\right\}dt$$

$$X(f) = \frac{1}{2}\left[\frac{1}{a + j2\pi(f - f_c)} + \frac{1}{a + j2\pi(f + f_c)}\right]$$

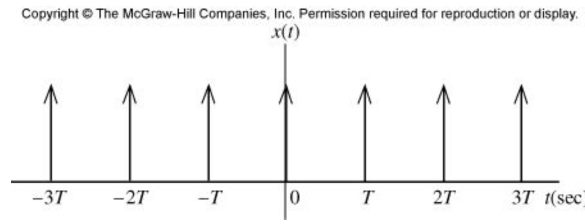


Problem 16.8

Solution:

Known quantities:

Functional form of an ideal sampling function of frequency $1/T_0$ Hz as shown in Figure 16.xx and having mathematical equation: $\delta_{T_0} = \sum_{m=-\infty}^{\infty} \delta(t - mT_0)$



Find:

a. The Fourier transform for the periodic signal.
b. Frequency spectrum of the signal for $T_o = 0.01\,\text{sec}$.

Analysis:

a. In a limiting sense, Fourier transforms can be defined for periodic signals. Therefore, it is reasonable to represent that a periodic signal can be represented in terms of a Fourier transform, provided that this transform is permitted to include delta functions. An ideal sampling function consists of an infinite sequence of uniformly spaced delta functions. We observe that the generating function for the ideal sampling function is simply a delta function $\delta(t)$. The periodic signal can be represented in terms of the complex exponential Fourier series:

$$\delta_{T_0} = \sum_{m=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t)$$

where c_n is the complex Fourier series coefficients defined as:

$$c_n = \frac{1}{T_0} \int_{-\infty}^{\infty} \delta(t) \exp(-j2\pi n f_0 t) dt$$
$$= f_0 G(nf_0)$$

where $G(nf_0)$ is the Fourier transform of $\delta(t)$ evaluated at the frequency nf_0 . For the delta function:

$$G(nf_0) = 1 \qquad \text{for all n}$$

Therefore, using the relation for Fourier transform pair for a periodic signal $g_{T_0}(t)$ with a generating function $g(t)$ and period T_0 :

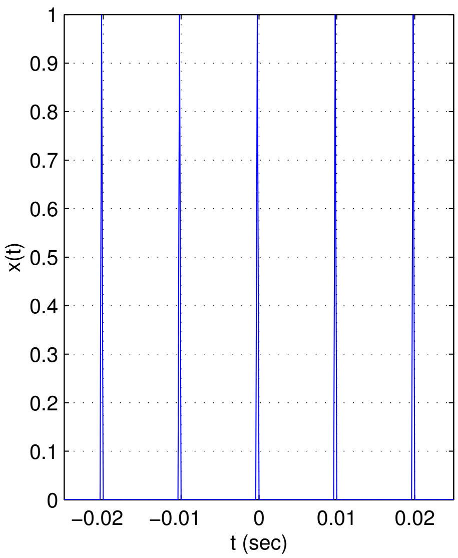
$$\sum_{m=-\infty}^{\infty} g(t - mT_0) \Leftrightarrow f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0)$$

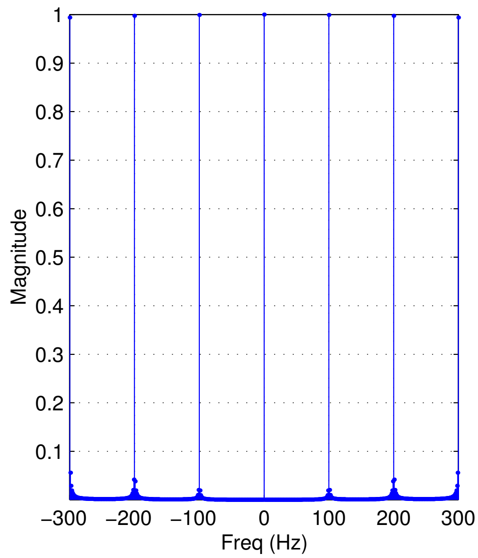
We get the Fourier transform pair for the ideal sampling function as:

$$\sum_{m=-\infty}^{\infty} \delta(t - mT_0) \Leftrightarrow f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$

We can see that the Fourier transform of a periodic train of delta functions, spaced T_0 seconds apart, consists of another set of delta functions weighted by a factor $f_0 = \dfrac{1}{T_0}$ and regularly spaced f_0 Hz apart along the frequency axis.

b.


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x(t)
−0.02
−0.01
0
0.01
0.02
t (sec)


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Magnitude
−300
−200
−100
0
100
200
300
Freq (Hz)

Problem 16.9

Solution:

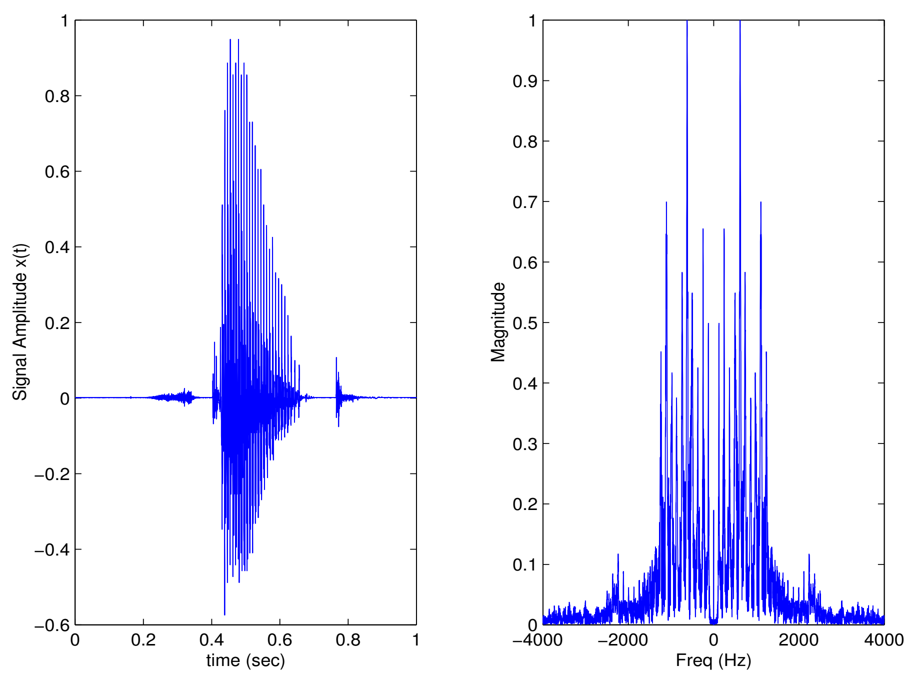
Known quantities:
Numerical form of an utterance signal as an audio file utter.au

Find:
The frequency components of the signal using the FFT tools of Matlab.

Analysis:
The Matlab program to identify the frequency components of the signal is:

```
sp = auread('utter.au');            % Loads the utterance file
l = length(sp);
t = linspace(0,1,l);
f = linspace(-l/2, l/2, l);
sp_f = abs(fftshift(fft(sp,l)));    % Frequency spectrum of the signal
sp_f = sp_f/max(sp_f);              % Normalize the frequency signal
```

The time and frequency domain signal is shown below.



Problem 16.10

Solution:

Known quantities:

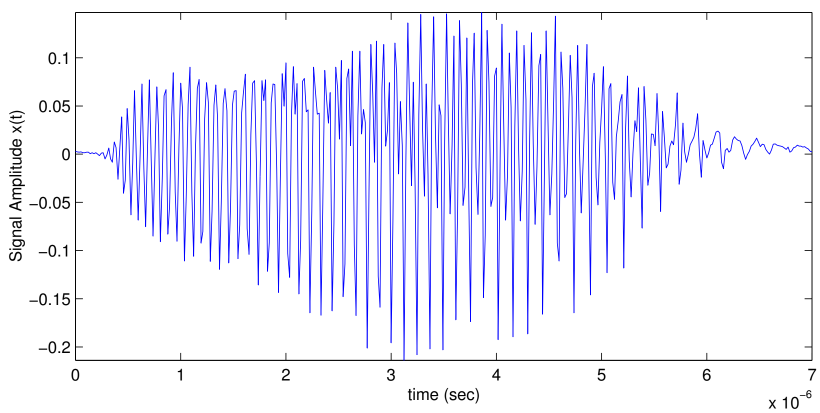
A bat echolocation chirp signal is provided.

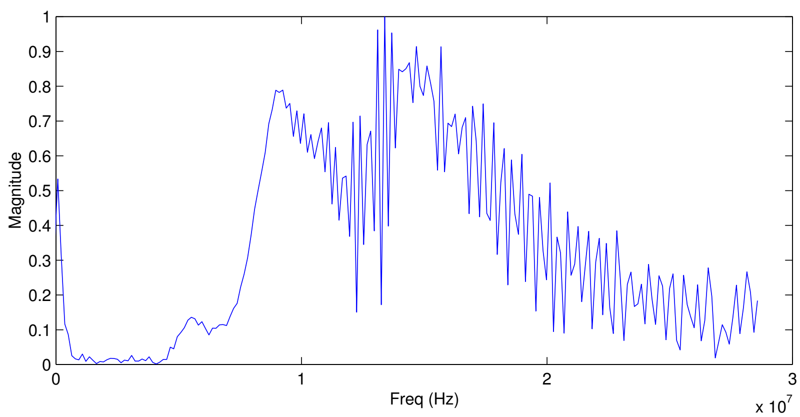
Find:

Frequency analysis of the signal and explain what you observe.

Analysis:

The time domain signal and its frequency spectrum is shown in the following figure. The message signal is a bat echolocation chirp which is $7\ \mu\text{sec}$ long and has 400 samples. From the single-sided frequency spectrum it can be observed that the signal energy is concentrated in the $9.0\ \text{MHz}$ to $19.0\ \text{MHz}$ range, the Ultra-sonic frequency band. Big brown bats produce high-frequency sound to locate their prey. Another feature of this signal is that it is a time and frequency varying signal and a very good example for Time-frequency representation analysis. The http://www-dsp.rice.edu/software/TFA/tfa.html has the TFR for this signal.





Section 16.3: Amplitude Modulation and Demodulation

Problem 16.11

Solution:

Known quantities:

Carrier signal amplitude $A_c = 1.0$, maximum modulating signal amplitude $A_{\max} = 3.0$, minimum modulating signal amplitude $A_{\min} = 0.6$

Find:

The modulation index μ for an AM modulated signal.

Analysis:

The modulation index for the signal is defined as $\mu = \frac{A_m}{A_C} = \frac{A_{\max} - A_{\min}}{2A_C} = \frac{3.0 - 0.6}{2} = 1.2$

Note: This signal is an over-modulated signal as $\mu > 1$. Hence the modulating signal amplitudes are not permissible. With $A_{\max} = 2.0$, we obtain a modulation index of 0.7.

Problem 16.12

Solution:

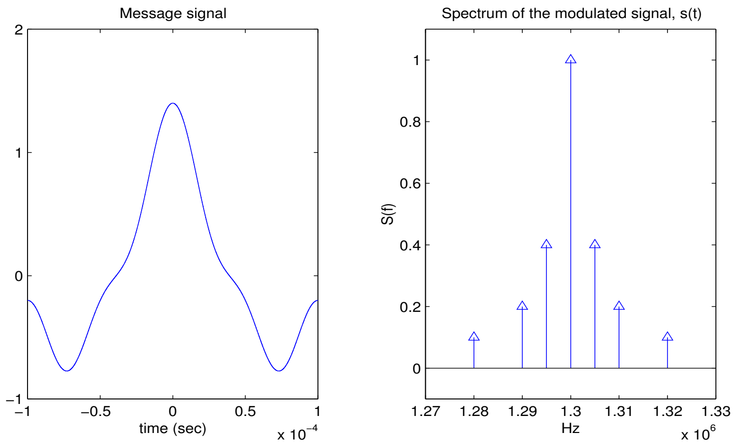
Known quantities:

Modulating signal $m(t) = 0.8\sin(2\pi 5000t) + 0.4\sin(2\pi 10000t) + 0.2\sin(2\pi 20000t)$ is amplitude modulated with a carrier signal of amplitude $A_c = 1.0$ and frequency $f_C = 1.3 \text{ MHz}$, and a modulation index $(\mu = 1)$.

Find:

Plot the frequency spectrum of the AM modulated signal.

Analysis:



Problem 16.13

Solution:

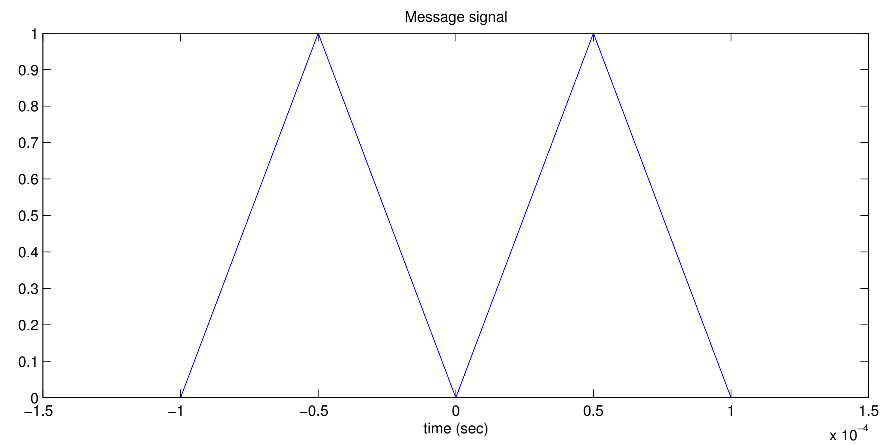
Known quantities:

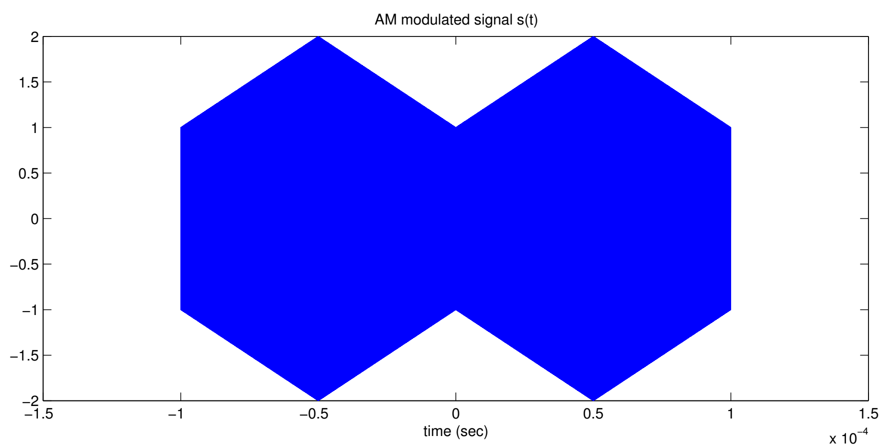
Time limited message signal $m(t) = A\left[1 - \frac{|t|}{T}\right]$ for $T = 0.0001$ sec, is AM modulated $(\mu = 1)$ with a carrier signal of unity amplitude and frequency $f_c = 10$ MHz

Find:

Plot the frequency spectrum of the AM modulated signal.

Analysis:





Comment: The time T is changed from $T = 0.01$ sec to $T = 0.0001$ sec in order to make the representation of the signal viable in Matlab.

Problem 16.14

Solution:

Known quantities:

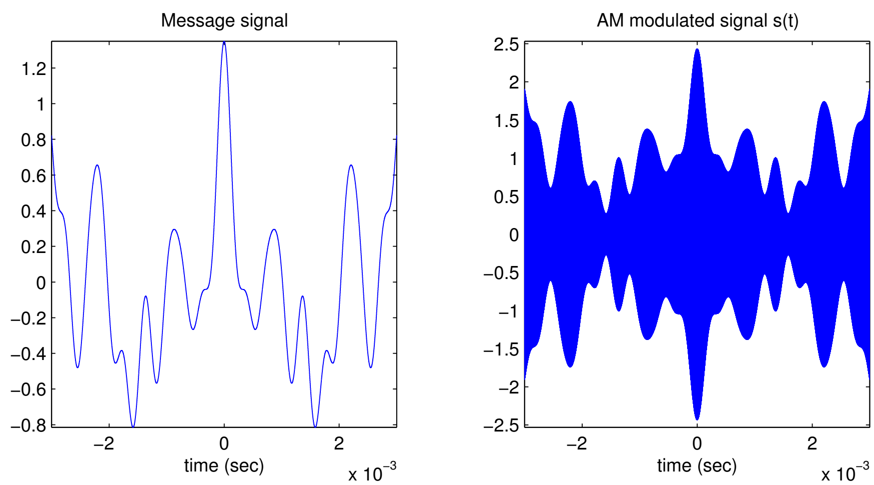
Voice signal, $m(t)$ is aired by an AM station with a carrier signal of unity amplitude and frequency $f_C = 1.6\ \text{MHz}$. Modulation index $(\mu = 1)$.

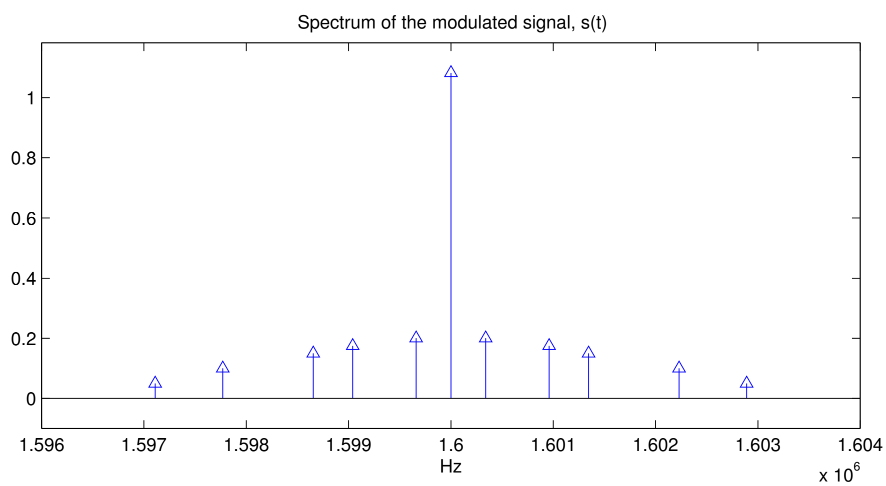
$$m(t) = 0.4\sin(2\pi 340t) + 0.35\sin(2\pi 960t) + 0.3\sin(2\pi 1345t) + 0.2\sin(2\pi 2230t) + 0.1\sin(2\pi 2890t)$$

Find:

Plot the time domain signal and the frequency spectrum of the AM modulated signal.

Analysis:





Problem 16.15

Solution:

Known quantities:

Non-periodic message signal, $m(t)$ is aired by an AM station with a carrier signal of unity amplitude and frequency $f_c = 0.5\ \text{MHz}$. Modulation index $(\mu = 1)$.

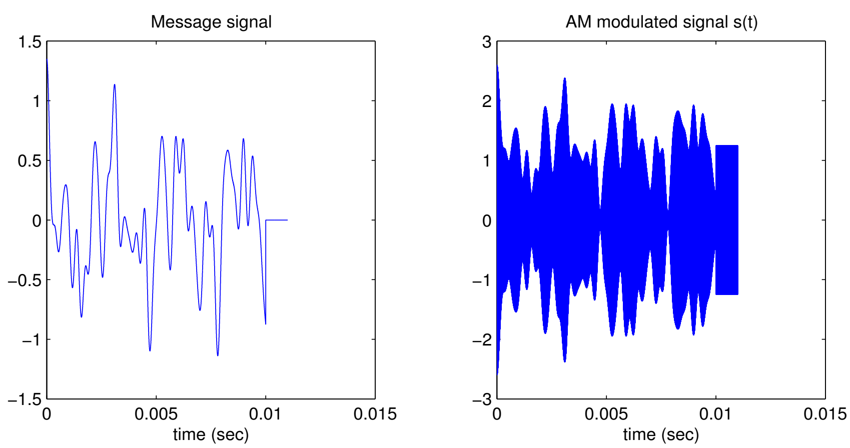
$$m(t) = 0.4\sin(2\pi 340t) + 0.35\sin(2\pi 960t) + 0.3\sin(2\pi 1345t) + 0.2\sin(2\pi 2230t) + 0.1\sin(2\pi 2890t) + u(t)$$

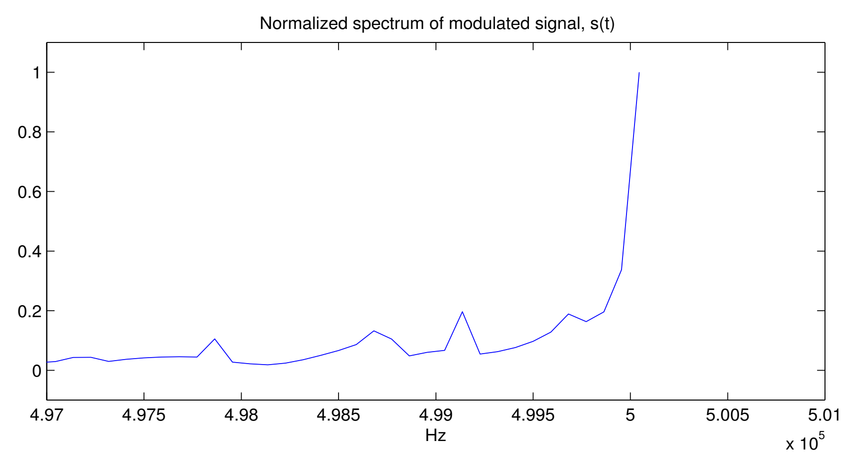
where $u(t) = \begin{cases} 1 & \text{for} \quad t \le 0.01 \\ 0 & \text{otherwise} \end{cases}$

Find:

Plot the time domain signal and the frequency spectrum of the AM modulated signal.

Analysis:





Problem 16.16

Solution:

Known quantities:

Functional form of the modulating signal $m(t)$, the carrier signal $c(t)$, and the modulation index μ.

$c(t) = A_c \cos(2\pi f_c t)$

$m(t) = A_m \cos(2\pi f_m t)$

Find:

The average power delivered to a 1-ohm resistor.

Analysis:

The AM signal is given by: $s(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$

Expressing the product of two cosines as the sum of sinusoidal waves, we get:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos\left[2\pi(f_c + f_m)t\right] + \frac{1}{2}\mu A_c \cos\left[2\pi(f_c - f_m)t\right]$$

The Fourier transform of $s(t)$ is therefore:

$$\begin{aligned} S(f) &= \frac{1}{2}A_c\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ &+ \frac{1}{4}\mu A_c\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right] \\ &+ \frac{1}{4}\mu A_c\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] \end{aligned}$$

Thus the spectrum of an AM wave, for sinusoidal modulation, consists of delta functions at $\pm f_c, f_c \pm f_m, -f_c \pm f_m$ as seen from its Fourier transform.

In practice, the AM wave $s(t)$ is a voltage or current wave. In either case, the average power delivered to a 1-ohm resistor by $s(t)$ is comprised of three components: $\text{Average power} = \frac{1}{T}\int_T x^2(t)dt$

Using Parsevals energy relation we can find average power in frequency domain as:

$\text{Average power} = |X(f)|^2 \quad \text{at } f = 0$

Hence the carrier frequency, upper side-frequency and lower side-frequency power is:

$$\text{Carrier Power} = \left|\frac{1}{2}A_c\left[\delta(-f_c) + \delta(f_c)\right]\right|^2 = \frac{1}{2}A_c^2$$

$$\text{Upper side - frequency power} = \left|\frac{1}{4}A_c\left[\delta(-f_c - f_m) + \delta(f_c + f_m)\right]\right|^2 = \frac{1}{8}\mu^2 A_c^2$$

$$\text{Lower side - frequency power} = \left|\frac{1}{4}A_c\left[\delta(-f_c + f_m) + \delta(f_c - f_m)\right]\right|^2 = \frac{1}{8}\mu^2 A_c^2$$

For a load resistor R different from 1-ohm, which is usually the case in practice, the expression for carrier power, upper side-frequency power, and lower side-frequency power are merely scaled by the factor $1/R$ or R, depending on whether the modulated wave $s(t)$ is a voltage or current, respectively.

Problem 16.17

Solution:

Known quantities:

Carrier signal frequency, $f_c = 0.82$ MHz , upper side-band frequency components at frequencies $f_{s1} = 0.825$ MHz, $f_{s2} = 0.83$ MHz, $f_{s3} = 0.84$ MHz , their amplitudes and the modulation index $\mu = 1$.

Find:

a. Modulating signal equation.
b. Plot spectrum of the modulating signal.
c. Plot the spectrum of the AM signal including the lower side-band.

Analysis:

a. We know from the theory for AM that the upper side-band frequency has frequency components at frequencies:

$$f_{sn} = f_c + f_{mn} \quad \text{where n is the number of frequency components in the modulating signal}$$

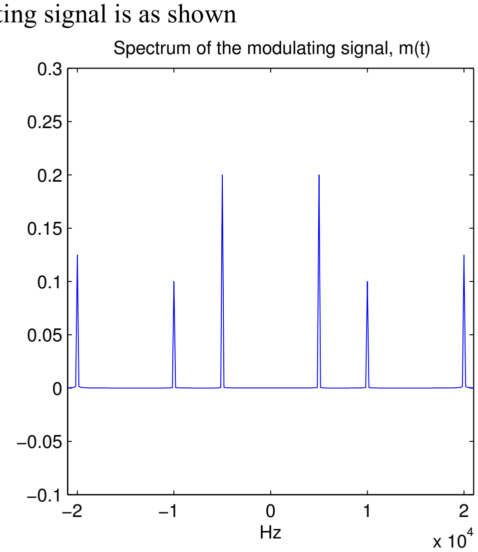
and, their amplitudes in the AM signal are $\frac{1}{2}$ times the original amplitude of the modulating signal for a modulation index $\mu = 1$. Hence, we can find the modulating signal components to be:

$$m_1(t) = 0.8\sin(2\pi 5000t)$$
$$m_2(t) = 0.4\sin(2\pi 10000t)$$
$$m_3(t) = 0.5\sin(2\pi 20000t)$$

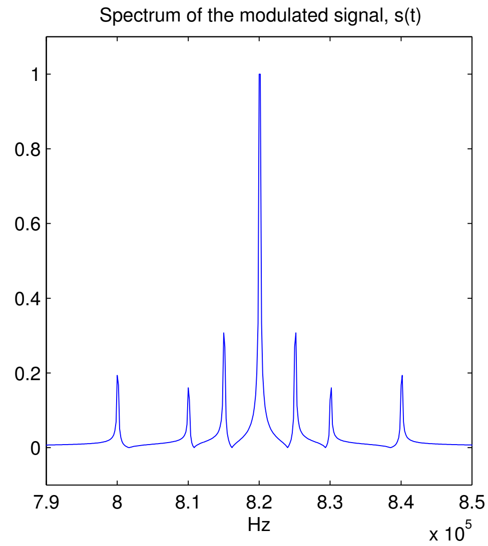
Hence the modulating signal is:

$$m(t) = 0.8\sin(2\pi 5000t) + 0.4\sin(2\pi 10000t) + 0.5\sin(2\pi 20000t)$$

b. The spectrum for the modulating signal is as shown



c. The spectrum for the AM modulated signal with the lower side-band is shown below



Problem 16.18

Known quantities:

AM frequency spectrum from $525\,\text{kHz}$ to $1.7\,\text{MHz}$, bandwidth for each channel is $10\,\text{kHz}$

Find:

a. Number of channels that can be transmitted in the given frequency range
b. The maximum modulating frequency that can be transmitted without overlap.

Analysis:

Assume: No guardband between channels.

a. The frequency range allocated for AM broadcast is

$$f_R = 1700 - 525 = 1175\,\text{kHz}$$

This range is partitioned to allow $10\,\text{kHz}$ of separation between each channel; therefore, the total number of channels, N is

$$N = \frac{1175}{10} \approx 118\,\text{channels}$$

b. The carriers of two separate channels are separated by $10\,\text{kHz}$. If we let the maximum frequency of the message signal increase, the outer edges of both sidebands move away from the carrier frequency and into each other, thereby increasing the bandwidth of each AM channel. The maximum allowable message frequency will occur at the midpoint of the spacing between the carriers. Hence, the maximum message frequency is half the frequency spacing between the carriers, $f_{m(\max)} = 5\,\text{kHz}$.

Problem 16.19

Known quantities:

A speech signal utter.au is given to you. Any US commercial AM channel can be used for transmission. Modulation index $(\mu = 1)$.

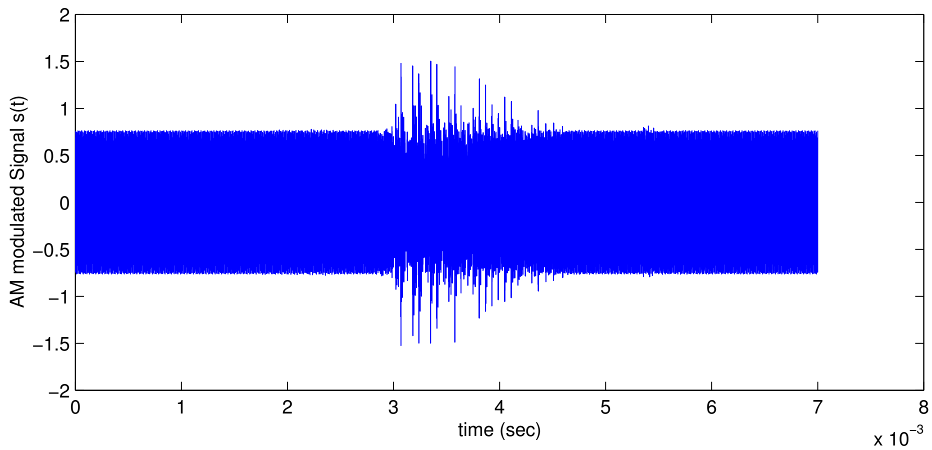
Find:

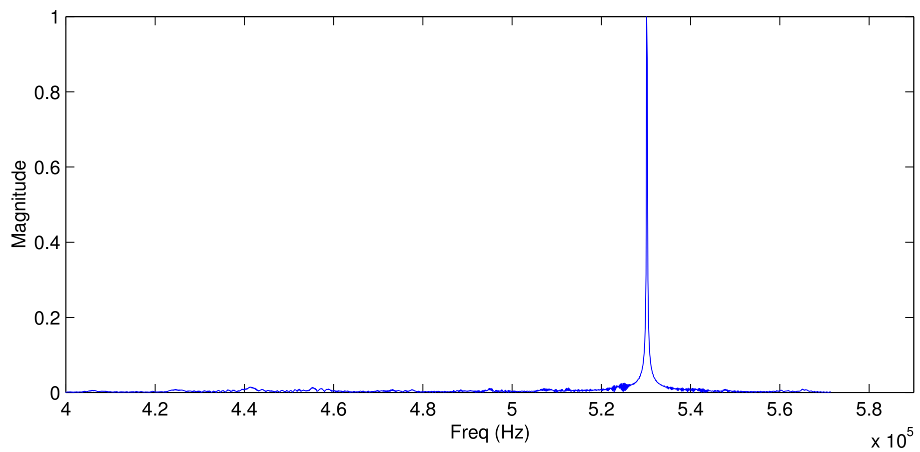
Frequency spectrum of the AM signal.

Analysis:

The speech signal utter.au is for a period of $7.0\,\text{msec}$. With a modulation index $(\mu = 1)$ the carrier signal amplitude is calculated as: $A_c = \dfrac{A_{m_{\max}} - A_{m_{\min}}}{2} = \dfrac{0.9491 + 0.5741}{2} = 0.7616$

We will use the first frequency band of the commercial AM, which is at $f_c = 0.53\,\text{MHz}$.





Section 16.4: Frequency Modulation and Demodulation

Problem 16.20

Known quantities:
Message signal, $V_m = 5\cos(750\pi t)$, carrier frequency $f_c = 105 * f_m = 105 * 375 = 39.375\,\text{kHz}$, modulation constant $K_f = 1005$.

Find:
Bandwidth of the message signal.

Analysis:
The bandwidth of the message signal is

$$\Delta f = K_f A_m$$
$$\therefore \Delta f = 5025\,\text{rad/sec}$$
$$\Delta f = \frac{2000}{2\pi}\,\text{Hz} \approx 799.8.5\,\text{Hz}$$
$$B_m = 2(f_m + \Delta f)$$
$$B_m = 2 * (375 + 799.8) = 2350\,\text{Hz}$$

Problem 16.21

Known quantities:
Message signal, $V_m = 2\cos(360\pi t)$, carrier frequency $f_c = 100 * f_m = 100 * 180 = 18.00\,\text{kHz}$, modulation constant $K_f = 1000$.

Find:
Bandwidth of the modulated signal.

Analysis:
The message signal is $V_m = 2\cos(360\pi t)$. Hence:

$$f_m = 180\,\text{Hz}$$
$$A_\text{m} = 2$$
$$f_c = 100 f_m = 18\,\text{kHz}$$

The maximum frequency deviation is given by:

$$\Delta f = K_f A_m$$
$$\therefore \Delta f = 2000\,\text{rad/sec}$$
$$\Delta f = \frac{2000}{2\pi}\,\text{Hz} \approx 318.5\,\text{Hz}$$

Bandwidth of the modulated signal is given by:

$$B_c = 2(\Delta f + f_m)$$
$$B_c = 2(318.5 + 180) = 997\,\text{Hz}$$

Problem 16.22

Known quantities:

Message signal, $V_m = 2\cos(360\pi t)$, carrier frequency $f_c = 100 * f_m = 100 * 180 = 18.00\ \text{kHz}$, modulation constant $K_f = 1000$.

Find:

Band of frequencies occupied this signal.

Analysis:

The carrier frequency is $f_c = 18\ \text{kHz}$. From Problem 16.22 we have the bandwidth of the modulated signal is $B_c = 997\ \text{Hz}$. The frequency band is centered about the carrier frequency. Therefore, the band of frequencies occupied spans from $\left(f_c - \frac{B_c}{2}\right)$ to $\left(f_c + \frac{B_c}{2}\right)$. Hence the frequency band is $\left(18000 - \frac{997}{2}\right)$ to $\left(18000 + \frac{997}{2}\right)$, which equals a band from $17.5015\ \text{kHz}$ to $18.4985\ \text{kHz}$.

Problem 16.23

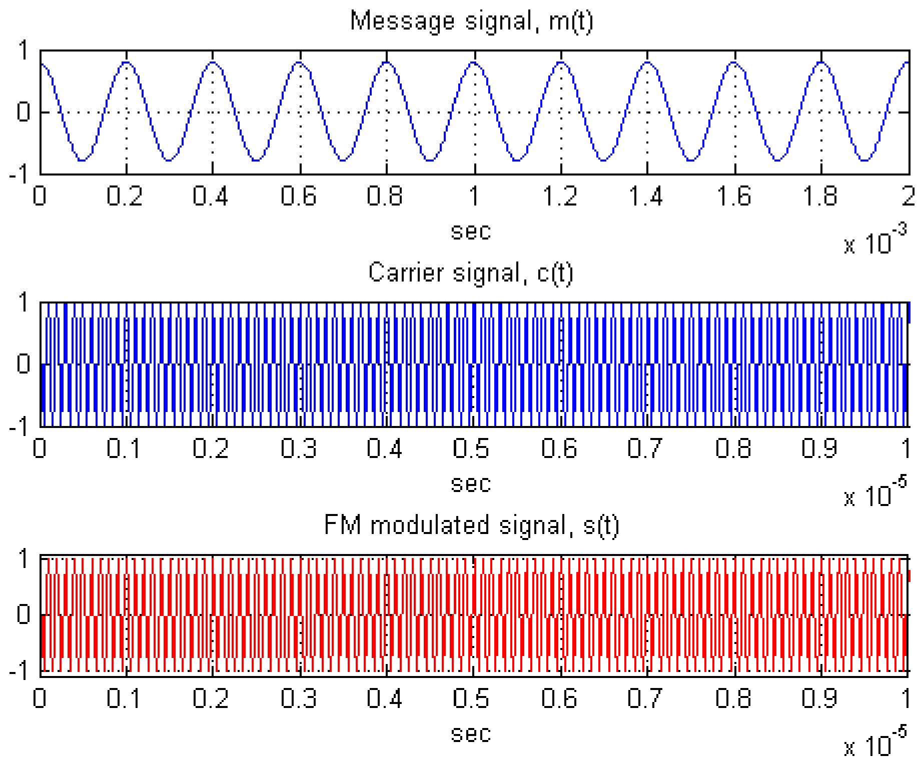
Known quantities:

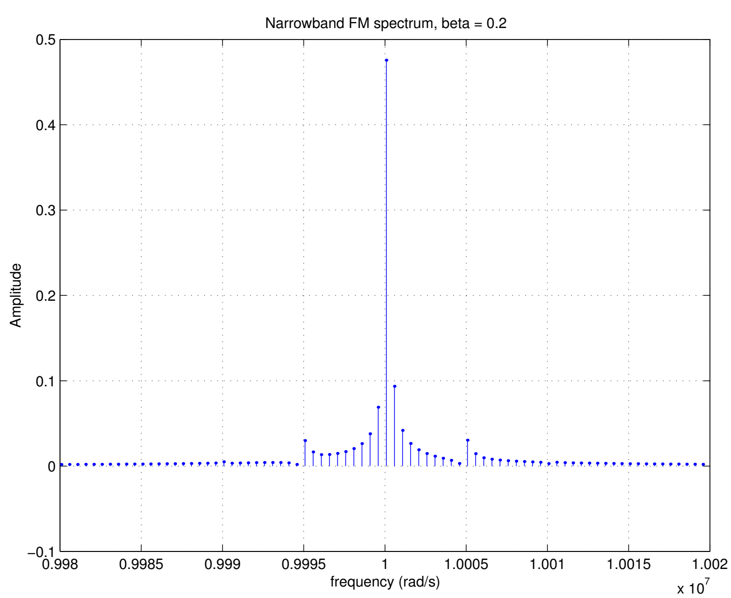
Message signal $m(t) = 0.8\sin(2\pi 5000t)$ is FM modulated by a carrier of unity amplitude and frequency $f_c = 10.0\ \text{MHz}$, with modulating constant $k_f = 1000$.

Find:

Plot the time and frequency domain FM modulated signal.

Analysis:


Message signal, m(t)
1
0
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
sec
x 10^-3
Carrier signal, c(t)
1
0
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
sec
x 10^-5
FM modulated signal, s(t)
1
0
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
sec
x 10^-5


Narrowband FM spectrum, beta = 0.2
Amplitude
0.5
0.4
0.3
0.2
0.1
0
−0.1
0.998
0.9985
0.999
0.9995
1
1.0005
1.001
1.0015
1.002
frequency (rad/s)
x 10^7

Problem 16.24

Known quantities:

A packet of information $m(t)$ is sent on a FM channel of frequency $f_c = 15.0$ MHz that uses a modulating constant $k_f = 6000$.

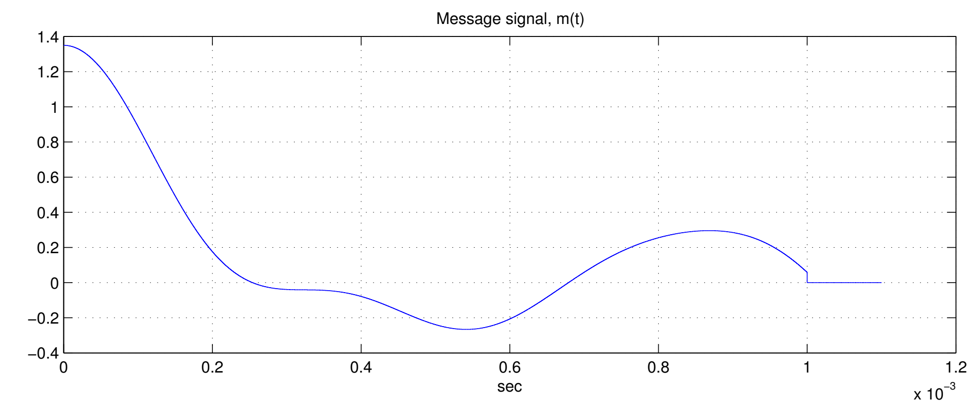
$$m(t) = 0.4\sin(2\pi 340t) + 0.35\sin(2\pi 960t) + 0.3\sin(2\pi 1345t) + 0.2\sin(2\pi 2230t) + 0.1\sin(2\pi 2890t) + u(t)$$

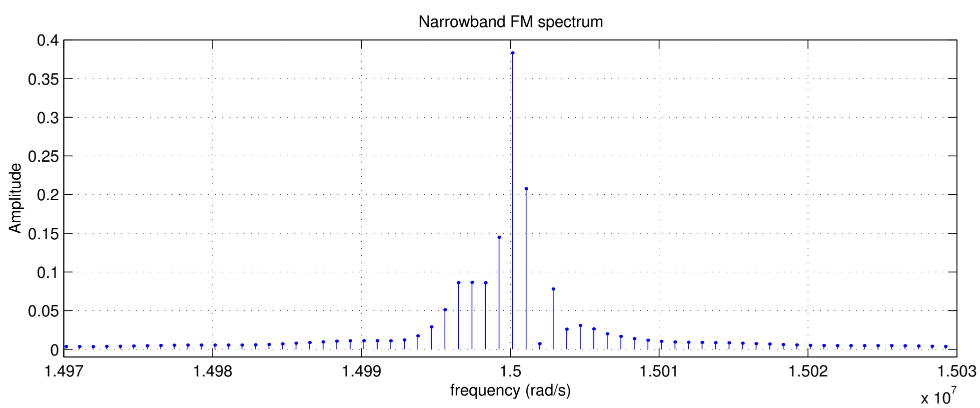
$$u(t) = \begin{cases} 1 & \text{for} \quad t \le 0.001 \\ 0 & \text{otherwise} \end{cases}$$

Find:

Plot the frequency spectrum of the FM modulated signal.

Analysis:





Problem 16.25

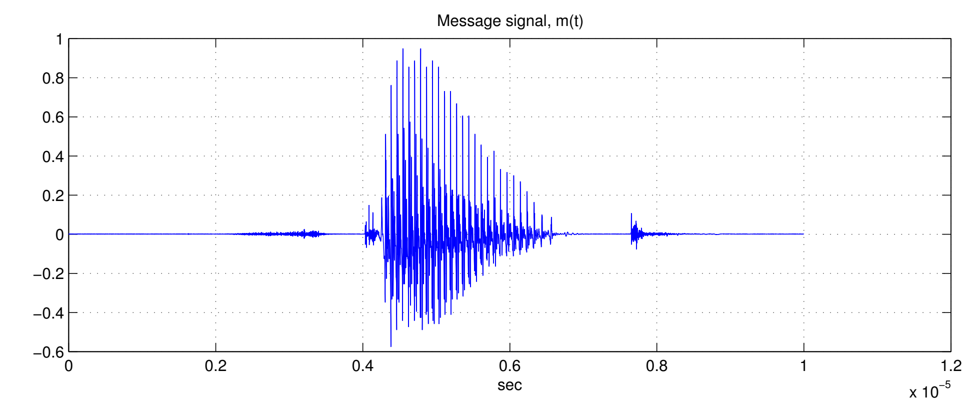
Known quantities:

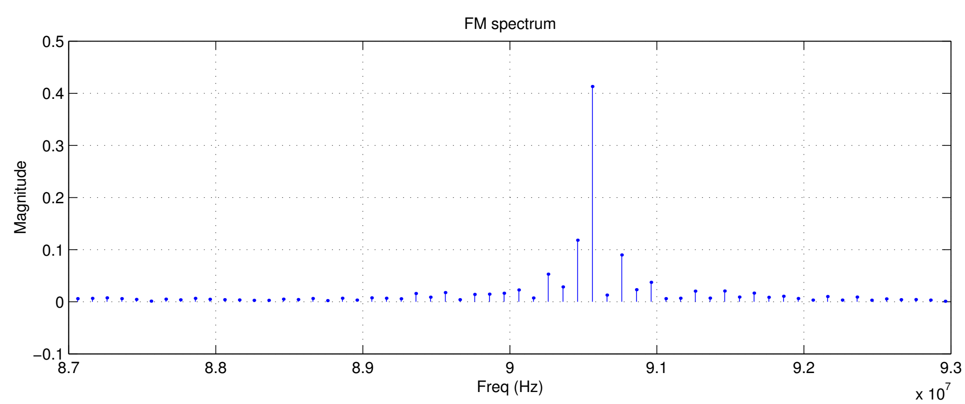
A speech signal utter.au is transmitted over National Public Radio, which uses a carrier frequency $f_c = 90.5\ \text{MHz}$ and a modulating constant $K_f = 66000$.

Find:

Plot the frequency spectrum of the FM modulated speech signal.

Analysis:





Problem 16.26

Known quantities:

Channel 2 of FM commercial broadcast in US is used for country music. The message signal is $V_m(t) = 10\cos(2\pi 10^3 t)$.

Find:

a. The carrier frequency.
b. The value of K_f

Analysis:

a. Commercial FM broadcast in US occupies frequencies from $88.0\ \text{MHz}$ to $108.0\ \text{MHz}$ with 100 possible channels. The separation between the channels is $B = 200\ \text{kHz}$. Hence channel 2 is transmitted at carrier frequency $f_c = 88.2\ \text{MHz}$.
b. To calculate the value of K_f we will use the relation of bandwidth to K_f which is:

$$B = 2\left(\Delta f + f_m\right) \text{ and } K_f = \frac{2\pi\Delta f}{A_m}$$

$$\Delta f = \frac{200000}{2} - 1000 = 99\ \text{kHz}$$

$$K_f = \frac{2\pi 99000}{10} = 62203$$