$$v^+ - v^-$$

$\pm V_{cc}$

$$V_{CC} = 15$$

r_{in}

$$r_{in} \to \infty$$

$10^{-9} \sim 10^{-12}\ A$

$$I^+ = I^- = 0$$

r_{out}

$$r_{out} \approx 0$$

V_{out}

R_L

$$1 \sim 20MHz$$

$$10^6 \sim 10^{12}\,\Omega$$

$6\ M\Omega$

Ω

A

v^-

v^{+}

$$v_{out} = A_d(v^+ - v^-) + A_c \frac{1}{2}(v^+ + v^-) \approx A_d(v^+ - v^-)$$

A_d

A_c

$$A_d \to \infty$$

$$A_c \to 0$$

$$CMRR = 20\ \log_{10}\left(\frac{A_d}{A_c}\right) > 100\ dB, \qquad (A_d > 10^5 A_c)$$

$$v_{out} = A(v^+ - v^-)$$

$$-V_{CC}$$

V_{CC}

$$A > 10^5$$

$$v^+ - v^- = v_{out}/A$$

$$v^- \approx v^+$$

$$v^{+} = 0$$

$$v^- \approx v^+ = 0$$

$$V_{out} = A(v^+ - v^-)$$

v_{out}

R_{in}

R_{out}

$$G_{oc}$$

$$A(v^+ - v^-) = A\ i_{in} r_{in}$$

$$v_{in} = i_{in}(r_{in} + r_{out}) + A\ (v^+ - v^-) = i_{in}(r_{in} + r_{out}) + A\ i_{in} r_{in} = i_{in}((A+1)r_{in} + r_{out})$$

$$i_{in}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = (A+1)r_{in} + r_{out} \approx Ar_{in}$$

$$v_{out} = v_{oc} = A(v^+ - v^-) + i_{in}r_{out} = Ai_{in}r_{in} + i_{in}r_{out} = i_{in}(Ar_{in} + r_{out})$$

$$G = \frac{v_{out}}{v_{in}} = \frac{Ar_{in} + r_{out}}{(A+1)r_{in} + r_{out}} \approx 1$$

$$(A+1)r_{in} \gg r_{out}$$

$$A \gg 1$$

$$v_{out} = v_{oc} \approx v_{in}$$

$$v^+ - v^- = v_{in}$$

$$i_{sc} = \frac{v_{in}}{r_{in}} + \frac{A(v^+ - v^-)}{r_{out}} = \frac{v_{in}}{r_{in}} + \frac{Av_{in}}{r_{out}} = v_{in}\left(\frac{r_{out} + Ar_{in}}{r_{in}r_{out}}\right) \approx v_{in}\frac{A}{r_{out}}$$

$$v_{oc} = v_{out} \approx v_{in}$$

$$R_{out} = \frac{v_{oc}}{i_{sc}} \approx \frac{v_{in}}{i_{sc}} = \left(\frac{r_{in} r_{out}}{r_{out} + A r_{in}}\right) \approx \frac{r_{out}}{A}$$

$$Ar_{in} \gg r_{out}$$

$$R_{in} \approx Ar_{in}$$

$10^{10}\Omega$

$$R_{out} \approx r_{out}/A$$

$10^{-3}\Omega$

$$R_{in} = \infty$$

$$R_{out} = 0$$

V_s

R_s

$$v_{out} = v_s \; \frac{R_L}{R_L + R_s} = v_s - v_s \left(\frac{R_s}{R_L + R_s} \right) < v_s$$

$$v_s R_s/(R_L + R_s)$$

$$v_{out} = G_{oc} v_{in} \left(\frac{R_L}{R_{out} + R_L} \right) = G_{oc} v_s \left(\frac{R_{in}}{R_s + R_{in}} \right) \left(\frac{R_L}{R_{out} + R_L} \right) \approx v_s$$

$$v_{out} \approx v_s$$

$$G_{oc} \approx 1$$

$$R_{in} \gg R_s \implies R_{in}/(R_s + R_{in}) \approx 1$$

$$R_{out} \ll R_L \implies R_L/(R_{out} + R_L) \approx 1$$

$$(R_s = 0,\ R_L = \infty)$$

$$r_{in} \gg R_1,\ R_f$$

$$r_{out} \ll R_1,\ R_f$$

$$r_{in} \approx \infty$$

$$v^{-} = v_s - i_{in} R_1$$

$$v^+ - v^- = -v^- = i_{in}R_1 - v_s$$

$$i_{in} = \frac{v_s - A(v_+ - v_-)}{R_1 + R_f} = \frac{v_s - A(i_{in}R_1 - v_s)}{R_1 + R_f}$$

$$i_{in} = \frac{(A+1)v_s}{(A+1)R_1 + R_f}$$

$$v_{out} = v_s - (R_1 + R_f)i_{in} = v_s \left[1 - \frac{(A+1)(R_1 + R_f)}{(A+1)R_1 + R_f}\right] = v_s \frac{-AR_f}{(A+1)R_1 + R_f}$$

$$G_{oc} = \frac{v_{out}}{v_s} = \frac{-AR_f}{(A+1)R_1 + R_f} \approx -\frac{R_f}{R_1}$$

$$R_s = 0$$

v_s

$$\frac{v^- - v_s}{R_1} + \frac{v^- - (-Av^-)}{R_f} = 0$$

$$v^{-} = v_s \frac{R_f}{R_f + (A+1)R_1}$$

i_{in}

$=$

$$\frac{v_s - v^-}{R_1} = \frac{v_s}{R_1}\left[1 - \frac{R_f}{R_f + (A+1)R_1}\right]$$

$$v_s \frac{1}{R_1} \frac{(A+1)R_1}{R_f + (A+1)R_1} = v_s \ \frac{A+1}{R_f + (A+1)R_1}$$

$$R_{in} = \frac{R_f + (A+1)R_1}{A+1} \approx R_1$$

$$i_{sc} = \frac{-Av^{-}}{r_{out}} + \frac{v^{-}}{R_f} = v^{-} \left(\frac{r_{out} - AR_f}{r_{out}R_f} \right)$$

$$v^- = v_s \, R_f / (R_s + R_1 + R_f)$$

$$i_{sc} = v_s \left(\frac{R_f}{R_s + R_1 + R_f} \right) \left(\frac{r_{out} - AR_f}{r_{out} R_f} \right) = v_s \, \frac{r_{out} - AR_f}{(R_s + R_1 + R_f) r_{out}}$$

$$\frac{v_s - v^-}{R_s + R_1} + \frac{(-Av^-) - v^-}{R_f + r_{out}} = 0$$

$$v^{-} = v_s \left(\frac{R_f + r_{out}}{(A+1)(R_s + R_1) + R_f + r_{out}} \right)$$

v_{OC}

$$[v^- - (-Av^-)]\frac{r_{out}}{R_f + r_{out}} - Av^- = v^-\left(\frac{r_{out} - AR_f}{R_f + r_{out}}\right)$$

$$v_s \; \frac{r_{out} - AR_f}{(A+1)(R_s + R_1) + R_f + r_{out}}$$

R_{out}

$$\frac{v_{oc}}{i_{sc}} = \frac{(R_s + R_1 + R_f)r_{out}}{(A+1)(R_s + R_1) + R_f + r_{out}}$$

$\approx$

$$\frac{(R_s + R_1 + R_f)r_{out}}{A(R_s + R_1)} \approx \left(\frac{R_1 + R_f}{R_1}\right)\frac{r_{out}}{A}$$

$R_s \ll R_1,\ R_f$

$$G_{oc} \approx -\frac{R_f}{R_1}$$

$$R_{in} \approx R_1$$

$$R_{out} \approx \left(\frac{R_1 + R_f}{R_1} \right) \frac{r_{out}}{A}$$

$$G_{oc} \approx \frac{R_1 + R_f}{R_1} > 1$$

$$R_{in} \approx \left(\frac{R_1}{R_1 + R_f} \right) A r_{in} \approx \frac{A}{G_{oc}} r_{in} < A r_{in}$$

$$R_{out} \approx \left(\frac{R_1 + R_f}{R_1} \right) \frac{r_{out}}{A} \approx \frac{G_{oc}}{A} r_{out} > r_{out}/A$$

$$G_{oc} > 1$$

$$R_f = 0$$

$$V_{out} = A(V^+ - V^-)$$

$\pm 15\,V$

$$A \to \infty$$

$$V^+ - V^- = V_{out}/A \to 0$$

$$V^{-} \approx V^{+}$$

$$V_{out} = V^- \approx V^+ = V_{in}$$

$$V_{out} = V_{in}$$

$$V^- \approx V^+ = 0$$

V^-

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_f} = 0,$$

$$V_{out} = -\frac{R_f}{R_1} V_{in}$$

R_1

R_2

Z_1

Z_2

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)}$$

$$V_{in} = V^+ \approx V^- = V_{out} \frac{R_1}{R_1 + R_f},$$

$$V_{out} = \left(\frac{R_1 + R_f}{R_1} \right) V_{in}$$

$$\sum_{k=1}^{n} \frac{V_k}{R_k} + \frac{V_{out}}{R_f} = 0,$$

$$V_{out} = -R_f \sum_{k=1}^{n} \frac{V_k}{R_k} = -\sum_{k=1}^{n} \frac{R_f}{R_k}\ V_k$$

$$V_{out} = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2 + \left(\frac{R_f}{R_1} + \frac{R_f}{R_2} + 1\right)\left(\frac{R_4}{R_3 + R_4}V_3 + \frac{R_3}{R_3 + R_4}v_4\right)$$

$$V = V^{-} \approx V^{+}$$

V^+

$$\frac{V_1 - V}{R_1} + \frac{V_{out} - V}{R_2} = 0,$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(1 + \frac{R_2}{R_1}\right)V$$

$$V \approx V^{+} = \frac{R_4}{R_3 + R_4} V_2$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2$$

V_{ref}

$$V_2 = V_{ref}$$

V_1

V_{shift}

$$V_{out} = -\frac{R_2}{R_1} V_1 + V_{shift},$$

$$V_{shift} = \left(\frac{R_1 + R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} V_{ref}$$

$$V_1 = V_{ref}$$

V_2

$$V_{out} = \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2 - V_{shift}$$

$$V_{shift} = \frac{R_2}{R_1} V_{ref}$$

$$R_1 = R_3$$

$$R_2 = R_4$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2 = \frac{R_2}{R_1}\ (V_2 - V_1)$$

$$R_4 = \infty$$

R_3

$$V = V_2$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(1 + \frac{R_2}{R_1}\right)V_2$$

$$R_1 = R_4 = \infty$$

$$V_{out} = V_2$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2 = V_2$$

$$R_4 = 0$$

$$R_3 = \infty$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2 = -\frac{R_2}{R_1}V_1$$

$R_3 = 0$

$$V_1 = 0$$

$$V_{out} = -\frac{R_2}{R_1}V_1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{R_4}{R_3 + R_4}V_2 = \left(1 + \frac{R_2}{R_1}\right)V_2$$

$n(t)$

$$V_1' = V_1 + n, \qquad V_2' = V_2 + n$$

$$V_{out} = \frac{R_2}{R_1}\ (V_2' - V_1') = \frac{R_2}{R_1}\ (V_2 - V_1)$$

$$R_3 + R_4$$

$$V_1' = V_1 \left(1 + \frac{R_f}{R_1}\right), \qquad V_2' = V_2 \left(1 + \frac{R_f}{R_1}\right)$$

$$V_{out} = \frac{R_4}{R_3}(V_2' - V_1') = \frac{R_4}{R_3}\left(1 + \frac{R_f}{R_1}\right)(V_2 - V_1)$$

$$R_0 = 2R_1$$

$$R_1 = R_0/2$$

$$V_{out} = \frac{R_4}{R_3}\left(1 + \frac{2R_f}{R_0}\right)(V_2 - V_1)$$

V_1'

$$V_2'$$

$$R_f$$

$$\frac{V_1' - V_1}{R_f} = \frac{V_1 - V_0}{R_1} = \frac{V_0 - V_2}{R_1} = \frac{V_2 - V_2'}{R_f}$$

$$V_1' = \left(1 + \frac{R_f}{R_1}\right) V_1 - \frac{R_f}{R_1} V_0$$

$$V_2' = \left(1 + \frac{R_f}{R_1}\right) V_2 - \frac{R_f}{R_1} V_0$$

$$R_3 = R_4 = R_f = 50\,k\Omega$$

$$R_1 = \infty$$

R_G

$2R_1$

V_{in}

Input voltage	0	1	2	3
Op-amps Outputs	000	001	011	111
Binary Representation	00	01	10	11

$$v^- = v^+ = 0$$

$$i_R + i_C = 0$$

$$i_R + i_C = \frac{v_i}{R} + C\frac{dv_{out}}{dt} = 0,$$

$$v_{out} = -\frac{1}{\tau}\int v_i dt$$

$$\tau \triangleq RC$$

$$H(j\omega) = -\frac{Z_2(j\omega)}{Z_1(j\omega)} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega\tau}$$

$$i_R + i_C = \frac{v_{out}}{R} + C\frac{dv_i}{dt} = 0,$$

$$v_{out} = -RC\frac{dv_i}{dt} = -\tau\frac{dv_i}{dt}$$

$$H(j\omega) = -\frac{Z_2(j\omega)}{Z_1(j\omega)} = -\frac{R}{1/j\omega C} = -j\omega\tau$$

$$v_{out}(t) = c_p v_{in}(t) + c_i \int v_{in}(t) dt + c_d \frac{d\, v_{in}}{dt}$$

$$R_2/R_1 = R_4/R_3$$

$$I_L = \frac{V^+ - V^-}{R_3} = \frac{V}{R_L}$$

$$v_{out} \approx C \ \exp(v_{in}/a), \qquad v_{out} = D \ \ln(v_{in}/b)$$

$C,\ D$

a, b