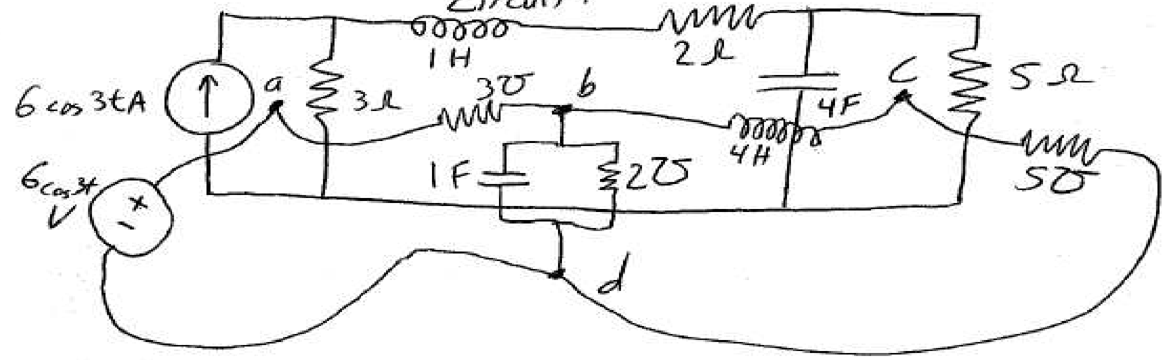
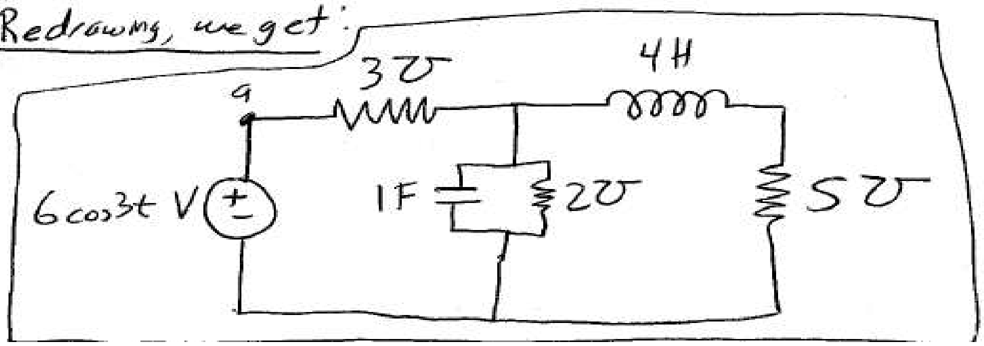
5pts.

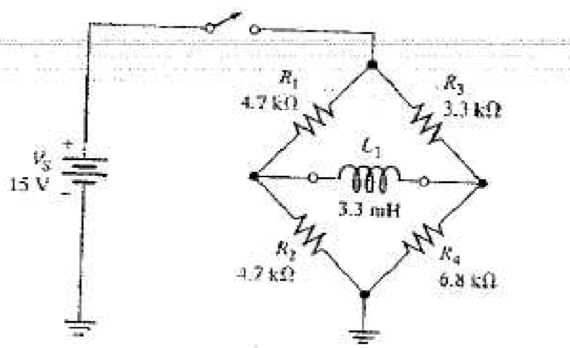
1. FoEE 3.25 : Find the dual of this circuit:




Redrawing, we get:




2. (15 points) What is the current through the inductor 1.0μs after the switch closes?




If we use Thevenin, we can reduce this to a simple RL circuit with a source.

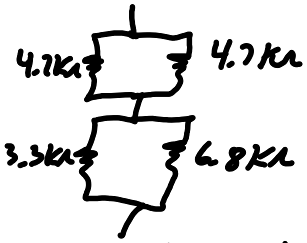
First, $v_{oc} = v_{ab} = v_a - v_b \Rightarrow v_a = \left(\frac{4.7}{4.7+4.7}\right)15 = 7.5V$

$$v_b = \left(\frac{6.8}{3.3+6.8}\right)15 = 10.1V$$

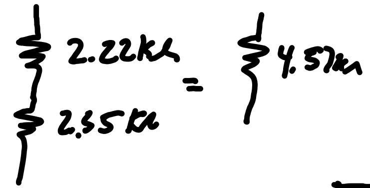
So, $v$ between a & b is $10.1 - 7.5V = 3.6V$

Note since we just asked for the current through the inductor → direction is arbitrary and $v_{oc}$ can equal $v_{ba}$. .

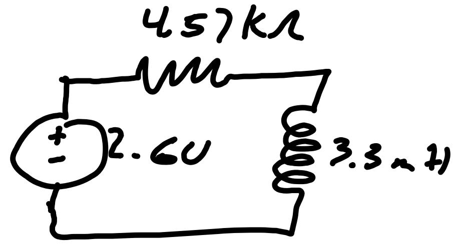
$R_{eq}$:




$\Rightarrow$




So, an equivalent ckt looks like this: (after $t=0$)




Now, it should be simple to see that:

$$i_L(1\mu s) = \frac{V_s}{R}\left(1 - e^{-tR/L}\right) = \frac{2.6V}{4.57k\Omega}\left(1 - e^{-\frac{(1\mu s)(4.57K)}{3.3m}}\right)$$

$$i_L(1\mu s) = 0.426\ mA$$

3) 10 pts.

**3.29** For the circuit shown in Fig. P3.28, replace the capacitor with a 5-H inductor. For the resulting circuit, the switch opens at time $t = 0$ s. Write a differential equation in $i(t)$ for $t \geq 0$ s. Find $i(t)$ and $v(t)$ for all time and sketch these functions.

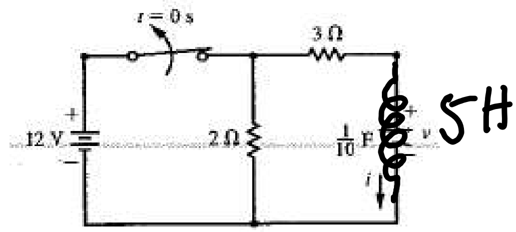
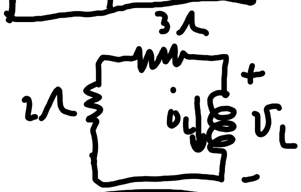



**Fig. P3.28**

For $t<0$

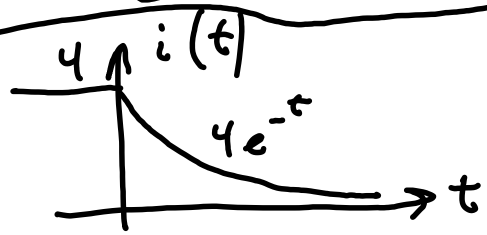

$i_L(0^-) = 4A = i_L(0^+)$

For $t>0$:



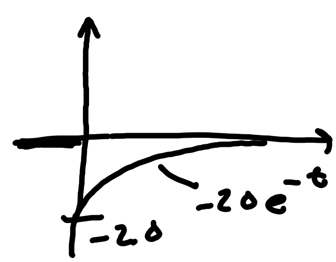

RL discharge circuit:

$$i_L(t) = i_L(0)e^{-t\frac{R}{L}}$$




$$= \begin{cases} 4e^{-t}\ A & t \geq 0 \\ 4 & t < 0 \end{cases}$$

$$v(t) = 5\frac{di}{dt} = \begin{cases} -20e^{-t}\ V. & t \geq 0 \\ 0 & t < 0 \end{cases}$$



4) 15 pts

**3.37** For the circuit shown in Fig. P3.36, replace the inductor with a $\frac{1}{8}$-F capacitor. For the resulting circuit, the switch opens at time $t = 0$ s. Write a differential equation in $v(t)$ for $t \geq 0$ s. Find $v(t)$ and $i(t)$ for all time and sketch these functions.

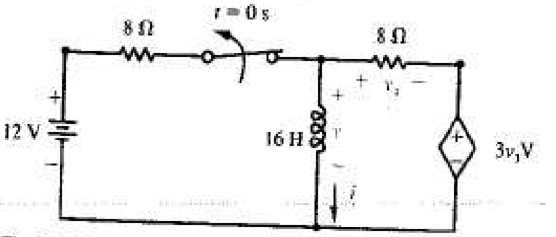
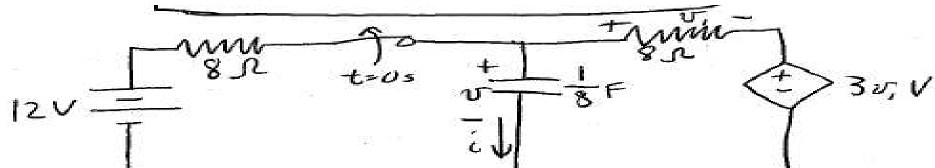
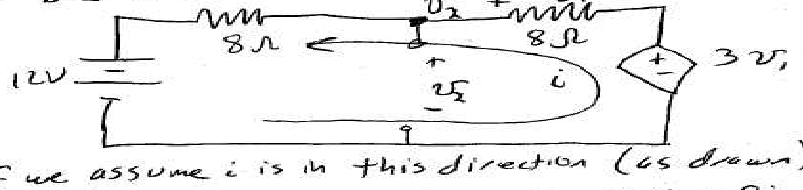



Fig. P3.36




First, we want to analyze this circuit for all $t<0$. That's simply a DC circuit where the capacitor is an open circuit.




If we assume $i$ is in this direction (as drawn), we know

$3v_1 - v_2 = 8i$ and $v_2 - 12V = 8i$ & $v_2 = 4v_1$

So, $3v_1 - v_2 = v_2 - 12V$

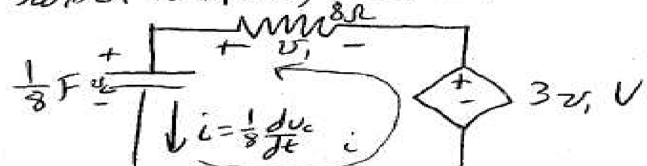
$3v_1 - 4v_1 = 4v_1 - 12V \Rightarrow -5v_1 = -12V$

$v_1 = +\frac{12}{5}V$

So, $v_2$ or $v_c(0^-) = \frac{12}{5} \cdot 4 = 9.6V$

We know voltage can't change instantaneously across a capacitor so we start with 9.6V.

After the switch is opened, we are left with:




$i = \frac{1}{8}\frac{dv_c}{dt}$

$v_c = 4v_1 = 4(8 \cdot -i) = -32i = -32 \cdot \frac{1}{8}\frac{dv_c}{dt}$

So, $+4\frac{dv_c}{dt} + v_c = 0 \Rightarrow \boxed{\frac{dv_c}{dt} + \frac{1}{4}v_c = 0}$ Diff. eq.

pg. 143 & 144: $\frac{dx(t)}{dt} + ax(t) = b$

$x(t) = \frac{b}{a} + Ae^{-at}$

$\boxed{\therefore\ v_c(t) = v_c(0)e^{-t/4}\ V}$

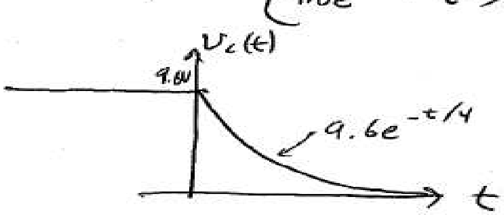
$= 9.6e^{-t/4}\ V$

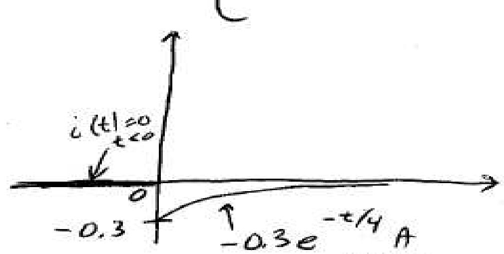
So, $i(t) = \frac{1}{8}\frac{dv_c}{dt} = \frac{1}{8}\frac{d}{dt}(9.6e^{-t/4})$

$\boxed{i_c(t) = \frac{-9.6}{8 \times 4}e^{-t/4} = -0.3e^{-t/4}A}$

$$v_c(t) = \begin{cases} 9.6 & t<0s \\ 9.6e^{-t/4} & t \geq 0s \end{cases}$$

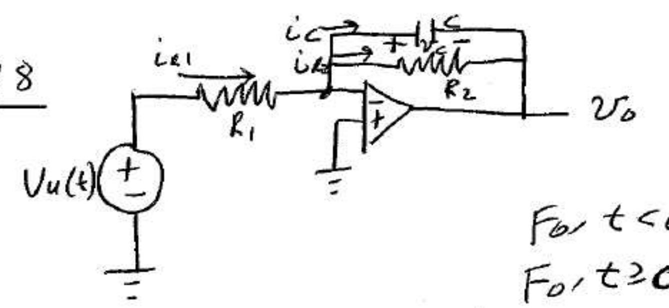
$$i_c(t) = \begin{cases} 0 & \text{for } t<0 \\ -0.3e^{-t/4} & t \geq 0 \end{cases}$$



. FoEE 3.48

10 pts.




For $t<0$ $v_o = 0$
For $t \geq 0$ ???

At $0^+$, let's see how the circuit behaves:

$v_c(0^+) = 0V$ (because $V_c(0^-) = 0V$)

$V_R(0^+) = 0V$ (because $V_c(0^+) = 0V$)

$i_{R1}(0^+) = \frac{(V-0)}{R_1} = \frac{V}{R_1}$

$i_{R2}(0^+) = 0A$ (because $V_{R2}(0^+) = 0V$)

$i_c(0^+) = \frac{V}{R_1}$ (by KCL it's the only place the current can go)

For $t \geq 0s$

By KCL at $v_-$: $\frac{V-0}{R_1} = i_c + i_{R2}$

$$\frac{V}{R_1} = C\frac{dv_c}{dt} + \frac{v_c}{R_2} \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_2 C} = \frac{V}{R_1 C}$$

So, the solution to this diff. eq. is:

$$v_c(t) = \frac{\frac{V}{R_1 C}}{\frac{1}{R_2 C}} + Ae^{-\frac{t}{R_2 C}} = \frac{VR_2}{R_1} + Ae^{-t/R_2 C}$$

$A = -\frac{VR_2}{R_1}$ so $v_c(0^+)$ is right.

$$\therefore v_c(t) = \frac{VR_2}{R_1}\left(1 - e^{-t/R_2 C}\right)$$

$$v_o(t) = -v_c(t) = -\frac{VR_2}{R_1}\left(1 - e^{-t/R_2 C}\right)u(t) \text{ Volts}$$

Optional Problems

3. (10 points) FoEE 3.31

**3.31** For the circuit shown in Fig. P3.30, replace the inductor with a 0.1-F capacitor. Suppose that $i_s(t) = 10$ A for $t < 0$ s and $i_s(t) = 0$ A for $t \geq 0$ s. Write a differential equation in $v(t)$ for $t \geq 0$ s. Find $v(t)$ and $i(t)$ for all time and sketch these functions.

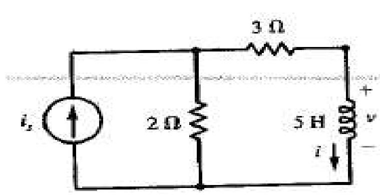
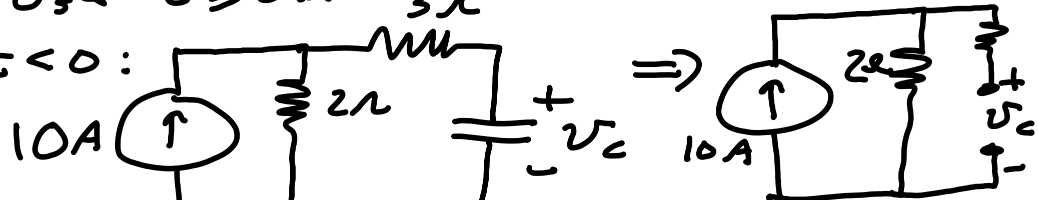



Fig. P3.30

First lets redraw the circuit with the capacitor for $t<0s$ & $t \geq 0s$.

For $t<0$:

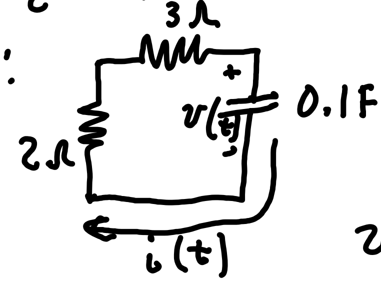


Since we have been in this situation for a *very* long time AND have a DC source, we use the DC replacement and turn the capacitor into an open ckt.

$\therefore v_c(0^-) = v_{2\Omega}(0^-) = 10 \cdot 2 = 20\,V$

$v_c(0^+) = v_c(0^-)$ because of instantaneous change rules.

For $t>0$:



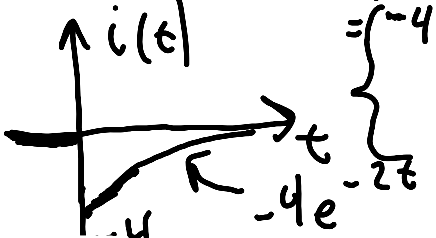

$v(t) + 5i = 0$

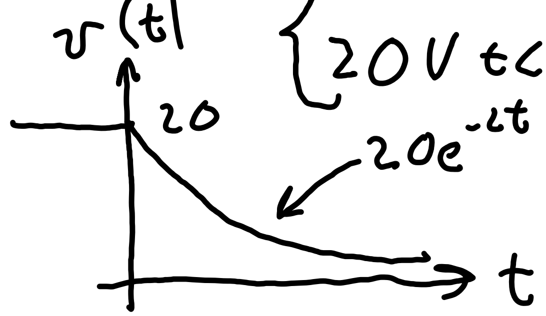
$5(0.1)\frac{dv}{dt} + v = 0 \Rightarrow \frac{dv}{dt} + 2v = 0$

$v(t) = v(0)e^{-t/RC} = \begin{cases} 20e^{-2t}\,V & t>0 \\ 20\,V & t<0 \end{cases}$

So, $i(t) = C\dot{v}(t) = \frac{1}{10} \cdot 20(-2)e^{-2t}$

$= \begin{cases} -4e^{-2t}\,A & t \geq 0 \\ 0\,A & t<0 \end{cases}$

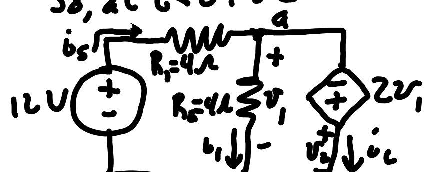



FoEE 3.35

$v_s(t) = 12$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. ~~Write a differential equation in $i(t)$ for $t \geq 0$ s.~~ ~~Find $i(t)$ and $v(t)$ for all time and sketch these functions.~~

So, at $t<0$: DC: L→short




The only way this works is if $v_1 = 0V$. So, $i_1 = 0A$. So, by KVL, all 12 V drop across $R_1$. So, $i_s = 3A$ and by KCL at "a", $i_L = 3A$.

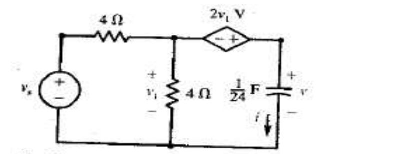
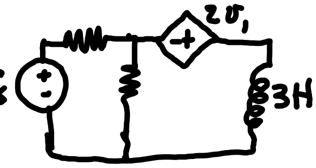
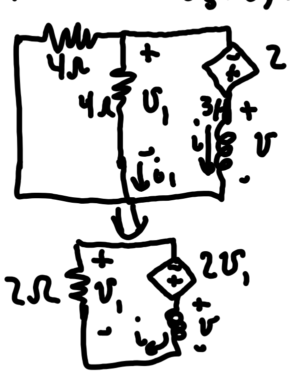



**Fig. P3.34**

**3.35** For the circuit shown in Fig. P3.34, replace the capacitor with a 3-H inductor. Suppose that




For $t>0$: $v_s = 0$, so source is short

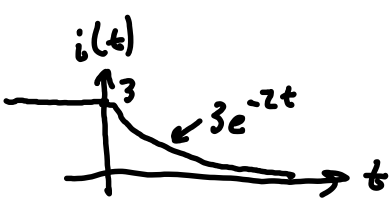



$v = 3v_1$ (KVL)

$3\frac{di}{dt} = 3(2)(-i)$

so, $\frac{di}{dt} + 2i = 0$

$i(t) = i(0)e^{-2t} = \begin{cases} 3e^{-2t}\,A & t \geq 0 \\ 3\,A & t<0 \end{cases}$




$v(t) = L\frac{di}{dt} = 3(-2)3e^{-2t} = \begin{cases} -18e^{-2t}\,V & t \geq 0 \\ 0 & t<0 \end{cases}$