(1)

4.8 Connect a 5-Ω resistor in parallel with the inductor in the circuit shown in Fig. P4.6. Suppose that $v_s(t) = 13\cos(2t - 22.6°)$ V. Find the voltage $v_o(t)$ across the inductor by using nodal analysis. Draw a phasor diagram. Is this circuit a lag network or a lead network?

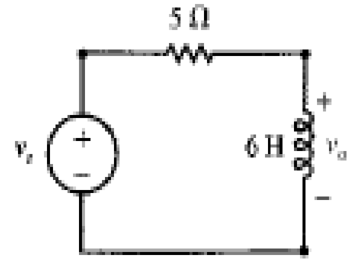
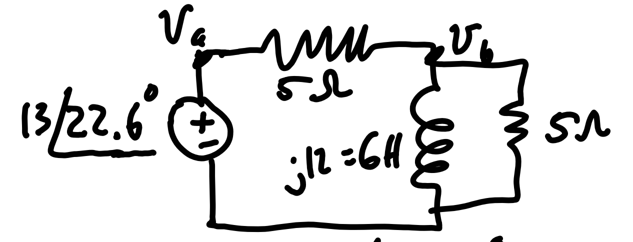


Fig. P4.6



$v_s(t) = 13\cos(2t - 22.6)$

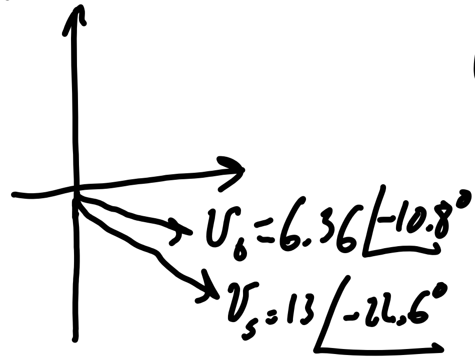
$$V_b = \frac{156\angle -22.6°}{\sqrt{601}\angle -11.8°} = 6.36\angle -10.8° = 6.36\cos(2t - 10.8)$$

$V_a = 13\angle -22.6° =$

$$\frac{V_a - V_b}{5} = \frac{V_b}{5} + \frac{V_b}{j12}$$

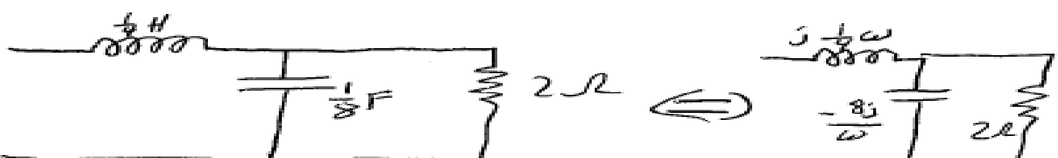
$12V_a - 12V_b = 12V_b - 5jV_b$

$(24 - j5)V_b = 12V_a = 12 \cdot 13\angle -22.8°$



Output leads source so this is a lead network.

(2) FoEE 4.13



$$\text{So, } Z = \underbrace{\left(\frac{1}{\frac{1}{\left(-\frac{8j}{\omega}\right)} + \frac{1}{2}}\right)}_{\text{Cap and resistor in parallel}} + \underbrace{\frac{j\omega}{4}}_{\text{in series with inductor}}$$

4.13 For the *RLC* connection given in Fig. P4.13, find the impedance **Z** when ω is (a) 2, (b) 4, and (c) 8 rad/s.

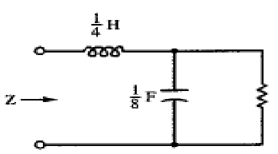


Fig. P4.13

$$Z = \frac{1}{\frac{1}{2} + \frac{j\omega}{8}} + \frac{j\omega}{4} = \frac{8}{4 + j\omega} + \frac{j\omega}{4}$$

$$= \frac{8(4 - j\omega)}{(4 + j\omega)(4 - j\omega)} + \frac{j\omega}{4} = \frac{32 - j\omega}{16 - j^2\omega^2} + \frac{j\omega}{4}$$

$$= \frac{32 - 8j\omega}{16 + \omega^2} + \frac{j\omega(16 + \omega^2)}{4(16 + \omega^2)}$$

$$= \frac{32}{16 + \omega^2} + \frac{j\omega[\omega^2 + 16 - 32]}{4(16 + \omega^2)}$$

$$= \frac{32}{16 + \omega^2} + j\frac{\omega[\omega^2 - 16]}{4(\omega^2 + 16)}$$

a) $\omega = 2\frac{rad}{s}$ $Z = \frac{32}{20} + j\frac{-24}{80} = (1.6 - j0.3)_\Omega = 1.63\angle -10.62°\ \Omega$

b) $\omega = 4\frac{rad}{s}$ $Z = \frac{32}{32} + j\frac{0}{128} = (1 + j0)_\Omega = 1\angle 0°\ \Omega$

c) $\omega = 8\frac{rad}{s}$ $Z = \frac{32}{80} + j\frac{384}{320} = (0.4 + j1.2)_\Omega = 1.26\angle 71.57°\ \Omega$

(3)

4.18 For the circuit shown in Fig. P4.17, find the Thévenin equivalent of the circuit in the shaded box when $v_s(t) = 4\cos(2t - 60°)$ V. Use this to determine $v_o(t)$.

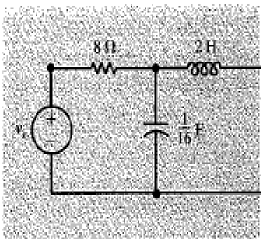
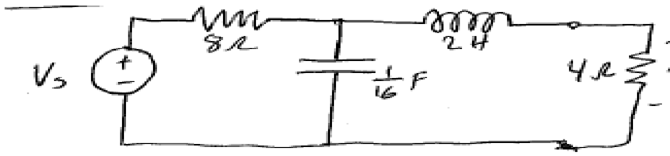


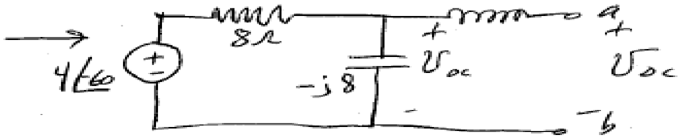
Fig. P4.17



$v_s = 4\cos(2t-60°)$ V

So, $\omega = 2 \frac{rad}{s}$

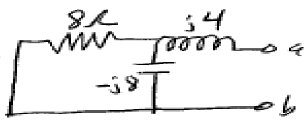
So, $V_{oc} \longrightarrow$



$$V_{oc} = \frac{-j8}{8-j8}\, 4\angle{-60} = \frac{8\angle{-90°}\;4\angle{-60}}{8\sqrt{2}\angle{-45°}}$$

$$V_{oc} = 2\sqrt{2}\angle{-105°} = 2\sqrt{2}\cos(2t-105°)\text{ V}$$

$Z_o =$



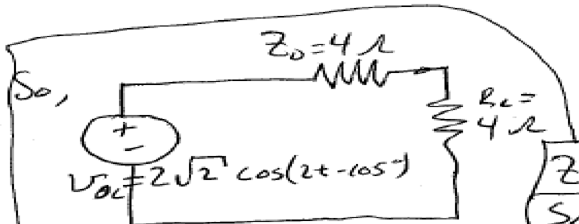
$$Z_o = j4 + \left(\frac{-64j}{8-j8}\right)$$

$$= j4 + \left(\frac{-8j}{1-j}\right) = j4 - \frac{8j(1+j)}{(1+j)(1-j)}$$

$$= j4 - \frac{(j8-8)}{1-j^2} = j4 - \frac{(j8-8)}{2}$$

$$\boxed{Z_o = j4 - j4 + 4 = 4\,\Omega}$$

So,



$$\boxed{\text{So, } V_o(t) = \sqrt{2}\cos(2t-105°)\text{ V}}$$

(4) 4.21

4.21 For the op-amp circuit shown in Fig. P4.21, find $v_o(t)$ when $v_s(t) = 6\sin 2t$ V.

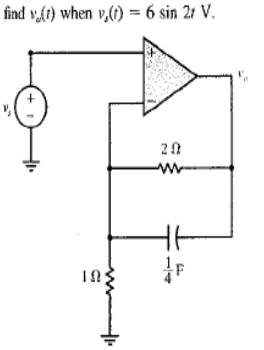
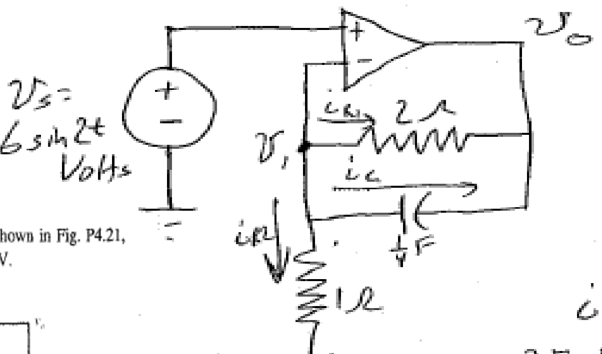


Fig. P4.21



Ideal op-amp: so $V_+ = V_-$

$V_1 = 6\sin 2t$

Let's write a KCL equation at V_1:

$$i_{R1} + i_c + i_{R2} = 0$$

$$\frac{V_1 - V_o}{2} + \frac{V_1 - V_o}{Z_c} + \frac{V_1}{1} = 0$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j \cdot 2 \cdot \frac{1}{4}} = -j2\,\Omega$$

$$V_1 = 6\angle{0°}$$

$$\text{So, } \frac{V_1}{2} - \frac{V_o}{2} + \frac{V_1}{-j2} + \frac{V_o}{j2} + V_1 = 0$$

$$V_1\left(\frac{1}{2} - \frac{1}{j2} + 1\right) = V_o\left(\frac{1}{2} - \frac{1}{j2}\right)$$

$$V_o = \frac{V_1\left(\frac{3}{2} + \frac{1}{2}j\right)}{\left(\frac{1}{2} + \frac{1}{2}j\right)} = \frac{6\angle{0°}\;1.58\angle{18.43°}}{0.707\angle{45°}}$$

$$= 13.4\angle{-26.57°}$$

$$\boxed{V_o(t) = 13.4\sin(2t - 26.57°)\text{ Volts}}$$

(5)

4.25 Use mesh analysis to find I_1 and I_2 for the circuit given in Fig. P4.25 when $V_{s1} = 250\sqrt{2}\angle -30°$ V, $V_{s2} = 250\sqrt{2}\angle -90°$ V, and $Z = 26 - j15\ \Omega$.

$Z = 26 - j15 = 30\angle -30°$

$V_{s1} = 250\sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{j}{2}\right)$

$V_{s2} = 250\sqrt{2}(-j)$

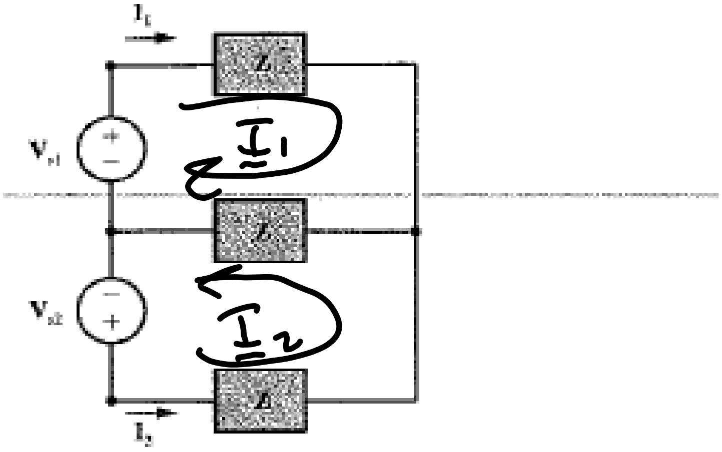


Fig. P4.25

KVL around 1st (top) loop:

$V_{s1} = I_1 Z + (I_2 + I_1)Z$

$V_{s1} = 2I_1 Z + I_2 Z$

$2V_{s1} = 4I_1 Z + 2I_2 Z$

$-V_{s2} = -I_1 Z - 2I_2 Z$

$2V_{s1} - V_{s2} = 3I_1 Z \qquad I_1 = \dfrac{500\sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{j}{2}\right) + 250\sqrt{2}j}{3 \cdot 30\angle -30°}$

KVL around bottom loop:

$V_{s2} = I_2 Z + (I_1 + I_2)Z$

$V_{s2} = 2I_2 Z + I_1 Z$

$$\text{So, } I_1 = \frac{250\sqrt{2}(\sqrt{3})}{90\angle -30°} = 6.8\angle 30°\text{ A}$$

From mesh analysis:

$Z I_2 = V_{s1} - 2I_1 Z$

$I_2 = \dfrac{V_{s1} - 2I_1}{Z} = \dfrac{V_{s1}}{Z} - \dfrac{2I_1}{Z} = \dfrac{250\sqrt{2}\angle -30°}{30\angle -30°} - 2(6.8)\angle 30°$

$= \dfrac{25\sqrt{2}}{\sqrt{3}} - 13.6\angle 30 = 11.8 - (11.8 + j6.8) = -j6.8$

$$\boxed{I_2 = 6.8\angle -90°\text{ A}}$$

(6)

4.32 For the op-amp circuit given in Fig. P4.22, when $v_s(t) = 3\cos 2t$ V, then the output voltage $v_o(t) = 10.6\cos(2t + 135°)$ V. Find the average power absorbed by each element.

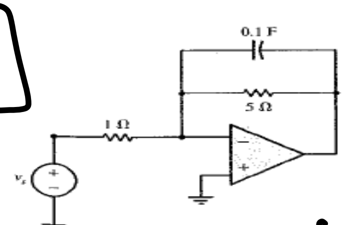
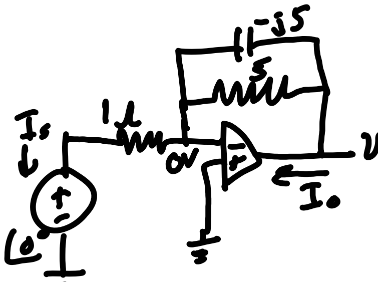


Fig. P4.22



$V_o = 10.6\cos(2t + 135°) \qquad \omega = 2$

$\underline{V_s} = 3\angle 0°$

$1\Omega: P_1 = \frac{1}{2}\frac{|V_s|^2}{R_1} = \frac{1}{2}\cdot\frac{9}{1} = \frac{9}{2} = 4.5\,W$

$5\Omega: P_2 = P_{5\Omega} = \frac{1}{2}\frac{|V_{oc}|^2}{R_5} = \frac{1}{2}\frac{(10-6)^2}{5} = 11.2\,W$

Capacitor: $P_{avg} = 0\,W$ (by inspection)

$$I_o = \frac{0-\underline{V}_o}{5} + \frac{0-\underline{V}_o}{-j5} = -\frac{1}{5}\left[\underline{V}_o + \frac{-\underline{V}_o}{j}\right] = \frac{-1}{5}\left[\underline{V}_o + j\underline{V}_o\right]$$

$$I_o = -\frac{1}{5}\left[10.6\angle 135^\circ\right](1+j)$$

$$= \frac{10.6}{5}\angle -45^\circ \sqrt{2}\angle 45^\circ = 3\angle 0^\circ$$

Opamp: $P_o = \frac{1}{2}|V_o||I_o|\cos(\text{ang}(V_o) - \text{ang}(I_o))$

$$= \frac{1}{2}(10.6)(3)\cos(135^\circ) = -11.2\,W$$

$$I_s = \frac{0-\underline{V}_s}{1} = -3\angle 0^\circ = 3\angle 180^\circ$$

For the voltage source:

$$P_s = \frac{1}{2}|\underline{V}_s||\underline{I}_s|\cos[\text{ang}(\underline{V}_s) - \text{ang}(\underline{I}_s)]$$

$$P_s = \frac{1}{2}(3)(3)\cos(0-180^\circ)$$

$$P_s = -\frac{9}{2} = -4.5\,W$$

(7) **4.42** A 115-V rms, 60-Hz electric hair dryer absorbs 500 W at a lagging pf of 0.95. What is the rms value of the current drawn by this dryer?

$$\overline{P} = V_e I_e \cos\theta = 115(I_e)\cos\theta$$

$$500 = 115(I_e)\,0.95$$

$$\boxed{\therefore I_e = 4.58\,A_{rms}}$$

Optional Problems

8. (0 points) FoEE 4.6

4.6 For the ac circuit shown in Fig. P4.6, suppose that $v_s(t) = 13\cos(2t - 22.6°)$ V. Find $v_o(t)$ by using voltage division. Draw a phasor diagram. Is this circuit a lag network or a lead network?

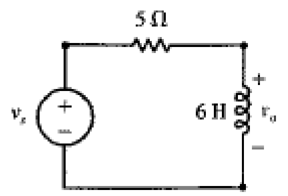
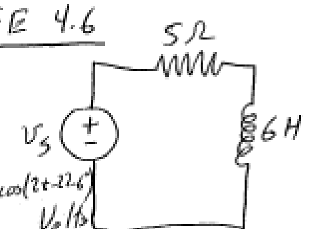


Fig. P4.6

FoEE 4.6



$v_s(t) = 13\cos(2t-22.6°)$ Volts

$Z_L = j\omega L = j \cdot 2 \cdot 6 = j12$

$$\therefore\ v_o(t) = \frac{Z_L}{Z_R + Z_L}\, v(t)$$

$$V_o = \frac{j12}{5+j12}\, 13\angle{-22.6°}$$

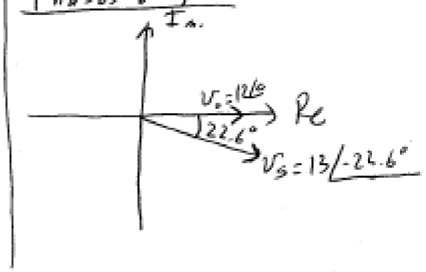
$$= \frac{12\angle 90 \quad 13\angle{-22.6}}{\sqrt{5^2+12^2}\ \angle \tan^{-1}\left(\frac{12}{5}\right)}$$

$$= \frac{12 \cdot 13}{13}\ \angle 90 - 22.6° - 67.4°$$

$$= 12\angle 0°$$

$$\boxed{v_o(t) = 12\cos(2t)\ \text{Volts}}$$

Phasor Diagram



Phase of $v_s = -22.6°$
Phase of $v_o = 0°$

$\boxed{\therefore\ \text{Lead Network}}$

9. (0 points) FoEE 4.17

4.17 For the circuit shown in Fig. P4.17, find the Thévenin equivalent of the circuit in the shaded box when $v_s(t) = 4\cos(4t - 60°)$ V. Use this to determine $v_o(t)$.

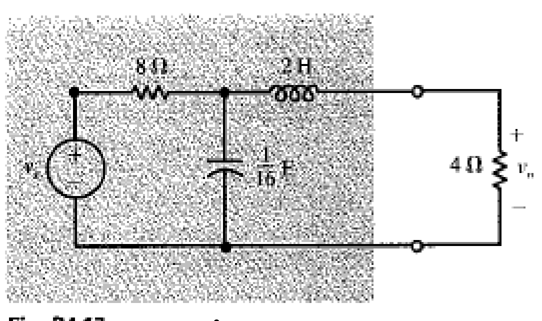
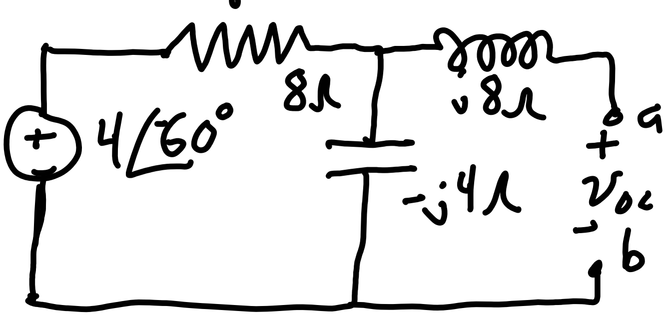


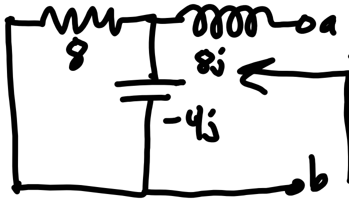
Fig. P4.17

First, we should find the freq. domain circuit



$$V_{oc} = \frac{-j4\ \ 4\angle{-60°}}{8 - j4} = \frac{-j\ 4\angle 60°}{2 - j} = \frac{1\angle{-90°}\ 4\angle{-60°}}{\sqrt{5}\ \angle{-26.6°}}$$

$$V_{oc} = 1.79\angle{-123°}\ \text{Volts}$$

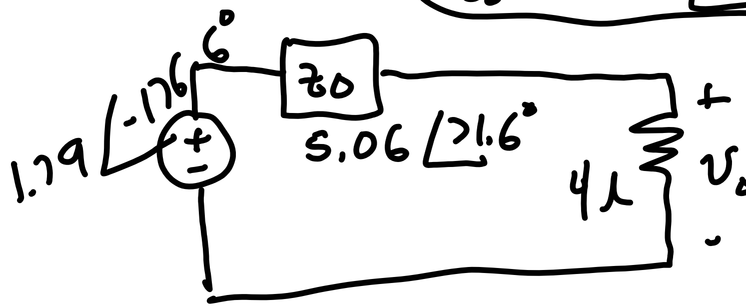


$$Z_0 = j8 + \frac{8(-j4)}{8-j4}$$

$$= j8 - \frac{j8}{2-j}$$

$$= \frac{j8(2-j)-j8}{2-j} = \frac{16j+8-j8}{2-j} = \frac{8j+8}{2-j} = \frac{8(j+1)}{2-j} = \frac{8\sqrt{2}\angle 45^\circ}{\sqrt{5}\angle -26.6^\circ}$$

$$Z_0 = 5.06\angle 71.6^\circ = 1.6 + j4.8$$



$$V_0 = \frac{4}{4+(1.6+j4.8)} \cdot 1.79\angle -123^\circ = 0.97\angle -164^\circ$$

$$V_0(t) = 0.97\cos(4t - 164^\circ)$$

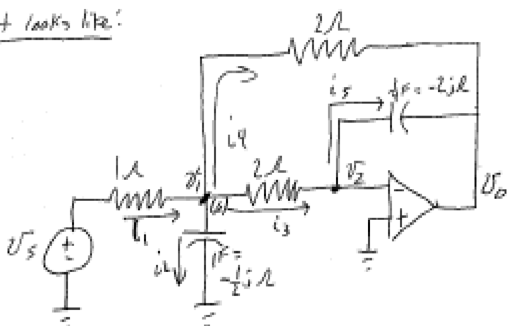
10. (0 points) FoEE 4.23

4.23 For the op-amp circuit shown in Fig. P4.23, find $v_o(t)$ when $v_s(t) = 4\cos(2t - 30^\circ)$ V. (See p. 258.)

FoEE 4.23 & find power absorbed by 1Ω resistor, each capacitor and for the op-amp.

$V_s(t) = 4\cos(2t - 30^\circ)$ So $V_s = 4\angle -30^\circ$

Circuit looks like:



KCL at node (a): $i_1 = i_2 + i_3 + i_4$ (1)

KCL at node (b): $i_3 = i_5$ (2)

Using Ohm's Law (2) becomes: $\frac{V_1 - V_2}{2} = \frac{V_2 - V_0}{-2j}$

Since op-amp is ideal $V_2 = 0V$ So, $\frac{V_1}{2} = \frac{-V_0}{-2j} \Rightarrow V_1 = -jV_0$

Now, expanding (1) with Ohm's Law:

$V_1 = -jV_0$
$V_2 = 0$

$$\frac{V_s - V_1}{1} = \frac{V_1}{-\frac{1}{2}j} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_0}{2}$$

$$V_s + jV_0 = 2V_0 + \frac{-jV_0}{2} - \frac{jV_0}{2} - \frac{V_0}{2}$$

$$V_s = V_0\left[\frac{3}{2} - 2j\right] \Rightarrow V_0 = \frac{4\angle -30^\circ}{2.5\angle -53.13^\circ}$$

So, $V_0 = 1.6\angle 23.13^\circ$

$\boxed{\therefore V_0(t) = 1.6\cos(2t + 23.13^\circ) \text{ Volts}}$

Avg. Power absorbed by 1Ω resistor:

$P_R = \frac{1}{2}\frac{V^2}{R} = \frac{1}{2}V^2$ Here, V is the amplitude of the voltage across the resistor.

So $V_R = V_s - V_1$

$V_s = 4\angle -30^\circ$ $V_1 = -jV_0 = 1.6\angle -66.87^\circ$

When adding or subtracting phasors, we should have them in rectangular form.

So, $V_R = (3.46 - 2j) - (0.6285 - 1.47j)$

$= 2.83 - 0.53j$

$V_R = 2.88\angle -10.61^\circ$

$\boxed{\text{So } P_R = \frac{1}{2}(2.88)^2 = 4.15\text{ W absorbed}}$

$\boxed{\text{Avg. Power absorbed by each capacitor} = 0\text{ W}}$

Avg. Power is ALWAYS 0 across a capacitor or inductor.

$\boxed{P_{1F} = 0W, \quad P_{\frac{1}{4}F} = 0W}$

Avg. Power absorbed by op-amp: direction depends on I_o

Eqn for avg. power: $P_A = \frac{1}{2}|V_o||I_o|\cos(\arg(V_o) - \arg(I_o))$

$$I_o = \frac{V_o}{-2j} + \frac{V_o - V_1}{2} = \frac{V_o j}{2} + \frac{V_o}{2} + \frac{jV_o}{2} = V_o\left(\frac{1}{2} + j\right)$$

So, $I_o = 1.6\angle 23.13^\circ \; 1.12\angle 63.43^\circ$

$= 1.79\angle 86.56^\circ$ Amps.

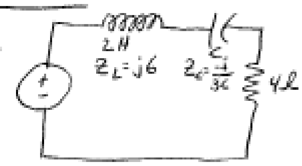
So $P_A = \frac{1}{2}|1.6||1.79|\cos(23.13^\circ - 86.56^\circ) = 1.43\cos(-63.43^\circ)$

$\boxed{P_A = -0.6383\text{ W}}$

This is negative power absorbed because the current is delivered by the op-amp!

11. (0 points) FoEE 4.28

FoEE 4.28



$P_{avg\ of\ resistor} = \frac{1}{2} I_m^2 R$

$I_m = \frac{V_m}{Z_m}$ & $Z_T = j6 + 4 - \frac{j}{C \cdot 3}$

$$= 4 + j\left(6 - \frac{1}{3C}\right)$$

$$Z_m = \sqrt{16 + \left(6 - \frac{1}{3C}\right)^2}$$

So, $P_{avg\ of\ resistor} = \frac{1}{2} \frac{V_m^2}{Z_m^2} R = \frac{1}{2} \cdot \frac{100}{\left(16 + \left(6 - \frac{1}{3C}\right)^2\right)} \cdot 4$

$$= \frac{200}{\left(16 + \left(6 - \frac{1}{3C}\right)^2\right)}$$

So, (a) $C = \frac{1}{6}F$ $\quad P = \frac{200}{16 + \left(6 - \frac{1}{3 \cdot \frac{1}{6}}\right)^2} = \frac{200}{32} = 6.25W$

(b) $C = \frac{1}{18}F$ $\quad P = \frac{200}{16 + \left(6 - \frac{1}{3 \cdot \frac{1}{18}}\right)^2} = \frac{200}{16} = 12.5W$

(c) $C = \frac{1}{30}F$ $\quad P = \frac{200}{16 + \left(6 - \frac{1}{3 \cdot \frac{1}{30}}\right)^2} = \frac{200}{32} = 6.25W.$