E84: HW1 Solutions

Handout #6
E84: Fall 2007
9/17/07

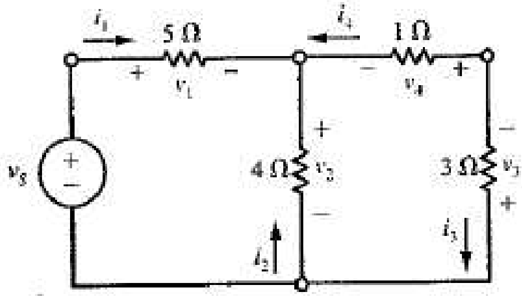
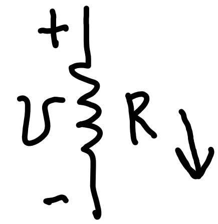


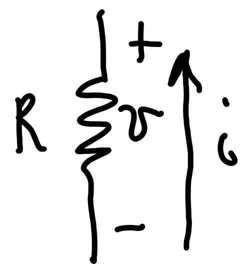
Fig. P1.7



$v = i \cdot R$

$i = \frac{v}{R}$

Know these



$v = -iR$

$i = \frac{-v}{R}$

1.8 Consider the circuit in Fig. P1.7. (a) Given $v_1 = 30$ V, find i_1. (b) Given $v_2 = 12$ V, find i_2. (c) Given $v_3 = -9$ V, find i_3. (d) Given $v_4 = -3$ V, find i_4.

c) $v_3 = -9\,V$

$i_3 = \frac{-\,-9V}{3\Omega} = 3A$

d) $v_4 = -3\,V$

$i_4 = \frac{v_4}{R_4} = \frac{-3V}{1\Omega} = -3A$

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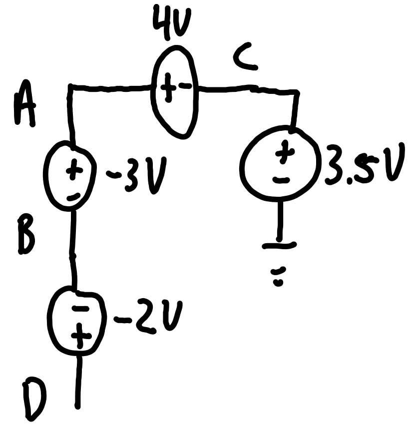
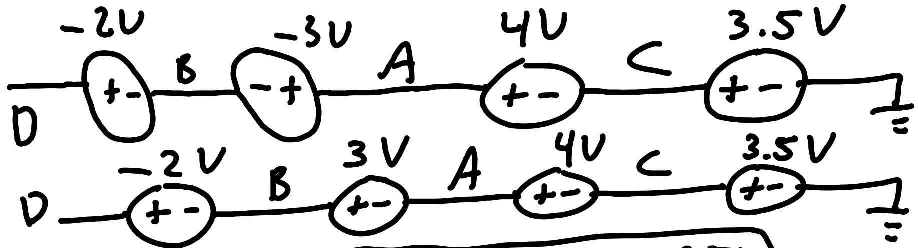
2. (15 points) In this problem, assume that voltages are present because of independent voltage sources. In other words, $V_{AB} = 2$ Volts would mean that the voltage at node A is 2 volts higher than the voltage at node B.
   a) Suppose $V_{AB} = -3$ Volts, $V_{AC} = 4$ Volts, and $V_{DB} = -2$ Volts. If Point C is at 3.5 Volts with respect to ground, what is the voltage at point D with respect to ground.
   b) Now suppose you close the circuit by connecting point D to a 17MOhm resistor. The other end of this resistor is connected to ground. What is the current through the resistor?
   c) What if you take the closed circuit in (b) and you cut the line at point A and insert another 17MOhm resistor (thus, closing the loop again). What is the current through the new 17MOhm resistor? What is the current through the first one?
   d) Instead of doing what you did in (c), now suppose you clip the circuit at point B and again insert a 17MOhm resistor. What is the current across this new resistor?
   e) Finally, assume that you take the same circuit from (d) and add another 17MOhm resistor in parallel to each of the existing (both) 17MOhm resistors. Now, what is the current through the voltage source that connects points B and D?

a) $V_{AB} = -3\,V$ $\qquad V_{AB} = V_A - V_B$

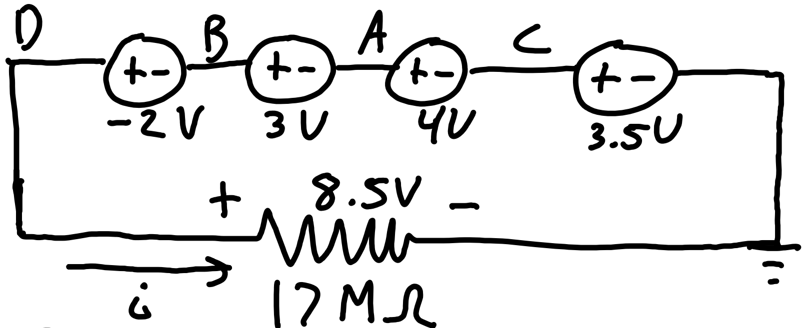
$V_{AB} = -V_{BA}$





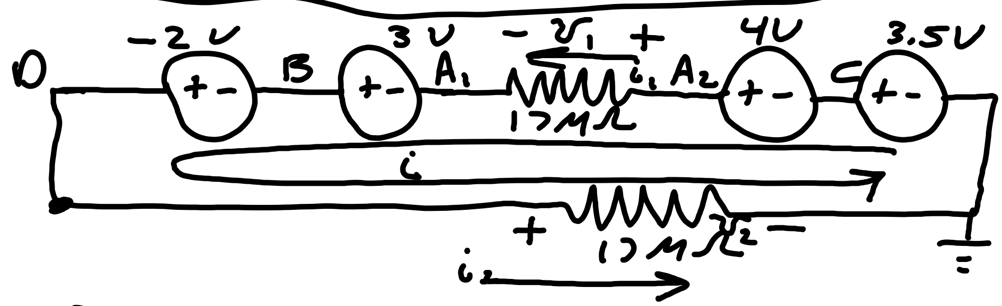
$\boxed{\text{Voltage D w.r.t GND} = -2+3+4+3.5 = +8.5\,V}$

b)



$$\boxed{i = \frac{8.5\,V}{17\,M\Omega} = 0.5\,\mu A = 500\,nA}$$

c)



$$- {}^-2V - 3V + v_1 - 4V - 3.5V + v_2 = 0$$

$$v_1 + v_2 = 8.5\,V$$

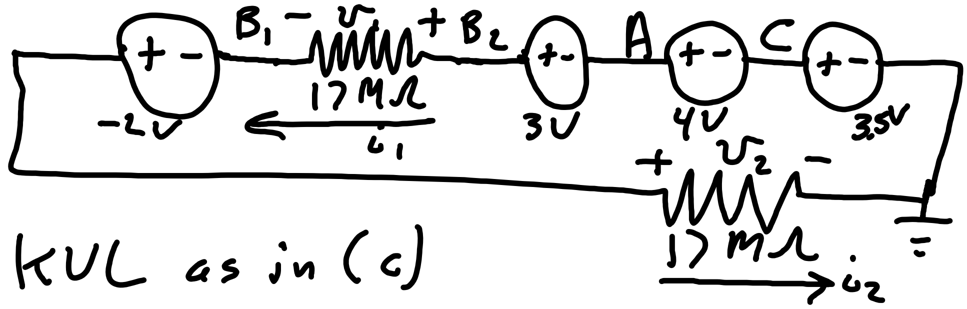
$$i_1 = i_2 = i \qquad v_1 = v_2 = i \cdot 17\,M\Omega$$

$$v_1 + v_1 = 2v_1 = 2\,i\,17\,M\Omega = 8.5\,V$$

$$\boxed{i_1 = i_2 = i = .25\,\mu A = 250\,nA}$$

Same for both resistors.

d)



Use KVL as in (c)

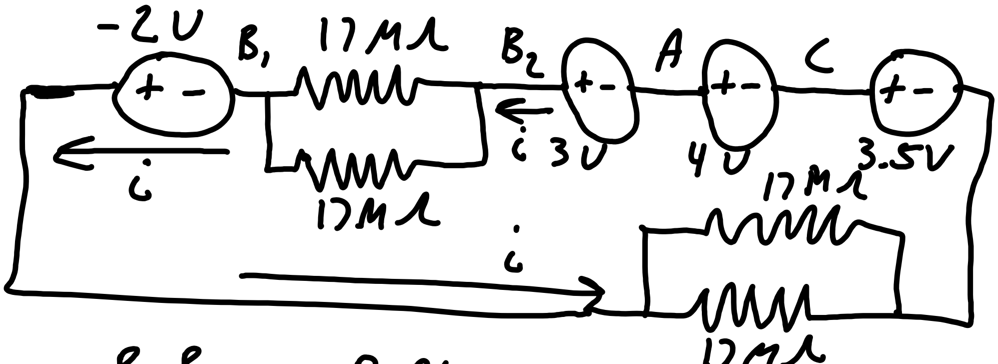
$$- -2V + v_1 - 3 - 4 - 3.5 + v_2 = 0$$

$$v_1 = v_2 = i \cdot 17\,M\Omega$$

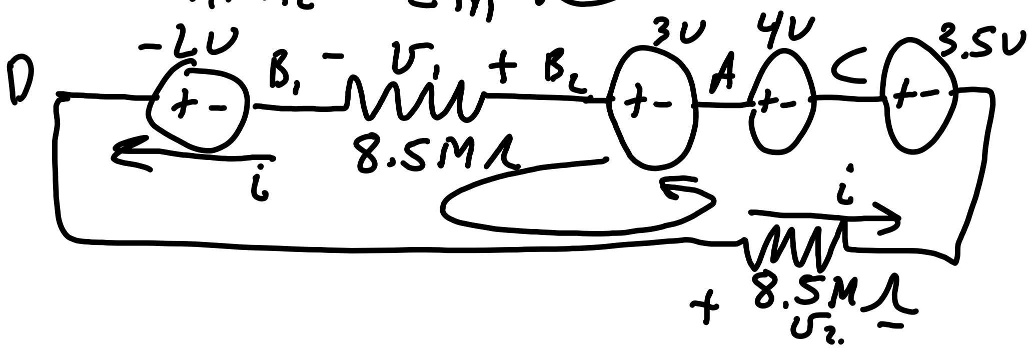
$$2\, i\, 17\,M\Omega = 8.5V$$

Like above $i_1 = i = \dfrac{8.5V}{34M\Omega} = .25\mu A$

e)



$$R_{\parallel} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \cancel{R_1}}{2\cancel{R_1}}$$ when $R_1 = R_2$



$$- -2V + v_1 - 3 - 4 - 3.5 + v_2 = 0$$

$$v_1 = v_2 = i \cdot 8.5\,M\Omega$$

$$2\,(i\ 8.5\,M\Omega) = 8.5V$$

$$i = \frac{8.5V}{2 \cdot 8.5M\Omega} = 0.5\mu A = 500nA$$

3)

1.14 Consider the circuit in Fig. P1.14. (a) Given $i_1 = 3$ A and $v_1 = 6$ V, find R_1. (b) Given $i_2 = 3$ A and $v_2 = -15$ V, find R_2. (c) Given $i_3 = -2$ A and $v_3 = 6$ V, find R_3. (d) Given $i_4 = -1$ A and $v_3 = 6$ V, find R_4.

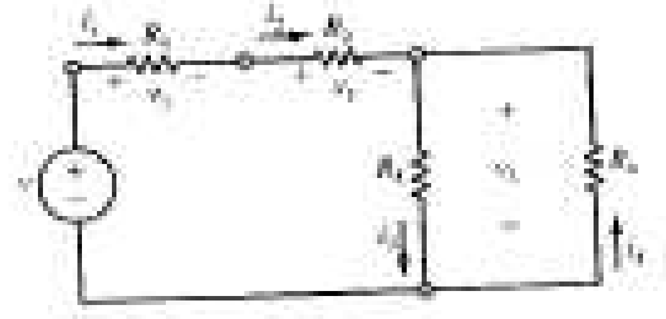


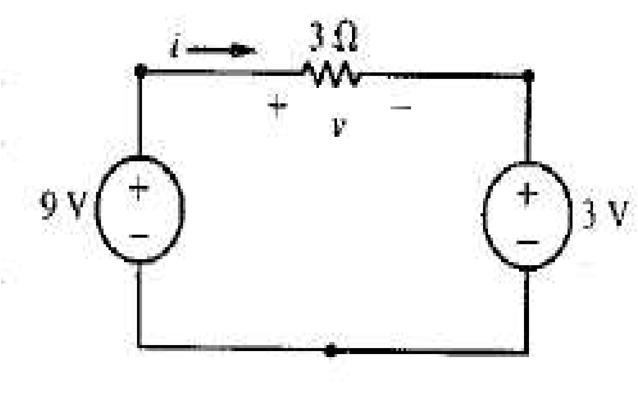
Fig. P1.14

a) $i_1 = \frac{v_1}{R_1}$, so $R_1 = \frac{v_1}{i_1} = \frac{6V}{3A} = 2\,\Omega$

b) An email clarified a typo: $v_2 = \pm 15$ V.

$R_2 = \frac{v_2}{i_2} = \frac{15V}{3A} = 5\,\Omega$

4) **1.25** Find the variables indicated for the circuits shown in Fig. P1.25.

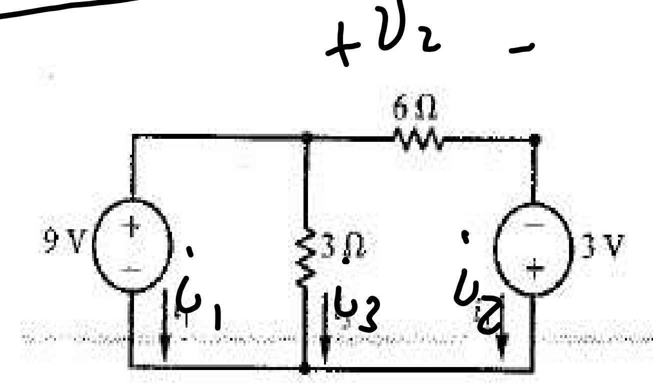


(a)

a) KVL: $9 = v + 3$

$v = 3i$

$\boxed{\text{So, } v = 6V,\ i = 2A}$



(d)

d) KVL around outside:

$9 = v_2 - 3 \Rightarrow v_2 = 12V$

$i_2 = \frac{v_2}{6} = \frac{12}{6} = 2A$; $i_3 = \frac{9V}{3\Omega} = 3A$

KCL $i_1 + i_3 + i_2 = 0$ so, $i_1 = -5A$

5) 1.33 c, d (for part c)

1.33 Consider the series-parallel circuit shown in Fig. P1.33. (a) Find V_s when $v_1 = 2$ V. (b) Find V_s when $i_3 = 3$ A. (c) Find V_s when $i_5 = 4$ A. (d) What is the resistance $R_{eq} = V_s/i$ loading the battery for part (a)? For part (b)? For part (c)?

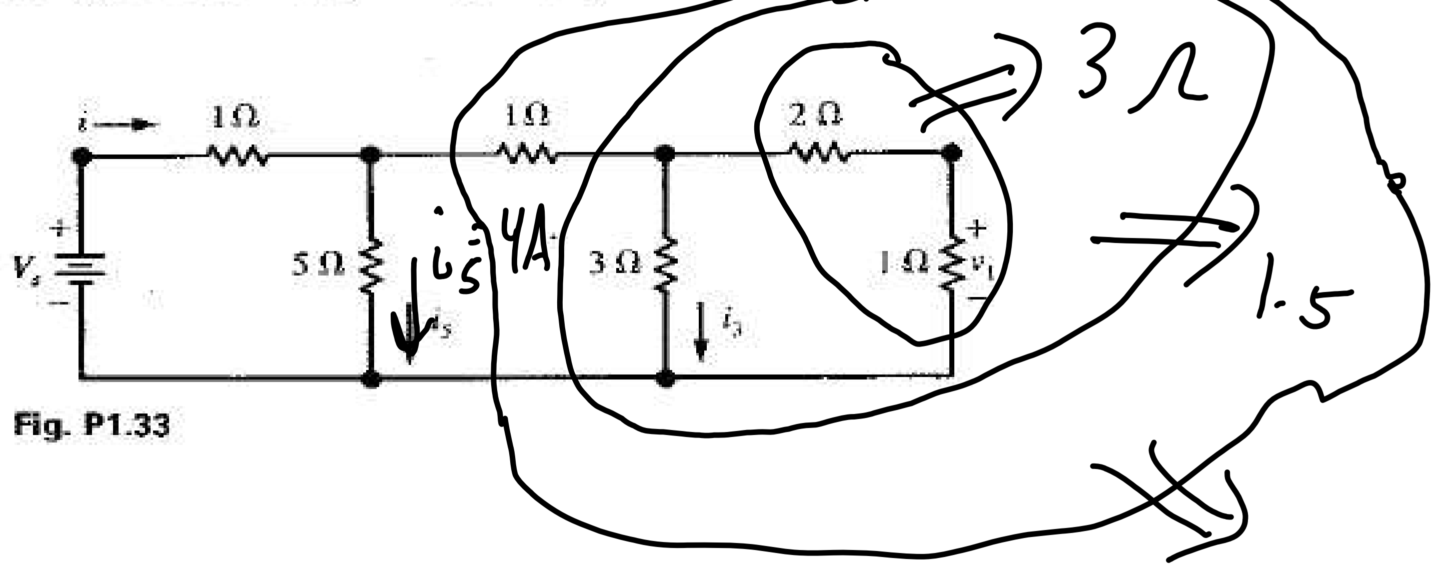
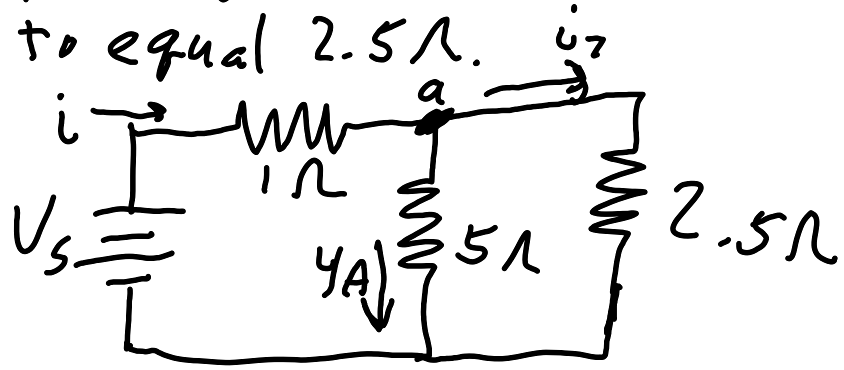


Fig. P1.33

The right network can combine to equal 2.5Ω.



$(4A)(5\Omega) = (2.5\Omega)(i_7)$

So, $i_7 = 8$ A

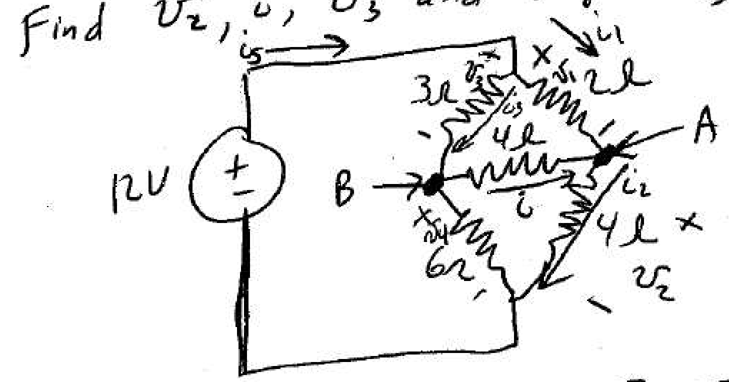
KCL@ a: $i = 4A + i_7 = 12$ A

KVL@ left loop $\boxed{V_s = (1\Omega)(i) + (4A)(5\Omega) = 32V}$

$R_{eq} = \frac{32}{12} = 8/3\ \Omega$

6) FoEE 1.40

Find v_2, i, v_3 and $R_{eq} = V_S / i_S$



Given: $v_1 = 4V$

By KVL around outer loop: $12 = v_1 + v_2$. $\therefore$ $\boxed{v_2 = 8V.}$

By Ohm's Law: $i_1 = \frac{v_1}{2\Omega} = \frac{4}{2} = 2A$ & $i_2 = \frac{v_2}{4\Omega} = \frac{8}{4} = 2A.$

By KCL: $i_1 + i = i_2$ $\therefore$ $\boxed{i = 0}$

By Ohm's Law, Since there is no current going through the "bridge resistor" $(i=0)$ there is no voltage drop across it.

So, the voltage at point A = voltage at point B.

$\therefore$ $v_4 = 8V$

By KVL on the "inner" loop: $12 = v_3 + v_4 = v_3 + 8$

$\boxed{\therefore v_3 = 4V.}$

Finally, $i_S = i_1 + i_3$ (KCL)

By Ohm's Law $i_3 = \frac{v_3}{3\Omega} = \frac{4}{3} A.$

So, $i_S = 2A + \frac{4}{3}A = \frac{10}{3} A$

So, $\boxed{R_{eq} = \frac{V_S}{i_S} = \frac{12V}{\frac{10}{3}A} = 3.6\Omega}$

7) FoEE 1.46

1.46 Consider the circuit shown in Fig. P1.46. (a) Find the resistance $R_{eq} = v_1/i_1$. (b) Find the voltage v_2 in terms of the applied voltage v_1.

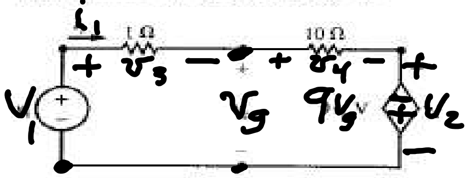


Fig. P1.46

KVL:

$v_1 - v_3 - v_4 - v_2 = 0$

Ohm's Law:

$v_3 = i_1 \cdot 1\Omega \quad ; \quad v_4 = i_1 \cdot 10\Omega$

Dep. source: $v_2 = -9 v_g$

KVL loop around right half:

$v_g - v_4 + 9 v_g = 0$

$\Rightarrow v_4 = 10 v_g$

$v_g = \frac{v_4}{10}$

$v_1 - v_3 - v_4 - v_2 = 0$

$v_1 - i_1 \cdot 1\Omega - i_1 \cdot 10\Omega - (-9 v_g) = 0$

$v_1 - i_1 - 10 i_1 + 9 v_g = 0$

$\longrightarrow v_g = \frac{v_4}{10} = \frac{i_1 10}{10} = i_1$

$v_1 - i_1 - 10 i_1 + 9 i_1 = 0$

$v_1 = 2 i_1$

a) $R_{eq} = \frac{v_1}{i_1} = \frac{2 i_1}{i_1} = 2\Omega$

b) $v_2 = -9 v_g = -9 i_1$

$i_1 = \frac{v_1}{R_{eq}} = \frac{v_1}{2}$

$v_2 = -\frac{9 v_1}{2} = -4.5 v_1$

Optional Problems:

8) FoEE 1.7

1.7 Consider the circuit shown in Fig. P1.7. (a) Given $i_1 = 4$ A, find v_1. (b) Given $i_2 = -2$ A, find v_2. (c) Given $i_3 = 2$ A, find v_3. (d) Given $i_4 = -2$ A, find v_4.

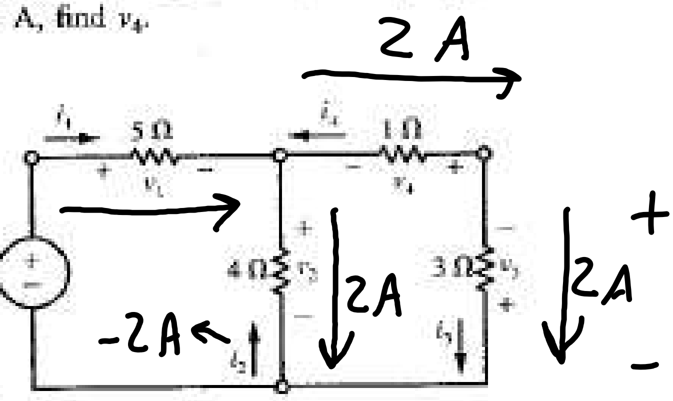


Fig. P1.7

a) $i_1 = 4A$

$v_1 = i_1 \cdot R_1 = 4 \cdot 5 = 20V$

b) $i_2 = -2A$

$v_2 = -i_2 \cdot R_2 = -{-2A} \cdot 4\Omega$

$v_2 = 8V$

c) $i_3 = 2A$ $\quad v_3 = -i_3 \cdot R_3 = -2A \cdot 3\Omega$

$v_3 = -6V$

d) $i_4 = -2A$

$v_4 = i_4 \cdot R_4 = -2A \cdot 1\Omega = -2V$

9) FoEE 1.24 a

1.24 Consider the circuit shown in Fig. P1.23. Find v when (a) $i_1 = 12$ A and $i_2 = 6$ A, (b) $i_1 = 6$ A and $i_2 = 6$ A, (c) $i_1 = 6$ A and $i_2 = 12$ A.

$G = \frac{1}{R}$

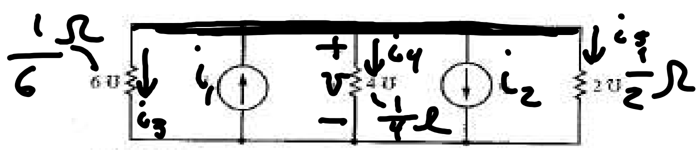


Fig. P1.23

KCL at top node:

$-i_3 + i_1 - i_4 - i_2 - i_3 = 0$

Ohm's Law: $\quad -\frac{v}{1/6} + i_1 - \frac{v}{1/4} - i_2 - \frac{v}{1/2} = 0$

$-6v + 12A - 4v - 6A - 2v = 0$

$6A = 12v$

$v = \frac{6A}{12} = \frac{1}{2}V$

10) FoEE 1.48

1.48 For the circuit shown in Fig. P1.48, suppose that $R = 10\ \Omega$. Determine (a) v_s, and (b) $R_{eq} = v_s/i_s$.

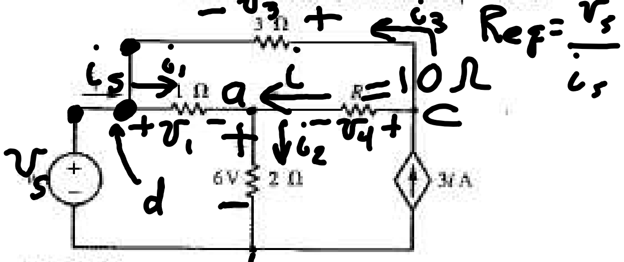


Fig. P1.48

$R_{eq} = \frac{v_s}{i_s}$

Ohm's Law:

$i_2 = \frac{6V}{2\Omega} = 3A$

KCL at node c:

$3i = i + i_3 \Rightarrow i_3 = 2i$

KCL at node a: $i_1 + i = i_2 = 3$

$i_1 = 3 - i$

KVL around top loop: $v_3 - v_4 + v_1 = 0$

$3\Omega \cdot i_3 - i \cdot 10\Omega + i_1 \cdot 1\Omega = 0$

$3\Omega(2i) - 10i + (3 - i) = 0$

$6i - 10i + 3 - i = 0$

$-5i = -3 \Rightarrow i = \frac{3}{5}A$

$i_1 = 3 - i = 2\frac{2}{5}A$

KVL around lower left loop:

$v_s = v_1 + 6V$

$v_s = i_1 \cdot 1\Omega + 6V = 2\frac{2}{5} + 6 = 8.4V$

b) $i_s + i_3 = i_1$

$i_s + 2i = 2.4 \qquad i_3 = 2i = 2 \cdot \frac{3}{5} = \frac{6}{5} = 1.2A$

$i_s = 2.4 - 1.2 = 1.2A$

$R_{eq} = \frac{v_s}{i_s} = \frac{8.4V}{1.2A} = 7\Omega$