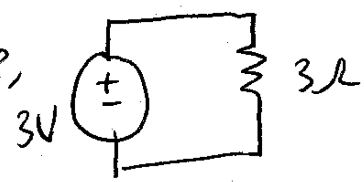
Practice Solutions

Handout #17
E84; Fall '07
1/10/07

① Here, we simply need to combine resistors:

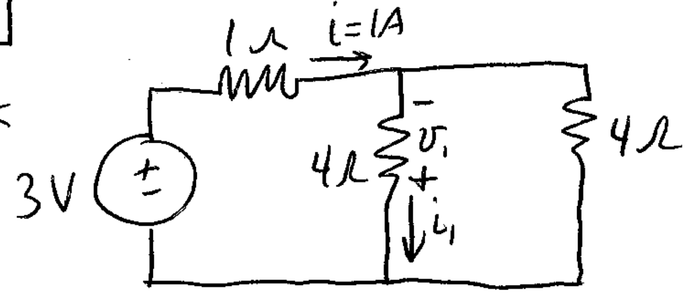
$$\underbrace{\left(\underbrace{\left(\underbrace{\left(\underbrace{\left(\underbrace{\left(\underbrace{(3+1)}_{4}/\!/4\right)}_{2}+1+1\right)}_{4}/\!/4\right)}_{2}+2\right)}_{4}/\!/4\right)}_{2}+1}_{3}$$

So,



$\therefore\ i = 1A$

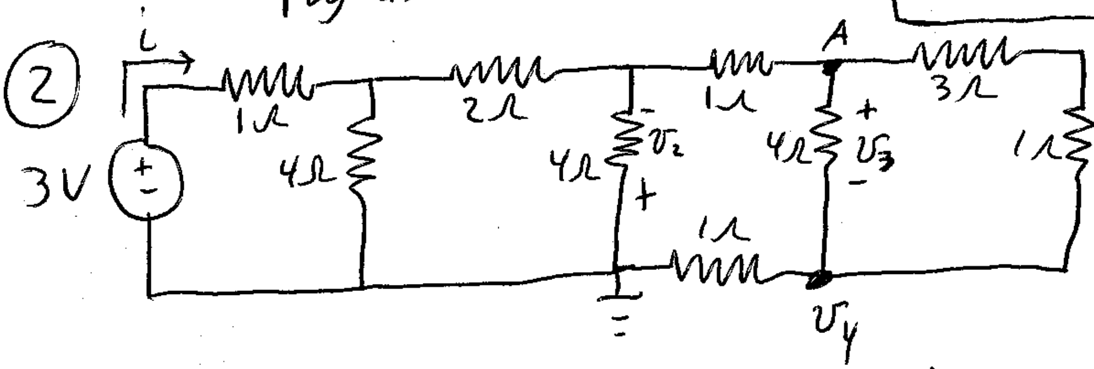
One step back



$$\boxed{\therefore\ i_1 = \frac{4}{4+4} = \frac{1}{2}A}$$

Pay attention to polarity: $\boxed{v_1 = -i_1 R = -\frac{1}{2}\cdot 4 = -2V}$

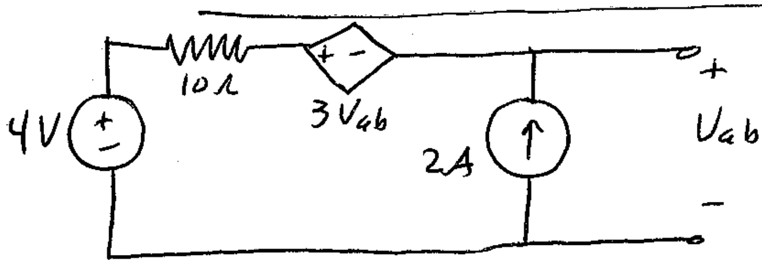
②



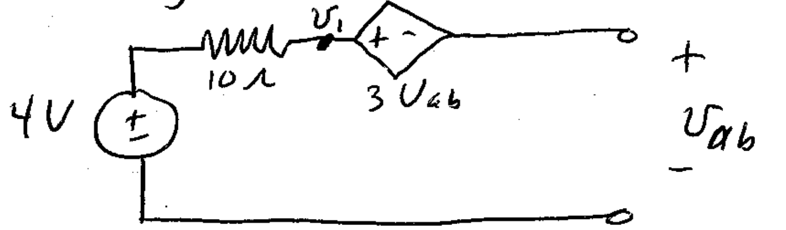
KCL at A: $\dfrac{-v_2 - (v_3 + v_4)}{1\Omega} = \dfrac{v_3}{4\Omega} + \dfrac{v_3}{4\Omega}$

This reduces to: $v_2 + \frac{3v_3}{2} + v_4 = 0$

(3)



Using superposition to find V_{ab} we have:

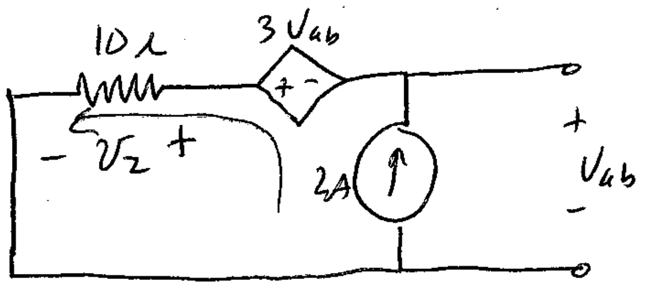


Since no current can flow through the open circuit

$v_1 = 4V$

$4 = 3V_{ab} + V_{ab}$

$\therefore V_{ab_{4V\,source}} = 1V$



The 2A has to flow around the closed loop.

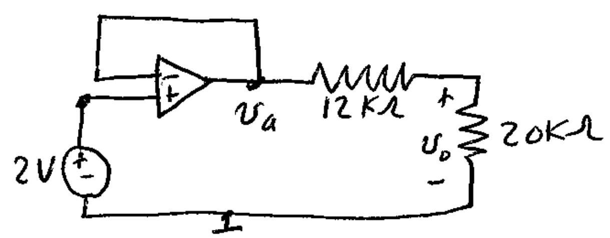
So $v_2 = 10 \cdot 2 = 20\ V$.

$20\ V = 3V_{ab} + V_{ab} = 4V_{ab}$

$\therefore V_{ab_{2A\,source}} = 5V$

$\boxed{\therefore V_{ab} = 6V}$

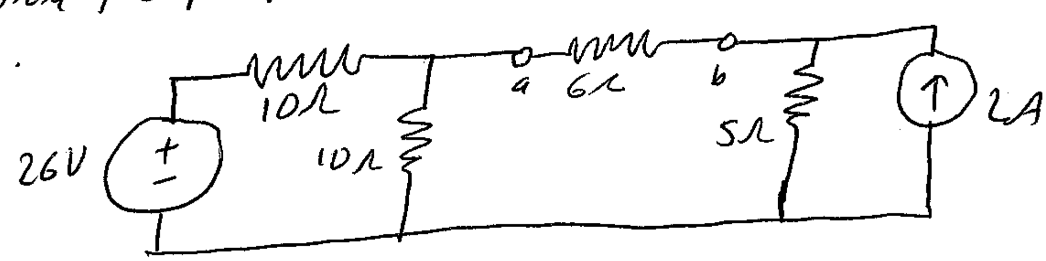
(4)



Since $v_- = v_+$, $v_a = 2V$.

So, $\boxed{v_o = \frac{20}{32} \cdot 2V = \frac{5}{4}V}$

(5) Finding the Thevenin & Norton equivalents of this circuit from the perspective of a & b, we first should find v_{oc}.



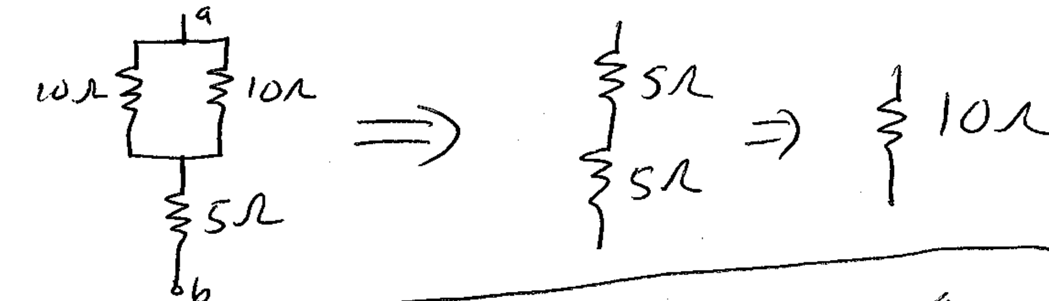
By opening the circuit at a & b, we simply need to find v_a and v_b.

$$v_a = 26\,V \cdot \frac{10}{20} = 13\,V.$$

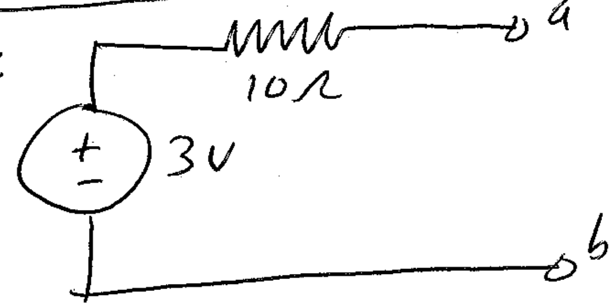
$$v_b = 2\,A \cdot 5\,\Omega = 10\,V$$

$$\therefore \underline{v_{oc} = v_a - v_b = 3\,V}$$

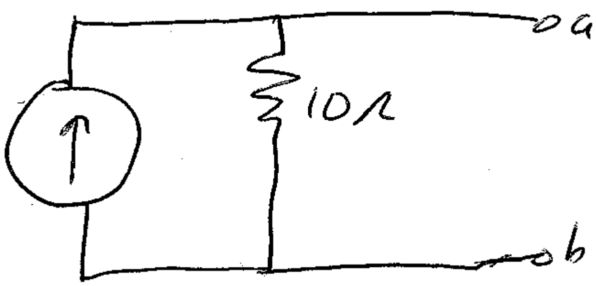
$R_{eq} =$



∴ Thevenin equivalent:

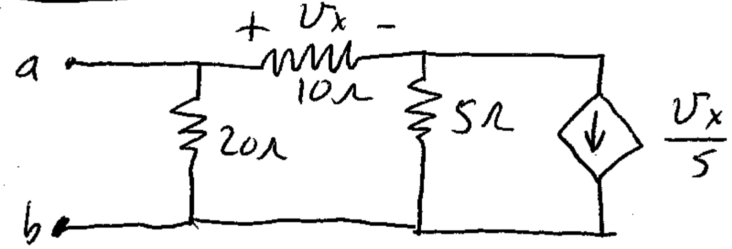


Norton Equivalent:

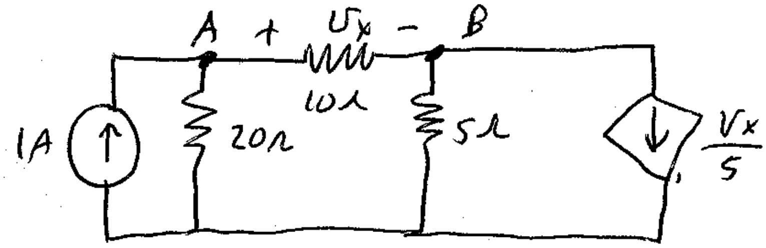


To get maximum power, $R_L = 10\,\Omega$.

(6)



To find the equivalent resistance, we're best off if we tie a current source of 1A between a & b and solve for the corresponding voltage.



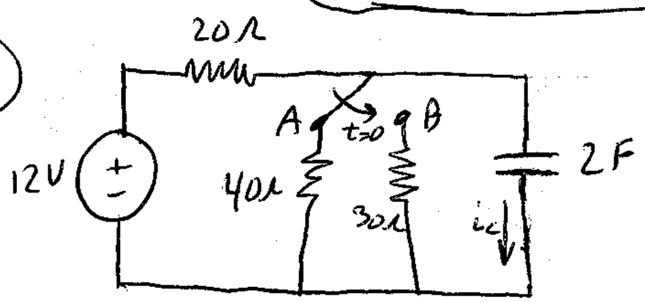
KCL at A: $1 = \frac{v_A}{20} + \frac{v_x}{10}$

KCL at B: $\frac{v_x}{10} = \frac{v_A - v_x}{5} + \frac{v_x}{5}$

Combining these equations (I'll leave the algebra to you)
you get: $v_x = 8\,V$
$v_A = 4\,V$

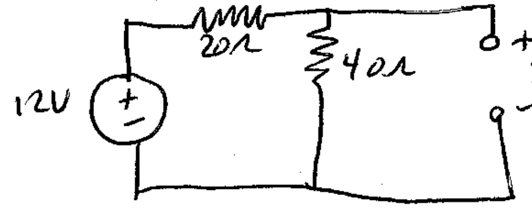
$$\therefore\ R_{eq} = \frac{v_A}{1A} = 4\,\Omega$$

(7)



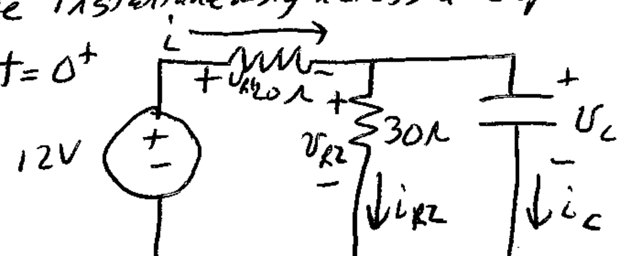
a) What is $i_c(0^+)$?

First let's see what the situation is before $t=0$.



$$v_c(0^-) = \frac{40}{60} \cdot 12 = 8V$$

Since voltage cannot change instantaneously across a capacitor, $v_c(0^+) = 8\,V$. So, at $t=0^+$



If $v_c(0^+) = 8V$, then $v_{R2}(0^+) = 8\,V$.

So, $i_{R2}(0^+) = \frac{8}{30} A$.

By KVL, $12 = v_{R1} + v_{R2}$

So, $v_{R1}(0^+) = 4V$

$\therefore i = \frac{4V}{20\Omega} = \frac{1}{5} A$

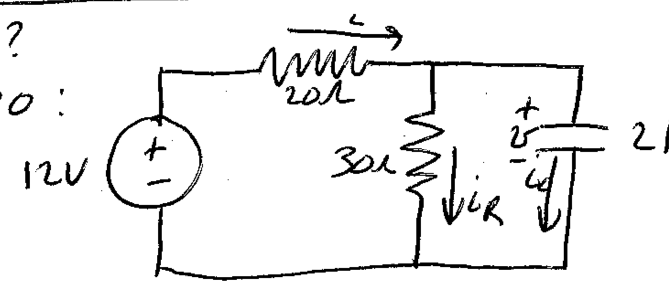
So, $i_c(0^+) = i - i_{R2}(0^+)$

$= \frac{1}{5} - \frac{8}{30}$

$\boxed{\therefore i_c(0^+) = \frac{-2}{30} = \frac{-1}{15} A}$

b) What is $v_o(t)$ for $t \geq 0$?

Again, at $t \geq 0$:



KVL: $12 = 20i + 30i_R$

KCL: $i = i_R + i_c$

$i_c = C\frac{dv}{dt}$, $v = 30i_R$

$i = \frac{12 - v}{20}$

So, combining the above: $i_c = i - i_R$

$$2\frac{dv}{dt} = \frac{12-v}{20} - \frac{v}{30}$$

$$\frac{dv}{dt} = \frac{3}{10} - \frac{v}{40} - \frac{v}{60}$$

$$\frac{dv}{dt} + \frac{v}{24} = \frac{3}{10}$$

Solution: $\frac{3/10}{\frac{1}{24}} + Ae^{-t/24} = 7.2 + Ae^{-t/24}$

Remember $v_c(0^+) = 8V$

$$\therefore \quad 7.2 + A = 8V$$

$$\text{So, } A = 0.8V$$

$$\boxed{\therefore \; v(t) = 7.2 + 0.8e^{-t/24}}$$

As a check $i_c(0^+)$ should equal $\frac{-1}{15}$ A.

Let's see: $i_c(t) = C\frac{dv}{dt} = 2 \cdot 0.8 \cdot \frac{-1}{24} e^{-t/24}$

$$= \frac{-1.6}{24} e^{-t/24}$$

So, $i_c(0) = \frac{-1.6}{24} = \frac{-1}{15}$ A ✓

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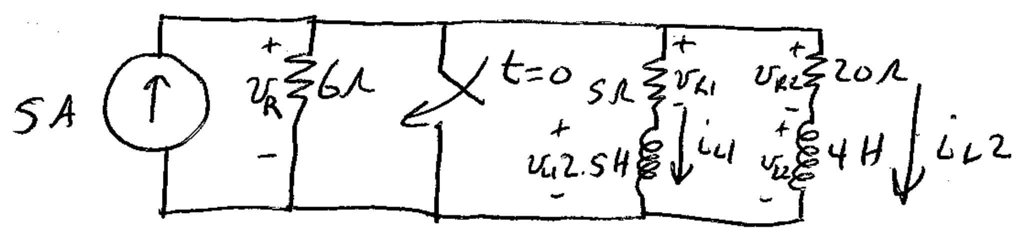
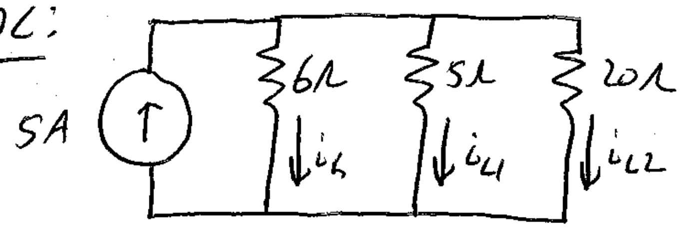
(8) Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit:



At DC:



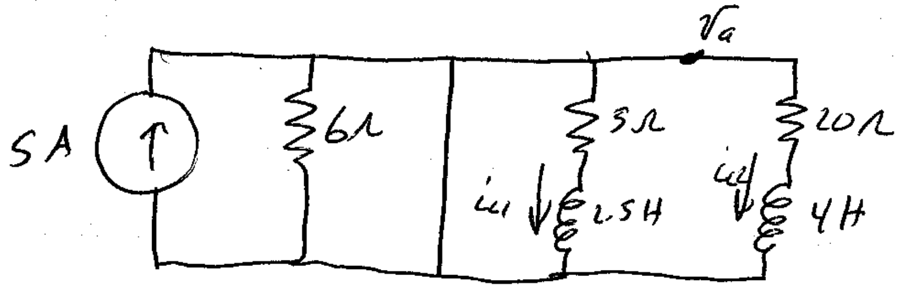
The equivalent resistance as seen by the source is 2.4 Ω. So, the voltage drop across each resistor is 12V.

So, $i_R(0^-) = 2A$

$i_{L1}(0^-) = \frac{12}{5}$ A

$i_{L2}(0^-) = \frac{12}{20} = \frac{3}{5}$ A

At $t=0$ the switch closes and we have:



Because of the short circuit, each RL combination discharges as if they were isolated.

This is because $v_a = 0V$.

$$\therefore\ 5 i_{L1} + 2.5 \frac{di_{L1}}{dt} = 0$$

$$\frac{di_{L1}}{dt} + 2 i_{L1} = 0$$

$$i_{L1}(t) = A_1 e^{-2t}$$

$$i_{L1}(0^+) = \frac{12}{5}$$

$$\text{So, } i_{L1}(t) = 2.4 e^{-2t}\ A \text{ for } t>0$$

By the same logic:

$$20 i_{L2} + 4 \frac{di_{L2}}{dt} = 0$$

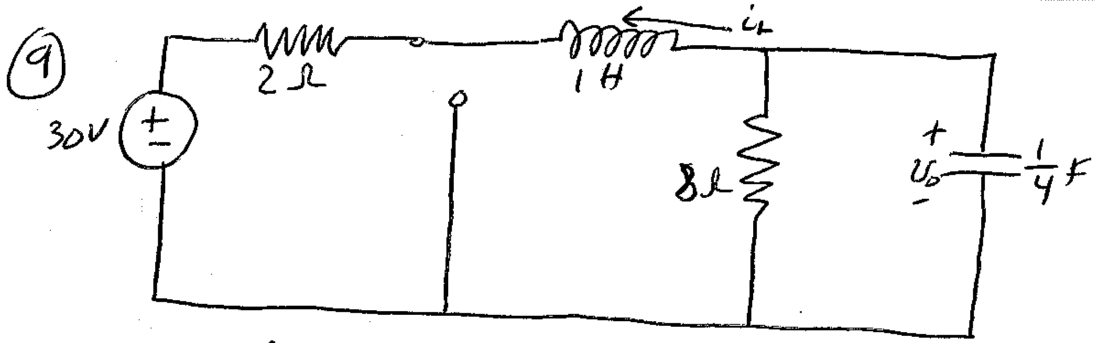
$$\frac{di_{L2}}{dt} + 5 i_{L2} = 0$$

$$i_{L2}(t) = A_2 e^{-5t}$$

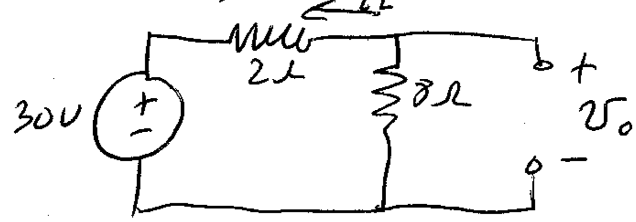
$$i_{L2}(0^+) = \frac{3}{5}$$

$$\text{So, } i_{L2}(t) = 0.6 e^{-5t}\ A \text{ for } t>0$$

(9)



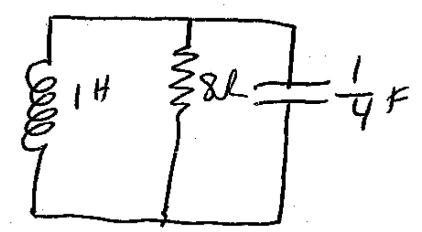
As always, we look at $t = 0^-$



$$v_o(0^-) = \frac{8}{10} \cdot 30 = 24\,V$$

$$i_L(0^-) = -3\,A$$

For $t \geq 0$



For an RLC circuit (parallel):

$$\frac{d^2 i}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha = \frac{1}{RC}} \frac{di}{dt} + \underbrace{\frac{1}{LC}}_{\omega_n^2 = \frac{1}{LC}} i = 0$$

$$\omega_d = \sqrt{4 - \frac{1}{16}} = 1.984 \Longleftarrow \alpha = \frac{1}{2 \cdot 8 \cdot \frac{1}{4}} = \frac{1}{4} \qquad \omega_n^2 = \frac{1}{\frac{1}{4}} = 4$$

So, this is underdamped:

$$i_L(t) = e^{-t/4}\left[A_1 \cos 1.984t + A_2 \sin 1.984t\right]$$

$$i_L(0) = \underline{A_1 = -3}$$

$$v_L(t) = v_c(t) = (1)\frac{di}{dt} = -\frac{1}{4}A_1 e^{-t/4}\cos 1.984t - \frac{1}{4}A_2 e^{-t/4} \sin 1.984t$$
$$+ -e^{-t/4}(1.984)A_1 \sin 1.984t$$
$$+ e^{-t/4}(1.984)A_2 \cos 1.984t$$

$$v_c(0) = 24 = -\frac{1}{4}A_1 + 1.984 A_2$$

$$A_2 = 11.717$$

So, $v_o(t) = \left[-\frac{1}{4}(-3) + 1.984(11.717)\right] e^{-t/4} \cos 1.984t$

$+ \left[-\frac{1}{4}(11.717) - 1.984(-3)\right] e^{-t/4} \sin 1.984t$

$$\boxed{v_o(t) = (24 \cos 1.984t + 3.024 \sin 1.984t) e^{-t/4}\ V}$$

$i_o(t) = C\frac{dv}{dt}$

$= \frac{1}{4}\left[24(-1.984) \sin 1.984t + (3.024)(1.984) \cos 1.984t\right] e^{-t/4}$

$+ \frac{1}{4}\left[\left(-\frac{1}{4}\right) e^{-t/4}\, 24 \cos 1.984t + \left(-\frac{1}{4}\right) e^{-t/4}\, 3.024 \sin 1.984t\right]$

$= \left[\frac{1}{4}(3.024)(1.984) - \frac{1}{4}\cdot\frac{1}{4}\cdot 24\right] e^{-t/4} \cos 1.984t +$

$\left[\frac{1}{4}\cdot 24\cdot(-1.984) + \frac{1}{4}\left(-\frac{1}{4}\right) 3.024\right] e^{-t/4} \sin 1.984t$

$$\boxed{i_o(t) = -12.095\, e^{-t/4} \sin 1.984t\ A}$$

To be critically damped: $\alpha^2 = \omega_n^2$

So $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$

$\frac{1}{4R^2C^2} = \frac{1}{LC} \Longrightarrow R^2 = \frac{L}{4C} = \frac{1}{1}$

So, if $R = 1$, this would be critically damped.