UNIVERSITY OF TORONTO | Statistical Sciences

Tutorial - Week 3

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Outline

- Estimators
- Maximum Likelihood Estimator (MLE)
- Bayesian Inference

Estimators

- Bias
- Consistency
- Efficiency
  - Variance of Estimator
  - Cramér-Rao lower bound (unbiased estimator)
- Mean Squared Error

Exercise - MIPS 20.9

Given a random sample $X_1, X_2, \ldots, X_n$ from a $Bern(p)$ distribution. One consider the estimators

$$T_1 = \frac{1}{n}(X_1 + \ldots + X_n) \text{ and } T_2 = \min\{X_1, \ldots, X_n\}$$

a. Are T_1 and T_2 unbiased estimators for p?

b. Get their Mean squared error

c. Which estimaor is more efficient when n=2?

MLE

- Likelihood or Log Likelihood function
- Properties:
  - Invariance
  - Asymptotically unbiased
  - Asymptotically minimum variance

Exercise - MIPS 21.7

Suppose that $x_1, x_2, \ldots, x_n$ is a dataset, which is a realization of a random sample from a Rayleigh distribution, which is a continuous distribution with probability density function given by

$$f_\theta(x) = \frac{x}{\theta} e^{-\frac{1}{2}x^2/\theta^2} \text{ for } x \geq 0.$$

In this case what is the maximum likelihood estimate for θ?

Bayesian Inference

- Bayes rule
- Law of total probability
- Prior and Posterior

Exercise - ER 7.1.4

Suppose that $(x_1, \ldots, x_n)$ is a sample from Poisson(λ) distribution with $\lambda > 0$ unknown. If the prior distribution of λ is Gamma(α, β),then obtain the form of the posterior density of λ.