HW2-Q3 √

2021年1月27日 星期三 14:08

Q3 maximize $z = 2 - 3x_1 + 3x_2 + 3x_3$

subject to $\begin{cases} 3x_1 + 3x_2 + x_3 \le 13 \\ -x_1 + x_2 + x_3 \le 1 \\ 3x_2 + x_3 \le 5 \\ x_1, x_2, x_3 \ge 0 \end{cases}$

(a) find maximum $z = 2 - 3x_1 + 3x_2 + 3x_3$

subject to: $\begin{cases} w_1 = 13 - 3x_1 - 3x_2 - x_3 \\ w_2 = 1 + x_1 - x_2 - x_3 \\ w_3 = 5 - 3x_2 - x_3 \\ x_1, x_2, x_3, w_1, w_2, w_3 \ge 0 \end{cases}$

We set $x_1 = 0$ since it has the negative sign. Then we got:

$$z = 2 + 3x_2 + 3x_3$$

subject to $\begin{cases} w_1 = 13 - 3x_2 - x_3 \\ w_2 = 1 - x_2 - x_3 \\ w_3 = 5 - 3x_2 - x_3 \\ x_1, x_2, x_3, w_1, w_2, w_3 \ge 0 \end{cases}$

The $M\min = \frac{1}{1}$ from w_2, so we flip x_3 with w_2, we got:

$\begin{cases} x_3 = 1 - x_2 - w_2 \\ w_1 = 13 - 3x_2 - 1 + x_2 + w_2 = 12 - 2x_2 + w_2 \\ w_3 = 5 - 3x_2 - 1 + x_2 + w_2 = 4 - 2x_2 + w_2 \end{cases}$

new $z = 2 + \cancel{3x_2} + 3 - \cancel{3x_2} - 3w_2$

new $z = 5 - 3w_2$

We set $w_2 = 0$ to get the maximum $z = 5$

$\Rightarrow \begin{cases} x_3 = 1 - x_2 \ge 0 \\ w_1 = 12 - 2x_2 \ge 0 \\ w_3 = 4 - 2x_2 \ge 0 \end{cases} \Rightarrow x_2 = 1$

$\therefore x_3 = 1 - 1 = 0, \quad w_1 = 12 - 2 = 10, \quad w_3 = 4 - 2 = 2.$

Thus, the admissible solution is:

$$\boxed{\begin{array}{l} x_1 = 0, \quad x_2 = 1, \quad x_3 = 0. \\ w_1 = 10, \quad w_2 = 0, \quad w_3 = 2. \\ \text{The maximum } z = 5 \end{array}}$$

OR: we can flip x_2 with w_2, and follow the same procedures above. We got another admissible solution:

$$\boxed{\begin{array}{l} x_1 = 0, \quad x_2 = 0, \quad x_3 = 1 \\ w_1 = 12, \quad w_2 = 0, \quad w_3 = 4. \\ \text{The maximum } z = 5 \end{array}}$$

(b) From part (a) we got two answers for the optimal solution:

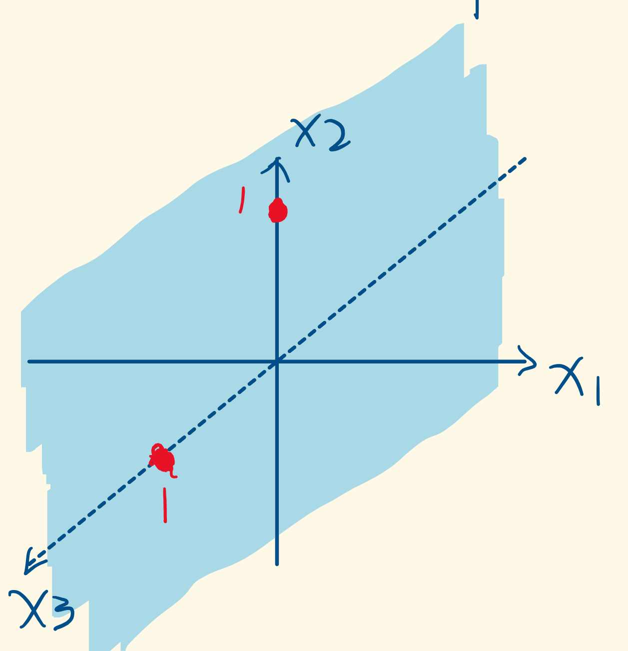
① $\begin{cases} x_1 = 0, x_2 = 1, x_3 = 0 \\ \max z = 5 \end{cases}$ and ② $\begin{cases} x_1 = 0, x_2 = 0, x_3 = 1 \\ \max z = 5. \end{cases}$

Also, we have the relationship between x_2 and x_3, which is $x_3 = 1 - x_2$ and $x_3, x_2 \ge 0$.

Solution set: $\mathbf{x}_1 = (0, 1, 0)$

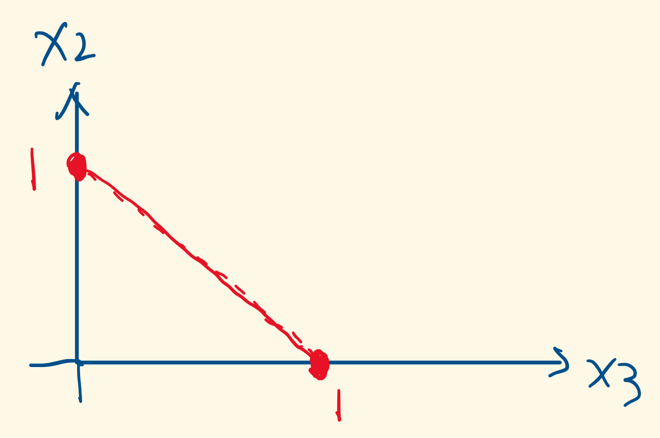
$\mathbf{x}_2 = (0, 0, 1)$

in a 3D plane:



The solution set is on the x_2-x_3 plane since x_1 is always 0.

To make it clear, we convert it to a 2-D plane:



From the figure, the constraint is the red line segment.

Meanwhile, the solutions which maximum the z are the corner points of the line segment, (any points of x_2, x_3 on the line will sum to one.) which follow the behavior we have discussed in the previous lecture.