Lecture 3-W3

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max $z = -3x_3 + x_4$ ← objective function

subject to:

$$\begin{cases} -3x_3 + 3x_4 \le 6 \\ -8x_3 + 4x_4 \le 4 \\ x_3 . x_4 \ge 0 \end{cases}$$

(?)

max $z = -3x_3 + x_4$ non-basic variables

subject to: basic

$$\begin{cases} x_1 = 6 + 3x_3 - x_4 \\ x_2 = 4 + 8 . x_3 - 4x_4 \\ \underbrace{x_1 \ldots x_4 \ge 0}_{\text{admissibility}} \end{cases} \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

augmented problem, a dictionary

Step 0: moving to the augmented form

Once we get here, a few things can happen.

ex1.

max $z = -3x_3 - x_4$ ← objective function

subject to:

$$\begin{cases} -3x_3 + x_4 \le 6 \\ -8x_3 + 4x_4 \le 4 \\ x_3 . x_4 \ge 0 \end{cases} \Rightarrow \begin{cases} x_1 = 6 + 3x_3 - x_4 \\ x_2 = 4 + 8x_3 - 4x_4 \\ x_1 \ldots x_4 \ge 0 \end{cases}$$

All variables appear in the objective function have a negative sign.

$\therefore$ $x_3 = x_4 = 0$, $x_1 = 6$, $x_2 = 4$ ← admissible solution ($z = 0$)

But since x_3 and x_4 appear with negative sign, even slightly increasing x_3, x_4, the z would drop value. $\Rightarrow$ $\boxed{\max z = 0}$

ex2.

max $z = -3x_3$

$x_1 = 6 + 3x_3 - x_4$

In this case, I'm forced to take $x_3 = 0$, leading to max $z = 0$.

$$\begin{cases} x_1 = 6 + 3x_3 - x_4 \\ x_2 = 4 + 8x_3 - 4x_4 \\ x_1 \dots x_4 \geq 0 \end{cases}$$

max $z = 0$.

I have multiple max points. $x_3 = 0$, $x_4 \geq 0$. Provided that $x_1, x_2 \geq 0$

$$\Rightarrow \begin{array}{l} x_1 = 6 - x_4 \\ x_2 = 4 - 4x_4 \end{array} \Rightarrow \begin{cases} 6 - x_4 \geq 0 \\ 4 - 4x_4 \geq 0 \end{cases} \Rightarrow 0 \leq x_4 \leq 1$$

$$\vec{x}_i = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 6-t \\ 4-4t \\ 0 \\ t \end{bmatrix} \quad 0 \leq t \leq 1$$

primily of admissible vectors giving the maximum value.

ex3.

$$\max \ z = -3x_3 + x_4$$

$$\begin{cases} x_1 = 6 + 3x_3 - 3 \cdot x_4 \\ x_2 = 4 + 8x_3 - 4x_4 \\ x_1 \dots x_4 \geq 0 \end{cases}$$

In this case, x_4 appears with the positive sign in the objective function, so I can increase the value of x_4 in the initial admissible solution: $x_3 = x_4 = 0$, $x_1 = 6$, $x_2 = 4$.

and increase the value of z.

How much to increase x_4?

$$\begin{cases} x_1 = 6 - 3x_4 \geq 0 \\ x_2 = 4 - 4x_4 \geq 0 \end{cases} \Rightarrow \begin{cases} \frac{6}{3} \geq x_4 \\ 1 \geq x_4 \end{cases} \Rightarrow \begin{cases} 2 \geq x_4 \\ 1 \geq x_4 \end{cases} \Rightarrow \boxed{0 \leq x_4 \leq 1}$$

$x_3 = 0$, $x_4 = 1$, $x_1 = 6 - 3 = 3$, $x_2 = 4 - 4 = 0$

max x_3, $x_2 = 0$, So I can use these as independent variables.

$$x_2 = 4 + 8x_3 - 4x_4$$

$$\Rightarrow x_4 = 1 + 2 \cdot x_3 - \frac{x_2}{4} = \boxed{1 - 0.25x_2 + 2x_3}$$

$$x_1 = 6 + 3 \cdot x_3 - 3 \cdot x_4$$

$$\Rightarrow = 6 + 3x_3 - 3(1 - 0.25x_2 + 2x_3)$$

$$= \boxed{3 + 0.75x_2 - 3x_3}$$

new objective function $z = -3x_3 + x_4$

new objective function $z = -3x_3 + x_4$

$z = 1 - 0.25x_2 + \cancel{2}x_3 - \cancel{3}x_3$

$z = 1 - 0.25x_2 - x_3$ ← all variables have negative sign.

subject to: $\begin{cases} x_4 = 1 - 0.25x_2 + 2x_3 \\ x_1 = 3 + 0.75x_2 - 3x_3 \\ x_1 \cdots x_4 \geq 0 \end{cases}$

So $\boxed{\text{in this case, the maximum } z = 1 \text{ and } x_2, x_3 = 0,\ x_4 = 1,\ x_1 = 3}$

↑ admissible vector that gives the max value.

ex4. max $z = +3x_3 - x_4$

subject to:

$\begin{cases} x_1 = 6 + 3x_3 - 3x_4 \\ x_2 = 4 + 8x_3 - 4x_4 \\ x_1 \cdots x_4 \geq 0 \end{cases}$

$x_3 = x_4 = 0$ admissible

$x_1 = 6,\ x_2 = 4$ before

In this case if increase the value of x_3, I improve the objective function.

$\begin{cases} x_1 = 6 + 3x_3 \geq 0 \\ x_2 = 4 + 8x_3 \geq 0 \end{cases}$

⇓ ← move negative, change signs.

$\begin{cases} 6 \geq -3x_3 \\ 4 \geq -8x_3 \end{cases} \Rightarrow \begin{cases} -2 \leq x_3 \\ -\frac{1}{2} \leq x_3 \end{cases} \Rightarrow$ no problem since $x_3 \geq 0$.

Thus, any positive x_3 is okay.

$\therefore \boxed{\max z = \infty}$

ex 5 max: $z = -2x_1 - x_2 + 1$

subject to:

$\begin{cases} -x_1 + x_2 \leq -1 \\ -x_1 - 2x_2 \leq -2 \\ x_2 \leq 1 \\ x_1, x_2 \geq 0 \end{cases} \Rightarrow \begin{cases} w_1 = -1 + x_1 - x_2 \\ w_2 = -2 + x_1 + 2x_2 \\ w_3 = 1 - x_2 \\ w_1, w_2, w_3, x_1, x_2 \geq 0 \end{cases}$

max $z = 1$? no. if $x_1, x_2 = 0$, $w_1, w_2 < 0$

ex 5' max $\zeta = -\lambda$ **Auxiliary Problem** (for initial solutions, always set the independent variables 0) — S.R. Liu

$\{ w_1 = -1 + x_1 - x_2 + \lambda = -1$

$$\begin{cases} w_1 = -1 + x_1 - x_2 + \lambda & = -1 \\ w_2 = -2 + x_1 + 2x_2 + \lambda & = -2 \\ w_3 = 1 - x_2 + \lambda \\ w_1, w_2, w_3, x_1, x_2 \geq 0 \end{cases}$$

set the indepedent variables 0) —S.R. Liu

We pick the WORST condition.

- setting $\lambda = 0$ in any dictionary of this problem I get back a dictionary for the original problem.
- max $\zeta = 0 \Leftrightarrow$ there is one admissible solution for the original problem.

$$w_2 = -2 + x_1 + 2x_2 + \lambda$$

$$\Downarrow$$

$$\boxed{\lambda = 2 - x_1 - 2x_2 + w_2}$$

$O.F = \zeta = -2 + x_1 + 2x_2 - w_2$

subject to:

$$\begin{cases} w_1 = -1 + \cancel{x_1} - x_2 + (2 - \cancel{x_1} - 2x_2 + w_2) \\ \lambda = 2 - x_1 - 2x_2 + w_3 \\ w_3 = 1 - x_2 + (2 - x_1 - 2x_2 + w_3) \\ w_i \geq 0,\ x_i \geq 0,\ \lambda \geq 0 \end{cases}$$

Maximum $-2 + x_1 + 2x_2 - w_2$

subject to:

$$\begin{cases} w_1 = 1 - 3x_2 + w_2 \\ \lambda = 2 - x_1 - 2x_2 + w_2 \\ w_3 = 3 - x_1 - 3x_2 + w_2 \\ w_i \geq 0,\ x_i \geq 0,\ \lambda \geq 0 \end{cases}$$