Lecture 4-W3

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Simplex algorithm

$$\max \ z = c_0 + c_1 \cdot x_1 + \cdots + c_n \cdot x_n .$$

subject to
$$\begin{cases} a_{11} x_1 + \cdots a_{1n} \cdot x_n \leq b_1 \\ a_{21} x_1 + \cdots a_{2n} \cdot x_n \leq b_2 \\ \vdots \\ a_{m1} x_1 + \cdots a_{mn} \cdot x_n \leq b_m \\ x_1 \cdots x_n \geq 0 \end{cases}$$

Step 0: introduce aditional variables $w_1 \cdots w_m$.

$$\begin{cases} w_1 = b_1 - a_{11} x_1 - \cdots a_{1n} \cdot x_n \\ w_2 = b_2 - a_{21} x_1 - \cdots a_{2n} \cdot x_n \\ \vdots \\ w_m = b_m - a_{m1} x_1 - \cdots a_{mn} \cdot x_n \\ x_1 \cdots x_n \geq 0 \end{cases}$$

Step 1: if some of the b_i's is negative.
solve the auxiliury problem:

$$\max: -\lambda$$
$$\begin{cases} w_1 = b_1 - a_{11} x_1 - \cdots a_{1n} \cdot x_n + \lambda \\ w_2 = b_2 - a_{21} x_1 - \cdots a_{2n} \cdot x_n + \lambda \\ \vdots \\ w_m = b_m - a_{m1} x_1 - \cdots a_{mn} \cdot x_n + \lambda \\ w_i \geq 0, \ x_1 \cdots x_n \geq 0, \ \lambda \geq 0 \end{cases}$$

form a new formulation of the problem where all the b_i's are non-negative.

Step 2: now $b_i \geq 0$ for $i = 1 \cdots m$

- if all the variables appear with negative coefficients in z. the $x_1 = \cdots = x_n = 0$. is optimal $\boxed{\max \ z = c_0}$. END

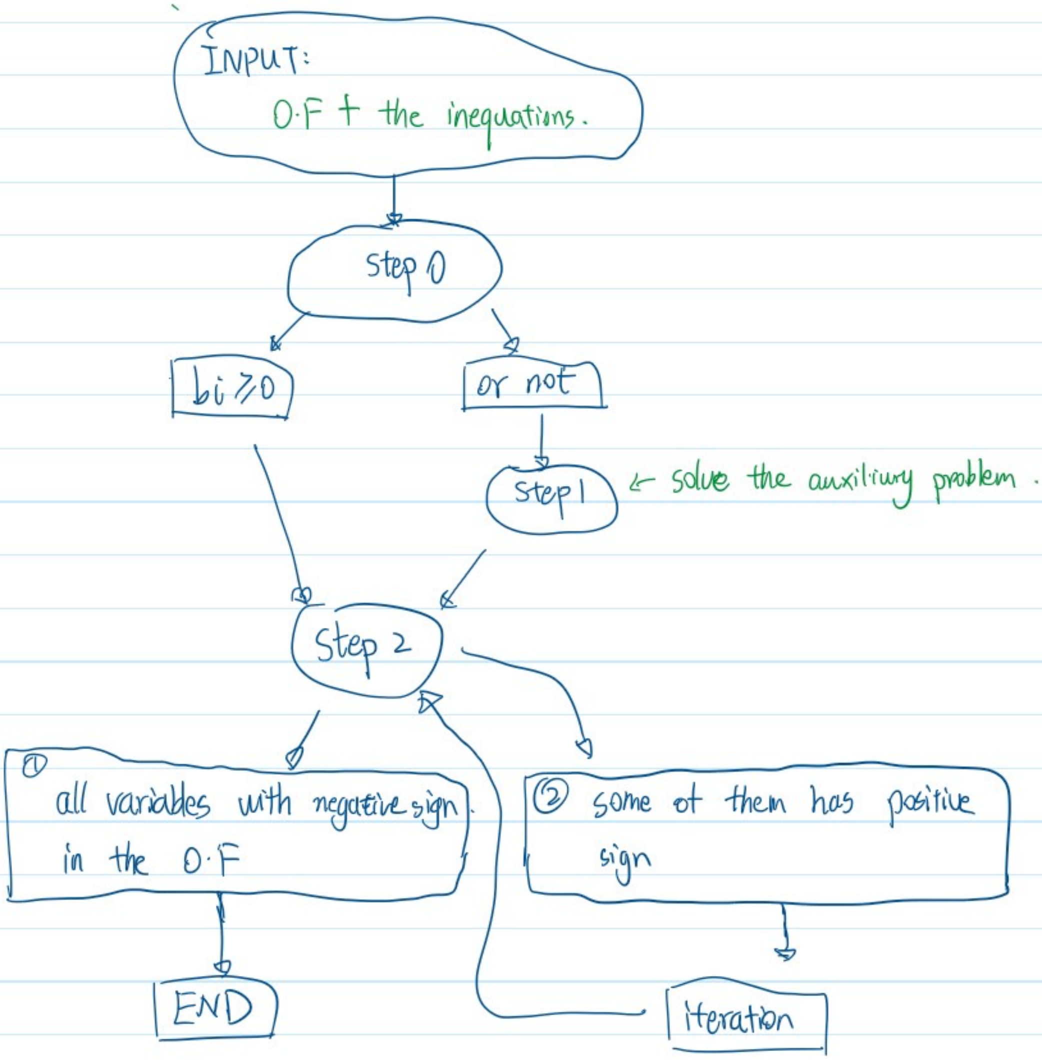
negative ? NOT SURE

in z. the $x_1 = \cdots = x_n = 0$. is optimal $\boxed{\max z = 0}$. END

negative? NOT SURE.

- if some of the variable do not appear in the objective function, then there are several optimal points:
  To find all of them, we set equal to zero. the variables that appear in the O.F (− v in O.F)
  This leads to the constraints for the parameter space of the multiple solution.

If one of the variables x_i appears with positive coefficient in the O.F, DO on iteration of the simplex method.



Iteration

$$\max z = c_0 + c_1 x_1 + \cdots + c_n x_n.$$

$$\begin{cases} w_1 = b_1 - a_{11} x_1 - \cdots a_{1n} \cdot x_n \\ w_2 = b_2 - a_{21} x_1 - \cdots a_{2n} \cdot x_n \\ \vdots \\ w_m = b_m - a_{m1} x_1 - \cdots a_{mn} \cdot x_n \\ w_i \geq 0, \ x_1 \cdots x_n \geq 0, \ b_i \geq 0 \end{cases}$$

We choose a variable x_i with positive coefficient in the O.F.

Computer $\left\{ \frac{b_1}{a_{1i}}, \frac{b_2}{a_{2i}}, \ldots, \frac{b_m}{a_{mi}} \right\}$

IF they are all negative, the problem is umbonded. $\Rightarrow$ $\boxed{\max z = \infty}$ END

ELSE compute

$$M = \min \left\{ \frac{b_1}{a_{1i}}, \ldots \frac{b_m}{a_{mi}} \right\}$$

we choose j in

$$J = \left\{ j \in \{1 \cdots m\} \text{ such that } \frac{b_j}{a_{ji}} = M \right\}$$ and flip x_i and x_j.

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