HW2-Q4 ✓

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Q4 maximize $z = 4x_1 - 3x_2 + 2x_3 - x_4$

subject to: $\begin{cases} -x_1 - x_2 - x_3 + x_4 \leq 5 \\ -x_1 + x_2 + x_3 - x_4 \leq 3 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$

(a) The reason of this problem doesn't have a maximum.

Since $z = 4x_1 - 3x_2 + 2x_3 - x_4$ and $x_1, x_2, x_3, x_4 \geq 0$.
To maximize z, we set $x_2, x_4 = 0$. We obtain:

$$\max z = 4x_1 + 2x_3, \text{ subject to: } \begin{cases} -x_1 - x_3 \leq 5 & ① \\ -x_1 + x_3 \leq 3 & ② \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

To maximize z, we need to increase x_1 and x_3. According to the constraints ① and ②, we can infinitly increase x_1 and x_3 as long as we keep x_1 is no more than 3 less than x_3 to satisfy the constraints ②. (In the other word, $x_3 \leq x_1 + 3$).
Therefore, we cannot have a maximum z.

(b). Find a solution with $z \geq 1000$.
As mentioned in part (a), to get the maximum z, we set $x_2 = x_4 = 0$. Then we have:

$$\max z = 4x_1 + 2x_3$$

$$\text{subject to: } \begin{cases} -x_1 - x_3 \leq 5 \\ -x_1 + x_3 \leq 3 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

The boundary is $x_3 = x_1 + 3$. We choose a solution on the boundary ☺. To achieve $z \geq 1000$, we pick:

$$\begin{cases} x_1 = 500 \\ x_3 = 503. \end{cases} \Rightarrow \begin{array}{l} -x_1 - x_3 = -500 - 503 = -1003 \leq 5 \ ✓ \\ -x_1 + x_3 = -500 + 503 = 3 \leq 3 \ ✓ \end{array}$$

check: $z = 4 \times 500 + 503 \times 2 = 2106 \geq 1000$

Therefore, the particular solution with $z \geq 1000$ is:

$$\boxed{\begin{cases} x_1 = 500 \\ x_2 = 0 \\ x_3 = 503 \\ x_4 = 0 \end{cases}}$$