HW2-Q2 ✓

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Q2 maximize $z = 6x_1 + 8x_2 + 5x_3 + 9x_4$

subject to:
$$\begin{cases} 2x_1 + x_2 + x_3 + 3\cdot x_4 \le 5 \\ x_1 + 3\cdot x_2 + x_3 + 2\cdot x_4 \le 3 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

convert to:
$$\begin{cases} w_1 = 5 - 2x_1 - x_2 - x_3 - 3x_4 \\ w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases}$$

$$M_1 = \left[\frac{5}{2}, \frac{5}{1}, \frac{5}{1}, \frac{5}{3}\right]. \quad M_2 = \left[\frac{3}{1}, \frac{3}{3}, \frac{3}{1}, \frac{3}{2}\right].$$

The minimum is $\frac{3}{3}$, which is from M_2. So, we flip x_4 with w_2 since x_4 has the largest coefficient in the objective function.

$$\begin{cases} x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}w_2 \\ w_1 = \frac{1}{2} - \frac{1}{2}x_1 + \frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{3}{2}w_2 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases}$$
$$\text{new } z = \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}w_2$$

To maximum z, we set $x_2 = w_2 = 0$. Now we get:

$$\Rightarrow \text{new } z = \frac{27}{2} + \frac{3}{2}x_1 + \frac{1}{2}x_3$$
$$\begin{cases} x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{1}{2}x_3 \\ w_1 = \frac{1}{2} - \frac{1}{2}x_1 + \frac{1}{2}x_3 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases}$$

Now the M min is $\left(\frac{1/2}{-\frac{1}{2}}\right)$ from w_1. Thus flip x_3 with w_1:

$$\Rightarrow \begin{cases} x_3 = -1 + x_1 + 2w_1 \\ x_4 = 2 - x_1 - w_1 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases}$$
$$\text{new } z = \frac{27}{2} + \frac{3}{2}x_1 + \frac{1}{2}x_1 + w_1 - \frac{1}{2}$$
$$\text{new } z = \frac{26}{2} + 2\cdot x_1 + w_1$$

Then we flip w_1 with x_4:

$$\Rightarrow \begin{cases} w_1 = 2 - x_1 - x_4 \\ x_3 = -1 + x_1 - 4 + 2x_1 + 2x_4 = -5 + 3x_1 + 2x_4 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases}$$
$$\text{new } z = 26/2 + 2x_1 + 2 - x_1 - x_4$$
$$\text{new } z = 15 + x_1 - x_4$$

To maximum z, we set $x_4 = 0$. Then we got:

$$\begin{cases} w_1 = 2 - x_1 \ge 0 \\ x_3 = -5 + 3x_1 \ge 0 \\ x_1, x_2, x_3, x_4, w_1, w_2 \ge 0 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \end{cases}$$

$\therefore$ $x_3 = -5 + 3\cdot 2 = 1$, $z = 15 + 2 = 17$, $w_1 = 2 - 2 = 0$

Then we got the admissible solution:

$$\boxed{\begin{array}{l} x_1 = 2,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0 \\ w_1 = 0,\ w_2 = 0, \\ \text{the maximum } z = 17 \end{array}}$$