PHYS

⇒ Final 8 soru

⇒ Bütün konular

⇒ Sample questions of 1st midterm

⇒ Vektör çarpımları

⇒ Rolling problemleri iyi çözmeniz

⇒ Geçen yıldan

→ İş Pulley makara

→ Angular momentum

① Space (L) ③ Mass (M)
② Time (T) ④ Charge (Q)

4 fundamental dimensions

$\{L\}$ → Only geometry $\{L, T\}$ kinematics $\{L, T, m\}$ Dynamics $\{L, T, m, Q\}$ → Electrodynamics

SI

Lenght → m

Time → Secs

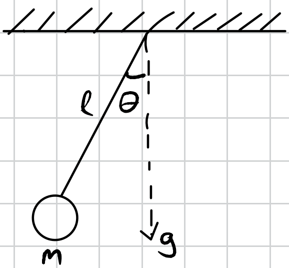
Mass → kg

Charge → Coulomb

Power $= \frac{kg \cdot m^2}{s^3}$ (W)

Frequency = Hertz $1/s$

Dimensional Analysis



$$[\tau] = f(l, m, g, \theta)$$

$$[\tau] = [l]^a [m]^b [g]^c$$

$$[\tau] = [L]^a [m]^b [L]^c [T^{-2}]^c$$

$$g = m/s^2 \,/\, \frac{[L]}{[T]^2}$$

$1 = -2c$

$c = -1/2$

$b = 0$

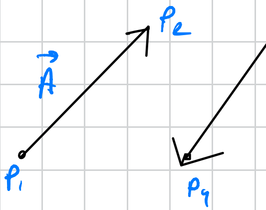
$a + c = 0$

$a = 1/2$

Vectors

Scalar → Physical quantity without direction

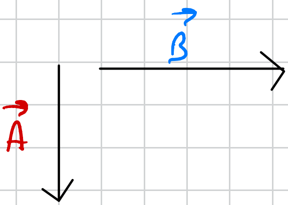
Vector → Both magnitude and Direction



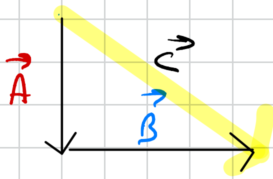
$\vec{A}' = \vec{B} = -\vec{A}$

Magnitude of A $= |\vec{A}| = A$

Addition and Subtraction

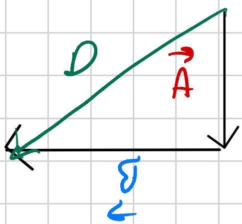


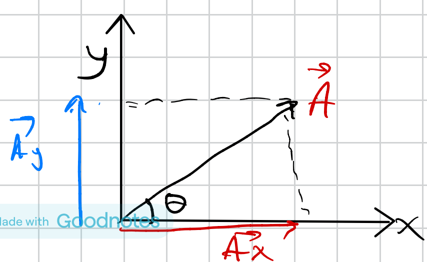
$\vec{A} + \vec{B} = C$



$\vec{A} - \vec{B} = \vec{D}$

$\vec{A} + (-B) = \vec{D}$





$\vec{A}_x + \vec{A}_y = \vec{A}$

$\cos\theta = \dfrac{A_x}{A}$

$\sin\theta = \dfrac{A_y}{A}$

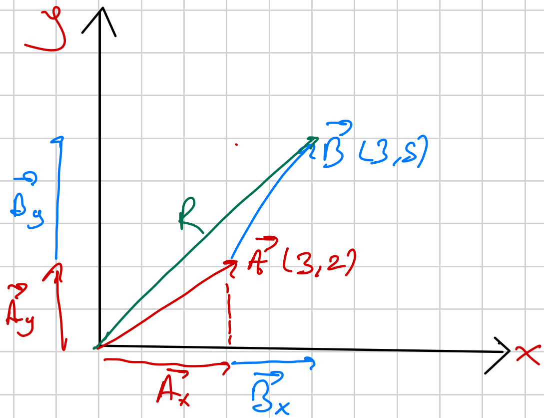
$\sin^2 x + \cos^2 x = 1$

$A_x^2 + A_y^2 = 1$

$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}$

$\theta = \arctan\left(\dfrac{A_y}{A_x}\right)$

$$c\vec{A} = c(\vec{A}_x + \vec{A}_y) = c\vec{A}_x + c\vec{A}_y$$ } Vector multiplication



Unit Vectors

1. Magnitude = 1
2. Carry no units
3. Represented by $\hat{i}, \hat{j}, k$

$|\hat{i}| = |\hat{j}| = |\hat{k}|$

x y z

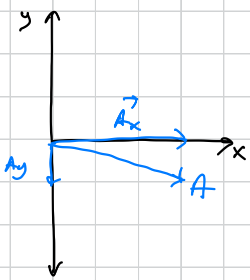
$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$ > magnitude

Example

$$\vec{A} = 3\hat{i} - \hat{j}$$



$$\boxed{\hat{A} = ? \quad \frac{3\hat{i} - \hat{j}}{\sqrt{10}}}$$

$$\hat{A} \neq A$$

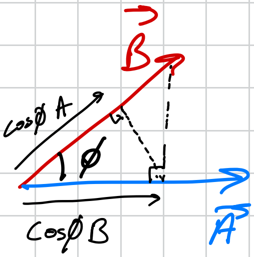
$$\hat{A} = \frac{3\hat{i}}{\sqrt{10}} - \frac{\hat{j}}{\sqrt{10}}$$

$$\sqrt{\left(\frac{3}{\sqrt{10}}\right)^2 + \left(\frac{1}{\sqrt{10}}\right)^2} = 1$$ {UNIT}

Product of vector

(1) Scalar (dot) product

$$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos\phi$$



$$= A.(\underbrace{B.\cos\phi}_{\text{magnitude component of } B \text{ in the direction of } \vec{A}}) = B.(\underbrace{A.\cos\theta}_{\text{ponent of } A \text{ in the direction of } B})$$

if ϕ is between 0 and 90 $\vec{A}.\vec{B} = +$

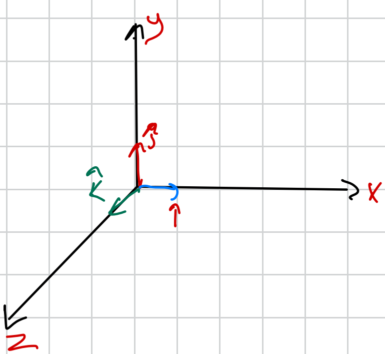
if ϕ is between 90 and 180 $\vec{A}.\vec{B} = +$

$$\vec{a} \perp \vec{b}$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\,\underbrace{\cos 90}_{0}$$

$$\vec{A}\cdot\vec{B} = \vec{B}\,\vec{A}$$

Commutative



$$\hat{i}\cdot\hat{j} = 0$$
$$\hat{i}\cdot k = 0$$
$$\hat{j}\cdot\hat{j} = 1$$

Example

$\vec{A} = 3\hat{i} + 2\hat{j}$ $\quad B = 4\hat{k}$ $\quad C = -\hat{j} + 2\hat{k} + 4\hat{i}$

$\vec{A}\,\vec{B} = (3\hat{i} + 2\hat{j})(4\hat{k})$

$\underbrace{12\hat{i}\hat{k}}_{0} + \underbrace{8\hat{j}\hat{k}}_{0} = 0$

$\vec{A}\vec{C} = (3\hat{i} + 2\hat{j})(-\hat{j} + 2\hat{k} + 4\hat{i})$

0 0 0 0

$12\hat{i}^2 - 2\hat{j}^2$

$12 - 2 = 10$

$\vec{B}\cdot\vec{C} = 4\hat{k}(-\hat{j} + 2\hat{k} + 4\hat{i})$

0 0

$8\hat{k}^2 = 8$

Example

$4 + 9 + 1$ $\quad 1 + 4$

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\quad \vec{b} = \hat{i} + 2\hat{j}$ $\quad |\vec{a}| = \sqrt{14}$

Find angle between $\vec{a}$ and $\vec{b}$ $\quad |\vec{b}| = \sqrt{5}$

$\vec{a}.\vec{b} = 2 + 6$

$\vec{a}.\vec{b} = 8$

$$\cos\phi = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\phi = \frac{8}{\sqrt{14}.\sqrt{5}}$$

$$\phi = \arccos\left(\frac{8}{\sqrt{14}.\sqrt{5}}\right)$$

Example

$\vec{A} = a\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = -2\hat{i} + \hat{j}$

$A \perp B$ what's $= a$

$\vec{A} \cdot \vec{B} = 0$

$-2a + 3 = 0$

$a = 3/2$

Ex

$\vec{a} = 4\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} - 3\hat{j} + 2k$

$\vec{a} + \vec{b} = 3\hat{i} + 0 + 5\hat{k}$

$\vec{a} + \vec{b} = 3\hat{i} + 5\hat{k}$

$9 + 25$

Find the angle that a+b makes with x ~~axis~~ axis

angle between x axis $\vec{a} + \vec{b}$

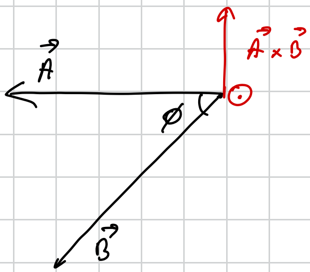
$$\underbrace{(\vec{a} + \vec{b}) \cdot \hat{i}}_{3} = |\vec{a} + \vec{b}| \, \underbrace{|\hat{i}|}_{x \text{ axis}} \cdot \cos\theta$$

$$3 = \sqrt{34} \cdot 1 \cdot \cos\theta$$

Cross Product (vector)

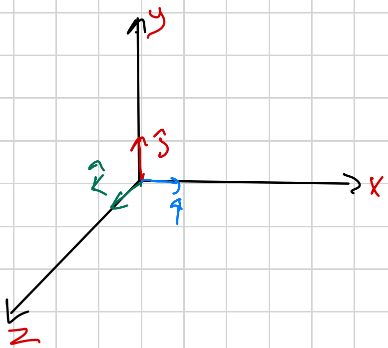
Right Hand Rule

$\vec{A} \times \vec{B} = \vec{C}$ $\qquad$ $|\vec{C}| = |\vec{A}|\,|\vec{B}| \sin\phi$



$C \rightarrow \perp \vec{A}$ and $\vec{B}$

$B \times A = -A \times B = -C$ (⊗)



$\hat{i} \times \hat{j} = \hat{k}$ $\qquad$ $\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{k} = \hat{i}$ $\qquad$ $\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{i} = \hat{j}$ $\qquad$ $\hat{k} \times \hat{k} = 0$

$$\left.\begin{aligned} \vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ \vec{B} &= B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \end{aligned}\right\} \vec{C} = A \times B = \begin{aligned} &(A_y B_z - A_z B_y)\,\hat{i} \\ &+ \\ &(A_z B_x - A_x B_z)\,\hat{j} \\ &+ \\ &(A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Determinant Method

$$\vec{A} \times \vec{B} = \begin{array}{c} + \quad - \quad + \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}_{3\times 3} \end{array}$$

$$\hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}_{2\times 2} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\boxed{D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad \det(D) = ad - bc)}$$

Ex:

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = \hat{i} + 2\hat{j}$

$\vec{A} \times \vec{B} = \vec{C}$

Find the unit vector to Both $\perp$ A and B $\rightarrow A\times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 3 & -1 \\ 1 & 2 & 0 \end{vmatrix} = \hat{i} \begin{vmatrix} 3, & -1 \\ 2, & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2, & 3 \\ 1, & 2 \end{vmatrix}$$

$$\hat{i}(0+2) - \hat{j}(0+1) + \hat{k}(4-3)$$

$$2\hat{i} - \hat{j} + \hat{k}$$

$$\hat{C} = \frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}}$$

$$\hat{C} = \frac{2\hat{i}}{\sqrt{6}} - \frac{\hat{j}}{\sqrt{6}} + \frac{\hat{k}}{\sqrt{6}}$$

Solving Determinant

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \\ \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \end{vmatrix}$$

sum_1

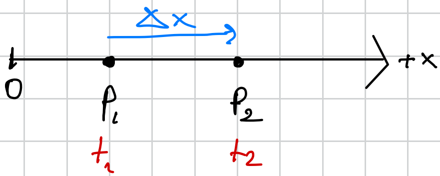
sum_2

$$\left((\hat{i}\, A_y\, B_z) + (A_x\, B_y\, \hat{k}) + (B_x\, \hat{j}\, A_z) \right)$$

$$-$$

$$\left((\hat{k}\, A_y\, B_x) + (A_z\, B_y\, \hat{i}) + (B_z\, \hat{j}\, A_x) \right)$$

One Dimensional Motion

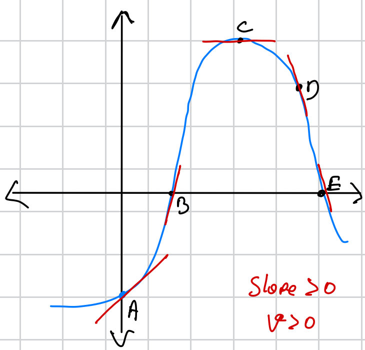


Average velocity $v_{ave} = \frac{\Delta x}{\Delta t}$

$$v_{ave} = \frac{x_2 - x_1}{t_2 - t_1} \qquad SI = m/s$$

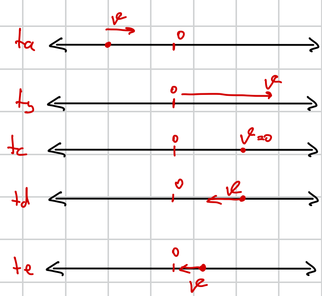
$$\vec{v} = v_x \hat{i}$$

$$\text{Instantaneous velocity} = v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



Slope > 0

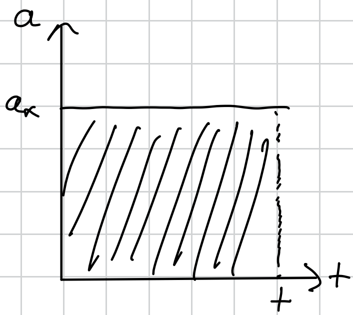
$v > 0$



Acceleration → the rate of change of velocity with time

$$a_{ave} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt} \cdot \left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Constant Acceleration



$t_i = 0\,s \qquad t_f = t$

$v = v_{0x} \qquad v = v_x$

$a_x = \dfrac{v_x - v_{0x}}{t - 0} \qquad v_x = v_{0x} + a_x t$

$x - x_0 = v_{0x} t + \frac{1}{2} a t^2$

$x = v_{0x} t + \frac{1}{2} a t^2 + x_0$

$a = \dfrac{dv}{dt} \qquad \int_{v_{0x}}^{v_x} dv = \int_{t=0}^{t} a\,dt$

$v \Big|_{v_{0x}}^{v_x} = at \Big|_0^t \longrightarrow v_x - v_{0x} = at$

$v_x = v_{0x} + at$

$v_x = \dfrac{dx}{dt}$

$\int_{x_0}^{x} dx = \int_{t=0}^{t} v_x\,dt = \int_{t=0}^{t} v_{0x} + at\,dt$

$x \Big|_{x_0}^{x} = v_{0x} + \dfrac{at^2}{2} \Big|_0^t$

$x - x_0 = v_{0x} t + \dfrac{at^2}{2}$

$x = x_0 + v_{0x} t + \frac{1}{2} a t^2$

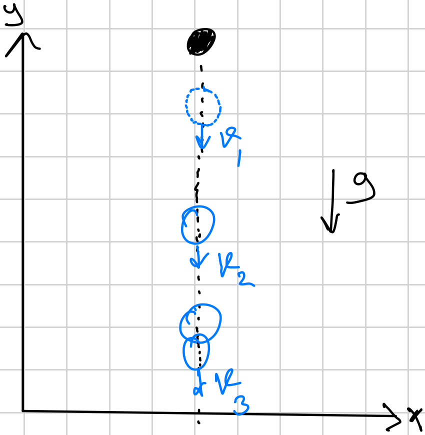
Example (HOMEWORK)

For a nonconstant acceleration given by

$$a = 4 \sin(2t)$$

Initial: $t=0$ $x_i = 0$ and $v_i = 0$

$\boxed{① \; v(t)}$ $\boxed{② \; x(t)}$



$$\vec{g} = -g\hat{j}$$

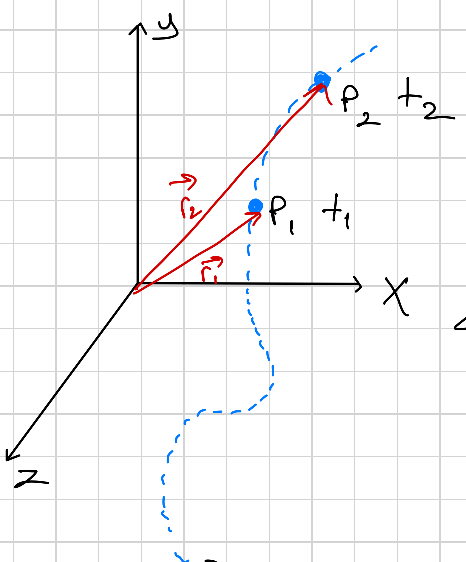
$$g = 9{,}81 \ m/s^2$$

$$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$y = \cancel{y_0} + V_{y_0} t + \frac{1}{2} a_y t^2$$

$$y = -\frac{1}{2} g t^2$$

$$V_y = \frac{dy}{dt} = -gt$$



$$\vec{V_{ave}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{r_2 - r_1}{t_2 - t_1}$$

$$P_1(x_1, y_1, z_1) \quad P_2(x_2, y_2, z_2)$$

$$\Delta r = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{V_{ave}} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

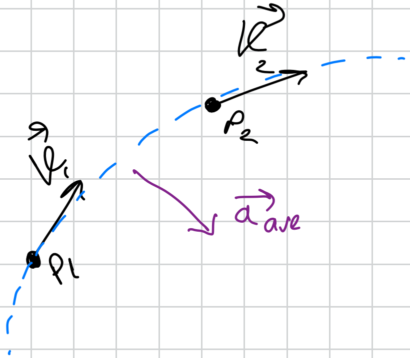
$$\vec{V_{ave}} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$v = \frac{d\vec{r}}{dt}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

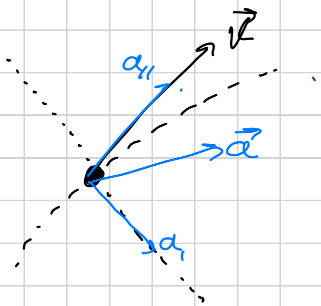
v = speed

$\vec{v}$ = velocity



$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

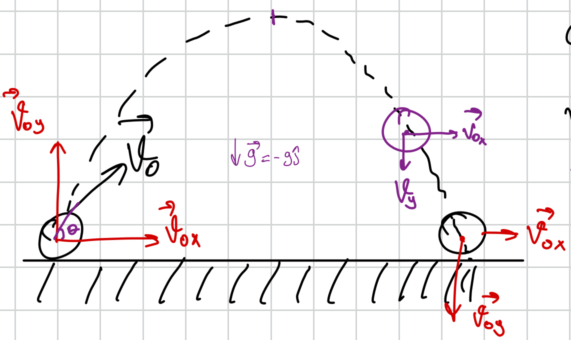
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}_x}{dt}\hat{i} + \frac{d\vec{v}_y}{dt}\hat{j} + \frac{d\vec{v}_z}{dt}\hat{k}$$



$$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

$\vec{a}_{\parallel}$ = magnitude, speed

$\vec{a}_{\perp}$ = direction



$a_x = 0 \qquad a_y = -g$

$v_x = v_{0x} t + a_x t$

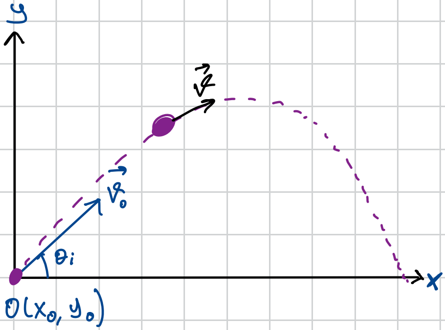
$\vec{v}_y = \vec{v}_{0y} + \vec{a}_y t = \vec{v}_y - gt$

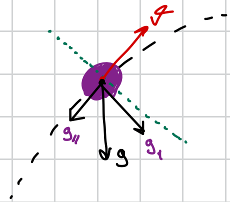
$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_0 \sin\theta\, t - \frac{1}{2} g t^2$$

$\vec{v}_{0x} = \vec{v}_0 \cos\theta$

$\vec{v}_{0y} = \vec{v}_0 \sin\theta$





$$V_{0x} = V_0 \cos\theta$$

$$V_{0y} = V_0 \cdot \sin\theta$$

$$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$$

$a_x = 0$

$a_y = -g$

No air resistance!

Initial Conditions

	x	y
$t=0$	$x_0 = 0$	$y_0 = 0$
	$a_x = 0$	$a_y = -g$
	V_{0x}	V_{0y}

$\Longrightarrow$

$$x = V_{0x} t$$

$$V_x = \frac{dx}{dt} = V_0 \cos\theta$$

$$y = V_{0y} t - \frac{1}{2} g t^2$$

$$V_y = \frac{dy}{dt} = V_{0y} - g t$$

$$x = V_0 \cos\theta_0 \cdot t$$

$$t = \frac{x}{V_0 \cdot \cos\theta}$$

$$\Longrightarrow y = V_{0y} \left(\frac{x}{V_0 \cos\theta_0} \right) - \frac{1}{2} g \left(\frac{x}{V_0 \cos\theta} \right)^2$$

$$y = x \tan\theta_0 - \frac{g x^2}{2 V_0^2 \cos^2\theta} \Longrightarrow \text{Parabola}$$

Time of Flight (τ)

$$y = 0 \quad \Big| \quad y = y_0 + V_0 t + \frac{1}{2} g t^2$$

$$\boxed{\text{Plug in } \tau} \Longrightarrow 0 = 0 + V_0 \tau - \frac{g \tau^2}{2} \Longrightarrow \tau = \frac{2 V_0}{g}$$

$$V_0 \tau = \frac{g \tau^2}{2} \qquad V_0 = \frac{g \tau}{2}$$

Time of Max Height

$V_y(t=t_{max}) = 0$

$V_y = 0 = V_{oy} - g t_{max}$

$V_{oy} = g t_{max}$

$t_{max} = \frac{V_{oy}}{g}$ or $\frac{V_o \sin\theta_o}{g} = \frac{T}{2}$

Maximum Height (H)

$H = y(t_{max})$

$y = y_o t_{max} - \frac{1}{2} g t_{max}^2 \qquad t_{max} = \frac{V_{oy}}{g}$

$y = V_{oy} \cdot \frac{V_{oy}}{g} - \frac{1}{2} g \cdot \left(\frac{V_{oy}}{g}\right)^2$

$= \frac{(V_{oy})^2}{2g} = \frac{(V_o \cdot \sin\theta)^2}{2g}$

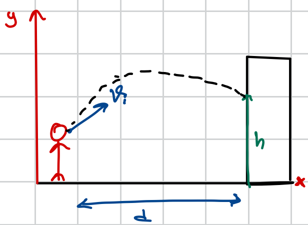
Max Distance in X axis Range

$R = x(T) = V_{ox} \cdot T = \frac{V_{ox} \cdot 2 V_{oy}}{g} = \frac{2 \cdot V_o^2 \sin\theta \cdot \cos\theta}{g}$

$= \frac{\sin 2\theta \cdot V_o^2}{g}$

Exercise 1

A fire fighter, a distance d from a burning building, directs a stream of water from a water hose at angle θ_i above the horizontal. If the initial speed of stream is V_i, at what height h does the water strike the building



fire fighter $= O(0,0)$

$x = x_o + V_{xo} t + \frac{1}{2} a t^2$

$x = 0 + V_{xo} t + 0$

$x = V_{ix} t$

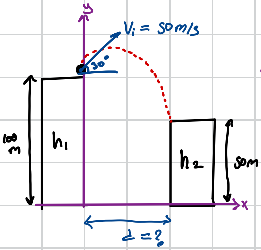
$t = \frac{d}{V_i \cos\theta_i}$

$y = y_o + V_{yi} t - \frac{1}{2} g t^2$

$y = 0 + V_{yi} t - \frac{1}{2} g t^2 = h$

$h = V_{yi}\left(\frac{d}{V_i \cos\theta_i}\right) - \frac{1}{2} g \left(\frac{d}{V_i \cos\theta_i}\right)^2$

Exercise 2



$d = x = x_i + V_{xi}t + \frac{1}{2}at^2$

$d = x = x_i + V_{xi}t$

$d = x = 50 . \cos 30$

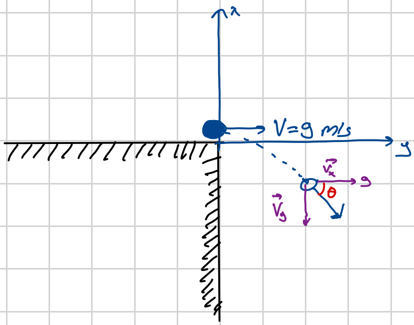
$t = \frac{d}{V_i . \cos(\theta)}$

$y = y_0 + V_{0y} . t - \frac{1}{2}gt^2$

$50 = 100 + V_0 \sin t - \frac{1}{2}gt^2$

$-50 = V_0 . \sin\left(\frac{d}{V_i . \cos(\theta)}\right) - \frac{1}{2}g\left(\frac{d}{V_i . \cos(\theta)}\right)^2$

Ex 3: r, V after t=0,5 secs



$x = x_0 + V_x t$

$x = 0 + 9 . 0,5$

$x = 4,5 m$

$y = y_0 + V_y t - \frac{1}{2}gt^2$

$y = 0 + 0 - \frac{1}{2}gt^2$

$y = -\frac{1}{2}gt^2$

$y = -\frac{1}{2} . 9,8 . \frac{1}{4}$

$y = -1,2 m$

$\vec{r} = (4,5, -1,2)$

$r = 4.7M$

$V_x = 9$

$V_y = \frac{dy}{dt} = -\frac{1}{\cancel{2}} g \cancel{2} t$

$V_y = -gt$

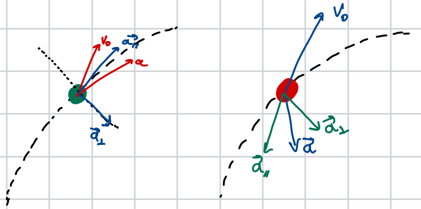
$V_y = -4.9 m/s$

$\vec{V} = 9\hat{i} - 4.9\hat{j}$

$V = \sqrt{9^2 + (4.9)^2}$

$V = 4,7 m/s$

Uniform Circular Motion

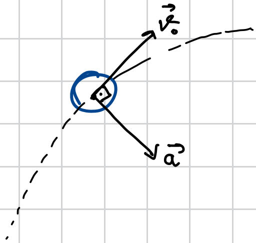


$\vec{V_0} \cdot a > 0$

Velocity increases

$a \cdot \vec{V_0} < 0$

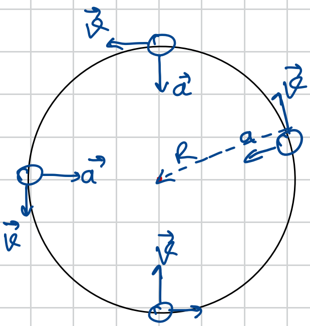
Velocity decreases

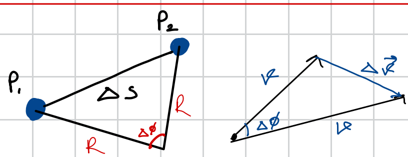


$\vec{a} \cdot \vec{V_0} = 0$

$\vec{a} \cdot \vec{V_0} = |\vec{a}| . |\vec{V_0}| . \cos 90$
$= 0$

$\vec{V} \to$ changes $\quad |\vec{V}| \to$ constant





$$|\vec{V_1}| = |\vec{V_2}| = V$$

$$\frac{|\Delta v|}{V} = \frac{\Delta s}{R} \longrightarrow |\Delta v| = \frac{V . \Delta s}{R}$$

$$a_{ave} = \frac{|\Delta V|}{\Delta t} \qquad a = \lim_{\Delta t \to 0} \frac{|\Delta V|}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{V . \Delta s}{\Delta t . r} = \frac{V}{R} \cdot \left[\lim_{\Delta t \to 0} \overbrace{\left(\frac{\Delta s}{\Delta t} \right)}^{V} \right]$$

$$a_{rad} = a_c = \frac{V^2}{R}$$

$$V = \frac{2\pi R}{T}$$

$$a = \frac{4\pi^2 R}{T^2}$$

$$F_c = \frac{mV^2}{R}$$

Non Uniform Circular Motion

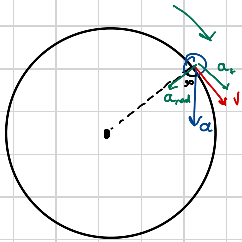
$a_{\parallel} = a_{t} \rightarrow$ tangential

$|\vec{a}_{rad}| = \dfrac{v^2}{R}$

$|a_{tan}| = \dfrac{d}{dt} v$

$a = \left|\dfrac{dv}{dt}\right| = \sqrt{a_{rad}^2 + a_t^2}$

Ex: Figure represents the total acceleration of a particle moving clockwise in a circle of radius 2.5 m a) radial acceleration b) speed of particle c) $a_t =$



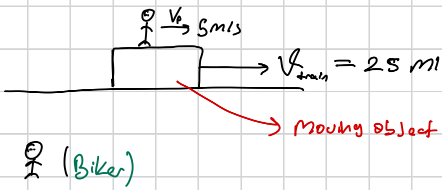
$|\vec{a}| = 15\ m/s^2$

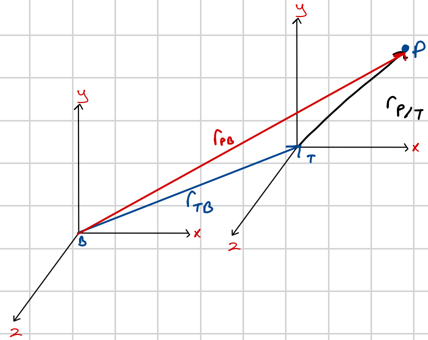
a) $a.\cos 30 = 13\ m/s^2$

b) $v^2 = a_{rad}.R$
$v = \sqrt{a_{rad}.R} \rightarrow \sqrt{13.(2,5)} = 5.7\ m/s$

c) $a_t = a.\sin 30 = 7.5\ m/s$

Relative Velocity





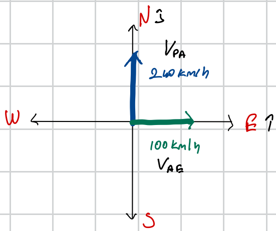
$$\boxed{\vec{r}_{PB} = \vec{r}_{PT} + \vec{r}_{TB}}$$

$$\frac{d\,r_{PB}}{dt} = \frac{d\,r_{PT}}{t} + \frac{d\,r_{TB}}{t}$$

$$\vec{V}_{PB} = \vec{V}_{PT} + \vec{V}_{TB}$$

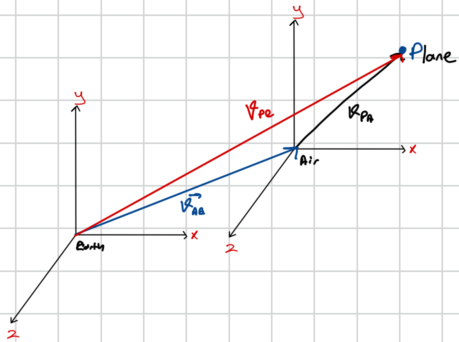
$$\vec{a}_{PB} = \vec{a}_{PT} + \vec{a}_{TB}$$

Example: An airplane's compass indicates that its headed due North, and its airspeed indicator shows that its moving through air at 240 km/h. If there's 100 km/h wind from west to east what's the velocity of plane relative to earth.



$$\vec{V}_{PE} = 240\hat{j} + 100\hat{i}$$

$$|\vec{V}_{PE}| = 260 \text{ km/h}$$



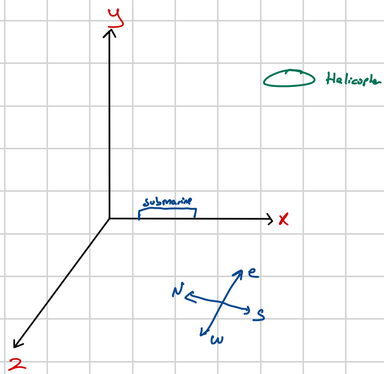
$$\vec{V}_{AE} = -\vec{V}_{EA}$$

Example 2

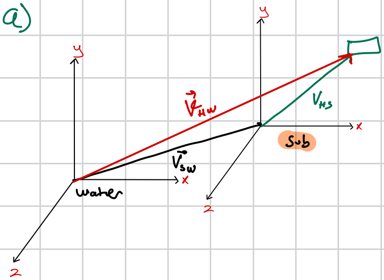
A helicopter is trying to land on a submarine deck which is moving ↓ south at 4 m/s. A 6 m/s wind is blowing into the west. Relative to submarine crew, the helicopter is descending vertically at 3 m/s

a) Find the velocity V_{hw} relative to water in terms of unit vectors.

b) Find the velocity V_{ha} of helicopter relative to air.



a)

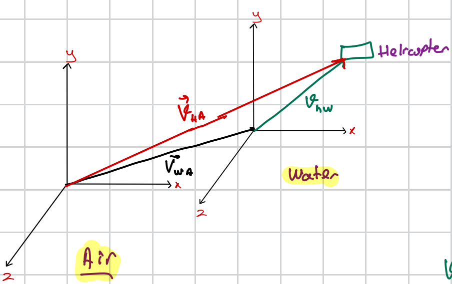


$$\vec{V}_{HW} = \vec{V}_{HS} + \vec{V}_{SW}$$

$$\vec{V}_{HW} = -3\hat{j} + 4\hat{i}$$

$$|\vec{V}_{HW}| = 5$$

b) Helicopter relative to air



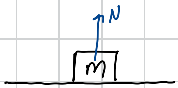
$$V_{HA} = V_{hw} + V_{wA}$$

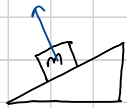
$$= -3\hat{j} + -6K + 4\hat{i}$$

$$V_{HA} = 4\hat{i} - 3\hat{j} - 6\hat{k}$$

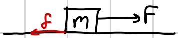
Force : $(\vec{F})$

① Normal Force

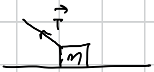




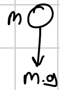
② Friction Force $(\vec{f})$



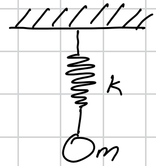
③ Tension Force $(\vec{T})$



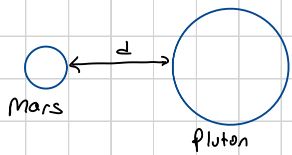
Weight



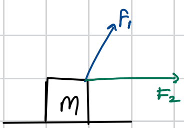
$\downarrow g$



$F = -kx$



$F = G \cdot \frac{m.M}{d^2}$



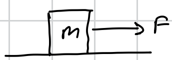
$$\vec{F} = \sum_{n=1}^{2} F_n$$

Newtons 1st law

$$\sum F = 0$$

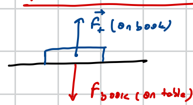
(body in equilibrium)

Newton's 2nd law



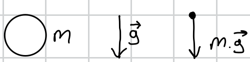
$$\vec{F} = m.\vec{a}$$

Newton's 3rd Law



$(F_{T(\text{on book})} = -F_{T(\text{on table})})$

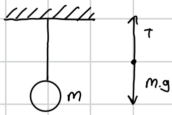
Free Fall Problem



$$m.\vec{a} = m.\vec{g}$$

$$\vec{a} = \vec{g}$$

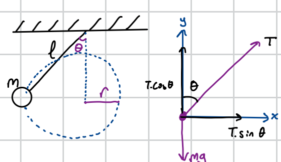
Hung Mass



$$\vec{T} = m.\vec{g}$$

Conical Pendulum

$$\boxed{a_c = \frac{v^2}{r}}$$



$\sum F_y = 0$ $\quad a_y = 0$ $\qquad \sum F_x = ma$

$$\frac{\cancel{T}.\sin\theta}{\cancel{T}.\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\tan\theta = \frac{v^2}{rg}$$

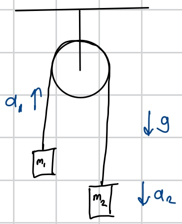
$$V = \sqrt{\tan\theta\, gr}$$

$$\frac{X}{t} = V = \frac{2\pi r}{T} \quad / \quad \frac{2\pi r}{T} = \sqrt{\tan\theta\, gr}$$

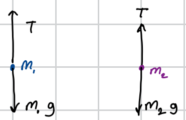
$$2\pi r = T.\sqrt{\tan\theta\, gr}$$

$$T = \frac{2\pi r}{\sqrt{\tan\theta\, gr}}$$

Atwood Machine



Moving down $a_1, a_2, T = ?$



$a_2 = -a_1$

$T - m_1 g = m_1 a_1$

$(T - m_2 g = m_2 a_2)$

$\downarrow$

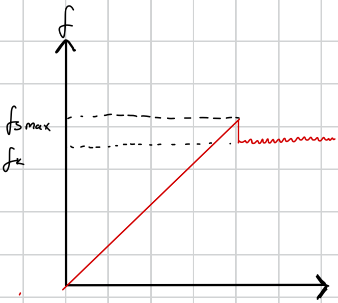
$m_2 g - T = m_2 a_1$

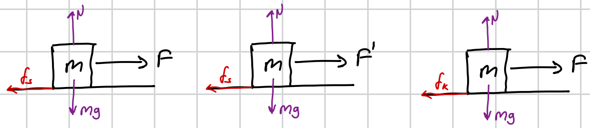
$m_2 g - m_1 g = m_2 a_1 + m_1 a_1$

$g(m_2 - m_1) = a_1 (m_2 + m_1)$

$\dfrac{g(m_2 - m_1)}{m_2 + m_1} = a_1$

$$\boxed{f = \mu . N}$$

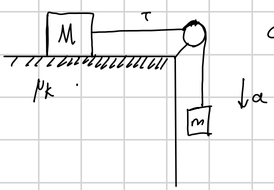




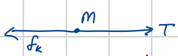
$f_s < \underbrace{\mu_s N}_{\text{Max}}$

$f_s = \mu_s . N$ (about to move)

Moving

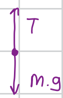


$a = ?$ and $T = ?$



$T - \mu_k N = M . a$

$T - \mu_k . M . g = M . a$

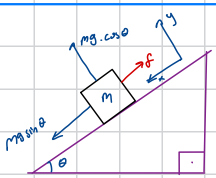


$mg - T = ma$

$T - mg = -ma$

$T = mg - ma$

$$a = \frac{g(m - \mu_k M)}{M + m}$$



$m.g.\sin\theta - m.g.\cos\theta \, \mu_k = m.a$

$g(\sin\theta - \cos\theta \, \mu_k) = a$

for $\mu = 0$

$mg \sin\alpha = ma$

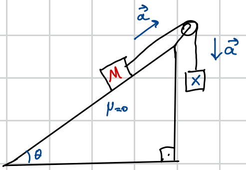
$g \sin\alpha = a$

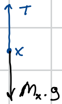
$$f_s = \mu_s N = mg \sin\theta$$

$mg = \dfrac{N}{\cos\theta}$ / $\mu_s . \cancel{N} = \dfrac{\cancel{N}}{\cos\theta} . \sin\theta$

$\mu_s = \tan\theta$

Inclined Plane for 2 Masses

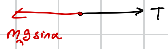




$M_x g - T = M_x a$

$M_x g - M_x a = T$

$M_x(g-a) = T$



$T - M_m g \sin\alpha = M_m a$

$T = M_m a + M_m g \sin\alpha$

$M_m(a + g\sin\alpha)$

$M_m(a + g\sin\alpha) = M_x(g-a)$

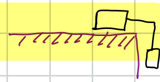
$M_m a + M_m g \sin\alpha = M_x g - M_x a$

$M_m a + M_x a = M_x g - M_m g \sin\alpha$

$a(M_m + M_x) = g(M_x - M_m \sin\alpha)$

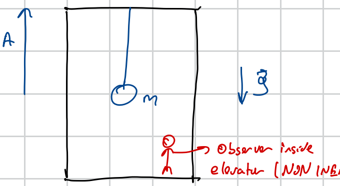
$$a = \frac{g(M_x - M_m \sin\alpha)}{(M_m + M_x)}$$

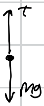
Negative acceleration = other direction

if $\alpha = 0$ 

if $\alpha = 90$ = Atwood

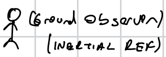
Relative Acceleration

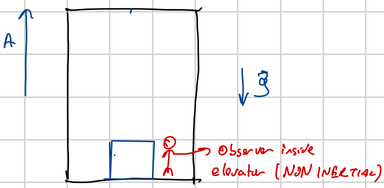


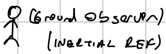


$F = T - mg = mA$

$T - mg - \underbrace{mA}_{\text{Fictious}} = 0$

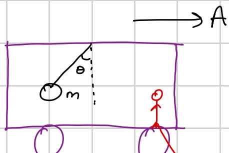


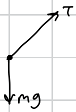




$N - mg = mA$

$N - mg - mA = 0$

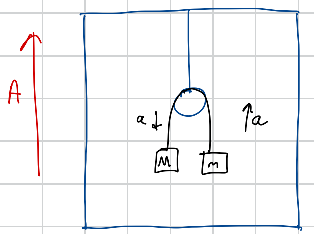




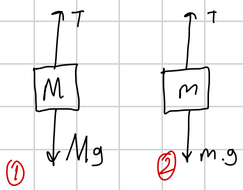
$= \boxed{T.\sin\theta = m.A}$

$T.\sin\theta - \underbrace{mA}_{\text{Fictious}} = 0$

Atwood Machine in elevator



$M > m$



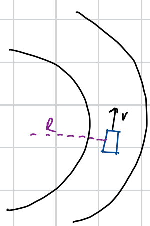
(1) $T - Mg = M(A - a)$

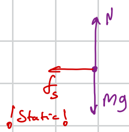
(2) $T - mg = m(a + A)$

$mg - T = M(-a - A)$

$$a = \frac{(g + A)(M - m)}{m + M}$$

For ground observer





$V_{max} = ?$

$\sum F_x = m.a$

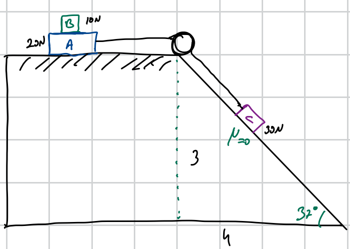
$f_s = ma$

$\frac{mv^2}{r} = m.g.\mu_s$

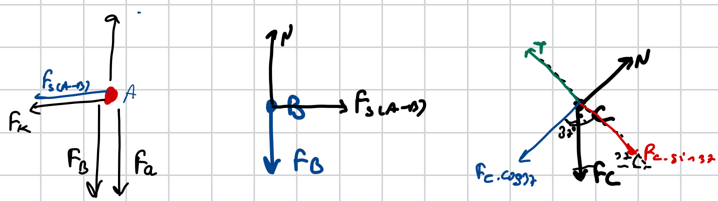
$\cancel{m}v^2 = \cancel{m} g \mu_s . R$

$v^2 = g \mu_s R$

$V_{max} = \sqrt{\mu_s R g}$



a) Freebody diagrams



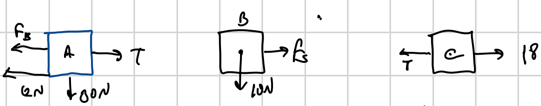
b) what is the max a without B sliding in terms of g and μ_s

$f_{smax} = \mu_s . m_B g$

$\mu_s . \cancel{m_B} g = \cancel{m_B} a_{max}$

$a_{max} = g . \mu_s$

c) Whats the min μ_s when $\mu_k = 0.4$ so A and B move together (a)



$T - \mu_s . 10 - 12 = m_a . a$

$+\mu_s . 10 = m_B a$

$18 - T = m_c . a$

$F_s = m a$

$6 = a . (m_a + m_B + m_c)$

$6 = 6a$

$\boxed{a = 1}$

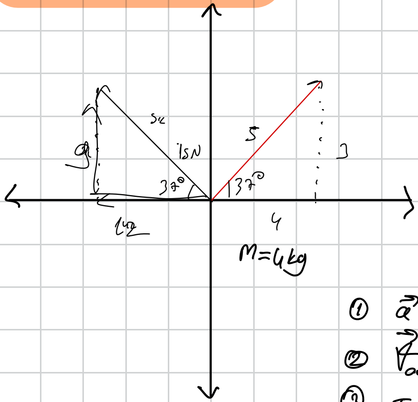
$\cancel{m}.a \leq \mu_s . \cancel{m} g$

$a \leq \mu_s . g$

$0{,}1 \leq \mu_s$

$\mu_{s\,min} = 0.1$

EXAMPLE



$t=0 \quad (x_0, y_0)$

$\vec{V_0}(2\hat{i}+\hat{j})\ m/s$

$\vec{F}_{net} = (-8\hat{i} + 12\hat{j})$

① $\vec{a}$

② $\vec{V}_{ave}$ $t=2$ t_2 4

③ Time to max x

④ θ-R , $\vec{v}$

$a_x = \frac{-8}{4}$ $\quad a_x = -2\ m/s^2$

$a_y = \frac{12}{4} = \quad a_y = 3\ m/s^2$

① $a = \sqrt{\square}$

② $V_{ave} = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{\Delta t}$

③ 1sec. / Find derivative

$\vec{r} = \vec{r_0} + \vec{V_0}t + \frac{1}{2}at^2$

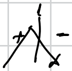
$\vec{r} = 0 + (2\hat{i}+\hat{j})t + \frac{1}{2}(-2\hat{i}+3\hat{j})t^2$

$r_x = (2t - t^2)\hat{i}$

$2 - 2t = 0$

$2 = 2t$

$t=1$ $\quad t1 = max$



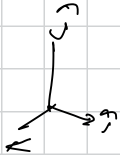
④ $\boxed{\text{angle between } r \text{ and } v \quad t=2}$

$\vec{r} \cdot \vec{v} = |r| \cdot |v| \cdot \cos\alpha$

OR

$\vec{A} \times \vec{B} = \vec{C}$

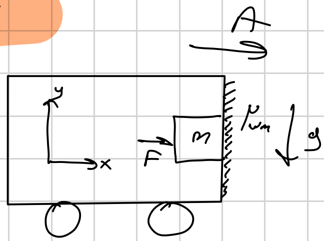
$|C| = |\vec{A}||\vec{B}| \sin\alpha$



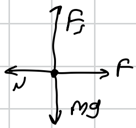
$\vec{r}(2) = 8\hat{j}$

$\vec{V}(t) = (2-2t)\hat{i} + (1+3t)\hat{j}$

Ex



F_{min} such that block won't slide



$$F - N - mA = 0 \quad \rightarrow N = F - mA$$

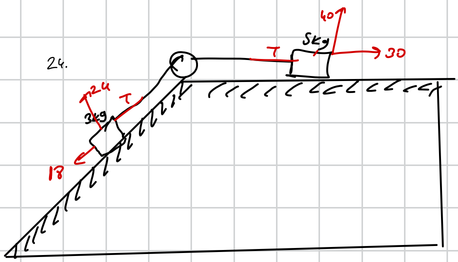
in y: $f_s - mg = 0$

$$f_s \leq \mu_s N$$

$$mg \leq \mu_s (F - mA)$$

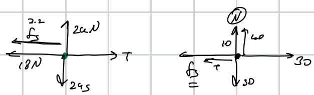
$$\frac{mg}{\mu_s} \leq F - mA$$

$$\frac{mg}{\mu_s} + mA \leq F_{min}$$



$\mu_s = 0.3$

$\mu_k = 0.2$



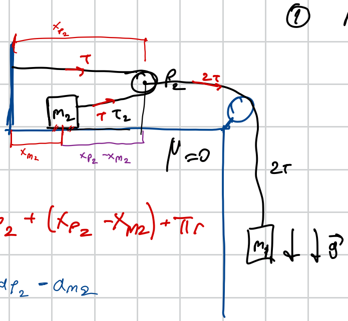
$$\Sigma + > \Sigma -$$

$$F \cdot \cos 53 > \underline{m_1 g \sin 37} + f_{s1} + f_{s2}$$

$$30N > 28.2 N$$

① $a = 0$

② distance travelled by m_1 in 2 secs



① relationships between a_1 and a_2

$$2T = 2m_2 a_2$$

$$m_1 g - 2T = m_1 \cdot a_1$$

$$L = x_{p_2} + (x_{p_2} - x_{m_2}) + \pi r$$

$$0 = 2a_{p_2} - a_{m_2}$$

$$2a_{p_2} = a_{m_2} = 2a_{m_1}$$

② T_1 vs T_2 $\quad T_2 = \dfrac{2m_1 m_2 g}{m_1 + 2m_2}$

③ $a_1 = a_2 =$ $\boxed{a_2 = 2a_1}$

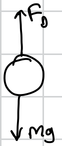
$$\cancel{m_2} a_2 = \frac{2m_1 \cancel{m_2} g}{m_1 + 2m_2}$$

$$a_2 = \frac{2m_1 g}{m_1 + 2m_2} \qquad a_1 = \frac{m_1 g}{m_1 + 2m_2}$$

Made with Goodnotes

$$\int \frac{dx}{x} = \ln x + c$$

Drag Force



Case 1 (smol)

$F_D \sim v$

$F_d = -b.v$

$\sum F_y = mg - bv = ma$

$$g - \frac{bv}{m} = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{g - \frac{bv}{m}}$$

$$t = \int_0^v \frac{dv}{g - \frac{bv}{m}}$$

$$g - \frac{b}{m}v = U$$

$$-\frac{b}{m}dv = dU$$

$$\int \frac{-\frac{m}{b}\,du}{U} = -\frac{m}{b}\int \frac{du}{U} \Rightarrow \boxed{-\frac{m}{b}\ln U}$$

$$t = -\frac{m}{b}\ln\left(g - \frac{bv}{m}\right)\Big|_0^v$$

$$t = -\frac{m}{b}\left[\ln\left(g - \frac{bv}{m}\right) - \ln g\right)$$

$$t = -\frac{m}{b}\left(\ln\left(1 - \frac{bv}{mg}\right)\right)$$

$$-\frac{bt}{m}$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bv}{mg}$$

$$\boxed{v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)}$$

Terminal Velocity $= \dfrac{mg}{b}$

$$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right) = V_T \cdot \left(1 - e^{-\frac{bt}{m}}\right)$$

$$a(t) = \frac{d}{dt}\left(V_T\left(1 - e^{-\frac{bt}{m}}\right)\right)$$

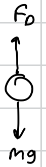
$$\frac{d}{dt}\left(V_T - V_T e^{-\frac{bt}{m}}\right)$$

$$= 0 - V_T\left(-\frac{bt}{m}\right)' \cdot e^{-\frac{bt}{m}}$$

$$= -V_T \cdot \frac{-b}{m} \cdot e^{-\frac{bt}{m}}$$

$$a(t) = V_T \cdot \frac{b}{m} \cdot e^{-\frac{bt}{m}}$$

Case 2 (Large object, high velocity)



$$F_D = \frac{1}{2} \cdot D \cdot \rho \cdot A \cdot v^2$$

Dimensionless drag coefficent

Density

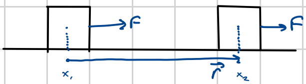
Cross Sectional Area

Velocity

$$V_T = \sqrt{\frac{2mg}{D\rho A}}$$

Work and Energy

$$V_2^2 = V_1^2 + 2ar$$



$W = \vec{F} \cdot \vec{r}$ $\qquad$ $\vec{F} = F\hat{r}$

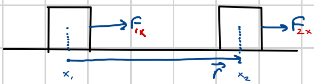
$\vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\theta$

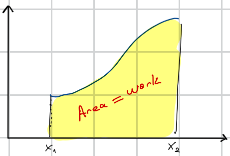
$$W = F \cdot r$$

$$W = m \cdot a \cdot r$$

$$W = m \cdot \left(\frac{V_2^2 - V_1^2}{2\cancel{r}} \right) \cdot \cancel{r} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2}$$

$W = \Delta$ Kinetic Energy

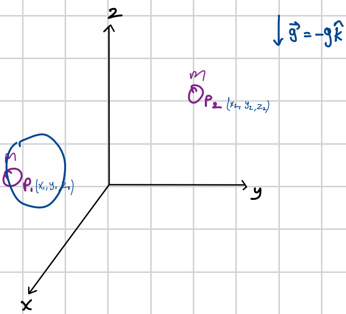




$$W = \int_{x_1}^{x_2} F(x)\,dx$$

$$F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

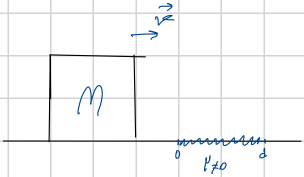
$$W = \int_{P_1}^{P_2} \vec{F}\, d\vec{r} = \int_{x_1}^{x_2} F_x\, dx + \int_{y_1}^{y_2} F_y\, dy + \int_{z_1}^{z_2} F_z\, dz$$



$$W = \int_{z_1}^{z_2} (-mg)\, dz$$

$$W = -mg \int_{z_1}^{z_2} dz$$

$$W = -mg\,(z_2 - z_1)$$



$$\frac{mv^2}{2} = m \cdot g \cdot \mu_k \cdot d$$

$$\frac{V^2}{2} = mg\mu_k d$$

$$\frac{V^2}{2 \cdot m \cdot g \cdot \mu_k} = \boxed{d}$$

$$mV = F \cdot t$$

$$mV = m \cdot g \mu_s \cdot t$$

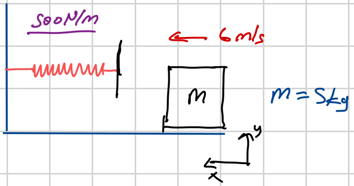
$$V = g\mu_s t$$

$$\frac{V}{g\mu_s} = t$$

$$\frac{1}{2} a t^2 = d$$

$$a t^2 = 2d$$

$$a = \frac{2d}{\left(\frac{V}{g\mu_s}\right)^2}$$



$m = 5kg$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

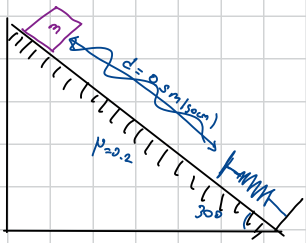
$$mv^2 = kx^2$$

$$5 \cdot 36 = 500x^2$$

$$\frac{5 \cdot 36}{500}$$

$$\sqrt{\frac{36}{100}} = x$$

$$\boxed{0{,}6\ m = x}$$



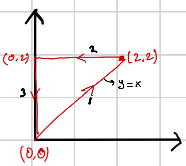
$m = 5kg$
$k = 15\ N/cm$
$1500\ N/m$

$$(mg\sin\alpha - mg\cos\alpha \cdot 0.2) \cdot d = \frac{1}{2}mv^2$$

$$\boxed{V_2 = 1{,}29\ m/s}$$

$$(mg\sin\alpha - f_k) \cdot X - \frac{1}{2}kx^2 = -\frac{1}{2}mV_2^2$$

$$\boxed{11.5\ cm}$$



W_1, W_2, W_2 ?

$$\boxed{F = 3\hat{i} + 4x\hat{j}}$$

$$W = \int \vec{F}\, d\vec{r}$$

$$W = \int (3\hat{i} + 4x\hat{j})(dx\hat{i} + dy\hat{j})$$

$$W_1 = \int_0^2 3\,dx + \int_0^2 4y\,dy = \Big|_0^2 3x + \Big|_0^2 2y^2$$

$$6 + 8 = 14\,J$$

$$W_2 = \int_2^0 3\,dx = \Big|_2^0 3x$$

$$0 - 6 = -6\,J$$

$$W_3 = \int_2^0 4x\,dy = \Big|_2^0 4xy = 0$$

Gravitational Potential Energy

$$W_{ext} = \vec{F}_{ext}\,\Delta\vec{r} = F_{ext} \cdot (y_f - y_i)\,\hat{j}$$

$$\vec{F}_{ext} = -\vec{F}_g \qquad W_g = (-F_{ext}) \cdot (y_f - y_i)\hat{j}$$

$$mg(y_f - y_i) = \Delta U$$

$$W = \Delta K = -\Delta U$$

$$K_2 - K_1 = -(U_2 - U_1)$$

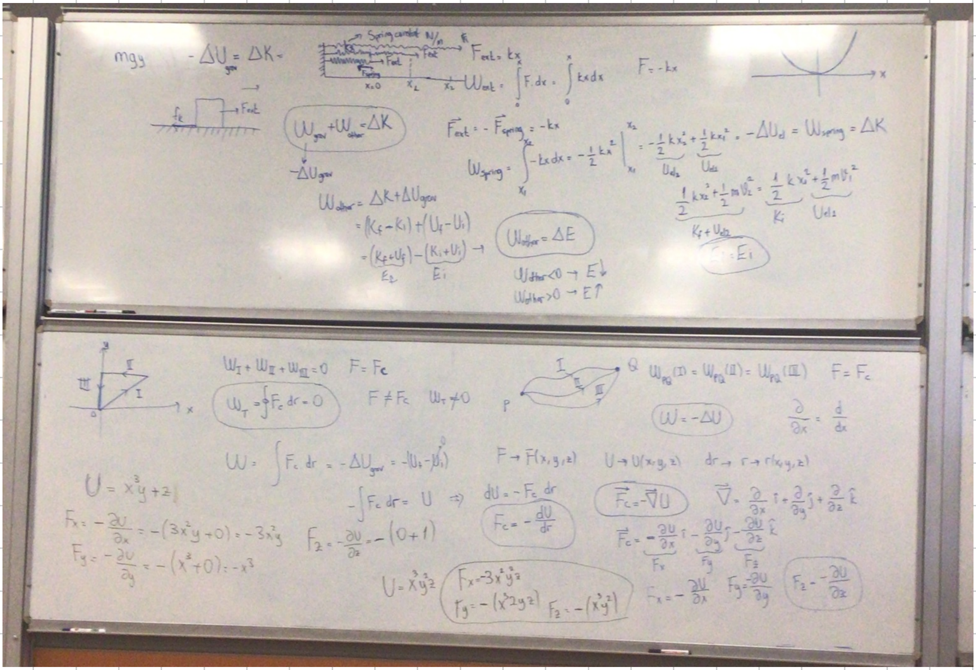
$$\underbrace{K_2 + U_2}_{E = K + U} = K_1 + U_1$$

$$\frac{1}{2}mV_2^2 + mgy_f = \frac{1}{2}mV_1^2 + mgy_i$$

$$V_2 = V_1^2 - 2g(y_f - y_i)$$

Made with Goodnotes

15/11/2023 Dersinden



$W_{fric} = \Delta E = E_f - E_i$

$(K_f + U_f) - (mgh)$

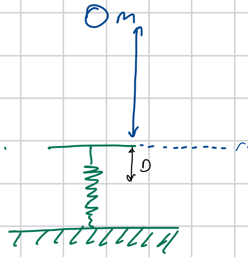
$\frac{1}{2}mv^2 - mgh = -\frac{mg\cos\theta \cdot \mu_K h}{\sin\theta}$

$W_F = \mu_K \cdot s$

$\boxed{mg\cos\theta \cdot s}$

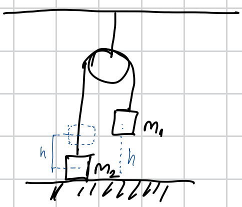
$\sin\theta = \frac{h}{s}$

$s = \frac{h}{\sin\theta}$



k, m, h given Max compression

$mg(h+d) = \frac{kD^2}{2}$



velocity of m_1 when it hits

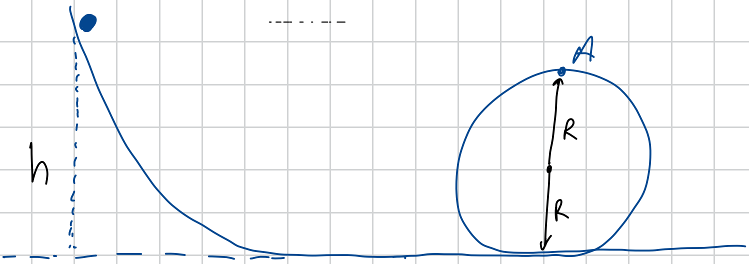
$E_i = E_f$

$U_{gi} + \cancel{K_i} = U_{gf} + K_f$

$m_1 gh = m_2 gh + \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_1^2$

$(m_1 - m_2)(gh) = \frac{1}{2}v^2(m_1 + m_2)$

$\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}$



$a = V_c = ?$

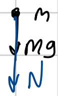
$b = N_A = ?$

$E_i = E_f$

b)

$\sum F = m.a$

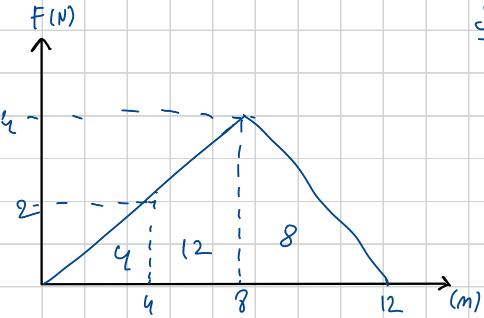
$U_{gi} + K_i = U_{gf} + K_f$



$N_A + mg = \frac{V^2}{R} \cdot m$

$g(h-2R) = \frac{V_A^2}{2}$

$N_A = \frac{mV^2}{R} - mg$



3kg Particle

Origin +3 m/s

$\frac{3 \cdot 9}{2} + 24 = \frac{3v^2}{2}$

$\frac{27}{2} + 24 = \frac{3v^2}{2}$

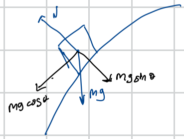
$\frac{9}{2} + 8 = \frac{v^2}{2}$

$9 + 16 = v^2$

$25 = v^2$

$v = 5 m/$

Calculate θ when object loses contact



$E_i = E_f$

$mg\frac{R}{2} = \frac{1}{2} m V_c^2 + R \sin\theta \, mg$

$\frac{mgR}{2} = \frac{1}{2} mgR \sin\theta + R\sin\theta \, mg$

$\frac{1}{2} = \frac{\sin\theta}{2} + \sin\theta$

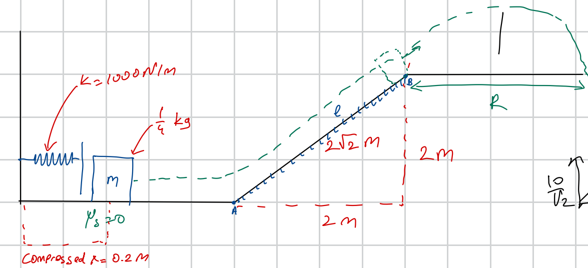
$\frac{1}{2} = \frac{3\sin\theta}{2} \quad \frac{1}{3} = \sin\theta$

$mg \sin\theta - \cancel{N} = m.a_{rad}$

$mg \sin\theta = m.a_{rad}$

$g \sin\theta = \frac{V^2}{R}$

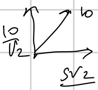
$V^2 = \sqrt{gr \sin\theta}$



$g = 10\ m/s^2$

$\theta = 45°$

$\boxed{\frac{8\sqrt{2}}{10\sqrt{2}}}$



$y = 0 + 5\sqrt{2}t - 5t^2$ $\quad \frac{5\sqrt{2}\cdot\sqrt{2}}{2}$

$y = 5\sqrt{2}t - 5t^2$ $\quad 5 - 5\frac{2}{4}$

$t(5\sqrt{2} - 5t)$

$t = 0, \quad 5\sqrt{2} = 5t$

$t = \sqrt{2}$ secs

$5\sqrt{2}\cdot\sqrt{2} = 10\,m$ Range

$$\frac{Kx^2}{2} = mgh + mg\sin\alpha \cdot 2\sqrt{2}\cdot\mu + \frac{mv^2}{2}$$

$$20 = 10\cdot\frac{1}{4}\cdot 2$$

$$5 + \frac{1}{4}\cdot 10 \cdot \cancel{\frac{\sqrt{2}}{2}} \cdot \cancel{2\sqrt{2}} \cdot \frac{1}{2}$$

$$20 = 5 + \frac{10}{4} + \frac{mv^2}{2}$$

$$15 - \frac{10}{4}$$

$$\frac{50}{4} = \frac{mv^2}{2}$$

$$\frac{100}{4} = \frac{mv^2}{4}$$

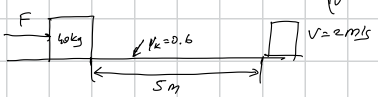
$$100 = v^2$$

$$\boxed{v = 10}$$

Energy At C

$$W_{AC} = K_C - K_A$$

$\frac{400.6}{10}$



$$(F - 240)\cdot 5 = \frac{1}{\cancel{2}}\, 40 \cdot 4^2$$

$$F - 240 = 80$$

$$F - 240 = 16$$

$$\boxed{F = 256}\,.\,5 = 1280\,J$$

$$P_{ins} = \frac{dW}{dt} = \frac{d}{dt}(F_{net}x)$$

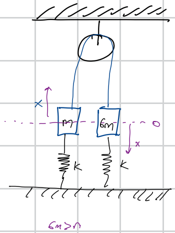
$$F_{net}\cdot\frac{dx}{dt} = F_{net}\cdot v$$

$$16\cdot 2 = 32$$

Made with Goodnotes

22/11/2023

Max compression for right spring



$W = \Delta E = E_f - E_i$

$E_i = E_f$

$U_g + U_{eli} = U_{gf} + U_{elf}$

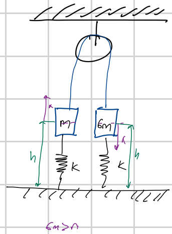
$0 + 0 = (mgx - 6mgx) + \left(\frac{kx^2}{2} \cdot 2\right)$

$0 = -5mgx + kx^2$

$kx^2 = 5mgx$

$kx = 5mg$

$x_{max} = \frac{5mg}{k}$



$mgh + 6mgh + 0$
$=$
$mg(h+x) + 6mg(h-x) + kx^2$

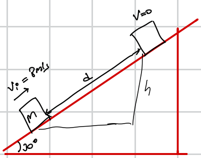
$7mgh = mgh + mgx + kx^2$
$6mgh - 6mgx$

$5mgx = kx^2$

$5mg = kx$

$\frac{5mg}{k} = x$

$m = 5kg$



a) $\Delta K = K_f - K_i = 0 - \frac{1}{2} \cdot 5 \cdot 64 = \boxed{-160J}$

b) $\Delta U = U_f - U_i = 1.5 \cdot 5 \cdot 10 = 75J$

c) $f_k = W_{fk} = \Delta E = \Delta K + \Delta U = -85J$; $-160J + 75J$

$W_{fk} = -f_k \cdot d$; $+85 = +f_k \cdot 3$; $f_k = \frac{85}{3}$

$d = \mu_k = f_k = \mu_k \cdot mg\cos\alpha$

$\frac{85}{3} = \mu_k \cdot 30 \cdot \frac{\sqrt{3}}{2}$

$\frac{17}{9\sqrt{3}} = \mu_k$

$\frac{h}{d} = \frac{1}{2}$ / $\boxed{h = \frac{d}{2}}$ $h = 1.5$

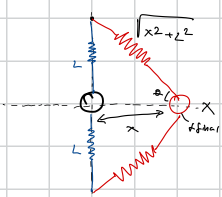
$m = 3kg$ $\quad V_i = (6i - 2j)$ $\Rightarrow KE = \frac{1}{2} \cdot 3 \cdot (6\hat{i} - 2\hat{j}) \cdot (6\hat{i} - 2\hat{j})$

$\frac{1}{2} \cdot 3 \cdot (40) = 60 J$

a) $KE = ?$

b) $W \to V_f = (8\hat{i} + 4\hat{j})$

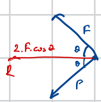
$b \Rightarrow K_f = \frac{1}{2} \cdot 3 \, (64 + 16) = 120J$ $\quad / \; 120 - 60 = \boxed{60J}$



$\cos\theta = \frac{x}{\sqrt{x^2+L^2}}$

$m = 1.18 kg$

$F = -2kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)\hat{i}$



$\sum \vec{F}_{net_x} = 2\vec{F} \cdot \cos\theta$

$\sum F_{net_x} = 2 \cdot F \cdot \frac{x}{\sqrt{x^2+L^2}}$

$F = -k.$(change in position)

$\vec{F} = -k \cdot \left(\sqrt{x^2+L^2} - L\right)\hat{i}$

$\vec{F}_{net} = -2 \cdot k \cdot \frac{x}{\sqrt{x^2+L^2}} \left(\sqrt{x^2+L^2} - L\right)$

$F_{net} = -2kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right)$

$$U(x) = kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)$$

$$\vec{F} = -\nabla U$$

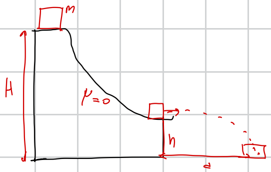
$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{F} = \frac{-\partial U}{\partial x}\hat{r}$$

$$\int_0^U dU = \int_0^x -F\,dx$$

$$U = -\int_0^x F\,dx$$

$$U = -\int_0^x \left(-2kx\left(1 - \frac{L}{\sqrt{L^2+x^2}}\right)\right)$$



$$\frac{1}{2}mv^2 = (H-h)m.g$$

$$\frac{1}{2}v^2 = (H-h).g$$

$$v = \sqrt{2g(H-h)}$$

$$y = \cancel{v_{y0}t} + h - \frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$$

$$\boxed{d = v_x t}$$

$$h = \frac{1}{2}gt^2$$

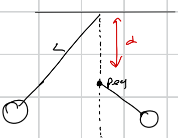
$$\sqrt{\frac{2h}{g}} = t$$

$$\boxed{d = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}}$$

$$d^2 = 4hH - 4h^2$$

$$d^2 + 4h^2 = 4hH$$

$$H = \frac{d^2+4h^2}{4h}$$



$\theta = 90°$ released

1) speed when lowest point

2) highest point after peg

3) minimum d in terms of L to full rotation

1) $E_i = E_f$

$$m.g.L = \frac{mv^2}{2}$$

$$2g.L = v^2$$

$$V = \sqrt{2gL}$$

2) $\ell = L - d$

$E_i = E_f$

$$m.g.L = \frac{1}{2}mv^2 + mg2R$$

$$gL = \frac{1}{2}v^2 + g2R$$

$$gL = \frac{1}{2}v_2^2 + 2g(L-d)$$

$$gL - 2gL + 2gd$$

$$2gd - gL = \frac{1}{2}v_2^2$$

$$\sqrt{4gd - 2gL} = V_2$$

c) $T \geq 0$

$T \downarrow \downarrow mg$

$$T + mg = m.a_{rad}$$

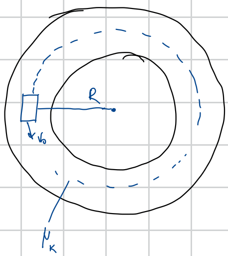
$$T + mg = \frac{mV_2^2}{R}$$

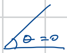
$$T = \frac{mv_2^2}{R} - mg$$

$$T = \frac{m(4gd - 2gL)}{R} - mg \geq 0$$

$$\frac{4gd - 2gL}{L-d} - g \geq 0$$

$$5d \geq 3L \qquad d \geq \frac{3}{5}L$$





After 1 revolution $v_f = \frac{2\sqrt{2}}{3}$

i) $W_{friction}$

ii) $\mu_K = ?$

iii) n revs to stop

$W_{friction} = \Delta E = E_f - E_i$

$W_f = \Delta K$

$W_{friction} = K_f - K_i$

$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$\frac{1}{2} m \left(\frac{8}{9} - 1\right) v^2$

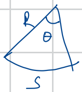
$-\frac{m v_0^2}{18} = W_{fric}$

$W_f = -f_K . 2\pi R$

$+\mu_K \cdot mg \underbrace{2\pi R}_{d} = +\frac{m v_0^2}{18}$

$\mu_K \cdot g . d = \frac{v_0^2}{18}$

$\boxed{\mu_K = \frac{v_0^2}{36 . \pi R}}$



$S = R . \theta$

$ds = R d\theta$

$W_{friction} = \int -f_K . ds = -\int f_K R\, d\theta$

$W_{friction} = \int_0^{2\pi} -f_K R\, d\theta$

$= -f_K R . 2\pi$

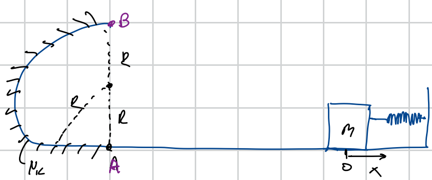
$W_{friction} = \Delta K = K_f - K_i$

$-K_i = n . W_{friction}$

$\frac{1}{2} m v^2 = n . \mu_K . mg . 2\pi R$

$\frac{1}{2} \cancel{m} v^2 = n . \frac{v^2}{36 g \cancel{\pi R}} . \cancel{mg} \cancel{2\pi R}$

$n = 9$



a) $x = ?$ b) $v_B = ?$ c) $W_{friction} = ?$ $\mu_K = ?$

$m = 0.5$ kg

$R = 1$ m

$k = 450$ n/m

$v_A = 12$ m/s

$f_{K avg} = 7$ N

a

$W_{friction} = \Delta E$

$\cancel{\frac{1}{2}} k x^2 = \cancel{\frac{1}{2}} m v^2$

$450 . x^2 = \frac{1}{2} . 12^2$

$x^2 = \frac{72}{450}$

$\boxed{x = 0.4 \text{ m}}$

c

$\boxed{W_{friction}} = \Delta E = E_B - E_0$

$\boxed{-f_K . \pi R} = \left(mg 2R + \frac{m v_B^2}{2}\right) - \left(\frac{1}{2} k x^2\right)$

$-7 \pi R = 2 mgR + \frac{1}{2} m \frac{v_B^2}{2} - \frac{1}{2} k x^2$

b $\boxed{v_B = \sqrt{76} \approx 8.7 \text{ m/s}}$

$f_K = N . \mu_K$

$\frac{7}{mg} = \mu_K$

Time dependent displacement $x(t) = t + 2t^3\,m$ For given time t

a) $K(t) = \frac{1}{2} m (v(t))^2$ $\quad v(t) = \frac{dx}{dt}$ $\quad \boxed{v(t) = 1+6t^2}$ $\quad \Big\rangle \; \boxed{K(t) = \frac{1}{2} m \cdot (1+6t^2)^2}$

b) $a(t) = \frac{dv}{dt} = \boxed{a = 12t}$

c) $F(t) = m.a(t) = \boxed{(12t).m}$

d) $W(t) = (12mt).(t+2t^3)$

e) $p(t) = \frac{dW}{dt} = 24mt + 96mt^3$

Linear Momentum

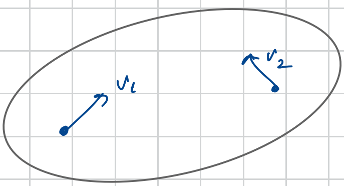
$\vec{p} = m.\vec{v}$

$\vec{p} \Rightarrow \left[kg \frac{m}{s} \right]$

$$\boxed{\begin{array}{l} F = m.a \qquad \vec{a} = \frac{dv}{dt} \\ F = m.\frac{dv}{dt} \Longrightarrow \frac{d}{dt}(\underbrace{m.v}_{\vec{p}}) = \frac{dp}{dt} \end{array}}$$

$$\vec{F} = \frac{d}{dt}(m.\vec{v}) = \frac{dm}{dt}.\vec{v} + m.\frac{dv}{dt}$$

$$\boxed{\vec{F} = \frac{dv}{dt}}$$ if m is constant



Exterior System

two forces $\begin{cases} F_{ext} \\ \text{Action Reaction Pair} \end{cases}$

$$m_1 \frac{dv_1}{dt} = \vec{F}_{ext_1} + \vec{F}_{1 on 2}$$

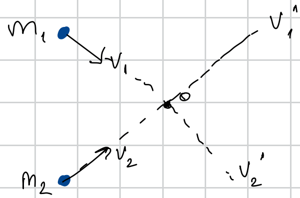
$$m_2 . \frac{dv_2}{dt} = \vec{F}_{ext_2} + \vec{F}_{2 on 1}$$

$F_{1on2} = -F_{2on1}$

$+$

$$\frac{d}{dt}(\underbrace{m_1V_1 + m_2V_2}_{P_{system}}) = F_{ext}$$

$\frac{dP}{dt} = \underbrace{F_{ext}}_{0}$) Conservation of MOMENTUM



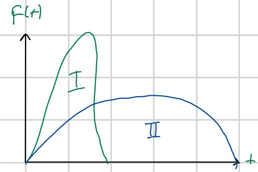
$P_i = P_f$

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$

$F_{ext} = \frac{dP}{dt} \rightarrow \int_{t_i}^{t_f} F_{ext}\,dt = \int_{P_i}^{P_f} dP$

$\Delta P = \underbrace{\int_{t_i}^{t_f} F_{ext}\,dt}$

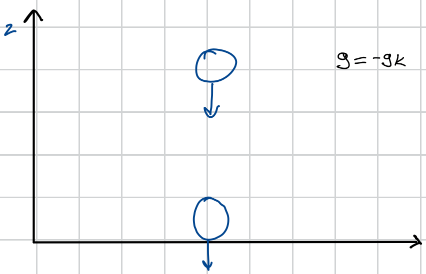
$J \Rightarrow$ impulse $\rightarrow$ [N.s] $\rightarrow$ [km/s]



$A_I = A_{II}$

$J_I = J_{II}$

$J_{ave} = \frac{F_{ave}}{\Delta t} = \frac{\Delta P}{\Delta t}$



$g = -gk$

$m_{ball} = 0.2\ kg$

$v_{ball} = 8\ m/s$

$t = 10^{-3}\ m$

$P_{before} = -m.v$

$-\frac{2}{10}.8 \Rightarrow -1.6\,\hat{k}\ \frac{kgm}{s}$

$P_{after} = m.v$

$0.2.8 = 1.6\,\hat{k}\ \frac{kgm}{s}$

$F_{ave} = \frac{\Delta P}{\Delta t} = \frac{3.2}{10^{-3}} = 3200N$

$J = \Delta P$

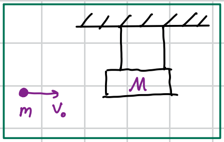
① Elastic Collision : $\Delta k = 0$ / $\Delta P = 0$

② Inelastic Collision : $k_f < k_i$ / $\Delta P = 0$

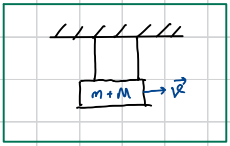
③ Completely Inelastic Collision : $\Delta P = 0$ / $k_f < k_i$

④ Superelastic Collision : $k_f > k_i$ | $\Delta P = 0$

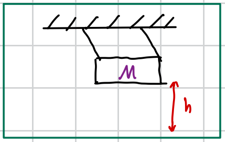
①



②



③



①-②

$mv_0 + M.0 = (m+M)v$

$\vec{v} = \frac{mv_0}{m+M}$

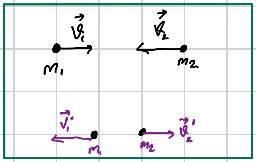
②-③

$E_i = E_f$

$\frac{1}{2}(m+M)v^2 = (M+m)gh$

$v^2 = 2gh$

$\frac{m^2 v_0^2}{(m+M)^2} = 2gh \Rightarrow h = \frac{m^2 v_0^2}{2g(m+M)^2}$



$m_1 \neq m_2$

$\Delta P = 0$

$\Delta K = 0$

$$\cancel{\tfrac{1}{2}} m_1 V_1^2 + \cancel{\tfrac{1}{2}} m_2 V_2^2 = \cancel{\tfrac{1}{2}} m_1 V_1'^2 + \cancel{\tfrac{1}{2}} m_2 V_2'^2$$

$$m_1 \vec{V_1} + m_2 \vec{V_2} = m_1 \vec{V_1}' + m_2 \vec{V_2}'$$

$$\text{Eq 1} \Rightarrow \quad m_1 (\vec{V_1}^2 - \vec{V_1}'^2) = m_2 (\vec{V_2}'^2 - \vec{V_2}^2)$$

$$\text{Eq 2} \Rightarrow \quad m_1 (\vec{V_1} - \vec{V_1}') = m_2 (\vec{V_2}' - \vec{V_2})$$

$$V_1 - \vec{V_2} = \vec{V_2}' - \vec{V_1}'$$

if $m_1 = m_2 \implies$ Exchange Velocities

$$\cancel{m_1} V_1 + \cancel{m_1} V_2 = \cancel{m_1} V_1' + \cancel{m_1} V_2'$$

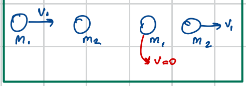
$$V_1 + V_2 = V_1' + V_2'$$

$$+\quad V_1 - V_2 = V_2' - V_1'$$

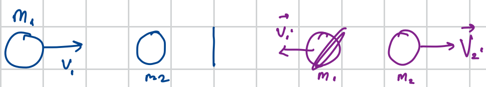
$$2\vec{V_1} = 2\vec{V_2}'$$

$$\vec{V_1} = \vec{V_2}' \quad \Big| \quad \vec{V_1}' = \vec{V_2}$$

If $V_2 = 0$



$M_1 \neq M_2 \quad V_2 = 0 \quad V_1' = ? \quad V_2' = ?$



① $m_1 \vec{V_1} + m_2\, 0 = m_1 \vec{V_1}' + m_2 \vec{V_2}'$

② $\vec{V_1} = \vec{V_2}' - \vec{V_1}'$

$\vec{V_1}' = \vec{V_2}' - \vec{V_1}$

$\vec{V_2}' = \vec{V_1} + \vec{V_1}'$

$$m_1 \vec{V_1} = m_1 (\vec{V_2}' - \vec{V_1}) + m_2 \vec{V_2}'$$

$$m_1 \vec{V_1} = m_1 \vec{V_2}' - m_1 \vec{V_1} + m_2 \vec{V_2}'$$

$$\frac{2 m_1 \vec{V_1}}{m_1 + m_2} = \vec{V_2}'$$

$$\vec{V_1}' = \frac{(m_1 - m_2)}{(m_1 + m_2)} \cdot V_1$$

$m_1 \gg m_2$

1000   0.5

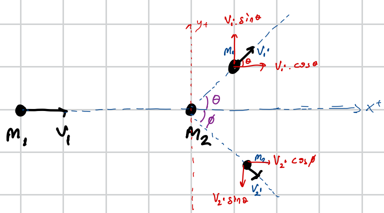
$\vec{V_1} \approx \vec{V_1}'$

$\vec{V_2}' \approx 2\vec{V_1}$

HW

$m_1 = m_2 \rightarrow \theta + \phi = \frac{\pi}{2}$

Elastic Collisions in 2D



$$\tfrac{1}{2} m_1 V_1^2 + \tfrac{1}{2} m_2\, 0 + \tfrac{1}{2} m_1 V_1'^2 + \tfrac{1}{2} m_2 V_2'^2$$

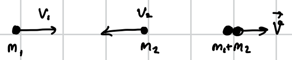
$\Delta K = 0$

$\Delta P = 0$

$$M_1 V_1 + M_2 \cdot 0 = M_1 V_1' \cos\theta + M_2 V_2' \cos\phi$$

$$\Delta P_y = 0 = M_1 V_1' \sin\theta - M_2 V_2' \sin\phi$$

Completely Inelastic



Prove that $K_f > K_i$ → $V_2 = 0$

$$m_1\vec{V_1} + m_2\vec{V_2} = (m_1+m_2)\vec{V}$$

$$m_1\vec{V_1} = (m_1+m_2)\vec{V}$$

$$\vec{V} = \frac{m_1 V_1}{m_1+m_2}$$

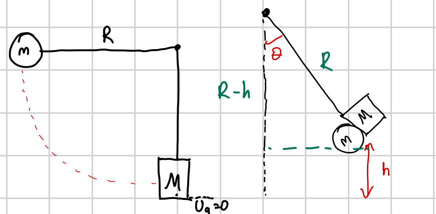
$$\Longrightarrow \boxed{K_i = \frac{1}{2} m_1 V_1^2}$$

K_f

$$\frac{1}{2}(m_1+m_2)V^2$$

$$\frac{1}{2}\frac{(m_1+m_2)(m_1)^2(V_1^2)}{(m_1+m_2)(m_1+m_2)}$$

$$\frac{K_f = \frac{m_1^2 \cdot V_1^2}{2(m_1+m_2)}}{K_i = \frac{m_1 V_1^2}{2}} = \frac{m_1}{m_1+m_2}$$



Before Collision After Collision

$h = ?$ | θ in terms of m, M

$$h = R - R\cos\theta$$

$$\frac{1}{2} m g R = \frac{1}{2} m V_1^2$$

$$\boxed{V_1 = \sqrt{2gR}}$$

m M m M

$$mV_1 + 0 = (m+M)V$$

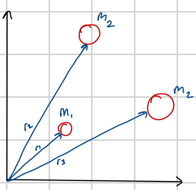
$$V = \frac{m\sqrt{2gR}}{m+M}$$

$$(m+M) g h = \frac{1}{2}(m+M)(V^2)$$

$$2 \cdot g \cdot R(1-\cos\theta) = \frac{m^2 2gR}{(m+M)^2}$$

$$\cos\theta = 1 - \frac{m^2}{(m+M)^2}$$

$$\theta = \arccos\left(1 - \frac{m^2}{(m+M)^2}\right)$$



$m_1 \to (x_1, y_1)$

$m_2 \to (x_2, y_2)$

$m_3 \to (x_3, y_3)$

$$\Delta P = 0 \Longrightarrow P_i = P_f$$

$$\frac{dP}{dt} = \sum F_{ext}$$

$$\frac{dP_1}{dt} + \frac{dP_2}{dt} = \sum \vec{F}_{ext}$$

$$\frac{d}{dt}\left(m_1 \cdot \vec{V_1} + m_2 \cdot \vec{V_2}\right) = \sum \vec{F}_{ext}$$

$$\frac{d}{dt}\left(m_1 \frac{dr_1}{dt} + m_2 \frac{dr_2}{dt}\right) = \sum \vec{F}_{ext}$$

$$\frac{d^2}{dt^2}\left(m_1\vec{r_1} + m_2\vec{r_2}\right) = \sum F_{ext}$$

$$\frac{d^2}{dt^2}\underbrace{\left(\frac{m_1\vec{r_1} + m_2\vec{r_2}}{(m_1+m_2)}\right)}_{R_{cm}}\overbrace{(m_1+m_2)}^{\boxed{M_{total}}} = \sum F_{ext}$$

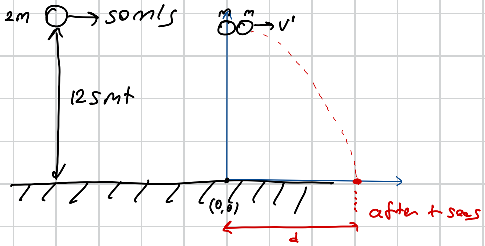
$$\frac{d^2}{dt^2} R_{cm} \cdot M_{total} = \sum F_{ext}$$

$$R_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r_i}}{\sum_{i=1}^{n} m_i}$$

$$V_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{V_i}}{\sum_{i=1}^{n} m_i}$$

$$V_{cm} = \frac{\sum_{i=1}^{n} P_i}{\sum_{i=1}^{n} m_i}$$

$$M \cdot \vec{a}_{cm} = \sum \vec{F}$$



SUPERELASTIC

a) $d = ?$

b) Velocity of Center of mass when $t=3$

c) Find R_{cm} at $t=3$

$\Delta P = 0$

$P_i = P_f$

a) $2mv = \cancel{m \cdot 0} + m\vec{v}'$

$v' = 2v$

$v' = 100 \, m/s$

$$x = \underbrace{x_0}_{0} + \underbrace{V_{0x}t}_{v't} + \underbrace{\tfrac{1}{2}a_x t^2}_{0}$$

$\boxed{d = v't}$ ⟸ $\boxed{500m}$

$$y = y_0 + V_{0y}t - \tfrac{1}{2}gt^2$$

$0 = 125 + 0 - \frac{1}{2}gt^2$

$t^2 = \frac{250}{10}$

$t^2 = 25 \Rightarrow \boxed{t = 5}$

b) Vcm when $t=3$

$V_{cm} = V_{cm}(0) + at$

$V_{cm} = V_{cm}(0) - gt\hat{j}$

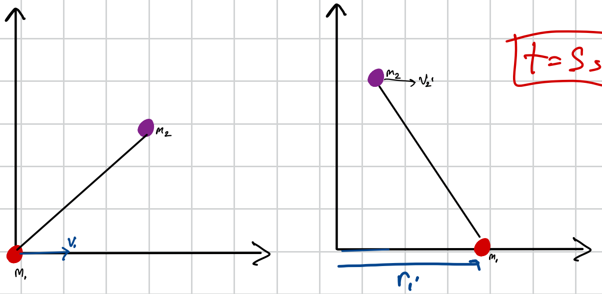
$V_{cm} = 50\hat{i} - 30\hat{j} \; m/s$

c) Rcm at $t=3$

$R_{cm} = R_{cm}(0) + V_{cm}(0)t + \frac{1}{2}at^2$

$R_{cm} = 125\hat{j} + (50.3)\hat{i} - 45\hat{j}$

$R_{cm} = 80\hat{j} + 150\hat{i}$



$t = 5s$

$\vec{r}_1 = 0\ m$

$\vec{r}_2 = 0.3\hat{i} + 0.6\hat{j}\ m$

$\vec{r}_1' = 10.6\hat{i}\ m$

$\vec{V}_1' = 3\hat{i} + 2\hat{j}\ m/s$

$\vec{r}_2$ and $\vec{V}_2'$?

$\vec{V}_1 = 6\hat{i}\ m/s$

$\vec{V}_2 = 0\ m/s$

$$R_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{0.5 \cdot 0 + 1(0.3\hat{i} + 0.6\hat{j})}{1.5} = 0.2\hat{i} + 0.4\hat{j}\,m$$

$$V_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2} = \frac{(0.5)\cdot(6\hat{i}) + 0.1}{1.5} = \boxed{2\hat{i}\ m/s}$$

$$V_{cm}' = \frac{m_1 V_1' + m_2 V_2'}{m_1 + m_2}$$

$$R_{cm}' = R_{cm} + V_{cm} \cdot t$$

$$R_{cm}' = (0.2\hat{i} + 0.4\hat{j}) + 10\hat{i}$$

$$R_{cm}' = 10.2\hat{i} + 0.4\hat{j}$$

$$\boxed{V_{cm} = V_{cm}'}$$

$$R' = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \Rightarrow \frac{0.5 \cdot 10.6\hat{i} + \vec{r}_2 \cdot 1}{1.5} =$$

$$\vec{r}_2' = 10\hat{i} + 0.6\hat{j}$$

$$V_2 = 1.5\hat{i} - 3\ m/s$$

Continuous Systems

! Not in textbook !

$$\vec{R}_{cm} = \frac{\int \vec{r}\, dm}{\int dm}$$

dm $\begin{cases} \text{1-D} & \lambda = \frac{dm}{dL} \Rightarrow \text{Linear Mass Density} \\ \text{2-D} & \sigma = \frac{dm}{dA} \Rightarrow \text{Surface Mass Density} \\ & \rho = \frac{dm}{dV} \Rightarrow \text{Volume Mass Density} \end{cases}$

$\left.\begin{array}{l} \lambda = \frac{m}{\ell} \\ \sigma = \frac{m}{A} \\ \rho = \frac{m}{V} \end{array}\right\}$ Constant For Uniform Objects

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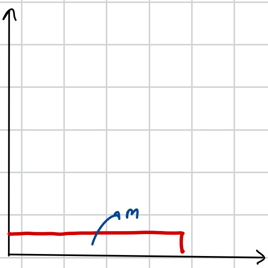


$R_{cm} = ?$

$$R_{cm} = \frac{\int r\, dm}{\int dm \; M} = \frac{1}{M}\int r\, dm$$

$\lambda = \frac{dm}{d\ell} \longrightarrow \begin{array}{l} dm = \lambda . dl \\ dm = \lambda . dx \end{array}$

$$\frac{1}{M}\int_0^\ell x\, \lambda\, dx = R_{cm}$$

$$\frac{1}{M}\int_0^\ell x \frac{M}{\ell}\, dx$$

$$\frac{1}{\ell}\int_0^\ell x\, dx = \boxed{\frac{\ell}{2}}\,\hat{i}$$

$$\lambda = \alpha x$$

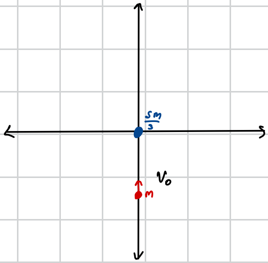
$$\vec{R}_{cm} = \frac{\int r\, dm}{\int dm}$$

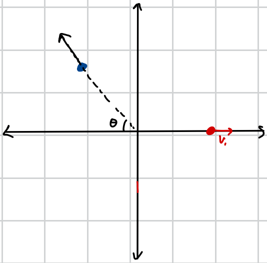
$$dm = \lambda\, d\ell = \lambda\, dx = \alpha x\, dx$$

$$\int dm = \int_0^\ell \alpha x\, dx$$

$$R_{cm} = \frac{2\ell}{3}\hat{i}$$

Elastic Collision in 2D Question





a) V_1, V_2 in terms of V_0

b) $V_{cm} =$

c) $\vec{J}$ on $\frac{5}{3}m$

$\Delta P = 0$

$P_{xi} = P_{xf} \mid P_{iy} = P_{fy}$

$$m V_1 - \frac{5m}{3} V_2 \cos\theta = 0$$

$$\cancel{m} V_1 = \frac{5}{3} \cancel{m} V_2 \cos\theta$$

$$V_1 = \frac{5}{3} \underbrace{V_2 \cos\theta}_{V_{2x}}$$

$\Delta P_y = 0$

$$m . V_0 = \frac{5}{3} m . \sin\theta . V_2$$

$$V_0 = \frac{5}{3} . \underbrace{\sin\theta \, V_2}_{V_{2y}}$$

$$\Delta K = 0 \Rightarrow K_i = K_f$$

$$\cancel{\tfrac{1}{2}} \cancel{m} V_0^2 = \cancel{\tfrac{1}{2}} \cancel{m} V_1^2 + \cancel{\tfrac{1}{2}} \frac{5}{3} \cancel{m} V_2^2$$

$$V_0^2 = V_1^2 + V_2^2$$

$$V_0^2 = V_1^2 + \frac{5 . 9}{3 . 25} (V_1^2 + V_0^2) \Rightarrow V_1 = \frac{V_0}{2}$$

$$\vec{V_2} = -V_{2x} \hat{i} + V_{2y} \hat{j}$$

$$\vec{V_2} = \frac{-3}{5} \frac{V_0}{2} \hat{i} + \frac{3 V_0}{5} \hat{j}$$

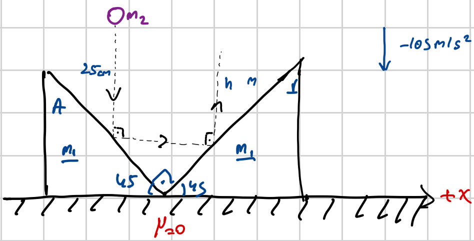
$$\boxed{V_1 = \frac{V_0}{2} \hat{i}}$$

$$\vec{V}_{cm} = \frac{m . V_0 + \cancel{\frac{5}{3} m . 0}}{\frac{8}{3} m} \Longrightarrow V_{cm} = \frac{3 V_0}{8} \hat{j}$$

$$J_{\frac{5m}{3}} = \Delta P_{\frac{5m}{3}} = m . (V_f - V_i)$$

$$\frac{5m}{3} (V_f - V_i) \Longrightarrow \boxed{\frac{V_2 . 5m}{3}}$$

Problem



$M_1 = 4m$

$m_2 = m$

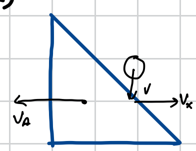
Calculate ⇒

$V_A =$

$V_B =$

$h_{max} =$

a)



$\Delta P_x = 0$

$P_{ix} = P_{fx}$

$0 = m_2 V_x - m_1 V_A$

$m_1 V_A = V_x . m_2$

$4m V_A = m V_x$

$4 V_A = V_x$

$\boxed{4 = \frac{V_x}{V_A}}$

$\Delta E = 0$

$m_2 g h = m_2 v^2 \frac{1}{2}$

$v^2 = 2 g h$

$v^2 = 2 . 10 . 0.25$

$v^2 = 5 \Rightarrow v = \sqrt{5}$

$\Delta E = 0$

$\frac{1}{2} m v^2 = \frac{1}{2} m_2 V_x^2 + \frac{1}{2} m_1 V_A^2$

$m v^2 = m V_x^2 + 4m V_A^2$

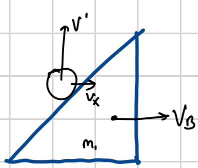
$5 = V_x^2 + 4 V_A^2$

$5 = 16 V_A^2 + 4 V_A^2$

$5 = 20 V_A^2$

$\boxed{\frac{1}{2} = V_A}$ $\boxed{V_x = 2}$

b)



$P_i = P_f$

$m_2 V_x = V_B . m_1$

$m V_x = 4 m V_B$

$V_x = 4 V_B$

$2 = 4 V_B$

$\boxed{V_B = \frac{1}{2} m/s}$

$h_m = \frac{V'^2}{2g}$

$\frac{1}{2} m_2 V_x^2 = \frac{1}{2} m_2 v'^2 + \frac{1}{2} m_1 V_B^2$

$m V_x^2 = m v_1^2 + 4m v_B^2$

$V_x^2 = V_1^2 + 4 V_B^2$

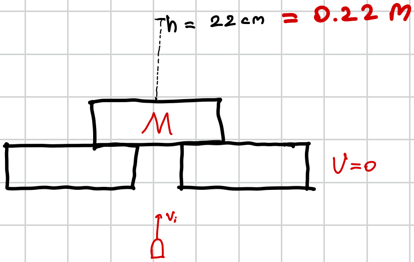
$4 = V_1^2 + 1$

$V_1 = \sqrt{3}$

c) $h = \frac{v^2}{2g}$

$h = \frac{3}{20} = 0{,}15m = 15cm$

Problem



$m = 5g = 5.10^{-3}$ kg

$M = 1.25$ kg

Calculate V_i

$$m_1 \cdot V_i = (m+M)\, v$$

$$\Delta E = 0 \implies \frac{1}{2}\cancel{(m+M)}\, v^2 = \cancel{(m+M)}\, g h$$

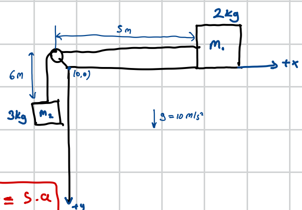
$$v^2 = 2gh \implies \frac{m^2 \cdot V_i^2}{(m+M)^2} = 2gh$$

$$V_i = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

$V_i = 1{,}05$ m/s

Another Problem

$t = 0s$



$\boxed{30 = 5.a \quad a = 6}$

a) Find $\vec{R}_{cm}$ at $t=0$ ∧ $t=1$

b) Find $A_{cm} = ?$

c) $F = ?$

a) $$r_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$r_1 = (5\hat{i})$ m
$r_2 = (6\hat{j})$ m

$$\implies \frac{2.(5\hat{i}) + 3.(6\hat{j})}{5} = \boxed{2\hat{i} + 3.6\hat{j}}$$

R_{cm} at $t=0$

r_{cm} at $t=1 \implies$

$x = \frac{1}{2} a t^2$

$x = \frac{1}{2}.(6)\, t^2$

$\boxed{x = 3.t^2}$ $x = 3m$

$r_1' = (2\hat{i})$ m
$r_2' = (9\hat{j})$ m

$$\frac{2.(2\hat{i}) + 3.(9\hat{j})}{5} = 0.8\hat{i} + 5.4\hat{j}$$

$$r_{cm}(t) = r_{cm}(0) + V_{cm}(0) + \frac{1}{2} a_{cm} t^2$$

$$r_{cm}(1) = r_{cm}(0) + V_{cm}(0) + \frac{1}{2} a_{cm}$$

$$0.8\hat{i} + 5.4\hat{j} = 2\hat{i} + 3.6\hat{j} + \frac{1}{2} a_{cm}$$

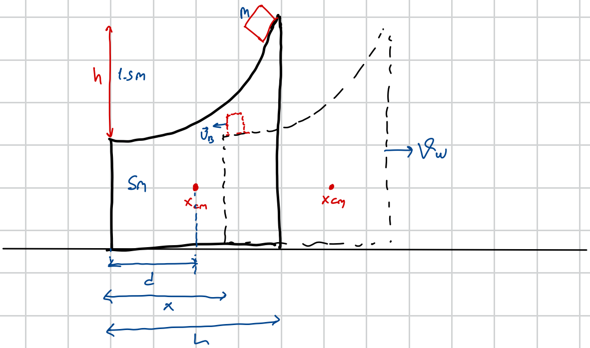
$$-1.2\hat{i} + 1.8\hat{j} = \frac{1}{2} a_{cm}$$

$$-2.4\hat{i} + 3.8\hat{j} = a_{cm}$$

$F = \sum m . a_{cm}$

$F = 5.(-2.4\hat{i} + 3.6\hat{j})$

Problem



a) $\frac{V_B}{V_w} =$

b) $V_B =$ $V_w =$

c = x → L

a) $\Delta P = 0$

$P_{ix} = P_{fx}$

$0 = 5m \cdot V_w - m \cdot V_B$

$5 \cdot V_w = V_B$

$\boxed{5 = \frac{V_B}{V_w}}$

b)

$\Delta E = 0$

$m g h = \frac{1}{2} m V_B^2 + \frac{1}{2} 5m V_w^2$

$15 = \frac{25 V_w^2 + 5 V_w^2}{2}$

$30 = 30 V_w^2 \Rightarrow \boxed{\begin{array}{l} V_w = 1 \; m/s \\ V_B = 5 \; m/s \end{array}}$

c =)

$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

$\frac{m \cdot L + 5m \cdot d}{6m} \Rightarrow \frac{L + 5d}{6}$

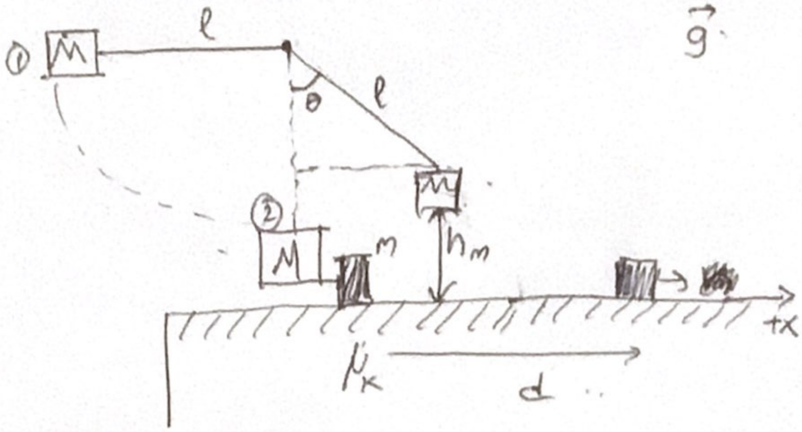
$r_{cm}' = \frac{m \cdot x + 5m \cdot (x + d)}{6m}$

$F_{ext} = 0 \Rightarrow x + 5(x + d) = L + 5d$

$\boxed{6x = L}$

08/12/2023

$\vec{g}$



$M = 0{,}25 kg$

$l = 0{,}8 m$

$m = 0{,}1 kg$

$\mu_k = 0{,}25$

$V_{M}' = 2 m/s$

$g = 10 m/s^2$

a) Velocity Before colision M b) $V_{m'}$ just after colision

c) M → determine θ at h_{max} d) fbd for m after colision

e) d = ?

$V_f^2 = V_0^2 + 2ax$

---

a) ① - ② $\Delta E = 0$

$\Delta K + \Delta U_g = 0$

$(K_f - K_i) + (\cancel{U_{gf}} - U_{gi}) = 0$

$K_f - 0 - U_{gi}$

$K_f = U_{gi}$

$\frac{1}{2} \cancel{M} V_M^2 = \cancel{M} \cdot g \cdot l$

$V_M^2 = 2gl$

$\boxed{V_M = \sqrt{2gl}}$

$V_M = \sqrt{2 \cdot 10 \cdot 0{,}8}$

a) $\boxed{V_M = 4 m/s}$

b)

$p_i = p_f$

$M V_M = M V_M' + m \cdot V_m'$

$0{,}25 \cdot 4 = 0{,}25 V_M' + 0{,}1 \cdot V_m'$

$1 = 0{,}5 + 0{,}1 \cdot V_m'$

$0{,}5 = 0{,}1 \cdot V_m'$

$\boxed{V_m' = 5 m/s}$

c

l - h_m

l

h_m

$\frac{l - h_m}{l} = \cos\theta$

$\frac{0{,}8 - h_m}{l}$ $\frac{0{,}6}{0{,}8}$

$\frac{1}{2} \cdot \frac{1}{4} 4 = \frac{1}{4} 10 \cdot \boxed{h_m = 0{,}2 m}$

d)

N

f_k — m

mg

e) ΔE

$W_{fric} = \Delta K + \cancel{\Delta U_g}$

$W_f = K_f - K_i$
0

$- f_k d$

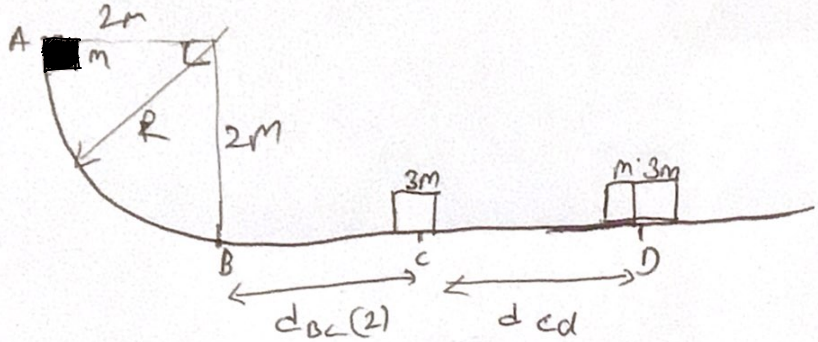
$+ f_k d = + K_i$

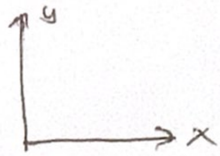
$\mu_k \cancel{m} g\, d = \frac{1}{2} \cancel{m} V_m'^2$

$0{,}25 \cdot 20 \cdot d = \frac{1}{2} V_m'^2$

d = 5 m

1





$R = 2M$

$d_{BC} = 2M$

$g = 10 m/s^2$

$\pi = 3$

$M = 15 kg$

Constent friction $\boxed{F = 30N}$

a) V_m first cart → second card

b) dcd =

c) acceleration of center of mass ~~first~~ before and after collision

---

a) $W_f = \Delta E = K_f - \cancel{K_i} + \cancel{U_{gf}} - U_{gi} \Rightarrow \frac{1}{2} m V_m^2 - m g R = -f_k \cdot \Delta cl$

$\left(\frac{2\pi R}{4} + 2\right) \Rightarrow \frac{2 \cdot 3 \cdot \cancel{2}}{\cancel{4}} + 2 = 5 =$ $\frac{1}{2} m V_m^2 - m \cdot 10 \cdot 2 = -30 \cdot 5$

$V_m = 2\sqrt{5}$

---

b) > d CD

$P_i = P_f$

$m \cdot V_m = (4m) \cancel{V'}$

$2\sqrt{5} = 4V'$

$\frac{\sqrt{5}}{2} = V'$

$W_{fric} = \Delta E$

$+f_K \cdot d_{CD} = +K_i$

$\cancel{30}_6 \cdot d_c = \frac{1}{\cancel{2}} \cdot \cancel{60}^{30} \cdot \frac{\cancel{5}}{4}$

$d_c = \frac{30}{6 \cdot 4}$

$d = \frac{5}{4} = 1.25 m$

c)

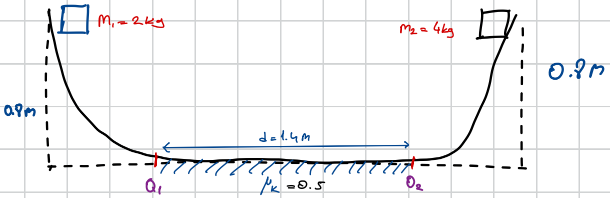
$\sum F_{ext} = M \vec{a}_{cm}$

$-30 = 60\, a_{cm}$

$-\frac{1}{2}\, m/s^2 = a_{cm}$

After collision and Before

Problem



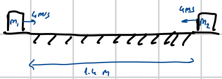
a) Position of collision

b) Inelastic ⇒ distance they will stop

a) At O_1 and O_2 $\Delta E = 0$

$$m_1 g h = \frac{1}{2} m_1 V_1^2$$

$$\boxed{\sqrt{2gh} = V_1 \text{ and } V_2}$$



$x_1 + x_2 = 1.4\,m$

$x_1 = x_0 + V_1(t) + \frac{1}{2}at^2 \Rightarrow V_1 t + \frac{1}{2}at^2$

$x_2 = -x_0 + V_2(t) + \frac{1}{2}at^2 \Rightarrow V_2 t + \frac{1}{2}at^2$

$8t + at^2 = 1.4$

$8t - \mu_k g = 1.4$

$8t - 5t^2 = 1.4$

$a = -\mu_k \cdot g$

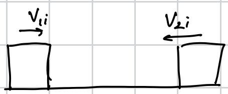
$a = -5\,m/s^2$

$\boxed{t = 1.4\,s}$

$x_1 = 4 \cdot 1.4 - \frac{1}{2} \cdot 5 \cdot (1.4)^2$

$x_1 = 0.7\,m$ / $x_2 = 0.7\,m$

b) Before collision



$V_{1i}^2 = V_1^2 + 2ax_1 = \boxed{3\,m/s}$

$V_{2i}^2 = V_2^2 + 2ax_2 = 3\,m/s$

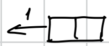
$P_i = P_f$

$m_1 \cdot \vec{V_{1i}} + m_2 \vec{V_{2i}} = (m_1 + m_2) V_f$

$2 \cdot (3\hat{i}) - 4 \cdot (3\hat{i}) = (6)\, V_f$

$-6\hat{i} = 6\vec{V}$

$\boxed{-\hat{i} = \vec{V}}$ $\quad |\vec{V}| = 1\,m/s$



$V_f^2 = V_i^2 + 2ad$

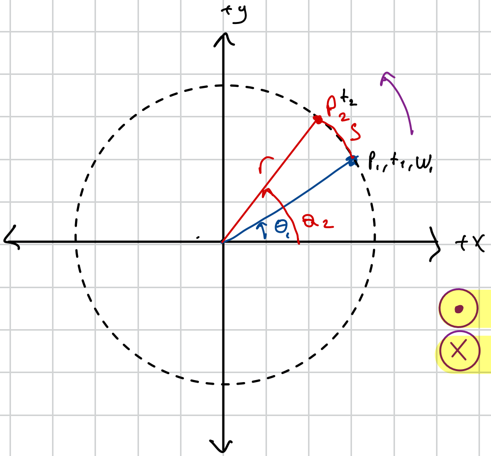
$0 = 1 - 2 \cdot 5d$

$d = 0.1\,m$

c) a_{cm} after collision

$$\sum F_{ext} = M \vec{a}_{cm}$$

Rotation Of Rigid Bodies



$s = r\theta$

$\Delta\theta = \theta_2 - \theta_1$

$\Delta t = t_2 - t_1$

$\frac{\Delta\theta}{\Delta t} = w_{avg} = [rad/s]$

$\theta = [rad]$

1 rev = 2π radians

$\frac{d\theta}{dt} = w_{ins}$

⊙ z +

⊗ z -

Angular Acceleration Avg

$\frac{\Delta w}{\Delta t} = \frac{w_2 - w_1}{t_2 - t_1} = \alpha_{avg}$

Ins Angular Acceleration

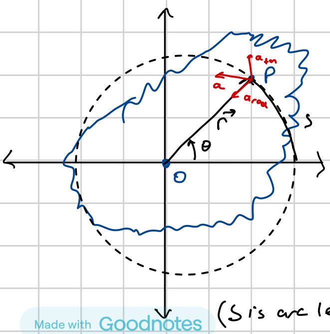
$\frac{dw}{dt} = \alpha_{ins} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) \Rightarrow \left(\frac{d^2\theta}{dt^2}\right)$

α ins + → $w\uparrow$

α ins - → $w\downarrow$

Translational Motion		Rotational Motion
$v = v_0 + at$	$\Longrightarrow$	$w = w_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\Longrightarrow$	$\theta = \theta + w_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2ax$	$\Longrightarrow$	$w^2 = w_0^2 + 2\alpha\theta$
$x - x_0 = \frac{1}{2}(v_x + v_0)t$	$\Longrightarrow$	$\theta - \theta_0 = \frac{1}{2}(w_2 + w_0)t$

a AND α are CONSTANT



(S is arc lenght)

$S = R\theta$

$\frac{ds}{dt} = \frac{d}{dt}(R\theta)$

$\frac{ds}{dt} = \cancel{\frac{dR}{dt}}^{\,0} \cdot \theta + \frac{d\theta}{dt} \cdot R$

$v = \frac{ds}{dt} = R \cdot \frac{d\theta}{dt}$

$v = Rw$

$\frac{dv}{dt} = R \cdot \frac{dw}{dt}$

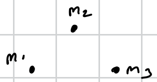
$a_{tangential} = R \cdot \alpha$

$a_{rad} = w^2 R$

$\vec{a}_{tan} + \vec{a}_{rad} = \vec{\alpha}$

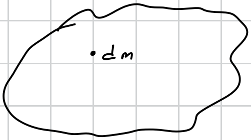
Made with Goodnotes

Moment of Inertia



$$KE = \frac{1}{2} m_i (w_i r_i)^2 = \frac{1}{2} w^2 \underbrace{(m_1 r_1^2 + m_2 r_2^2 + m_i r_i^2)}_{I}$$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

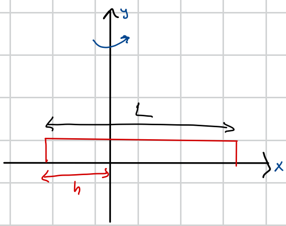


$$I = \int r^2 dm$$

For 1D $\Rightarrow dm = \lambda dr$ | λ = linear mass density

2D $\Rightarrow dm = \sigma dA$ | σ = area mass d.

3D $\Rightarrow dm = \rho dV$ | ρ = Volume mass d.



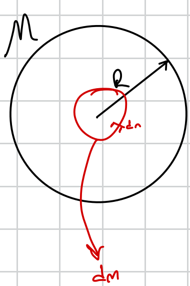
$$I_H = \int r^2 dm = \int x^2 \lambda dx = \int_{-h}^{L-h} x^2 \frac{M}{L} . dx$$

$$= \frac{1}{3} M (L^2 - 3Lh + 3h^2)$$

h should be 0 For Left
$= \frac{1}{3} M L^3$

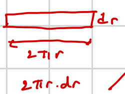
h should be L for Right
$= \frac{1}{3} M L^3$

h should be $\frac{L}{2}$ for middle



$$I_{disk} = \int r^2 dm \Rightarrow \int r^2 \sigma dA \Rightarrow \int r^2 . \frac{M}{\pi r^2} dA$$

$dm = \sigma dA$



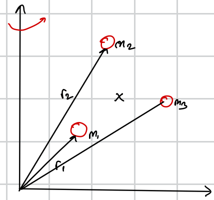
$dA = 2\pi r dr$

$$\frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \frac{r^4}{4} \Big|_0^R$$

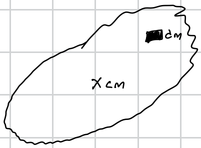
$$= \boxed{\frac{1}{2} M R^2}$$

$\sigma = A x^2$

$$\int r^2 . \sigma . dA = \int_0^R x^4 A \, 2\pi r dr$$



$$\vec{R}_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} \qquad I_y = \sum_{i=1}^{n} m_i r_i^2$$



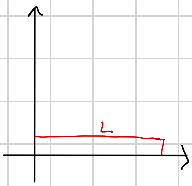
$$\vec{R}_{cm} = \frac{\int \vec{r}\, dm}{\int dm} \qquad I_y = \int r^2 dm$$

1D $\Rightarrow dm = \lambda\, dr$ $\qquad \lambda = \frac{M}{\ell}$

2D $\Rightarrow dm = \sigma\, dA$ $\qquad \sigma = \frac{M}{A}$

3D $\Rightarrow dm = \rho\, dV$ $\qquad \rho = \frac{M}{V}$

1st case uniform Rod



$$\frac{\int r\, dm}{\underbrace{\int dm}_{M}} \Rightarrow \frac{\int r \lambda\, dr}{M} \Rightarrow \frac{\frac{M}{L}\cdot \int x\, dx}{M} \Rightarrow \frac{1}{L}\int_0^L x\, dx$$

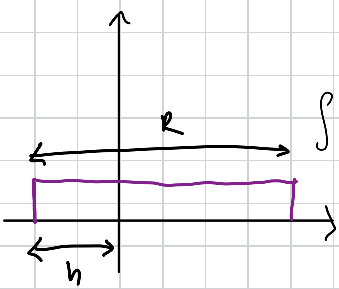
$$\frac{1}{L}\cdot\left[\frac{L^2}{2}\right] = \frac{L}{2}$$

Non uniform case

$dm = \lambda dx$

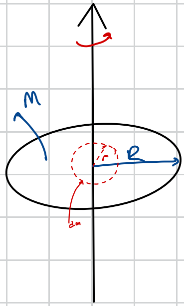
$\lambda = \alpha x$

$$R' = \frac{\int r\, dm}{\int dm} = \frac{\int x \lambda\, dx = \alpha \int_0^L x^2 dx = \frac{\alpha x^3}{3}\Big]_0^L = \boxed{\frac{\alpha L^3}{3}}\ N}{\int_0^L \alpha x\, dx \Rightarrow \frac{\alpha x^2}{2} \Rightarrow \boxed{\frac{\alpha L^2}{2}}\ D} \Rightarrow \frac{\frac{\cancel{\alpha} L^3}{3}}{\frac{\cancel{\alpha} L^2}{2}} = \boxed{\frac{2L}{3}}$$



$I_h =$ For uni.

$$\int r^2 dm = \int x^2 \lambda\, dx = \int_{-h}^{L-h} \frac{M}{L} x^2 dx = \frac{M}{L}\cdot \frac{x^3}{3}\Big]_{-h}^{L-h} = \frac{M}{3}\left[L^2 - 3LH + 3h^2\right]$$



$$I_{disk-s} =$$

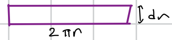
$dm = \sigma \, dA$

$dm = \frac{M}{A} \cdot dA$

$$I = \int r^2 \, dm$$

$$I = \int R^2 \frac{M}{\pi R^2} \, dA$$

dA

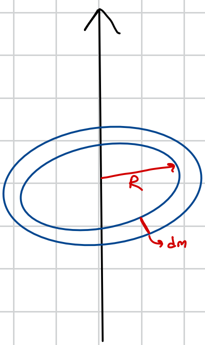


$\boxed{2\pi r \, dr = dA}$

$$I = \int r^2 \frac{M}{\cancel{\pi} R^2} \, 2 \cancel{\pi} r \, dr$$

$$I = \frac{M}{\pi} \int_0^R 2r^3 \, dr$$

$$= \frac{1}{2} M R^2$$



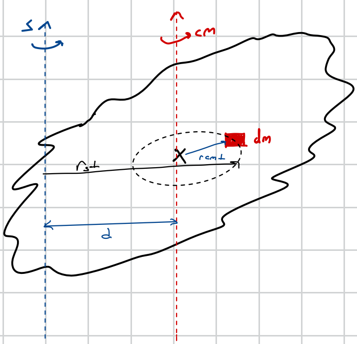
$$I_{ring} =$$

$dm = \lambda \, dr \qquad \lambda = \frac{M}{L} \qquad \lambda = \frac{M}{2\pi R}$

$$I = \int r^2 \, dm$$

$$I = \int r^2 \lambda \, dr \Rightarrow \int \underbrace{r^2}_{R^2 = \text{same distance}} \frac{M}{2\pi R} \, dr \Rightarrow \int_0^{2\pi R} R^2 \frac{M}{2\pi R} \, dr \Rightarrow \frac{MR^2}{2\pi R} \int_0^{2\pi r} dr$$

$$\cancel{2\pi R} \cdot \frac{MR^2}{\cancel{2\pi R}} = MR^2 \qquad R_{cm} = 0$$



$$I_s = \int r_{s\perp}^2 \, dm$$

$$I_s = \int (d + r_{cm})^2 \, dm$$

$$I = \underbrace{\int d^2 \, dm}_{\text{body}} + \underbrace{\int 2d \, r_{cm\perp} \, dm}_{\text{body}} + \underbrace{\int r_{cm\perp}^2 \, dm}_{\text{body}}$$

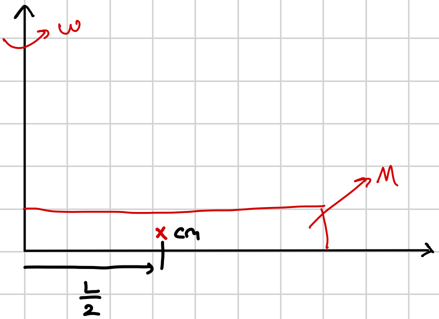
$d^2 \int dm \qquad 0 = \text{cancels out} \qquad I_{cm}$

$r_{s\perp} = d + r_{cm\perp}$

$$\boxed{d^2 M} + I_{cm} = I_s$$

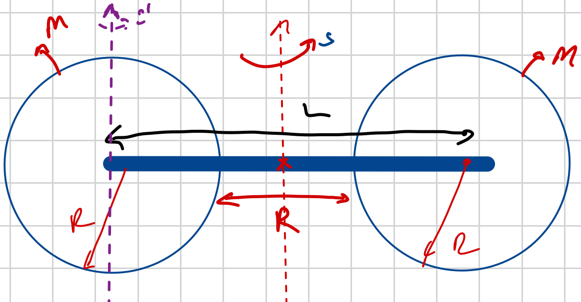
Made with Goodnotes

$R_{cm} = L/2$ $\quad$ $I_y = Md^2 + I_{cm}$



$M\cdot\left(\frac{L}{2}\right)^2$ $\qquad$ $\frac{1}{12}ML^2$

$$\frac{3ML^2}{12} + \frac{ML^2}{12} = \frac{4ML^2}{12} = \boxed{\frac{ML^2}{3}}$$



$L = 3R$

$$I_s = 2I_{disk} + \underbrace{I_{rod}}_{\frac{1}{12}ML^2 \;\Downarrow\; \frac{1}{12}M9R^2}$$

$$I_{disk} = I_{cm} + M\left(\frac{3}{2}R\right)^2$$

$$I_d = \frac{1}{2}MR^2 + M\frac{9}{4}R^2$$

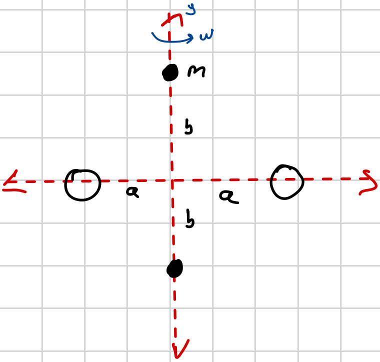
$$I_{s'} = \underbrace{I_{rod}} + I_{disk-2} + I_{disk\,\ell}$$

$$\frac{1}{3}ML^2 + \left[\frac{MR^2}{2} + M9R^2\right] + \frac{MR^2}{2}$$

20/12/2023

!! Point !!

Find I and KE_{rot}



a) Rotate abt. y axis

b) Rotate about origin

$$I = \sum_{i=1}^{n} m_i r_i^2$$

a) $I_y = Ma^2 + Ma^2 = 2Ma^2$

$K_{Rot} = \frac{1}{2} I_y w^2 = Ma^2 (w^2)$

b) $I_o = 2Ma^2 + 2mb^2$

$K_{rot,o} = \frac{1}{2} I w^2 \Rightarrow (Ma^2 + Mb^2) w^2$

c) If M is uniform sphere

$I_{sphere_{cm}} = \frac{2}{5} MR^2$

$I_y = 2(Ma^2 + \frac{2}{5} MR^2)$

$\alpha_z(t) = 8.6 \text{ rad/s}^2 - 2.3t \text{ rad/s}^3$

a = At rest $t=0$, $w = 5s$

b = θ = $t = 0, 5$

$\frac{dw}{dt} = \alpha$

$dw = \alpha \, dt$

$\int dw = \int_0^5 \alpha \, dt$

$w = 8.6t - \frac{2.3t^2}{2} \Big]_0^5$

$43 - 28.75 = 14.25 \text{ rad/s}$

$\frac{d\theta}{dt} = w$

$d\theta = dt \, w$

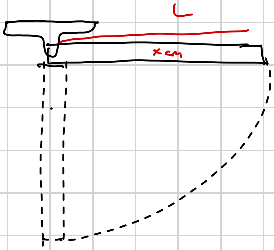
$\Delta\theta = \int w \, dt$

$\Delta\theta = \int_0^5 8.6t - \frac{2.3t^2}{2}$

$\Delta\theta = 4.3t^2 - \frac{2.3t^3}{6} \Big]_0^5$

$\Delta\theta = 60$ radians

Made with Goodnotes



a) w

b) V_{cm} , V_L

$\int r^2 dm$

a) $m \cdot g \cdot \frac{L}{2} = \frac{1}{2} I w^2$

I_{rod}

$\int r^2 \lambda \, dr$

$\int r^2 \frac{M}{L} dr$

$m \cdot g \cdot \frac{L}{2} = \frac{1}{2} \frac{1}{3} M L^2 w^2$

$\frac{M}{L} \int x^2 dx$

$\cancel{m} \cdot g \cdot \cancel{L} = \frac{1}{3} \cancel{M} L^{\cancel{2}} w^2$

$g = \frac{1}{3} L w^2$

$\frac{M}{L} \int_0^L x^2 dx$

$\sqrt{\frac{3g}{L}} = w$

$\frac{M}{L} \left. \frac{x^3}{3} \right]_0^L$

$\boxed{\frac{ML^2}{3}}$

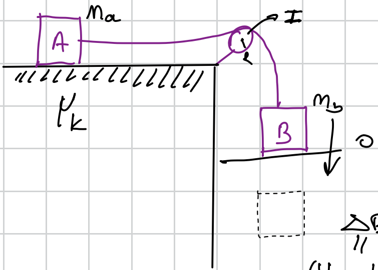
b) $= V_{cm} = W \cdot r_{cm} = V_{cm}$

$W \frac{L}{2} = V_{cm}$

$\sqrt{\frac{3g}{L} \frac{L^2}{4}} = \frac{\sqrt{3gL}}{2} = V_{cm}$

$$\boxed{\begin{gathered} V_L = W \cdot L = V_L \\ \sqrt{\frac{3g}{L}} \cdot L = \sqrt{\frac{3g \cdot L^2}{L}} \Longrightarrow V_{cm} = \sqrt{3gL} \end{gathered}}$$

Made with Goodnotes



⇒ use Energy methods

$V_b = d, g, m_a, m_b, I, R$

$\Delta E = -W_{friction}$

$\Delta E = -m_a \cdot g \cdot \mu_k \cdot d$

$$\Delta E = (U_{gf}-U_{gi}) + (K_{Ef}-K_{Ei}) + (K_{E\,rot\,f} - K_{E\,rot\,i})$$

$$(-m_b \cdot g \cdot d) + \left(\frac{1}{2} m_A v^2 + \frac{1}{2} m v_B^2\right) + \left[\frac{1}{2} I \omega^2\right]$$

I_{disk}

$$I_{disk} = \int r^2 dm$$

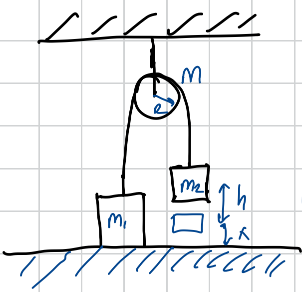
$$\int r^2 \frac{M}{\pi R^2} \cdot dA$$

$$\frac{2M}{R^2} \int_0^R r^2 \; r dr$$

$$\frac{2M}{R^2} \left.\frac{r^4}{4}\right]_0^R$$

$$\frac{M}{R^2} \frac{R^4}{2} = \boxed{\frac{MR^2}{2}}$$

!GÖZ!



$\Delta E = 0$

$$(U_f - U_i) + (K_{Ef} - K_{Ei}) + (K_{rot\,f} - K_{rot\,i})$$

$$(m_1 g h + m_2 g x)$$

$$- \cancel{m_1 g 0} - m_2 g h - m_2 g x$$

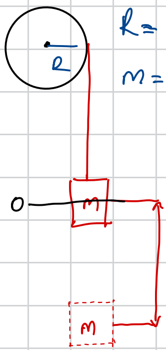
$$(m_1 g h - m_2 g h) + \left(\frac{1}{2}(m_1+m_2) v^2\right) + \left(\frac{1}{2} I \omega^2\right) = 0$$

$$\left(\frac{1}{2} \frac{v^2}{\cancel{R^2}} \cdot \frac{M \cancel{R^2}}{2}\right)$$

$$(m_1 - m_2) g h + \frac{1}{2}(m_1+m_2) v^2 + \left(\frac{1}{4} v^2 M\right) = 0$$

2)

Question



$R = 0.28\,m$

$m = 4.2\,kg$

$t = 0$

$t = 2$ seconds

$\boxed{M \text{ of wheel}}$

$\Delta E = 0$

$E_i = E_f$

$\cancel{U_{gi}}_0 + \cancel{K_i}_0 = U_{gf} + K_f$

$0 = -3mg + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$M = 47.6\,kg$

$y(t) = \cancel{y_0} + \cancel{V_y t} + \frac{1}{2}at^2$

$3 = \frac{1}{2}a\,4$

$\boxed{1.5\,m/s^2 = a}$

$V(t) = at$

$V(2) = 1.5 \cdot 2$

$V(2) = 3\,m/s$

Q Rot Mot

! I need more boullets !

A boullet of mass m moving with the speed V_0 hits rod of mass 5m and lenght L which is pinned at its top end. The boullet hits the rod at a distance of $\frac{2}{3}L$ from the top of the rod. After the collision, the boullet becomes embedded in the rod and the 2 of them rotates until they reach a maximum angle θ (For rod of mass M and lenght L rotating about its cm is given by $(\frac{1}{12}ML^2)$)

a) find moment of inertia of boullet rod system when boullet hits the rod

b) find angular valocity of rod-boullet immediately after collision

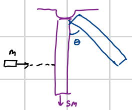
c) find Max θ

$[V = \omega . R_{cm}]$

$\frac{V_0}{6} = \omega . \frac{19L}{36}$

$\omega = \frac{6V_0}{19L}$

A



a) $I_{rod\,end} = d^2 m + I_{cm\,rod}$

$I_{rod\,end} = \frac{L^2}{4} . M + \frac{1}{12} ML^2$

$\boxed{I_{rod\,end} = \frac{1}{3}ML^2} \rightarrow I_{rod} = \boxed{\frac{1}{3} 5mL^2}$

$I_{rod+bullet} = I_{rod\,end} + I_{boullet}$ $= \boxed{\frac{19ML^2}{9}}$

B

$mV_0 + 0 = 6m . V_{2L/3}$

$m . V_0 = 6m . V$

$V_0 = 6V$

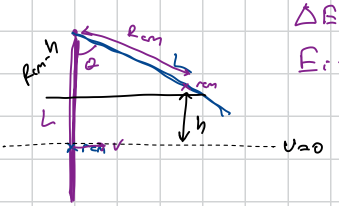
$\frac{V_0}{6} = V$

$R_{cm} = \frac{\sum m_i r_i}{\sum m_i}$

$R_{cm} = \frac{5m . \frac{L}{2} + m . \frac{2L}{3}}{6m}$

$R_{cm} = \frac{19L}{36}$

C



$\Delta E = 0$

$E_i = E_f$

$\Rightarrow \frac{1}{2}I\omega^2 = 6mgh$

$\frac{1}{2} \cdot \left[\frac{19ML^2}{9}\right] \cdot \left[\frac{6V_0}{19L}\right]^2 = 6mgh$

$\boxed{h = \frac{V_0^2}{3 \cdot 19g}}$

$\frac{R_{cm}-h}{R_{cm}} = \cos\theta$

$1 - \frac{h}{R_{cm}} = \cos\theta$

$1 - \frac{V_0^2}{3 \cdot 19g} \cdot \frac{36}{19L} = \cos\theta$

$1 - \frac{12V_0^2}{19^2 L} = \cos\theta$

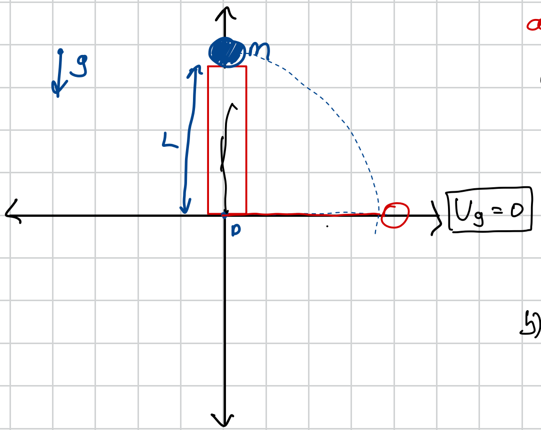
Last Question Exam 2

A block of mass m is attached to a rod of mass m and lenght L where the other end of the rod is pinned to a point that is the origin of 2D xy coordinate system. The system is initially at rest t=0 and move about a fixed axis of P in clockwise direction. ($I_{rod,cm} = \frac{1}{12} mL^2$)

a) I_{system} with respect to P

b) Find R_{cm} of system when block-mass system passes through x axis

c) Calculate angular velocity when block mass system passes x axis



a) $\underbrace{I_{rod}}_{\frac{1}{12}mL^2 + md^2} + I_{mass} = I_{system}$

$\frac{1}{3} ML^2 + ML^2 = I_{system}$

$\boxed{\frac{4}{3} ML^2 = I_{system}}$

b) $R_{cm\ when\ x} = \dfrac{m \cdot \frac{L}{2} + Lm}{m+m}$ $\qquad \dfrac{\frac{3}{2} \cancel{m} L}{2\cancel{m}}$

$R_{cm} = \left(\frac{3}{4} L\right)\hat{\imath}$

c) $\Delta E = 0$

$E_i = E_f$

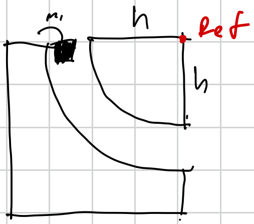
$\frac{1}{2} I w^2 = 2m \cdot g \cdot \frac{3}{4} L$

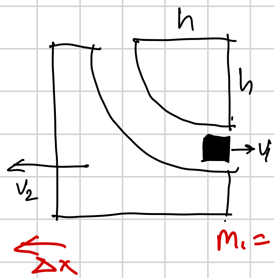
$\left(\frac{4}{3} L\right) w^2 = 3g$

$w = \frac{3}{2}\sqrt{\frac{g}{L}}$

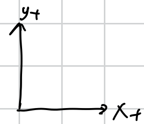
Made with Goodnotes

Consider a block of mass m_2 with a curved tunnel of height of h. A small mass m_1 is released from the top of the tunnel. Masses are initially at rest. Assume all surfaces are frictionless. Find speed of the masses m_1 and m_2 at the instant that m_1 leaves the tunnel. the tunnel. Find total displacement of m_2 at that instant. Find Relative velocity of m_1 with respect to m_2 at at instant using unit vectors.





$m_1 = h - \Delta x$



$\Delta P = 0$

$0 = \vec{V_2} m_2 + \vec{V_1} m_1$

$V_2 m_2 = V_1 m_1$

$V_2 = V_1 \cdot \frac{m_1}{m_2}$

$V_2 = \sqrt{\frac{2gh}{1 + \frac{m_1}{m_2}}} \cdot \frac{m_1}{m_2}$

$m_1 g h = \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_1 v_1^2$

$2 m_1 g h = m_1 V_1^2 + m_2 V_2^2$

$2 m_1 g h = m_1 V_1^2 + m_2 \left(\frac{V_1 m_1}{m_2}\right)^2$

$2 m_1 g h = m_1 V_1^2 + \frac{V_1^2 m_1^2}{m_2}$

$2gh = V_1^2 + \left(\frac{m_1}{m_2}\right) V_1^2$

$2gh = V_1^2 \left(1 + \frac{m_1}{m_2}\right)$

$V_1 = \sqrt{\frac{2gh}{1 + \frac{m_1}{m_2}}}$

b) $R_{cm} = R_{cm}'$

$\frac{m_1 (h - \Delta x) - m_2 \Delta x}{m_1 + m_2} = \Delta x = \boxed{\frac{m_1 h}{m_1 + m_2}}$

$\vec{V}_{m_1 m_2} = \vec{V}_{m_1} - \vec{V}_{m_2}$

$V_{m_1} \hat{\imath} - (-V_2 \hat{\imath})$ → $(V_{m_1} + V_{m_2}) \hat{\imath}$

Made with Goodnotes

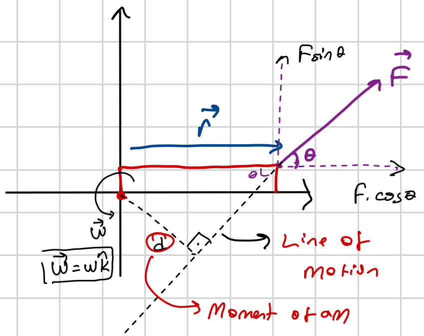
27/12/2023

Rotational Dynamics

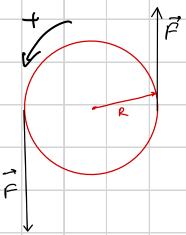
Torque → τ

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = |\vec{r}| . |\vec{F}| . |\sin\theta|$

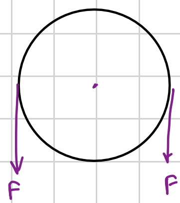


$\vec{r} = r . \hat{i}$



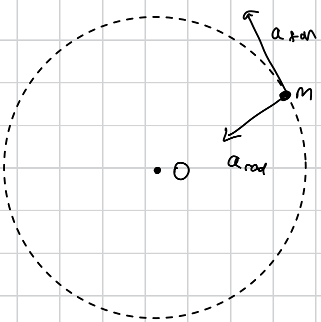
$\sum F = 0$

$\sum \tau = 2Fr$



$\sum F = 2F$

$\sum \tau = 0$



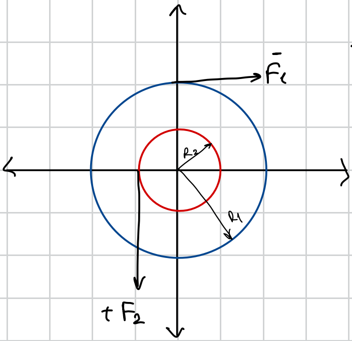
$\sum F_t = m . a_t = m . (\alpha . R)$

$R . \sum F_t = m . \alpha . R^2$

$\boxed{\tau = I \alpha}$ → Rotational Analogue of Newtons 2nd Law

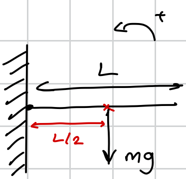
Made with Goodnotes

Example



$$\boxed{\sum \tau = -F_1 R_1 + F_2 R_2}$$

Ex



Calculate α

$$\tau = I \cdot \alpha$$

$$\vec{\tau} = mg \cdot \frac{L}{2}$$

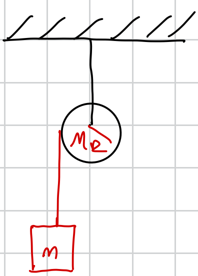
$$\frac{mgL}{2} = \frac{mL^2}{3} \cdot \alpha$$

$$\frac{3\,mgL}{2\,mL^2} = \alpha$$

$$\boxed{\frac{3}{2}\,\frac{g}{L} = \alpha}$$

$$a = \alpha \cdot r$$

$$a = \frac{3gr}{2L} \quad \text{if } r > \frac{2L}{3} \quad a > g$$



calculate acceleration of rod

$$I_{cyl} = \frac{1}{2} M R^2$$

$$\tau = I\alpha$$

$$mg - T = ma$$

$$mg - \frac{1}{2} Ma = ma$$

$$mg = \left(\frac{M}{2} + m\right) . a$$

$$a = \frac{mg}{m + \frac{M}{2}}$$

$$\tau = T . R$$

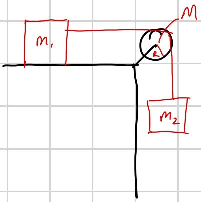
$$T.R = \frac{1}{2} MR^2 \alpha$$

$$T = \frac{1}{2} M.(R.\alpha)$$

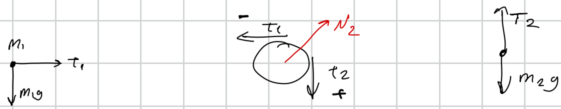
$$T = \frac{M.a}{2}$$

Example

$$\boxed{I_{cyl} = MR^2}$$



Calculate T_1, T_2 and a



$$\boxed{T_1 = m_1 . a}$$

$$\sum \tau = I\alpha$$

$$T_2 . R - T_1 R = MR^2 . \alpha$$

$$T_2 - T_1 = MR . \alpha$$

$$\boxed{T_2 - T_1 = M . a}$$

$$\boxed{m_2 g - T_2 = m_2 a}$$

$$m_2 g - m_2 a = T_2$$

$$T_1 = m_1 a$$
$$m_2 g - T_2 = m_2 a$$
$$+ \quad T_2 - T_1 = Ma$$

$$m_2 g = m_1 a + m_2 a + Ma$$
$$m_2 g = a(m_1 + m_2 + M)$$
$$a = \frac{m_2 g}{(m_1 + m_2 + M)}$$

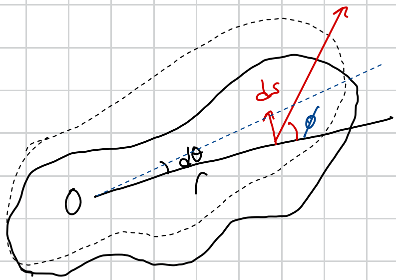
$$T_1 = \frac{m_1 . m_2 . g}{(m_1 + m_2 + M)}$$

$$T_2 = m_2 g - \frac{m_2^2 g}{(m_1 + m_2 + M)}$$

Torque Energy

$$ds = r . d\theta$$

$$dW = \vec{F} . d\vec{s} = (F . \sin\theta) . r . d\theta$$



$$dW = \tau . d\theta$$

$$\int_0^W dW = \tau \int_{\theta_1}^{\theta_2} d\theta \implies \boxed{W = \tau . \Delta\theta}$$

$$P_{ower} = \frac{dW}{dt} = \tau . \frac{d\theta}{dt} = \tau . w_{angular}$$

$$\tau = I . \alpha$$

$$\tau = I . \frac{dw}{dt}$$

$$\tau = I . \frac{dw}{d\theta} . \frac{d\theta}{dt}$$

$$\tau = I . w . \frac{dw}{d\theta}$$

$$\tau . d\theta = I . w . dw$$

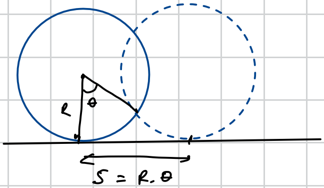
$$\int_{\theta_1}^{\theta_2} \tau . d\theta = \int_{w_1}^{w_2} I . w . dw$$

$$\tau . (\theta_2 - \theta_1) = \frac{1}{2} I w^2 \Big]_{w_1}^{w_2}$$

$$\tau . \Delta\theta = W$$

$$\boxed{W = \Delta k}$$

Rolling Without Slipping

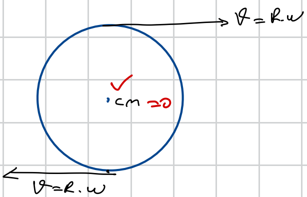


$$V_{cm} = \frac{ds}{dt} = \frac{d}{dt}(R\theta) = \frac{d\theta}{dt}\cdot R = \omega \cdot R$$

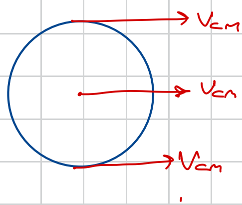
$$a_{cm} = \frac{dV_{cm}}{dt} = R\cdot\frac{d\omega}{dt} = R\alpha$$

$$K_E = K_{E_t} + K_{E_{rot}}$$

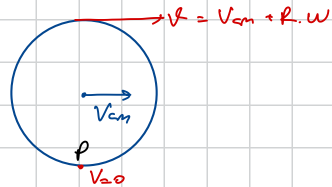
Rolling Motion = Translational + Rotational



Pure Rotation



Pure Linear



Rolling motion

Conditions for Rolling

⇒ Point P is at rest w r Ground

$V_{cm} = \omega r$

friction = Static

$a_{cm} = R.\alpha$

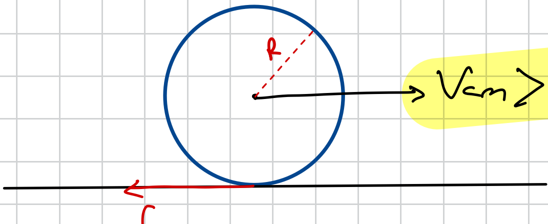
1 question in Final

Rolling With Slipping

* $V_{cm} \neq \omega . R$

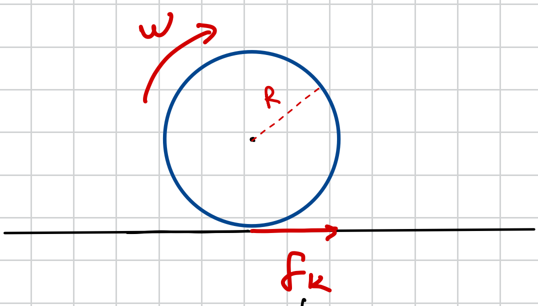
* $a_{cm} \neq \alpha . R$

① Bowling ball without initial rotation



f_k → increase angular velocity / decrease V_{cm}

② Bowling ball with inital rotation

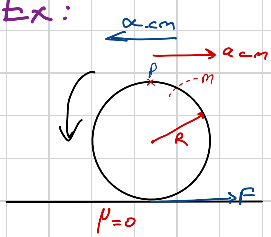


$$\omega > \frac{V_{cm}}{R}$$

f_k → decrease angular velocity / increase V_{cm}

AAA Final

Ex:



Disk is being pulled by a constant F
The disk slides on ice without friction.

Find a_p in terms of m and F

$I_d = \frac{1}{2} M R^2$

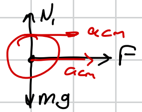
Rolling with sliding

$V_{cm} \neq \omega R$

$a_{cm} \neq \alpha \cdot R$

Translational Part

$\sum F_{net} = M \cdot a_{cm}$



$F = m \cdot a_{cm}$

$\boxed{\frac{F}{m} = a_{cm}}$

Rotational Part

$\sum \tau = I \cdot \alpha_{cm}$

$F \cdot R = \frac{1}{2} M R^2 \cdot \alpha_{cm}$

$\boxed{\alpha_{cm} = \frac{2F}{M \cdot R}}$

$a_P = -\alpha_{cm} R + a_{cm}$

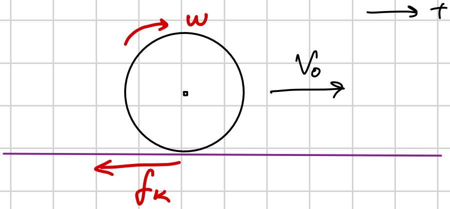
$\frac{F}{m} - \frac{2F}{mR} \cdot R = \frac{-F}{m}$

Made with Goodnotes

Ex:

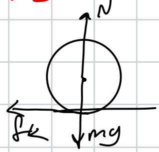
A rolling ball slips. (There is a friction between object and horizontal surface)

A bowling ball of mass m and radius R is thrown along a flat surface so that it initially slides with a linear speed V_0 but does not rotate. As it slides it begins to spin and eventually rolls without slipping. How long does it take to begin rolling without slipping. ($I_{sphere} = \frac{2}{5} mR^2$)



$\rightarrow t$

① FBD



$$-f_k = m \cdot a_{cm}$$

$$-\mu \cdot mg = m \cdot a_{cm}$$

$$\boxed{-\mu g = a_{cm}}$$

$$\boxed{V_{cm} = V_0 - \mu_k g t}$$

②

$$\sum \tau = I_{cm} \alpha_{cm}$$

$$f_k \cdot R = \frac{2}{5} MR^2 \cdot \alpha_{cm}$$

$$\mu_k M g R = \frac{2}{5} mR^2 \alpha_{cm}$$

$$\alpha_{cm} = \frac{5 \cdot \mu_k \cdot g}{2R}$$

③

$[V_{cm} = w R]$ for

$$w = \cancel{w_0} + \alpha_{cm} t$$

$$w = \alpha_{cm} t$$

$$W = \frac{5 \mu_k \cdot g}{2R} \cdot t$$

$$V_0 - \mu_k \cdot g \cdot t = \left(\frac{5}{2} \mu_k g t\right)$$

$$V_0 = \frac{7 \mu_k g t}{2}$$

$$t = \frac{2 \cdot v_0}{7 \cdot \mu_k \cdot g}$$

Problem

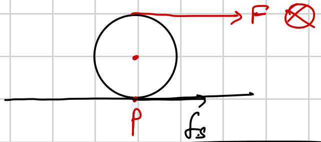
A string is wound around a solid cylinder of Radius $\boxed{R = 0.1\ m}$ mass of $\boxed{m = 16kg}$. The cylinder is then unwound under a constant force $\boxed{F = 60\ N}$. Assume that the cylinder starts from rest and rolls without slipping on the horizontal surface. $\left[I_{cylinder}\ \frac{1}{2} MR^2 \right]$

✓ a) Calculate the acceleration of center of mass of the cylinder

✓ b) Calculate the force of static friction f_s which is necessary to prevent sliding.

c) Calculate angular speed of cylinder after it rolled through an angle of 25 radians.

d) Calculate work done on the cylinder during the turn to 25 radians.



① Translational motion

$$\Sigma F_{net} = m \cdot a$$

$$\boxed{F + f_s = m \cdot a_{cm}} \quad ①$$

② Rotational Motion

$$\Sigma \tau = I \cdot \alpha_{cm}$$

$$F \cdot R - f_s \cdot R = I \cdot \alpha_{cm}$$

$$(F - f_s) R = \frac{1}{2} MR^2 \cdot \alpha_{cm}$$

$$(F - f_s) = \frac{1}{2} MR \cdot \alpha_{cm}$$

③

$$V_{cm} = w \cdot R$$

$$\underset{0.1}{\alpha_{cm}} \cdot R = \underset{5}{a_{cm}}$$

$$F + f_s = m \cdot a_{cm}$$

$$+ \quad F - f_s = \frac{1}{2} m \cdot a_{cm}$$

$$a_{cm} = \frac{4F}{3m}$$

$$a_{cm} = \frac{4 \cdot 60}{16 \cdot 3} \Rightarrow \boxed{\boxed{5}\ m/s^2}$$

A

④

$$60 + f_s = 80$$

$$\boxed{f_s = 20\ N}\ B$$

⑤ $\boxed{\alpha_{cm}}$

$$w^2 = \cancel{w_0^2}^{\,0} + 2\alpha_{cm}\theta$$

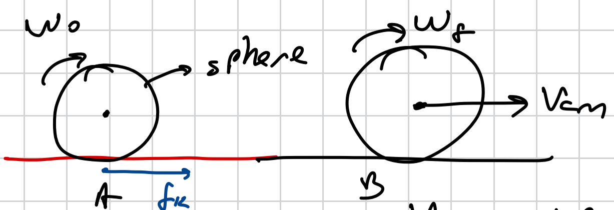
$$w^2 = 2 \cdot 50 \cdot 25$$

$$\boxed{w = 50\ rad/s}$$

⑥ $\Delta K = \text{Work done}$

$$\frac{1}{2} I w^2 + \frac{1}{2} m v^2$$

A uniform metal sphere of mass $M = 0.5\,kg$ is spinning at $700\ rad/s$ about horizontal axis through its center when it is placed on a horizontal surface with its center of mass stationary. Assume that sphere always maintains contact with the surface as it slides and its translational speed increases. The coefficient of kinetic friction between sphere - surface $\mu_k = 0.2$. What is the distance that sphere has travelled at the time it stops. Sliding and pure roling occurs. $I = \frac{2}{5} MR^2$



$A - B = d$

$V_{cm} = w \cdot R$

$w > \frac{V_{cm}}{R}$

Translational Motion

$\underline{A - B}$

a_{cm}

$\underbrace{\sum F_{net}}= m.\ a_{cm}$

$f_k = m.\ a_{cm}$

$\cancel{m} g\ \mu_k = \cancel{m}.\ a_{cm}$

$(10.\ 0.2) = a_{cm}$

$2 = a_{cm}$

$V_{cm} = \cancel{V_{cm0}} + a_{cm} t$

0 at A

$V_{cm} = (a_{cm}) t = \boxed{2t}$

Rotational Motion

$\sum \tau = I \alpha$

$f_k . R = \frac{2}{5} m R^2 \alpha$

$\cancel{m} . g . \mu_k . \cancel{R} = \frac{2}{5} \cancel{m} R^{\cancel{2}} \alpha$

$\frac{-5 g \mu_k}{2\ R} = \alpha_{cm}$

$\alpha_{cm} = -250\ Rad/s^2$

$W(t) = W_0 - \alpha t$

$w(t) = 700 - 250t$

$700 - 250t = 100t \qquad 350t = 700$

$\boxed{t = 2\ secs}$

$r(t) = \cancel{r_0} + \cancel{V_0 t} + \frac{1}{2} a t^2$

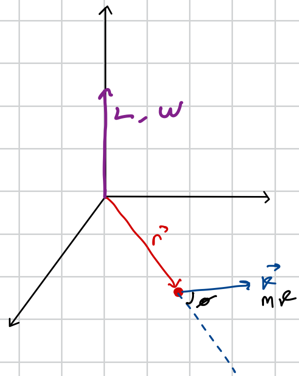
$r(2) = \frac{1}{\cancel{2}} . \cancel{2} . 4$

$r(2) = 4\ meters$

Angular Momentum $\vec{L}$

$$L = \vec{r} \times \vec{p} = \vec{r} \times m \cdot \vec{v}$$

$$|L| = r \cdot p \cdot \sin\phi$$



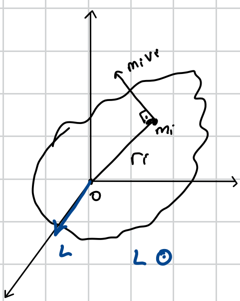
$$\frac{dL}{dt} = \frac{d}{dt}(\vec{r} \times m \cdot \vec{v})$$

$$= \frac{dr}{dt} \times mv + r \times \frac{dv}{dt} \cdot m$$

$$= \underbrace{\cancel{\vec{v} \times m\vec{v}}}_{0} + r \times \underbrace{m\vec{a}}_{F}$$

$$\frac{dL}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

Angular motion of Rigid Body



$$\vec{L} = L \cdot \hat{k}$$

$$L_i = \vec{r_i} \times m_i \vec{V_i}$$

$$L_i = r_i \cdot m_i \cdot V_i \cdot \sin 90$$

$$L_i = m_i \cdot v_i \cdot V_i$$

$$L_i = \underbrace{m_i \cdot r_i^2}_{I} \, w.$$

$$L_i = I_i \cdot W$$

$$L = \sum L_i$$

$$\sum \tau = I \cdot \alpha$$

$$\frac{dL}{\cancel{dt}} = I \cdot \frac{dw}{\cancel{dt}}$$

$$\int dL = I \int dw$$

Made with Goodnotes

Conservation of Momentum

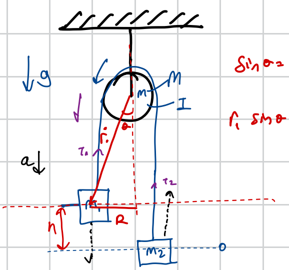
$F_{ext} \rightarrow \frac{dP}{dt} = 0 \quad \Delta P = 0$

$\sum \tau = 0 \quad \frac{dL}{dt} = 0 \quad \Delta L = 0 \quad | \quad L_f - L_i = 0 \quad | \quad I_f \cdot W_f = I_i \cdot W_i$

$m \cdot r_f^2 \cdot w_f = m \cdot r_i^2 \cdot w_i$

$W_f = \frac{W_i \cdot r_i^2}{r_f^2}$

Revisiting Atwood Machine



$\sin\theta = \frac{R}{r_i}$

$r_i \sin\theta = R$

Dynamics

$\sum F_{net} = ma$

$\alpha \cdot r = a$

$\alpha = \frac{a}{r}$

$m_1 g - T_1 = m_1 a$

$T_2 - m_2 g = m_2 a$

$+ \quad (T_1 - T_2) \cdot = I \cdot \frac{\alpha}{r}$

$\sum \tau = I \cdot \alpha$

$(m_1 - m_2) g = I \frac{a}{r^2} + (m_1 + m_2) a$

$(m_1 - m_2) g = a \left(\frac{I}{r^2} + m_1 + m_2 \right)$

$\frac{(m_1 - m_2) g}{\frac{I}{r^2} + m_1 + m_2} = a$

Energy

$\Delta E = 0$

$U_{gi} + K_i = U_{gf} + U_{gi}$

$w \cdot r = v$

$w = \frac{v}{r}$

$m_1 g h = m_2 g h + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I w^2$

$(m_1 - m_2) g h = \frac{1}{2} v^2 \left(m_1 + m_2 + \frac{I}{R^2}\right)$

$(m_1 g - m_2 g) \underbrace{\frac{dh}{dt}}_{v} = \frac{1}{2} 2v \underbrace{\frac{dv}{dt}}_{a} \left(m_1 + m_2 + \frac{I}{R^2}\right)$

$a = \frac{(m_1 - m_2) g}{m_1 + m_2 + \frac{I}{R^2}}$

Angular momentum method

$(L = m \cdot r \cdot v \sin\alpha)$

$\frac{dL}{dt} = \sum \tau$

$L_{total} = L_{m_1} + L_{m_2} + L_{pulley}$

$w \cdot r = v$

$w = \frac{v}{R}$

$L_{total} = m_1 \cdot v \cdot R + m_2 v \cdot R + I \cdot \frac{v}{R}$

$(m_1 - m_2) g R =$

$\frac{dL_{total}}{dt} = \underbrace{\frac{dv}{dt}}_{a} \left(m_1 R + R_2 r + \frac{I}{R} \right)$

Question

Sınavda çıkabilir

A wad of sticky clay of mass m velocity $\vec{V_i}$ is fired at a solid cylinder of mass M and the radius is R. The cylinder is initially at the rest and its mounted on a fixed horizontal axel that runs through its center of mass. The line of motion of the projectile is perpendicular to the axel and at a distance $d<R$ from the center

a) Find angular speed that just after clay strikes $\boxed{I_{cm} = \frac{1}{2}MR^2}$

b) Is mechanical energy of system conserved in this process

$$L_i = L_f$$

$$m.d.V_i = I_f . w_f$$

$$m.d.V_i = \left(\frac{1}{2}MR^2 + mR^2\right) w_f$$

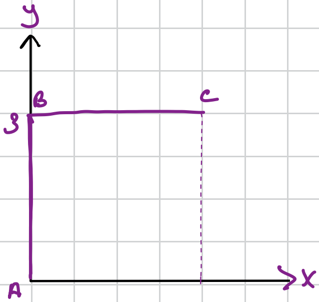
$$w_f = \frac{m.d.V_i}{\left(\frac{1}{2}MR^2 + mR^2\right)}$$

w_f

$$\Delta E \stackrel{?}{=} 0$$

$$K_i + \cancel{U_{gi}} = K_f + \cancel{U_{gf}}$$

$$\frac{1}{2} m V_i^2 \neq \frac{1}{2} I_f W_f^2$$



A potential is defined by $U = xy^2 \, J$ x, y meters

a) Find F

b) Find work done by F along the path ABC

$c = U_c - U_A = -W$ for the path

a) $F = -\nabla U = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} = -y^2\hat{\imath} - 2xy\,\hat{\jmath}$

$F = (-y^2\hat{\imath} - 2xy)\hat{\jmath}$

b) $\int(-y^2\hat{\imath} - 2xy\hat{\jmath})\,(dx\,\hat{\imath} + dy\,\hat{\jmath}) \Rightarrow \overline{\int -y^2\,dx + \int -2xy\,dy}$

$W_{ABC} = W_{AB} + W_{BC}$

$W_{AB} = \int_0^3 -2xy\,dy = \underbrace{-\overset{0}{x}y^2\Big]_0^3}_{0}$

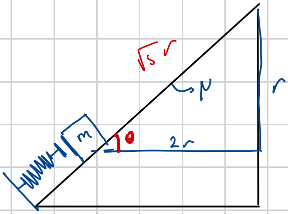
$W_{BC} = \int_0^3 -y^2 dx \qquad -y^2 x\Big]_0^3 = -27$ joules

---

$U_c \Rightarrow C(3,3) = 27\,J \qquad 27 - 0 = --27$

$U_A = A(0,0) = 0\,J \qquad 27 = 27$ ✓

A block of mass m is initially at rest near the bottom of an incline. A massless spring of force constant k initially compressed by d, The mass starts to move up with friction on incline when the spring is released. Find V_f of mass m in terms of m, k, d, r, μ, g when it reaches the top.



$$\Delta E = -W_{friction}$$

$$\Delta E = -mg\,\cancel{\frac{2R}{\sqrt{5}}}\cdot\mu\cdot\sqrt{5}r$$

$$\Delta E = -mg\cdot 2r^2\mu$$

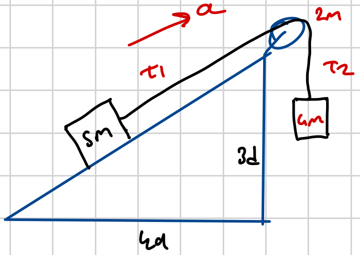
$$(U_{gf} - \cancel{U_{gi}}^{\,0}) + (K_f - \cancel{K_i}^{\,0}) + (\cancel{U_{sf}}^{\,0} - U_{si})$$

$$(mgr) + \frac{1}{2}mv^2 - \frac{1}{2}kd^2 = -2mgr^2\mu$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kd^2 - 2mgr^2\mu - mgr$$

$$mv^2 = \frac{kd^2}{m} - \frac{4mgR^2\mu}{m} - \frac{2mgr}{m}$$

$$v^2 = \frac{kd^2}{m} - 4gr\mu - 2g$$



$I_{disk} = \frac{1}{2} MR^2$

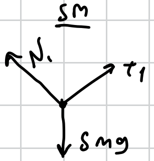
a) Draw FBD, show forces on only

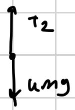
b) T_1, T_2 ? → m, g

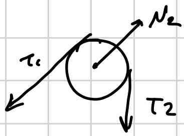
c) calculate a in terms of

$\alpha . R = a$

a)







$\Sigma \tau = I \alpha$

$T_2 R - T_1 R = MR^2.$

$T_2 - T_1 = ma$

b) $T_1 - 5mg . \sin\theta = 5ma$

$4mg - T_2 = 4ma$

$T_2 - T_1 = Ma.$

$4mg - 3mg = 10ma$

$mg = 10ma$

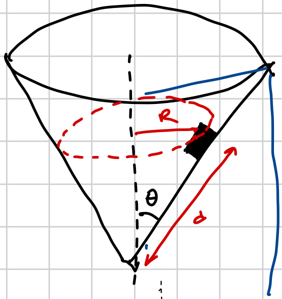
$g = 10a$

$\boxed{\frac{g}{10} = a}$

$T_1 =$

$T_2 = \frac{18mg}{5}$

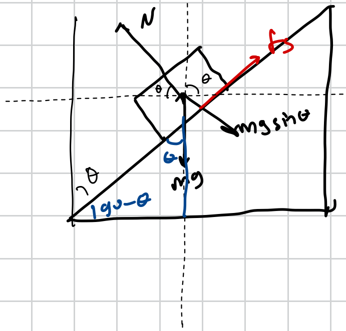
! ÖNEMLİ ! ! FİNAL !



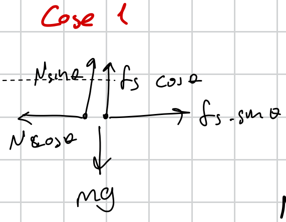
Static friction = μ

find range of ω for which car does not slip

draw fbd)



Case 1



$$N\cos\theta - f_s \cdot \sin\theta = m \cdot w^2 \cdot R$$

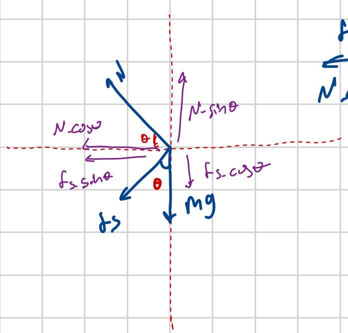
$$N\sin\theta + f_s \cdot \cos\theta = m \cdot g$$

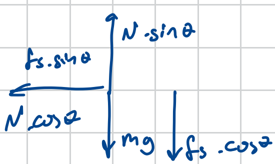
$$\frac{N \cdot \cos\theta - \mu_s \cdot N\sin\theta = m \cdot w^2 \cdot R}{N \cdot \sin\theta + \mu_s \cdot N\cos\theta = mg}$$

$$\sin\theta = \frac{r}{d}$$

$$r = d\sin\theta$$

$$\sqrt{\frac{g}{d\sin\theta}\left(\frac{\cos\theta - \mu_s \cdot \sin\theta}{\sin\theta + \mu_s \cdot \cos\theta}\right)} = w_{MIN}$$





$$W_{2\,MAX} = \sqrt{\frac{g}{d \cdot \sin\theta}\left(\frac{\cos\theta + \mu_s \cdot \sin\theta}{\sin\theta - \mu_s \cdot \cos\theta}\right)}$$