

MODELING THE CAT-FIV INTERACTION

Cautionary note: building a mathematical model is not an end by itself. It is mainly a tool we use try to understand how complex biological processes work, as well as the key parameters that drive the system. A model is based on a biological reality that we need to simplify in order to try to answer biological questions.

1 A rural cat population

The Saint-Just-Chaleyssin cat population is made up of approximately a hundred of individuals. The population is stabilized by cat owners who control litter sizes through kitten euthanasia.

Demographic and behavioral parameters

	Juveniles [0,1[Adults (>1)
Proportion	28%	72%
Annual fecundity	0	7.6
Annual mortality	0.35*	0.35
Sex-ratio	0.5	0.5

** mortality rate of kitten who escape euthanasia*

Adult male cats are highly aggressive (frequent fights) and the strongest ones tend to monopolize fertile female during the oestrus period.

Epidemiologic parameters of FIV

There is no recovery from FIV. Once infected, cats carry and excrete the virus for the rest of their life.

The virus can be found in:

- **Saliva**
- Sperm
- Milk
- ...

Prevalences: Juveniles: 0%
Adult males: 24.4%
Adult females: 10%

On average there are 2.35 new cases of infection by FIV every years, mainly in adult males.

Risk factor: **sex**, age, origin.

Annual mortality of infected cats: 0.45

2 A few additional pieces of information about FIV

In urban cat populations, there are almost no aggressive contacts between individuals outside the reproductive period. All sexually mature males can manage to reproduce. The prevalence of FIV in adult males is only around 10%. In females, compared to rural area, despite the higher frequency of mating prevalence of FIV is not higher (also around 10%).

In rural cat populations, dominant individuals, who are generally older and more aggressive, are the ones who move to find females and hence enter in contact with other males from other populations. They also are the individuals that are the most infected by FIV.

3 Studying the transmission of FIV in the Saint-Just-Chaleyssin population using a modeling approach

Question 0: From what you read, what seems to be the transmission mode of FIV? Do female play an important role in the transmission of the virus? How can that be used to simplify the model?

Question 1: draw the flow diagram of the compartmental model with no age structure representing the spread of FIV in our cat population. We will use a proportionate mixing model for the incidence function.

Question 2: write the system of ordinary differential equations that corresponds to this model. You will use a brake term h , which will represent the euthanasia of kitten, such that the population is stable at $K=100$ individuals.

Question 3: what is the expression of h ?

Question 4: determine R_0 as a function of the model parameters.

Question 5: Under which condition on R_0 can the virus spread in the population?

Question 6: calculate the expression of all the equilibrium points of the model.

Question 7: under which condition is the endemic equilibrium stable? (we will assume that this happens if, and only if, the disease free equilibrium is unstable)

Question 8: Using the pieces of information provided in section 1, provide an estimate of the model parameters (all but the transmission rate)

Question 9: what value of the transmission rate gives us the observed value of the disease prevalence? (adults and juveniles confounded since the model has no age structure)

Question 10: using all the model parameters, estimate the predicted number of new cases of infection per year (ie, the incidence)