Studying the persistence of FECV in catteries

The Feline Enteric Corona Virus (FECV) is a virus of domestic cats, which produces a relatively benign infection leading to strong diarrhea. The virus is transmitted through the direct contact with an infected cat or through infected faces. FECV prevalence is usually high and depends on the size and density of cats in the cattery. Once infected, cats are infectious during a couple of weeks to months. Recovery does not lead to immunization, so recovered cats can rapidly get reinfected. Even though it is possible to measure anti-FECV antibody titers from cat blood samples, this method is not reliable to diagnose recent infections because antibody titers remain high over several months. For cats that have already been infected by the virus, researching virus RNA in cat faces using a PCR is hence much more reliable.

The objective of this practical exercise is to study the transmission of FECV in catteries. The model will be parameterized using the experimental results of Foley et al. («The persistence of SIS disease in a metapopulation » (1999) *Journal of Applied Ecology*, 36 p 555-563).

Table 1 : number of FECV infected catteries according to the cattey size.

nb of cats in the cattery	1	2	3	4	5
nb of observations	12	2	6	6	4
nb of infected catteries	1	0	3	4	4

In order to estimate the transmission rate of FECV, the authors put in contact susceptible cats with **one** infected cat. Each day, cat faces are analyzed and newly infected cats are removed from the cattery (so that there is always one and only one infected cat in the cattery). Results are summarized in table 2 (prop stands for the cumulated proportion of infected cats since the beginning of the experiment) and Figure 1.

day	prop	day	prop	day	prop	day	prop
0	1	20	0.4	40	0.18	60	0.1
1	1	21	0.35	41	0.18	61	0.1
2	0.92	22	0.35	42	0.18	62	0.1
3	0.92	23	0.35	43	0.18	63	0.1
4	0.76	24	0.35	44	0.18	64	0.05
5	0.76	25	0.29	45	0.18	65	0.05
6	0.76	26	0.23	46	0.18	66	0
7	0.76	27	0.23	47	0.18		
8	0.76	28	0.23	48	0.18		
9	0.76	29	0.23	49	0.18		
10	0.76	30	0.23	50	0.18		
11	0.72	31	0.23	51	0.18		
12	0.47	32	0.23	52	0.18		
13	0.47	33	0.23	53	0.18		
14	0.47	34	0.23	54	0.18		
15	0.47	35	0.23	55	0.18		
16	0.47	36	0.23	56	0.18		
17	0.4	37	0.18	57	0.1		
18	0.4	38	0.18	58	0.1		
19	0.4	39	0.18	59	0.1		

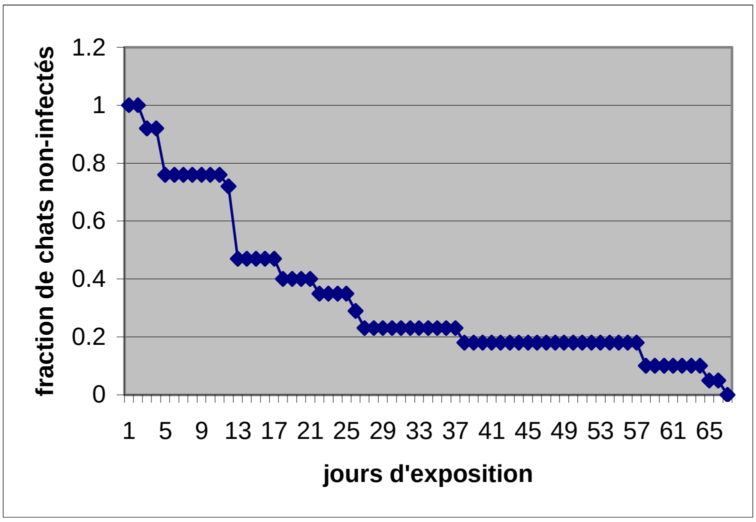

Figure 1 : proportion of cats that were exposed to one infected cat without being infected in time.

Last, by keeping infected cats isolated (so that they can't get reinfected), the authors showed that the mean duration of the infection is 37.2 days.

Question 1 :

Using data presented above, estimate the recovery and transmission rates of FECV.

Question 2 :

Build a mathematical model for the transmission of FECV in cattery. What hypothesizes have been made? (demographic processes will be neglected)

Question 3 :

Model study: calculate the R_0 and the equilibrium prevalence of FECV. Compare to observations of Table 1. Conclude. Draw some curves representing the number of infected individuals in time.

Question 4 :

What is the stochastic process that corresponds to the model developed in question 2. Run a couple of simulations of the process. What do yo conclude?

Question 5 :

If we repeat a large number of times the simulation of the process, we can estimate the time before the extinction of the virus from the cattery. Write a program doing this calculation and compare it to the following theoretical formula:

$$\text{Text} = \sum_{i=1}^{N} \frac{1}{\gamma} \left[\frac{\beta}{\gamma}\right]^{i-1} \frac{(N-1)!}{i(N-i)!}$$

How do you explain the persistence of the virus in some catteries?

Question 6 :

We now study a metapopulation made up of a large number of catteries (with N cats per cattery). We denote *e* the rate at which the virus goes extinct from one cattery and *m* the reintroduction rate of the virus within virus free catteries.

a) What differential equation gives the proportion *p* of infected catteries (according to *e* and *m*).

b) What is the value p^* of *p* at the equilibrium.

Question 7 :

We give *m* for different cattery sizes. Complete the table. Conclude.

nb of cats per cattery (N)	1	2	3	4	5
nb of observations	12	2	6	6	4
nb of infected catteries	1	0	3	4	4
$m(N)$ (year^{-1})	0.606	1.212	1.818	2.424	3.03
$e(N)$ (year^{-1})					
p^*					
Expected nb of infected catteries					

Question 8 :

Inspiring from the stochastic process of question 4, simulate a model representing a metapopulation made up of n connected catteries of size N with, inside each cattery, a SIS type stochastic dynamic. Remark: the state of the population will then be characterized by the S_k and I_k (k=1…N), which represents the number of susceptible and infected cats within cattery k, respectively.