Math 302 Theorem 2.7

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Proof. Suppose $gcd(a, b) = 1$ and $gcd(b, c) = 1$.

Case 1. $b = 0$.

Because $gcd(b, c) = 1$ and $b = 0$, c must equal 1. Thus $gcd(ab, c) = gcd(0, c) = gcd(0, 1) = 1$.

Case 2. $b \neq 0$.

Since $gcd(a, c) = 1$, then by Theorem 2.3, we can write $ax_1 + cy_1 = 1$ for some integers x_1 and y_1. So, $abx_1 + cy_1b = b$. Since $gcd(b, c) = 1$, we can also write $bx_2 + cy_2 = 1$ for some integers x_2 and y_2, so $abx_2 + cy_2a = a$.

We then have $(abx_2 + cy_2a)bx_1 + cy_1b = b$. So, $(abx_2 + cy_2a)x_1 + cy_1 = 1$. Then, $ab(x_1x_2) + c(y_2ax_1 + y_1) = 1$.

Because $abk_1 + ck_2 = 1$ for some integers k_1 and k_2, $gcd(ab, c) = 1$ by Corollary 2.4.

Therefore $gcd(ab, c) = 1$. $\square$