Assignment 3
Introduction to Robotics
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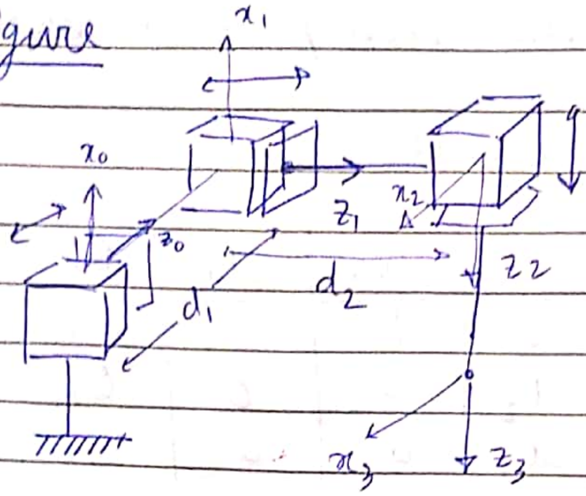
Question 1

Workspace of a manipulator is a the collection of all the points the end effector of the manipulator can reach . Singularities are those points in the workspace where the end-effector cannot move in certain directions. The configurations of the robot when it attains singularity is called a Singular configuration. The DOF of the freedom gets reduced in these configurations and cannot move freely and cause the system to fail because of the high constraint and instability. One must keep a close eye on the singularity conditions while trajectory planning. Mathematically, the inverse of Jacobian matrix ceases to exist and the matrix loses it's rank.

Yes a configuration can be checked to be near to singularity configuration by checking the rank of the Manipulator Jacobian in some configuration, it is possible to detect whether that configuration is close to singularity or not.

Q5) Figure



Since there are no fixed links, we take values of a_i's as 0. All the joints are prismatic joints.

DH parameters

** link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	$90°$	d_2	$90°$
3	0	0	d_3	$-90°$

Thus,

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$T_0^3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q6) For the manipulator mentioned,

DH params:

link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	-90°	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

$$\therefore A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

Direct Drive

Direct Drive was first developed by Carnegie Mellon University in the year 1981.
Direct driven robots are basically manipulators in which every joint is equipped with motors and encoders i.e. the motors moves with the movement of the links and are not grounded. These equations governing the manipulator have to take into account coriolis forces. The following are the equations governing the manipulator:

$$\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + g_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + g_2 &= \tau_2 \end{aligned} \qquad (6.90)$$

Direct-drive robot, the problems of backlash, friction, and compliance due to the gears are eliminated.

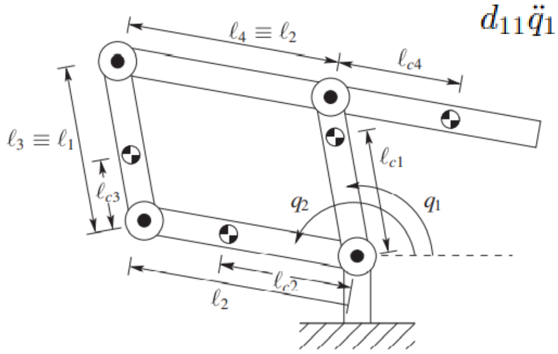
Remotely Driven

$$\begin{aligned} d_{11}\ddot{p}_1 + d_{12}\ddot{p}_2 + c_{221}\dot{p}_2^2 + g_1 &= \tau_1 \\ d_{21}\ddot{p}_1 + d_{22}\ddot{p}_2 + c_{112}\dot{p}_1^2 + g_2 &= \tau_2 \end{aligned} \qquad (6.99)$$

This is similar to Direct driven manipulator. The major exception being the fact that the motors are now grounded. This means that the equations dont have to take into account the coriolis forces. The equation become simplified. It is also to be noted that because the links are driven by motors which are on the ground, this means that the movement of one link does not change the orientation of other. Making the manipulator is a complex job because it requires highly precise timing belts and one must ensure that torques must be in viable ranges to prevent the slipping of the timing belts over the pulleys.

Five-Bar Linkage

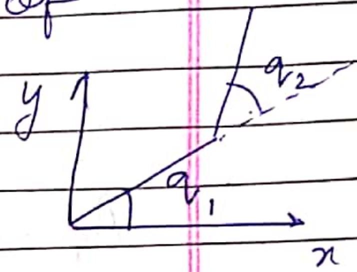
Five-bar linkage configuration helps in solving the problems of direct driven and remotely driven manipulators.



$$d_{11}\ddot{q}_1 + g_1(q_1) = \tau_1, \qquad d_{22}\ddot{q}_2 + g_2(q_2) = \tau_2 \qquad (6.111)$$

q1 and q2 are decoupled as seen in the equation and can be controlled separately using the two equation. The setup is very complex & difficult to execute. It is easy to control the dynamics and controls of the five-bar linkage i.e. simulation is easy but the manufacture and design is tough. More joints leads to more friction and more precision for design considerations, assembly and tolerances.

Q8) Using DH parameters and Euler Lagrangian to find EOM of 2R manipulator



For link 1,

$$J_{v_1} = \begin{bmatrix} -l_1/_2 \sin q_1 & 0 \\ l_1/_2 \cos q_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$V_{c_1} = J_{v_1} \dot{q}$$

$$V_{c_2} = J_{v_2} \dot{q}$$

Jacobians

$$J_{v_2} = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \\ 0 & 0 \end{bmatrix}$$

$$K_{trans} = \frac{1}{2} m_1 V_{c_1}^T V_{c_1} + \frac{1}{2} m_2 V_{c_2}^T V_{c_2}$$

$$= \frac{1}{2} \dot{q} \left[m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} \right] \dot{q}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = (\dot{q}_1 + \dot{q}_2) \hat{k}$$

↳ z direction

$v_{c_1} \rightarrow$ velocity of COM of link 1
$l_{c_1} \rightarrow$ length till COM of link 1

$$K_{rot} = \frac{1}{2}\dot{q}^T \left[I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right] \dot{q}$$

moment of inertia of the links.

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$d_{11} = m_1 l_{c_1}^2 + m_2 (l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2} \cos q_2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2 (l_{c_2}^2 + l_1 l_{c_2} \cos q_2) + I_2$$

$$d_{22} = m_2 l_{c_2}^2 + I_2$$

Now we calculate the Christoffel symbols:

$$C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \partial d_{11} / \partial q_2 = -m_2 l_1 l_{c_2} \sin q_2 = h.$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \cdot \frac{\partial d_{22}}{\partial q_1} = h.$$

Scanned with CamScanner

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$$

$$c_{122} = c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential Energies.

$$P_1 = m_1 g l_{c_1} \sin q_1$$

$$P_2 = m_2 g (l_1 \sin q_1 + l_{c_2} \sin(q_1 + q_2))$$

$$P = P_1 + P_2$$

$$g_1 = \frac{\partial P}{\partial q_1} = (m_1 l_{c_1} + m_2 l_1) g \cos q_1 + m_2 l_{c_2} g \cos(q_1 + q_2)$$

$$g_2 = \frac{\partial P}{\partial q_2} = m_2 l_{c_2} g \cos(q_1 + q_2)$$

Euler Lagrangian ↓

$$\sum_{j=1}^{n} d_{kj}(q)\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k$$

Thus,

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + g_1 = \tau_1 \quad (1)$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + g_2 = \tau_2 \quad (2)$$

Q10) Given $D(q)$ and $V(q)$,

Kinetic Energy K

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

$$= \frac{1}{2}\sum_{i,j} d_{ij}(q)\dot{q}_i\dot{q}_j$$ } matrix multiplication

$$L = K - P = \frac{1}{2}\sum_{i,j} d_{ij}(q)\dot{q}_i\dot{q}_j - P(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}\dot{q}_j$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{ij}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial P}{\partial q_k}$$

k^{th} joint

$\therefore$ For each of link $k \in (1, \ldots n)$

$$\sum_j d_{kj}\ddot{q}_j + \sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial P}{\partial q_k} = \tau_k$$

Rearranging and ~~sym~~ simplifying we write in terms of christofel's symbols.

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

Thus EL J

$$\sum_{j=1}^{n} d_{kj}(q)\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}(q)\dot{q}_i\dot{q}_j + g_k(q) = \tau_k$$