

Question 1

$$1) \quad m(x) = \frac{1}{n} \sum_{i=1}^n x_i$$

$$m(a + b x) = \frac{1}{n} \sum_{i=1}^n (a + b x_i)$$

$$m(a + b x) = \frac{1}{n} \sum_{i=1}^n a + \frac{1}{n} \sum_{i=1}^n b x_i$$

$$= a + b \cdot \left(\frac{1}{n} \sum_{i=1}^n x_i \right)$$

$$= a + b \cdot m(x)$$

$$2) \quad \text{cov}(x, y) = \frac{1}{n} \sum_{i=1}^n (x_i - m(x)) \cdot (y_i - m(y))$$

$$\text{cov}(x, a + b y) = \frac{1}{n} \sum_{i=1}^n (x_i - m(x)) \cdot ((a + b y_i) - m(a + b y))$$

$$= \frac{1}{n} \sum_{i=1}^n (x_i - m(x)) \cdot (a + b y_i - a - b m(y))$$

$$= \frac{1}{n} \sum_{i=1}^n (x_i - m(x)) \cdot (b y_i - b m(y))$$

$$= b x \left(\frac{1}{n} \sum_{i=1}^n (x_i - m(x)) \cdot (y_i - m(y)) \right)$$

$$= b x \text{cov}(x, y)$$

$$3) \quad \text{cov}(a + b x, a + b x) = b x \text{cov}(x, a + b x)$$

$$= b x b x \text{cov}(x, x)$$

$$= b^2 \text{cov}(x, x)$$

$$= b^2 s^2$$