Instagram Questions

roshawn wright

June 2024

Questions

1. $$\int \frac{2}{\sqrt{x} - \sqrt[3]{x}}\, dx$$

2. $$\lim_{x \to \infty} \sqrt{x}(\sqrt{x+8} - \sqrt{x-4})$$

3. $$\int \frac{1 - sin^2(x)}{3\ cos^2(x)}\, dx$$

4. $$\int_{\int_0^3 x^2\ dx}^{\int_0^2 x^3\ dx} 2x\, dx$$

Question 1

We must first handle the denominator we have $\sqrt{x}$ and $\sqrt[3]{x}$. We can turn the first $\sqrt{x}$ into $x^{\frac{1}{2}}$ and we can also turn $\sqrt[3]{x}$ into $x^{\frac{1}{3}}$. by doing so we now end up with the integral

$$\int \frac{2}{\sqrt[6]{x}}\, dx$$

.

Step by Step solution

Substitute $u = \sqrt[6]{x}$ (i.e., $u^6 = x$):

$$dx = 6u^5\, du$$

Also, $\sqrt{x} = u^3$ and $\sqrt[3]{x} = u^2$.
Rewrite the integrand in terms of u:

$$\int \frac{2}{\sqrt{x} - \sqrt[3]{x}}\, dx = \int \frac{2}{u^3 - u^2} \cdot 6u^5\, du$$

Simplify the integrand:

$$\int \frac{2 \cdot 6u^5}{u^3 - u^2} \, du = \int \frac{12u^5}{u^3 - u^2} \, du = \int \frac{12u^5}{u^2(u-1)} \, du = \int \frac{12u^3}{u-1} \, du$$

Simplify further by polynomial division:

$$\frac{12u^3}{u-1} = 12u^2 + 12u + 12 + \frac{12}{u-1}$$

So, the integral becomes:

$$\int (12u^2 + 12u + 12 + \frac{12}{u-1}) \, du$$

Integrate term by term:

$$\int 12u^2 \, du + \int 12u \, du + \int 12 \, du + \int \frac{12}{u-1} \, du$$

Let's solve each integral separately:

$$\int 12u^2 \, du = 12 \cdot \frac{u^3}{3} = 4u^3$$

$$\int 12u \, du = 12 \cdot \frac{u^2}{2} = 6u^2$$

$$\int 12 \, du = 12u$$

$$\int \frac{12}{u-1} \, du = 12 \ln |u - 1|$$

Combining these results:

$$4u^3 + 6u^2 + 12u + 12 \ln |u - 1| + C$$

Substitute $u = \sqrt[6]{x}$ back into the solution:

$$4(\sqrt[6]{x})^3 + 6(\sqrt[6]{x})^2 + 12\sqrt[6]{x} + 12 \ln |\sqrt[6]{x} - 1| + C$$

Simplify the exponents:

$$4\sqrt{x} + 6\sqrt[3]{x} + 12\sqrt[6]{x} + 12 \ln |\sqrt[6]{x} - 1| + C$$

Final Answer:

$$\int \frac{2}{\sqrt{x} - \sqrt[3]{x}} \, dx = 4\sqrt{x} + 6\sqrt[3]{x} + 12\sqrt[6]{x} + 12 \ln |\sqrt[6]{x} - 1| + C$$

Question 2

Rewrite the expression inside the limit:

$$\sqrt{x}\left(\sqrt{x+8}-\sqrt{x-4}\right)$$

Rationalize the expression:
Multiply and divide by the conjugate of $\sqrt{x+8}-\sqrt{x-4}$, which is $\sqrt{x+8}+\sqrt{x-4}$:

$$\sqrt{x}\left(\sqrt{x+8}-\sqrt{x-4}\right)\cdot\frac{\sqrt{x+8}+\sqrt{x-4}}{\sqrt{x+8}+\sqrt{x-4}}$$

Simplify the numerator:

$$\sqrt{x}\cdot\frac{\left(\sqrt{x+8}-\sqrt{x-4}\right)\left(\sqrt{x+8}+\sqrt{x-4}\right)}{\sqrt{x+8}+\sqrt{x-4}}$$

The numerator simplifies as follows:

$$\left(\sqrt{x+8}\right)^2-\left(\sqrt{x-4}\right)^2=(x+8)-(x-4)=x+8-x+4=12$$

So, the expression becomes:

$$\sqrt{x}\cdot\frac{12}{\sqrt{x+8}+\sqrt{x-4}}$$

Simplify further:

$$12\cdot\frac{\sqrt{x}}{\sqrt{x+8}+\sqrt{x-4}}$$

Divide the terms in the denominator by $\sqrt{x}$:

$$12\cdot\frac{\sqrt{x}}{\sqrt{x}\sqrt{1+\frac{8}{x}}+\sqrt{x}\sqrt{1-\frac{4}{x}}}$$

$$=12\cdot\frac{1}{\sqrt{1+\frac{8}{x}}+\sqrt{1-\frac{4}{x}}}$$

Evaluate the limit as $x\to\infty$:
As $x\to\infty$, $\frac{8}{x}\to 0$ and $\frac{4}{x}\to 0$. Therefore:

$$\sqrt{1+\frac{8}{x}}\to\sqrt{1+0}=1$$

$$\sqrt{1-\frac{4}{x}}\to\sqrt{1-0}=1$$

So, the expression simplifies to:

$$12\cdot\frac{1}{1+1}=12\cdot\frac{1}{2}=6$$

Final Answer

$$\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x+8}-\sqrt{x-4}\right)=6$$

Question 3

Knowing your Trig Identities is key in answering questions involving anything trig related.

In question 3 we can tell that the numerator is $cos^2(x)$. How? by knowing the simple fact below

To express $\cos^2\theta$ in terms of $\sin^2\theta$ we do some algebra:

1. Start with the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

2. Subtract $\sin^2\theta$ from both sides:

$$\cos^2\theta = 1 - \sin^2\theta$$

So the integral becomes:

$$\int \frac{\cos^2(x)}{3\cos^2(x)}\, dx$$

This simplifies to:

$$\int \frac{1}{3}\, dx$$

Now the integral is straightforward:

$$\int \frac{1}{3}\, dx = \frac{1}{3}\int dx = \frac{x}{3} + C$$

where C is the constant of integration.

Thus, the solution to the integral is:

$$\frac{x}{3} + C$$

Question 4

Evaluate the First Integral:

$$\int_0^3 x^2\, dx$$

The integral of x^2 is $\frac{x^3}{3}$. So, we have:

$$\left.\frac{x^3}{3}\right|_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = \frac{27}{3} - 0 = 9$$

Evaluate the Second Integral:

$$\int_0^2 x^3\, dx$$

The integral of x^3 is $\frac{x^4}{4}$. So, we have:

$$\left.\frac{x^4}{4}\right|_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} - 0 = 4$$

Use the Results as Limits for the Integral of $2x$: Now, we use the results of the first two integrals (9 and 4) as the lower and upper limits for the integral of $2x$:

$$\int_4^9 2x\, dx$$

The integral of $2x$ is x^2. So, we have:

$$x^2\big|_4^9 = 9^2 - 4^2 = 81 - 16 = 65$$

Therefore, the final result of the given expression is 65.