clep questions

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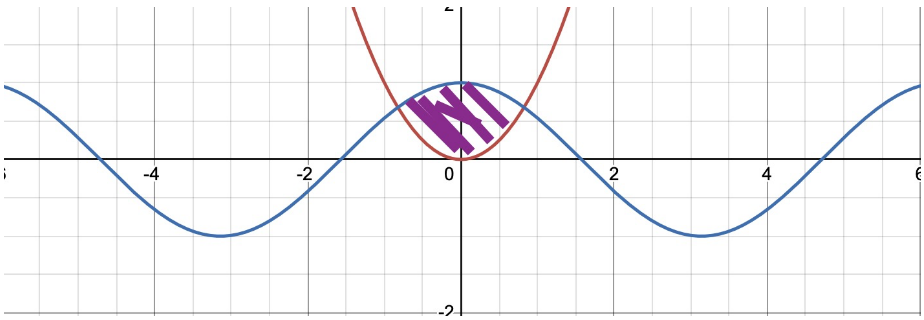
1 Introduction

Questions which we were trivial but i was just blind. some genius huhhhh.

2 Questions Listed

1. solve for the area of the shaded region between the curves $y = x^2$ and $y = cos(x)$
2. find the third derivative of the function $f(x) = x^2 + cos(2x)$
3. Integrate $(2x - 3)^4$
4. Integrate $5cos(1 - 5x)$
5. $lim_{x\to\infty} \frac{ln(2x)}{x}$
6. $lim_{x\to1} \frac{e^x - e}{x-1}$
7. $lim_{x\to0}(e^x + 2sin(x) + 4cos(2x) + x^2)$
8. $$\frac{d}{dx}\int_1^x (t^3 + 4)\,dt = (x^3 + 4)$$
9. $lim_{x\to2} \;\; x(x - 2)e^x$
10. $$\int \frac{x^2 + 3}{2x}\,dx$$
11. $$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{n}\sin\left(\frac{\pi}{n}k\right)\right)$$
12. $x + y^2 = 5$ differentiate with respect to y.

Question 1.



We need to solve question 1 by viewing the graph above and noticing the points of intersection.

The points of intersection approximately occur at $x = -\pi/2$ and $x = \pi/2$.

1. Set up the integral:

The area A between the curves from $x = -a$ to $x = a$ (in this case, $a = \pi/2$) is given by:

$$A = \int_{-a}^{a} (\cos x - x^2)\, dx$$

Substituting the limits:

$$A = \int_{-\pi/2}^{\pi/2} (\cos x - x^2)\, dx$$

2. Calculate the integral:

This integral can be split into two separate integrals:

$$A = \int_{-\pi/2}^{\pi/2} \cos x\, dx - \int_{-\pi/2}^{\pi/2} x^2\, dx$$

For the first integral:

$$\int_{-\pi/2}^{\pi/2} \cos x\, dx = [\sin x]_{-\pi/2}^{\pi/2} = \sin(\pi/2) - \sin(-\pi/2) = 1 - (-1) = 2$$

For the second integral:

$$\int_{-\pi/2}^{\pi/2} x^2\, dx = 2\int_{0}^{\pi/2} x^2\, dx = 2\left[\frac{x^3}{3}\right]_0^{\pi/2} = 2\left(\frac{(\pi/2)^3}{3} - 0\right) = \frac{\pi^3}{12}$$

3. Combine the results:

$$A = 2 - \frac{\pi^3}{12}$$

Therefore, the area of the shaded region between the curves $y = x^2$ and $y = \cos x$ from $x = -\pi/2$ to $x = \pi/2$ is $2 - \frac{\pi^3}{12}$.

Question 2

First Derivative

$$f(x) = x^2 + \cos(2x)$$

$$f'(x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\cos(2x))$$

$$f'(x) = 2x - 2\sin(2x)$$

Here, we used the chain rule for the second term:

$$\frac{d}{dx}(\cos(2x)) = -\sin(2x) \cdot \frac{d}{dx}(2x) = -2\sin(2x)$$

Second Derivative

$$f''(x) = \frac{d}{dx}(2x - 2\sin(2x))$$

$$f''(x) = 2 - 4\cos(2x)$$

Again, using the chain rule for the second term:

$$\frac{d}{dx}(-2\sin(2x)) = -2 \cdot \cos(2x) \cdot \frac{d}{dx}(2x) = -4\cos(2x)$$

Third Derivative

$$f'''(x) = \frac{d}{dx}(2 - 4\cos(2x))$$

$$f'''(x) = 0 + 8\sin(2x)$$

Using the chain rule for the second term:

$$\frac{d}{dx}(-4\cos(2x)) = -4 \cdot (-\sin(2x)) \cdot \frac{d}{dx}(2x) = 8\sin(2x)$$

So, the third derivative of $f(x) = x^2 + \cos(2x)$ is:

$$f'''(x) = 8\sin(2x)$$

Question 3

To integrate $(2x-3)^4$, we can use substitution. Let $u = 2x - 3$. Then, we have $du = 2dx$, or $dx = \frac{du}{2}$.

Now, we rewrite the integral in terms of u:

$$\int (2x-3)^4 \, dx = \int u^4 \cdot \frac{du}{2} = \frac{1}{2} \int u^4 \, du$$

Next, we integrate u^4:

$$\frac{1}{2} \int u^4 \, du = \frac{1}{2} \cdot \frac{u^5}{5} = \frac{u^5}{10}$$

Finally, substitute $u = 2x - 3$ back into the expression:

$$\frac{u^5}{10} = \frac{(2x-3)^5}{10}$$

So, the integral of $(2x-3)^4$ with respect to x is:

$$\int (2x-3)^4 \, dx = \frac{(2x-3)^5}{10} + C$$

Question 4

To integrate $5\cos(1-5x)$ with respect to x, we can use substitution.

Let $u = 1 - 5x$. Then, we have $du = -5dx$, or $dx = -\frac{du}{5}$.

Now, rewrite the integral in terms of u:

$$\int 5\cos(1-5x) \, dx = 5 \int \cos(u) \left(-\frac{du}{5}\right) = -\int \cos(u) \, du$$

Next, integrate $\cos(u)$:

$$-\int \cos(u) \, du = -\sin(u)$$

Finally, substitute $u = 1 - 5x$ back into the expression:

$$-\sin(1 - 5x)$$

So, the integral of $5\cos(1 - 5x)$ with respect to x is:

$$\int 5\cos(1 - 5x)\,dx = -\sin(1 - 5x) + C$$

Question 5

To find the limit of $\frac{\ln(2x)}{x}$ as x approaches infinity, we can use L'Hôpital's Rule.

First, we confirm that this limit is of the form $\frac{\infty}{\infty}$:

$$\lim_{x\to\infty} \frac{\ln(2x)}{x}$$

As x approaches infinity, both $\ln(2x)$ and x approach infinity. Thus, we can apply L'Hôpital's Rule by taking the derivatives of the numerator and the denominator:

$$\lim_{x\to\infty} \frac{\ln(2x)}{x} = \lim_{x\to\infty} \frac{\frac{d}{dx}[\ln(2x)]}{\frac{d}{dx}[x]}$$

The derivative of $\ln(2x)$ with respect to x is:

$$\frac{d}{dx}[\ln(2x)] = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

The derivative of x with respect to x is:

$$\frac{d}{dx}[x] = 1$$

Now, apply these derivatives:

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

Therefore, the limit is:

$$\lim_{x\to\infty} \frac{\ln(2x)}{x} = 0$$

Question 6

To find the limit of $\frac{e^x - e}{x-1}$ as x approaches 1, we can use L'Hôpital's Rule.

First, confirm that this limit is of the form $\frac{0}{0}$:

$$\lim_{x \to 1} \frac{e^x - e}{x - 1}$$

As x approaches 1, both the numerator $e^x - e$ and the denominator $x - 1$ approach 0.

Since it is a $\frac{0}{0}$ form, we apply L'Hôpital's Rule by taking the derivatives of the numerator and the denominator:

$$\lim_{x \to 1} \frac{e^x - e}{x - 1} = \lim_{x \to 1} \frac{\frac{d}{dx}[e^x - e]}{\frac{d}{dx}[x - 1]}$$

The derivative of $e^x - e$ with respect to x is e^x (since e is a constant, its derivative is 0). The derivative of $x - 1$ with respect to x is 1.

Now, apply these derivatives:

$$\lim_{x \to 1} \frac{e^x}{1} = \lim_{x \to 1} e^x = e$$

Therefore, the limit is:

$$\lim_{x \to 1} \frac{e^x - e}{x - 1} = e$$

Question 7

To find the limit of $e^x + 2\sin(x) + 4\cos(2x) + x^2$ as x approaches 0, we can evaluate each term in the expression individually at $x = 0$:

$$\lim_{x \to 0} e^x = e^0 = 1$$

$$\lim_{x \to 0} 2\sin(x) = 2\sin(0) = 2 \cdot 0 = 0$$

$$\lim_{x \to 0} 4\cos(2x) = 4\cos(2 \cdot 0) = 4\cos(0) = 4 \cdot 1 = 4$$

$$\lim_{x \to 0} x^2 = 0^2 = 0$$

Now, sum these results:

$$\lim_{x \to 0} (e^x + 2\sin(x) + 4\cos(2x) + x^2) = 1 + 0 + 4 + 0 = 5$$

Therefore, the limit is:

$$\lim_{x \to 0} (e^x + 2\sin(x) + 4\cos(2x) + x^2) = 5$$

Question 8

To differentiate the integral

$$F(x) = \int_1^x (t^3 + 4)\, dt$$

with respect to x, we can use the Fundamental Theorem of Calculus, Part 1. The Fundamental Theorem of Calculus states that if

$$F(x) = \int_a^x f(t)\, dt$$

then

$$\frac{d}{dx} F(x) = f(x)$$

Here, $f(t) = t^3 + 4$. Therefore,

$$\frac{d}{dx} \left(\int_1^x (t^3 + 4)\, dt \right) = t^3 + 4 \Big|_{t=x} = x^3 + 4$$

So, the derivative of the given integral with respect to x is:

$$\frac{d}{dx} \left(\int_1^x (t^3 + 4)\, dt \right) = x^3 + 4$$

Question 9

To find the limit of $x(x-2)e^x$ as x approaches 2, we can directly substitute $x = 2$ into the expression:

$$\lim_{x \to 2} x(x-2)e^x$$

Substitute $x = 2$:

$$2(2-2)e^2 = 2 \cdot 0 \cdot e^2 = 0$$

Therefore, the limit is:

$$\lim_{x \to 2} x(x-2)e^x = 0$$

Question 10

To integrate the function $\frac{x^2+3}{2x}$ with respect to x, we can first simplify the integrand:

$$\frac{x^2+3}{2x} = \frac{x^2}{2x} + \frac{3}{2x} = \frac{x}{2} + \frac{3}{2x}$$

Now, we can integrate each term separately:

$$\int \left(\frac{x}{2} + \frac{3}{2x} \right) dx$$

Integrate $\frac{x}{2}$:

$$\int \frac{x}{2}\, dx = \frac{1}{2} \int x\, dx = \frac{1}{2} \cdot \frac{x^2}{2} = \frac{x^2}{4}$$

Integrate $\frac{3}{2x}$:

$$\int \frac{3}{2x}\, dx = \frac{3}{2} \int \frac{1}{x}\, dx = \frac{3}{2} \ln |x|$$

Combining these results, we get:

$$\int \left(\frac{x}{2} + \frac{3}{2x} \right) dx = \frac{x^2}{4} + \frac{3}{2} \ln |x| + C$$

Question 11

To evaluate the limit of the sum

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{n}\sin\left(\frac{\pi}{n}k\right)\right),$$

we can interpret this as a Riemann sum. The sum

$$\sum_{k=1}^{n}\left(\frac{\pi}{n}\sin\left(\frac{\pi}{n}k\right)\right)$$

approximates the integral

$$\int_0^{\pi}\sin(x)\,dx$$

as n approaches infinity.

To see this more clearly, rewrite the sum in a form that resembles a Riemann sum. Notice that

$$\frac{\pi}{n}=\Delta x$$

and

$$\frac{\pi}{n}k=x_k.$$

Thus, the sum can be written as:

$$\sum_{k=1}^{n}\sin(x_k)\Delta x.$$

As $n\to\infty$, this Riemann sum converges to the integral:

$$\int_0^{\pi}\sin(x)\,dx.$$

Now, let's evaluate the integral:

$$\int_0^{\pi}\sin(x)\,dx.$$

The antiderivative of $\sin(x)$ is $-\cos(x)$, so:

$$\int_0^{\pi}\sin(x)\,dx=\left[-\cos(x)\right]_0^{\pi}.$$

Evaluating this, we get:

$$-\cos(\pi)-(-\cos(0))=-(-1)-(-1)=1-(-1)=1+1=2.$$

Therefore, the limit is:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{\pi}{n} \sin\left(\frac{\pi}{n}k\right)\right) = 2.$$

Question 12

To differentiate the equation $x + y^2 = 5$ with respect to y, we'll use implicit differentiation.

Treat x as a function of y (i.e., $x = x(y)$) and differentiate both sides of the equation with respect to y.

Starting with the original equation:

$$x + y^2 = 5$$

Differentiate both sides with respect to y:

$$\frac{d}{dy}(x) + \frac{d}{dy}(y^2) = \frac{d}{dy}(5)$$

Since x is a function of y, we use the chain rule for $\frac{d}{dy}(x)$:

$$\frac{dx}{dy} + 2y = 0$$

This simplifies to:

$$\frac{dx}{dy} = -2y$$

So, the derivative of $x + y^2 = 5$ with respect to y is:

$$\frac{dx}{dy} = -2y$$

One should always learn from their mistakes & I have LEARN!