Taking Laplace Transform: (Lab:Supp: Q4c)
(Test Q4B/Q4A)

$\therefore$

$$X(s) = LC\left[s^2Y(s) - sy(0) - \dot{y}(0)\right] + RC\left[sY(s) - y(0)\right] + Y(s)$$

$$\therefore Y(s)\left[s^2LC + RCs + 1\right] \underbrace{- sy(0)LC - LC\dot{y}(0) - y(0)RC}_{\text{due to initial conditions}} = \underbrace{X(s)}_{\text{due to input.}}$$

$\div$ Left/right by LC.

$$Y(s)\left[s^2 + \frac{R\cancel{C}}{L\cancel{C}}s + \frac{1}{LC}\right] = \frac{X(s)}{LC} + sy(0) - \frac{\cancel{LC}}{\cancel{LC}}\dot{y}(0) - y(0)\frac{R\cancel{C}}{L\cancel{C}}$$

$$\therefore Y(s) = \frac{1}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}\cdot\left[\frac{X(s)}{LC} + sy(0) + \dot{y}(0) + \frac{R}{L}y(0)\right].$$

Sub: $y(0) = 1.5V$.

$\dot{y}(0) \Rightarrow \frac{dy(t)}{d(t)}$; from $i(t) = C\frac{dy(t)}{d(t)}$

at $i(t=0) = 2A$, $\therefore i(0) = 2 = C\frac{dy(t)}{d(t)}$

$$\Rightarrow \frac{dy(0)}{d(t)} = \frac{2}{C} = \frac{2}{(1/6)}$$

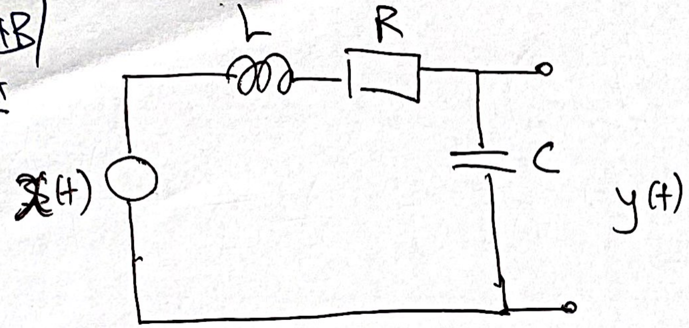
$$\dot{y}(0) = 12 \quad \boxtimes$$

and $X(s) = 0$

$$\therefore Y(s) = \left[\frac{1}{s^2 + \underbrace{\frac{5}{1}}_{\left(\frac{R}{L}\right)}s + \underbrace{\frac{1}{(1/6)}}_{\left(\frac{1}{LC}\right)}}\right]\left[\underbrace{\frac{0}{(LC)}}_{X(s)} + s\underbrace{1.5}_{y(0)} + \underbrace{12}_{\dot{y}(0)} + \underbrace{\frac{5}{1}}_{\frac{R}{L}}\underbrace{(1.5)}_{y(0)}\right]$$

$$= \left[\frac{1}{s^2 + 5s + 6}\right]\left[1.5s + 19.5\right] \quad \boxtimes$$

Tut 4B/
Q4A



Find $y(t)$, given $L = 1$ Farad, $R = 5\,\Omega$, $C = 1/6\,F$

at $t = 0$, $i(0) = 2A$
$y(0) = 1.5V$.
$\Rightarrow$ initial condition is not relax!

Key idea: a) KVL = sum of voltage = 0.

b) Voltage across $-\!\!\text{ooo}\!\!-$ = $L\frac{di(t)}{dt}$ — change of current across inductor.

voltage across $-\!\square\!-$ = $i(t)R$

relationship of current across capacitor $-\!||\!-$ $\Rightarrow$ $i(t) = C\frac{dy(t)}{d(t)}$ — change of voltage across capacitor.

Voltage across capacitor $= \frac{1}{C}\int i(t)\,dt$.

① KVL

$$x(t) = V_L + V_R + V_C$$

$$= L\frac{di(t)}{dt} + i(t)R + y(t).$$

Key idea, remove $i(t)$, $di(t)$ and change to $y(t)$.

By $i(t) = C\frac{dy(t)}{d(t)}$

$$\frac{di(t)}{d(t)^2} = C\frac{d^2y(t)}{d(t)^2}.$$

$$\therefore x(t) = L\left(C\frac{d^2y(t)}{d(t)^2}\right) + C\frac{dy(t)}{d(t)}R + y(t)$$

Now we have a 2nd order eq? based only on $y(t)$.

Take Laplace T.

$$\frac{dy(t)}{d(t)} \overset{L}{\Longleftrightarrow} sY(s) - Y(0)$$

$$\frac{d^2y(t)}{dt^2} \overset{L}{\Longleftrightarrow} s^2Y(s) - \underbrace{sY(0)}_{\text{initial condition of } Y(t)} - \dot{Y}(0)$$

initial condition of $\dot{y}(t)$

Example: when $x(t) = 10\,u(t) \xleftrightarrow{\text{Laplace}} \frac{10}{s}$

(i/p = unit step.)

$y(0) = 0$

$\dot{y}(0) = 0.$

$R = 1$, $L = 1$, $C = 1/6.$

∴ substitute into :

$$Y(s) = \left(\frac{1}{s^2 + 1s + 6}\right)\left[\frac{10}{(s)LC}\right] + \underbrace{\text{initial condition}}_{=0}$$

$$= \left[\frac{1}{s^2 + 1s + 6}\right]\left[\frac{10}{(\frac{1}{6}s)}\right].$$

$10 \leftarrow K$ ($x(t) = Ku(t)$).

$\left(\frac{R}{L}\right)$ $\left(\frac{1}{LC}\right)$ LC

$$Y(s) = \frac{10}{\frac{1}{6}s^3 + \frac{1}{6}s^2 + 1s + 0}$$

PFE

$$\Rightarrow r_1 = \begin{bmatrix} -5 + 1.0426j \\ -5 - 1.0426j \\ 10 \end{bmatrix} \begin{matrix} \leftarrow r_1(1) \\ \leftarrow r_1(2) \\ \leftarrow r_1(3) \end{matrix}$$

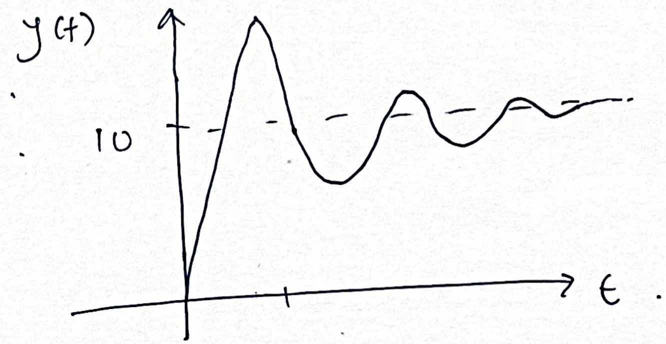
$$p_1 = \begin{bmatrix} -0.5 + 2.3979j \\ -0.5 - 2.3979j \\ 0 \end{bmatrix} \begin{matrix} \leftarrow p_1(1) \\ \leftarrow p_1(2) \\ \leftarrow p_1(3) \end{matrix}$$

$k_1 = 0.$

$$\therefore y(t) = r_1(1) * \exp(p_1(1) .* t) + r_1(2) * \exp(p_1(2) * t) + r_1(3) * \exp(p_1(3) * t).$$

see L06_Laplace-Sup-Q4.m.

∴



will become 10 V at $t \to \infty$.

Using final value theorem.

$$y(t) \to \infty \Rightarrow \lim_{s \to 0} sY(s) = \left.\frac{10}{\frac{1}{6}s^2 + \frac{1}{6}s + 1}\right|_{s \to 0}$$

$$= \frac{10}{1} = 10\text{ V} \boxtimes$$