HW1

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W1

The probability of A is the summation of the joint probability of A with each section of partition.

$P(\mathrm{A}) = \sum_{i=1}^{N} P(\mathrm{A}|\mathrm{B}_i)P(\mathrm{B}_i)$

W2

(a)

There are two conditions that p_{ij} must satisfy:

1. $\sum_{j=1}^{N} p_{ij} = 1$
2. $(p_{ij})_{1\le i,j\le N} \ge 0$

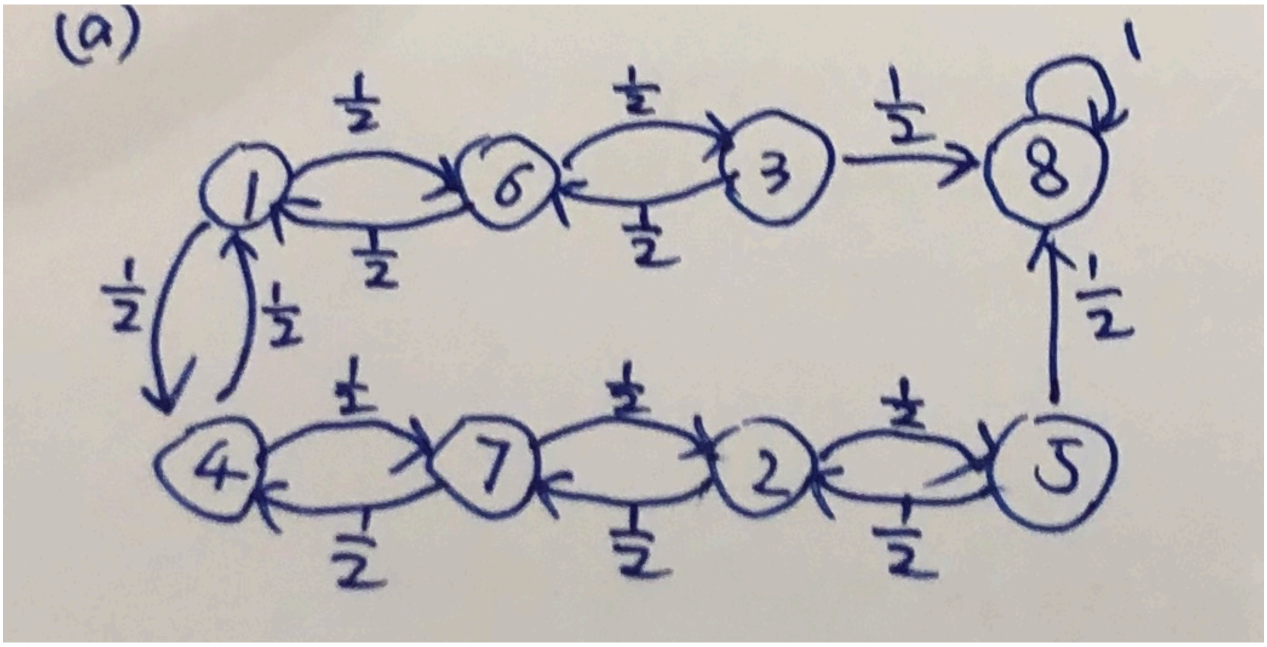
(b)

Let $P^k = (p_{ij}^k)_{1\le i,j\le N}$

p_{ij}^k refers to the probability of a transition that starts at state i and ends at state j after k steps.

P1

1.



2.

$$P = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

P2

(a) $\binom{9}{0} + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \binom{9}{5} + \binom{9}{6} + \binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 512$

(b) $9! = 362880$

(c) The minimal number of steps to guarantee a win is 7.

(d) To win the game, it requires at least 3 turns. That being said, in 2 turns there is no chance to win. So,

$u_2 = 0$

There are 8 ways to form a line in the grid, and there are $\binom{9}{3}$ ways to fill the grid in 3 turns. So,

$u_3 = \frac{8}{\binom{9}{3}} = \frac{2}{21}$

To win within 4 steps, a line need to be formed and a random cell will be formed from the other 6 cells. So, there are $8 * \binom{6}{1}$ ways to win the game within 4 steps. And there are $\binom{9}{4}$ ways to fill the grid in 4 turns. So,

$u_4 = \frac{8*\binom{6}{1}}{\binom{9}{4}} = \frac{8}{21}$

There are 2 ways we could not win within 6 turns. And there are $\binom{9}{6}$ ways to fill the grid in 4 turns. So,

$u_6 = 1 - \frac{2}{\binom{9}{6}} = \frac{41}{42}$

(e)

Yes, I think the sequence is the markov chain for the following 2 reasons:

1. The transformation probability of future state is conditionally independent of past state, given the current state.
2. For the entries p_{ij} in the transition matrix, we have :
   - $\sum_{j=1}^{9} p_{ij} = 1$
   - $(p_{ij})_{1 \le i,j \le 9} \ge 0$

(f)

i. Given X_2, there are 7 cells to fill, and generating X_3 takes a specific cell. So, $P(X_3|X_2) = \frac{1}{7}$

ii. X_1 as a past state, would not influence the probability of each future state. So, $P(X_3|X_2, X_1) = P(X_3|X_2) = \frac{1}{7}$

iii. $P(X_1, X_2, X_3) = P(X_1) * P(X_2|X_1) * P(X_3|X_2, X_1) = \frac{1}{9} * \frac{1}{8} * \frac{1}{7} = \frac{1}{504}$

iiii. $P(X_3 = (d)) = \frac{1}{\binom{9}{3}} = \frac{1}{84}$

v. Given current state to be $X_3 = (d)$, there are 6 cells can be filled and 3 ways to win in next turn, so $P(\text{Win} \mid \text{X\_3}) = \frac{3}{6} = \frac{1}{2}$