Trajectory Of A Golf Ball

Johannes C.

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Contents

1 Abstract

Displacement of the ball due to wind in Nintendo's "Switch Sports" golf can feel arbitrary sometimes and seemingly differs from hole to hole. The goal of this project is to find out which physical model lies behind the simulation of the golf ball and the wind physics used in the game, thus eliminating any feeling of randomness that often arises while playing.

Chapter 2 will present an analytical solution to the problem with respect to air resistance but no wind.

Wind will be added to the problem numerically in chapter 3. This chapter will also introduce a number of constants chosen to fit the parameters in the game and will also go in detail on how these constants were determined.

Lastly, different wind models will be tested to see which one best fits the one chosen in the game. For this, each of the courses will be mapped to provide the most accurate results.

2 Analytical Solution with Drag and no Wind

2.1 Differential Equations and Boundary Conditions

The underlying differential equations of the problem are given by

$$m\ddot{x} = -\mu m\dot{x} \quad , \quad m\ddot{z} = -mg - \mu m\dot{z} \quad . \tag{1}$$

Here, updraft forces as well as the Magnus effect are ignored. The characteristic dimples of the ball enhance both the air resistance as well as the effectiveness of the Magnus effect making the ball easier to control.

The boundary conditions are given by

$$x(0) = 0 \quad , \quad z(0) = 0 \quad , \quad \dot{x}(0) = v_x = v_0\cos\varphi \quad , \quad \dot{z}(0) = v_z = v_0\sin\varphi \quad . \tag{2}$$

Here, we use μ as the drag constant between air and the ball. It has the dimension of $[\mu] = 1\,\mathrm{s}^{-1}$

2.2 Solution

The solutions for $x(t)$ and $z(t)$ using the boundary conditions (2) are

$$x(t) = \frac{v_0\cos\varphi}{\mu}\left(1 - e^{-\mu t}\right) \quad , \tag{3}$$

$$z(t) = -\frac{g}{\mu}t - \left(\frac{v_0\sin\varphi}{\mu} + \frac{g}{\mu^2}\right)\left(e^{-\mu t} - 1\right) \quad . \tag{4}$$

To get the trajectory $z(x)$ we solve (3) for t and plug it into (4). We get

$$z(x) = \frac{g}{\mu^2}\ln\left(1 - \frac{\mu}{v_0\cos\varphi}x\right) + \left(\tan\varphi - \frac{g}{\mu v_0\cos\varphi}\right)\cdot x \quad \text{for} \quad x < \frac{v_0\cos\varphi}{\mu} \quad . \tag{5}$$

3 Numerical Solution with Wind

3.1 Differential Equations and Boundary Conditions

By expanding on our analytical approach, we can calculate our trajectory in respect to wind. Here, we keep in mind that wind can only blow in the x-y plane. Thus, we get the following equations:

$$m\ddot{x} = -c_x\dot{x} + ma_{\text{Wind},x} \tag{6}$$
$$m\ddot{y} = -c_y\dot{y} + ma_{\text{Wind},y} \tag{7}$$
$$m\ddot{z} = -c_z\dot{z} - mg \tag{8}$$
$$\tag{9}$$

Here we once again ignore updraft, the Magnus effect as well as the dimples of the ball. The added term describes the force F_{Wind} of the wind applied to the ball in the x and y direction. This Force can be calculated using the simplified drag model for balls [???]. It yields the following result:

$$F_{\text{Wind}} = ma_{\text{Wind}} = \frac{1}{2}C_{\text{D}}A\rho v_{\text{Wind}}^2 \quad . \tag{10}$$

Here, C_{D} is some drag coefficient between ball and wind, $A = \pi R^2$ is the area of the circular crosssection, ρ is the air density and v_{Wind} is the wind velocity. This velocity can be split into its horizontal and lateral components which yields the force in the respective direction. The boundary conditions are the same as in section 2.1.

This approach however is a vast simplification as it doesn't account for the effectiveness of the wind being dependent on z, as wind affects the ball more at further distances from the ground, as wind doesn't get obstructed as much by trees and other objects. This will be accounted for in a later model.

3.2 Choice of Constants

Since Nintendo is a Japanese Studio, constants were chosen based on geographical conditions and characteristics of coastal Japan. The drag constants have placeholder values for now but will be adjusted later on. The values were chosen as follows:

Tab. 1: Chosen Constants and their Values.

Constant [Unit]	**Value**
m [kg]	0.045
g [$\text{m} \cdot \text{s}^{-2}$]	9.798
$c_{x,y,z}$ [weird units]	0.004
C_{D}	0.5
ρ [$\text{kg} \cdot \text{m}^{-3}$]	1.184
A [m^2]	0.00143

3.3 Plotting and Verification

Next, a 3D plot is created to depict the trajectory. To simulate a full power driver tee-shot, the values $v_0 = 90\,\text{m} \cdot \text{s}^{-1}$ and $\varphi = 9°$ were chosen. The wind was set to $v_{\text{Wind}} = v_{\text{Wind},y} = 15\,\text{m} \cdot \text{s}^{-1}$, which is the strongest crosswind (east to west) possible in-game. The resulting plot can be seen in 1.

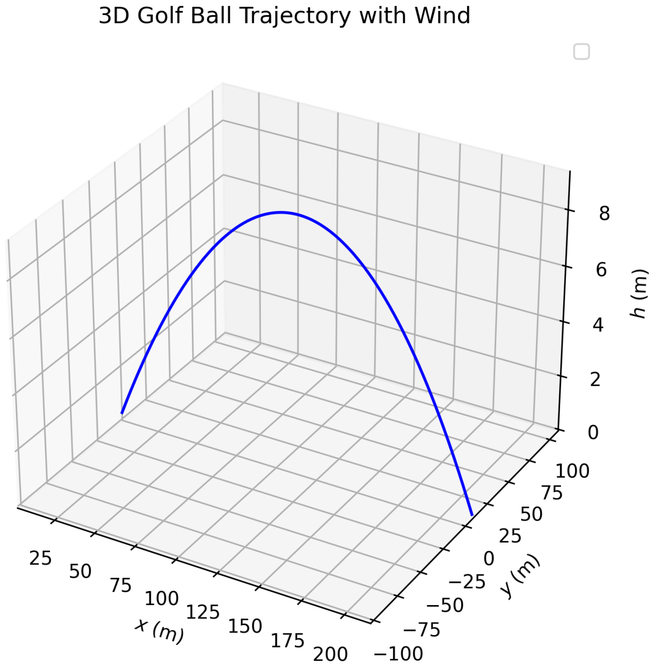

Fig. 1: Simplified trajectory for a tee-shot. The ball hits the ground at roughly $x = 225\,\mathrm{m} \approx 246\,\mathrm{yd}$ which points to our constants being in the correct regime. The lateral displacement on contact with the ground is roughly $y = 7{,}5\,\mathrm{m}$ which seems a little on the lower side, most likely due to the chosen simplifications or due to the drag coefficient C_D being too small.

To make sure that the wind force is applied correctly the ball, the velocity vector of the ball along different points of the trajectory was plotted. The result should be roughly parabolic, though the air resistance should distort it a little bit. This can be seen in 2.

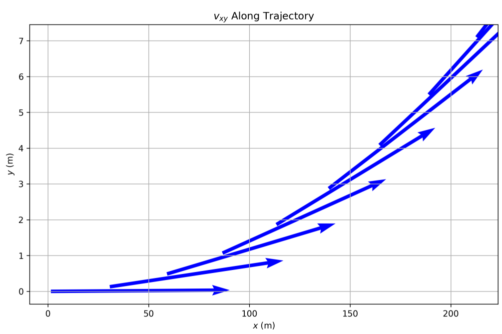

Fig. 2: Velocity vectors along the trajectory. The curvature of the vector does indeed seem to be parabolical, meaning the wind velocity is correctly transferred to the ball.