

COMP 3804/Math 3804: Assignment 2

Due Date: Sunday, October 23rd at 11:59PM

School of Computer Science

Carleton University

Your assignment should be submitted online on Brightspace as a single .pdf file. The filename should contain your name and student number. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded.

Question 1 [15 marks]

Consider a 4-heap to be a heap-ordered tree in which each internal node has 4 children, all leaves are on at most two adjacent levels, and, on the last level, all leaves are as much to the left as possible. Let n be the total number of nodes in the tree and assume that $n = 4 * k + 1$, for some positive integer k .

1. State precisely:
 - how many leaf-nodes a 4-heap on n nodes can have minimally and maximally,
 - how many nodes there are on each level, i , (excluding the lowest level),
 - an expression, phrased in terms of n , for the height of the tree,
 - how many nodes there are on all levels together (excluding the lowest level),
 - a relation between the total number of internal nodes and the number of leaves.
2. Assume the tree is stored in an array $H[1 \dots n]$. (Note: we start with index 1 not 0.) Give address formulae for: the children of an internal node stored at $H[i]$ and the parent node of a node stored at $H[i]$.

Question 2 [11 marks]

1. For standard binary heaps, we also store the heap in an array $H[1 \dots n]$. Now, state parent/child index relations, but use the bit representation of the array indices. So, state (child to parent) and (parent to child) array index relations using the binary representation of indices. So, we get e.g., the children of the root are stored at $\text{binary}(10)$ and $\text{binary}(11)$ and the parent of $\text{binary}(10)$ and $\text{binary}(11)$ is the root stored at $\text{binary}(1)$.
2. Let the i^{th} parent(node) be recursively defined as follows:
 - for $i = 0$ the i^{th} parent(node) is the node itself.
 - for $i > 1$, i^{th} parent(node) is the parent of $(i - 1)^{\text{st}}$ parent(node).Now, state a simple formula (in binary) for the i^{th} parent of a node stored at $\text{binary}(b_1 \dots b_m)$.

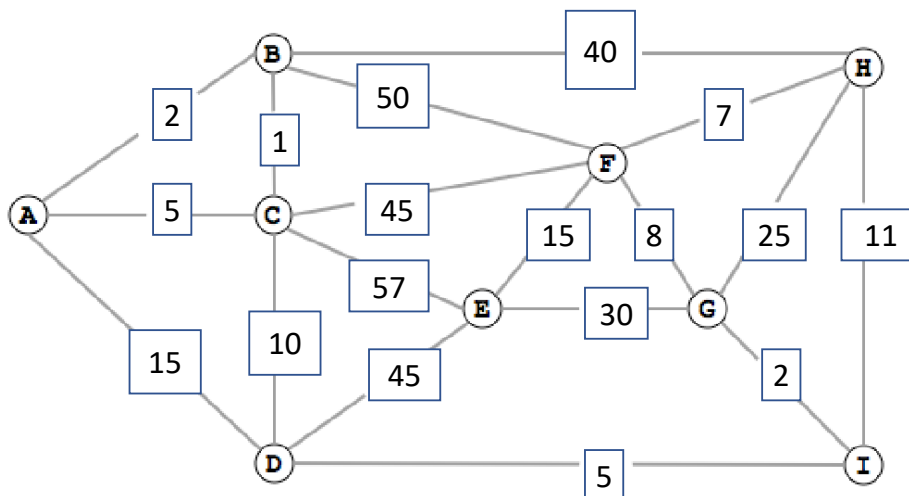


Figure 1: The input graph for Question 3.

Question 3 [20 marks]

Suppose that we want to find a minimum spanning tree of the graph shown on Figure 1.

- Run Prim's algorithm on this graph. Start from vertex A and whenever there is a choice of vertices, always select in the alphabetic order. Draw a table showing the intermediate values at each step.
- Run Kruskal's algorithm on the same graph. Clearly show the disjoint-set data structure (i.e., the structure of the directed trees) at every intermediate step, assuming union using path compression.

Question 4 [20 marks]

Let $G = (V, E)$ be a graph with distinct weights. Let $e_i, i = 1 \dots |E|$ be the list of edges, sorted by weight. Let $j < k$, be two indices such that e_j is not in the minimum spanning tree of G , but e_k is. Prove or disprove that the removal of e_i (from the graph) cannot disconnect G .