

Assignment 1

COMP2804 Winter 2020

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1 ID

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2 Non-Local Strings

1. What is the number of dec-strings of length n ?
by the Product Rule, the number of blocks is

$$10^n$$

2. What is the number of dec-strings d_1, \dots, d_n of length n such that $d_1d_2 \neq 00$. In other words, what is the number of dec-strings of length n that don't begin with 00?

Therefore, by the Product Rule, then number of squarefree blocks is

$$(10^2 - 1) \times 10^{n-2}$$

$$10^n - 10^{n-2}$$

3. What is the number of dec-strings d_1, \dots, d_n of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 11$?

Therefore, by the Product Rule, then number of non-local blocks is

$$10^n - 10^{n-2} - 10^{n-2}$$

$$(10^3 - (2 \times 10)) \times 10^{n-3}$$

4. What is the number of dec-strings d_1, \dots, d_n of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 01$?

$$10^n - 10^{n-2} - 10^{n-2} - 2$$

5. What is the number of dec-strings d_1, \dots, d_n of length n such that $d_1d_2 \neq 00$ or $d_1d_2d_3 = 111$?
6. What is the number of dec-strings d_1, \dots, d_n of length $n \geq 4$ such that $d_1d_2 \neq 00$ or $d_3d_4 \neq 11$.
7. A dec-string d_1, \dots, d_n is *bad* if $d_i = d_i + 1$ or $d_i + d_i + 1 = 9$ for at least one $i \in \{1, \dots, n-1\}$ and it is good otherwise. What is the number of good dec-strings of length n ?
8. A dec-string d_1, \dots, d_n is *2-bad* if, $d_i = d_j$ or $d_i + d_j = 9$ for some $i < j \leq i+2$ and it is *2-good* otherwise. What is the number of *2-good* dec-strings?
9. A dec-string d_1, \dots, d_n is *k-bad* if $d_i = d_j$ or $d_i + d_j = 9$ for some $i < j \leq i+k$ and is *k-good* otherwise. What is the number of *k-good* dec-strings of length n ?
10. By now the pattern should become clear.

For a k -non-local block s_0, s_1, \dots, s_{19} we have $26-i$ choices for s_i for each $i \in \{0, \dots, k-1\}$ and we have $26-k$ choices for s_i for each $i \in \{k, \dots, 19\}$. Therefore, by the Product Rule, then number of k -non-local blocks is

$$\left(\prod_{i=0}^{k-1} (26-i) \right) \times (26-k)^{20-k} = (26 \times 25 \times \dots \times (26-k+1)) \times (26-k)^{20-k}$$

3 Collective Arts

Collective Arts Brewing currently makes 30 types of IPA and 6 types of Lager.

1. The manager at Mike's Place needs to choose 4 types of IPA and 4 types of Lager. How many options does the manager have?

$$30 \times 29 \times 28 \times 27 \times 6 \times 5 \times 4 \times 3$$