Assignment 1

COMP2804 Winter 2020

January 14, 2020

1 ID

Name: Ryan Lo
Student ID: 101117765

2 Non-Local Strings

1. What is the number of dec-strings of length n?

   by the Product Rule, the number of blocks is

   $$10^n$$

2. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d1d2 \neq 00$. In other words, what is the number of dec-strings of length n that don't begin with 00?

   Therefore, by the Product Rule, then number of squarefree blocks is

   $$(10^2 - 1) \times 10^{n-2}$$

   $$10^n - 10^{n-2}$$

3. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 11$?

   Therefore, by the Product Rule, then number of non-local blocks is

   $$10^n - 10^{n-2} - 10^{n-2}$$

   $$(10^3 - (2 \times 10)) \times 10^{n-3}$$

4. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 01$?

   $$10^n - 10^{n-2} - 10^{n-2} - 1$$

   $$(10^3 - (2 \times 10) - 1) \times 10^{n-3}$$

5. What is the number of dec-strings $d_1, \ldots, d_n$ of lenght n such that $d_1 d_2 \neq 00$ or $d_1 d_2 d_3 = 111$?

$$(10^3 - (10+1)) \times 10^{n-3}$$

6. What is the number of dec-strings $d_1, \ldots, d_n$ of length $n \geq 4$ such that $d_1 d_2 \neq 00$ or $d_3 d_4 \neq 11$.

$$(10^4 - ((4 \times 10) - 1)) \times 10^{n-4}$$

7. A dec-string $d_1, \ldots, d_n$ is *bad* if $d_i = d_i + 1$ or $d_i + d_i + 1 = 9$ for at least one $i \in \{1, \ldots, n-1\}$ and it is good otherwise. What is the number of good dec-strings of length n?

8. A dec-string $d_1, \ldots, d_n$ is $2 - bad$ if, $d_i = d_j$ or $d_i + d_j = 9$ for some $i < j \leq i+2$ and it is $2 - good$ otherwise. What is the number of $2 - good$ dec-strings?

9. A dec-string $d_1, \ldots, d_n$ is $k - bad$ if $d_i = d_j$ or $di + dj = 9$ for some $i < j \leq i + k$ and is $k - good$ otherwise. What is the number of $k - good$ dec-strings of length n?

10. By now the pattern should become clear.

    For a k-non-local block $s_0, s_1, \ldots, s_{19}$ we have $26 - i$ choices for s_i for each $i \in \{0, \ldots, k-1\}$ and we have $26 - k$ choices for s_i for each $i \in \{k, \ldots, 19\}$. Therefore, by the Product Rule, then number of k-non-local blocks is

$$\left(\prod_{i=0}^{k-1} (26 - i)\right) \times (26-k)^{20-k} = (26 \times 25 \times \cdots \times (26 - k + 1)) \times (26-k)^{20-k}$$

3 Collective Arts

Collective Arts Brewing currently makes 30 types of IPA and 6 types of Lager.

1. The manager at Mike's Place needs to choose 4 types of IPA and 4 types of Lager. How many options does the manager have?

$$30 \times 29 \times 28 \times 27 \times 6 \times 5 \times 4 \times 3$$

2. The 8 beers (4 IPA and 4 Lager) selected in the previous question must be placed in a line on a display shelf so that no two IPA are adjacent and no two Lager are adjacent. How many ways are there to do this?

3. Continuing from the previous question, suppose that two of the beers selected were All Together Now (an IPA) and Hot Pink (a Lager). Since both cans are pink, the manager doesn't want to place them adjacent to each other. How many ways are there to do this (while still alternating between IPA and Lager)?

4. How many of the arrangements from the previous question have the All Together Now among from the 4 leftmost bottles and the bottle of Hot Pink among the 4 rightmost bottles?

4 Restricted Permutations

Consider all permutations of the integers 1,...,1000.

1. In how many of these permutations do 1,2,3,4 appear consecutively and in this order?
2. In how many of these permutations do 1,2,3,4 appear consecutively, but not necessarily in order? (For example, they may appear as 1,2,3,4, or 4,2,3,1, or 3,1,2,4, or so on.)
3. In how many of these permutations does 1 appear before 2, 2 appear before 3, and 3 appear before 4? (In other words, 1,2,3,4 appear in order, but not necessarily consecutively.)
4. In how many of these permutations do 1,2,3,4 appear in order but no two are adjacent?

5 Drug Trials

A certain friend of mine has spent the better part of a lifetime testing recreational drugs. After thorough testing, this friend has identified 20 recreational drugs $D_1, \ldots, D_{20}$ and determined (experimentally) that any 3 of these drugs can be taken simultaneously with no adverse effects.

1. Assuming my friend determined this entirely by testing, how many experiments did my friend have to perform?
2. A new designer drug called D_{21} has just hit the streets and my friend wants to know if D_{21} can be added to their list. That is, can any triple of $D_1, \ldots, D_{21}$ be safely taken together? How many additional experiments does my friend need to determine this?
3. Suppose my friend survives the experience and D_{21} makes it onto the list. My friend takes scrupulous notes about all experiments and notices something peculiar about the answers to the preceding two questions. What combinatorial identity did my friend just discover?
4. Suppose that an impatient novice wants to repeat my friends discovery using fewer experiments. This novice is willing to take 5 drugs at a time. What is the fewest number of experiments this novice can perform so that for any triple $D_i, D_j, D_k \in \{D_1, \ldots, D_{21}\}$, at least one of this novice's experiments includes D_i, D_j, and D_k?