Part 2 Extra Solutions

Monday, October 30, 2023 1:43 PM

Example 1.
Define X to be the event "murder by poison".
Define Y to be the event "murder by blunt-force".
Define Z to be the event "there was blood".

We want to compute P(X|Z) and P(Y|Z).

We are given:
P(X) = 0.6
P(Y) = 0.4
P(Z|X) = 0.1
P(Z|Y) = 0.9

We can compute:
P(Z) = P(Z|X) * P(X) + P(Z|Y) * P(Y)
= 0.1 * 0.6 + 0.9 * 0.4
= 0.42

Using Bayes Theorem we get:

P(X|Z) = P(Z|X) * P(X) / P(Z)
= 0.1 * 0.6 / 0.42
= 0.14

P(Y|Z) = P(Z|Y) * P(Y) / P(Z)
= 0.9 * 0.4 / 0.42
= 0.86

Thus, it is most likely the murder was by blunt force.

Example 2.

$$P(M=T \mid C=T) = \frac{P(M=T \wedge C=T)}{P(C=T)}$$

$$= \frac{1}{P(C=T)} \sum_{s \in T,F} \sum_{g \in T,F} P(C=T \mid M=T \wedge G=g) P(G=g \mid S=s) \, P(M=T) P(S=s)$$

$$= \frac{1}{P(C=T)} P(M=T) \sum_{s \in T,F} P(S=s) \sum_{g \in T,F} P(C=T \mid M=T \wedge G=g) P(G=g \mid S=s)$$

Define $f_1(s) = \sum_{g \in T,F} P(C=T \mid M=T \wedge G=g) P(G=g \mid S=s)$

$$= \frac{1}{P(C=T)} P(M=T) \sum_{s \in T,F} P(S=s) f_1(s)$$

Define $f_2 = \sum_{s \in T,F} P(S=s) f_1(s)$

$$= \frac{1}{P(C=T)} P(M=T) f_2$$

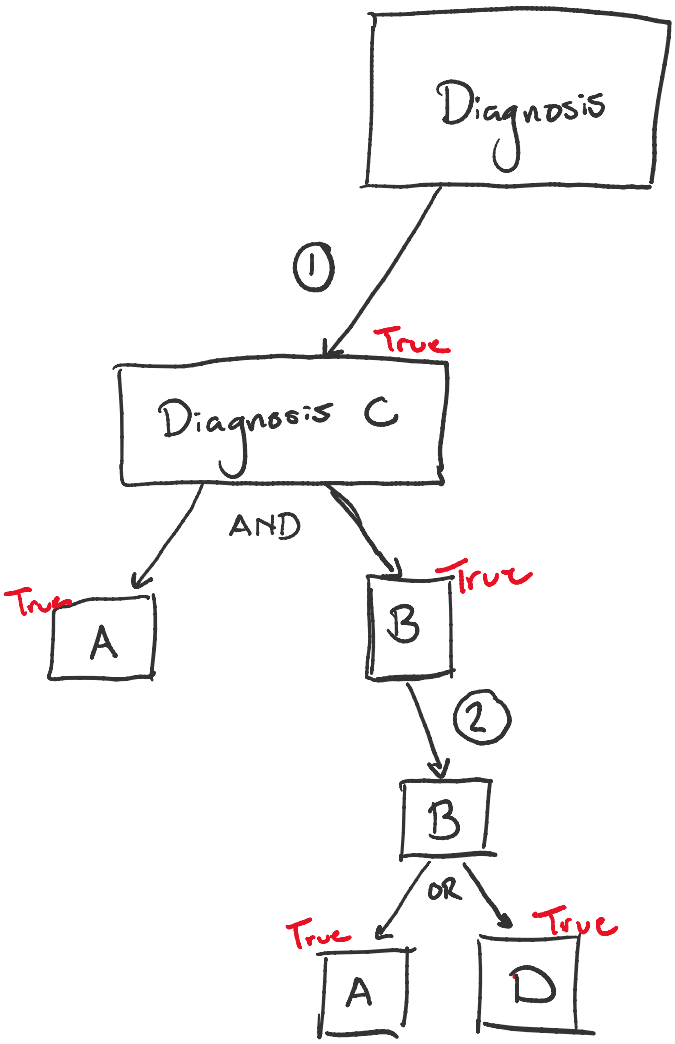
Example 3.

a) Rules examined (in order): 1, 2, 3, 4, 5, 1, 2, 3, 4, 5

Working memory:
A is true
D is true
B is true
H is true
I is false
diagnosis is C

b) Rules examined (in order; see diagram): 1, 2

Working memory:
A is true
D is true
B is true
diagnosis is C



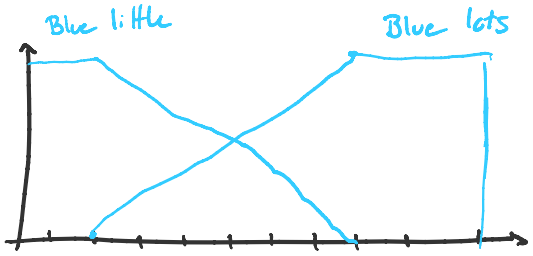
Example 4.

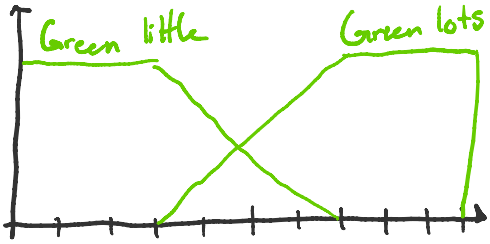
We must compute s(t(A, B), s(C, D)), where t(x, y) = x * y and s(x, y) = x + y - x * y.

s(t(A, B), s(C, D))
= s(t(0.4, 0.5), s(0.2, 0.1))
= s(0.2, 0.28)
= 0.424

Example 5.

Draw our membership functions below.





First, we must compute the output membership functions for each rule.
To do so, we must first compute the fuzzy truth value of each proposition.
m_BlueLittle(x) = (0.8 - 0.4) / (0.8 - 0.2) = 0.6667
m_BlueLots(x) = (0.4 - 0.2) / (0.8 - 0.2) = 0.3333
m_GreenLittle(x) = (0.7 - 0.6) / (0.7 - 0.3) = 0.25
m_GreenLots(x) = (0.6 - 0.3) / (0.7 - 0.3) = 0.75
Then, we must compute the rule strengths for each rule.

1. t(m_BlueLots(x), m_GreenLittle(x)) = 0.083
2. t(m_BlueLittle(x), m_GreenLots(x)) = 0.5

Then, we compute the output membership functions.
m_Output1(x) = 0.083 if x is ocean; 0 otherwise
m_Output2(x) = 0.5 if x is forest; 0 otherwise

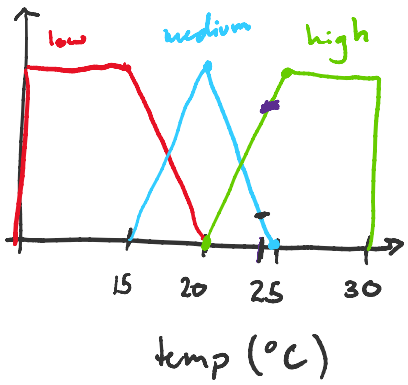
Next, we compute the combined output membership function from all rules.
m_Combined(x) = 0.083 if x is ocean; 0.5 if x is forest; 0 otherwise

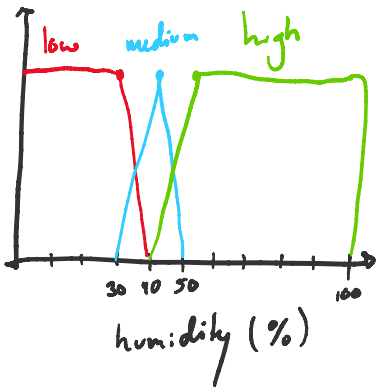
Finally, we defuzzify the outputs by determining the value of x that leads to the largest value.
argmax(m_Combined(x)) = forest

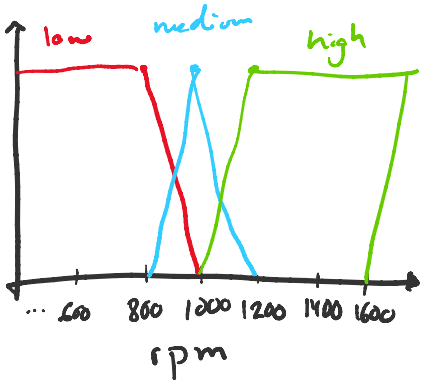
Thus, the output is forest.

Example 6.

Diagrams of the relevant membership functions are below.





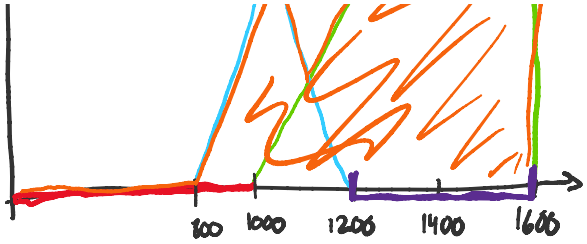


We observe temperature is 24°C and humidity is 43%. What should the fan speed be?

A)
First, we must compute the output membership functions for each rule.
To do so, we must first compute the fuzzy truth value of each proposition.
m_TempHigh(x) = 0.8
m_TempMedium(x) = 0.2
m_TempLow(x) = 0
m_HumdiityHigh(x) = 0.3
m_HumidityMedium(x) = 0.7
m_HumidityLow(x) = 0
Then, we must compute the rule strengths for each rule us the Lukasiewicz t-norm and s-norm.

1. s(0.8, 0.3) = 1
2. S(0.2, 0.7) = 0.9
3. t(0, 0) = 0

Below, we have illustrated the output membership functions for each rule (rpm medium in blue and rpm high in green) and the combined output membership function (orange).

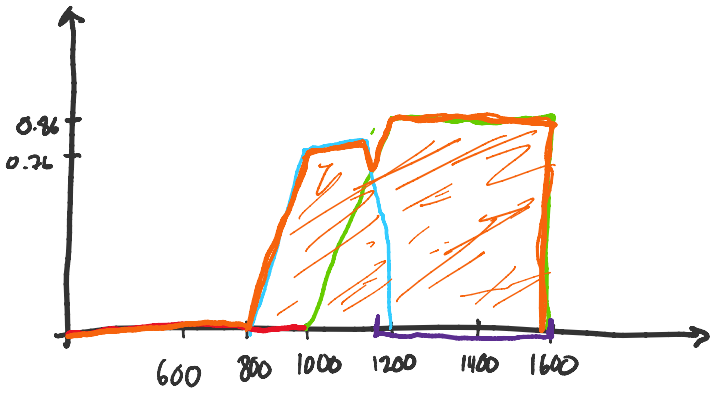


Defuzzifying using mean of maxima yields a crisp output value 1400rpm.

B)
The fuzzy truth values of propositions are the same as for part A) above.
m_TempHigh(x) = 0.8
m_TempMedium(x) = 0.2
m_TempLow(x) = 0
m_HumdiityHigh(x) = 0.3
m_HumidityMedium(x) = 0.7
m_HumidityLow(x) = 0
Then, we must compute the rule strengths for each rule us the Goguen t-norm and s-norm.

1. s(0.8, 0.3) = 0.86
2. S(0.2, 0.7) = 0.76
3. t(0, 0) = 0

Below, we have illustrated the output membership functions for each rule (rpm medium in blue and rpm high in green) and the combined output membership function (orange).



Defuzzifying using mean of maxima yields a crisp output value 1386rpm.