Assignment 1

COMP2804 Winter 2020

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1 ID

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2 Non-Local Strings

1. What is the number of dec-strings of length n?

   by the Product Rule, the number of blocks is

   $$10^n$$

2. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d1d2 \neq 00$. In other words, what is the number of dec-strings of length n that don't begin with 00?

   Therefore, by the Product Rule, then number of squarefree blocks is

   $$10^n - 10^{n-2} when n > 1 when n = 0, 10$$

3. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 11$?

   Therefore, by the Product Rule, then number of non-local blocks is

   $$26 \times 25 \times 24^{18} = 4\,536\,445\,601\,729\,847\,895\,680\,614\,400 \ .$$

4. What is the number of dec-strings $d_1, \ldots, d_n$ of length n such that $d_1d_2 \neq 00$ and $d_2d_3 \neq 01$?

5. What is the number of dec-strings $d_1, \ldots, d_n$ of lenght n such that $d_1d_2 \neq 00$ or $d_1d_2d_3 = 111$?

6. What is the number of dec-strings $d_1, \ldots, d_n$ of length $n \geq 4$ such that $d_1d_2 \neq 00$ or $d_3d_4 \neq 11$.

7. A dec-string $d_1, \ldots, d_n$ is *bad* if $d_i = d_i + 1$ or $d_i + d_i + 1 = 9$ for at least one $i \in \{1, \ldots, n-1\}$ and it is good otherwise. What is the number of good dec-strings of length n?

8. A dec-string $d_1, \ldots, d_n$ is $2 - bad$ if, $d_i = d_j$ or $d_i + d_j = 9$ for some $i < j \leq i+2$ and it is $2 - good$ otherwise. What is the number of $2 - good$ dec-strings?

9. A dec-string $d_1, \ldots, d_n$ is $k - bad$ if $d_i = d_j$ or $di + dj = 9$ for some $i < j \leq i + k$ and is $k - good$ otherwise. What is the number of $k - good$ dec-strings of length n?

10. By now the pattern should become clear.

    For a k-non-local block $s_0, s_1, \ldots, s_{19}$ we have $26 - i$ choices for s_i for each $i \in \{0, \ldots, k-1\}$ and we have $26 - k$ choices for s_i for each $i \in \{k, \ldots, 19\}$. Therefore, by the Product Rule, then number of k-non-local blocks is

$$\left(\prod_{i=0}^{k-1}(26-i)\right) \times (26-k)^{20-k} = (26 \times 25 \times \cdots \times (26-k+1)) \times (26-k)^{20-k}$$

3 Collective Arts

1.