QUIZ #2 – COMP 3106 INTRODUCTION TO ARTIFICIAL INTELLIGENCE

NOVEMBER 6, 2023
8:35AM – 9:55AM (80 MINUTES)
SOUTHAM HALL 416

Instructions

The quiz is open-book, and you may consult your notes and the textbook during quizzes. You may not use electronic devices (except non-programmable scientific calculators) during quizzes.

You must complete the quiz individually.

The quiz includes five multiple-choice questions and four written answer questions. Multiple-choice questions are worth 2 marks each. Written answer questions are worth 10 marks each.

For the multiple-choice questions, circle the correct answer. Justification of your answers is not required and will not be considered.

For the written answer questions, write your answer in the space provided below the question. You are not required to provide justification (unless specifically requested). However, if your final answer is incorrect, partial credit may be awarded for justification. If your final answer is incorrect and you do not provide justification, you may receive a grade of zero for that question.

Name:
Student ID:

Question 1 [2 marks]

Suppose we have a Naïve Bayes classifier whose performance we wish to measure on a binary classification task. The confusion matrix for this classifier on the task is given below. What is the F1 score for the classifier on this task?

		Prediction	
		positive	negative
Ground-Truth	positive	15	10
	negative	35	40

a) 0.400
b) 0.450
c) 0.550
d) 0.640
e) 0.667

Question 2 [2 marks]

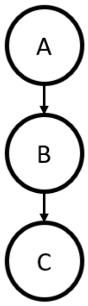
Suppose we have a fuzzy rule-based system whose performance we wish to measure on a regression task. The true labels and the predicted labels by the system are given below. What is the mean absolute error for the system on this task?

Ground-Truth	Predicted
3.2	3.1
7.5	7.3
1.3	1.7
4.5	4.9
2.7	2.2
6.1	6.0
5.9	5.7

a) 0.043
b) 0.200
c) 0.271
d) 0.355
e) 0.400

Question 3 [2 marks]

Consider the Bayesian network given below. Which of the following is a correct expression for the probability $P(A \wedge B \wedge C)$?



a) $P(A)$
b) $P(C)$
c) $P(A)P(B)P(C)$
d) $P(A)P(B|A)P(C|B)$
e) $P(C)P(B|C)P(A|B)$

Question 4 [2 marks]

Consider goal-based reasoning on the rule-based system with given below. The goal is to determine diagnosis. Assume the A, B, C are askable, and their truth values are A = True, B = False, C = False; assume the D and diagnosis are not askable. Suppose the user asks for a "why" explanation on the proposition C. Which of the following is a reasonable "why" explanation?

IF A and D THEN diagnosis is X.
IF B or C THEN D.

a) C follows from the rule IF A and D THEN diagnosis is X.
b) C follows from the rule IF B or C THEN D.
c) C is needed to determine the value of "A and D" in the rule IF A and D THEN diagnosis is X.
d) C is needed to determine the value of "B or C" in the rule IF B or C THEN D.
e) C is queried from the user.

Question 5 [2 marks]

Given a t-norm t and an s-norm s, what is the value of $t(s(x, 0), 1)$?

a) 0
b) x
c) $1 - x$
d) $x/2$
e) 1

Question 6 [10 marks]

Suppose we are interested in predicting who is at your front door. We shall assume that only neighbours or salespeople ever come to your front door. Based on your prior experiences opening your front door, you make the following observations.

30% of the time it is a neighbour.
70% of the time it is a salesperson.

If a neighbour comes to your front door, they come before noon 10% of the time.
If a salesperson comes to your front door, they come before noon 30% of the time.

If a neighbour comes to your front door, they ring the doorbell 40% of the time.
If a salesperson comes to your front door, they ring the doorbell 80% of the time.

Now, suppose you hear a knock at your front door (i.e. they did not ring the doorbell), and it is after noon. Use Bayes theorem with the naïve Bayes assumption to compute who is most likely to be at the front door and with what probability.

In your answer, include the following:

a) The probability the person at your front door is a neighbour.
b) The probability the person at your front door is a salesperson.
c) The most likely person at your front door given the evidence.

Define the following events:
N is the event neighbour is at the door
S is the event salesperson is at the door
M is the event someone is at the door before noon
D is the event the doorbell is rung

$P(N) = 0.3;\ P(S) = 0.7$
$P(M|N) = 0.1;\ P(M|S) = 0.3$
$P(D|N) = 0.4;\ P(D|S) = 0.8$

First, we compute the probability of the evidence

$$
\begin{aligned}
&P(not\ D\ and\ not\ M) \\
&= P(not\ D\ and\ not\ M|N)P(N) + P(not\ D\ and\ not\ M|S)P(S) \\
&= P(not\ D|N)P(not\ M|N)P(N) + P(not\ D|S)P(not\ M|S)P(S) \\
&= 0.6 * 0.9 * 0.3 + 0.2 * 0.7 * 0.7 \\
&= 0.26
\end{aligned}
$$

Now, compute the probability of each class given the evidence

$$
\begin{aligned}
&P(N|not\ D\ and\ not\ M) \\
&= P(not\ D\ and\ not\ M|N)P(N)/P(not\ D\ and\ not\ M) \\
&= P(not\ D|N)P(not\ M|N)P(N)/P(not\ D\ and\ not\ M) \\
&= 0.6 * 0.9 * 0.3\ /\ 0.26 \\
&= 0.62
\end{aligned}
$$

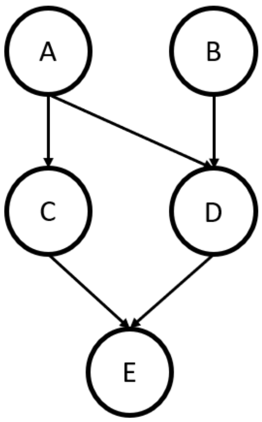
$$
\begin{aligned}
&P(S|not\ D\ and\ not\ M) \\
&= P(not\ D\ and\ not\ M|S)P(S)/P(not\ D\ and\ not\ M) \\
&= P(not\ D|S)P(not\ M|S)P(S)/P(not\ D\ and\ not\ M)
\end{aligned}
$$

$= 0.2 * 0.7 * 0.7 / 0.26$
$= 0.38$

It is most likely your neighbour.

Question 7 [10 marks]

Consider the Bayesian network illustrated below.



Write an expression for P(A = True, B = True | C = True). Assume all variables take on the values true or false. Use the variable elimination algorithm to eliminate repeated calculations in your expression.

$P(A = T, B = T | C = T)$
$= P(A = T, B = T, C = T) \;/\; P(C = T)$

First, consider the numerator:

$$
\begin{aligned}
&P(A = T, B = T, C = T) \\
&= \sum_d \sum_e P(A = T, B = T, C = T, D = d, E = e) \\
&= P(A = T)P(B = T)P(C = T|A = T) \sum_d P(D = d|A = T, B = T) \sum_e (E = e|C = T, D = d) \\
&= P(A = T)P(B = T)P(C = T|A = T) \sum_d P(D = d|A = T, B = T) \\
&= P(A = T)P(B = T)P(C = T|A = T)
\end{aligned}
$$

We have noted the two terms in the sums over d and e are both equal to one.

Next, consider the denominator:

$$
\begin{aligned}
&P(C = T) \\
&= \sum_a \sum_b \sum_d \sum_e P(A = a, B = b, C = T, D = d, E = e) \\
&= \sum_a P(A = a) \sum_b P(B = b) * P(C = T|A = a) \sum_d P(D = d|A = a, B = b) \sum_e P(E = e|C = T, D = d) \\
&= \sum_a P(A = a) * P(C = T|A = a) \sum_b P(B = b) \\
&= \sum_a P(A = a) * P(C = T|A = a) * f1 \\
&= f2
\end{aligned}
$$

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Where:

$$f1 = \sum_{b} P(B = b)$$

$$f2 = \sum_{a} P(A = a) * P(C = T|A = a) * f1$$

$$P(A = T, B = T|C = T) = \frac{P(A = T)P(B = T)P(C = T|A = T, B = T)}{f2}$$

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Question 8 [10 marks]

Consider a rule-based system for deciding what type of pizza to order. The rule-based system has the following rules.

IF size is large AND vegetarian is false THEN topping is pepperoni.
IF crust is thin AND vegetarian is true THEN topping is mushroom.
IF crust is thick THEN topping is extra-cheese.
IF appetite is low OR budget is low THEN size is small.
IF appetite is high THEN crust is thick.

Assume the propositions can take on the following values.
Topping is in {pepperoni, mushroom, extra-cheese}
Size is in {small, large}
Crust is in {thin, thick}
Vegetarian is in (true, false}
Appetite is in {low, high}
Budget is in {low, high}

The following propositions are askable and if asked, the user will provide the following values.
Vegetarian = false
Appetite = high
Budget = high

Perform data-based reasoning on this rule-based system to determine as much information as possible about the system.

In your answer, include the following:

a) The order in which the rules were applied.
b) The list of the values of all propositions, in the order they were determined.

Order rules were examined: 1, 2, 3, 4, 5; 1, 2, 3, 4, 5; 1, 2, 3
(no new information after an iteration over entire set of rules)

Working memory (in order added)
Appetite is high
Budget is high
Crust is thick (from rule 5)
Vegetarian is false
Topping is extra-cheese (from rule 3)

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Question 9 [10 marks]

Consider a fuzzy rule-based system for automatically controlling the windshield wipers in a car. In this example, assume the window wipers only have three discrete settings: high, low, intermittent. The following fuzzy rules are used. Use the Godel t-norm t(x, y) = min(x, y) and the Godel s-norm s(x, y) = max(x, y).

IF speed is high OR rain is heavy THEN setting is high.
IF speed is high AND rain is light THEN setting is low.
IF speed is low OR rain is light THEN setting is intermittent.

The fuzzy membership functions for speed is low and speed is high are trapezoidal (units are km/h), given by:

Speed is low: a = 0, b = 0, c = 30, d = 60
Speed is high: a = 30, b = 60, c = 100, d = 100

Rain intensity can be measured in millimetres per hour. The fuzzy membership functions for rain is light and rain is heavy are trapezoidal, given by:

Rain is light: a = 0, b = 0, c = 1, d = 3
Rain is heavy: a = 1, b = 3, c =10, d = 10

Suppose we are driving with speed 40km/h and rain intensity is 2 millimeters per hour. Determine the discrete setting for the windshield wipers.

In your answer, include the following:

a) The fuzzy truth value of each proposition.
b) The rule strength for each rule.
c) The output membership function for each rule.
d) The combined output membership function.
e) The crisp output value (i.e. high, low, or intermittent).

a)

$$T(\textit{speed is high}) = 0.33$$
$$T(\textit{speed is low}) = 0.67$$
$$T(\textit{rain is heavy}) = 0.5$$
$$T(\textit{rain is light}) = 0.5$$

b)
Rule 1 strength: 0.5
Rule 2 strength: 0.33
Rule 3 strength: 0.67

c)

$$m_{output1}(x) = \begin{cases} 0.5 \text{ if } x \text{ is high} \\ 0 \text{ otherwise} \end{cases}$$
$$m_{output2}(x) = \begin{cases} 0.33 \text{ if } x \text{ is low} \\ 0 \text{ otherwise} \end{cases}$$
$$m_{output3}(x) = \begin{cases} 0.67 \text{ if } x \text{ is intermittent} \\ 0 \text{ otherwise} \end{cases}$$

d)

$$m_{combined}(x) = \begin{cases} 0.5 \ if \ x \ is \ high \\ 0.33 \ if \ x \ is \ low \\ 0.67 \ if \ x \ is \ intermittent \end{cases}$$

e) Windshield wipers should be intermittent.