COMP 1805 - Winter 2021 Supplemental Sheet

Standard Sums

$$\sum_{i=1}^{n} k = kn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=0}^{n} ax^i = \frac{ax^{n+1}-a}{x-1}, x \neq 0, x \neq 1$$

Definition of O, Ω and Θ

$f(n) \in O(g(n))$ provided that $f(n) \leq cg(n), \forall n \geq n_0$, for constants $c, n_0 > 0$.

$f(n) \in O(g(n))$ provided that $\lim_{n\to\infty} f(n)/g(n) \leq c$ for constant $c > 0$.

$f(n) \in \Omega(g(n))$ provided that $f(n) \geq cg(n), \forall n \geq n_0$, for constants $c, n_0 > 0$.

$f(n) \in \Omega(g(n))$ provided that $\lim_{n\to\infty} f(n)/g(n) \geq c$ for constant $c > 0$.

$f(n)$ is $\Theta(g(n))$ provided that $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$.

Logic and Set Rules

$T =$ True $F =$ False		$\mathcal{U} =$ Universe $\emptyset = \{\}$ (empty set)
$A \wedge T \equiv A$ $A \vee F \equiv A$	Identity Rules	$A \cap \mathcal{U} = A$ $A \cup \emptyset = A$
$A \wedge F \equiv F$ $A \vee T \equiv T$	Domination Rules	$A \cap \emptyset = \emptyset$ $A \cup \mathcal{U} = \mathcal{U}$
$A \vee A \equiv A$ $A \wedge A \equiv A$	Idempotent	$A \cup A = A$ $A \cap A = A$
$\neg(\neg A) \equiv A$	Double Negation	$\overline{\overline{A}} = A$
$A \vee \neg A \equiv T$	Law of excluded middle	$A \cup \overline{A} = \mathcal{U}$
$A \wedge \neg A \equiv F$	Contradiction	$A \cap \overline{A} = \emptyset$
$\neg(A \vee B) \equiv \neg A \wedge \neg B$ $\neg(A \wedge B) \equiv \neg A \vee \neg B$	De Morgan's Laws	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
$A \vee B \equiv B \vee A$ $A \wedge B \equiv B \wedge A$	Commutative Laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
$((A \vee B) \vee C) \equiv (A \vee (B \vee C))$ $((A \wedge B) \wedge C) \equiv (A \wedge (B \wedge C))$	Associativity Laws	$((A \cup B) \cup C) = (A \cup (B \cup C))$ $((A \cap B) \cap C) = (A \cap (B \cap C))$
$(A \wedge (B \vee C)) \equiv (A \wedge B) \vee (A \wedge C)$ $(A \vee (B \wedge C)) \equiv (A \vee B) \wedge (A \vee C)$	Distributivity Laws	$(A \cap (B \cup C)) = (A \cap B) \cup (A \cap C)$ $(A \cup (B \cap C)) = (A \cup B) \cap (A \cup C)$
$A \to B \equiv \neg A \vee B$	Implication Relation	
$A \to B \equiv \neg B \to \neg A$	Contrapositive	
$A \vee (A \wedge B) \equiv A$ $A \wedge (A \vee B) \equiv A$	Absorption Laws	$(A \cap (A \cup B)) = A$ $(A \cup (A \cap B)) = A$
$A \leftrightarrow B \equiv (A \to B) \wedge (B \to A)$	Bidirection	
	Difference Equivalence	$A \setminus B = A \cap \overline{B}$ $A - B = A \cap \overline{B}$
$A \oplus B \equiv (A \wedge \neg B) \vee (\neg A \wedge B)$	Exclusive OR	

Inference Rules

$$\left.\begin{array}{l} A \\ \hline \therefore\ A \lor Q \end{array}\right\}$$ Addition

$$\left.\begin{array}{l} A \land B \\ \hline \therefore\ A \end{array}\quad \begin{array}{l} A \land B \\ \hline \therefore\ B \end{array}\right\}$$ Simplification

$$\left.\begin{array}{l} A \to B \\ A \\ \hline \therefore\ B \end{array}\right\}$$ Modus ponens

$$\left.\begin{array}{l} A \to B \\ \neg B \\ \hline \therefore\ \neg A \end{array}\right\}$$ Modus tollens

$$\left.\begin{array}{l} A \lor B \\ \neg A \\ \hline \therefore\ B \end{array}\right\}$$ Disjunctive Syllogism

$$\left.\begin{array}{l} A \\ B \\ \hline \therefore\ A \land B \end{array}\right\}$$ Conjunction

$$\left.\begin{array}{l} A \to B \\ B \to C \\ \hline \therefore\ A \to C \end{array}\right\}$$ Transitivity (Hypothetical Syllogism)

$$\left.\begin{array}{l} A \lor B \\ \neg A \lor C \\ \hline \therefore\ B \lor C \end{array}\right\}$$ Resolution

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$$\left.\begin{array}{l} \forall x\ P(x) \\ \hline \therefore\ P(c) \\ \quad \text{for } \textbf{any}\ c \in U \end{array}\right\}$$ Universal instantiation

$$\left.\begin{array}{l} \exists x\ P(x) \\ \hline \therefore\ P(c) \\ \quad \text{for } \textbf{some}\ c \in U \end{array}\right\}$$ Existential instantiation

$$\left.\begin{array}{l} P(c) \text{ for } \textbf{any}\ c \in U \\ \hline \therefore\ \forall x\ P(x) \end{array}\right\}$$ Universal generalization

$$\left.\begin{array}{l} P(c) \text{ for } \textbf{some}\ c \in U \\ \hline \therefore\ \exists x\ P(x) \end{array}\right\}$$ Existential generalization