New Jersey Institute of Technology
DEPARTMENT OF MATHEMATICAL SCIENCES
Math 111-029 Quiz 1

Your Name: Exam Key   PROF. ALLAIRE

1. **(Winter is Coming)** In Season 7 of HBO show Game of Thrones, Gendry departed his friends from some location, point **A**, and ran to another place, point **B**, through some treacherous terrain to call for backup. Suppose the distance from **A** to **B** is roughly more than a marathon, say **27 miles**. The fastest marathon time is around 2 hours. Lets cut Gendry some slack and say it took him **3 hours**. What was his average speed? For those who have watched this episode, do you think this speed is feasible?

Avg Speed = $\frac{\text{Distance Traveled}}{\text{Time Elapsed}} = \frac{27 \text{ miles}}{3 \text{ hours}} = 9$ mph.

(+2)

2. Find the equation of the **tangent line** to the function $\mathbf{f(x) = 3x^2}$ at $\mathbf{x = 1}$ by first approximating the slope of the secant line using an interval $[1, 1+h]$ and then investigating h at 0. Write the final answer in slope-intercept form ($y = mx + b$). Hint: Avg. rate of change in interval $[x, x+h]$: $\frac{f(x+h)-f(x)}{h}$.

+2

m_{sec} $\frac{f(1+h) - f(1)}{h} = \frac{3(1+h)^2 - 3(1)^2}{h} = \frac{3(1+2h+h^2) - 3}{h}$

$= \frac{6h + 3h^2}{h} = 6 + 3h$. As $h \to 0$ $m_{sec} \to 6 = m_{tan}$

$y - 3 = 6(x-1) \Rightarrow y = 6x - 3$

3. Evaluate the following limits. If the limit does not exist, explain why.

(+2) (a) $\lim_{x \to 1} \frac{x^2+2}{x+1} = \frac{1^2+2}{1+1} = \frac{3}{2}$

+2

(b) $\lim_{x\to 0} \frac{\frac{1}{x+4}+\frac{1}{x-4}}{x}$ $= \lim_{x\to 0} \frac{\frac{1}{x+4}+\frac{1}{x-4}}{x} \cdot \frac{(x+4)(x-4)}{(x+4)(x-4)}$ +1

$= \lim_{x\to 0} \frac{(x-4)+(x+4)}{x(x+4)(x-4)} = \lim_{x\to 0} \frac{2}{(x+4)(x-4)}. = \frac{-2}{16} = \frac{-1}{8}$

+1

(c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim_{x\to 4} \frac{(x-4)}{(x-4)(\sqrt{x}+2)}$ +1

$= \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{2+2} = \frac{1}{4}$ +1

+2