Math 111 Exam #1
September 28, 2016

Time: 1 hour and 25 minutes
Instructions: Show all work for full credit. No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

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ID #: Exam Solutions!

Instructor/Section: ____________

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

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Problem(s)	Score	Total

1. Show that the graph of $f(x) = 3x^3 + 5x - 11 = 0$ has a solution between x=1 and x=2. State which theorem you use to show this. **(7 points)**

(i) $f(1) = 3(1)^2 + 5(1) - 11 = -3 < 0$

(ii) $f(2) = 3(2)^2 + 5(2) - 11 = 11 > 0$

(iii) f is continuous since it is a polynomial.

(iv) By IVT there is a point c between $x=1$ and $x=2$ such that $f(c) = 0$

$\boxtimes$.

2. Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why. Show all work. **(10 points)**

a. $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}\left(\frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}\right) = \lim_{x \to 0} \frac{(x^2+9)-(9)}{x^2(\sqrt{x^2+9}+3)}$$

$$= \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)} = \lim_{x \to 0} \frac{1}{(\sqrt{x^2+9}+3)}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$

b. $\lim_{x \to 0} \frac{-x^2+5x-6}{x^2-4} = \lim_{x \to 0} \frac{-(x^2-5x+6)}{(x-2)(x+2)}$

$$= \lim_{x \to 0} \frac{-(x-3)(x-2)}{(x-2)(x+2)} = \frac{-6}{-4} = \frac{3}{2}$$

Note: we could have just plugged it in!

3. Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why. Show all work. **(10 points)**

a. $\lim_{x\to 0} \frac{\tan(2x)}{x}$

$$\lim_{x\to 0} \frac{\tan(2x)}{x} = \lim_{x\to 0} \frac{\sin(2x)}{x} \cdot \frac{1}{\cos(2x)}$$

$$= \lim_{x\to 0} \frac{\sin(2x)}{2x} \cdot \frac{2}{\cos(2x)} = 1 \cdot \frac{2}{1} = 2$$

Since $\lim_{x\to 0} \frac{\sin x}{x} = 1$

b. $\lim_{x\to\infty} \frac{x^2\sqrt{x}-4x+8}{3x^3}$

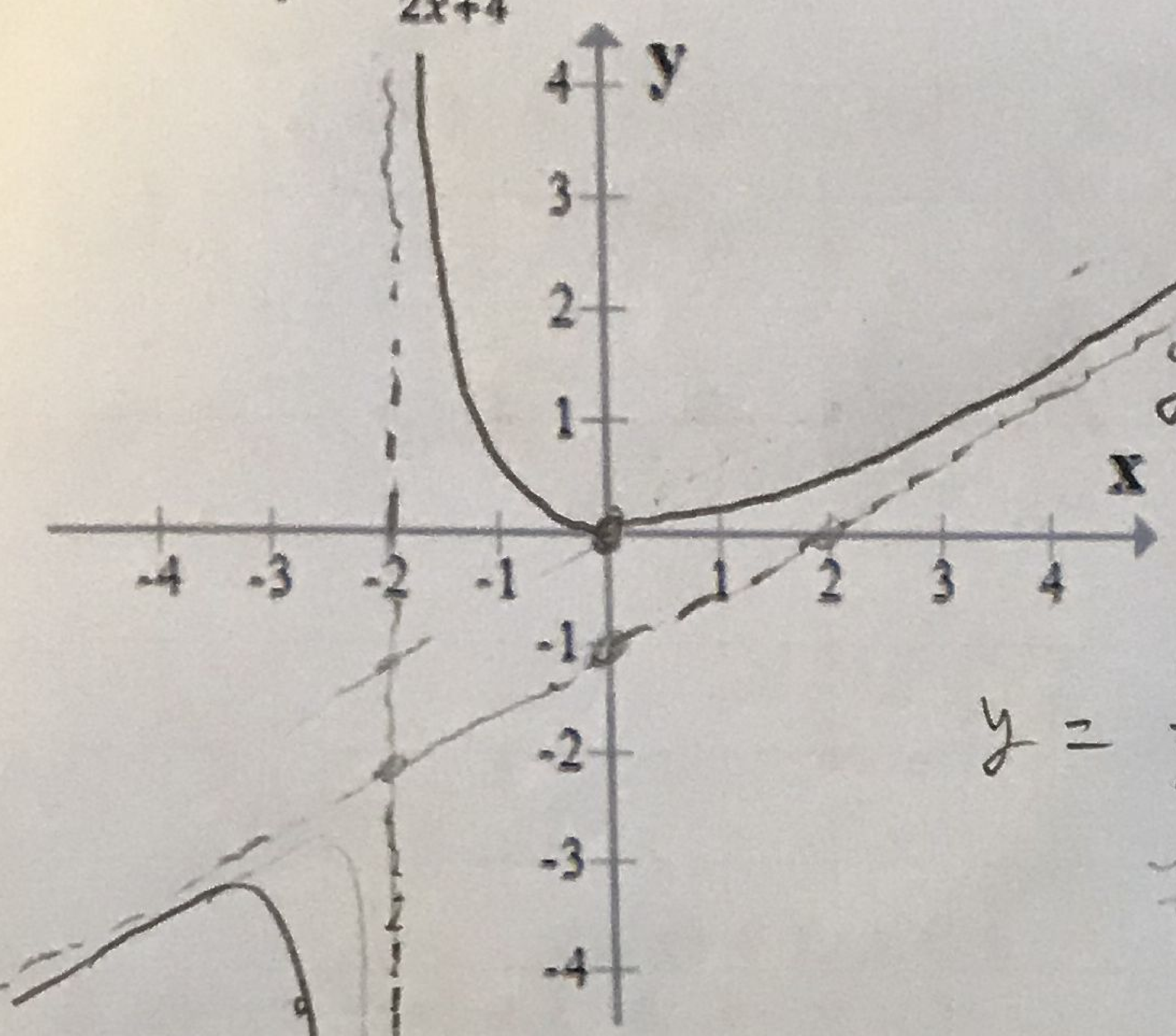
$$\lim_{x\to\infty} \frac{x^2\sqrt{x}-4x+8}{3x^3} = \lim_{x\to\infty} \frac{x^{5/2}-4x+8}{3x^3}$$

$$= \lim_{x\to\infty} \frac{x^{5/2}/x^3 - \frac{4}{x^2} + \frac{8}{x^3}}{3} = \lim_{x\to\infty} \frac{\left(\frac{1}{x^{1/2}} - \frac{4}{x^2} + \frac{8}{x^3}\right)}{3}$$

$$= \frac{0-0+0}{3} = 0$$

4. Graph the following rational functions. Find the equations of any asymptotes and include them on the graph, along with any x and y intercepts (**16 points**)

a. $y = \frac{x^2}{2x+4}$



Step 1: Division

$$\begin{array}{r} \frac{1}{2}x - 1 \\ 2x+4 \overline{) x^2} \\ -(x^2+2x) \\ \hline -2x \\ -(-2x-4) \\ \hline 4 \end{array}$$

$$y = \frac{x^2}{2x+4} = \left(\frac{x}{2}-1\right) + \frac{4}{2x+4}$$

$f(x) = y$

x	y
-3	$\frac{9}{-2} = -4.5$

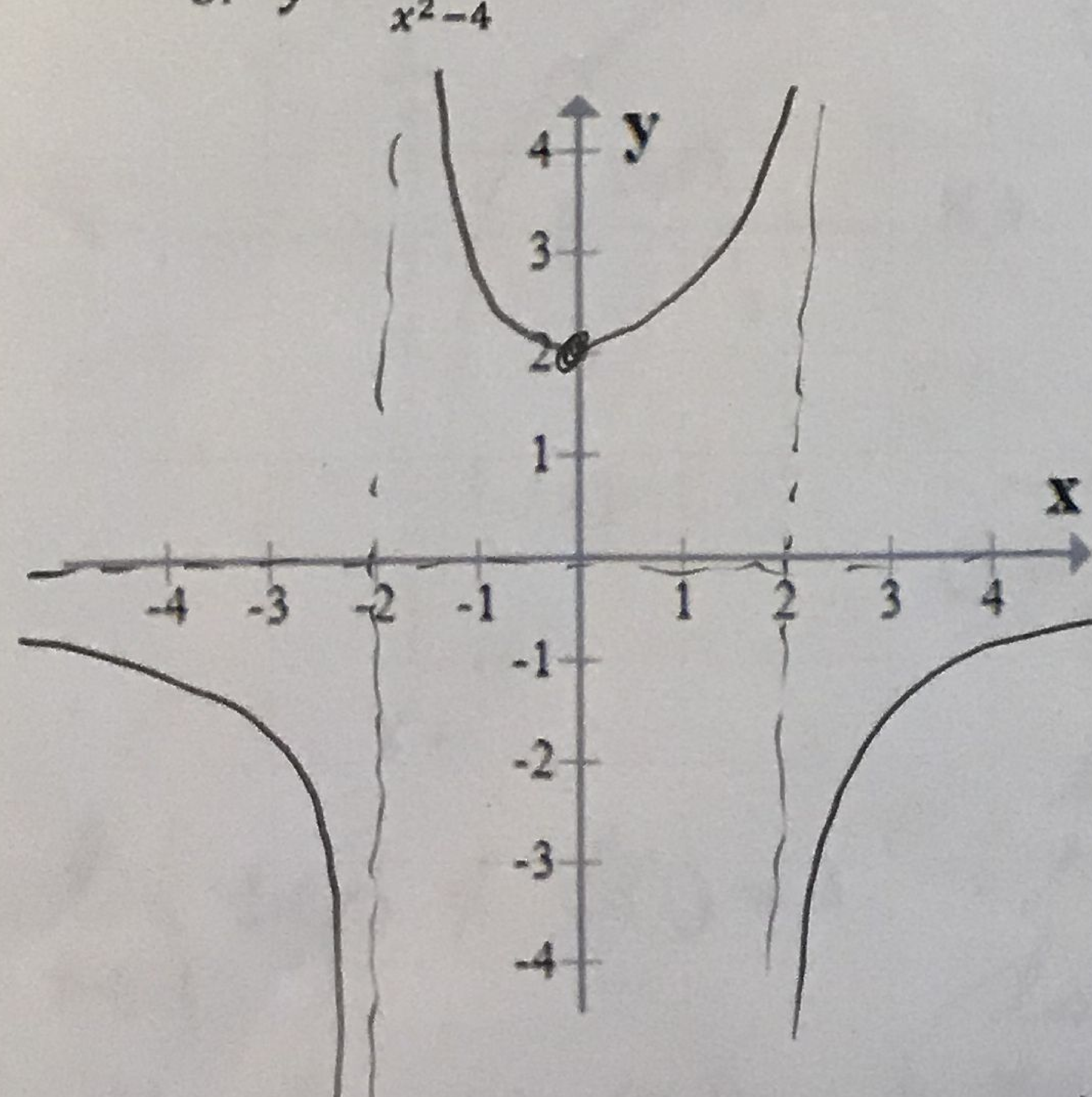
$\boxed{H.A}$ None since $\lim_{x \to \pm\infty} f(x) = \pm\infty$ (Num. is higher than denom by 1 power)

$\boxed{V.A}$: $x = -2$ since $\lim_{x\to -2^+} f(x) = 0 + \lim_{x \to -2^+} \frac{4}{2x+4} = 0 + \infty = +\infty$.

$\boxed{\text{oblique}}$: $y = \left(\frac{x}{2} - 1\right)$ Since for large x $\frac{4}{2x+4}$ is small.

y-intercept: $(0,0)$ (also x-intercept)

b. $y = \frac{-8}{x^2-4}$



V.A. $x = 2$ $\lim_{x \to 2^+} \frac{-8}{x^2-4} = -\infty$

$\lim_{x \to 2^-} \frac{-8}{x^2-4} = +\infty$

$x = -2$ $\lim_{x \to -2^+} \frac{-8}{x^2-4} = +\infty$

$\lim_{x \to -2^-} \frac{-8}{x^2-4} = -\infty$

H.A.: $\boxed{y=0}$, since $\lim_{x \to \pm\infty} \frac{-8}{x^2-4} = 0$

No oblique.

Intercepts.

y-int $(0,2)$

x-int none.

$\frac{-8}{x^2-4} \neq 0$ for any x

5. For what value(s) of the constant 'a' would f(x) be continuous at every x? **(5 points)**

$$f(x) = \begin{cases} a^2x - 2a & x \ge 2 \\ 12 & x < 2 \end{cases}$$

(i) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\lim_{x\to 2} (a^2x-2a) = 2a^2 - 2a = \lim_{x\to 2^+}(12) = 12$

$2a^2 - 2a = 12$

$a^2 - a = 6$

$a^2 - a - 6 = 0$

-6

$-3 \quad 2$

$(a-3)(a+2) = 0$

$a = 3, -2$

f is cont. everywhere besides 2 already.

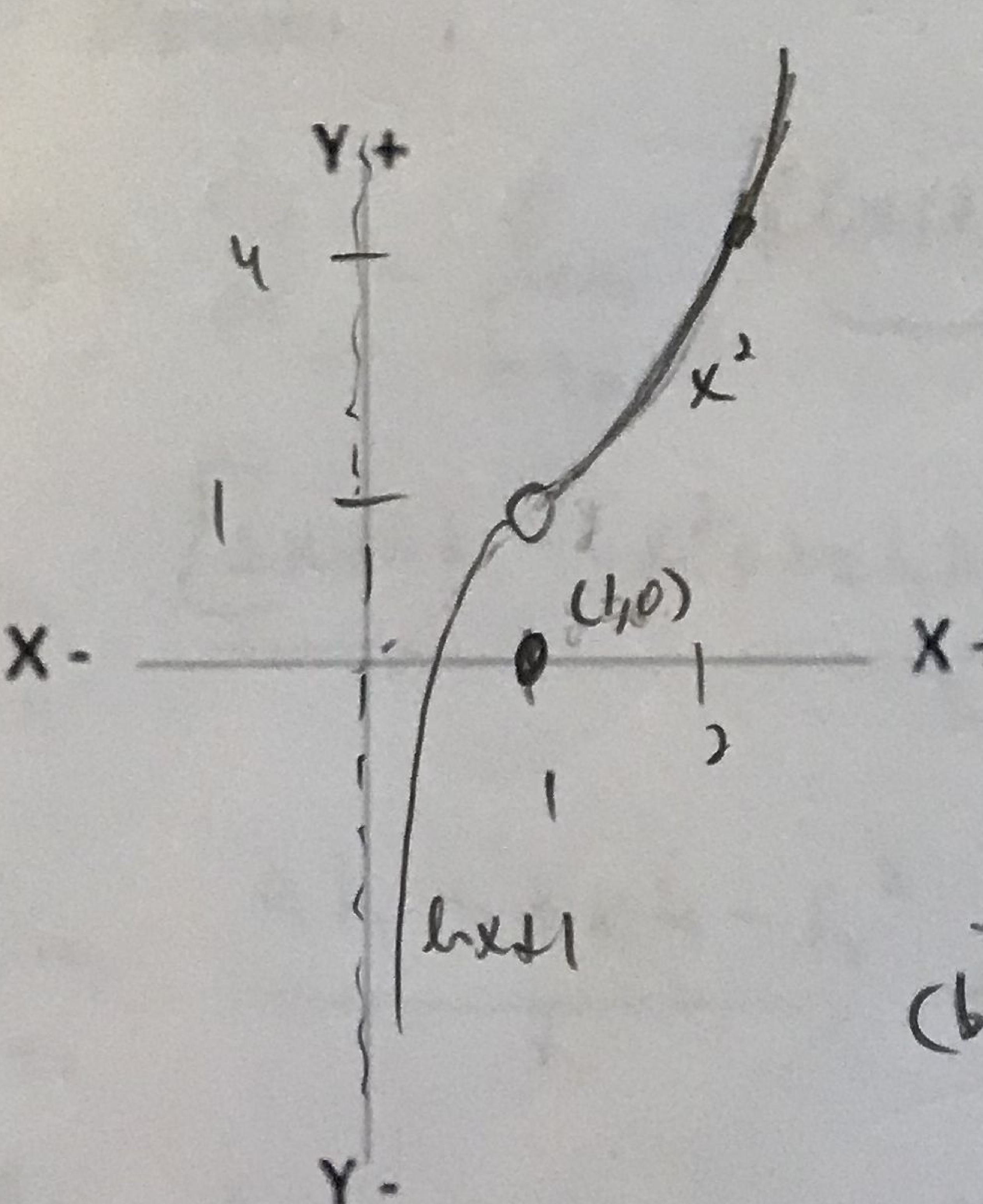
6. Given $f(x) = \begin{cases} \ln(x) + 1, & x < 1 \\ 0, & x = 1 \\ x^2, & x > 1 \end{cases}$

a. Sketch f(x) **(4 points)**

b. Find $\lim_{x\to 1} f(x)$. Show all work, including left and right limits. **(4 points)**

c. Determine if the graph is continuous at x=1 **(4 points)**

x	$x<1$	$x=1$	$x>1$
x	$\ln(x)+1$	0	x^2
1	1 (open)	0 (closed)	1 (open)



(a)

(i) $\ln(x)+1$ has V.A. at $x=0$.

(ii) $f(2) = 2^2 = 4$

(b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} \ln(x)+1 = 0+1 = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} x^2 = 1^2 = 1$

$\lim_{x\to 1} f(x) = 1$

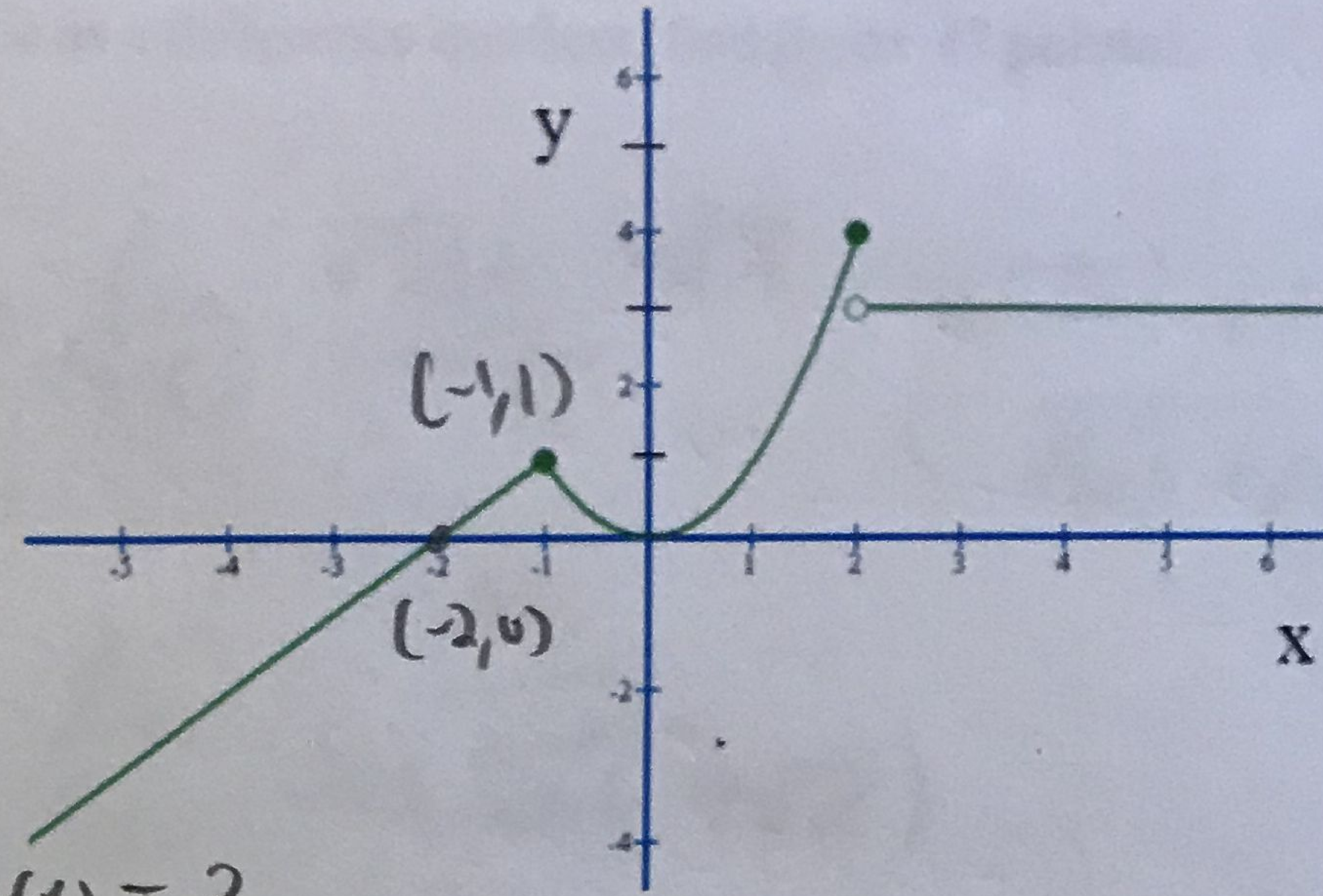
(c) $\lim_{x\to 1} f(x) \ne f(1) = 0$

$\therefore$ f is not continuous!

7. Given the graph of the piecewise function f(x), answer the following **(6 points)**:

a. Find $\lim_{x \to 2} f(x)$ or explain why it does not exist (Show all work, including left and right limits)

b. Find f'(-4), the derivative of the function at x = – 4



(a) $\lim_{x \to 2} f(x)$ DNE since

$\lim_{x \to 2^-} f(x) = 4$ and $\lim_{x \to 2^+} f(x) = 3$

(b) for $x < -1$ $f(x)$ is a straight line w/ slope $\frac{1-0}{-1-(-2)} = \frac{1}{1} = 1$

Therefore $f'(-4) = 1$ Since $-4 < -1$.

8. Using the definition of the derivative as a difference quotient, find dy/dx for $y = 2x - x^2$ **(7 points)**

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[2(x+h) - (x+h)^2] - (2x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{[2x + 2h - (x^2 + 2xh + h^2)] - (2x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{2h - 2xh - h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2 - 2x - h = 2 - 2x$$

9. Consider the function $y = \sqrt{x}$

a. Using the definition of the derivative as a difference quotient, find dy/dx **(7 points)**

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \lim_{h\to 0} \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{x\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

b. Find the equation of the tangent line to this curve at x = 4 **(5 points)**

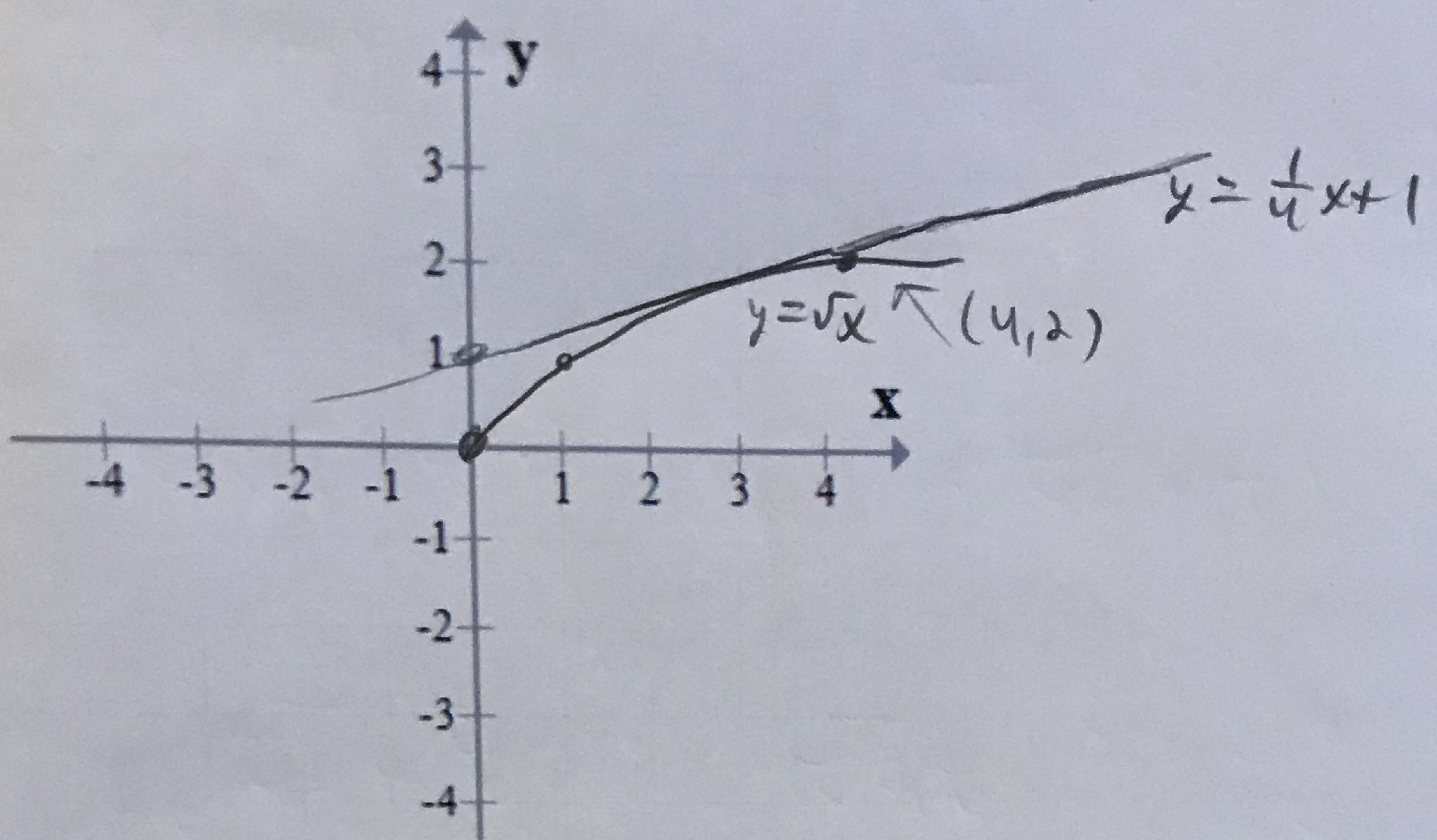
$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4} = m$$

Use point $(4, 2)$ since $f(4) = \sqrt{4} = 2$.

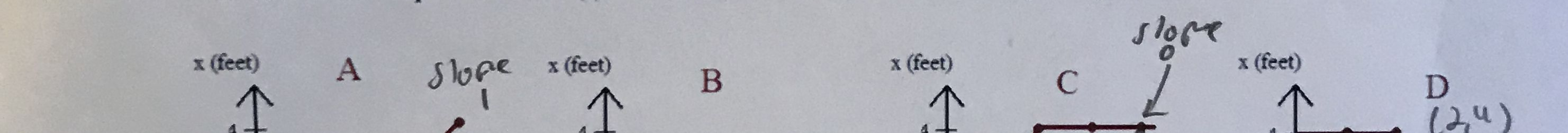
$$y - 2 = \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x + 1$$

c. (continued from previous page): Graph both the function and its tangent line on the following xy-axis (show all x and y-intercepts): **(5 points)**



10. The graphs below show the motion of four particles (A, B, C, and D) and their position x(t) in feet with respect to time (t) in seconds. Answer the following: **(2 points each)**



a. What is the average speed of particle C between 0 and 4 seconds? $4 \text{ ft}/4 \text{ sec} = 1 \text{ ft}/\text{sec}$

b. Which particle is moving fastest at t=1 second? D velocity $= -4/2$ ft/sec $= -2$ ft/sec

c. What is the velocity of the particle from part (b) at t=1 second? -1 ft/sec

d. Which particle(s) have a velocity of 0 feet/sec at t=3 seconds? (C)

e. Which particle(s) are moving during the entire time between 0 and 4 seconds? A, B, D.