New Jersey Institute of Technology
DEPARTMENT OF MATHEMATICAL SCIENCES
Math 111-029 Quiz 6

Your Name: Exam Solutions PROF. ALLAIRE

+2 1. (a) Find the linear approximation, $L(x)$ of the function $f(x) = \ln(x-2)$ at $x = 3$.

Step 1: $f'(x) = \frac{1}{x-2}$

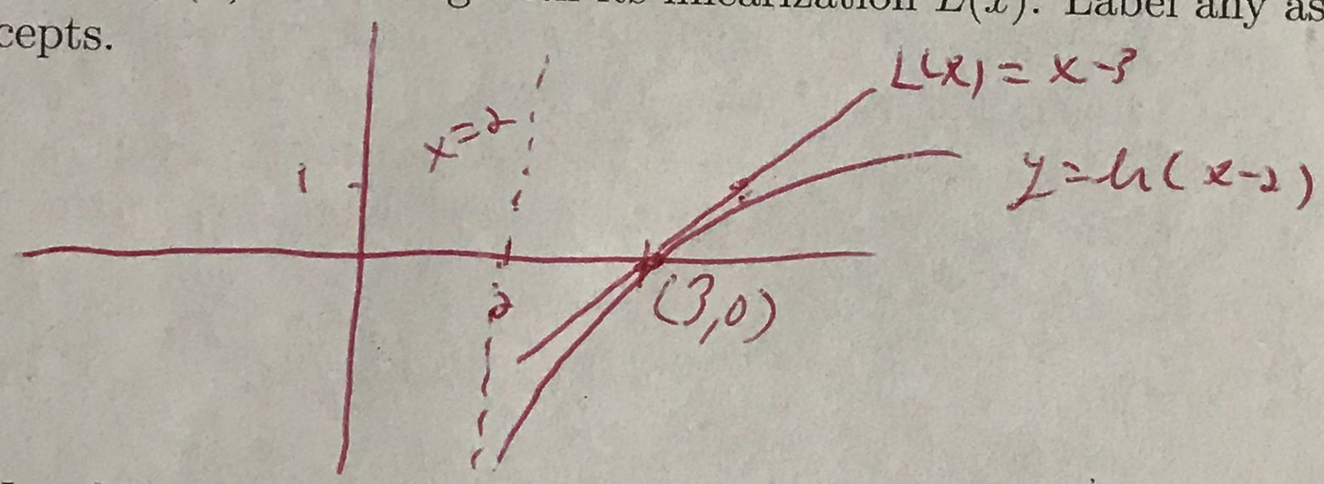
$m = f'(3) = \frac{1}{3-2} = 1$

Step 2: Pt. $(3, 0)$

Step 3: $y - 0 = 1(x-3)$

$L(x) = x - 3$

+1 (b) Sketch $f(x)$ and along with its linearization $L(x)$. Label any asymptotes and intercepts.



+1 (c) Use this approximation to estimate $\ln(1.1)$.

$\ln(3.1-2) = \ln(1.1) = f(3.1) \approx L(3.1) = 3.1 - 3 = 0.1$

+1 (d) Use a calculator to provide the true value of $\ln(1.1)$. What is the **relative** error in this approximation (rel. err = (approx. - true)/true)?

True $\approx$ 0.0953

Rel. err. $= \frac{0.1 - 0.0953}{0.0953} \approx 0.04921$

+1 (e) What is the **percentage** error in this approximation?

% error = rel. err $\times$ 100% = 4.92%

(+2) 2. The height and radius of a right circular cylinder are equal, so the cylinder's volume is $V = \pi h^3$. The volume is to be calculated with an error of no more than 1% of the true value. Find approximately the greatest error that can be tolerated in the measurement of h, expressed as a percentage of h. (Hint: dV/V measures the relative error in Volume; $100\% \times dV/V$ measures the percentage error in volume; What does dh/h measure?)

$V = \pi h^3$

$dV = 3\pi h^2 dh$

$\frac{dV}{V} = \frac{3\pi h^2 dh}{V} = \frac{3\pi h^2 dh}{\pi h^3} =$

$= 3\frac{dh}{h}$

$\frac{dh}{h} = \frac{1}{3}\frac{dV}{V} \leq \frac{1}{3}(0.01)$

$= 0.00\overline{3}$

(.33%)

(+2) 3. The formula $V = kr^4$, discovered by the physiologist Jean Poiseuille (1797-1869), allows us to predict how much the radius of a partially clogged artery of radius r has to be expanded in order to restore normal blood flow. The formula says that the volume V of blood flowing through the artery in a unit of time at a fixed pressure is a constant k times the radius of the artery to the fourth power. How will a 10% increase in r affect V?

$V = kr^4$

$dV = 4kr^3 dr$

$\frac{dV}{V} = \frac{4kr^3 dr}{V} = \frac{4kr^3 dr}{kr^4}$

$= 4\frac{dr}{r}$

A 10% increase to r makes $\frac{dr}{r} = 0.1$

and $\frac{dV}{V} = 4(0.1) = 0.4$ or

a (40%) increase to V.