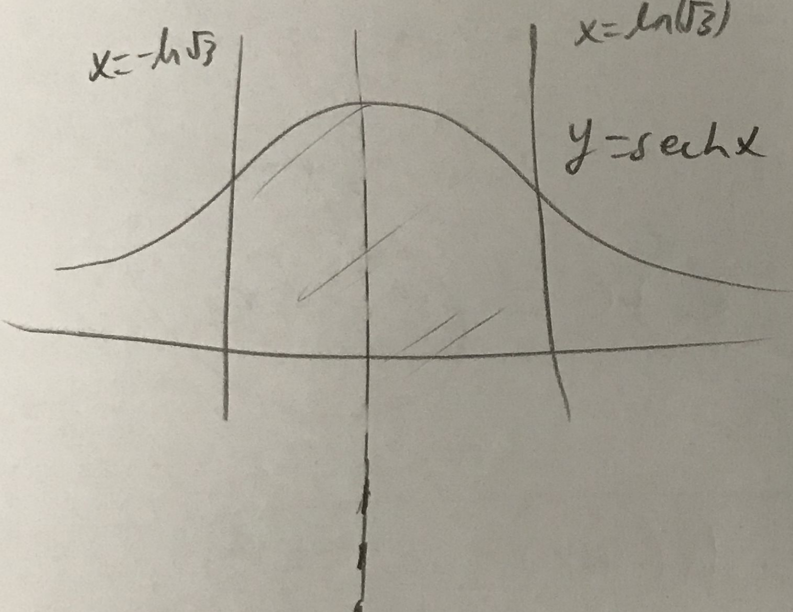
Math 112 HW #3

Section 7.3 (+4)

#80: Volume The region enclosed by the curve $y = \operatorname{sech} x$, the x-axis, and the lines $x = \pm \ln(\sqrt{3})$ is revolved around about the x-axis to generate a solid. Find the volume of the solid



Disks.

$$V = \pi \int_{-\ln\sqrt{3}}^{\ln\sqrt{3}} \operatorname{sech}^2 x \, dx = \pi \tanh x \Big|_{-\ln\sqrt{3}}^{\ln\sqrt{3}}$$

$$= \pi \left[\tanh(\ln\sqrt{3}) - \tanh(-\ln\sqrt{3}) \right]$$

$$= \pi \left[\frac{e^{\ln\sqrt{3}} - e^{-\ln\sqrt{3}}}{e^{\ln\sqrt{3}} + e^{-\ln\sqrt{3}}} - \frac{e^{-\ln\sqrt{3}} - e^{\ln\sqrt{3}}}{e^{-\ln\sqrt{3}} + e^{\ln\sqrt{3}}} \right]$$

$$= \pi \left[\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{3} + \frac{1}{\sqrt{3}}} - \frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{\frac{1}{\sqrt{3}} + \sqrt{3}} \right]$$

1

$$\pi\left[\frac{2}{4} - \frac{(-2)}{4}\right] = \pi$$

Section 8.1: (+3)

#34 $\int e^{z+e^z}\,dz$ Let $u = e^z$, $du = e^z\,dz$

$$= \int e^z \cdot e^{e^z}\,dz$$

$$= \int e^u\,du = e^u + C = e^{e^z} + C$$

#43 Arc length $y = \ln(\cos x)$ $0 \le x \le \pi/3$ (+3)

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x$$

$$L = \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\,dx = \int_0^{\pi/3} \sec x\,dx.$$

$$= \ln|\sec x + \tan x|\Big|_0^{\pi/3} = \ln|\sec(\pi/3) + \tan(\pi/3)| - \ln|\sec(0) + \tan(0)|$$

$$= \ln|2+\sqrt{3}| - \ln|1| = \ln(2+\sqrt{3})$$