Math 112 - HW #4 (1pt)

Section 8.2:

#28 $\int \ln(x+x^2)\,dx = \int \ln(x(1+x))\,dx$

$= \underbrace{\int \ln x\,dx}_{\text{Parts}} + \int \underbrace{\ln(1+x)}_{\text{Sub.}}\,dx$

$u = \ln x \quad dv = 1\,dx \qquad$ Let $z = 1+x$

$du = \frac{1}{x}dx \quad v = x \qquad dz = dx$

$x\ln x - \int 1\,dx + \int \ln z\,dz$

$= x\ln x - x + (z\ln z - z)$ same as other one.

$= x\ln x - x + (1+x)\ln(1+x) - (1+x)$

$= x\ln(x(x+1)) + \ln(1+x) - 2x \underbrace{-1}_{\text{call } c} + c$

$= x\ln(x+x^2) + \ln(1+x) - 2x + c$

(1 pt)

#38 $\int x^5 e^{x^3}\,dx = \int x^3 \cdot x^2 e^{x^3}\,dx$ Let $z = x^3$

$dz = 3x^2\,dx$

$= \frac{1}{3}\int z e^z\,dz = \frac{1}{3}\left[ze^z - \int e^z\,dz\right]$

$u = z \quad dv = e^z$
$du = dz \quad v = e^z$

$= \frac{1}{3} z e^z - \frac{1}{3} e^z + C$

$= \frac{1}{3} x^3 e^{x^3} - 1/3\, e^{x^3} + C$

#46 $\int \sqrt{x}\, e^{\sqrt{x}}\,dx = \int \frac{x e^{\sqrt{x}}}{\sqrt{x}}\,dx$ Let $z = \sqrt{x}$ (2pts.)

$dz = \frac{1}{2\sqrt{x}}\,dx$

$= 2\int z^2 e^z\,dz$ Let $u = z^2 \quad dv = e^z\,dz$
$du = 2z\,dz \quad v = e^z$

$= 2\left[z^2 e^z - 2\int z e^z\,dz\right]$ $u = z \quad dv = e^z\,dz$
$du = dz \quad v = e^z$

$= 2\left[z^2 e^z - 2\left[ze^z - \int e^z\,dz\right]\right] =$

$2\left[z^2 e^z - 2ze^z + 2e^z\right] = 2z^2 e^z - 4ze^z + 4e^z + C$

$= \boxed{2xe^{\sqrt{x}} - 4\sqrt{x}\, e^{\sqrt{x}} + 4e^{\sqrt{x}} + C}$

(4)

(1)

(2pts.)

#53 $y = x\sin x$ $\quad$ x-axis $\quad$ Area enclosed.

(a) $0 \le x \le \pi$

(b) $\pi \le x \le 2\pi$

(c) $2\pi \le x \le 3\pi$

(a)
$$A = \int_0^{\pi} x\sin x \, dx \qquad u = x \quad dv = \sin x\, dx$$
$$du = dx \quad v = -\cos x$$

$$= -x\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x \, dx = -x\cos x + \sin x \Big|_0^{\pi}$$

$$= \pi$$

(TOP curve is $y=0$)

(b)
$$A = -\int_{\pi}^{2\pi} x\sin x \, dx = -\left[-x\cos x + \sin x\right]_{\pi}^{2\pi}$$

$$= x\cos x - \overset{0}{\sin x}\Big|_{\pi}^{2\pi} = 2\pi + (\pi(-1)) = 3\pi$$

(c)
$$A = \int_{2\pi}^{3\pi} x\sin x \, dx = -x\cos x + \overset{0}{\sin x}\Big|_{2\pi}^{3\pi} =$$

$$= 3\pi - (-2\pi) = 5\pi$$

(5)

(d) pattern $\quad n = 0, 1, 2, \ldots$

$$A = \int_{n\pi}^{(n+1)\pi} |x\sin x - 0| \, dx = (-1)^n \int_{n\pi}^{(n+1)\pi} x \sin x \, dx$$

notice this

$$(-1)^n \left[-x\cos x + \cancel{\sin x}^{\,0} \Big|_{n\pi}^{(n+1)\pi} \right] = (-1)^n \left[-(n+1)\pi \cos((n+1)\pi) + n\pi \cos(n\pi) \right]$$

Note $\cos(n\pi) = (-1)^n$ &

$$A = (-1)^n \left[-(n+1)\pi (-1)^{n+1} + n\pi (-1)^n \right]$$

$$= (-1)^n \left[(n+1)\pi (-1)^{n+2} + n\pi (-1)^n \right]$$

Note $(-1)^{n+2} = (-1)^n$ for $n = 0, 1, 2, \ldots$

$$= (-1)^n \left[(2n+1)\pi (-1)^n \right] = (-1)^{2n} (2n+1)\pi$$

Note $(-1)^{2n} = 1$ since $2 \cdot n$ is even.

$A = (2n+1)\pi \quad n = 0, 1, 2, \ldots$

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Section 8.3 (1 pt)

#63

$$\int \frac{\sec^3 x}{\tan x} = \int \frac{\sec^2 x \sec x}{\tan x}\, dx$$

$$= \int \frac{(1+\tan^2 x)\sec x}{\tan x}\, dx = \int \frac{\sec x}{\tan x} + \sec x \tan x \, dx.$$

$$= \int \csc x \, dx + \sec x$$

$$= -\ln|\csc x + \cot x| + \sec x + C$$

①

(1 pt)

#64 $\int \frac{\sin^3 x}{\cos^4 x}\,dx = \int \frac{\sin^2 x \cdot \sin x}{\cos^4 x}\,dx$

$= \int \frac{(1-\cos^2 x)\sin x}{\cos^4 x}\,dx = \int \frac{\sin x}{\cos^4 x}\,dx - \int \frac{\sin x}{\cos^2 x}\,dx$

Let $u = \cos x$

$du = -\sin x\,dx$

$= \int -u^{-4}\,du + \int u^{-2}\,du$

$= \frac{u^{-3}}{3} - u^{-1} = \frac{1}{3}\sec^3 x - \sec x + C$

(2 pts)

#68

$$\int x\cos^3 x\,dx = \int x \cos^2 x \cdot \cos x \, dx$$

$$= \int x\,(1-\sin^2 x)\cdot \cos x \, dx$$

$$= \int x\cos x\,dx - \int x\cos x \sin^2 x\,dx.$$

$u = x$ $\quad dv = \cos x\,dx$

$du = dx$ $\quad v = \sin x$

$u = x$ $\quad dv = \cos x \sin^2 x\,dx$

$du = dx$ $\quad v = \frac{1}{3}\sin^3 x$

$$= x\sin x - \int \sin x\,dx - \left[\frac{1}{3}x\sin^3 x - \frac{1}{3}\int \sin^3 x\,dx\right]$$

$$= x\sin x + \cos x - \frac{1}{3}x\sin^3 x + \frac{1}{3}\int \sin^2 x \sin x\,dx$$

$$= x\sin x + \cos x - \frac{1}{3}x\sin^3 x + \frac{1}{3}\int (1-\cos^2 x)\sin x\,dx.$$

$u = \cos x$

$du = -\sin x\,dx.$

$$= x\sin x + \cos x - \frac{1}{3}x\sin^3 x - \frac{1}{3}\int (1-u^2)\,du$$

$$= x\sin x + \cos x - \frac{1}{3}x\sin^3 x \cdot \frac{1}{3}\left[u - u^3/3\right]$$

$$= x\sin x + \cos x - \frac{1}{3}x\sin^3 x - 1/3\cos x + 1/9\cos^3 x + C$$

$$= x\sin x + 2/3\cos x - 1/3\, x\sin^3 x + 1/9\cos^3 x + C$$

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