HW #10

(+2)

Section 10.7

#22 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n} (x-2)^n}{3n} \overset{\text{note}}{=} \sum_{n=1}^{\infty} \frac{(-1)^n 9^n (x-2)^n}{3n}$

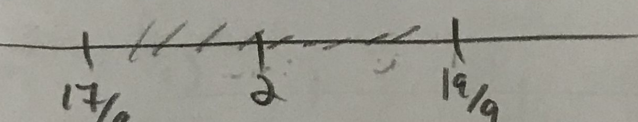
Ratio test

$\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{9^{n+1}(x-2)^{n+1}}{3(n+1)} \cdot \frac{3n}{9^n (x-2)^n} \right|$

$\displaystyle= \lim_{n\to\infty} \left|\frac{n}{n+1}\right| \cdot \left|\frac{9}{1}\right| \cdot \left| x-2 \right|$

$= 9\,|x-2| < 1 \implies |x-2| < 1/9.$

$R = 1/9$



Endpts. If $x = 19/9 = 2 + 1/9$

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 9^n (1/9)^n}{3n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$ → Converges only conditionally. Alt. Harmonic

If $x = 17/9 = 2 - 1/9$

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 9^n (-1/9)^n}{3n} = \sum_{n=1}^{\infty} \frac{1}{3n}$ diverges.

(a) Interval of convergence $\left(\frac{17}{9}, \frac{19}{9}\right]$

$R = 1/9$

(b) Converges absolutely from $\left(\frac{17}{9}, \frac{19}{9}\right)$

& conditionally @ $x = 19/9$.

#24 $\sum_{n=1}^{\infty} \ln(n)\, x^n$ Ratio (+2)

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\ln(n+1)\, x^{n+1}}{\ln(n)\, x^n}\right| = \lim_{n\to\infty}\left|\frac{\ln(n+1)}{\ln(n)}\right| \cdot |x|$$

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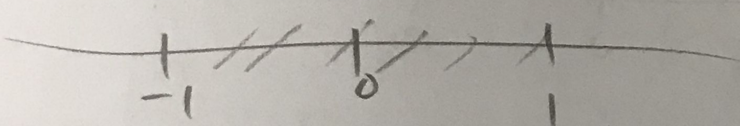
$$= \lim_{n\to\infty}\left|\frac{n}{n+1}\right| \cdot |x| = |x| < 1 \Rightarrow R = 1$$



Endpts.

If $x = 1$ $\sum_{n=1}^{\infty} \ln(n)$ diverges by nth term.

If $x = -1$ $\sum_{n=1}^{\infty} (-1)^n \ln(n)$ div. by nth term.

I.O.C. $(-1, 1)$ converges abs. here & $R = 1$

(b).

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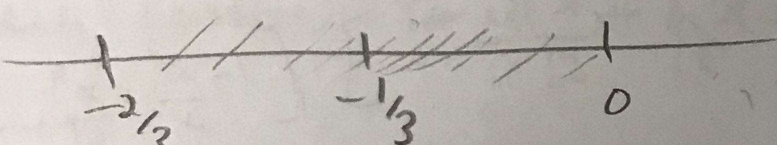
#32 $\displaystyle\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2} = \sum_{n=1}^{\infty} \frac{3^{n+1}(x+\frac{1}{3})^{n+1}}{2n+2}$

Ratio

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{3^{n+2}(x+\frac{1}{3})^{n+2}}{2(n+1)+2}\cdot\frac{2n+2}{3^{n+1}(x+\frac{1}{3})^{n+1}}\right|$$

$$= \lim_{n\to\infty} 3\left|\frac{2n+2}{2n+4}\right|\cdot\left|x+\tfrac{1}{3}\right| = \lim_{n\to\infty} 3\left|x+\tfrac{1}{3}\right| < 1$$

$|x+\frac{1}{3}| < \frac{1}{3}$ $\Rightarrow$ $R=\frac{1}{3}$ Center $a=-\frac{1}{3}$.



Endpts.

If $x=0 \rightarrow \displaystyle\sum_{n=1}^{\infty}\frac{3^{n+1}(\frac{1}{3})^{n+1}}{2n+2} = \sum_{n=1}^{\infty}\frac{1}{2n+2}$ diverges by DCT to Harmonic.

If $x=-\frac{2}{3}$ $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n+1}(-\frac{1}{3})^{n+1}}{2n+2} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n+2}$ Converges conditionally by AST.

I.o.C. $[-\frac{2}{3}, 0)$

Conv. abs on $(-\frac{2}{3}, 0)$ & cond. at $x=-\frac{2}{3}$.

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#55 $\sin x = x - x^3/3! + \cdots$

(a) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$

converges for all x.

. get this by differentiating!

(b) $\sin(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$

$= 2x - \frac{8x^3}{3!} + \cdots$

(c) $2\sin x \cos x = 2\left[x - x^3/3! + \frac{x^5}{5!} - \cdots\right]\cdot\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right]$

$= 2\left[0 + 1x + 0x^2 + \left(-\frac{1}{6} - \frac{1}{2}\right)x^3 + 0x^4 + \left(\frac{1}{120} + \frac{1}{24} + \frac{1}{2!\cdot 3!}\right)x^5\right]$

$= 2\left(x - \frac{2}{3}x^3 + \frac{16}{120}x^5 + \cdots\right)$

= same as $\sin(2x)$ ✓

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Section 10.8 :

(+2)

#34 $f(x) = (1-x+x^2)e^x$

Note: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all $x \in \mathbb{R}$

Thus,

$f(x) = (1-x+x^2)(1+x+x^2/2!+\cdots)$

$= (1-x+x^2) + (x-x^2) + x^2/2 + (x^3 - x^3/2 + x^3/6) + \cdots$

$= 1 + \frac{x^2}{2} + 2x^3/3 + \cdots$

which converges for all x in $(-\infty, \infty)$.