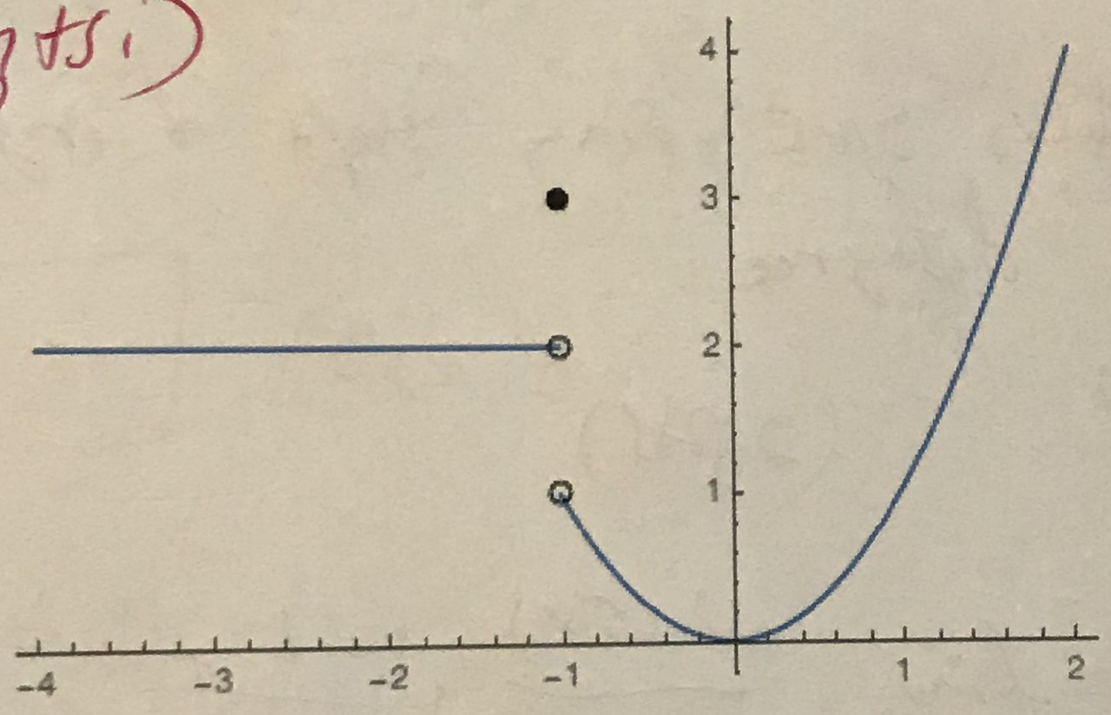
New Jersey Institute of Technology
DEPARTMENT OF MATHEMATICAL SCIENCES
Math 111-029 Quiz 2

Your Name: PROF. ALLAIRE

1. Let $f(x) = \begin{cases} 2\,, & x < -1 \\ 3\,, & x = -1 \\ x^2\,, & x > -1 \end{cases}$. Using the following graph answer the questions below. If a limit does not exist, **explain why**.

(2 pts) (3 pts)



Rubric
1-2 wrong (-0.5)
3-4 -1
-5 ~~-1.5~~ -2

(a) $\lim_{x \to -1^-} f(x)$ = 2

(b) $\lim_{x \to -1^+} f(x)$ = 1

(c) $\lim_{x \to -1} f(x)$ DNE since

$\lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x)$

(d) $f(-1) =$ 3

(e) Is f continuous at $x = -1$? If not, what type of discontinuity is it? (Removable, Infinite, Jump, or Oscillating)

NO, f is discontinuous and has a jump discontinuity.

2. Find $\lim_{x \to -2} f(x)$ or if the limit does not exist, explain why. **Show all work.**

(2 pts)

$$f(x) = \frac{|x+2|}{x+2}$$

(a) $\lim_{x \to -2^+} \frac{|x+2|}{x+2} = \lim_{x \to -2} \frac{(x+2)}{(x+2)} = \lim_{x \to -2} (1) = 1$

(b) $\lim_{x \to -2^-} \frac{|x+2|}{x+2} = \lim_{x \to -2} \frac{-(x+2)}{(x+2)} = \lim_{x \to -2} (-1) = -1$

Therefore, $\lim_{x \to -2} f(x)$ DNE since left & right-hand limits disagree.

3. Find $\lim_{x \to 0} \frac{\tan(5x)}{\sin(x)}$ (2 pts)

$$\lim_{x \to 0} \frac{\tan(5x)}{\sin x} = \lim_{x \to 0} \frac{\sin(5x)}{1} \cdot \frac{1}{\cos(5x)} \cdot \frac{1}{\sin x}$$

$$= \lim_{x \to 0} \left(\frac{\sin(5x)}{5x} \cdot \left(\frac{5}{\cos(5x)} \right) \cdot \left(\frac{x}{\sin x} \right) \right)$$

$$= (1) \cdot (5) \cdot (1) = 5$$

4. For what value(s) of a and b is the following function continuous? **Show all work.**

(3 pts)

$$f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & x < 2 \\ ax + b & 2 \leq x \leq 3 \\ 0 & x > 3 \end{cases}$$

(i) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\downarrow$

$\lim_{x \to 2} \frac{x^2+3x-10}{x-2} = \lim_{x \to 2} \frac{(x+5)(x-2)}{(x-2)} = \lim_{x \to 2} (x+5) = 7 = \lim_{x \to 2} (ax+b)$

$= 2a+b$

$\boxed{2a+b = 7}$ (eqn 1).

(ii) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\downarrow$

$\lim_{x \to 3} ax+b = 3a+b = \lim_{x \to 3} (0) = 0$

$\boxed{3a+b = 0}$ (eqn 2)

$\downarrow$

$\boxed{b = -3a}$

$2a+(-3a) = 7 \iff \boxed{a = -7}$, $\boxed{b = 21}$