(+1)

#50. $a_n = \dfrac{\ln(n)}{\ln(2n)}$

$$\lim_{n\to\infty} \frac{\ln(n)}{\ln(2n)} = \lim_{n\to\infty} \frac{(1/n)}{(2/2n)} = 1$$ converges.

— Math 112 HW #7

§ 10.1

#48 $a_n = \frac{3^n}{n^3}$ (+1)

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{3^n \cdot \ln 3}{3n^3} = \lim_{n\to\infty} \frac{3^n(\ln 3)^2}{6n}$$

$$= \lim_{n\to\infty} \frac{3^n(\ln 3)^3}{6} = +\infty$$ diverges!

#50 $a_n = \frac{\ln(n)}{\ln(2n)}$ (+1)

$$\lim_{n\to\infty} \frac{\ln(n)}{\ln(2n)} = \lim_{n\to\infty} \frac{(1/n)}{(2/2n)} = 1$$ converges

Homework Week #7

Section 10.1:

(+2)

#70 $a_n = \left(\frac{n}{n+1}\right)^n = \left(\frac{n+1}{n}\right)^{-n} = \left(1+\frac{1}{n}\right)^{-n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{-n\ln(1+\frac{1}{n})} = e^{\lim_{n\to\infty} -n\ln(1+\frac{1}{n})}$

$= e^{\lim_{n\to\infty} \frac{-\ln(1+1/n)}{1/n}} = e^{\lim_{n\to\infty} -\left(\frac{1}{1+1/n}\right)}$ L'Hospital's Rule

$= \boxed{e^{-1}}$

#74 (+1)

$a_n = \frac{(10/11)^n}{(9/10)^n + (11/12)^n}$

Divide everything by $(11/12)^n$

$\lim_{n\to\infty} \frac{\left(\frac{120}{121}\right)^n}{\left(\frac{108}{110}\right)^n + 1} = \frac{0}{0+1} = \boxed{0}$

#80 $a_n = (3^n + 5^n)^{1/n}$ (+2)

$$= \lim_{n\to\infty} e^{\frac{1}{n}\ln(3^n+5^n)} \overset{L'H}{=} \lim_{n\to\infty} e^{\frac{3^n\ln 3 + 5^n \ln 5}{3^n+5^n}}$$

$$= \lim_{n\to\infty} e^{\frac{(3/5)^n \ln 3 + \ln 5}{(3/5)^n + 1}} = e^{\ln 5} = 5$$

Section 10.2. (+1)

#63 $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n} = \sum_{n=1}^{\infty} (1/2)^n + \sum_{n=1}^{\infty} (3/4)^n$

$$= \frac{1/2}{1-1/2} + \frac{3/4}{1-3/4} = 1 + 3 = 4$$

converses. By Geometric series.

#64 $a_n = \frac{2^n + 4^n}{3^n + 4^n}$ (+2)

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(1/2)^n + 1}{(3/4)^n + 1} = \frac{0+1}{0+1} = 1 \neq 0.$$

Diverses Since $a_n \not\to 0$.

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