9/26/17

Math 112-011 HW #2

Section 6.3 : Arc length

(+2)

#26 The length of an Astroid

$$x^{2/3} + y^{2/3} = 1$$

$y = (1-x^{2/3})^{3/2}$

$y' = 3/2\,(1-x^{2/3})^{1/2} \cdot$

$2/3\, x^{-1/3}$

$= x^{-1/3}(1-x^{2/3})^{1/2}$

$$S = 8 \cdot \int_{\sqrt{2}/4}^{1} \sqrt{1+(y')^2}\, dx$$

$$= 8 \int_{\sqrt{2}/4}^{1} \sqrt{1 + x^{-2/3}(1-x^{2/3})}\, dx$$

$$= 8 \int_{\sqrt{2}/4}^{1} \sqrt{x^{-2/3}}\, dx = 8 \int_{\sqrt{2}/4}^{1} x^{-1/3}\, dx$$

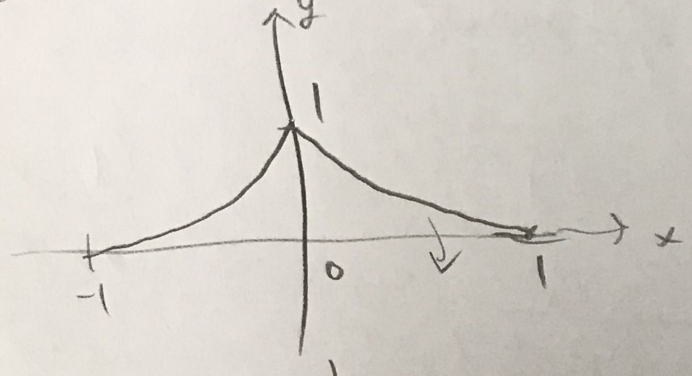
$$= 8\left(\tfrac{3}{2}\right) x^{2/3} \Big|_{\sqrt{2}/4}^{1}$$

$$= 12 \cdot \left(1 - \tfrac{1}{2}\right) = 12\left(\tfrac{1}{2}\right) = 6$$

$S = 6$

Section 6.4: Area of Surface of Revolution (+2)

#32 The surface of an Astroid



$y = (1 - x^{2/3})^{3/2}$

$SA = 2 \int_0^1 2\pi y \sqrt{1+(y')^2}\, dx$

$= 4\pi \int_0^1 (1-x^{2/3})^{3/2}\, x^{-1/3}\, dx$

Let $u = 1 - x^{2/3}$

$du = -2/3\, x^{-1/3}\, dx$

$4\pi\left(-3/2\right) \int_1^0 u^{3/2}\, du$

$= -6\pi\left(\frac{2}{5}\right) u^{5/2} \Big|_1^0$

$= -\frac{12\pi}{5}(0-1) = \frac{12\pi}{5}$

2

Section 6.5: Work

(+3)

#8 Leaky Sandbag

- Bag weight: 144 lb
- Lifted at constant rate
- Sand leaks out at constant rate
- Sand half gone when lifted to 18 ft

How much work was done lifting the sand this far?

We have two points (x, F) x = distance off ground.

$(0, 144)$ & $(18, 72)$

$m = \frac{144-72}{0-18} = -4$ lb/ft

Then $F - 144 = -4(x-0)$

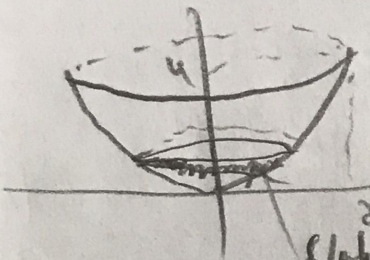
$F(x) = 144 - 4x$

$W = \int_0^{18} F(x)\,dx = \int_0^{18} 144 - 4x\,dx = 144x - 2x^2 \Big|_0^{18}$

$= 144(18) - 2(18)^2 = 1944$ ft-lb

(+3)

#19 Graph of $y=x^2$ $0 \le x \le 2$ revolved about y-axis to form tank that is then filled w/ salt water from Dead sea (weight $73 lb/ft^3$)



$F =$ weight/volume $\times$ volume.

$dV = \pi r^2 dy$

$r = \sqrt{y} - 0 = \sqrt{y}$.

use

$dV = \pi y \, dy$

$F(y) = 73 lb/ft^3 \cdot dV$

$F(y) = 73\pi y \, dy$

distance to cover $d = (4-y)$

$W = \int_0^4 F \cdot d \, dy = \int_0^4 73\pi y (4-y) \, dy$

$= 73\pi \int_0^4 (4y - y^2) \, dy = 73\pi \left(2y^2 - y^3/3 \Big|_0^4 \right)$

$= \frac{2336\pi}{3}$ ft. lb

≈ 2446.25 ft-lb.

4