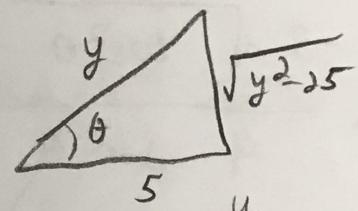
CALC 2 HW #5

Section 8.4

(~~2pts~~) (+1)

#12

$$\int \frac{\sqrt{y^2-25}}{y^3}\,dy \quad , \quad y > 25$$




Let $y = 5\sec\theta$

$dy = 5\sec\theta\tan\theta\,d\theta$

$\sec\theta = \frac{y}{5}$

$\cos\theta = \frac{5}{y}$

$$\rightarrow \int \frac{5}{125}\,\frac{\tan\theta\,(5\sec\theta\tan\theta)}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{5}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{5}\int \sin^2\theta\,d\theta$$

$$= \frac{1}{5}\int \frac{1}{2} - \frac{\cos(2\theta)}{2}\,d\theta = \frac{1}{5}\left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right]$$

$$= \frac{1}{5}\left[\frac{\sec^{-1}(y/5)}{2} - \frac{2\sin\theta\cos\theta}{4}\right] =$$

$$\frac{1}{10}\sec^{-1}(y/5) - \frac{1}{10}\,\frac{\sqrt{y^2-25}}{y}\left(\frac{5}{y}\right)$$

$$= \left[\frac{1}{10}\sec^{-1}(y/5) - \frac{\sqrt{y^2-25}}{2y^2}\right] + c$$

9

$\sin^2 + \cos = 1$

$1 + \cot^2 = \csc^2$

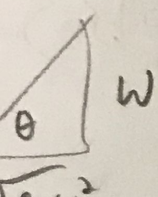
$$\int \frac{\sqrt{9-w^2}}{w^2}\, dw$$

Let $w = 3\sin\theta$

$dw = 3\cos\theta \, d\theta$

$$\int \frac{3\sqrt{\cos^2\theta}\,(3\cos\theta)\, d\theta}{9\sin^2\theta} = \int \frac{\cos^2\theta}{\sin\theta}\, d\theta = \int \cot^2\theta \, d\theta$$

$$\int \csc^2\theta - 1 \, d\theta = -\cot\theta - \theta$$




$$= -\frac{\sqrt{9-w^2}}{w} - \sin^{-1}\left(\frac{w}{3}\right) + C$$

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(44) $$\int \frac{\sqrt{1-(\ln x)^2}}{x \ln x}\, dx$$

Let $u = \ln x$

$du = \frac{1}{x}\, dx$

(~~4~~) (~~12~~) (41)

$$\int \frac{\sqrt{1-u^2}}{u}\, du$$

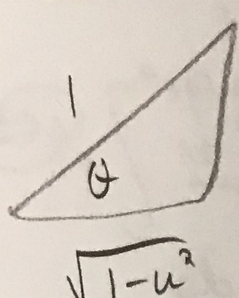
Let $u = \sin\theta$

$du = \cos\theta \, d\theta$

$$= \int \frac{\cos^2\theta}{\sin\theta}\, d\theta = \int \frac{1-\sin^2\theta}{\sin\theta}\, d\theta = \int \csc\theta \, d\theta - \int \sin\theta \, d\theta$$

$$= -\ln|\csc\theta + \cot\theta| + \cos\theta + C$$

44 (cont)




$$= -\ln\left|\frac{1}{u}+\frac{\sqrt{1-u^2}}{u}\right| + \sqrt{1-u^2} +$$

$$= -\ln\left|\frac{1}{\ln x}+\frac{\sqrt{1-(\ln x)^2}}{\ln x}\right| + \sqrt{1-(\ln x)^2} +$$

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#57 Evaluate $\int x^3\sqrt{1-x^2}\,dx$ using. (2pts.)

(a) I.B.P.

$$\int x^3\sqrt{1-x^2}\,dx = \int x^2\cdot\left(x\sqrt{1-x^2}\right)dx$$

Let $u = x^2$ $\quad dv = x\sqrt{1-x^2}\,dx$

$du = 2x\,dx$

$v = \int x\sqrt{1-x^2}\,dx \quad (z = 1-x^2,\ dz = -2x\,dx)$

$= -\frac{1}{2}\int z^{1/2}\,dz = -\frac{1}{3}z^{3/2} = -\frac{1}{3}(1-x^2)^{3/2}$

$$= -\frac{1}{3}x^2(1-x^2)^{3/2} + \frac{2}{3}\int x\,(1-x^2)^{3/2}\,dx \qquad \text{Let } u = 1-x^2$$

$du = -2x\,dx$

$$= -\frac{1}{3}x^2(1-x^2)^{3/2} + \frac{1}{3}\int u^{3/2}\,du = -\frac{1}{3}x^2(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} + C$$

* 57 cont.

(b) Substitution

$\int x^3 \sqrt{1-x^2}\, dx$ Let $u = 1-x^2$ $x^2 = 1-u.$

$du = -2x\, dx$

$= -\frac{1}{2} \int (1-u)\sqrt{u}\, du$

$= -\frac{1}{2} \int u^{1/2} - u^{3/2}\, du = -\frac{1}{2}\left[\frac{2u^{3/2}}{3} - \frac{2}{5}u^{5/2}\right]$

$= \frac{-(1-x^2)^{3/2}}{3} + \frac{1}{5}(1-x^2)^{5/2} + C$

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(c) Trig. sub. $\int x^3 \sqrt{1-x^2}\, dx$ Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

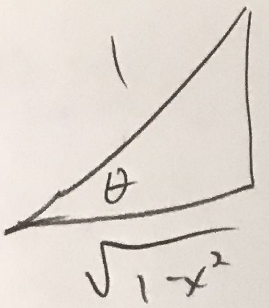
$\int \sin^3\theta \cos\theta \underbrace{(\cos\theta\, d\theta)}_{dx} = \int \sin^3\theta \cos^2\theta\, d\theta.$

$= \int (1-\cos^2\theta)\cos^2\theta \sin\theta\, d\theta$ Let $u = \cos\theta$

$du = -\sin\theta\, d\theta$

$= -\int u^2 - u^4\, du = -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] =$

$= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5} + C$




$$\sin\theta = \frac{x}{1}$$

$$\cos\theta = \sqrt{1-x^2}$$

$$= -\frac{(1-x^2)^{3/2}}{3} + \frac{(1-x^2)^{5/2}}{5} + C$$

Juz!

Calc 2 HW # 5

Section 8.5 (~~2 pts~~) (+1)

#1 $\int_{-1}^{0} \frac{x^3}{x^2-2x+1}$

(1) Long division

$$\begin{array}{r} x+2 \\ x^2-2x+1 \overline{) \; x^3 \qquad\qquad} \\ -(x^3-2x^2+x) \\ \hline 2x^2-x \\ -(2x^2-4x+2) \\ \hline 3x-2 \end{array}$$

$$\frac{x^3}{x^2-2x+1} = x+2+\frac{3x-2}{x^2-2x+1}$$

$$\int_{-1}^{0} \frac{x^3}{x^2-2x+1}\,dx = \int_{-1}^{0} x+2\,dx + \int_{-1}^{0} \frac{3x-2}{(x-1)^2}\,dx$$

(This can be done via u-sub.)

(2) $\frac{3x-2}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} \Rightarrow$

$3x-2 = A(x-1)+B$

If $x=1$, $B=1$

If $x=0$ $\quad -A+1=-2 \Rightarrow A=3$

$\therefore$

$$\text{Int} = \int_{-1}^{0} x+2\, dx + \int_{-1}^{0} \frac{3}{(x-1)}\, dx + \int_{-1}^{0} \frac{1}{(x-1)^2}\, dx$$

$$= \frac{x^2+2x}{2} + 3\ln|x-1| - \frac{1}{(x-1)} \Big|_{-1}^{0}$$

$$= 1 - \left(\frac{1}{2} - 2 + 3\ln 2 + \frac{1}{2}\right)$$

$$= 2 - 3\cdot\ln 2$$

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#30 $\displaystyle\int \frac{x^2+x}{x^4-3x^2-4}\, dx = \int \frac{x(x+1)}{(x^2-4)(x^2+1)}\, dx = \int \frac{x(x+1)}{(x+2)(x-2)(x^2+1)}\, dx$

(+1) ($\cancel{2}$)

$$\frac{x(x+1)}{(x+2)(x-2)(x^2+1)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+1}$$

$\rightarrow$ $x(x+1) = A(x-2)(x^2+1) + B(x+2)(x^2+1) + (Cx+D)(x+2)(x-2)$

If $x=2$ $\quad 6 = 20B \Rightarrow B = \frac{6}{20} = \frac{3}{10}$

If $x=-2$ $\quad 2 = -20A \Rightarrow A = \frac{-1}{10}$

If $x=0$ $\quad 0 = -2A + 2B - 4D = \frac{1}{5} + \frac{3}{5} - 4D = 0$

$4D = \frac{4}{5}$

$D = \frac{1}{5}$

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if $x=-1$

$0 = -6A + 2B + 3C - 3D = 0$

$-\frac{3}{5} + \frac{3}{5} + 3C - \frac{3}{5} = 0$

$3C = \frac{-3}{5}$

$C = \frac{-1}{5}$

Then,

$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{-4/10}{x+2}\,dx + \int \frac{3/10}{x-2}\,dx + \frac{1}{5}\int \frac{x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}$

$u = x^2+1$

$\cancel{du = 2x\,dx}$

$= -\frac{4}{10} \ln|x+2| + \frac{3}{10} \ln|x-2| - \frac{1}{10} \ln|x^2+1| + \frac{1}{5} \tan^{-1}(x) + C$

3

(~~2 pts.~~)

(2 pts.)

③

$$\int \frac{2\theta^3+5\theta^2+8\theta+4}{(\theta^2+2\theta+2)^2}\,d\theta$$

$$\rightarrow \frac{2\theta^3+5\theta^2+8\theta+4}{(\theta^2+2\theta+2)^2} = \frac{A\theta+B}{\theta^2+2\theta+2} + \frac{C\theta+D}{(\theta^2+2\theta+2)^2}$$

$$2\theta^3+5\theta^2+8\theta+4 = (A\theta+B)(\theta^2+2\theta+2)+(C\theta+D)$$

$$= A\theta^3+(2A+B)\theta^2+(2A+2B+C)\theta+(2B+D)$$

$\therefore$ $A=2$

$2A+B=5$

$2A+2B+C=8$

$2B+D=4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 & 5 \\ 2 & 2 & 1 & 0 & 8 \\ 0 & 2 & 0 & 1 & 4 \end{array}\right] \rightarrow \text{Linear System!}$$

$A=2$, $B=1$, $C=2$, $D=2$

$$\rightarrow \int \frac{2\theta+1}{\theta^2+2\theta+2}\,d\theta + \int \frac{2\theta+2}{(\theta^2+2\theta+2)^2}\,d\theta \rightarrow \begin{array}{l} u=\theta^2+2\theta+2 \\ du=2\theta+2\,d\theta \end{array}$$

$$= \int \frac{2\theta+2-1}{\theta^2+2\theta+2}\,d\theta + \int \frac{1}{u^2}\,du$$

$\ln|u| - \frac{1}{u} - \int \frac{1}{\theta^2+2\theta+2}\, d\theta$

$\ln|u| - \frac{1}{u} - \int \frac{1}{(\theta+1)^2+1}\, d\theta$ $\quad u=\theta+1$, $du = d\theta$

$|u| - \frac{1}{u} - \int \frac{1}{u^2+1}\, du$

$\ln|\theta^2+2\theta+2| - \frac{1}{\theta^2+2\theta+2} - \tan^{-1}(\theta+1) + C$

(1) Division

38) $\int \frac{2y^4}{y^3-y^2+y-1}\, dy$

$$\begin{array}{r} 2y+2 \\ y^3-y^2+y-1 \,\overline{)\,2y^4} \\ -(2y^4-2y^3+2y^2-2y) \\ \hline 2y^3-2y^2+2y \\ -(2y^3-2y^2+2y-2) \\ \hline 2 \end{array}$$

(+1)

$\int 2y+2 + \frac{2}{(y^2+1)(y-1)}\, dy$

$\frac{2}{(y^2+1)(y-1)} = \frac{A}{y-1} + \frac{By+C}{y^2+1}$

$2 = A(y^2+1) + (By+C)(y-1)$

If $y=1$ $\quad 2A = 2 \Rightarrow A=1$

If $y=0$ $\quad A - C = 1 - C = 2 \Rightarrow C = -1$

If $y=-1$, $\quad 2 = 2A + 2B + 2C = 2 + 2 + 2B = 2$ $\Rightarrow B = -1$

5

$$= \int 2y+2\,dy + \int \frac{1}{y-1}\,dy + \int \frac{-y-1}{y^2+1}\,dy$$

$$= y^2+2y + \ln|y-1| - \int \frac{y}{y^2+1}\,dy - \int \frac{1}{y^2+1}\,dy$$

$u = y^2+1$

$du = 2y$

$$= y^2+2y+\ln|y-1| - \frac{1}{2}\ln|y^2+1| - \tan^{-1}(y) + C$$