Math 112 · Calc 2 HW #8

Section 10.3 # 20, 34, 36

#20 $\sum_{n=2}^{\infty} \frac{\ln(n)}{\sqrt{n}}$ Integral test, (+2)

$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}\,dx$ let $z = \ln x \iff x = e^{z} \iff \sqrt{x} = e^{z/2}$

$dz = \frac{1}{x}\,dx$

$\Rightarrow \int_2^{\infty} \frac{\ln x}{\sqrt{x}}\,dx = \int_{\ln 2}^{\infty} z e^{z/2}\,dz$ Int by parts.

$u = z$ $dv = e^{z/2}\,dz$

$du = dz$ $v = 2e^{z/2}$

$= 2ze^{z/2}\Big|_{\ln 2}^{\infty} - 2\int_{\ln 2}^{\infty} e^{z/2}\,dz$ Diverges

$= \lim_{z\to\infty} (2z-4)\,e^{z/2} - (2z-4)\,e^{z/2}\Big|_{\ln 2} = \infty$

#34 $\sum_{n=1}^{\infty} n\tan(\frac{1}{n})$ $\lim_{n\to\infty} n\tan(\frac{1}{n}) = \lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{1/n} \cdot \frac{1}{\cos(\frac{1}{n})}$

$= 1 \cdot 1 = 1 \neq 0.$

Diverges by nth term test (divergence test). (+2)

D

(+2)

#36 $\sum_{n=1}^{\infty} \frac{2}{1+e^n}$ Int test

$\int_1^{\infty} \frac{2}{1+e^x}\,dx$ let $u = e^x$

$du = e^x\,dx = u\,dx$

$dx = \frac{du}{u}$

$= \int_e^{\infty} \frac{2}{u(1+u)}\,du$ PFD.

$\frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u}$

$2 = A(1+u) + Bu$ if $u=0$ $A=2$ if $u=-1$ $B=-2$

$= \int_e^{\infty} \frac{2}{u} - \frac{2}{1+u}\,du$

$= 2\ln|u| - 2\ln|1+u| \Big|_e^{\infty}$

$= 2\ln\left|\frac{u}{u+1}\right| \Big|_e^{\infty}$

$= \lim_{u\to\infty} 2\ln\left|\frac{u}{u+1}\right| - 2\ln\left|\frac{e}{e+1}\right|$

$= 2\ln(1)^{\to 0} - 2[\ln(e) - \ln(e+1)]$

$= 2\ln(e+1) - 2$ converges!

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Section 10.4:

#4 $\displaystyle\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$

(+1)

$$\sum \frac{1}{n} \leq \sum \frac{n}{n^2-n} \leq \sum \frac{n+2}{n^2-n}$$

Since $\sum \frac{1}{n}$ diverges (Harmonic series) so does $\displaystyle\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$.

$\square$.

#36 $\displaystyle\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{n2^n} + \frac{1}{n^2}\right)$ $\rightsquigarrow$ you can actually find this #!

$$\leq \sum_{n=1}^{\infty} \frac{1}{2^n} + \frac{1}{n^2} \rightarrow \text{converges!}$$

(+1)

#40 $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$

(+1)

$$\lim_{n\to\infty} \frac{\left(\frac{2^n+3^n}{3^n+4^n}\right)}{(3/4)^n} = \lim \frac{8^n+12^n}{9^n+12^n} = \lim \frac{(8/12)^n+1}{(9/12)^n+1} = 1 > 0$$

Since $\sum (3/4)^n$ conv. so does ours. $\square$

#4b $\sum_{n=1}^{\infty} \tan(1/n)$ LCT! (+1)

$$\lim_{n\to\infty} \frac{\tan(1/n)}{1/n} = \lim_{n\to\infty} \frac{\sin(1/n)}{(1/n)} \cdot \frac{1}{\cos(1/n)} = 1 > 0.$$

By LCT since $\sum 1/n$ diverges so does $\sum \tan(1/n)$