New Jersey Institute of Technology
DEPARTMENT OF MATHEMATICAL SCIENCES
Math 111-029 Quiz 7

Your Name: PROF. ALLAIRE

1. Consider the function $f(x) = e^{-x^2}$

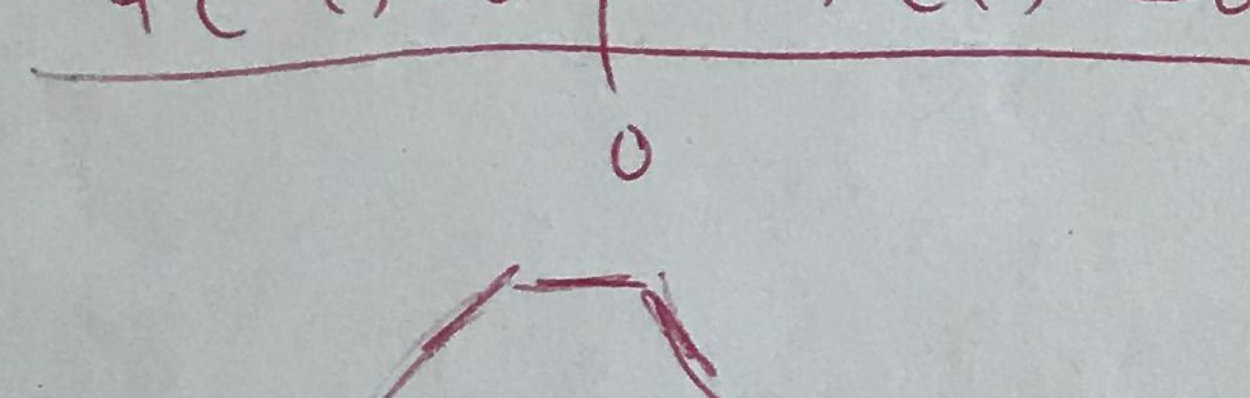
   (a) Find the critical point(s) of $f(x)$.

   $f'(x) = -2x e^{-x^2} = 0$ when $x = 0$.

   (+1) $(0, 1)$

   (b) Using the first derivative test, find the open intervals where f is increasing and decreasing, along with any local (relative) max or min.

   (+2) $f'(-1) > 0$, $f'(1) < 0$

   Inc: $(-\infty, 0)$

   Dec: $(0, \infty)$

   local max $x = 0$ $(0, 1)$

   (c) Which theorem guarantees that f attains its extreme values on $[-1, 1]$?

   (+1) Extreme Value theorem

   (d) What is the absolute maximum and absolute minimum of f on $[-1, 1]$ and where do they occur?

   (+2)

   $f(0) = 1$

   $f(-1) = e^{-1} = \frac{1}{e}$

   $f(1) = e^{-1} = \frac{1}{e}$

   Note: $1/e < 1$

   Abs. max is $\underline{1}$ which occurs at $x = 0$

   Abs. min is $1/e$ which occurs at $x = 1$ and $x = -1$.

2. Consider $f(x) = e^{-x^2}$ as before. Since f is continuous on $[-1,1]$ and differentiable on $(-1,1)$ and $f(1) = f(-1)$, Rolle's theorem guarantees us that there is a point c in $(-1,1)$ such that $f'(c) = 0$. With this knowledge, find the value of c in the conclusion of the mean value theorem $\frac{f(b)-f(a)}{b-a} = f'(c)$ for the given function f and the interval $[-1,1]$.

$$f'(c) = \frac{f(1) - f(-1)}{1-(-1)} = \frac{0}{2} = 0$$

+1

$f'(c) = 0$

$c = 0$ from #1

3. Find the critical point(s) of $f(x) = 2x(x-4)$ and identify the open intervals where f is increasing and decreasing. $f(x) = 2x^2 - 8x$

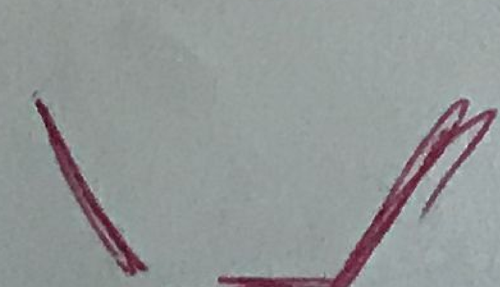
+3

$f'(x) = 4x - 8 = 4(x-2) = 0$

when $x = 2$

$f'(0) < 0$, $f'(3) > 0$

$\ominus$ 2 $\oplus$



Inc: $(2, \infty)$

Dec: $(-\infty, 2)$

Crit pt: $(2, -8)$