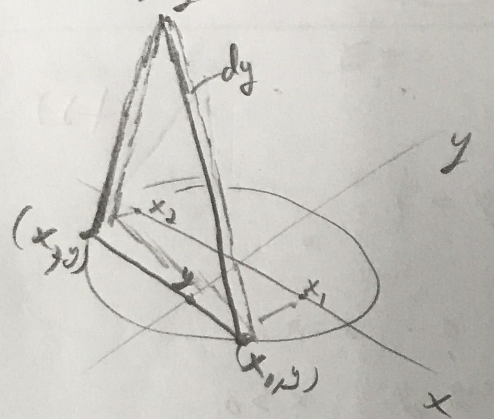
Math 112 - HomeWork # 1 Due 9/13.

Section 6.1

10 Base is a disk $x^2+y^2 \le 1$. (+2)

Cross Sections by planes perpendicular to the y-axis between $y=-1$ and $y=1$ are isosceles triangles w/ one leg in the disk



- We will add slices along y-axis, which means we want $\int A(y)dy$.
- Note that $A(y) = \frac{1}{2}bh$ for a triangle.
- For an isosceles triangle $b=h$ $\Rightarrow A(y) = \frac{1}{2}b^2$.

- Suppose we place a representative triangle at a location y on the y-axis. This y value would correspond to 2 x-values x_1, x_2 defined by

$$x_1 = \sqrt{1-y^2}$$
$$x_2 = -\sqrt{1-y^2}$$

Then the base of the triangle can be measured as

$$b = x_1 - x_2 = 2\sqrt{1-y^2}$$
$$\Rightarrow \boxed{b^2 = 4(1-y^2)}$$

Then

$$V = \int_{-1}^{1} A(y)\,dy = \frac{1}{2}\int_{-1}^{1} b^2\,dy = \frac{1}{2}\int_{-1}^{1} 4(1-y^2)\,dy$$

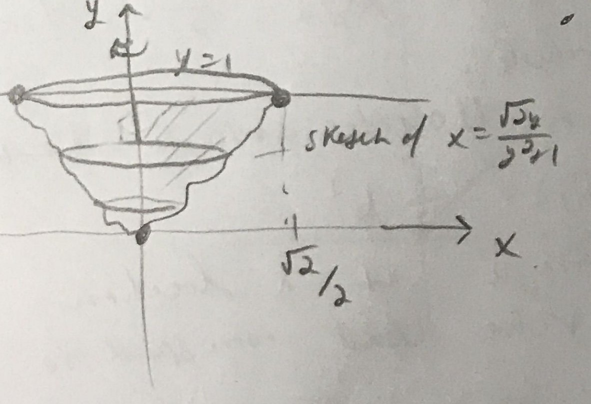
$$= 2\int_{-1}^{1} 1-y^2\,dy = 2\left(y - y^3/3\right)\Big|_{-1}^{1} =$$

$$2\left((1-1/3) - (-1+1/3)\right) = 2(4/3) = 8/3$$

$$\boxed{V = 8/3}$$

#36 Volumes about y-axis (+2)

$$x = \frac{\sqrt{2y}}{y^2+1} \qquad x=0, \quad y=1.$$



- when $y=0$ $x=0$.
- for any y $x>0$.

if we use $y=1$ we get $x = \frac{\sqrt{2}}{2}$

we use a disk

$R(y)$ = Right curve − Left curve.

$$= \frac{\sqrt{2y}}{y^2+1} - 0 = \frac{\sqrt{2y}}{(y^2+1)}$$

$$R^2 = \frac{2y}{(y^2+1)^2}$$

$$V = \pi\int_0^1 R^2\,dy = \pi\int_0^1 \frac{2y}{(y^2+1)^2}\,dy.$$

Let $u = (y^2+1)$
$du = 2y\,dy$

$$= \pi\int_1^2 u^{-2}\,du = \frac{-\pi}{u}\Big|_1^2$$

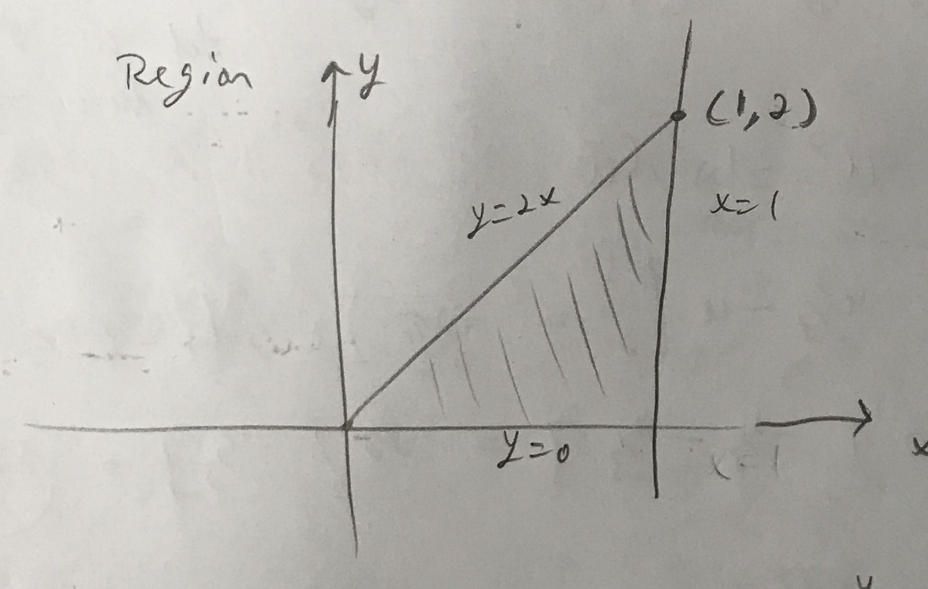
1

(36 continued)

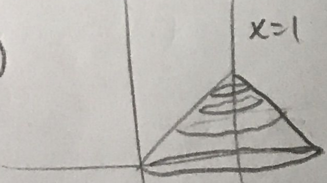
$$= -\pi\left[\frac{1}{2} - 1\right] = \frac{\pi}{2}$$

#52) Find the volume of the solid generated by revolving the triangular region bounded by the lines $y=2x$, $y=0$, and $x=1$ about

(a) the line $x=1$ (b) the line $x=2$



(a)



Disks $R(y) = 1 - \frac{y}{2}$ (+1)

$$V = \pi\int_0^2 \left(1 - y/2\right)^2 dy$$

Let $u = 1 - y/2$

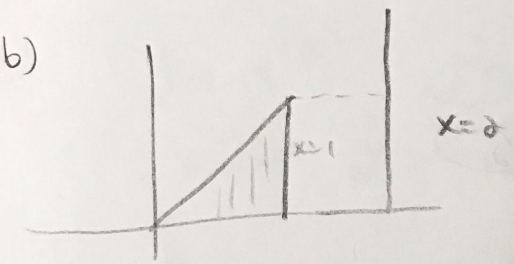
$du = -\frac{1}{2}\,dy$

$dy = -2\,du$

$$= -2\pi \int_1^0 u^2\,du = -2\pi\left[u^3/3\right]_1^0$$

$$= -2\pi\left[-1/3\right] = 2\pi/3$$

(b)



(+1)

Use washers $R(y) = (2 - y/2)$

$r(y) = (2 - 1)$

$$V = \pi \int_0^2 R^2(y) - r^2(y)\, dy$$

$$= \pi \int_0^2 (2 - y/2)^2 - (1)^2 \, dy$$

let $u = 2 - y/2$

$du = -\frac{1}{2} dy$

$dy = -2\, du$

$$= -2\pi \int_2^1 u^2 - 1 \, du$$

$$= -2\pi \left[u^3/3 - u \right|_2^1 = -2\pi \left[(1/3 - 1) - (8/3 - 2) \right]$$

$$= -2\pi \left[-2/3 - 2/3 \right] = -2\pi \left[-4/3 \right] = 8\pi/3$$

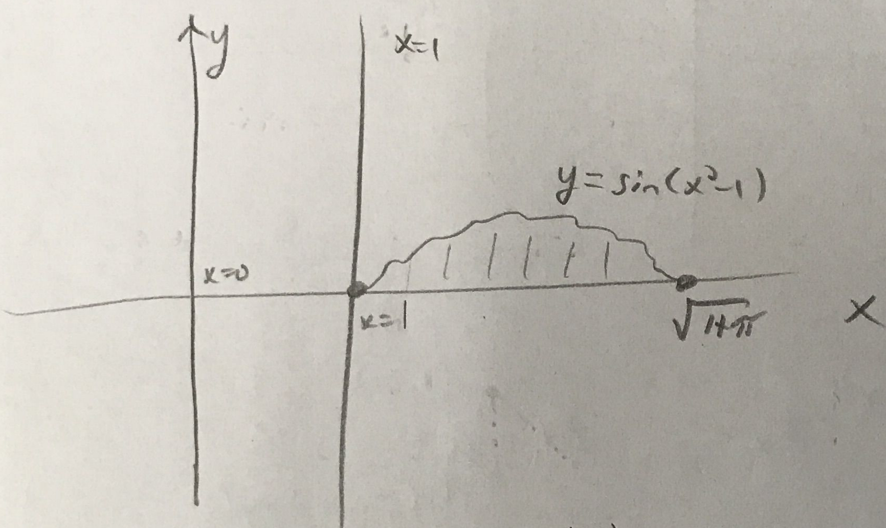
$$V = 8\pi/3$$

Section 6.2

#42 Bundt Cake around y-axis (+2)

$y = \sin(x^2-1)$ and x-axis

over $1 \le x \le \sqrt{1+\pi}$



Use shells. TOP $= \sin(x^2-1)$

Bottom $= 0$

$f(x) =$ Top $-$ Bottom $= \sin(x^2-1)$

Radius $= x$. (Since y-axis)

$V = 2\pi \int_1^{\sqrt{1+\pi}} x \sin(x^2-1)\,dx$. Let $u = x^2-1$, $du = 2x\,dx$

$= \pi \int_0^{\pi} \sin u \, du = -\pi \cos u \Big|_0^{\pi}$

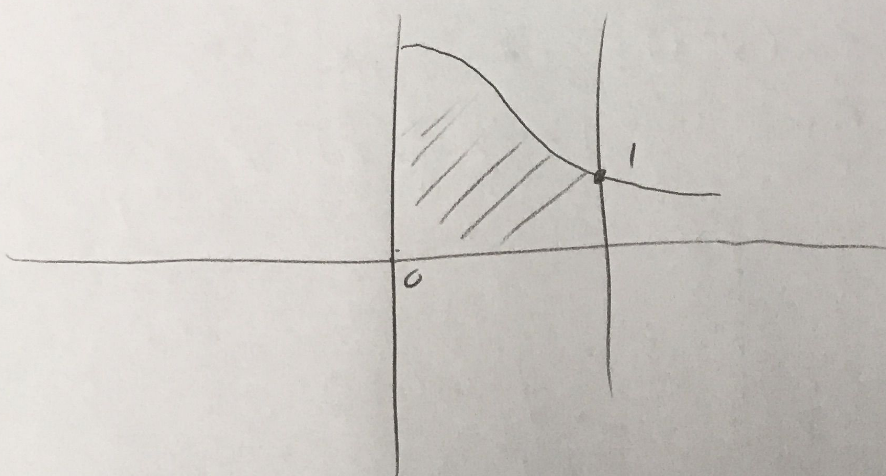
$= -\pi[-1 - \cos(0)]$

$= \pi[1 + \cos(1)]$.

$= 2\pi$

#47 $y = e^{-x^2}$ $y=0$, $x=0$, $x=1$

about y-axis (+1)



shells.

$V = 2\pi \int_0^1 x e^{-x^2} \, dx$ let $u = -x^2$, $du = -2x\,dx$

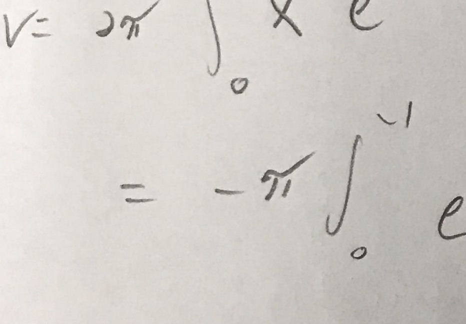
$= -\pi \int_0^{-1} e^u \, du = -\pi e^u \Big|_0^{-1}$

$= -\pi \left[e^{-1} - 1\right]$

$= \pi \left[1 - \frac{1}{e}\right]$

#48 $y = e^{x/2}$ $y = 1$ $x = \ln 3$ about x-axis

$e^{\ln 3/2} = e^{\ln(\sqrt{3})} = \sqrt{3}$



shells.

$V = 2\pi \int_1^{\sqrt{3}} y\,(\ln 3 - 2\ln y)\,dy$ (+1)

Washers

$V = \pi \int_0^{\ln 3} (e^{x/2})^2 - (1)^2 \, dx = \pi \int_0^{\ln 3} (e^x - 1)\,dx$

$= \pi \left[e^x - x \Big|_0^{\ln 3}\right] = \pi\left[(3 - \ln 3) - (1)\right]$

$= \pi\left[2 - \ln 3\right]$