New Jersey Institute of Technology
DEPARTMENT OF MATHEMATICAL SCIENCES
Math 111-029 Quiz 8

Your Name: PROF. ALLAIRE

1. Suppose we are given the following information:

$$f(x) = \frac{x}{x^2+1} - \tan^{-1}(x) \qquad f(0) = 0 \tag{1}$$

$$f'(x) = \frac{-2x^2}{x^2+1} \qquad \lim_{x\to\infty} \tan^{-1}(x) = \pi/2 \approx 1.5 \tag{2}$$

$$f''(x) = \frac{4x\left(x^2-1\right)}{\left(x^2+1\right)^3} \qquad \lim_{x\to-\infty} \tan^{-1}(x) = -\pi/2 \approx -1.5 \tag{3}$$

Note that $\tan^{-1}(1) = \pi/4 \approx 0.8$ and $\tan^{-1}(-1) = -\pi/4 \approx -0.8$ and therefore $f(1) \approx -0.3$ and $f(-1) \approx 0.3$.

(a) Find the critical point(s) of $f(x)$. Write both "$x =$" as well as the coordinate "$(x, f(x))$".

(+2)

(a) $x = 0$ or $(0,0)$ when $f'(x) = 0$.

(+2)

(b) Using the first derivative test, find the open intervals where f is increasing and decreasing, along with any local (relative) max or min. Use interval notation!

(b) $f'(-1)$ | $f'(1)$ — always decreasing

(−) 0 (−)

Dec: $(-\infty, 0) \cup (0, \infty)$ No rel. min or max.

Inc: Never

(+2)

(c) Find the possible points of inflection using $f''(x)$. Determine the open intervals where f is concave up and concave down. Specify whether the "possible" points of inflection are indeed, by definition, inflection points, based on the concavity test.

$f''(x) = 0$ at $x = 0, 1, -1$ Possible inflection pt.

$f''(-2)$ $f''(-\frac{1}{2})$ $f''(\frac{1}{2})$ $f''(2)$

−1 0 1

(−) (+) (−) (+)

C.D: $(-\infty, -1) \cup (0, 1)$

C.U: $(-1, 0) \cup (1, \infty)$

Inflection pts.

$(-1, 0.3)$

$(0, 0)$

$(1, -0.3)$

(+2)

(d) Identify any asymptotes of the function.

No vertical

Horizontal $\lim_{x\to\infty} \frac{x}{x^2+1} - \tan^{-1}x = 0 - \pi/2 = -\pi/2 \approx -1.5$

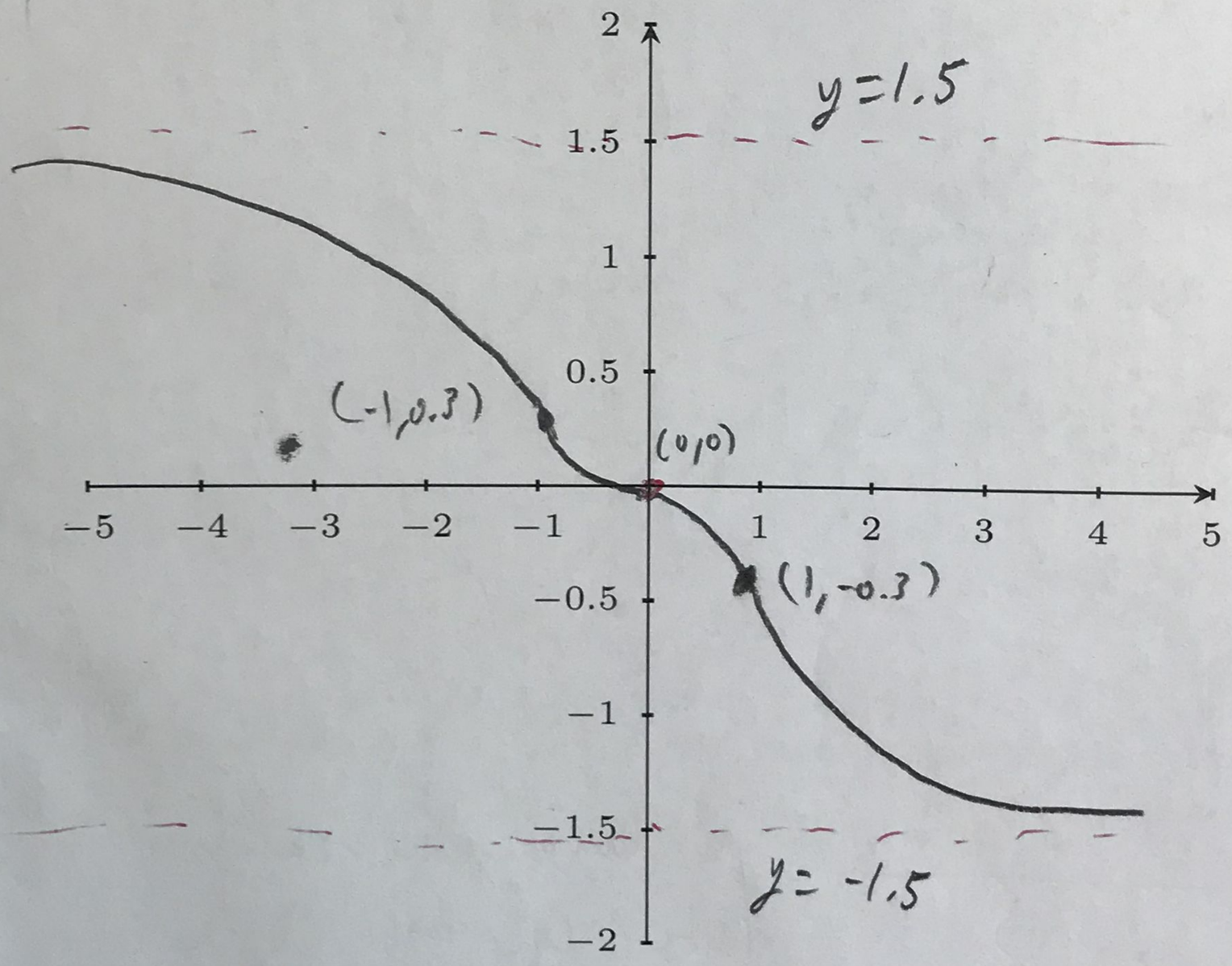
$x = \pm 1.5$

$\lim_{x\to-\infty} \frac{x}{x^2+1} - \tan^{-1}x = 0 + \pi/2 = \pi/2 \approx 1.5$

(e) Use the information above to sketch a graph of the function, labeling any (i) critical points, (ii) inflection points, (iii) intercepts, and (iv) asymptotes. Since some of the values are decimals, simply approximate their position.

(+2)

x/y - Intercept
(0,0)



Crit pt.
(0,0)

Inflection
(-1,0.3)
(0,0)
(1,-0.3)