Calc 2 ~~H~~ Hw # 6

Section 8.8 (+2)

#16 $\displaystyle\int_0^2 \frac{s+1}{\sqrt{4-s^2}}\,ds = \underbrace{\int_0^2 \frac{s}{\sqrt{4-s^2}}\,ds}_{} + \int_0^2 \frac{1}{\sqrt{4-s^2}}\,ds$

$u = 4-s^2$ $\quad$ $s = 2\sin\theta$

$du = -2s\,ds$ $\quad$ $ds = 2\cos\theta\, d\theta$

$$-\frac{1}{2}\int_4^0 \frac{1}{\sqrt{u}}\,du + \int_0^{\pi/2} \frac{2\cos\theta\, d\theta}{2\cos\theta} = \lim_{R\to 0} -\sqrt{u}\Big|_4^R + \left(\pi/2 - 0\right)$$

$$= 2 + \pi/2 = 4+\pi/2$$

#28 $\displaystyle\int_0^1 \frac{4r}{\sqrt{1-r^4}}\,dr$ $\quad$ $u = r^2$, $du = 2r\,dr$ (+2)

$$\lim_{R\to 1} 2\int_0^R \frac{1}{\sqrt{1-u^2}}\,du = \lim_{R\to 1} 2\sin^{-1}(r^2)\Big|_0^R = 2(\pi/2 - 0)$$

$$= \pi$$

(+2)

#54 $\displaystyle\int_2^{\infty} \frac{x\,dx}{\sqrt{x^4-1}} \geq \int_2^{\infty} \frac{x}{\sqrt{x^4}}\,dx = \int_2^{\infty} \frac{1}{x}\,dx \to \infty.$

since $p=1$. Thus $\displaystyle\int_2^{\infty} \frac{x\,dx}{\sqrt{x^4-1}}$ diverges.

(+2)

#64 $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{e^x+e^{-x}} = \lim_{R\to\infty} \int_{-R}^{R} \frac{e^x}{e^{2x}+1}\,dx$ let $u=e^x$, $du=e^x\,dx$

$$\lim_{R\to\infty} \int_{e^{-R}}^{e^R} \frac{1}{u^2+1}\,du = \lim_{R\to\infty} \tan^{-1} u \Big|_{e^{-R}}^{e^R}$$

$$= \lim_{R\to\infty} \left[\tan^{-1}(e^R) - \tan^{-1}(e^{-R})\right]$$

$$= \pi/2 - 0 = \pi/2$$

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or

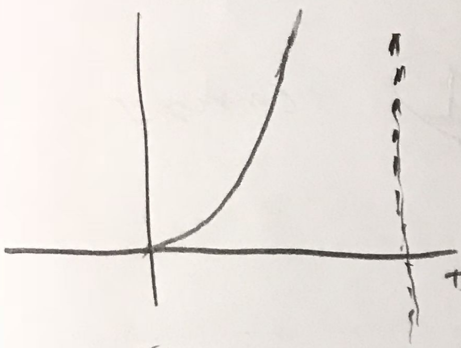
$\displaystyle\int_{-\infty}^{\infty} \frac{dx}{e^x+e^{-x}}\,dx = 2\int_0^{\infty} \frac{dx}{e^x+e^{-x}}$, $0 < \frac{1}{e^x+e^{-x}} < \frac{1}{e^x}$ for $x>0$.

$\displaystyle\int_0^{\infty} \frac{dx}{e^x}$ conv $\Rightarrow 2\int_0^{\infty} \frac{dx}{e^x+e^{-x}}$ conv. $\square$

(8)

#71 Area between curves (-2)

$y = \sec x$ and $y = \tan x$ from $x=0$ to $x = \pi/2$.



$$\int_0^{\pi/2} (\sec x - \tan x)\, dx = \lim_{b \to \pi/2^-} \left[\ln|\sec x + \tan x| - \ln|\sec x| \right]_0^b$$

$$= \lim_{b \to \pi/2^-} \left[\ln[1 + \sin b] \right] = \ln 2.$$

Section 10.1

#48 $a_n = \dfrac{3^n}{n^3} =$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{3^n \cdot \ln 3}{3n^2} = \lim_{n\to\infty} \frac{3^n (\ln 3)^2}{6n}$$

$$= \lim_{n\to\infty} \frac{3^n (\ln 3)^3}{6} = +\infty. \text{ diverge!}$$

(+1)

#50. $a_n = \dfrac{\ln(n)}{\ln(2n)}$

$$\lim_{n\to\infty} \frac{\ln(n)}{\ln(2n)} = \lim_{n\to\infty} \frac{(1/n)}{(2/2n)} = 1$$ converges.