1. Classify the surface and state the salient features of $4x^2 - 3y^2 + 12z^2 + 8x - 12y + 28 = 0$.

2. Classify the surface and state the salient features of $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.

Answers:

1. Use completing the square on $4x^2 - 3y^2 + 12z^2 + 8x - 12y + 28 = 0$ to get $4(x^2 + 2x) - 3(y^2 + 4y) + 12z^2 = -28 \rightarrow 4(x+1)^2 - 3(y+2)^2 + 12z^2 = -28 + 4 * (1) - 3 * (4) = -36$, which can be rewritten as $-\frac{(x+1)^2}{9} + \frac{(y+2)^2}{12} - \frac{z^2}{3} = 1$. This is a hyperboloid of two sheets (b/c there are two negative signs) that opens in the y direction (b/c the y term is positive) with center $(-1, -2, 0)$. There will be two vertices along the y-axis that are a distance of $\sqrt{12} = 2\sqrt{3}$ away from the center, i.e. the vertices are $\left(-1, -2 \pm 2\sqrt{3}, 0\right)$.

2. Use completing the square on $4y^2 + z^2 - x - 16y - 4z + 20 = 0$ to get $-x + 4(y^2 - 4y) + (z^2 - 4z) = -20 \rightarrow -x + 4(y-2)^2 + (z-2)^2 = -20 + 4 * (4) + (4) = 0$, which can be rewritten as $(y-2)^2 + \frac{(z-2)^2}{4} = \frac{x}{4}$. This is an elliptic paraboloid (b/c there are two positive squared terms and one linear term) that opens in the positive x direction (b/c the x term is linear) with vertex $(0, 2, 2)$.