1. Evaluate the surface integral $\iint_S (2xyz)\, dS$ where S is the portion of the plane $2x + 3y + z = 6$ above the first quadrant.

2. Set up, but do not evaluate, the surface integral $\iint_S \langle z, -y^2, x\rangle \cdot d\mathbf{S}$ where S is the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane and with upward oriented normal vectors.

Answers:

1. This is a scalar function surface integral of an explicitly defined surface $z = g(x, y)$, so we can use $\iint_S f(x, y, z)\, dS = \iint_R f(x, y, g(x, y))\sqrt{z_x^2 + z_y^2 + 1}\, dA$. The surface $z = 6 - 2x - 3y$ will intersect the xy-plane along the line $y = 2 - \frac{2}{3}x$ and it lies over a triangular region in the xy-plane bounded by this line and $x, y = 0$. Restricting the given function to the surface results in $2xyz = 2xy(6 - 2x - 3y)$, and the partial derivatives are $z_x = -2$ and $z_y = -3$. Putting this all into the integral gives $\iint_S (2xyz)\, dS = \int_0^3 \int_0^{2-\frac{2}{3}x} 2xy(6 - 2x - 3y)\sqrt{(-2)^2 + (-3)^2 + 1}\, dy\, dx = \frac{18\sqrt{14}}{5} \approx 13.47$.
2. This is a vector function surface integral (flux) of a surface $z = g(x, y)$, so we can use $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_R \mathbf{F} \cdot \langle -z_x, -z_y, 1\rangle\, dA$. The surface $z = 9 - x^2 - y^2$ will intersect the xy-plane along the circle $x^2 + y^2 = 9$ and R is the region bounded within. The normal vector is $\mathbf{n} = \langle -z_x, -z_y, 1\rangle = \langle 2x, 2y, 1\rangle$, and the function restricted to the surface will be $\mathbf{F} = \langle z, -y^2, x\rangle = \langle 9 - x^2 - y^2, -y^2, x\rangle$. Since the region R is a circle, we can convert $\mathbf{F}$ and $\mathbf{n}$ into polar coordinates. Then $\mathbf{F} \cdot \mathbf{n} = \langle 9 - r^2, -r^2 \sin^2\theta, r\cos\theta\rangle \cdot \langle 2r\cos\theta, 2r\sin\theta, 1\rangle = (18r - 2r^3)\cos\theta - 2r^3\sin^3\theta + r\cos\theta$ and $\iint_S \langle z, -y^2, x\rangle \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^3 \left((18r - 2r^3)\cos\theta - 2r^3\sin^3\theta + r\cos\theta\right) r\, dr\, d\theta$.