1. Determine the outward flux of the vector field $\mathbf{F} = \langle xyz, x^2, z\rangle$ through the solid bounded by the coordinate planes and the plane $3x + y + z = 6$.

2. Determine the outward flux through the surface bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane of the vector field $\mathbf{F} = \langle x^2 + \sin(yz), 2y + z, 3y^2 - 2z\rangle$.

Answers:

1. Outward flux through the surface S is $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV$. The curl of $\mathbf{F}$ is

$$\nabla \cdot \mathbf{F} = \frac{\partial(xyz)}{\partial x} + \frac{\partial(x^2)}{\partial y} + \frac{\partial(z)}{\partial z} = yz + 0 + 1.$$

The triangular region in the xy-plane below the surface $z = 6 - 3x - y$ is bounded by $0 \le x \le 2$ and $0 \le y \le 6 - 3x$. Then the net flux is

$$\int_0^2 \int_0^{6-3x} \int_0^{6-3x-y} (yz + 1)\, dz\, dy\, dx = \frac{168}{5} = 33.6.$$

2. The divergence of F is $\nabla \cdot \mathbf{F} = \frac{\partial(x^2+\sin(yz))}{\partial x} + \frac{\partial(2y+z)}{\partial y} + \frac{\partial(3y^2-2z)}{\partial z} = 2x + 2 - 2 = 2x$. The region in the xy-plane is a circle centered out the origin with radius 2. Then the net outward flux is

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV = \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} (2x)\, dz\, dy\, dx = \int_0^{2\pi} \int_0^2 \int_0^{4-r^2} (2r\cos\theta)\, r\, dz\, dr\, d\theta = 0.$$