MAT273 **Equations of Lines and Planes 12.5**

1. Determine the equation of the line through the point $(-2, 0\ ,4)$ and parallel to the vector $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

2. Determine the equation of the line through the points $(-3, 2, -3)$ and $(1, -1, 4)$.

3. Determine the equation of the plane containing the points $(1, -3, 6)$, $(0, 4, 4)$ and $(5, 8, 1)$.

4. Find the equation of the line of intersection between the planes $x - 2y + z\ = 2$ and $2x + 3y - 2z = 4$.

Answers:

1. $\vec{l}(t) = \langle -2,0,4\rangle + \langle 2,4,-2\rangle t$
2. The direction vector is $\langle 1-(-3), -1-2, 4-(-3)\rangle = \langle 4,-3,7\rangle$, so $\vec{l}(t) = \langle -3,2,-3\rangle + \langle 4,-3,7\rangle t = \langle 1,-1,4\rangle + \langle 4,-3,7\rangle t$.
3. Let $\vec{A} = \langle 0-1, 4-(-3), 4-6\rangle = \langle -1,7,-2\rangle$ and $\vec{B} = \langle 5-1, 8-(-3), 1-6\rangle = \langle 4,11,-5\rangle$. Then $\vec{n} = \langle a,b,c\rangle = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 7 & -2 \\ 4 & 11 & -5 \end{vmatrix} = \langle -35+22, -(5+8), -11-28\rangle = \langle -13,-13,-39\rangle = -13\langle 1,1,3\rangle$. Use $\langle 1,1,3\rangle$ and a point $(1,-3,6)$ to get $1(x-1) + 1(y+3) + 3(z-6) = 0 \rightarrow x+y+3z = 16$.
4. The line of intersection will be parallel to the vector $\vec{v} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \langle 1,-2,1\rangle \times \langle 2,3,-2\rangle = \langle 1,4,7\rangle$ and will pass through the point where $z = 0$ and the solution to $\begin{cases} x - 2y = 2 \\ 2x + 3y = 4 \end{cases}$, which is $(2,0,0)$. Therefore the resulting line is $\vec{l}(t) = \langle 2,0,0\rangle + \langle 1,4,7\rangle t$.