MAT273 — Final Exam Review

Show all your work, justify your answers and interpret any applicable solutions using complete sentences. You may use a scientific calculator like the Ti-36x Pro, but not a graphing calculator. (These problems are intended to help you prepare for the exam. Do not assume that these are the only problems you need to do. You should also review your notes, homework, quizzes, and other exercises from the text.)

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1. A box is dragged along the floor by a rope that applies a force of 50 pounds at an angle of 60° with the floor. How much work is done in moving the box 15 feet?

2. Find the area of the triangle with endpoints $(1, 5, -2)$, $(0, 0, 0)$ and $(3, 5, 1)$.

3. Find a unit vector that is perpendicular to the two vectors $\langle 2, 6, -3\rangle$ and $\langle 4, -3, 2\rangle$.

4. Find the equation of the line that is parallel to $\langle 1 + 2t, 4 - t, 6 + 2t\rangle$and passes through the point $(-2, 0, 5)$.

5. Find the intersection of the lines $\langle 2 + t, 2 + 3t, 3 + t\rangle$ and $\langle 2 + t, 3 + 4t, 4 + 2t\rangle$.

6. Find the plane whose points are equidistant from the points $(2, -1, 1)$ and $(3, 1, 5)$.

7. Find the line of intersection of the two planes $2x - 3y - 7z = 2$ and $x + 2y - 3z = -5$.

8. Identify the surface and make a rough sketch of $9x^2 - 4y^2 - 9z^2 = 36$.

9. Change the equation $x^2 + y^2 + z^2 = 2z$ into an equation in cylindrical and spherical coordinates.

10. Find a vector equation of the tangent line to the function $\mathbf{r}(t) = \langle t^2, -\frac{1}{t+1}, 4 - t^2\rangle$ at $t = 2$.

11. Find the arc length of the helix $\mathbf{r}(t) = \langle a\cos t, a\sin t, ct\rangle$ for $0 < t < T$.

12. Find the curvature and radius of curvature for $x = e^t\cos t$, $y = e^t\sin t$, $z = e^t$ when $t = 0$.

13. Given the acceleration of a particle in motion $\mathbf{a}(t) = \langle -\cos t, -\sin t \rangle$ with an initial velocity of $\langle 1, 0 \rangle$ and initial position of $\langle 0, 1 \rangle$, find the position vector function of the particle.

14. Find the angle between the velocity and acceleration vectors when $t = 1$ for the position function $\mathbf{s}(t) = \langle t^3, t^2 \rangle$.

15. Find the scalar tangential and normal components of acceleration at $t = 1$ of the particle whose position is given as $\mathbf{r}(t) = \langle t^3 - 2t, t^2 - 4 \rangle$.

16. A shell is fired from ground level with a muzzle speed of 320 ft/sec at an elevation of 60°. Find the position and speed at impact with the ground.

17. Sketch the domain of the function $z = \ln(x + y + 2)$.

18. Sketch the level curves of the function $z = x^2 + \frac{y^2}{4}$.

19. Find $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$.

20. The pressure, temperature, and volume of a an "ideal" gas behave according to the ideal gas law $PV = nRT$ where n is the number of moles of the gas, R is the gas constant in Joules per Kelvin (J/K), P is the absolute pressure in Pascals (Pa), V is the volume in cubic meters (m^3) , and T is the absolute temperature in Kelvin (K). When the gas is contained, then n is also a constant. Find the instantaneous rate of change of the volume with respect to the pressure if the gas is contained with $n = 4.00$ moles, the volume is 2.50 m^3, the temperature remains fixed at 281 K and $R = 8.31$ J/mol-K.

21. The area of a triangle is to be computed from the formula $A = \frac{1}{2}ab\sin\theta$, where a and b are side lengths and θ is the included angle between the sides. Suppose a, b, and θ are measured to be 40 ft, 50 ft, and 30°, respectively. Use the total differential of the area to calculate the maximum error in area if the maximum error in a, b, and θ are ½ ft, ¼ ft, and 2°, respectively.

22. Find the gradient of the function $f = y\ln(x + y + z)$ at the point $(-3, 4, 0)$.

23. Find (a) the unit vector of the direction that the directional derivative of the function $f = \sqrt{x^2 + y^2}$ at the point $(4, -3)$ is a maximum, (b) the rate of change in f in that direction, and (c) the direction in which the rate of change in the function be a minimum.

24. Find the equation of the tangent plane to the surface $z = e^{3y}\sin(3x)$ at the point $(\frac{\pi}{6}, 0, 1)$.

25. Find all relative maximum, relative minimum, and saddle points for the function $z = x^2 + y^2 + \frac{2}{xy}$.

26. Find the absolute maximum for the function $z = x^2 + 2y^2 - x$ within the circular boundary $x^2 + y^2 = 4$.

27. Find the volume of the wedge cut from the cylinder $4x^2 + y^2 = 9$ by the planes $z = 0$ and $z = y + 3$.

28. Use polar coordinates to find the volume in the first octant bounded above by the surface $z = r\sin\theta$, below by $z = 0$, and laterally by the plane $x = 0$ and the surface $r = 3\sin\theta$.

29. Find the surface area of the sphere of radius 4 centered at $(0, 0, 0)$ that lies between the planes $z = 1$ and $z = 2$.

30. Find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4$ and $z = 0$.

31. Find the center of mass of the solid bounded by the surface $z = 1 - y^2$, $z = 0$, $y \geq 0$, $x = -1$, $x = 1$.The density is given by $\delta(x, y, z) = yz$.

32. Find the volume of the solid in the first octant bounded by the sphere of radius 2 and the cones $\varphi = \frac{\pi}{6}$ to $\varphi = \frac{\pi}{3}$.

33. Find the work done by the force $\mathbf{F} = \langle xy, yz, xz\rangle$ along the curve $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ for $0 < t < 1$.

34. Confirm that $\mathbf{F} = \langle ye^{xy}, xe^{xy}\rangle$ is conservative and then compute the work done by $\mathbf{F}$ along any smooth curve from point $(-1, 1)$ to $(2, 1)$.

35. Evaluate (a) $\oint_C (x^2 - y)dx + x\,dy$ where C is the circle $x^2 + y^2 = 4$ traversed once counterclockwise, and (b) $\oint_C \ln(1+y)\,dx - \frac{xy}{1+y}dy$ where C is the triangle with vertices $(0,0)$, $(2,0)$, and $(0,4)$ traversed once counterclockwise.

36. Find the mass of the lamina that is the portion of the surface $z = 4 - y^2$ between the planes $x = 0$, $x = 3$, $y = 0$, and $y = 3$, if the density is $\delta(x,y,z) = y$.

37. Find the flux of $\mathbf{F} = \langle x^3, x^2y, xy \rangle$ over the surface of the solid bounded by $z = 4 - x^2$, $y + z = 5$, $z = 0$, and $y = 0$ with outward normal vector.

38. Find the volume of the portion of the sphere of radius 4 centered at $(0,0,0)$ that lies between the planes $z = 1$ and $z = 2$.

Answers:

1. 375 ft-lb
2. 9.67
3. $\langle 0.088, -0.469, -0.879\rangle$
4. $\langle 2t-2, -t, 5+2t\rangle$
5. $(1, -1, 2)$
6. $x+2y+4z=14.5$
7. $\langle -\frac{11}{7}-23t, -\frac{12}{7}+t, -7t\rangle$
8. Hyperboloid of 2 sheets, opens in x-directions, vertices $(\pm 2, 0, 0)$
9. $r^2+(z-1)^2=1$, $\rho=2\cos\varphi$
10. $\langle 4+4t, -\frac{1}{3}+\frac{1}{9}t, -4t\rangle$
11. $T\sqrt{a^2+c^2}$
12. $\kappa=0.4714, \frac{1}{\kappa}=2.1213$
13. $\langle t+\cos t-1, \sin t-t+1\rangle$
14. 15°
15. $a_N=\frac{10}{\sqrt{5}}, a_T=\frac{10}{\sqrt{5}}$
16. x = 2771.28 ft, v = 320 ft/s
17. Region above the line $y=-x-2$
18. Ellipses getting larger as z increases
19. DNE
20. -0.0006691 m^3/Pa
21. 39 ft^2
22. $\langle 4, 4, 4\rangle$
23. (a) $\nabla f=\langle\frac{4}{5}, -\frac{3}{5}\rangle$; (b) $|\nabla f|=1$; (c) $-\nabla f=\langle -\frac{4}{5}, \frac{3}{5}\rangle$
24. $3y-z=-1$
25. $(-1,-1,4)$ and $(1,1,4)$ are both relative minimums
26. $\left(-\frac{1}{2}, \pm\frac{\sqrt{15}}{2}, \frac{33}{4}\right)$ are absolute maximums, $\left(\frac{1}{2}, 0, -\frac{1}{4}\right)$ is an absolute minimum
27. 27π/2
28. 27π/16
29. 8π
30. 256/15
31. (0, 16/35, 1/2)
32. 1.533
33. 27/28
34. $\varphi=e^{xy}$, so $\varphi(2,1)-\varphi(-1,1)=7.02$
35. (a) 8π; (b) -4
36. 56.016
37. 4608/35
38. $\frac{41}{3}\pi$