**MAT 273** **Formula Sheet**

$$|\mathbf{v}| = \sqrt{a^2+b^2+c^2}$$

$$\mathbf{u}\cdot\mathbf{v} = u_1v_1+u_2v_2+u_3v_3 = |\mathbf{u}||\mathbf{v}|\cos\theta$$

$$W = \mathbf{F}\cdot\mathbf{d} = |\mathbf{F}||\mathbf{d}|\cos\theta$$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = |\mathbf{u}|\cos\theta\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix} = \begin{vmatrix}u_2 & u_3\\ v_2 & v_3\end{vmatrix}\mathbf{i} - \begin{vmatrix}u_1 & u_3\\ v_1 & v_3\end{vmatrix}\mathbf{j} + \begin{vmatrix}u_1 & u_2\\ v_1 & v_2\end{vmatrix}\mathbf{k}$$

$$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin\theta$$

$$\tau = \mathbf{r}\times\mathbf{F}$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = \langle x_0+at, y_0+bt, z_0+ct\rangle$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$$

$$L = \int_a^b |r'(t)|dt = \int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2+[h'(t)]^2}dt = \int_\alpha^\beta \sqrt{[f(\theta)]^2+[f'(\theta)]^2}d\theta$$

$$s(t) = \int_a^t |\mathbf{r}'(u)|du$$

$$\kappa(s) = \left|\frac{d\mathbf{T}}{ds}\right|$$

$$\kappa(t) = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{\mathbf{a}\cdot\mathbf{N}}{|\mathbf{v}|^2} = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$$

$$\mathbf{N}(s) = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{1}{\kappa}\frac{d\mathbf{T}}{ds}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \mathbf{N}\times\mathbf{B}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \mathbf{B}\times\mathbf{T}$$

$$\mathbf{B}(t) = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|} = \mathbf{T}\times\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T}$$

$$\mathrm{a}_N = \kappa|\mathbf{v}|^2 = \mathbf{a}\cdot\mathbf{N} = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|}$$

$$a_T = \frac{d^2s}{dt^2} = \mathbf{a}\cdot\mathbf{T} = \frac{\mathbf{v}\cdot\mathbf{a}}{|\mathbf{v}|}$$

$$\tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N} = -\frac{1}{|\mathbf{v}|}\frac{d\mathbf{B}}{dt}\cdot\mathbf{N} = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{a}'}{|\mathbf{v}\times\mathbf{a}|^2}$$

$$f(a,b) \approx L(x,y) = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

$$\Delta z \approx dz = f_x(a,b)dx + f_y(a,b)dy$$

$$D_{\mathbf{u}}f(a,b) = \nabla f(a,b)\cdot\mathbf{u} = |\nabla f(a,b)|\cos\theta$$

$$\nabla f(x,y,z) = \langle f_x, f_y, f_z\rangle$$

$$D(x,y) = f_{xx}(a,b)f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$$

$$x^2+y^2 = r^2 \qquad x^2+y^2+z^2 = \rho^2$$

$$x = r\cos\theta = \rho\sin\varphi\cos\theta \qquad y = r\sin\theta = \rho\sin\varphi\sin\theta \qquad z = \rho\cos\varphi$$

$$\iint_R f(x,y)\,dA = \int_a^b\int_{g(x)}^{h(x)} f(x,y)\,dy\,dx = \int_\alpha^\beta\int_{g(\theta)}^{h(\theta)} f(r,\theta)\,r\,dr\,d\theta$$

$$\iiint_D f(x,y,z)\,dV = \int_a^b\int_{g(x)}^{h(x)}\int_{G(x,y)}^{H(x,y)} f(x,y,z)\,dz\,dy\,dx = \int_a^b\int_{g(\theta)}^{h(\theta)}\int_{G(r,\theta)}^{H(r\,\theta)} f(r,\theta,z)dz\,r dr\,d\theta = \int_\alpha^\beta\int_a^b\int_{G(\varphi,\theta)}^{H(\varphi,\theta)} f(\rho,\varphi,\theta)\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$A_R = \iint_R dA \qquad V_D = \iiint_D dV$$

$$\bar{f} = \frac{1}{A_R}\iint_R f(x,y)\,dA = \frac{1}{V_D}\iiint_D f(x,y,z)\,dV$$

$$m = \iint_D \rho(x,y)dA = \iiint_D \delta(x,y,z)dV \qquad \bar{x} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$\langle \bar{x}, \bar{y} \rangle = \frac{1}{m} \langle \iint_D x\, \rho(x,y)dA\, , \iint_D y\, \rho(x,y)dA \rangle$$

$$\langle \bar{x}, \bar{y}, \bar{z} \rangle = \frac{1}{\mathrm{m}} \langle \iiint_D x\, \delta(x,y,z)dV\, , \iiint_D y\, \delta(x,y,z)dV\, , \iiint_D z\, \delta(x,y,z)dV \rangle$$

$$A = \iint_R \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}\, dA = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

$$I_x = \iint_D y^2\, \rho(x,y)dA \qquad I_y = \iint_D x^2\, \rho(x,y)dA \qquad I_0 = I_x + I_y = \iint_D (x^2 + y^2)\, \rho(x,y)dA$$

$$I_x = \iiint_E (y^2 + z^2)\rho(x,y,z)dV \qquad I_y = \iiint_E (x^2 + z^2)\rho(x,y,z)dV \qquad I_z = \iiint_E (x^2 + y^2)\rho(x,y,z)dV$$

$$\int_C f\, ds = \int_a^b f\big(x(t), y(t)\big)\, |\mathbf{r}'(t)|\, dt$$

$$\int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\, dx + Q\, dy + R\, dz = \int_a^b \mathbf{F} \cdot \mathbf{r}'(t)\, dt = f(B) - f(A)$$

$$\int_C \mathbf{F} \cdot \mathbf{n}\, ds = \int_C f\, dy - g\, dx = \int_a^b \big(f(t)y'(t) - g(t)x'(t)\big)\, dt$$

$$\nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

$$\iint_S f(x,y,z)\, dS = \iint_R f\big(x(u,v), y(u,v), z(u,v)\big)\, |\mathbf{r}_u \times \mathbf{r}_v|\, dA = \iint_R f\big(x, y, g(x,y)\big)\sqrt{g_x^2 + g_y^2 + 1}\, dA$$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_R \mathbf{F} \cdot \mathbf{n}\, dA = \iint_R \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA = \iint_R \mathbf{F}(x,y,g) \cdot \langle -g_x, -g_y, 1 \rangle\, dA$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C P\, dx + Q\, dy = \iint_R (Q_x - P_y)\, dA \qquad \oint_C \mathbf{F} \cdot \mathbf{n}\, ds = \oint_C f\, dy - g\, dx = \iint_R (f_x + g_y)dA$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS \qquad \iint_{\boldsymbol{S}} \boldsymbol{F} \cdot \boldsymbol{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV$$