1. Given the sinusoidal cylinder $y = \sin(x)$ where $0 \leq x \leq \pi$ and $0 \leq z \leq 4$.
   a) Determine the surface area.
   b) Determine the equation of the tangent plane at $\left(\frac{\pi}{6}, \frac{1}{2}, 2\right)$.

2. Given the cubic cylinder $y = x^3$ where $0 \leq x \leq 2$ and $0 \leq z \leq 6$ with normal vector in the positive x-direction, set up the flux integral for the function $\mathbf{F}(x, y, z) = yz\mathbf{i} + x\mathbf{j} - z^2\mathbf{k}$.

Answers:

1. This surface would be the graph of $y = \sin(x)$ in the xy-plane extended upwards and parallel to the z-axis. It cannot be described explicitly as $z = g(x, y)$, so we will need to determine a parameterization. Let $u = x$ and $v = z$. Then we have the function $\mathbf{r}(u, v) = \langle u, \sin u, v\rangle$ over the region $D = \{(u, v) : 0 \le u \le \pi, 0 \le v \le 4\}$.
   a) To find surface area, we need $\mathbf{r}_u = \langle 1, \cos u, 0\rangle$ and $\mathbf{r}_v = \langle 0, 0, 1\rangle$. So $\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle \cos u, -1, 0\rangle$ and $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{\cos^2 u + 1}$. Then the area is $A = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA = \int_0^4 \int_0^\pi \sqrt{\cos^2 u + 1}\, du\, dv = 15.28$.
   b) To write the equation of a plane, $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, we need a normal vector and a point. We are given the point $\left(\frac{\pi}{6}, \frac{1}{2}, 2\right)$ which corresponds to $\mathbf{r}\left(\frac{\pi}{6}, 2\right) = \langle \frac{\pi}{6}, \sin\frac{\pi}{6}, 2\rangle = \langle \frac{\pi}{6}, \frac{1}{2}, 2\rangle$, and the normal vector will be $\mathbf{n} = \mathbf{r}_u \times \mathbf{r}_v$. At the point $\mathbf{r}\left(\frac{\pi}{6}, 2\right)$, we get $\mathbf{n} = \langle \cos\frac{\pi}{6}, -1, 0\rangle = \langle \frac{\sqrt{3}}{2}, -1, 0\rangle$. Thus the equation of the tangent line will be $\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \left(y - \frac{1}{2}\right) = 0$.
2. This surface would be the graph of $y = x^3$ in the xy-plane stretched upward parallel to the z-axis. Since it cannot be explicitly expressed as $z = g(x, y)$, we will use the parameterization $u = x$ and $v = z$ to get the function $\mathbf{r}(u, v) = \langle u, u^3, v\rangle$ over the region $D = \{(u, v) : 0 \le u \le 2, 0 \le v \le 6\}$. The flux of $\mathbf{F}(x, y, z) = yz\mathbf{i} + x\mathbf{j} - z^2\mathbf{k}$ is $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$. We get $\mathbf{r}_u = \langle 1, 3u^2, 0\rangle$ and $\mathbf{r}_v = \langle 0, 0, 1\rangle$, so $\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3u^2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 3u^2, -1, 0\rangle$. Also, by restricting $\mathbf{F}$ to $\mathbf{r}$, we get $\mathbf{F}(u, v) = \langle u^3 v, u, -v^2\rangle$. Thus the flux of $\mathbf{F}$ over the surface is $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA = \iint_D \langle u^3 v, u, -v^2\rangle \cdot \langle 3u^2, -1, 0\rangle\, dA = \int_0^6 \int_0^2 3u^5 v - u\, du\, dv = 564$.