1. Determine $\int_C y^2 x\, dx + x\, dy$ where C is travelling once around the circle $x^2 + y^2 = 9$.

2. Determine $\int_C \langle 3xy, -y^3 \rangle \cdot d\mathbf{r}$ where C is the curve that starts at $(-3, 9)$ and goes to $(3, 9)$ along $y = x^2$ and then goes from $(3, 9)$ to $(-3, 9)$ along $y = 9$.

Answers:

1. Given $\mathbf{F}(x,y) = \langle y^2x, x\rangle = \langle f, g\rangle$, we get $\frac{\partial g}{\partial x} = 1, \frac{\partial f}{\partial y} = 2xy$. Then $\int_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA =$ $\iint_R (1 - 2xy)dA = \int_0^{2\pi}\int_0^3 (1 - 2r\cos\theta\, r\sin\theta)r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 (r - 2r^3\cos\theta\sin\theta)dr\,d\theta = 9\pi$.
2. Given $\mathbf{F}(x,y) = \langle 3xy, -y^3\rangle = \langle f, g\rangle$, we get $\frac{\partial g}{\partial x} = 0, \frac{\partial f}{\partial y} = 3x$. Then $\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA = \int_{-3}^{3}\int_{x^2}^{9} (-3x)dy\,dx = 0$.