MTH201 Chapter 2 Practice Test

Give yourself 75 minutes to complete this test. I recommend spending at most 10-15 minutes on the multiple choice ($<$ 1.5 minutes per question). Then you will have 60 minutes to do the free response questions (about 6 minutes per question), which is more than enough considering this test was going to be administered during the 50 minute period.

1. What concept is shown in the following integral?

$$\int_0^1 \sqrt{1+9^2}\ dx$$

A. Area
B. Volume - Disk
C. Volume - Shells
D. Arc Length
E. Surface Area
F. Work

2. What is the best method to solve the problem:
"Find the volume of a solid formed by revolving the region between $y = \sec(x)$, $y = \tan(x)$, and $x = 0$ about the x-axis."

A. Disk
B. Washer
C. Shells
D. Neither

3. What does the underlined part of the integral represent?

$$\int_0^5 2\pi\,(\underline{x+2})\,x\ dx$$

A. The width of a shell.
B. The outer radius of a washer.
C. The inner radius of a washer.
D. The radius of a shell.

4. A Spring exerts a force of 5 N when stretched 1 m. How much work is required to stretch the spring 1.8 m?.
A. 9 N
B. 4 N
C. 8.1 N
D. 16.2 N

5. Which integral is an example of surface area?
   A. $\int_0^1 2\pi x^3\sqrt{1+9x^4}\ dx$
   B. $\int_1^2 \sqrt{1+\frac{9}{4}x}\ dx$
   C. $\int_1^4 2\pi x\sqrt{1+x}\ dx$
   D. $\int_0^1 2\pi x\left(x-x^2\right)dx$

6. Which integral represents the volume revolving $y = 2\cos(3x)$ about the y-axis?
   A. $\frac{\pi}{3}\int_0^2 \arccos\left(\frac{y}{2}\right)dy$
   B. $2\pi\int_0^{\frac{\pi}{6}} x\left(2\cos(3x)\right)dx$
   C. $\pi\int_0^{\frac{\pi}{6}} 2\cos^2(3x)dx$
   D. $2\pi\int_0^2 y\left(\frac{1}{3}\arccos\frac{y}{2}\right)dy$

7. Which integral represents the volume revolving the region enclosed by $x = \sqrt{y}$ and $x = \frac{y}{4}$ across the x-axis?
   A. $\pi\int_0^{16}\left(\left(\sqrt{y}\right)^2 - \left(\frac{y}{4}\right)^2\right)dy$
   B. $2\pi\int_0^{16}\left(\sqrt{y}-\frac{y}{4}\right)^2 dx$
   C. $2\pi\int_0^4 x\left((4x)^2-\left(x^2\right)^2\right)dx$
   D. $\pi\int_0^4\left((4x)^2 - x^4\right)dx$

8. Which integral is an example of arc length?
   A. $\int_1^2 \sqrt{1+81y^4}\ dy$
   B. $\pi\int_1^2 y^3\sqrt{1+9y^4}\ dy$
   C. $2\pi\int_1^2 x\sqrt{x}\ dx$

9. What does the underlined part of the integral represent?

$$2\pi\int_1^2 (x+\underline{1})\,x\ dx$$

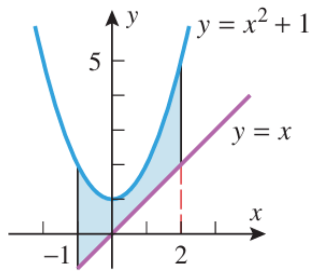
   A. Height of shell
   B. Distance of disk to y-axis
   C. The axis or line of rotation
   D. Radius of shell.

10. What does the integral represent?

$$2\pi\int_0^1 x^3\left(1+9x^4\right)^{\frac{1}{2}}dx$$

   A. Volume (shell method)
   B. Volume (disk method)
   C. Surface area
   D. Mass of circular object

11. Find the area of the shaded region



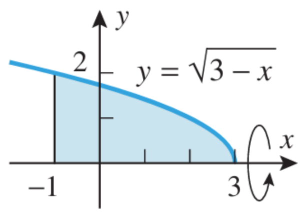
12. A spring exerts a force of 100 N when it is stretched 0.2 m beyond its natural length. How much work is required to stretch the spring 0.8 m beyond its natural length?

13. Use the shell method to find the volume of the solid when the region enclosed by $y = x^3$, $x = 1$ and $y = 0$ is revolved about the y-axis.

14. Set up, but do not evaluate, an integral representing the surface area when $y = \sqrt{4 - x^2}$ is revolved about the x-axis on $-1 \leq x \leq -1$.

15. A cylindrical tank of radius 5 ft and height 9 ft is half-filled with water. Find the work required to pump all the water over the upper rim. (The weight density of water is $62.4\frac{lb}{ft^3}$

16. Find the volume of a solid that results when the shaded region is revolved about the x-axis.



17. Find the volume of the solid generated when the region enclosed by $x = \sqrt{y}$ and $x = \frac{y}{4}$ is revolved about the x-axis.

18. A rectangular dam is 40 ft high and 60 ft wide. Assume the weight density of water is $62.5\frac{lb}{ft^3}$. Compute the total force F on the dam when the surface of the water is halfway down the dam.

19. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

20. Use shells to find the volume of the solid that is generated when the region that is enclosed by $y = \frac{1}{x^3}$, $x = 1$, $x = 2$, and $y = 0$ is revolved about the line $x = -1$.

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I evaluated all the integrals mostly manually, but to save time you can plug the entire integral in your calculator without π, turn the decimal into a fraction (if exact answer is required), and add the π after.

1. What concept is shown in the following integral?

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D. **Arc Length**
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C. $\pi\int_0^{\frac{\pi}{6}} 2\cos^2(3x)dx$
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A. $\pi\int_0^{16}\left(\left(\sqrt{y}\right)^2-\left(\frac{y}{4}\right)^2\right)dy$
B. $2\pi\int_0^{16}\left(\sqrt{y}-\frac{y}{4}\right)^2 dx$
C. $2\pi\int_0^4 x\left((4x)^2-\left(x^2\right)^2\right)dx$
D. $\boldsymbol{\pi\int_0^4\left((4x)^2-x^4\right)dx}$

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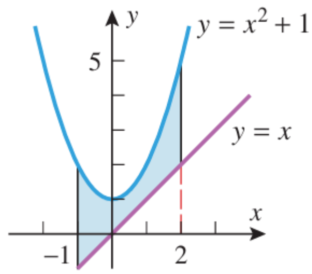


The top curve is $y = x^2 + 1$ and the bottom is $y = x$.

$$A = \int_{-1}^{2} \left[\left(x^2 + 1\right) - x \right] dx$$

$$= \int_{-1}^{2} \left(x^2 - x + 1\right) dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + x \right]_{-1}^{2}$$

$$= \left(\frac{8}{3} - \frac{-11}{6} \right)$$

$$= \boldsymbol{\frac{9}{2}} \textbf{\textit{ units}}^{\mathbf{2}}$$

12. A spring exerts a force of 100 N when it is stretched 0.2 m beyond its natural length. How much work is required to stretch the spring 0.8 m beyond its natural length?
Start by finding the spring constant with Hooke's Law $F = kx$.

$$F = kx$$
$$k = \frac{F}{x} = \frac{100\ N}{0.2\ m} = 500\ \frac{N}{m}$$

Now integrate $F(x) = kx$ over the distance the spring is stretched.

$$W = \int_0^{0.8} F(x)\, dx$$
$$= 500 \int_0^{0.8} x\ dx$$
$$= \mathbf{160\ N \cdot m}$$
$$= \mathbf{160\ J}$$

13. Use the shell method to find the volume of the solid when the region enclosed by $y = x^3$, $x = 1$ and $y = 0$ is revolved about the y-axis.
The radius of the shells is x and the heights are x^3.
This is being applied on $0 \leq x \leq 1$.

$$V = 2\pi \int_0^1 x\left(x^3\right)dx$$

$$= 2\pi \int_0^1 x^4 dx$$

$$= 2\pi \left[\frac{1}{5}x^5\right]_0^1 \qquad = \boldsymbol{\frac{2\pi}{5}\ units^3}$$

14. Set up, but do not evaluate, an integral representing the surface area when $y = \sqrt{4 - x^2}$ is revolved about the x-axis on $-1 \leq x \leq -1$. To find the surface area you need the derivative of the function.

$$\begin{aligned} y &= \left(4 - x^2\right)^{\frac{1}{2}} \\ \frac{dy}{dx} &= \frac{1}{2}\left(4 - x^2\right)^{-\frac{1}{2}} \cdot (-2x) \\ &= \frac{-2x}{2\sqrt{4 - x^2}} \\ &= \frac{-x}{\sqrt{4 - x^2}} \end{aligned}$$

This is the formula for surface area:

$$S = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Apply the formula:

$$\boldsymbol{S = 2\pi \int_{-1}^{1} \sqrt{4 - x^2} \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2}\, dx}$$

This is similar to problem 14c on the SI practice problems if you want to know how to evaluate it. It involves squaring the derivative, getting a common denominator in the square root, combining the terms in the square root, splitting it, and canceling terms.

15. A cylindrical tank of radius 5 ft and height 9 ft is half-filled with water. Find the work required to pump all the water over the upper rim. (The weight density of water is $62.4\frac{lb}{ft^3}$

You can memorize the formula in the notes but I will show how to do a rough derivation in case you forget.

To find work, our final answer has units of $lb \cdot ft$. So far, we know that the weight density needs to be in the integral, which has units of $\frac{lb}{ft^3}$.

What else do we need to add to the integral to get units of $lb \cdot ft$?

$$\frac{lb}{ft^3} \cdot \boxed{ft^4} = lb \cdot ft$$

We need something in terms of the geometry of the cylinder to get units of ft^4.

If we multiply ρ by the volume, we get lb, which is the weight (force) of all the water. To get the units we want, we have to multiply it by a distance unit (ft). We can think of this as the weight of the water applied (multiplied) over the distance the water travels. So, the whole result of the integral gets nice units of $lb \cdot ft$ and matches the definition of work (force applied over a distance).

This extra distance unit is represented by x. Now, we just need to find a formula for volume.

$$V = CSA \cdot h$$

The cross-sectional area is a constant $\pi\,(5)^2 = 25\pi\ units^2$. The height will be represented by x (note that I am defining the $+x$-axis going from the bottom to the top of the cylinder). To find the height, we don't want to multiply the cross-sectional area by the entire height but rather the thickness of an infinitesimal disk. This will be dx.

The limits of the integral are the positions on the x axis that the liquid will travel over. That will be $4.5 \leq x \leq 9$.

Put the whole integral together:

$$W = \int_{x_1}^{x_2} \rho \cdot CSA \cdot h\ dx$$

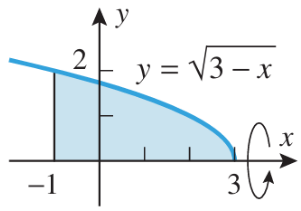
$$= \int_{4.5}^{9} 62.4 \cdot 25\pi \cdot x \cdot dx$$

$$= \boldsymbol{43,785\pi\ ft \cdot lb}$$

Do another unit check just in case:

$$\frac{lb}{ft^3} \cdot ft^2 \cdot ft \cdot ft = \frac{lb}{ft^3} \cdot ft^4 = lb \cdot ft$$

16. Find the volume of a solid that results when the shaded region is revolved about the x-axis.



Use the disk method.

$$V = \pi \int_{-1}^{3} \left(\sqrt{3-x}\right)^2 dx$$

$$= \pi \int_{-1}^{3} (3-x)\, dx$$

$$= \pi \left[3x - \frac{1}{2}x^2\right]_{-1}^{3}$$

$$= \pi \left(\frac{9}{2} + \frac{7}{2}\right)$$

$$= \boldsymbol{8\pi \; units^3}$$

17. Find the volume of the solid generated when the region enclosed by $x = \sqrt{y}$ and $x = \frac{y}{4}$ is revolved about the x-axis.

Find where the functions intersect to find the bounds of the integral

$$x^2 = 4x$$
$$x = 0,\ x = 4$$

Now, set up the washer method.

$$V = \pi \int_0^4 \left((4x)^2 - \left(x^2\right)^2\right) dx$$
$$= \pi \int_0^4 \left(16x^2 - x^4\right) dx$$
$$= \left[\frac{16}{3}x^3 - \frac{1}{5}x^5\right]_0^4$$
$$= \pi \left(\frac{2048}{15}\right)$$
$$= \boldsymbol{\frac{2048\pi}{15}\ units^3}$$

18. A rectangular dam is 40 ft high and 60 ft wide. Assume the weight density of water is $62.5\frac{lb}{ft^3}$. Compute the total force F on the dam when the surface of the water is halfway down the dam.

Find your width and depth functions:

$$w(x) = 60\ ft$$
$$s(x) = x$$

Set up and evaluate the integral:

$$F = \int_0^{20} (62.5 \cdot 60 \cdot x \cdot dx)$$

750,000 *lb*

Unit check:

$$\frac{lb}{ft^3} \cdot ft \cdot ft \cdot ft = lb$$

19. Find the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.
It is easy to use disks by rewriting the functions in terms of x and integrating on the x-axis.

$$y = \sqrt{x}$$

Set the functions equal to each other to find the bounds:

$$\sqrt{x} = x$$
$$x = 0, \ x = 1$$

The radius of the top and bottom disks will be lengthened by 1 because of the other axis of rotation.
Set up and evaluate the integral:

$$\begin{aligned} V = \pi \int_0^1 \left(\left(\sqrt{x} + 1\right)^2 - (x+1)^2 \right) dx \\ &= \pi \int_0^1 \left(x + 2\sqrt{x} + 1 - x^2 - 2x - 1 \right) dx \\ &= \pi \int_0^1 \left(-x^2 - x + 2\sqrt{x} \right) dx \\ &= \pi \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} \right]_0^1 \\ &= \pi \left(-\frac{1}{3} - \frac{1}{2} + \frac{4}{3} \right) \\ &= \pi \left(1 - \frac{1}{2} \right) \\ &= \boldsymbol{\frac{\pi}{2}} \ \textbf{\textit{units}}^{\mathbf{3}} \end{aligned}$$

20. Use shells to find the volume of the solid that is generated when the region that is enclosed by $y = \frac{1}{x^3}$, $x = 1$, $x = 2$, and $y = 0$ is revolved about the line $x = -1$.
The shell formula is $\int r \cdot h \cdot dx$, where r is the radius of the shells and h is the height of the shells.
In this case the height is $\frac{1}{x^3}$ and the radius is $x + 1$ because the axis of rotation is 1 unit away.
Set up and evaluate the integral:

$$V = 2\pi \int_1^2 (x+1)\left(\frac{1}{x^3}\right) dx$$

$$\begin{aligned}
&= 2\pi \int_1^2 \frac{x+1}{x^3} dx \\
&= 2\pi \int_1^2 \left(\frac{x}{x^3} + \frac{1}{x^3}\right) dx \\
&= 2\pi \int_1^2 \left(\frac{1}{x^2} + \frac{1}{x^3}\right) dx \\
&= 2\pi \int_1^2 \left(x^{-2} + x^{-3}\right) dx \\
&= 2\pi \left[\frac{x^{-1}}{-1} + \frac{x^{-2}}{-2}\right]_1^2 \\
&= 2\pi \left[\frac{-1}{x} - \frac{1}{2x^2}\right]_1^2 \\
&= 2\pi \left(\frac{-5}{8} - \frac{-3}{2}\right) \\
&= 2\pi \left(\frac{7}{8}\right) \\
&= \boldsymbol{\frac{7\pi}{4}\ units^3}
\end{aligned}$$

MTH201 Chapter 3 Practice Test

Give yourself 50 minutes to complete this test. You will have about 4.1 minutes per question.

1. (12 pts) Evaluate using Partial Fraction Decomposition.

$$\int \frac{2x^2+3}{x\left(x-1\right)^2}dx$$

2. (12 pts) Integrate using trig integration / trig reduction techniques.

$$\int \tan 4x \sec^4 4x dx$$

3. (12 pts) Evaluate using IBP.

$$\int_1^e x^2 \ln x dx$$

4. (12 pts) Evaluate using trig substitution.

$$\int \frac{\sqrt{x^2-9}}{x} dx$$

Evaluate using any method except tables (15 pts each).

1.

$$\int \frac{dx}{x^2 - 3x - 4}$$

2.

$$\int_1^\infty e^{-2x}\,dx$$

3.

$$\int \sin{(\ln x)} dx$$

4.

$$\int \sin^2 x \cos^3 x dx$$

5.

$$\int e^x \sqrt{1 - e^{2x}}\,dx$$

6.

$$\int_0^4 \frac{dx}{(x-4)^2}$$

Evaluate using any method, including tables.

1.

$$\int e^x \sqrt{3 - 4e^{2x}} dx$$

2.

$$\int \sqrt{4 - x^2} dx$$

MTH201 Chapter 3 Practice Test

Give yourself 50 minutes to complete this test. You will have about 4.1 minutes per question.

1. (12 pts) Evaluate using Partial Fraction Decomposition.

$$\int \frac{2x^2+3}{x\left(x-1\right)^2}dx$$

Use PFD.

$$\begin{aligned}\frac{2x^2+3}{x\left(x-1\right)^2} &= \frac{A}{x}+\frac{B}{x-1}+\frac{C}{\left(x-1\right)^2}\\ 2x^2+3 &= A\left(x-1\right)^2+B\left(x\right)\left(x-1\right)+C\left(x\right)\\ &= Ax^2-2Ax+A+Bx^2-Bx+Cx\end{aligned}$$

Set up the system and solve.

$$\begin{aligned}A+B&=2\\ -2A-B+C&=0\\ A&=3\end{aligned}$$

$$A=3,\ B=-1,\ C=5$$

Evaluate the integral.

$$\begin{aligned}\int \frac{2x^2+3}{x\left(x-1\right)^2}dx &= \int\left(\frac{3}{x}-\frac{1}{x-1}+\frac{5}{\left(x-1\right)^2}\right)dx\\ &= \boldsymbol{3\ln|x|-\ln|x-1|-\frac{5}{x-1}+C}\end{aligned}$$

2. (12 pts) Integrate using trig integration / trig reduction techniques.

$$\int \tan 4x \sec^4 4x dx$$

Split off a $\sec 4x$ so that you have $\tan 4x \sec 4x dx$ in the integrand, which is part of the derivative of $\sec 4x$. Then set $u=\sec 4x$, and find an expression for $\tan 4x \sec 4x dx$.

$$\int \tan 4x \sec^4 4x dx = \int \tan 4x \sec 4x \sec^3 4x dx$$

$$u=\sec 4x,\ du=4\sec 4x \tan 4x\ dx \Rightarrow \tan 4x \sec 4x\ dx=\frac{du}{4}$$

$$\begin{aligned}\int \tan 4x \sec 4x \sec^3 4x dx &= \frac{1}{4}\int u^3 du\\ &= \frac{1}{16}u^4+C\\ &= \boldsymbol{\frac{1}{16}\sec^4 4x+C}\end{aligned}$$

3. (12 pts) Evaluate using IBP.

$$\int_1^e x^2 \ln x dx$$

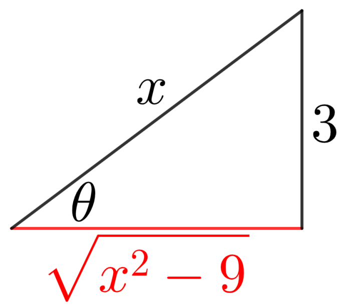
You only need IBP once. Set $u = \ln x$ according to LIATE.

$$\begin{aligned}
u = \ln x, \Rightarrow du &= \frac{1}{x}dx \\
dv = x^2 dx \Rightarrow v &= \frac{1}{3}x^3 \\
\int_1^e x^2 \ln x \; dx &= \frac{1}{3}x^3 \ln x \Big|_1^e - \int_1^e \frac{1}{3}x^3 \cdot \frac{1}{x}dx \\
&= \frac{1}{3}x^3 \ln x \Big|_1^e - \frac{1}{3}\int_1^e x^2 dx \\
&= \frac{1}{3}x^3 \ln x \Big|_1^e - \frac{1}{9}x^3\Big|_1^e \\
&= \left(\frac{1}{3}e^3 - 0\right) - \left(\frac{1}{9}e^3 - \frac{1}{9}\right) \\
&= \frac{2}{9}e^3 + \frac{1}{9} \\
&= \boldsymbol{\frac{2e^3+1}{9}}
\end{aligned}$$

4. (12 pts) Evaluate using trig substitution.

$$\int \frac{\sqrt{x^2-9}}{x}dx$$

Use trig sub.



$$\begin{aligned}
\cos\theta &= \frac{\sqrt{x^2-9}}{x} \\
\csc\theta &= \frac{x}{3} \Rightarrow x = 3\csc\theta \Rightarrow dx = -3\csc\theta\cot\theta \; d\theta \\
\int \frac{\sqrt{x^2-9}}{x}dx &= -3\int \cos\theta\csc\theta\cot\theta \; d\theta \\
&= -3\int \cot^2\theta \; d\theta
\end{aligned}$$

Use the reduction formula for cotangent.

$$\begin{aligned}
-3\int \cot^2\theta\, d\theta &= -3\left(-\frac{1}{1}\cot\theta - \int d\theta\right)\\
&= 3\cot\theta + 3\theta + C\\
&= 3\frac{\sqrt{x^2-9}}{3} + \csc^{-1}\frac{x}{3} + C\\
&= \boldsymbol{\frac{\sqrt{x^2-9}}{3} + \csc^{-1}\frac{x}{3} + C}
\end{aligned}$$

Evaluate using any method except tables (15 pts each).

1.
$$\int \frac{dx}{x^2-3x-4}$$

The denominator is factorable, so you can use PFD.

$$\begin{aligned}
\frac{dx}{x^2-3x-4} &= \frac{1}{(x-4)(x+1)}\\
&= \frac{A}{x-4} + \frac{B}{x+1}\\
1 &= A(x+1) + B(x-4)\\
&= Ax + A + Bx - 4B
\end{aligned}$$

Set up the system and solve.

$$\begin{aligned}
A + B &= 0\\
A - 4B &= 1
\end{aligned}$$

$$\Rightarrow A = \frac{1}{5},\ B = -\frac{1}{5}$$

Evaluate the integral.

$$\int \frac{1}{5(x-4)} - \frac{1}{5(x+1)} dx = \boldsymbol{\frac{1}{5}\ln|x-4| - \frac{1}{5}\ln|x+1| + C}$$

2.

$$\int_1^{\infty} e^{-2x}dx$$

Since one of the bounds is ∞, this is an improper integral.

$$\begin{aligned}
\int_0^{\infty} e^{-2x}dx &= \lim_{t\to\infty}\int_0^t e^{-2x}dx \\
&= \lim_{t\to\infty} -\frac{1}{2}e^{-2x}\Big|_0^t \\
&= \lim_{t\to\infty}\left(-\frac{1}{2}e^{-2t}+\frac{1}{2}\right) \\
&= \lim_{t\to\infty}\left(-\frac{1}{2e^{2t}}+\frac{1}{2}\right) \\
&= -\frac{1}{\infty}+\frac{1}{2} = 0+\frac{1}{2} \\
&= \mathbf{\frac{1}{2}}
\end{aligned}$$

3.

$$\int \sin(\ln x)dx$$

You can integrate this with IBP if you set $u = \sin(\ln x)$.

$$\begin{aligned}
u &= \sin(\ln x) \Rightarrow du = \frac{\cos(\ln x)}{x}dx \\
dv &= dx \Rightarrow v = x
\end{aligned}$$

$$\begin{aligned}
\int \sin(\ln x)dx &= x\sin(\ln x) - \int x\cdot\frac{\cos(\ln x)}{x}\cdot dx \\
&= x\sin(\ln x) - \int \cos(\ln x)dx
\end{aligned}$$

Do IBP again.

$$\begin{aligned}
u &= \cos(\ln x) \Rightarrow du = -\frac{\sin(\ln x)}{x}dx \\
dv &= dx \Rightarrow v = x
\end{aligned}$$

$$\begin{aligned}
x\sin(\ln x) - \int \cos(\ln x)dx &= x\sin(\ln x) - \left(x\cos(\ln x) - \int -\sin(\ln x)dx\right) \\
&= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)dx \\
2\int \sin(\ln x)dx &= x\sin(\ln x) - x\cos(\ln x) + C \\
\int \sin(\ln x)dx &= \mathbf{\frac{1}{2}(\sin(\ln x) - x\cos(\ln x) + C)}
\end{aligned}$$

4.

$$\int \sin^2 x \cos^3 x dx$$

This can be done with trig reduction tecnhiques. Split off a $\cos x$ and let $u = \sin x$. Use the Pythagorean Identity to write $\cos^2 x$ in terms of $\sin^2 x$.

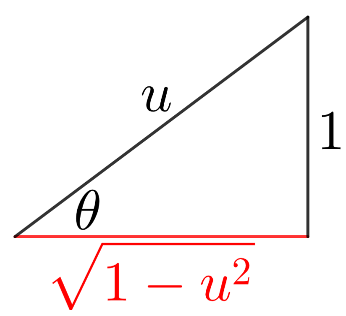
$$\begin{aligned}
\int \sin^2 x \cos^3 x dx &= \int \sin^2 x \cos^2 x \cos x \ dx \\
&= \int \sin^2 x \left(1 - \sin^2 x\right) \cos x \ dx \\
& u = \sin x \Rightarrow du = \cos x dx \\
\int \sin^2 x \left(1 - \sin^2 x\right) \cos x \ dx &= \int u^2 \left(1 - u^2\right) du \\
&= \int \left(u^2 - u^4\right) du \\
&= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C \\
&= \boldsymbol{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}
\end{aligned}$$

5.

$$\int e^x \sqrt{1 - e^{2x}} dx$$

Once you do u-substitution, you can integrate with trig sub.

$$\begin{aligned}
& u = e^x \Rightarrow du = e^x dx \\
\int e^x \sqrt{1 - e^{2x}} dx &= \int \sqrt{1 - u^2} du
\end{aligned}$$



$$\begin{aligned}
\cos\theta &= \sqrt{1 - u^2} \\
& u = \sin\theta \Rightarrow du = \cos\theta \ d\theta \\
\int \sqrt{1 - u^2} du &= \int \cos^2\theta \ d\theta
\end{aligned}$$

Use reduction formulas.

$$\begin{aligned}\int \cos^2\theta\, d\theta &= \frac{\sin\theta\cos\theta}{2} + \frac{1}{2}\int d\theta \\ &= \frac{\sin\theta\cos\theta}{2} + \frac{1}{2}\theta + C \\ &= \frac{u\sqrt{1-u^2}}{2} + \frac{1}{2}\arcsin u + C \\ &= \boldsymbol{\frac{e^x\sqrt{1-e^{2x}}}{2} + \frac{1}{2}\arcsin e^x + C}\end{aligned}$$

6.

$$\int_0^4 \frac{dx}{(x-4)^2}$$

The integrand is discontinuous at the top bound, so it is an improper integral.

$$\begin{aligned}\int_0^4 \frac{dx}{(x-4)^2} &= \lim_{t\to 4}\int_0^t \frac{dx}{(x-4)^2} \\ &= \lim_{t\to 4} -\frac{1}{x-4}\bigg|_0^t \\ &= \lim_{t\to 4}\left(-\frac{1}{t-4} - \frac{1}{4}\right) \\ &= -\frac{1}{0} - \frac{1}{4} \\ &= -\infty - \frac{1}{4}\end{aligned}$$

$\Rightarrow$ **The integral is divergent.**

Evaluate using any method, including tables.

1.

$$\int e^x\sqrt{3-4e^{2x}}dx$$

You can do this with tables, but it will take a bit of work. You have to make substitutions twice.

$$u = e^x \Rightarrow du = e^x dx$$

$$\int e^x\sqrt{3-4e^{2x}}dx = \int \sqrt{\left(\sqrt{3}\right)^2 - (2u)^2}dx$$

$$v = 2u \Rightarrow dv = 2du \Rightarrow du = \frac{1}{2}dv$$

$$\int \sqrt{\left(\sqrt{3}\right)^2 - (2u)^2}dx = \frac{1}{2}\int \sqrt{\left(\sqrt{3}\right)^2 - v^2}dx$$

Use Formula #74: $\int \sqrt{a^2-u^2}du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\left(\frac{u}{a}\right) + C$.

$$\begin{aligned}
\frac{1}{2}\int \sqrt{\left(\sqrt{3}\right)^2 - v^2}dx &= \frac{1}{2}\left[\frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2}\arcsin\left(\frac{v}{a}\right) + C\right] \\
&= \frac{1}{2}\left[\frac{2u}{2}\sqrt{3-4u^2} + \frac{3}{2}\arcsin\left(\frac{2u}{\sqrt{3}}\right) + C\right] \\
&= \frac{2e^x}{4}\sqrt{3-4e^{2x}} + \frac{3}{4}\arcsin\left(\frac{2e^x}{\sqrt{3}}\right) + C \\
&= \boldsymbol{\frac{e^x}{2}\sqrt{3-4e^{2x}} + \frac{3}{4}\arcsin\left(\frac{2e^x}{\sqrt{3}}\right) + C}
\end{aligned}$$

2.

$$\int \sqrt{4-x^2}dx$$

Since I did not design this test well, you can use Formula #74, but this time it's a lot easier.

$$\begin{aligned}
\int \sqrt{4-x^2}dx &= \int \sqrt{2^2-x^2}dx \\
&= \boldsymbol{\frac{x}{2}\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right) + C}
\end{aligned}$$

MTH201 Chapter 6 and 7 Practice Test

Give yourself 50 minutes to complete this test.

Multiple Choice

1. Convert the point $\left(6, \frac{\pi}{6}\right)$ to rectangular form.
   (a) $\left(3, 3\sqrt{3}\right)$
   (b) $\left(0, 2\sqrt{3}\right)$
   (c) $\left(3\sqrt{3}, 3\right)$
   (d) $\left(\sqrt{3}, \sqrt{3}\right)$

2. What is a set of parametric equations that best describes a circle of radius 5, centered at $(1, 1)$, oriented clockwise?
   (a) $(5\sin t + 1, 5\cos t + 1)$
   (b) $(-5\cos t + 1, 5\sin t - 1)$
   (c) $(5\sin t + 1, -5\cos t + 1)$
   (d) $(5\cos t + 1, 5\sin t + 1)$

3. What function is represented by the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k$?
   (a) $\ln(1 + x)$
   (b) $1 - e^{-x}$
   (c) $\arctan x$
   (d) $\ln(1 - x)$

4. Rewrite the rectangular point $\left(2\sqrt{3}, -2\right)$ into polar form.
   (a) $\left(-4, \frac{11\pi}{6}\right)$
   (b) $\left(-4, -\frac{\pi}{6}\right)$
   (c) $\left(4, \frac{\pi}{6}\right)$
   (d) $\left(4, \frac{11\pi}{6}\right)$

5. Which point is the same as the polar coordinate $\left(5, \frac{\pi}{4}\right)$?
   (a) $\left(-5, \frac{5\pi}{4}\right)$
   (b) $\left(5, \frac{5\pi}{4}\right)$
   (c) $\left(-5, \frac{9\pi}{4}\right)$
   (d) $\left(-5, \frac{\pi}{4}\right)$

6. Write the rectangular point $(1, 1)$ into polar form. Choose all that apply.
   (a) $\left(1, \frac{\pi}{4}\right)$
   (b) $\left(\sqrt{2}, \frac{\pi}{4}\right)$
   (c) $\left(\sqrt{2}, \frac{3\pi}{4}\right)$
   (d) $\left(\sqrt{2}, -\frac{7\pi}{4}\right)$
7. Find $\frac{d^2y}{dx^2}$ for the set of parametric equations at $t = 1$:

$$x(t) = \sqrt{t}$$
$$y(t) = 2t + 4$$

   (a) -4
   (b) 0
   (c) 4
   (d) 2

Free Response

1. Find the Maclaurin Series for $f(x) = \cos(\pi x)$. Then, find the interval of convergence.
2. Find the equation of the tangent line at $t = 1$ for the parametric curve $x = e^t,\ y = e^{-t}$.
3. Sketch the parametric curve and indicate its orientation.

$$x(t) = 3t - 4$$
$$y(t) = 6t + 2$$

4. Approximate $\sin 4°$ to 5 decimal places of accuracy using a Maclaurin Polynomial.
5. Find the radius of convergence for the power series $\sum_{k=0}^{\infty} \frac{x^k}{k+1}$.
6. Find the area of the region enclosed by the rose $r = 4\cos(3\theta)$.
7. Rewrite the function $f(x) = \frac{1}{x}$ as a power series centered at 1 and find the interval of convergence.
8. Find the arc length of the parametric equation from $0 \leq t \leq 1$.

$$x(t) = t^2$$
$$y(t) = \frac{1}{3}t^3$$

9. Find the area of the parametric equation on $0 \leq t \leq 2\pi$.

$$x(t) = 3\cos t$$
$$y(t) = 3\sin t$$

10. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the parametric equation at $t = \frac{\pi}{3}$

$$x(t) = \sec t$$
$$y(t) = \tan t$$

11. Find the area of one petal of $r = 3\cos(2\theta)$. Use reduction formulas.

Chapter 7 answers

1. $(3\sqrt{3}, 3)$ c

2. a. $(5\sin(t)+1, 5\cos(t)+1)$

3. a. $\ln(1+x)$

4. d: $(4, \frac{11\pi}{6})$

5. $(-5, \frac{5\pi}{4})$ a

6. b, c $(\sqrt{2}, \frac{\pi}{4})$ and $(\sqrt{2}, -\frac{7\pi}{4})$

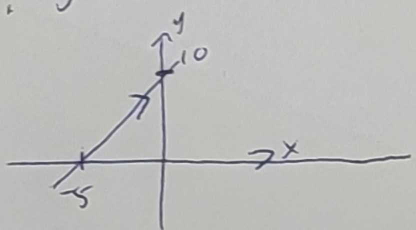
7. C: 4

FRQ

1. $\sum \frac{(-1)^k \pi^{2k}}{(2k)!}$

2. $y = e^{-2}x + 2e^{-1}$

3.



4. 0.069756

5. $R = 1$

6. $A = 4\pi$

7. $\frac{1}{x} = \sum_{n=0}^{\infty} (1-x)^n$, $I = (0, 2)$

8. $\frac{1}{3}(5\sqrt{5} - 8)$

9. radius is 3 $\rightarrow A = \pi r^2 = \pi(3)^2 = 9\pi$

10. $\frac{dy}{dx} = \frac{2}{\sqrt{3}}$, $\frac{d^2y}{dx^2} = \frac{-1}{3\sqrt{3}}$

11. $\frac{9\pi}{8}$