OUR TITLE

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Contents

1. Introduction to Planar Graphs

Definition 1.1. A graph G is called a *planar graph* if G can be drawn in the plane without any two of its edges crossing [1]. If G is already drawn in the plane without crossings, then G is a *plane graph*.

Importantly, any graph isometric to a plane graph is therefore planar.

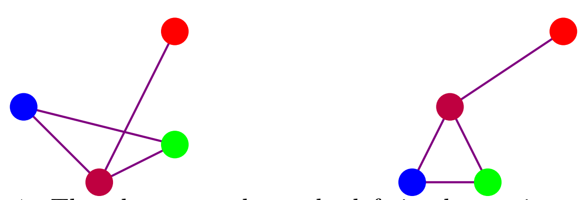

Figure 1. The planar graph on the left is planar since it is isomorphic to the plane graph on the right.

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From here on, when referring to planar graphs, we will be considering the plane graph that the graph is isomorphic to. Often, when working with planar graphs, one is concerned with whether or not a given graph is planar. This question appears often in contexts where there are connections on a $2D$ gid and intersections are impossible.

In order to solve this, we must discuss what properties define a planar graph. One important theorem is the Euler Identity.

Theorem 1.1 (The Euler Identity [1]). *For every connected plane graph of order n, size m and having r regions,*

$$n - m + r = 2.$$

In order to be able to understand this theorem, let us first discuss the regions of a graph. A *region* is an area bounded by the edges and vertices of a graph G. Additionally, there is an external region which is unbounded.

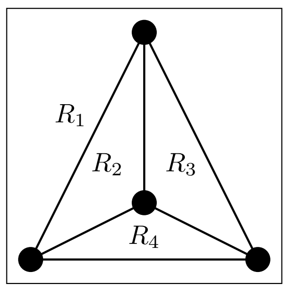

FIGURE 2. A planar graph with its regions denoted.

With this in mind, let us prove Euler's Identity.

Proof. Proceed by induction.

(1) *Base Case:* Consider the graph K_1, of order 1, size 0 and containing one region. Since there are no possible edges that could cross, the induction holds.
(2) *Inductive Hypothesis:* If we assume that the hypothesis holds for a graph of order n, size m and r regions, then we will show it holds for a graph of order n, size $m = 1$ and

$\square$

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References

[1] Gary Chartrand, Linda Lesniak, and Ping Zhang, *Graphs and Digraphs*, CRC Press, Boca Raton, 2016.