September 14th and 18th Lectures: Solving LPs via the Simplex Method.

References: Chapters 2 and 3 of Chvatal. Note that we discussed avoiding cycling via the smallest subscript rule rather than the perturbation rule so there is no need to read pages 34-36 or the part of page 37 before the paragraph beginning **Smallest-subscript rule.** Furthermore we did not discuss tableaux so there is no need to read the tableaux section on pages 23-25.

September 14th Lecture: We showed that every LP instance can be formulated as an instance of LP in standard form (for the proof see Chapter 1 of Chvatal). We showed how the simplex method worked using the example of Chapter 2 of Chvatal although we did a different set of pivots(our calculations are set out in the Powerpoint for the lecture). We defined *slack* and *decision* variables, *dictionaries, feasible dictionaries, basic* and *non-basic* variables, *pivot.* We implicitly set out the main steps of the Simplex Method

1) Find a feasible dictionary.

2) Determine if there is a non-basic variable with positive cost coefficient in the expression for z. If not, terminate the current solution is optimal

3) Choose a non basic variable x_e with positive cost coefficient in this expression (x_e is the entering variable).

4) Determine if there is a basic variable x_j whose nonnegativity gives an upper bound on the possible values of x_e if we increase it while leaving all other nonbasic variables at 0. If not, the problem is unbounded, return this fact.

5) Otherwise let x_l be (one of) the basic variable(s) whose nonnegativity gives the most stringent upper bound on the possible values of x_e. x_l is the leaving variable.

6) Rewrite the equation for x_l in the dictionary as an equation for x_e.

7) Replace x_e by the RHS of this equation in all the other equations of the dictionary.

8) Go back to Step 2.

We asked ourselves what could go wrong. We were concerned with whether we could find a feasible dictionary as required in Step 1 and whether the method would always terminate with an optimal solution or if it could, e.g., cycle through the same set of dictionaries endlessly

September 18th Lecture: : We continued our study of the Simplex Method for solving LPs. We proved (see page 33 of the text) that each choice of a basis yields a unique dictionary. It follows that there are at most $\binom{n+m}{m}$ different dictionaries. Thus, for any LP in standard form, provided we can find a feasible dictionary in Step 1, and that we never visit a dictionary twice, then the algorithm will either terminate with an optimal solution in Step 2 or because the problem is unbounded in Step 4. We next noted that the objective function can never decrease in an iteration and that it remains the same precisely if the leaving variable had value zero in the dictionary at the beginning of the iteration. We called such iterations degenerate and dictionaries with a zero valued basic variable degenerate. We observed that the only way that we could visit a dictionary twice was via a cycle consisting of a sequence of degenerate iterations. This involves staying at the same solution which has more than n zero valued variables. We discussed the example on pages 31-32 of Chavatal where such a cycle is traversed. We followed the proof in the text that if from amongst all choices for the entering variable we always chose that with the lowest subscript and from amongst all choices for the leaving variable we always choose the one with the lowest subscript then we would never repeat a dictionary and hence would eventually terminate provided we could find a feasible dictionary in Step 1.