Assignment 2 Part 2

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Question 1

In the two phase simplex method we must first find if the problem has any feasible solutions. In this phase we cannot pivot out the auxiliary variable, therefore can only choose between to constraints giving

C(n+m, m-1) = C(3 + 15, 2) = C(18, 2) + 1= 154

Now assuming the result from the the first phase gives us a feasible starting solution we can begin finding the optimal feasible solution. Since we know that the LP has an optimal feasible solution we can know there is a finite number of dictionaries. More specifically there are;

C(n+m, m) = C(3 + 15, 3) = C(18, 3) = 816

Therefore at the maximum iterations possible to find the optimal feasible solution is 970. This is assuming we avoid cycling using the smallest subscript rule. Taking this into account you should be wiling to pay is $109.70.

Question 2

The basic solution to the problem (4,0,0,7,0,0,11,15,10,0,0,3,4,6,0) is not possible. This is the case because the problem is formulated with only 3 constraints, and therefore only three basic variables. To find a feasible solution to a dictionary we must set all the non basic variable of the dictionary to 0 find values for basic variables. Therefore we could only solve for the values of three variables at a time because we only have three equations (minus the objective function) coming from the constraints. Therefore we should have a maximum of 3 non-zero variables in the basic solution.

Question 3

We use the fact that if the primal of an LP is unbounded then the dual of the LP is infeasible.
$x_6 = 7 - x_1 + 3x_4 + x_5$
$x_2 = 9 + 3x_1 + 3x_4 - 3x_5$
$x_3 = 12 - x_1 + 5x_4 - 11x_5$
$z = 562 + 24x_1 + 5x_4 - 7x_5$
By looking at the above dictionary we can deduce that the variable x_4 is unbounded. Since the dual is attempting to approximate an upper bound on the primal, if the primal is unbounded then the dual will have no solution.

Question 4

For the first phase of the simplex we have family F of the form:
$\{z, x_1,x_2,x_3,\ldots\}$
$\{z, x_1,x_5,x_3,\ldots\}$
$\{z, x_1,x_5,x_6,\ldots\}$
Where each entry is of size m with consisting of P+z, thus p is defined as:

$P_1=\{x_1,x_2,x_3,\ldots\}$
$P_2=\{x_1,x_5,x_3,\ldots\}$
$P_3=\{x_1,x_5,x_6,\ldots\}$
We are trying to find a bijection between this set and the degenerate sets created when solving a dictionary in the second phase of the simplex method:
$\{0, x_1,x_2,x_3,\ldots\}$
$\{0, x_1,x_5,x_3,\ldots\}$
$\{0, x_1,x_5,x_6,\ldots\}$
At maximum the number n of degenerate dictionaries is less than or equal to the the number of of dictionaries in F consisting of P. We get this by setting the values of z to 0 in the first form the simplex method.