COMP 360 Assignment 3

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Question 1

- Let S_m be a set consisting of 4m 1's and 2m 2's
- Let S_j be a subset of S_m where $2m \leq j \leq 4m$ and the Sum of the elements in S_j is equal to 4m
- Let $|S_j| = |S_1| + |S_2|$ where S_1 is the subset of S_j consisting of all 1's, S_2 the same for 2's
- From problem description: $|S_1| = 2j - 4m$; $|S_2| = 4m - j$
- Then, $|S_j| = 2j - 4m + 4m - j = j$

Proof:

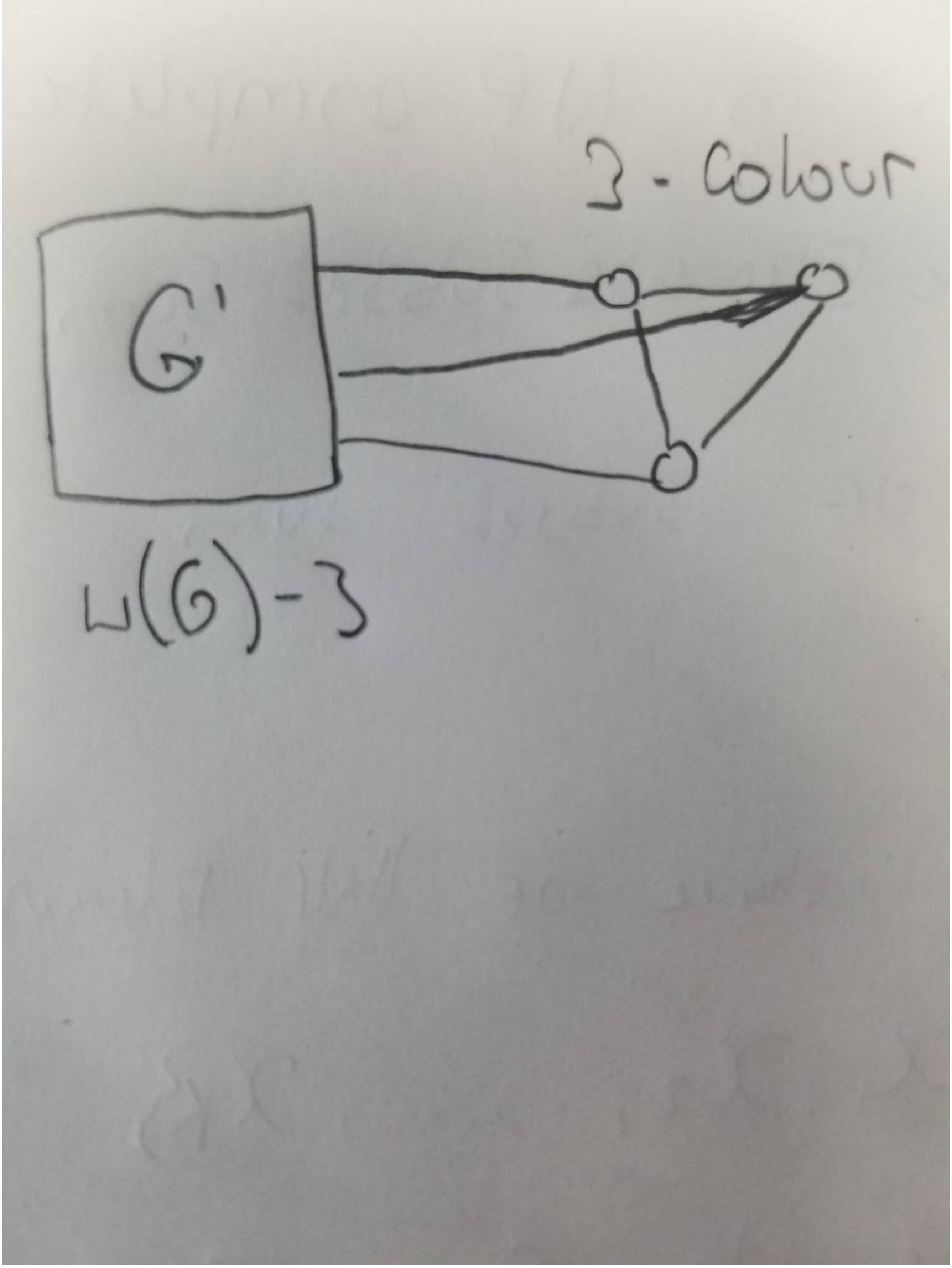
$\sum(S_j) = 1*|S_1| + 2*|S_2|$
$= 1*(2j - 4m) + 2(4m - j)$
$= 2j- 4m + 8m - 2j$
$= 4m$

- From this we have show that for any set S_j constrained by $2m \leq j \leq 4m$ there is a solution of size j where the sum of all elements of the set is equal to 4m

Question 2

- Will attempt to reduce subset sum problem to a problem in NP
- Prove: Subset-Sum ≤ Half The Elements Subset Sum
- Let S_k be a set with k-integers
  $S_k = \{c_1, c_2, \ldots, c_k\}$
- We then want to double the set to get a 2K set:
  $S_{2k} = \{c_1, c_2, \ldots, c_k, c_1, c_2, \ldots, c_k\}$
- Then we add an element to S_{2k}: $T-\sum S_k$, Where T is the target
- This element is chosen because as you can see if we take the original set S_k from S_k we can formulate that: $T=(T - \sum S_k) + \sum S_k$
- From this we know that the subset $S=\{S_k, (T - \sum S_k)\}$ has a solution T
- Thus we know that if the original set has a solution then so will the Half sum set problem
- The certificate for this is to take the solution subset, and sum the set and verify is equals the target which is in poly time

Question 3



- We prove this by reducing 3-colouring≤w(G)-colouring
- We first create a graph G` that is a complete subgraph and has a colouring w(G)-3
- We also want to create another graph G that is 3-coloured
- Connecting every node in the G to the graph G' we have reduced the problem from 3-

coloring≤w(G)-colouring

- To reduce w(G)-colouring≤3-SAT
  - For each node create a w(g) literals
  - Need a clause so that each node only has 1 colour, ie only one literal can be true
  - We also need clauses to verify that each node is not the same colou\r as a node adjacent to it
- The certificate of this problem to to iterate through the nodes and verify that no node has the same colour as an adjacent node

Question 4

x	y	{x OR not y} AND {not x OR y}
F	F	T
F	T	F
T	F	F
T	T	T

- In every instance of a True solution, if y is assigned true then so is x as you can see by the truth table

Question 5

- We want to reduce 3-SATy≤only 5 copies satisfiability
- If a literal appears only 5 times in the satisfiability instance then there is no issue
- If a literal appears more than 5 times we want to create a new set which we can call x_n where x_n corresponds to the the literal that appears more than five times, then for each literal above the upper bound create a new literal y_n. Thus we get: $x_n=\{y_1,y_2,\ldots,y_n\}$
- Now for this new literal we can create a new clause A such that:
  A={y_1 OR y_2 OR … OR y_n}
- Now we add two new clauses to the original set of clauses C_j:
  $C_{x'n}$ = x_n OR A OR A
  C_{xn} = x_n OR not A OR not A
- The certificate for this is to solve the satisfiability

Question 6

- Page 1091 he is proving the solution is NP-hard not NP-complete