Review of The Last Lecture

Linear Program:

Maximize or minimize a linear objective function subject to a set of linear constraints.

Integer Linear Program:

Maximize or minimize a linear objective function subject to a set of linear constraints and the constraint that all variables values are integers.

Feasible solution: assignment of values to variables satisfying all constraints.

Polly’s LP:

Minimize $3x_1+24x_2+13x_3+9x_4+20x_5+19x_6$

Subject to:

$$110x_1+205x_2+160x_3+160x_4+420x_5+260x_6 \geq 2000$$

$$4x_1+32x_2+13x_3+8x_4+4x_5+14x_6 \geq 55$$

$$2x_1+12x_2+54x_3+285x_4+22x_5+80x_6 \geq 800$$

$$x_1,x_2,x_3,x_4,x_5,x_6 \geq 0$$

Feasible solution $x_1=14,x_2=0,x_3=0,x_4=3,x_5=0,x_6=0$
Cost=3x14+3x9=69.

Lower bound: energy costs at least 3/110 cents per calorie so total cost $\geq$ 6000/110 cents

COMP 360 7/10/2018 Lecture
Formulating Problems as LPs I

Two Scheduling Problems

Chvatal Problem 1.6

[Adapted from Greene et al. (1959).] A meat packing plant produces 480 hams, 400 pork bellies, and 230 picnic hams every day; each of these products can be sold either fresh or smoked. The total number of hams, bellies, and picnics that can be smoked during a normal working day is 420; in addition, up to 250 products can be smoked on overtime at a higher cost. The *net* profits are as follows:

	Fresh	Smoked on regular time	Smoked on overtime
Hams	$8	$14	$11
Bellies	$4	$12	$7
Picnics	$4	$13	$9

For example, the following schedule yields a total net profit of $9,965:

	Fresh	Smoked	Smoked (overtime)
Hams	165	280	35
Bellies	295	70	35
Picnics	55	70	105

The objective is to find the schedule that maximizes the total net profit. Formulate as an LP problem in the standard form.

Vasek, its not an LP its an ILP, fix your book!

Maximize $8x_1+14x_2+11x_3+4x_4+12x_5+7x_6+4x_7+13x_8+9x_9$

Subject To:

$x_1+x_2+x_3 = 480$

$x_4+x_5+x_6 = 400$

$x_7+x_8+x_9 = 230$

$x_2+x_5+x_8 \leq 420$

$x_3+x_6+x_9 \leq 250$

$X_1,\ldots,x_9 \geq 0$

$X_1,\ldots,x_9$ integer

Chvatal Problem 1.8

An electronics company has a contract to deliver 20,000 radios within the next four weeks. The client is willing to pay \$20 for each radio delivered by the end of the first week, \$18 for those delivered by the end of the second week, \$16 by the end of the third week, and \$14 by the end of the fourth week. Since each worker can assemble only 50 radios per week, the company cannot meet the order with its present labor force of 40; hence it must hire and train temporary help. Any of the experienced workers can be taken off the assembly line to instruct a class of three trainees; after one week of instruction, each of the trainees can either proceed to the assembly line or instruct additional new classes.

At present, the company has no other contracts; hence some workers may become idle once the delivery is completed. All of them, whether permanent or temporary, must be kept on the payroll till the end of the fourth week. The weekly wages of a worker, whether assembling, instructing, or being idle, are \$200; the weekly wages of a trainee are \$100. The production costs, excluding the worker's wages, are \$5 per radio.

For example, the company could adopt the following program.

First week: 10 assemblers, 30 instructors, 90 trainees
Workers' wages: \$8,000
Trainees' wages: \$9,000
Profit from 500 radios: \$7,500
Net loss: \$9,500

Second week: 120 assemblers, 10 instructors, 30 trainees
Workers' wages: \$26,000
Trainees' wages: \$3,000
Profit from 6,000 radios: \$78,000
Net profit: \$49,000

Third week: 160 assemblers
Workers' wages: \$32,000
Profit from 8,000 radios: \$88,000
Net profit: \$56,000

Fourth week: 110 assemblers, 50 idle
Workers' wages: \$32,000
Profit from 5,500 radios: \$49,500
Net profit: \$17,500

This program, leading to a total net profit of \$113,000, is one of many possible programs. The company's aim is to maximize the total net profit. Formulate as an LP problem (not necessarily in the standard form).

Vasek, Its an ILP not an LP, fix your book!

Maximize $15r_1+13r_2+11r_3+9r_4-200w_1-200w_2-200w_3-200w_4-100t_1-100t_2-100t_3-100t_4$

Subject To:

$r_1+r_2+r_3+r_4= 20{,}000$

$r_1 \leq 50a_1$, $r_2 \leq 50a_2$, $r_3 \leq 50a_3$, $r_4 \leq 50a_4$

$t_1 \leq 3\mathrm{Ins}_1$, $t_2 \leq 3\mathrm{Ins}_2$, $t_3 \leq 3\mathrm{Ins}_3$, $t_4 \leq 3\mathrm{Ins}_4$

$a_1+\mathrm{Ins}_1 \leq w_1$, $a_2+\mathrm{Ins}_2 \leq w_2$, $a_3+\mathrm{Ins}_3 \leq w_3$, $a_4+\mathrm{Ins}_4 \leq w_4$

$w_1+t_1=w_2$, $w_2+t_2=w_3$, $w_3+t_3=w_4$

All variables ≥ 0

All variables integer

Knapsack

A Specific Knapsack Instance

A thief flies his helicopter into an isolated jungle temple, while all the monks are away on a pilgrimage. He will be able to steal statues whose total weight is at most 1000 pounds, as if the helicopter is loaded with more, it will be unable to take off. There are eight statues whose weights and values are given in the table below. The problem is to find the maximum total value he can steal.

	Weight	Value
Sun God	800	2000
Moon God	670	1300
Earth Goddess	550	1250
Emperor	250	750
Empress	250	750
Elephant	550	1200
Jackal	40	100
Panther	40	90

Cleaning Out The Temple

Maximize $2000x_1+1300x_2+1250x_3+750x_4+750x_5+1200x_6+100x_7+100x_8$

Subject To:

$800x_1+670x_2+550x_3+250x_4+250x_5+550x_6+40x_7+40x_8 \quad \leq 1000$

For i with $1 \leq i \leq 8$, $x_i \geq 0$, $x_i \leq 1$, x_i integer.

A General Knapsack Instance

An instance consists of a set $I_1,\ldots,I_n$ of items, where item I_j has real weight w_j and real value v_j, and a knapsack which can hold total weight W. A solution is a subset of the items which can fit in the knapsack (i.e. their combined weight is at most W). Our objective is to choose a solution whose total value is as large as possible.

Maximize $\sum_{i=1}^{n} v_i x_i$

Subject to:

$\sum_{i=1}^{n} w_i x_i \leq W$

For i with $1 \leq i \leq n$, $x_i \geq 0$, $x_i \leq 1$, x_i integer

Multi-Knapsack: You are at a store where you can pick as many copies of each item as you please.

Maximize $\sum_{i=1}^{n} v_i x_i$

Subject to:

$\sum_{i=1}^{n} w_i x_i \leq W$

For i with $1 \leq i \leq n$, $x_i \geq 0$, x_i integer

Fractional Multi-Knapsack: Youcan pick as many copies of each item as you please and you are choosing liquids so can pick e.g. 1.6457 litres.

Maximize $\sum_{i=1}^{n} v_i x_i$

Subject to:

$\sum_{i=1}^{n} w_i x_i \leq W$

For i with $1 \leq i \leq n$, $x_i \geq 0$,

Fractional Knapsack: there is only one item of each type but you are choosing liquids so can pick any fraction of that item.

Maximize $\sum_{i=1}^{n} v_i x_i$

Subject to:

$\sum_{i=1}^{n} w_i x_i \leq W$

For i with $1 \leq i \leq n$, $x_i \geq 0, x_i \leq 1$

Lower Bound On The Number of Comparisions to Build A Heap

Summer Research Project carried out by a McGill Undergraduate student in 2005 after his first year.

Resulted in a conference publication.

LP had 209 constraints and hundreds of variables,

Details here: https://www.cs.mcgill.ca/~zli47/heaps.html

(Just Provided as an example of a larger LP formulaton arising in a different area, you need not look at any of the details or remember anything about it)

STOPPED HERE, GO NO FURTHER,
ENTER AT YOUR OWN RISK

Finding The Maximum

Find the maximum of n reals: $b_1,\ldots,b_n$

Satisfiability

Boolean Variables: are TRUE or FALSE

The literals corresponding to a Boolean variable x are x and its negation not(x)- which is true precisely if x is false.

Satisfiability

Boolean Variables: are TRUE or FALSE

The literals corresponding to a Boolean variable x are x and its negation not(x)- which is true precisely if x is false.

A clause is the OR of literals.

Satisfiability

Boolean Variables: are TRUE or FALSE

The literals corresponding to a Boolean variable x are x and its negation not(x)- which is true precisely if x is false.

A clause is the OR of literals.

A satisfiability instance consists of a family X of Boolean variables and a boolean formula which is the AND of a family F of clauses such that each literal in these clauses corresponds to a variable in X.

Satisfiability

Boolean Variables: are TRUE or FALSE

The literals corresponding to a Boolean variable x are x and its negation not(x)- which is true precisely if x is false.

A clause is the OR of literals.

A satisfiability instance consists of a family X of Boolean variables and a boolean formula which is the AND of a family F of clauses such that each literal in these clauses corresponds to a variable in X.

We are asked to determine if we can choose a truth assignment to the variables of X so that the formula evaluates to true.

Satisfiability

Boolean Variables: are TRUE or FALSE

The literals corresponding to a Boolean variable x are x and its negation not(x)- which is true precisely if x is false.

A clause is the OR of literals.

A satisfiability instance consists of a family X of Boolean variables and a boolean formula which is the AND of a family F of clauses such that each literal in these clauses corresponds to a variable in X.

We are asked to determine if we can choose a truth assignment to the variables of X so that the formula evaluates to true.

Which Formulas are satisfiable?

$(x_1 \text{ OR } x_2)$ AND $(x_1 \text{ OR not}(x_2))$ AND $(\text{not}(x_1) \text{ OR not}(x_2))$ AND $(\text{not}(x_1) \text{ OR } x_2)$

$(x_1 \text{ OR } x_2)$ AND $(x_1 \text{ OR } x_3)$ AND $(x_1 \text{ OR } x_4)$ AND

$(\text{not}(x_2) \text{ OR not}(x_3))$ AND $(\text{not}(x_2) \text{ OR not}(x_4))$ AND $(x_3 \text{ OR } x_4)$ AND $(\text{not}(x_3) \text{ OR not}(x_4))$

$(x_1 \text{ OR not}(x_2))$ AND $(\text{not}(x_1) \text{ OR not}(x_2))$ AND
$(x_1 \text{ OR not}(x_3))$ AND $(\text{not}(x_1) \text{ OR not}(x_3))$ AND
$(x_1 \text{ OR not}(x_4))$ AND $(\text{not}(x_1) \text{ OR not}(x_4))$ AND
$(x_1 \text{ OR not}(x_5))$ AND $(\text{not}(x_1) \text{ OR not}(x_5))$ AND
$(x_1 \text{ OR not}(x_6))$ AND $(\text{not}(x_1) \text{ OR not}(x_6))$ AND
$(x_2 \text{ OR } x_3 \text{ OR } x_4 \text{ OR } x_5 \text{ OR } x_6)$