

# COMP 360 A2

Ryan Chalmers

# Part I

## Question 1

### Original LP

#### Variables

- Staple-poor =  $x_1$
- Staple-average =  $x_2$
- Staple-premium =  $x_3$
- Nail-poor =  $x_4$
- Nail-average =  $x_5$
- Nail-premium =  $x_6$
- Screw-poor =  $x_7$
- Screw-average =  $x_8$
- Screw-premium =  $x_9$

#### Objective

Maximize:

$$15x_1 + 25x_2 + 40x_3 + 20x_4 + 40x_5 + 50x_6 + 80x_7 + 110x_8 + 130x_9$$

#### Subject To

$$10x_1 + 15x_2 + 25x_3 + 15x_4 + 25x_5 + 30x_6 + 70x_7 + 100x_8 + 115x_9 \leq 1000,$$

$$x_1 + x_2 + x_3 \geq 80,$$

$$x_4 + x_5 + x_6 \geq 25,$$

$$x_7 + x_8 + x_9 \geq 20,$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \geq 0,$$

$$-10x_2 + x_3 \geq 0,$$

$$-10x_5 + x_6 \geq 0,$$

$$-10x_8 + x_9 \geq 0,$$

### Dual

#### Objective

Minimize:

$$100y_1 - 80y_2 - 25y_3 - 20y_4$$

#### Subject To

$$10y_1 - y_2 \geq 15$$

$$15y_1 - y_2 + 10y_5 \geq 25$$

$$25y_1 - y_2 - y_5 \geq 40$$

$$15y_1 - y_3 \geq 20$$

$$25y_1 - y_3 - 10y_6 \geq 50$$

$$70y_1 - y_4 \geq 80$$

$$100y_1 - y_4 - 10y_7 \geq 110$$

$$115y_1 - y_4 - y_y \geq 130$$

## Solution

$$y_1 = 2$$

$$y_2 = 5$$

$$y_3 = 10$$

$$y_4 = 60$$

$$y_5 = 5$$

$$y_6 = 0$$

$$y_7 = 0$$

$$z = -1650$$

## Rough Work

$$\begin{array}{rcl}
 \max & 15x_1 + 25x_2 + 40x_3 + 20x_4 + 40x_5 + 50x_6 + 80x_7 + 110x_8 + 130x_9 & \\
 & 10x_1 + 15x_2 + 25x_3 + 15x_4 + 25x_5 + 30x_6 + 70x_7 + 100x_8 + 115x_9 \leq 100 & \\
 2 & -x_1 - x_2 - x_3 & \leq -80 \\
 3 & & -x_4 - x_5 - x_6 \leq -25 \\
 4 & & -x_7 - x_8 - x_9 \leq -20 \\
 5 & 10x_2 - x_3 & \leq 0 \\
 6 & 10x_5 - x_6 & \leq 0 \\
 7 & 10x_8 - x_9 \leq 0 &
 \end{array}$$

Dual →

$$\begin{aligned}
 & (10y_1 - 1y_2)x_1 + (15y_1 - y_2 + 10y_5)x_2 + (25y_1 - y_2 - y_5)x_3 \\
 & + (15y_1 - y_3)x_4 + (25y_1 - y_3 - 10y_6)x_5 + (30y_1 - y_3 - y_6)x_6 \\
 & + (70y_1 - y_4)x_7 + (100y_1 - y_4 - 10y_7)x_8 + (115y_1 - y_4 - y_7)x_9
 \end{aligned}$$

Minimize  $100y_1 - 80y_2 - 25y_3 - 20y_4$

Subject to.

$$\begin{array}{rcl}
 1 & 10y_1 - 1y_2 & \geq 15 \\
 2 & 15y_1 - y_2 & + 10y_5 \geq 25 \\
 3 & 25y_1 - y_2 & - y_5 \geq 40 \\
 4 & 15y_1 & - y_3 \geq 20 \\
 5 & 25y_1 & - y_3 - 10y_6 \geq 40 \\
 6 & 30y_1 & - y_3 - y_6 \geq 50 \\
 7 & 70y_1 & - y_4 \geq 80 \\
 8 & 100y_1 & - y_4 - 10y_7 \geq 110 \\
 9 & 115y_1 & - y_4 - y_7 \geq 130
 \end{array}$$

Figure 1. Rough Work

## Question 2

### Row Player Optimal Solution

$$\begin{bmatrix} 2 & 6 & 0 \\ -5 & 8 & -1 \\ 0 & -5 & -5 \end{bmatrix}$$

maximize  $z$

Subject to  $z - 2x_1 + 5x_2 \leq 0$

$$z - 6x_1 - 8x_2 + 5x_3 \leq 0$$

$$z + x_2 + 5x_3 \leq 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Figure 2. Formulation

### Solution

$$x_1 = 1$$

$$x_2 = 0$$

$$x_3 = 0$$

### Column Player Optimal Solution

Minimize  $W$

$$\text{Subject to } -W - \sum_{j=1}^3 a_{ij} y_j \geq 0$$

$$-W + \sum_{j=1}^3 a_{ij} y_j \leq 0$$

$$-W + 2y_1 + 6y_2 \leq 0$$

$$-W - 5y_1 - 8y_2 - y_3 \leq 0$$

$$-W - 5y_2 - 5y_3 \leq 0$$

$$y_1 + y_2 + y_3 = 1$$

$$y_1, y_2, y_3 \geq 0$$

Figure 3. Formulation

### Solution

$$y_1 = 0$$

$$y_2 = 0$$

$$y_3 = 1$$

### Comments

If the row player is playing optimally and the column player is playing optimally then on average the row player will win 0.