Homework #2 of PH621, Due due on Friday, February 9th

1. Considerna solid homogeneous cylinder of radius α and mass m rolling without slipping on the inside of a stationary larger cylinder of radius R as shown in Figure 1.
   (a) Write down the Lagrangian for the system, determine the equation of motion of the inside cylinder.
   (b) Find the period of small oscillations about the stable equilibrium.

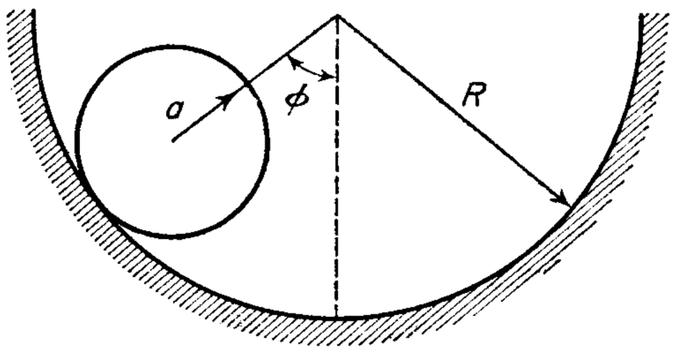

Figure 1

2. A double pendulum with equal lengths l and different masses m_1 and m_2 performs small oscillations in a plane. Introduce the transverse displacements of the first particle from the vertical η_1, and of the second particle from the first particle η_2.
   (a) Show that the Lagrangian is given by

$$L = \frac{1}{2}m_1\dot{\eta_1}^2 + \frac{1}{2}m_2(\dot{\eta_1} + \dot{\eta_2})^2 - \frac{g}{2l}[(m_1 + m_2)\eta_1^2 + m_2\eta_2^2]$$

   (b) Show that the normal-mode frequencies is

$$\omega^2 = (g/l)(1 \pm \gamma)^{-1},$$

   where

$$\gamma = [m_2(m_1 + m_2)^{-1}]^{1/2}$$

   (c) Construct the normal mode eigenvectors and describe the motions.
   (d) Verify that the modal matrix has the form

$$\underline{A} = (2m_1)^{-1/2}\begin{bmatrix} (1-\gamma)^{1/2} & -(1+\gamma)^{1/2} \\ \gamma^{-1}(1-\gamma)^{1/2} & \gamma^{-1}(1+\gamma)^{1/2} \end{bmatrix}$$

   and demonstrate explicitly that $\underline{A}$ diagonalizes the matrices $\underline{m}$ and $\underline{v}$

(e) Construct the normal coordinates.

(f) Assume that $m_2 \ll m_1$. If the upper mass is displaced slightly from the vertical and released from rest, show that the subsequent motion is such that at regular intervals one pendulum is stationary and the other oscillates with maximum amplitude.

3. Consider the case of a double pendulum shown in the Figure 2 where the top pendulum has length L_1 and the bottom length is L_2, and similarly the bob masses are m_1 and m_2. Assume small oscillations in the plane. Find and describe the normal modes and coordinates

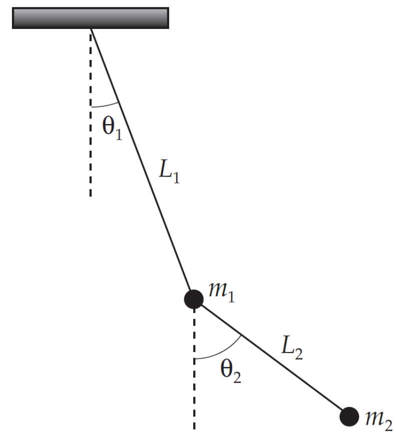

Figure 2

4. Find the characteristic frequencies and describe the normal modes of the system of three coupled pendula shown in Figure 3.

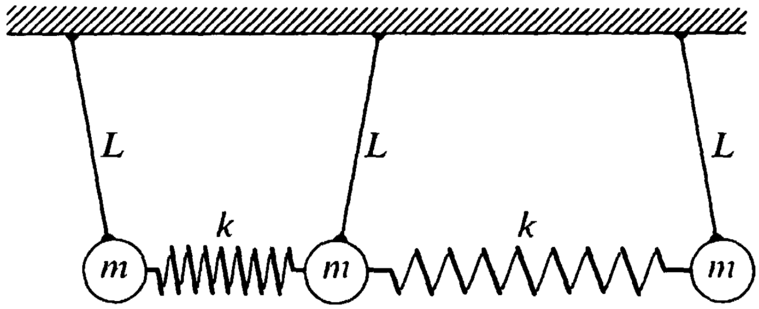

Figure 3