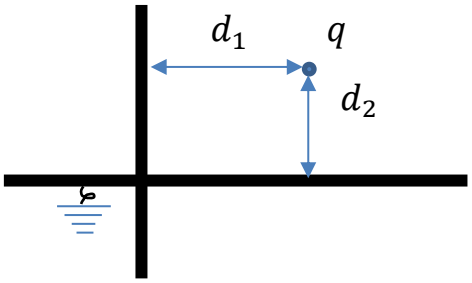
Assignment 1, total of 20 points

1. (5pt) A charge q is placed near two perpendicular grounded, infinitely large metal plates, what is the force exerted on q? Can you guess how the far field potential behaves?



2. (5pt) Prove the Green's reciprocity theorem: If a charge distribution ρ_1 generates a potential $\varphi_1(\boldsymbol{r})$, and some different charge distribution ρ_1 generates a different potential $\varphi_2(\boldsymbol{r})$. Then we have

$$\int_{\text{all space}} \rho_1 \, \varphi_2 d^3\boldsymbol{r} = \int_{\text{all space}} \rho_2 \, \varphi_1 d^3\boldsymbol{r}$$

Notice that we are talking about two independent configurations of $\rho - \varphi$.

(5pt) Now let us assume there are two separate, isolated conductors A and B. There is no constraint on their shapes and relative distance. If I put a total charge of Q on A, it generates electric field which has constant potential V_{AB} on B and V_{AA} on A (because B is a conductor). If I put the same total charge Q on B, while leaving A charged with Q, then A has potential V_{BA}. Prove that $V_{BA} = V_{AB} + V_{AA}$.

3. (5pt) A metal half-sphere conductor of radius a is placed in uniform electric field as shown below. If we define the potential of the conductor to be zero. What is the potential at any given point outside of the sphere? At far away of the conductor, can you express the electric field by a dipole approximation?