Homework #1 of PH621, Due due on Wednesday, January 24th

1. Consider a mechanical system of N particles. Let $q_1, q_2, \cdots, q_n$ be a set of generalized coordinates that completely specify the configuration of the system and $\vec{r_i}$ be the radius vector of the i^{th} particle. The fundamental relations between the radius vectors of all the particles and the generalized coordinates can be written as

$$\vec{r_i} = \vec{r_i}(q_1, q_2, \cdots q_n, t), \qquad i = 1, 2, \cdots, N.$$

Show that

$$\frac{d}{dt}\left(\frac{\partial}{\partial q_i}\vec{r_1}\right) = \frac{\partial}{\partial q_i}\left(\frac{d}{dt}\vec{r_1}\right)$$

2. Let $q_1, q_2, \cdots, q_n$ be a set of *independent* generalized coordinates for a system of n degrees of freedom, with a Lagrangian $L(q, \dot{(q)}, t)$. Suppose we transform to another set of independent coordinates $s_1, s_2, \cdots, s_n$ by means of transformation functions

$$q_i = q_i(s_1, s_2, \cdots, s_n, t), \qquad i = 1, 2, \cdots, n.$$

Such a transformation is called a *point transformation*. Show that if the Lagrangian function is expressed as a function of s_j, $\dot{s}_j$ and t through the equations of transformation, then L satisfies Lagrange's equations with respect to the s coordinates:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}_j}\right) - \frac{\partial L}{\partial s_j} = 0.$$

In other words, the form of the Lagrange's equations is invariant under a point transformation.

3. The point of support of a simple pendulum of length b moves on a massless rim of radius a with constant angular velocity ω (Figure 1). Determine the equation of motion for the angle θ.

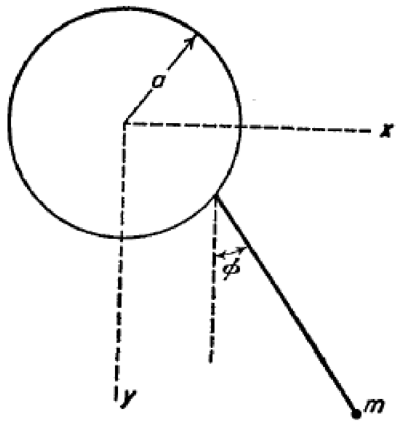

Figure 1

4. A simple planar pendulum of mass m_2 with a mass m_1 at the point of support that can move on a horizontal line in the plane in which m_2 moves (Figure 2). Determine the equation of motion for x and the angle ϕ.

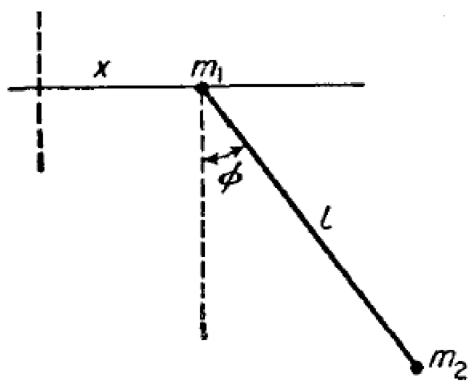

Figure 2

5. Consider a mechanical system that consists of a bead of mass m constrained to move along a vertical hoop of radius R (Figure 3). The hoop is rotating along a vertical axis through the center of the hoop, with constant angular velocity ω.

   (a) If the particle were unconstrained and free to move in a gravitational field, how many generalized coordinates would the system have?

   (b) How many constrains are there in the system? How many independent generalized coordinates do you need to specify the configuration of the mechanical system?

   (c) What are the forces acting on the mass? Which of the forces acting the mass can be derived from a potential, and which are constraint forces?

   (d) How would you find the constrain forces using the Lagrangian formulation?

   (e) Determine the Lagrangian of the system to describe the position of the mass on the hoop in terms of generalized coordinates.

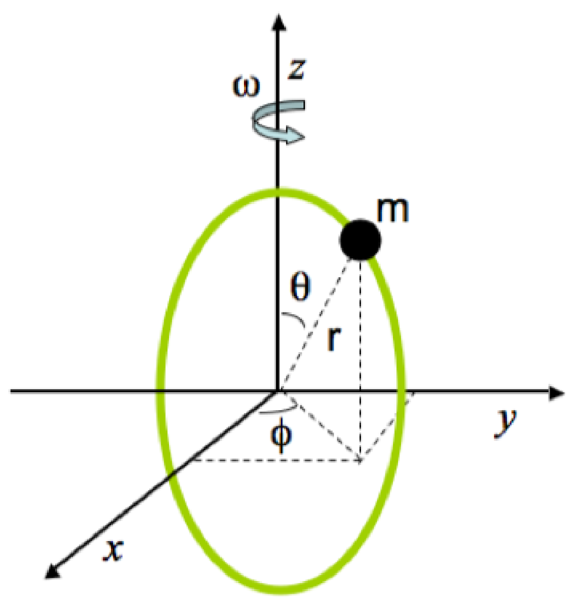

Figure 3

6. A simple pendulum of length b and bob with mass m is attached to a massless support moving horizontally with constant acceleration a.

   (a) Determine the equations of motion.

   (b) Determine the period for small oscillations.